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Assimilation of GPS Radio Occultation Observations

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By

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# TABLE OF CONTENTS

List of Tables .................................................. vii
List of Figures .................................................. viii
Abstract ....................................................... xviii

1. Introduction ................................................. 1
   1.1 A Brief Description of the Radio Occultation Techniques .... 1
   1.2 Atmospheric Variables Retrievable from GPS RO Measurements . 5
   1.3 Considerations for GPS RO Data Assimilations ................. 8
   1.4 Plan of Dissertation .................................... 11

2. Assimilation of GPS Bending Angle — An Accurate Method .... 12
   2.1 Introduction ............................................ 12
   2.2 A Global Data Assimilation System ......................... 14
   2.3 A Ray-tracing Method for Calculating Bending Angle ........ 15
   2.4 SSI/GPS Bending Angle Assimilation System and Observations . 17
   2.5 Impact of Observational Weighting ........................ 20
   2.6 Sensitivity to Variation of Gravity Acceleration ............ 37
   2.7 Impact of the Vertical Resolution of GPS Observations for Data Assimilation ............................................. 41
   2.8 Analysis Increments of Temperature, Moisture, Surface Pressure and Refractivity ........................................... 43
   2.9 Summary and Conclusions ................................ 52

3. Assimilations of GPS Refractivity — A Computationally Efficient Method ... 58
   3.1 Introduction ............................................ 58
   3.2 SSI/GPS Refractivity Assimilation System .................... 59
   3.3 Experimental Design and Observations ....................... 59
   3.4 Convergence and Computational Cost ........................ 61
   3.5 Impact Radius of GPS Observations ........................ 65
   3.6 Observation and Analysis Increments of Refractivity ......... 67
   3.7 Analysis Increments of Temperature, Moisture and Surface Pressure .... 72
   3.8 Comparison of GPS Bending Angle and Refractivity Assimilations within Cloud Regimes ................................... 74
4. Assimilation of GPS Excess Phase Delay — An Accurate and Computationally Efficient Method
4.1 Introduction ........................................... 88
4.2 Basic Concepts and Observation Operator .................. 89
4.3 Features of the Observation Operator ........................ 94
4.4 Experimental Design and Data Sets .......................... 103
4.5 Numerical Results from Forward Simulation .................. 105
4.6 Data Results from Assimilation ............................ 118
4.7 Assimilation of Symmetric GPS Excess Phase Delay as an Alternative 129
4.8 Summary and Conclusions ................................... 135
LIST OF TABLES

2.1 Geographical locations of 52 GPS/MET soundings and 56 nearby radiosonde stations. 18
2.2 The mean and r.m.s errors for temperature and specific humidity analyses above 850 hPa. Observations from the 56 radiosonde soundings are included in the calculations. 30
2.3 Same as Table 2.2, except for the surface pressure (hPa). 36
2.4 The analysis increments of the surface pressure (hPa) for GPS38, 47 and 03. 49
2.5 Same as Table 2.2, except that those soundings for which the differences between the GPS/MET observation and background have an opposite sign to the differences between radiosonde observation and background in most of vertical levels are excluded. 53
3.1 The number of outer and inner loops during the minimization of the cost function defined in Eq. (2.1) for each of iteration setup experiments. 62
3.2 The CHAMP soundings chosen for the single sounding analysis (see text in Section 3.8). 80
LIST OF FIGURES

1.1 Schematic illustration of the geometry of radio occultation measurement. \( \mathbf{V}_T \) and \( \mathbf{V}_R \) are the velocity vectors of the transmitter at transmitting time and the receiver at receiving time, respectively, \( \mathbf{u}_T \) and \( \mathbf{u}_R \) are unit vectors representing the ray directions at the transmitter and receiver, \( \phi_T (\phi_R) \) is the angle between the ray path and the transmitter (receiver) position vector, \( r_T \) and \( r_R \) are the radii of the transmitter and receiver from the Earth, \( \theta \) is the angle between the transmitter and receiver position vectors, \( \alpha \) is the bending angle, \( a \) is the impact parameter, which is the perpendicular distance from the center of the Earth to the ray incident to the atmosphere and \( P_t \) is the tangent point, which is the closest point on the ray to the center of the Earth. The positions of the transmitter and receiver and the local curvature center of the ray define a occultation plane. .......... 6

2.1 The total number of the radiosonde soundings within a five degree latitudinal band that are collocated with GPS/MET observations and are used in the evaluation of the GPS/MET bending angle assimilation results. ........... 19

2.2 Three estimates of the standard deviation of bending angle observational errors. 24

2.3 The 52 vertical profiles of the differences between the simulated \( N^{LOC} \) and GPS/MET observed refractivity \( (N_{obs}) \) before and after bending angle assimilation. The left panels show the profiles for soundings 1-28 and the right panels show those for soundings 29-56. (a,b) \( N^{LOC} - N_{obs} \) before assimilation. (c,d) \( N^{LOC} - N_{obs} \) from EXP1. (e,f) \( N^{LOC} - N_{obs} \) from EXP2. .............. 27

2.4 Same as Fig. 2.3, except for the differences between \( N^{GPS} \) and \( N_{obs} \). (a,b) \( N^{GPS} - N_{obs} \) before assimilation. (c,d) \( N^{GPS} - N_{obs} \) from EXP1. (e,f) \( N^{GPS} - N_{obs} \) from EXP2. ............ 28

2.5 The vertical distributions of the mean (solid curve) and the standard deviations (short lines) of the simulated and observed refractivity differences averaged from the 52 soundings before and after assimilation. (a) \( N^{LOC} - N_{obs} \) and (b) \( N^{GPS} - N_{obs} \) before assimilation. (c) \( N^{LOC} - N_{obs} \) and (d) \( N^{GPS} - N_{obs} \) from EXP1. (e) \( N^{LOC} - N_{obs} \) and (f) \( N^{GPS} - N_{obs} \) from EXP2. ............ 29
2.6 The vertical distributions of the mean errors and the RMS errors of the temperature and specific humidity fields before and after the assimilation averaged from the 52 soundings, verified with all 56 collocated radiosonde observations. (a) The mean errors of the temperature. (b) The RMS errors of the temperature. (c) The mean errors of the specific humidity. (d) The RMS errors of the specific humidity. 31

2.7 The vertical distributions of the mean errors (left) and the RMS errors (right) of the temperature before and after the assimilation averaged from the soundings within (a,b) 40°N-60°N, (c,d) 60°N-90°N and (e,f) 90°S-40°N. 32

2.8 Same as Fig. 2.7, except for the specific humidity. 33

2.9 (a) Differences of the absolute mean errors of temperature between the analysis and background (|EXP error| − |GES error|) at the 56 collocated radiosonde sounding locations averaged for all levels above 850 hPa. (b) Differences of the absolute RMS errors of temperature between analysis and background (|EXP RMS| − |GES RMS|) at the 56 collocated radiosonde sounding locations averaged for all levels above 850 hPa. (c) Same as (a) except for the specific humidity. (d) Same as (b) except for the specific humidity. 35

2.9 -continued. 36

2.10 The distribution of the differences of (a) $N^{GPS}$ and (b) $N^{LOC}$ between EXP1 and EXP1.G with height and latitude. 39

2.11 The vertical distributions of the differences between the simulated and observed (a) $N^{GPS}$ and (b) $N^{LOC}$. 40

2.12 The vertical distributions of the analysis differences between EXP1 and EXP1.G for (a) temperature and (b) specific humidity. 42

2.13 The vertical distributions of the mean (left) and the standard deviations (right) of the differences between the simulated and observed (a-b) $N^{GPS}$ and (c-d) $N^{LOC}$. 43

2.14 The vertical distributions of the mean and RMS errors of the temperature and specific humidity for EXP1 and EXP1.RES. (a) The mean errors of the temperature. (b) The RMS errors of the temperature. (c) The mean errors of the specific humidity. (d) The RMS errors of the specific humidity. 44

2.15 The vertical distributions of the relative sensitivities of refractivity to changes in temperature and specific humidity. 46

2.16 The vertical distributions of the refractivity analysis increments of (a) $N^{LOC}$ and (b) $N^{GPS}$. 47
2.17 The vertical distributions of the analysis increments of (a) temperature and (b) specific humidity. ........................................ 48

2.18 The vertical distributions of the analysis increments for the refractivity (left) and the specific humidity (right) at locations of (a,b) GPS38, (c,d) GPS47 and (e,f) GPS03. .................................................. 50

2.19 The vertical distributions of the specific humidity at locations of (a) GPS38 and (b) GPS03. .................................................. 52

2.20 The vertical distributions of the observational increments of $N^{LOC}$ in terms of (a-b) the GPS/MET observations and (c-d) radiosonde observations. Left panels are for GPS38 and right panels for GPS03. .................................................. 53

2.21 The vertical distributions of the analysis increments of $N^{LOC}$ in terms of (a-b) the GPS/MET observations and (c-d) radiosonde observations. Left panels are for GPS38 and right panels for GPS03. .................................................. 54

2.22 The vertical distributions of the observational increments of $N^{LOC}$ in terms of the GPS/MET observations (GES-GPS) and radiosonde observations (GES-RAD). .................................................. 55

3.1 (a) Distributions of the 434 CHAMP occultations within a 6-hour window centered at 06 UTC from 21 to 31 May 2002 and (b) the lowermost and uppermost altitudes of these CHAMP RO observation profiles. Distributions of the 34 CHAMP occultations within a 6-hour window centered at 06 UTC May 27, 2002 are marked by closed triangles. .................................................. 61

3.2 Estimates of (a-b) vertical covariances and (c-d) fractional standard deviations of GPS RO bending angle (a,c) and refractivity (b,d) by using CHAMP RO observations from May 2002. .................................................. 63

3.3 Estimates of vertical correlations of GPS RO (a) bending angle and (b) refractivity by using CHAMP RO observations from May 2002. .................................................. 64

3.4 Variations of (a) the cost function and (b) the norm of its gradient with the number of iterations from seven iteration setup experiments in Table 1. .................................................. 66

3.5 Final values of the norm of the cost function gradient (square point) from each of the experiments in Fig. 3.4 and the total wallclock time that experiment has taken (circle point) running on an IBM SP4 parallel computer system with 28 processors. An exponential fitting to the gradient norm (solid line) and a linear fitting to the wallclock time (dashed line) are also given. .................................................. 67
3.6 Distributions of the specific humidity analysis differences (a) between BA and NOGPS and (b) between REF and NOGPS at σ = 0.8585 level at 06UTC May 21, 2002. Unit: g kg$^{-1}$. The contour interval is 0.1 g kg$^{-1}$. (c) Distributions of the CHAMP occultations within the time window. ................. 68

3.7 σ-longitude cross-sections of the specific humidity analysis differences (a) between BA and NOGPS and (b) between REF and NOGPS, passing through one occultation point at (19.36°W, 52.64°N) at 06UTC May 21, 2002. (c) Zonal variations of the specific humidity differences at σ = 0.8585 level between BA and NOGPS (solid line) and between REF and NOGPS (dotted line). Unit: g kg$^{-1}$. ....................................................... 69

3.8 Zonal variations of the mean specific humidity analysis differences at σ = 0.8585 level between BA and NOGPS (solid line) and between REF and NOGPS (dotted line). The mean value is computed by averaging the differences over ±10 zonal model grids next to each of the 434 GPS occultation points. The closest model grid to the occultation point is indicated by 0 in the x-axis. Unit: g kg$^{-1}$. ....................................................... 70

3.9 Vertical profiles of (a-c) the mean observation increments (analysis minus observation) of $N^{GPS}$ and (d-f) their root mean square values (RMS) for BA (solid line), REF (dotted line) and NOGPS (star). Data are computed and plotted in three latitudinal bands: (a,d) 90°S−30°S, (b,e) 30°S−30°N and (c,f) 30°N−90°S. Unit: N unit. ....................................................... 71

3.10 Vertical profiles of (a-c) the mean analysis increments (analysis minus background) of $N^{GPS}$ and (d-f) their root mean square values (RMS) for BA (solid line), REF (dotted line) and NOGPS (star). Data are computed and plotted in three latitudinal bands: (a,d) 90°S−30°S, (b,e) 30°S−30°N and (c,f) 30°N−90°S. Unit: N unit. ....................................................... 72

3.11 Same as Fig. 3.10, except for temperature in the model space at σ levels. Unit: Kelvin. ....................................................... 74

3.12 Same as Fig. 3.11, except for specific humidity field. Unit: g kg$^{-1}$. ............ 75

3.13 (a) Zonal averaged analysis increments (analysis minus background) of surface pressure and (b) their root mean square values (RMS) for BA (solid line), REF (dotted line) and NOGPS (star). Unit: hPa. (c) The number of GPS occultations within each of 5° latitudinal bands used in the average. .............. 76
3.14 Distributions of occultations within (a) convective clouds, (b) cumulus clouds and (c) stratus clouds. The soundings in mid-latitudes (60°S-30°S and 30°N-60°N) are marked by closed circles and the soundings in tropics (30°S-30°N) are marked by closed triangles. There are 9 (30) GPS RO soundings within tropical (mid-latitude) convective clouds, 37 (17) soundings within tropical (mid-latitude) cumulus clouds and 6 (23) soundings within tropical (mid-latitude) stratus clouds.

3.15 Spaghetti plots of vertical profiles of temperature analysis differences between BA and REF (BA minus REF) under five categories of cloud systems: (a) tropical and (b) mid-latitude convective clouds, (c) tropical and (d) mid-latitude cumulus clouds and (e) mid-latitude stratus clouds. The individual profiles marked by closed circles are from the soundings listed in Table 3.2.

3.16 Vertical profiles of (a) the mean temperature analysis differences and (b) their root mean square values (RMS) between BA and REF (BA minus REF) under five categories of cloud systems in Fig. 3.15: (a) tropical (solid line) and (b) mid-latitude convective clouds (open circle), (c) tropical (dashed line) and (d) mid-latitude cumulus clouds (open square) and (e) mid-latitude stratus clouds (star). Unit: Kelvin.

3.17 Same as Fig. 3.15, except for specific humidity. Unit: g kg$^{-1}$.

3.18 Same as Fig. 3.16, except for specific humidity. Unit: g kg$^{-1}$.

3.19 (a) For GPS sounding 1 in Table 3.2, the horizontal distribution of the local refractivity analyses from the background (contour) at $\sigma = 0.7$ (Unit: N unit) and the averaged occultation point location (triangle), the longitude-$\sigma$ cross-sections of the analysis increments (analysis minus background) of $N^{LOC}$ for (b) BA and (c) REF and (d) the difference of $N^{LOC}$ analyses between BA and REF. Unit: N unit. The cross-sections cut across the model grid closest to the corresponding occultation point.

3.20 Same as Fig. 3.19, except for GPS sounding 2 in Table 3.2.

3.21 The differences between (a-c) $N^{LOC}$ and $N_{obs}$, (d-f) $N^{GPS}$ and $N_{obs}$ and (g-i) $N^{GPS}$ and $N^{LOC}$ for the background (left panels), BA (middle panels) and REF (right panels) analyses of two GPS soundings in Table 3.2. Solid lines indicate the results from GPS sounding 1 and dotted lines are from GPS sounding 2. Unit: N unit.

3.22 Vertical profiles of the mean differences (a-b) between $N^{LOC}$ and $N_{obs}$ and (c-d) between $N^{GPS}$ and $N_{obs}$ for the background (star), BA (solid line) and REF (dotted line). Unit: N unit. Panels (a) and (c) are averaged over the soundings within mid-latitude convective clouds and panels (b) and (d) are averaged over the soundings within mid-latitude cumulus clouds.
3.23 Vertical profiles of the root mean squares (RMS) of the differences (a-b) between $N^{LOC}$ and $N_{obs}$, (c-d) between $N^{GPS}$ and $N_{obs}$ for the background (star), BA (solid line) and REF (dotted line) and (e) between $N^{GPS}$ and $N^{LOC}$ for the background. Unit: N unit. Panels (a), (c) and the solid line in panel (e) are averaged over the soundings within mid-latitude convective clouds and panels (b), (d) and the dotted line in panel (e) are averaged over the soundings within mid-latitude cumulus clouds.

4.1 Schematic illustration of the definition of a tangent link. Assume the earth is an ellipsoid. Given a GPS measurement at a tangent point $(\lambda, \phi, h)$, a tangent line (referred to as the tangent link) is constructed so that it goes through the point $(\lambda, \phi, h)$, tangent to the local curvature and coplanar to the plane containing the GPS and LEO satellites (occultation plane).

4.2 Schematic illustration of the definition of the elements in the matrix $B$. Given a vertical profile of a GPS refractivity measurement, a tangent link $m$ is constructed through the tangent point $(P_t)$ at layer $m$ bounded by levels $m$ and $m+1$. The element $b_{m,l}$ in the matrix $B$ is defined as the total interception length of the link $m$ inside the layer $l$ bounded by levels $l$ and $l + 1$.

4.3 Schematic illustration of the definition of the elements in the matrix $A$. The element $a_{m,n}$ in the matrix is defined as the total interception length of the link $m$ inside the $n$th model grid box. (a) shows an example with the tangent link $m$ in a longitudinal plane. The grid box is indexed by counting the box number in the order of latitude, longitude and vertical levels. Therefore, the grid box $n+nlat$ is the one next to the grid box $n$ as shown. (b) is a generalized 3D view of the interception of the tangent link $m$ with the $n$th grid box. The local refractivity value in the grid box $n$, required by the excess phase delay observation operator, is given either by the value of the refractivity at the left-lower corner grid of the box (point A), or by interpolating the refractivities onto the middle point of the intercepted link by the model grids (point B).

4.4 The values of (a) the matrix $B$ and (b) the first $(\vec{b}_1)$, 11st $(\vec{b}_{11})$ and 21st $(\vec{b}_{21})$ rows of $B$. The index $m$ (x-axis in (a)) represents the index of the layer where the tangent point for a tangent link is located. The index $l$ (y-axis in (a) and x-axis in (b)) is the index of the layer which a tangent link goes through. The grid setup has $1^\circ$ horizontal resolution and $1$ km vertical resolution. The altitudes are from 0 km to 30 km. The tangent points of the GPS observation profile are assumed at $0^\circ$ longitude and $30^\circ$ latitude without vertical shifts. The tangent directions are parallel to the equator.

4.5 The values of the second row elements of the matrix $A$ ($\vec{a}_2$). Only non-zero values are shown in the figure. X-axis indicates the distance between the tangent point and the middle point of the tangent link within each of the interception grid boxes along the tangent link. The grid and observation setup are the same as in Fig. 4.4.
4.6 Kernel functions (solid lines) and their Gaussian function fittings (dotted) along the tangent links at 1 km tangent altitude as functions of distance. The grid and observation setup are the same as in Fig. 4.4, except that the latitudes of the tangent points are located at (a)10°N, (b)20°N, (c)30°N, (d)40°N, (e)50°N, (f)60°N, (g)70°N and (h)80°N. The x-axis is the same as in Fig. 4.5. ......................................................... 98

4.7 Gaussian function fittings in Fig. 4.6 as functions of latitude. .............. 99

4.8 Kernel functions under four model grid setups with: (a) and (e) 1° longitude resolution and 1 km vertical resolution; (b) and (f) 0.5° longitude resolution and 1 km vertical resolution; (c) and (g) 1° longitude resolution and 0.5 km vertical resolution; (d) and (h) 0.5° longitude resolution and 0.5 km vertical resolution. The tangent links are at 1 km tangent altitude, 0° longitude and 10°N latitude (left panels) or 50°N latitude (right panels). The x-axis is the same as in Fig. 4.5. ......................................................... 100

4.9 Tracks of the points where the refractivity are valued in the corner scheme (dotted line) and middle-point scheme (solid line) along a tangent link at 0.4 km tangent altitude. The vertical level structure follows the GPS sounding observed on 0400UTC May 21, 2002 at the occultation point (62.39°W, 54.29°S). ......................................................... 101

4.10 Differences and percentage differences between the simulated excess phase delays and their observation values at two GPS occultation locations: The occultation in (a) was observed at 0400UTC May 21, 2002 at the occultation point (62.39°W, 54.29°S) and the one in (b) was observed at 0327UTC May 21, 2002 located at (158.98°E, 3.92°S). Solid(plus) lines represent the difference (percentage difference) obtained from the corner scheme and dashed (dotted) lines represent the results obtained from the middle-point scheme. Difference values are obtained by subtracting the observations from the simulations \((L - L_{\text{obs}})\) and percentage values are obtained by dividing the differences over the observations \((\frac{L - L_{\text{obs}}}{L_{\text{obs}}} )\). ......................................................... 102

4.11 Global distributions of 1158 CHAMP RO occultations observed during May 24-31, 2002. ......................................................... 104

4.12 Vertical resolution of a smoothed CHAMP vertical profile (triangle) observed at \(262.3°E, 25.5°N\) on 0228UTC July 20, 2002 as functions of altitudes, compared with the model resolution at the grid \(262.5°E, 22.5°N\) (square). ......................................................... 105

4.13 Vertical error correlation of the GPS refractivity observations calculated by using CHAMP data during May 2002. ......................................................... 106

4.14 Vertical error correlation of GPS excess phase delay observations calculated by using CHAMP data during May 2002. ......................................................... 107
4.15 Spaghetti plots of $\triangle_{per}L$ (cyan) and $\triangle_{per}N^{LOC}$ (red). $\triangle_{per}L$ is plotted over $\triangle_{per}N^{LOC}$. There are a total 1158 soundings during May 24-31 2002.

4.16 Vertical profiles of mean and standard deviation (error bars) of $\triangle_{per}L$ (left panel) and $\triangle_{per}N^{LOC}$ (right panel). The quantities are calculated from 1158 simulated soundings during May 24-31 2002.

4.17 Comparison of the percentage differences between PHA and REF with $(\triangle_{per}L - \triangle_{per}N^{LOC})$ indicated in x-axis and $(\triangle_{per}N^{LOC})$ indicated in y-axis.

4.18 The global distribution of the NCEP refractivity background at the 700hPa pressure surface simulated on 06UTC May 30, 2002. Unit: N unit.

4.19 The global distribution of the vertical gradient of the NCEP refractivity background in Fig 4.18. 32 CHAMP occultation available in the time window are indicated by the triangles. Unit: N unit/km.

4.20 Same as Fig 4.19, except for the horizontal gradient of the refractivity. Unit: N unit/km.

4.21 The percentage difference between PHA and REF $(\triangle_{per}L - \triangle_{per}N^{LOC})$ (a,d), the horizontal refractivity gradient (b,d) and the vertical refractivity gradient (c,f) at the 700hPa surface as functions of latitude (left panels) and longitude (right panels).

4.22 The averaged occultation points and the tangent links for two single occultations RO1 and RO2. RO1 is at the GPS occultation (258.63°E, 23.05°N) observed at 0723UTC May 30, 2002 and RO2 is at the GPS occultation (81.87°E, 51.85°S) observed at 0643UTC May 30, 2002.

4.23 Cross-sections of the distributions of refractivity (contour) and its horizontal gradient(shaded) in the averaged occultation planes for RO1 (upper panel) and RO2 (lower panel). The tangent points in the vertical direction are marked by the solid dots.

4.24 Same as in Fig. 4.23 except that the shaded contours are the vertical gradients for each of occultations.

4.25 Vertical profiles of the vertical gradient (left panel) and the horizontal gradient of the refractivity (right panel) for RO1 (open circle/square) and RO2 (closed circle/square).

4.26 Vertical profiles of $\triangle_{per}L$ (solid line) and $\triangle_{per}N^{LOC}$ (dashed line) for (a) RO1 and (b) RO2.
4.27 The norm of the gradient of the cost functions varying with the iteration number in the minimization to assimilation GPS observations within ±3 hour time window centered at 06UTC May 30, 2002. Solid line is the result from NOGPS, circle line is from PHA and long dashed line is from REF. 120

4.28 Vertical profiles of the mean (left panels) and STD (right panels) of $\Delta_{per}L$ (a,b) and $\Delta_{per}N^{LOC}$ (c,d) before (solid lines) and after the GPS data assimilations (dashed lines). 121

4.29 Vertical profiles of the mean (a,b and c) and STD values (d, e, and f) of $\Delta N^{LOC}$ and $\Delta N^{GPS}$ for the experiment NOGPS. The left panels are for $\Delta N^{LOC}$ before (solid line) and after (dashed line) observation assimilations, the middle panels are for $\Delta N^{GPS}$ before (solid line) and after (square marked) assimilations and the right panels are for $\Delta N^{LOC}$ (dashed line) and $\Delta N^{GPS}$ (square marked) after assimilations. 123

4.30 Same as Fig. 4.29, except for the experiment PHA. 124

4.31 Same as Fig. 4.29, except for the experiment REF. 125

4.32 Vertical profiles of the mean (left) and STD (right) of $\Delta N^{GPS}$ background (square) and analyses from the experiment NOGPS (dotted line), PHA (star) and REF (solid line). 126

4.33 Spaghetti plots of $\Delta_{GPS}T$ for (a) PHA and (b) REF. The profiles have been color-coded according to their corresponding occultation latitudes: blue represents soundings in the tropics ($30^\circ S - 30^\circ N$), cyan represents soundings in mid-latitudes ($30^\circ S - 60^\circ S$ and $30^\circ N - 60^\circ N$) and magenta represents soundings in high-latitudes ($60^\circ S - 90^\circ S$ and $60^\circ N - 90^\circ N$). 126

4.34 Vertical profiles of the mean (upper panels) and STD values (low panels) of $\Delta_{GPS}T$ for PHA (stared lines) and REF (dashed lines). (a) and (d) are for soundings in the tropics ($30^\circ S - 30^\circ N$), (b) and (e) are for mid-latitude soundings ($30^\circ S - 60^\circ S$ and $30^\circ N - 60^\circ N$) and (c) and (f) are for high-latitude soundings ($60^\circ S - 90^\circ S$ and $60^\circ N - 90^\circ N$). 127

4.35 Same as Fig. 4.33, except for specific humidity. 128

4.36 Same as Fig. 4.34, except for specific humidity. 128

4.37 Global distribution of the 500hPa NCEP temperature background at 06UTC May 30, 2002. 129

4.38 Global distribution of the 500hPa temperature analysis increment from PHA at 06UTC May 30, 2002. 130
4.39 Distribution of the analysis increments of 500 hPa temperature in subdomain (10° – 140°E, 90° – 10°S) at 06UTC May 30, 2002 for (a) NOGPS, (b) REF and (c) PHA. The triangles indicate the GPS occultation points.

4.40 Global distribution of the 850hPa NCEP specific humidity background at 06UTC May 30, 2002.

4.41 Global distribution of the 850hPa specific humidity analysis increment from PHA at 06UTC May 30, 2002.

4.42 Distribution of the GPS increments of the 850hPa specific humidity subdomain (10° – 140°E, 90° – 10°S) at 06UTC May 30, 2002 for (a) REF and (b) PHA. The triangles indicate the GPS occultation points.

4.43 Spaghetti plots of $\Delta_{per L}$ and $\Delta_{per L^{sym}}$. $\Delta_{per L}$ (cyan) is plotted over $\Delta_{per L^{sym}}$ (blue). There are a total of 1158 soundings during May 24-31 2002.

4.44 Vertical profiles of mean and standard deviation (error bars) of $\Delta_{per L^{sym}}$ calculated from 1158 soundings shown in Fig. 4.43.

4.45 Comparison of the simulation difference between PHA-sym and PHA ($\Delta_{per L^{sym}} – \Delta_{per L}$) (y-axis) and the simulation difference between PHA-sym and REF ($\Delta_{per L^{sym}} – \Delta_{per NLOC}$) (x-axis) at the 700hPa surface.

4.46 Vertical profiles of $\Delta_{per L}$ (solid line) and $\Delta_{per L^{sym}}$ (dashed line) for (a) RO1 and (b) RO2.

4.47 Same as Fig. 4.32, except that the vertical profiles of the mean (left) and STD (right) values of $\Delta N_{GPS}$ analysis from PHA-sym (dashed line) are added.

4.48 Same as Fig. 4.33, except for results from PHA-sym.

4.49 Same as Fig. 4.34, except that results from PHA-sym are added (solid lines).

4.50 Same as Fig. 4.35, except for results from PHA-sym.

4.51 Same as Fig. 4.36, except that results from PHA-sym are added (solid lines).

4.52 Same as Fig. 4.39, except for the result from PHA-sym.

4.53 Same as Fig. 4.42, except for the result from PHA-sym.
Unlike conventional and satellite observations, the Global Positioning System (GPS) radio occultation (RO) techniques provide all-weather, high-vertical-resolution observations that require no calibration. In this dissertation, the assimilation of GPS RO data is studied using the National Centers for Environmental prediction (NCEP) three dimensional variational analysis system.

Three GPS data assimilation choices are considered and compared. A set of GPS bending angle assimilation (BA) experiments is first carried out and sensitivity of BA results to the observational weighting, the quality of the background fields, the variation of the gravity, and the vertical resolution of the GPS data are investigated. The GPS local refractivity assimilation (REF) is then conducted and compared with BA. Although REF is computationally cheaper than BA, the bias and root mean square errors of the background fields are more significantly reduced by BA than REF. Differences between GPS refractivity and bending angle assimilations are larger in thick-layered cloud systems (e.g., convective clouds in the mid-latitudes and cumulus clouds in the tropics) than in thin clouds and clear sky, which are found to be associated with the strength of horizontal gradient of the atmospheric refractivity.

Aiming at achieving both accuracy and computational efficiency, a new observation operator that simulates the GPS excess phase delay is proposed and tested for GPS RO data assimilation. Using the excess phase delay, the along-track refractivity and refractivity gradient information can be included while the computational cost is kept low. Numerical results from the forward simulation and data assimilation using the excess phase delay (PHA) are compared with those of REF. PHA tends to produce a warmer and wetter model atmosphere, with finer structures and larger radii of influence than REF. Compared to GPS
observations, simulations and analyses produced by PHA are more accurate than those of REF. It is also pointed out that under the assumption of the spherical symmetry of the local refractivity, the observation operator for the excess phase delay simplifies into a point scheme in which only a vertical profile of model refractivity is required.
CHAPTER 1

Introduction

1.1 A Brief Description of the Radio Occultation Techniques

A good understanding of the vertical profiles of temperature and humidity throughout the troposphere and the stratosphere is fundamental to our understanding of the present state of the atmosphere, and to our numerical weather prediction (NWP) and climate studies. For NWP, accurate forecasts require not only an accurate model representing the atmosphere’s dynamics and physics, but also an accurate description of the initial atmospheric state. In recent years, NWP models have developed rapidly, including more detailed descriptions of the atmospheric processes with finer resolutions. Uncertainties in the initial conditions become more critical to the NWP models’ capabilities of making realistic forecasts and simulations. Studies show that the errors of temperature, wind and humidity can rapidly grow to dominate errors in the subsequent forecast and therefore prohibit useful forecasts beyond 5-6 days. Having an accurate initial condition is particularly important in certain geographical areas, e.g. those in which small perturbations to the state of the atmosphere will grow rapidly. Many of these are found in the vicinity of atmospheric fronts and jet-streams in which cloud cover is often extensive. For climate research, long-term stable and consistent monitoring of atmospheric parameters with an absolute accuracy helps to obtain a good description of the mean state and variability of the atmosphere.

The best initial state of the atmosphere can be obtained by assimilating all available measurements from many different observing systems into a forecast model. At present, most of widely-used observations come from two types of systems, i.e. radiosondes and satellites. Radiosondes are balloon-borne instruments that are carried aloft to measure soundings of wind, temperature, moisture and geopotential height at different atmospheric
pressure levels up to 20–30 km height. The synoptic observing programs of the United States and other World Meteorological Organization (WMO) countries typically provide one to four soundings per day from stations mostly in the Northern Hemisphere. Worldwide, there are nearly 900 upper-air observation stations which are mostly confined to land. Satellites remote sensing measure radiances above clouds or at the Earth’s surface with radiometric instruments. Measured in a number of discrete wavelength bands, the satellite observations include information on the vertical profile of temperature and water vapor. The same technique can be applied to measuring other atmospheric constituents such as ozone, carbon dioxide, etc. The limitations of the satellite radiance measurements are their coarse vertical and temporal resolutions. Both radiosondes and satellites require constant system calibrations. Among other requirements, there is clear demand for higher accuracy, better vertical resolution and higher stability observations of temperature and humidity profiles.

The radio occultation (RO) remote sensing technique provides a new way to sense through the atmosphere. Primarily, when a radio signal wave is transmitted through the atmosphere, its ray path is bent over due to the atmosphere refraction. The refraction along the ray-path is due to the variation of the density of the atmosphere (and also other parameters, if there are, such as electron density), which in turn depends on the temperature, moisture and pressure of the atmosphere. Therefore, a measurement of such bending enables a retrieval of the atmospheric state along the ray-path.

Details on the historic development of the RO technique are discussed in many papers (e.g., Melbourne et al., 1994 [30]; Yunck et al., 2000 [44]). The RO technique was developed independently at the Stanford University and the Jet Propulsion Laboratory (JPL), and was first proposed for probing the Martian atmosphere in early 1960’s. Since then a pioneering collaboration of these two teams have made the RO technique widely used for inferring the structure of planetary atmospheres over the last 40 years (e.g., for Mars, Fjeldbo and Eshleman, 1968 [8]; for Venues, Fjeldbo and Kliore, 1971 [9]; for Jupiter, Kliore et al., 1975 [13]; for Saturn, Tyler et al., 1982 [42]; for Uranus, Lindal et al., 1987 [23]; for Neptune, Lindal et al., 1992 [24]). For these planets, the radio signal transmitter is on a spacecraft orbiting the planet and the receiver, which keeps track of the transmitted signals, is on the Earth. Although early suggestions for applying the RO technique to the Earth were made as far back as the 1960’s (Fishbach, 1965 [7]; Lusignan et al., 1969 [28]), the RO technique has not been applied to the study of the Earth’s atmosphere until the last 15
years. Part of the reasons is that the RO technique requires the signal transmitter and the receiver to be off the Earth and the positioning of the transmitter and receiver platforms to be sufficiently accurate. The development of the Global Positioning System (GPS) in 1980’s has enabled the RO technique to sense the Earth’s atmosphere. By placing a GPS receiver on a low Earth orbit (LEO) satellite, the signals transmitted from the existing constellations of higher orbiting GPS satellites can be tracked. When a LEO satellite and a GPS satellite are occulted by the Earth, the signal measurements between two satellites are related to the bending of the ray path and therefore the thermal dynamic information of the atmosphere of the Earth. The relative motion of the LEO and the occulted GPS satellite enable the measurements taken at different tangent heights.

The uniqueness of the GPS RO technique, unlike any other existing remote sensing technique, makes its observations attractive in many aspects. Following is a list of important aspects of the RO observations: (Yunck et al., 2000 [44], Kursinski et al., 1997 [19])

- The prospect of concurrently sampling the full global atmosphere at low cost with compact, low-power, low-data-rate sensors, easily embedded in spacecraft large and small;
- Self-calibrating stable profiles that never drift, can be compared between all occultation sensors over all time, and provide a calibration standard for other sensor types, because frequency is derived either from stable atomic reference clocks or from less stable clocks whose instabilities can be solved for and removed;
- Virtually unbiased measurements that can be averaged over days or weeks to yield normal points with an equivalent temperature accuracy of order 0.1 K;
- Fully independent measurement of pressure and height, permitting recovery of absolute geopotential heights with no external reference;
- An ability to sound the atmosphere from the stratopause to near the Earth’s surface;
- A vertical resolution of a few hundred meters in the troposphere, provided by limb-sounding geometry and a point-like field of view;
- All-weather observations made at radio wavelengths, insensitive to particulates which contaminate many other tropospheric remote sensing techniques;
• An extraordinary diversity of applications outside of atmospheric science.

Because of these unique characteristics, GPS RO measurements complement other observing systems very well and may contribute to improved global analyses and forecasts as well as climate studies.

The GPS constellation currently consists of 24 satellites at about 20,200 km, with about a 12-hour period, orbiting in six different planes inclined at 55°. The first proof-of-concept GPS mission for the RO sensing of the Earth’s atmosphere was the GPS/Meteorology (GPS/MET) mission in 1995 (Ware et al. 1996 [43]; Kursinski et al. 1996 [18]). A modified TurboRogue receiver was placed in the LEO onboard NASA’s MicroLab I spacecraft. Even though many GPS/MET soundings failed to penetrate the lowest 3-5 km of the troposphere in the presence of significant water vapor, the GPS observations made by the mission have demonstrated a better than 1 K mean temperature agreement between 1 and 40 km heights with the correlative data sets (Rocken et al., 1997 [37]; Kursinski et al., 1996 [18]).

Following GPS/MET, the German CHAllenging Minisatellite Payload (CHAMP) and the Argentine satelite de Aplicaciones Cientificas-C (SAC-C) were launched in 2000. The latter two both carry the improved occultation antenna and a new generation semi-codeless GPS flight receiver, which allow radio signals to be measured with higher signal to noise ratio (SNR). Currently, CHAMP and SAC-C are providing about 200 globally distributed RO measurements per day. The horizontal resolution of a single profile is about 200–300 km. By the end of 2005 or early 2006, another mission incorporating RO receivers, Constellation Observing System for Meteorology, Ionosphere & Climate/Formosa Satellite Mission #3 (COSMIC/FORMOSAT-3) (USA-Taiwan), will be launched. The COSMIC system will have a constellation of six low-Earth orbiting (LEO) satellites sampling the same region on the Earth every 100 min (Kuo et al., 2000 [15]). It can be expected that the atmosphere will be explored with more than 100,000 occultation soundings each day with a horizontal sampling resolution better than 100 km in the near future after the completion of COSMIC (Yunck et al., 2000 [44]).
1.2 Atmospheric Variables Retrievable from GPS RO Measurements

GPS satellites transmit two-frequency radio signals at L-band, $f_1 = 1575.42$ MHz and $f_2 = 1227.60$ MHz. The basic observables detected by the receiver installed on the LEO satellite are the phase measurements in the two frequencies. The phases change (Doppler shift) of the received signal from that of the transmitted signal originates from the relative motions of the LEO and the GPS satellite, and from the atmosphere refraction if the two satellites are occultated by the Earth. Using the geometry and notation of Fig 1.1, the Doppler shift $f_d$ is given by

$$f_d = \frac{f_T}{c} (V_T \cdot u_T + V_R \cdot u_R),$$

where $f_T$ is the transmitter frequency, $c$ is the light speed in vacuum, $V_T$ and $V_R$ are the velocity vectors of the transmitter (GPS satellite) at transmitting time and the receiver (LEO satellite) at receiving time respectively, and $u_T$ and $u_R$ are unit vectors representing the ray directions at the transmitter and receiver. Writing the ray-direction vectors into radial (superscript $r$) and tangential (superscript $\theta$) components, Eq. (1.1) becomes

$$f_d = -\frac{f_T}{c} (v_r^T \cos \phi_T + v_r^T \sin \phi_T + v_r^R \cos \phi_R - v_r^R \sin \phi_R),$$

where $\phi_T$ ($\phi_R$) are the angles between the ray path and the transmitter (receiver) position vector.

The transmission of the signals follow the Fermat’s principle globally and the Snell’s law locally. Under the spherical symmetry assumption, the Snell’s law is written in terms of Bouguer’s formula (Born and Wolf, 1980 [2]) as

$$nr \sin \phi = nr = \text{constant},$$

where $n$ is the refractivity index, $r$ is the radius of the ray from the local curvature center and $a = nr$ is the impact parameter. As $r \to \infty$, $n \to 1$. Following the above equation and the Earth/satellite geometry, we have

$$a = r_T \sin \phi_T = r_R \sin \phi_R,$$

$$\alpha = \phi_T + \phi_R + \theta - \pi,$$

where $\alpha$ is the angle between the ray path and the Earth/satellite geometry.
where $r_T$ and $r_R$ are the radii of the transmitter and receiver from the local curvature center, $\theta$ is the angle between the transmitter and receiver position vectors and $\alpha$ is the bending angle. Given Doppler shift measurements, transmitter and receiver positions, and velocity vectors and transmitter frequency, the bending angle of a ray path through the Earth’s atmosphere can be determined under the spherical symmetry assumption using Eqs. (1.2)–(1.4).

The retrieved $\alpha$ is a one-dimension (1D) quantity in the occultation plane as a function of $a$. It is impossible to derive the three-dimension (3D) variation of the refractivity from $\alpha(a)$ unless assuming that the local refractivity is spherically symmetrical around the tangent point. Under such an assumption, $n$ as a function of the radius $r$ can be inverted by an Abel inversion integral (Fjeldbo and Eshelman, 1968 [8]; Fjeldbo et al, 1971 [9]) as

$$n(r) = \exp\left(\frac{1}{\pi} \int_{a}^{\infty} \frac{\alpha(x)}{\sqrt{x^2 - a^2}} dx\right), \quad (1.5)$$
For completeness, the "forward" Abel integral is given as

\[ \alpha(a) = -2a \int_{\infty}^{\infty} \frac{d \ln n(x)}{dx} \frac{1}{\sqrt{x^2 - a^2}} dx, \]  

(1.6)

The refractivity of the atmosphere \((N)\) is

\[ N = (n - 1) \times 10^6. \]  

(1.7)

The local refractivity is related to the atmospheric parameters following the equation (Bean and Dutton, 1968 [35]; Hajj et al., 2002 [10])

\[ N = 77.6 \frac{p}{T} + 3.73 \times 10^5 \frac{p q}{T^2 (0.622 + 0.378 q)} - 40.3 \times 10^6 \frac{n_e}{f^2} + 1.4 W_w + 0.6 W_i \]  

(1.8)

where \(T\) is the air temperature \((K)\), \(q\) is the specific humidity \((kg/kg)\), \(p\) is the air pressure \((hPa)\), \(n_e\) is the electron density \((m^{-3})\), \(W_w\) and \(W_i\) are liquid water and ice content \((g/m^3)\), respectively. For realistic suspensions of water or ice, the last two terms are small in comparison with the other terms and are therefore neglected (Kursinski, 1997 [17]). In the ionosphere (tangent point altitude > 60 km), the first two terms can be ignored and therefore electron density can be derived directly from the refractivity measurement by the third term (ionospheric term). In the neutral atmosphere, an ionospheric calibration process is used to remove the ionospheric effect from data and therefore \(N\) below the ionosphere is related to the atmospheric state quantities in terms of the sum of the first two terms (dry term and moist term),

\[ N = 77.6 \frac{p}{T} + 3.73 \times 10^5 \frac{p q}{T^2 (0.622 + 0.378 q)}. \]  

(1.9)

In the regions where the humidity is not significant, the moist term can be removed from the equation, which becomes

\[ N = 77.6 \frac{p}{T}. \]  

(1.10)

Combining Eq.(1.10) with the equation of state,

\[ p = \rho RT, \]  

(1.11)

and the hydrostatic equilibrium equation

\[ \frac{\partial p}{\partial z} = -\rho g, \]  

(1.12)
where \( \rho \) is the air density \((kg/m^3)\), \( R \) is the dry gas constant and \( z \) is height, we can obtain the dry air pressure and temperature. In the areas where the moisture cannot be neglected, the derivation of \( T \), \( p \) and \( q \) from the measurement of \( N \) is an underestimated problem (three equations and four unknowns) and therefore an independent knowledge of one of the three state variables \((T, p \) and \( q \)) is required to solve for the other two variables.

### 1.3 Considerations for GPS RO Data Assimilations

As promising as they are, GPS RO observation data are expected to improve both the numerical forecast as well as climate studies. Assimilating the GPS RO data into the numerical model is the first step toward this goal. Due to its flexibility in assimilating either direct (the observation variable is also the model variable) or indirect observations, variational data assimilation method (Lorenc, 1986 [27]; LeDimet and Talagrand, 1986 [22]) is a good choice for the assimilation of GPS RO observations. Briefly to say, the variational data assimilation obtains the analysis in the model space by minimizing a cost function defined in terms of the deviations of the desired analysis from the first guess (background) field and from the observations, weighted by the inverse of the background and observation error matrices respectively. Specifically, one attempts to minimize the cost function

\[
J(x_a) = \frac{1}{2}(x_a - x_b)^T B^{-1}(x_a - x_b) + \frac{1}{2}(H(x_a) - d)^T R^{-1}(H(x_a) - d),
\]

where \( x_a \) is the analysis vector on the analysis/forecast grid, \( x_b \) is the forecast background vector, \( d \) is a vector of observations, \( R \) is the sum of the observation and observation operator error covariance matrices, \( B \) is the background error covariance matrix, and \( H \) is the forward (observation) operator which transforms model variables to observational types and locations. Minimizing the cost function requires the computation of the gradient of the cost-function at each step. The gradient of the cost-function with respect to the control variable \( x_a \) is written as

\[
\nabla_{x_a} J = B^{-1}(x_a - x_b) + H^T R^{-1}(H(x_a) - d),
\]

where the matrix \( H = \partial H(x)/\partial x \vert_{x=x_a} \) is the Jacobian matrix corresponding to the (nonlinear) observation operator and \( H^T \) is called the adjoint of the observation operator \( H \). For a specific observation quantity \( H \), \( H^T \) and \( R \) are given a prior. They are the major
components for consideration to select the assimilation variable for GPS RO data. More discussions on the options of the assimilation of the RO observations can be found in Eyre, 1994 [5] and Kuo, 2000 [15].

1.3.1 Phase Measurement and Doppler Shift

As the raw measurement of the GPS RO observation, the phases (and amplitudes) of the signals are the “raw” observations and therefore incorporate no assumptions. No further data processing is required for the observational data itself. However, the forward model of the phase measurement is the most complex one. It requires an accurate simulation of the signals transmitting from the GPS satellite to the LEO satellite, subject to the ionospheric effect, atmospheric attenuation, multi-path, and other possible effects along the ray-path (Kursinski et al., 1997 [19]). Even if a observation operator is developed, the computation of the forward model and its adjoint model will be tremendously expensive. The Doppler shift, derived from the phase measurement, is obtained under the assumption of the single path propagation and requires a bit more but not much on the observation error estimation. The Doppler shift is partially due to the relative motion of the GPS and LEO satellites and therefore its simulation requires the information of the positioning of the two satellites. Same as for the phase measurement, the effects along the ray-path of the signal have to be considered in the simulation. Therefore the computational cost of the observation operator for the Doppler shift will be large as well.

1.3.2 Bending Angle

The GPS bending angle is derived through a geometry relationship of the ray of the signal, the GPS satellite and the LEO satellite. The ionospheric effect can be removed. The only assumption made from the phase measurements is the spherical symmetry. Therefore, the characteristics of the observation error are still relatively simple. Being consistent to the observation procedure, a ray tracing observation operator is preferred to simulate the bending angle as a function of the impact parameter. Since the ray is only bent by the atmosphere, its simulation does not necessarily require the positioning of the GPS and LEO satellites as the phase measurements do, but does require the initial conditions at the tangent point of the ray instead. The same spherical symmetry assumption made along the whole ray-path (globally) will be made in the observation operator and therefore introduce no extra error.
if assimilating the GPS bending angle. Ionospheric modeling is not required. To simulate a ray-path, the ray-tracing equation is integrated. The observation operator for the bending angle assimilation is thus relatively expensive.

1.3.3 Refractivity

The GPS refractivity is derived from bending angle via the Abel inversion by assuming spherical symmetry locally around the tangent point of a specific ray. Such an assumption, plus the uncertainty in setting up of the upper limit of the Abel integration, introduce additional errors to the observation data. There are two ways to assimilate refractivity. One way is to simulate the refractivity following the bending angle assimilation procedure: simulate bending angle through a ray-tracing model and then compute the refractivity using the Abel inversion integral. This is similar to the bending angle assimilation with additional calculation involved in Abel inversion. Another way is to simply calculate the local refractivity at the observed tangent points via Eq. (1.8). The observation operator is simple and cheap. But it simulates the refractivity as a point value, representing the thermal-dynamic state at the specific tangent point, rather than an integrated value along the ray-path as the observed GPS refractivity does. The incomparability becomes more severe in regions with large refractivity gradients (usually in the regions with fronts, jet-stream, etc.), where observational information are crucial for the success of NWP.

1.3.4 Temperature, Moisture and Pressure

The direct assimilation of the GPS temperature and moisture at pressure (or height) levels only requires some interpolations from the model grids to the observation locations (also required by the above mentioned observation operators). It is straight-forward with almost no extra computational cost. However, the retrieval process of these observations accumulates all errors starting from the GPS raw phase measurements, and makes the estimation of the error characteristics much harder. Besides, observational errors of the GPS temperature and humidity may be correlated with the background errors of these two variables due to the use of auxiliary data used to separately derived temperature and moisture observations.
1.4 Plan of Dissertation

Based on the above theoretical discussions, GPS bending angle and refractivity appear to be the good choices for assimilating the GPS observations. However, the advantages for assimilating each of them counter-act the disadvantages in terms of accuracy and computational cost. Development and test of an alternative scheme with both computation efficiency and accuracy is to be made.

The dissertation is planned as follows: In Chapter 2, results of a set of the GPS bending angle assimilation experiments are discussed by incorporating the GPS data into the NCEP 3D variational assimilation system. By using the bending angle observation operator, a few sensitivity studies are also carried out in this chapter in aid of the development and improvement of the GPS observation observation operators (not only for the bending angle assimilation). In Chapter 3, a local refractivity assimilation observation operator is developed and implemented into the same system as Chapter 2. The GPS bending angle and refractivity assimilations are compared and their advantages and disadvantages are quantified for the first time in a real data framework. A new GPS observation scheme, the assimilation of the GPS derived “excess phase delay”, are proposed and tested in Chapter 4. The performance of the scheme is assessed in the context of the GPS refractivity assimilation. Chapter 5 summarizes the dissertation and briefly discusses the future work on the assimilation and expanded applications of the GPS observations.
CHAPTER 2

Assimilation of GPS Bending Angle — An Accurate Method

2.1 Introduction

As discussed earlier, GPS bending angle observation is more accurate and has simpler error characteristics than the GPS refractivity, temperature and humidity observations. There are two ways to simulate the bending angle on a model atmosphere. One way is to solve for a ray-tracing model, which simulates the observed ray-path of the signals between the signal transmitter on GPS satellite and the receiver on LEO satellite. A three-dimensional ray-tracing model is most accurate. One such model is the Radio Occultation Simulation for Atmospheric Profiling (ROSAP) described by Hoeg et al., 1995 [12]. Given simulated atmospheric refractivity and observed positions and velocities of occulted satellites, the Doppler shift of a specific signal ray can be obtained following a ray trajectory simulated in a 3D space and thereafter the bending angle of the ray-path can be derived that is physically consistent with GPS observations. The ray path that intersects both the GPS and LEO satellites is found using a ray-shooting method through an iterative procedure. The expected accuracy at the end point of the ray path, compared with the actual LEO satellite (receiver) position, is within 1 mm. Though accurate as it is, the 3D ray-shooting model is computationally expensive. Zou et al. (2002) [48] gave an example of the computational cost of running such a model. By simulating a single occultation with 380 rays on a Cray J90, the 3D model takes about 9 times more of CPU seconds than a two-dimensional (2D) ray-tracing model. The 2D ray-tracing model was developed and chosen as an observation operator for bending angle assimilation by Zou et al., 1999 [49]. It assumes that the ray-path of the signal is confined in a 2D occultation plane (defined by the positions of a LEO satellite and its occulted GPS satellite and the center of the local curvature at a specific tangent point of the
occultation) and thus ignores the gradient of the refractivity perpendicular to the occultation plane. The 2D ray-tracing model has been linked into the National Centers for Environmental Prediction (NCEP) spectral statistical interpolation (SSI) data analysis system and utilized for assimilating real GPS bending angle data from GPS/MET and CHAMP (Matsumura et al., 1999 [29]; Zou et al., 2000 [50]; Liu et al., 2001 [26]; Shao and Zou, 2002 [38]; Zou et al., 2004 [46]).

Another way to simulate GPS bending angle is via the forward Abel integral in Eq. (1.6), based on the local refractivity distribution from a background field (Eyre, 1994 [5]; Engeln, 2003 [4]). This method was applied to an atmosphere under the spherical symmetric assumption to assimilate GPS/MET bending angle by Palmer et al. (2000 [32]; 2001 [33]). Compared with the ray-tracing method, this observation operator has a much lower computational cost. However, it is inconsistent with the physics of observations and does not take into account the integrated information of the refractivity and its gradient along the ray-path.

This chapter describes the assimilation of GPS bending angle observations using a 2D ray-tracing model and evaluates the assimilation results using collocated radiosonde observations. Some issues which were not addressed for the optimal use of GPS observations in early papers are discussed here, including the sensitivity study of GPS bending angle assimilation to the specifications of observational errors, sensitivity to the variation of gravity, and sensitivity to the vertical resolution of the GPS observations. The impacts of GPS data assimilation on large-scale analysis are examined.

A global 3D variational assimilation system used in this dissertation is introduced in Section 2.2. The ray-tracing method for simulating the GPS bending angle is described in Section 2.3. The specific bending angle assimilation system and GPS/MET observations are described in Section 2.4. The impacts of observational weighting on the bending angle assimilations are presented in Section 2.5. A test of the sensitivity of the analysis to the variation of gravity is conducted in Section 2.6. Impact of the vertical resolution of the assimilated GPS observations is evaluated in Section 2.7. Analysis increments of temperature, specific humidity, surface pressure and refractivity are examined in Section 2.8. Finally, summary and conclusions are provided in Section 2.9. Most of the work in this chapter was published in Shao and Zou, 2002 [38].
2.2 A Global Data Assimilation System

For all the experiments conducted in this dissertation, the NCEP SSI analysis system (Parrish and Derber 1992 [34]; Rizvi and Parrish, 1995 [36]) is used. Following the same notation in Eq. (1.13), the cost function \( J \) is given as

\[
J(x_a) = \frac{1}{2} (x_a - x_b)^T B^{-1} (x_a - x_b) + \frac{1}{2} (H(x_a) - d)^T R^{-1} (H(x_a) - d) + \left( \frac{\partial D}{\partial t} - \frac{\partial D_b}{\partial t} \right)^T O_D^{-1} \left( \frac{\partial D}{\partial t} - \frac{\partial D_b}{\partial t} \right).
\] (2.1)

The last term imposes an additional dynamical balance constraint. The time tendency of the divergence increment \((D - D_b)\) is required to remain small during the minimization process, where \(D_b\) is a vector of the background divergence. The term \(O_D\) is the diagonal error covariance matrix of \(D_b\).

In order to obtain \(x_a\), which is the 3D-Var analysis vector, a new variable \(w\) is defined:

\[
w = C^{-1} (x_a - x_b),
\] (2.2)

where \(C\) is related to \(B\) by \(B = CC^T\). In terms of \(w\), Eq. (2.1) becomes

\[
J = \frac{1}{2} [w^T w + [d - H(x_b + Cw)]^T R^{-1} [d - H(x_b + Cw)]].
\] (2.3)

Here, the last term in Eq. (2.1) has been omitted for simplicity. At the minimum, \(J\) satisfies \(\frac{\partial J}{\partial w} = 0\), which yields from Eq. (2.3)

\[
\frac{\partial J}{\partial w} = w - C^T H^T R^{-1} [d - H(x_b + Cw)] = 0,
\] (2.4)

where \(H\) is the tangent linear operator of \(H\) given by \(H = \frac{\partial H}{\partial x}\). In the NCEP/SSI, Eq. (2.4) is solved for \(w\) using a perturbation method with two embedded loops. In the inner loop, the optimal increment of \(w\), \((\Delta w)\), is obtained using the conjugate gradient algorithm; In the outer loop, \(w\) is updated by \(\Delta w\). Each inner loop carries out a number of iterations to gain an acceptable \(w\) (and thus \(x_a\)). At each analysis time, data assimilation is completed by prescribed number of outer and inner loop.

The analysis variables used in the SSI are spectral coefficients at sigma levels of normalized vorticity, unbalanced divergence, unbalanced temperature, ozone, surface skin temperature, specific humidity and coefficients for the bias correction of the satellite radiance.
data (NCEP Office Note 443 [3]). They can be uniquely transformed into the model variables of vorticity, divergence, temperature, logarithm of surface pressure and specific humidity. For comparison with the observations, the variables are transformed to Gaussian grid and then linearly interpolated to observation locations.

For experiments conducted in this study, a 6 hour assimilation window is used. The observation types already in the system include (if not all) radiosonde/dropsonde, aircraft reports, GOES precipitable water, NOAA-14 and NOAA-15 HIRS 1b radiances, NOAA-15 and NOAA-16 AMSU-A 1b radiances, NOAA-15 and NOAA-16 AMSU-B 1b radiances, SBUV ozone profiles. Here, GOES represents the Geostationary Operational Environmental Satellite Program, NOAA represents the National Oceanic & Atmospheric Administration, HIRS represents High Resolution Infrared Radiation Sounder, AMSU represents the Advanced Microwave Sounding Units and SBUV represents Solar Backscatter Ultraviolet. The new observations incorporated are vertical profiles of the GPS observations at occultation points.

2.3 A Ray-tracing Method for Calculating Bending Angle

The observation operator for GPS bending angle consists of solving a ray-trajectory equation, which governs the behavior of the ray-path of a radio signal wave under the influence of a refractivity field. When expressed in a Cartesian coordinate, the equation is written as (Kravtsov and orlov, 1990 [14])

$$\frac{d^2x}{d\tau^2} = n\nabla n,$$

(2.5)

where $x$ is the position vector pointing from the Earth’s center to the ray trajectory in the Cartesian coordinate, $n$ is the refractivity index of the atmosphere, and $\tau$ is defined by $d\tau = \frac{ds}{n}$ where $s$ is the length of the ray. Eq.( 2.5) can be decomposed into two coupled first-order equations:

$$\frac{dx}{d\tau} = y(\tau),$$

$$\frac{dy}{d\tau} = n\nabla n.$$ (2.6)

The ray-trajectory can be numerically solved for any given 3D field of $n$, once either initial conditions (initial position and direction) or boundary conditions (two end point
positions) of the ray are prescribed. The boundary problem may require a (expensive) ray-
shooting method and is subject to multi-solutions due to multi-path propagations (Zou et
al., 1999 [49]). Therefore, Eq. (2.6) is solved as an initial value problem in this study.

The 2D ray-tracing model have gone through two versions since 1999. The first version
(V1.0) (Zou et al., 1999 [49]) integrates Eq. (2.6) starting from a “virtual” transmitter (GPS
satellite) position $x_T$ in the tangent ray direction $u_T^T$ (approximation of $u_T$ in Fig. 1.1) of at
$x_T$. $x_T$ and $u_T^T$ are approximated by the following equations:

$$u_T^T = \frac{x_P}{|x_P|} \times u_P^n,$$

(2.7)

$$x_0 = x_P - \lambda u_P^T,$$

(2.8)

where $x_P$ is the position vector of a specific tangent point, $u_P^T$ is the unit vector tangent to
the ray-path at the tangent point (positive for the direction pointing from the transmitter
toward the receiver), $u_P^n$ is the unit vector normal to the occultation plane at the tangent
point, and $\lambda$ is a constant whose values is found by solving the equation of $|x_T| = 20,200$ km
(i.e., seeking a virtual transmitter position in the opposite direction of the tangent vector at
a distance of 20,200 km from the tangent point). The values of the refractivity index ($n$)
and its gradient ($\nabla n$) are calculated from model variables $T$, $p$, and $q$ and interpolated to
points on the ray-path. The numerical integration stops when the simulated ray goes out of
the atmosphere (the height of the ray from the Earth’s surface $\geq$ 100 km). Then, at the end
point of the ray, the “virtual” receiver (LEO satellite) position ($x_R$)is defined and so is the
tangent direction at the receiver ($u_R^T$). The angle between the two tangent vectors $u_T^T$ and
$u_R^T$ is taken as the simulated bending angle:

$$\alpha(a) = (\hat{u}_T^T, \hat{u}_R^T),$$

(2.9)

where the hat represents the angle between two vectors, and $a$ is the impact parameter
calculated at the location of the receiver under the spherical symmetry assumption. The
V1.0 ray-tracing model has shown to simulate GPS bending angles with reasonable accuracy.

Several modifications were made to the ray-tracing model in 2003 (Liu and Zou,
2003 [25]). First, a new starting point scheme is introduced to the ray-tracing model. The
integration of Eq. (2.6) starts from an observed tangent point location and continues in two
opposite directions, one toward the receiver and another toward the transmitter. Secondly,
the computation procedure of the bending angle is revised. A variable integration step size is
used, with the step size above a prescribed constant height being larger than that below the height. Currently, the step size is 4 km above 60 km altitude and 2 km below 60 km. Other modifications can be found in Liu and Zou (2003 [25]. The modified model (V2.0) is superior to the original one in both accuracy and efficiency. In this dissertation, the ray-tracing model V1.0 was used for experiments in Chapter 2 and V2.0 is used by Chapters 3 and 4.

### 2.4 SSI/GPS Bending Angle Assimilation System and Observations

The SSI system used in this chapter has a spectral triangular truncation of 62 (T62), which is equivalent to $1.875 \times 1.875$ degree latitude/longitude resolution, and has 28 vertical layers, extending from near the Earth’s surface to a height of about 35 km. Each minimization contains two outer loops and 50 inner loops. Only one sounding of bending angle observation will be incorporated for each minimization. Since the cost function is minimized in terms of a summation of the misfit between the model analysis and observations at each of the GPS soundings, such a single-sounding minimization is a way to assure minimization is taken and therefore the optimal value is obtained at every sounding.

GPS/MET observations are used. In order to compare the analysis data from the assimilation experiment with the radiosonde (RAD) observations, 52 soundings are selected from a total of 837 GPS/MET occultations observed during the period of June 20 to 30, 1995. For each of them, there is at least one RAD profile that was observed within a ± 3 hour time window and a 200 km spatial distance and had more than 5 levels of data. There are a total of 56 stations collocated with the 52 GPS occultations. Their collocations of the GPS and RAD data are given in Table 2.1 and the number of the RAD data varying with latitude is shown in Fig. 2.1. The original vertical resolution of the GPS/MET bending angle profiles was 0.2 km. To be consistent with the model resolution (L28), the assimilated GPS/MET bending angles have been averaged into a vertical resolution of 0.4 km below 10 km, 2 km between 10-20 km, and 4 km above 20 km height.

Taking the RAD data as the truth, the mean and root mean square errors of the analysis are defined by:

$$\text{mean}(n) = \frac{1}{L_n} \sum_{k=1}^{L_n} (x_{a}^{(n,k)} - x_{RAD}^{(n,k)})$$
Table 2.1: Geographical locations of 52 GPS/MET soundings and 56 nearby radiosonde stations.

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\[
RM S(n) = \sqrt{\frac{1}{L_n} \sum_{k=1}^{L_n} (x_{a}^{(n,k)} - x_{RAD}^{(n,k)})^2}
\]  

(2.10)

where \( L_n \) is the total number of vertical levels \((k)\) of the \( n \)th radiosonde observations. They describe the error properties averaged vertically. The total errors averaged over all the 56
Figure 2.1: The total number of the radiosonde soundings within a five degree latitudinal band that are collocated with GPS/MET observations and are used in the evaluation of the GPS/MET bending angle assimilation results.

The collocated radiosonde soundings are expressed as:

$$VAR = \frac{1}{N} \sum_{n=1}^{N} VAR(n)$$

(2.11)

where “VAR” indicates the statistical quantity of mean or RMS, and $N = 56$ is the total number of radiosonde soundings. The vertical profiles of the three similar statistical measures averaged over 56 radiosonde soundings are calculated as

$$\text{mean}(k) = \frac{1}{N_k} \sum_{n=1}^{N_k} (x_{a}^{(n,k)} - x_{RAD}^{(n,k)})$$
\[ RMS(k) = \sqrt{\frac{1}{N_k} \sum_{n=1}^{N_k} (x_a^{(n,k)} - x_{RAD}^{(n,k)})^2} \]  

(2.12)

Here, \( N_k \) is the total number of radiosonde data at the \( k \)th level and may be less than \( N \) in levels where radiosonde observations are not available from all 56 radiosonde soundings. For convenience of comparison, the analyses and the radiosonde data have been vertically interpolated onto fixed \( p \) levels from 1000 hPa to 320 hPa for the specific humidity, and from 1000 hPa to 20 hPa for the temperature, with a 20 hPa interval between the adjacent levels.

### 2.5 Impact of Observational Weighting

#### 2.5.1 Theoretical Analysis

As mentioned, parameters related to observations in Eq. (2.1) must be determined independently for each individual observation type. The error covariance matrix \( \mathbf{R} \) is one of them. Its inverse, called observational weighting, determines the confidence to be placed in each observation. \( \mathbf{R} \) contains errors resulting from instrument errors, representativeness errors and observation operator errors. The representativeness error accounts for the fact that the instrument taking the observation usually observes at a scale much smaller than the scale of the analysis. Although well recognized (Lorenc, 1986 [27]), they are seldom treated explicitly and are instead lumped together with the instrument errors. The sum of instrument errors and representativeness errors are called observation errors whose covariance matrix will be denoted as \( \mathbf{O} \). We will use \( \mathbf{F} \) to represent the forward model error covariance matrix. And thus \( \mathbf{R} \) is given by \( \mathbf{R} = \mathbf{O} + \mathbf{F} \).

The role of observational weighting can be examined by expressing the analysis increment \((x_a - x_b)\) in terms of the observational increment \((\mathbf{d} - H(x_b))\). Assuming that the background field \( x_b \) is not too far from the optimal solution \( x_a \) (i.e., the final analysis field), then \( H(x_a) \) can be expressed by the first two terms of the Taylor expansion around \( x = x_b \):

\[ H(x_a) = H(x_b) + H(x_a - x_b) \]  

(2.13)

At the minimum of \( J \), \( \nabla J = 0 \), i.e. (see (1.14)),

\[ \mathbf{B}^{-1}(x_a - x_b) + \mathbf{H}^T\mathbf{R}^{-1}(H(x_a) - \mathbf{d}) = 0. \]  

(2.14)
Substituting (2.13) into (2.14) we obtain

$$x_a - x_b = \left( H^T R^{-1} H + B^{-1} \right)^{-1} H^T R^{-1} (d - H(x_b))$$ (2.15)

If the observation error covariance matrix $R$ is diagonal, an assumption made more frequently on $R$ than on the background error covariance matrix $B$, the background error statistics $B$, along with the tangent linear and adjoint operators $H$ and $H^T$, solely determines the spread of the increments brought by each observation to its nearby regions and to other model variables not directly associated to the observed quantity.

Assuming that $R$ is a diagonal matrix with the observational error variance $\sigma_o^2$ as its elements, then for a single observation $(d)$, $H$ contains only one row and $H^T$ is a vector. After some mathematical manipulations, one obtains from (2.15) an equivalent expression for the analysis increment

$$x_a - x_b = \frac{B H^T}{\sigma_o^2 + \sigma_b^2} (d - H(x_b))$$ (2.16)

where $\sigma_b^2 = H B H^T$ is a scalar and represents the implicitly specified error variance of the background. Applying $H$ to (2.16) we obtain the analysis increment at the observational point:

$$H(x_a - x_b) = \frac{\sigma_b^2}{\sigma_o^2 + \sigma_b^2} (d - H(x_b))$$ (2.17)

Equation (2.17) shows that one point observation will affect the analysis at the same grid point differently depending on the background and observation error variances, or more precisely the ratio of the two variances. If the background is far more accurate than the observation, then $x_a = x_b$. If the observation is more accurate than the background, the observation with a smaller error variance will have a larger impact on the analysis. For a fixed background error variance $\sigma_b^2$, an over-estimate of the observation error variance $\sigma_o^2$ (i.e., the estimated observation error variance is larger than the true variance) will reduce the impact of the observations.

### 2.5.2 Three Bending Angle Observational Weightings

The errors for bending angle observations are described by

$$\alpha_{\text{obs}} - \alpha^t$$ (2.18)
where $\alpha_{\text{obs}}$ and $\alpha^t$ are vectors representing the observed bending angle and the true bending angle, respectively. The main sources of difference between the observed and true bending angle include the measurement errors, such as clock errors, thermal noise, tropospheric noise, multi-path effects, and instrument errors (Syndergaard, 1999 [41]), as well as errors due to the assumption of spherical symmetry introduced in the bending angle retrieval procedure. Although the error due to the spherical symmetry assumption generally has the largest impact on the bending angle difference (Palmer et al., 2000 [32]; Healy, 2001a [11]), it has no effect on data assimilation if the assumption of spherical symmetry is also introduced in the forward ray-tracing model (Zou et al., 2002 [48]).

Two basic quantities describing the statistics of $\alpha_{\text{obs}} - \alpha^t$ are the mean error vector and the error covariance matrix:

$$\epsilon_{\text{obs}} = \alpha_{\text{obs}} - \alpha^t \quad (2.19)$$

and

$$O = (\alpha_{\text{obs}} - \alpha^t)(\alpha_{\text{obs}} - \alpha^t)^T \quad (2.20)$$

Similarly, the forward modeling errors are characterized by the following two quantities

$$\epsilon = H(x^t) - \alpha^t \quad (2.21)$$

and

$$F = (H(x^t) - \alpha^t)(H(x^t) - \alpha^t)^T \quad (2.22)$$

where $x^t$ is the true state vector.

The definitions of the true mean errors, the observation error covariance matrix, and the forward model error covariance matrix (see eqs. (2.19)-(2.22)) involve the truth $x^t$, which is never precisely known. In atmospheric data assimilation (see (2.1)), only the statistical properties of the observation and forward model errors including $O$ and $F$ are required. The mean errors are assumed to be removed before data assimilation. Although the truth $x^t$ for every realization is not known, it is hoped that the statistical properties of the observations and forward model errors can be estimated reasonably well through various simplifying assumptions. In this study, three different bending angle weightings will be used to assess their impacts on assimilation results.

In the first set of experiments (EXP1), the weightings for the observations are simply defined as the inverse of the mean square differences between the observed and modeled
bending angles based on the NCEP background field and GPS/MET observations (solid line in Fig. 2.2), i.e.,

\[ R_1 = (H(x_b) - \alpha_{\text{obs}})(H(x_b) - \alpha_{\text{obs}})^T \] (2.23)

Assuming the background errors, the observational errors, and the forward model errors are not correlated, and the observation operator is linear, one obtains

\[ R_1 = \frac{((H(x_b) - H(x^t)) + (H(x^t) - \alpha^t) - (\alpha_{\text{obs}} - \alpha^t))}{((H(x_b) - H(x^t)) + (H(x^t) - \alpha^t) - (\alpha_{\text{obs}} - \alpha^t))^T} \]

\[ = \text{HBH}^T + \text{F} + \text{O} \]

\[ = \text{HBH}^T + \text{R} \] (2.24)

Therefore, the calculated covariances based on \( R_1 \) include contributions of background error variances, and are therefore larger than the true observation error variances.

The second set of experiments (EXP2) derives weightings from the errors in the calculation of radio occultation bending angles caused by a 2D approximation of ray-tracing (dotted line in Fig. 2.2) [Zou et al., 2002], i.e.,

\[ R_2 = \frac{(\alpha^{3D,\text{SYM}} - \alpha^{2D}) (\alpha^{3D,\text{SYM}} - \alpha^{2D})^T}{(\alpha^{3D,\text{SYM}} - \alpha^{2D}) (\alpha^{3D,\text{SYM}} - \alpha^{2D})^T} \] (2.25)

\( \alpha^{3D,\text{SYM}} \) is the simulated bending angle using a 3D ray-tracing model with the spherical symmetry assumption, and \( \alpha^{2D} \) is the simulated bending angle using the 2D ray-tracing model with the spherical symmetry assumption (i.e., \( H \) in (2.1)). The 3D ray-tracing model used is called the ROSAP (Radio Occultation Simulation for Atmospheric Profiling) model developed by Hoeg et al. (1995 [12]). In this 3D ray-tracing model, both the along-track and across-track gradients of refractivity are included. The ray path that intersects both satellites (GPS and LEO) is found using the ray-shooting method through an iterative procedure. The iterative process is terminated when the end-point of the ray path is close to the actual LEO satellite position (within 1-mm distance).

Assuming that errors of the two forward ray-tracing models are not correlated with each other and with the background errors, \( R_2 \) can be expressed as

\[ R_2 = \frac{((H_{3D}(x_b) - H_{3D}(x^t)) - (H(x_b) - H(x^t)) + (H_{3D}(x^t) - \alpha^t) - (H(x^t) - \alpha^t))}{((H_{3D}(x_b) - H_{3D}(x^t)) - (H(x_b) - H(x^t)) + (H_{3D}(x^t) - \alpha^t) - (H(x^t) - \alpha^t))^T} \]

\[ = H_{3D} \text{BH}^T_{3D} + \text{HBH}^T + \text{F}_{3D} + \text{F} \] (2.26)
where \( H_{3D} \) represents the 3D ray-tracing model and \( H \) is the 2D ray-tracing model. If the 3D ray-tracing model is assumed perfect, \( F_{3D} = 0 \). Therefore, the observational error variances \( O \) are not included in the second estimate. This may be justified by the fact that below 40 km, the forward model errors are larger than any of the total measurement errors (i.e., observational noise including thermal noise, multi-path noise, satellite velocity error, and residual ionospheric error, as well as forward model parameter errors including uncertainties associated with the refractivity coefficients \( c_1 \) and \( c_2 \) in (1.9)) and the local radius of curvature) (Palmer et al., 2000 [32]). The background errors are accounted for twice in the second estimate.

The third set of experiments (EXP3) uses derived weightings based on the standard deviation of the total bending angle errors estimated by Palmer et al., (2000 [32]) (dashed line
in Fig. 2.2). The total bending angle error variance was constructed by adding the variances of the observation noise, the forward model parameter error and the forward modeling errors. The forward modeling errors included errors due to the spherical symmetry assumption and the representativeness errors.

Figure 2.2 shows the standard deviation of bending angle observation errors based on the above-mentioned three estimates. The bending angle errors estimated by $R_1$ (EXP1) is very close to Palmer’s estimate (EXP3) below 15 km, and is largest among the three estimates above 15 km. The standard deviation used in EXP2 is much smaller than those of EXP1 and EXP3 at almost all heights.

Errors in bending angle data may be vertically correlated due to the filtering of the phase measurements, satellite position and velocity errors, clock error, and incomplete calibration of the ionospheric bending (Kursinski et al., 1997 [19]; Feng and Herman, 1999 [6]). Since the vertical correlation was found not to be broad (Zou et al., 2002 [48]), bending angle observation errors are assumed uncorrelated and only the variances are considered in this study.

### 2.5.3 Comparison of Simulated and Observed Refractivity

Described in Chapter 1, the values of the neutral atmospheric refractivity can be obtained either from atmospheric state variables (Eq. (1.9)) or from bending angle (Eq. (1.6)). We will denote the refractivity calculated from Eq. (1.9) as $N^{\text{LOC}}$ and that from Eq. (1.6) as $N^{\text{GPS}}$. The values of $N^{\text{LOC}}$ and $N^{\text{GPS}}$ at the same tangent point can be different since $N^{\text{LOC}}$ reflects the atmospheric thermodynamic state ($T$, $p$ and $q$) at the tangent point while $N^{\text{GPS}}$ is a complicated nonlinear function of the atmospheric thermodynamic state along the ray path (mostly near the tangent point within ± 500 km distance). The GPS refractivity observations ($N_{\text{obs}}$), which are derived from bending angle observations, is in the same sense as $N^{\text{GPS}}$.

It is interesting to examine a set of figures showing the vertical profiles of $N^{\text{LOC}} - N_{\text{obs}}$ and $N^{\text{GPS}} - N_{\text{obs}}$ for all 52 GPS soundings before and after the bending angle assimilation (Figs. 2.3 and 2.4). The mean values and the standard deviations of these two quantities are shown in Fig. 2.5. The biases of $N^{\text{LOC}}$ and $N^{\text{GPS}}$ for EXP3 are very similar to EXP1, therefore only EXP1 and EXP2 are presented here. To guarantee the reliability of the statistical values, only the data above 1 km are shown, with 20 or more GPS data at each
level. We can find that the bending angle assimilation, which minimizes the differences of \( \alpha - \alpha_{\text{obs}} \), reduces the values of both \( N_{\text{LOC}} - N_{\text{obs}} \) and \( N_{\text{GPS}} - N_{\text{obs}} \). Before assimilation, the values for both \( N_{\text{LOC}} \) and \( N_{\text{GPS}} \) calculated from background fields are higher than the GPS/MET observations (i.e., positive background minus observation refractivity in Fig. 2.5a-b). This is consistent with the study by Rocken et al. (1997 [37]) who found a negative bias of GPS/MET refractivity from the conventional data. After the bending angle assimilation, the sign of the bias errors for \( N_{\text{GPS}} \), although much reduced, remains positive (Fig. 2.5d,f).

The sign of the bias errors for \( N_{\text{LOC}} \) has changed to a negative bias in the lower troposphere (Fig. 2.5c,e). The error reduction in terms of \( N_{\text{GPS}} \) is much more significant than that of \( N_{\text{LOC}} \) through the bending angle assimilation. This is to be expected given the physical nature of \( N_{\text{GPS}} \) which is more consistent with the GPS-derived bending angle. Both \( \alpha \) and \( N_{\text{GPS}} \) include the effect of atmospheric refractivity along the ray path, while \( N_{\text{LOC}} \) describes the property of the atmospheric refractivity in a grid box only. Comparing the left with the right panels of Fig. 2.5, we found that the differences between \( N_{\text{LOC}} \) and \( N_{\text{obs}} \) are different with those between \( N_{\text{GPS}} \) and \( N_{\text{obs}} \), both before and after the bending angle assimilation. Such a difference reflects the amount of information contained in \( N_{\text{GPS}} - N_{\text{LOC}} \) that may be difficult to account for in the GPS refractivity assimilation if \( N_{\text{LOC}} \) is directly compared with \( N_{\text{obs}} \) in refractivity assimilation.

2.5.4 Comparison of Temperature and Specific Humidity Analyses with Radiosonde Observations

Minimization of the misfit between simulated and observed bending angles directly causes changes in temperature, specific humidity and pressure. We first examine the mean errors and RMS errors of temperature and specific humidity above 850 hPa (Table 2.2). Their vertical profiles are shown in Fig. 2.6. The assimilation of the bending angle leads to a better fit of the atmospheric state analysis to radiosonde observations: Both mean and RMS errors from all three data assimilation experiments are smaller than those of GES (see Table 2.2). The mean errors are reduced by about 22% for temperature and 55% for specific humidity for EXP1 and EXP3, and 81% for temperature and 2% for specific humidity for EXP2. For most of the troposphere, the background temperature and moisture fields have positive biases from the radiosonde measurements (Fig. 2.6), implying that the background fields are probably too wet and too warm. After assimilation of the bending angle, both
Figure 2.3: The 52 vertical profiles of the differences between the simulated \( N^{LOC} \) and GPS/MET observed refractivity \( N_{\text{obs}} \) before and after bending angle assimilation. The left panels show the profiles for soundings 1-28 and the right panels show those for soundings 29-56. (a,b) \( N^{LOC} - N_{\text{obs}} \) before assimilation. (c,d) \( N^{LOC} - N_{\text{obs}} \) from EXP1. (e,f) \( N^{LOC} - N_{\text{obs}} \) from EXP2.
Figure 2.4: Same as Fig. 2.3, except for the differences between $N^{GPS}$ and $N_{obs}$. (a,b) $N^{GPS} - N_{obs}$ before assimilation. (c,d) $N^{GPS} - N_{obs}$ from EXP1. (e,f) $N^{GPS} - N_{obs}$ from EXP2.
Figure 2.5: The vertical distributions of the mean (solid curve) and the standard deviations (short lines) of the simulated and observed refractivity differences averaged from the 52 soundings before and after assimilation. (a) $N^{LOC} - \hat{N}_{obs}$ and (b) $N^{GPS} - \hat{N}_{obs}$ before assimilation. (c) $N^{LOC} - N_{obs}$ and (d) $N^{GPS} - N_{obs}$ from EXP1. (e) $N^{LOC} - N_{obs}$ and (f) $N^{GPS} - N_{obs}$ from EXP2.
Table 2.2: The mean and r.m.s errors for temperature and specific humidity analyses above 850 hPa. Observations from the 56 radiosonde soundings are included in the calculations.

<table>
<thead>
<tr>
<th></th>
<th>GES</th>
<th>EXP1</th>
<th>EXP2</th>
<th>EXP3</th>
<th>GES</th>
<th>EXP1</th>
<th>EXP2</th>
<th>EXP3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>T (K)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMS</td>
<td>1.83</td>
<td>1.69</td>
<td>1.70</td>
<td>1.71</td>
<td>0.91</td>
<td>0.82</td>
<td>0.84</td>
<td>0.82</td>
</tr>
<tr>
<td>mean</td>
<td>0.26</td>
<td>0.20</td>
<td>-0.05</td>
<td>0.20</td>
<td>0.13</td>
<td>-0.06</td>
<td>-0.13</td>
<td>-0.09</td>
</tr>
</tbody>
</table>

The bias and RMS errors are reduced for both temperature and specific humidity in most of the vertical levels. We notice the large deviation of the bias error of EXP2 temperature from those of the background, EXP1 and EXP3. Recall that the standard deviation of the observation errors in EXP2 is much smaller than that in EXP1 and EXP3. The contribution of the GPS/MET bending angle observations must have increased during the assimilation in EXP2 due to the larger observational weighting (inverse of the error covariance). This results in a larger analysis increment for EXP2 than in the other two experiments. It is seen that the differences between EXP1 (EXP3) and EXP2 are more significant for temperature than for specific humidity. The analysis increment of temperature (-0.31 K) from EXP2 is about 4 times larger than that of EXP1 and EXP3 (-0.06 K). Below 200 hPa, EXP2 produced a much colder atmosphere than EXP1 and EXP3, and a warmer atmosphere above this level.

The mean and RMS errors averaged in the different latitudinal areas (90°S-40°N, 40°N-60°N, and 60°N-90°N) are also examined (Figs. 2.7 and 2.8). The number of the radiosonde soundings in the three ranges is 10, 30 and 16, respectively. Both the mean errors and the RMS errors in mid-latitudes for the temperature (Fig. 2.7a-b) and the specific humidity (Fig. 2.8a-b) have been improved except in EXP2 where the mean errors of the temperature exhibit opposite sign with the largest magnitude. Most features seen in Fig. 2.6 represent mid-latitude (40°N-60°N) situations where the majority of the GPS/MET soundings were located (see Fig. 2.1). We also notice that the large negative moisture bias errors below 750 hPa in Fig. 2.6c come from soundings south of 40°N (Fig. 2.8e). Large uncertainty exists regarding the impact of observational weighting on the assimilation of GPS/MET bending angle in the high latitudes, tropics, and Southern Hemisphere due to the small sampling in these regions.

The differences of the absolute values of the mean and RMS error for each sounding
Figure 2.6: The vertical distributions of the mean errors and the RMS errors of the temperature and specific humidity fields before and after the assimilation averaged from the 52 soundings, verified with all 56 collocated radiosonde observations. (a) The mean errors of the temperature. (b) The RMS errors of the temperature. (c) The mean errors of the specific humidity. (d) The RMS errors of the specific humidity.
Figure 2.7: The vertical distributions of the mean errors (left) and the RMS errors (right) of the temperature before and after the assimilation averaged from the soundings within (a,b) 40°N-60°N, (c,d) 60°N-90°N and (e,f) 90°S-40°N.
Figure 2.8: Same as Fig. 2.7, except for the specific humidity.
(averaged in the vertical direction) between the analysis and the background are plotted in Fig. 2.9. The negative (positive) values indicate an improvement (degradation) of the GPS analysis over the background field. Due to smoothing effects of the vertical average, the latitudinal features are not as apparent as in Figs. 2.7 and 2.8. But we still find better analysis in the mid-latitudes 40°N-60°N. For most of the soundings, EXP1, EXP2 and EXP3 produced the same sign of the differences of the absolute values of the mean error and RMS error between the bending angle assimilation and the background. In other words, if EXP1 brings an improvement to the temperature analysis, the other two experiments do as well, and if EXP1 produces a degradation to the temperature analysis, as do the others. The magnitude of temperature adjustments produced by EXP2 are larger than those by EXP1 and EXP3 for most of the soundings. The differences between EXP1(3) and EXP2 for the specific humidity field are not as significant as for the temperature. These results suggest that the temperature analysis is more sensitive than the specific humidity analysis to the specification of the observational weightings of the bending angle data. It is also noticed that a large degradation in one field does not imply a degradation in another field. For example, while a large degradation of the specific humidity occurred at the radiosonde sounding 03 (GPS03), a slight improvement in the temperature analysis is observed in EXP2.

2.5.5 Comparison of Surface Pressure Analysis with Radiosonde Observations

Like temperature and specific humidity, the surface pressure $p_s$ is also an independent variable updated by the assimilation of the bending angle. However, the impact of bending angle assimilation on the surface pressure seems to be negative. Table 2.3 shows the mean and RMS errors of $p_s$. The background field has a negative bias (-0.81 hPa) from the radiosonde observation. The analyses from the three experiments do not change the sign of the bias error, but with a slight error increase in both mean and RMS quantities compared with the background.

One of the possible reasons for the degradation of the surface pressure analysis by bending angle assimilation, suggested by Matsumura et al. (1999 [29]), is the inconsistency between the data assimilation system and the ray-tracing observation operator in dealing with the latitudinal and vertical variation of the acceleration of gravity. A single value of the gravity constant $g_0$ is used in the NCEPS SSI system and a variable $g(\phi, z)$ is used in the ray-tracing...
Figure 2.9: (a) Differences of the absolute mean errors of temperature between the analysis and background (|EXP error| − |GES error|) at the 56 collocated radiosonde sounding locations averaged for all levels above 850 hPa. (b) Differences of the absolute RMS errors of temperature between analysis and background (|EXP RMS| − |GES RMS|) at the 56 collocated radiosonde sounding locations averaged for all levels above 850 hPa. (c) Same as (a) except for the specific humidity. (d) Same as (b) except for the specific humidity.
Table 2.3: Same as Table 2.2, except for the surface pressure ($hPa$).

<table>
<thead>
<tr>
<th></th>
<th>GES</th>
<th>EXP1</th>
<th>EXP2</th>
<th>EXP3</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS</td>
<td>3.72</td>
<td>3.90</td>
<td>4.44</td>
<td>3.86</td>
</tr>
<tr>
<td>mean</td>
<td>-0.81</td>
<td>-0.81</td>
<td>-2.43</td>
<td>-1.04</td>
</tr>
</tbody>
</table>

Figure 2.9: -continued.
observation operator, where $\phi$ and $z$ are latitude and height, respectively. Unfortunately, a test on the sensitivity of the surface pressure analysis to the gravity, carried out by repeating EXP1 for all the 52 GPS/MET soundings with a constant $g_0$ in the ray-tracing procedure did not produce a noticeable reduction in either mean or RMS values (see Section 2.6).

Before we offer any further explanation on the analysis from bending angle assimilation, some further examination of the assimilation results based on the analysis increments of $T$, $q$, and $p_s$ in terms of the analysis increment of $N$ has been given in the 2.8. We find that the refractivity is more sensitive to changes in temperature and surface pressure, and not as sensitive to the specific humidity. Larger sensitivities are found at lower heights. The sign of the analysis increment of the refractivity largely determines the signs of $T$, $q$ and $p_s$. The inconsistency between the GPS/MET and radiosonde observations, compared with the background fields, has influenced the evaluation of the analysis results.

### 2.6 Sensitivity to Variation of Gravity Acceleration

In Section 2.5.5, we mentioned that the inconsistency between the data assimilation system and the ray-tracing operator in dealing with the variation of the gravitational constant ($g$) with the latitude and height could have contributed to the degradation of the surface pressure analysis. To assess the sensitivity of the data assimilation results to $g$, we have repeated EXP1 with a constant gravity acceleration $g_0$ (EXP1.G). Numerical results from EXP1.G will then be compared with those of EXP1 in which the variation of $g$ with latitude and height is taken care of in the ray-tracing operator.

The theoretical gravity at any latitude $\phi$ is given by (Lambeck, 1988 [21])

$$g_{re} = g_e(1 + f_2\sin^2\phi - \frac{1}{4}f_4\sin^22\phi). \quad (2.27)$$

with $f_2 = -f + \frac{5}{2}m - \frac{17}{14}fm + \frac{15}{14}m^2$ and $f_4 = -\frac{1}{2}f^2 + \frac{5}{2}fm$. $m$ is the ratio between the centrifugal force at the equator and the gravity at the equator and is equal to 0.003468 (rad). $g_e$ is the theoretical gravity at the equator obtained by

$$g_e = \frac{GM}{R_e^2}(1 - f + \frac{3}{2}m - \frac{15}{14}mf)^{-1}. \quad (2.28)$$

$G$ is the gravitational constant and $M$ is the mass of Earth. $GM = 398600.4415 \times 10^9 (m^3 s^{-2})$. $R_e = 6378.1363 \times 10^3 (m)$ is the equatorial radius of the Earth, $f$ is the flattening ratio as \( f = \frac{R_e - R_p}{R_e} = \frac{1}{298.2564}, \) and $R_p$ is polar radius of the Earth.
Geopotential $\Phi$ is related to geometric height $z$ by

$$\Phi = \int_0^z g(\phi, z)dz.$$  \hspace{1cm} (2.29)

where $g$ is the gravity acceleration at a given altitude $z$ and latitude $\phi$:

$$g = \frac{g_{re}R^2}{(R + z)^2}.$$  \hspace{1cm} (2.30)

$R$ is the radius of the reference ellipsoid at latitude $\phi$. Performing integration (2.29) by using (2.30), we obtain

$$\Phi = \frac{g_{re}Rz}{(R + z)} = g_0 Z.$$  \hspace{1cm} (2.31)

and geopotential height $Z = \frac{\Phi}{g_0}$, where $g_0 = 9.80665(\text{ms}^{-2})$ is the global average of gravity at mean sea level. The difference between geometric ($z$) and geopotential height ($Z$) is zero at the surface and increases with height. The difference also varies with latitude.

Fig. 2.10 shows the distribution of the analysis differences between the two experiments (EXP1-EXP1.G) for $N^{GPS}$ and $N^{LOC}$ with height and latitude. The data have been averaged over 5° latitude bands. We find that the analysis differences of the refractivity (EXP1-EXP1.G) increase with height and decrease with latitude. There are positive differences in lower latitudes with a maximum at 17 km near the equator. At 60°N, the differences become negative below 13 km. The magnitudes of the differences are less than 1 (N unit). The vertical distributions of the differences between the simulated and GPS/MET observed refractivities are shown in Fig. 2.11. While $N^{GPS}$ from EXP1.G compares better with GPS/MET observations than EXP1, $N^{LOC}$ does not. Such differences result in a small bias of the vertical profiles of the temperature and specific humidity from the two experiments. The averaged vertical profiles of the differences between EXP1 and EXP1.G for the temperature and specific humidity are shown in Fig. 2.12. It seems that including a variable gravity acceleration produces a small negative bias in temperature and a small positive bias in specific humidity at almost all levels. The maximum temperature difference between two experiments is only about 0.03$K$, while the maximum difference for the specific humidity is about 0.01 g kg$^{-1}$. For the surface pressure, the mean and RMS errors of EXP1.G only differ on the order of $10^{-3}hPa$ from those of EXP1.

From the above analyses, we may find that the variation of gravity does influence the distribution of the analyses with height and latitude, but its impacts are not very significant.
Figure 2.10: The distribution of the differences of (a) $N^{GPS}$ and (b) $N^{LOC}$ between EXP1 and EXP1.G with height and latitude.
Figure 2.11: The vertical distributions of the differences between the simulated and observed (a) $N^{GPS}$ and (b) $N^{LOC}$. 
A complete study on the impact of gravity should include a set of experiments in which both the data assimilation system and the ray-tracing observation operator take into account the variation of gravity with height and latitude. Including the variation of gravity into the NCEP SSI system, however, is beyond the scope of this work.

## 2.7 Impact of the Vertical Resolution of GPS Observations for Data Assimilation

The vertical resolution of the bending angle profiles provided by GPS/MET is 0.2 km. For a data assimilation system using 28 vertical levels, we have chosen a vertical resolution of 0.4 km below 10 km, 2 km between 10-20 km and 4 km above 20 km height to save computational cost while still roughly matching the model resolution. The information content of the GPS/MET observation could be reduced after such data selection. Here, we increase the vertical resolution of the assimilated GPS/MET data to 0.2 km below 14 km, 0.6 km between 14-20 km, 1 km between 20-30 km and 2 km between 30-40 km. The model resolution is as before. The weighting in EXP1 is used. The new experiment is called as EXP1.RES.

Fig. 2.13 shows the differences of $N^{GPS}$ and $N^{LOC}$ between the analyses and GPS/MET observations for both experiments. We find that the increase of the vertical resolution of the GPS/MET observations provides a more accurate description of the vertical profile of the refractivities in EXP1.RES. The RMS error for both $N^{GPS}$ and $N^{LOC}$ have been reduced. For $N^{LOC}$, there is only slight bias between EXP1.RES and EXP1: EXP1.RES is more positive above 12 km and negative below, resulting from a larger adjustment of EXP1.RES from the background. The significant discrepancy between the two experiments exists in $N^{GPS}$. The analysis of EXP1.RES is much closer to the GPS/MET observations both in terms of mean and RMS errors.

The vertical profiles of the mean and RMS errors of temperature and specific humidity are shown in Fig. 2.14. The increase of the data resolution (EXP1.RES) further reduces the mean error of the temperature analysis in the middle and lower troposphere above 850 mb. Changes in the surface pressure (not shown) are relatively small.
Figure 2.12: The vertical distributions of the analysis differences between EXP1 and EXP1.G for (a) temperature and (b) specific humidity.
Figure 2.13: The vertical distributions of the mean (left) and the standard deviations (right) of the differences between the simulated and observed (a-b) $N_{GPS}$ and (c-d) $N_{LOC}$.

### 2.8 Analysis Increments of Temperature, Moisture, Surface Pressure and Refractivity

During the assimilation of the bending angle, the temperature, moisture and surface pressure are simultaneously adjusted in order to minimize the distance between the observed and simulated bending angles. Since a direct transform between the refractivity and the bending angle exists using the Abel transform and Abel inversion, we will give a discussion on the relationship between the variation of the refractivity, instead of the bending angle, and the analysis variables of $T$, $q$ and $p_s$.

Within the assimilation process, we first calculate the refractivity ($N_{LOC}$) from model
Figure 2.14: The vertical distributions of the mean and RMS errors of the temperature and specific humidity for EXP1 and EXP1.RES. (a) The mean errors of the temperature. (b) The RMS errors of the temperature. (c) The mean errors of the specific humidity. (d) The RMS errors of the specific humidity.

variables by using Eq. (1.9). Since the system is in the sigma (σ) vertical coordinate, Eq. (1.9) can be written as

\[ N = c_1 \frac{p_s \sigma}{T} + c_2 \frac{p_s \sigma q}{T^2 (0.622 + 0.378q)} . \]  

(2.32)

by substituting \( p \) with \( p_s \sigma \). Assuming the background is not too far from the analysis, we may express the variation of the refractivity purely due to a change in a particular dependent variable as follows:

\[ \frac{(\Delta N)^{(k)}}{\Delta p_s} \approx \frac{N}{p_s} \]  

(2.33)
\[
\frac{(\triangle N)_{T,ps}^{(k)}}{\triangle q^{(k)}} \approx c_2 \frac{T^2}{2} (0.622 + 0.378q)^2.
\]
\[
\frac{(\triangle N)_{q,ps, T}^{(k)}}{\triangle T^{(k)}} \approx -c_3 \frac{p_s \sigma q}{T^2} \frac{\triangle N^{(k)}}{N^{(k)}}
\]
\end{equation}

where the superscript \(k\) is the index of the vertical levels. We find both \(\triangle p_s\) and \(\triangle q\) have the same sign as \(\triangle N\), and \(\triangle T\) has an opposite one. That is, if the refractivity increases, both the surface pressure and moisture will increase and the temperature will decrease at the corresponding level. Since \(\triangle N\) at each level all contributes to the variation of the surface pressure, the total \(\triangle p_s\) should be an accumulation of the effects of \(\triangle N\), weighted by the different \(N\) at each level:

\[
\frac{\triangle p_s}{p_s} \approx \sum_k \frac{(\triangle N)_{T, q, p}^{(k)}}{N^{(k)}}
\]

When examining the analysis increments between \(N\) and any of \(T\), \(q\) or \(p_s\), these relationships may not work well since all the variables \(T\), \(q\) and \(p_s\) are simultaneously adjusted. In order to gain some insights into the relative importance of various variables to the bending angle and refractivity, we calculated the relative sentivities of refractivity to the three model variables at all levels and for all the 52 GPS soundings. Here, the relative sensitivity is defined as (Zou et al., 1993 [45])

\[
s_i = (\nabla N)^{(i)} \frac{x^{(i)}}{N}
\]

where, \(x\) is a vector of the atmospheric state (i.e., \(T\), \(q\) or \(p_s\)), \(x^{(i)}\) is its \(i\)th component, \((\nabla N)^{(i)}\) is the \(i\)th component of the gradient vector \(\nabla N\), and \(s_i\) is the \(i\)th component of the relative sensitivity of \(N\) to the changes of \(x\). Numerical results of the relative sensitivity, averaged over all the soundings, are shown in Fig. 2.15. Based on these results, we find that the refractivity is more sensitive to temperature (\(|s_T| > 1\)) and surface pressure (\(|s_{p_s}| = 1\)), and not as sensitive to the specific humidity (\(|s_q| < 1\)). Larger sensitivities are found at lower heights. The relative sensitivity parameter is negative to the temperature. This means increases (decreases) of the temperature result in decreases (increases) of the refractivity. Eyre (1994 [5]) shows the similar relationship between the temperature perturbation and the change in the refracted angle.

Such a simple theoretical analysis seems to be consistent with our numerical results. As shown in Fig. 2.5, both \(N^{LO}\) and \(N^{GPS}\) calculated from the background have positive biases
Figure 2.15: The vertical distributions of the relative sensitivities of refractivity to changes in temperature and specific humidity.

from $N_{\text{obs}}$ ($N^{GPS} - N_{\text{obs}} > 0$ and $N^{LOC} - N_{\text{obs}} > 0$). We also notice that the refractivity difference between the background and the observation ($N_{\text{obs}}$) has two maximum values at about 3.5 km and 1.5 km. The analysis increments of refractivity for the three experiments are all negative with a similar shape (Fig. 2.16). The analysis increment of the refractivity from EXP2 is largest due to the use of the largest observational weightings in this experiment.

Fig. 2.17 shows the analysis increments of temperature and specific humidity from the three experiments. As expected, the pattern of the temperature increment does not resemble that of the refractivity. There are positive increments above 300 hPa. Two maximum increments occur around 350 hPa and below 800 hPa. The increment of the specific humidity has the same variation trend as the refractivity.

The consistency between $\Delta N$ and $\Delta q$ is found in almost all of the soundings. Fig. 2.18 gives some examples. The left panels of the figure show the analysis increments of $N^{LOC}$ at three radiosonde locations GPS38, 47 and 03. $\Delta N$ is positive at each level for GPS38 and negative at each level for GPS03. For GPS47, $\Delta N$ is negative above 700 hPa and positive
Figure 2.16: The vertical distributions of the refractivity analysis increments of (a) $N^{\text{LOC}}$ and (b) $N^{\text{GPS}}$. 
Figure 2.17: The vertical distributions of the analysis increments of (a) temperature and (b) specific humidity.
Table 2.4: The analysis increments of the surface pressure (hPa) for GPS38, 47 and 03.

<table>
<thead>
<tr>
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<th>GPS38</th>
<th>GPS47</th>
<th>GPS03</th>
</tr>
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<tbody>
<tr>
<td><strong>EXP1</strong></td>
<td>4.38</td>
<td>0.36</td>
<td>-0.42</td>
</tr>
<tr>
<td><strong>EXP2</strong></td>
<td>0.02</td>
<td>-2.94</td>
<td>-0.79</td>
</tr>
<tr>
<td><strong>EXP3</strong></td>
<td>3.72</td>
<td>-0.10</td>
<td>-0.43</td>
</tr>
</tbody>
</table>

below 700 hPa. The patterns of moisture increments for all three experiments (right panels in Fig. 2.18) are almost identical to those of the refractivity.

For the surface pressure, the analysis increments of the three experiments are all negative (see Table 2.3), consistent with the above sensitivity analysis given the negative increment of refractivity at all levels. To see the vertically accumulated impacts of the refractivity increment $\Delta N$ on the increment of the surface pressure $\Delta p_s$, the three soundings shown in Fig. 2.18 can be examined. The $p_s$ increments for all three experiments for these three soundings are given in Table 2.4. For GPS38 (GPS03), $\Delta p_s$ is positive (negative) corresponding to a positive (negative) $\Delta N$ (left panels in Fig. 2.18) at each level. For GPS47, the contribution of $\Delta N$ on $\Delta p_s$ is changed by the relative magnitudes of the negative $\Delta N$ above 700 hPa and the positive $\Delta N$ below. For EXP1, the area of the positive part is larger, so the resulting $\Delta p_s$ is positive. A similar explanation can be used for the negative $\Delta p_s$ in EXP2. EXP3 has a smaller $\Delta p_s$ since the areas of the negative and the positive part of $\Delta N$ are close in size.

From the above analysis, we find that the analyses of temperature and specific humidity after GPS/MET data assimilation compare more favorably with radiosonde observations than the background fields. However, in the lower troposphere (below 850 hPa) and for the surface pressure, the GPS/MET analyses do not show an improved agreement with radiosonde data. One reason is the sparseness of the GPS/MET data in the lower levels. Among the 52 GPS/MET soundings, 24 soundings did not reach below 1.2 km. The other reason is probably the inconsistency between the GPS/MET and radiosonde observations. A GPS/MET observation measures an averaged effect of the refractivity over a ray path whereas a radiosonde observation is somewhat more like a point measurement. Inconsistency between the two data sets may result in uncertainties in the evaluation of the GPS/MET data assimilation analysis with radiosonde observations. Here we give two
Figure 2.18: The vertical distributions of the analysis increments for the refractivity (left) and the specific humidity (right) at locations of (a,b) GPS38, (c,d) GPS47 and (e,f) GPS03.
examples for GPS38 and GPS03. In Fig. 2.9c and d, we find that the assimilation of GPS03 and GPS38 produced a degradation and an improvement for the analysis of the specific humidity, respectively. Fig. 2.19 shows the vertical profiles of the specific humidity at the two radiosonde locations. For the background moisture, both GPS38 and GPS03 have a negative bias compared with the radiosonde observations and a wet adjustment is needed to bring an improvement to the moisture analyses at these two sounding locations. At GPS38, the moisture analysis increment is positive (i.e., a wet adjustment) and the analysis becomes closer to the observation after the assimilation. However, at GPS03, the moisture analysis increment is negative and results in an even larger dry bias after the data assimilation. Since $\Delta q$ has the same sign as $\Delta N$, we could find the reason for the difference of the analysis quality of the two single-sounding experiments by examining the vertical profiles of the observational increments of refractivity in terms of GPS/MET data and radiosonde data (Fig. 2.20). In Fig. 2.20a-b, the differences between $N_{\text{LOC}}$ (from background, EXP1, EXP2, and EXP3) and $N_{\text{obs}}$ at two soundings are shown. Minimization of the distance between the desired analysis and GPS/MET bending angle observations reduces the differences between $N_{\text{LOC}}$ and $N_{\text{obs}}$ as expected, but not necessarily the differences between the analysis and radiosonde observations, such as GPS03 (see Fig. 2.20d). Based on the background field, GPS38 (thick solid line in Fig. 2.20a) needs a positive increment of the refractivity and GPS03 (thick solid line in Fig. 2.20b) needs a negative one. From this point of view, assimilation of both GPS03 and GPS38 work well (shown in Fig. 2.21 of vertical profiles of the observational increments). On the other hand, the background bias of GPS38 to the radiosonde observation (Fig. 2.20c) is similar to the GPS/MET observation (Fig. 2.20a), but for GPS03, the bias errors of the background field with the radiosonde (Fig. 2.20d) and GPS/MET observations (Fig. 2.20b) have opposite signs in the lower levels. There is a negative bias between the background and radiosonde refractivity below 600 hPa. In other words, the inconsistency between the radiosonde and GPS/MET data has contributed to the “degradation” of GPS03 which may not be true.

In fact, we have examined each of the observational increments for refractivity for the 52 GPS/MET occultations (GES-GPS) and 56 radiosonde locations (GES-RAD). Fig. 2.22 shows 10 soundings with the most obvious inconsistency between the two types of observations compared to the NCEP background fields. The observational increment of the refractivity with respect to GPS observations has an opposite sign to that of radiosonde
Figure 2.19: The vertical distributions of the specific humidity at locations of (a) GPS38 and (b) GPS03.

data. Compared with Fig. 2.9, we could find such inconsistency bears a good correlation with the degradation of the analyses, especially the specific humidity. Table 2.5 shows the mean and RMS errors of the temperature and specific humidity after excluding these soundings from the evaluation of the bending angle assimilation results. We find that errors from all three data assimilation experiments are smaller than those including all soundings in the evaluation. The most significant differences occur in the mean errors of the specific humidity variable.

2.9 Summary and Conclusions

The sensitivity of the bending angle assimilation to the observational weighting is studied based on a series of single-occultation GPS/MET bending angle assimilation experiments.
Figure 2.20: The vertical distributions of the observational increments of $N^{LOC}$ in terms of (a-b) the GPS/MET observations and (c-d) radiosonde observations. Left panels are for GPS38 and right panels for GPS03.

Table 2.5: Same as Table 2.2, except that those soundings for which the differences between the GPS/MET observation and background have an opposite sign to the differences between radiosonde observation and background in most of vertical levels are excluded.

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<tr>
<td>EXP3</td>
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</table>
Figure 2.21: The vertical distributions of the analysis increments of $N^{LOC}$ in terms of (a-b) the GPS/MET observations and (c-d) radiosonde observations. Left panels are for GPS38 and right panels for GPS03.

Three different weightings and 52 GPS/MET occultations are used. The 52 GPS/MET occultations are chosen from a total of 837 GPS/MET occultations available during June 20-30, 1995. For each selected GPS/MET occultation, there is at least one radiosonde profile that had more than 5-levels of data and was observed within a $\pm 3$ h time window and a 200 km distance of the occultation. A single occultation experiment is carried out for each of these 52 GPS/MET soundings and each of the three observational weightings. The 56 collocated radiosonde profiles are used for the numerical evaluations of the GPS/MET data assimilation results.

It is found that the bending angle assimilation with any of the three weightings produced temperature and specific humidity analyses within the NCEPS SSI system which compare
Figure 2.22: The vertical distributions of the observational increments of $N^{LOC}$ in terms of the GPS/MET observations (GES-GPS) and radiosonde observations (GES-RAD).

favorably with independent radiosonde data. The temperature analysis is much more sensitive to the observational weighting than the specific humidity. The analysis increments of the specific humidity and the surface pressure show a similar tendency of changes to that of the refractivity. That is, the sign of the differences between the background refractivity and the GPS/MET observations determine the sign of the analysis increments for both fields. Thus the efficiency of the data assimilation is partly determined by the quality of the background fields. Such a one-to-one correspondence is not found between the analysis increment of the temperature and the observational increment of refractivity.

Moreover, the sensitivities of the bending angle assimilation to the gravity and the vertical data resolution are tested. Differences between the experiments including and excluding the variation of the gravity in the ray-tracing operator are rather small. The temperature analysis is found to be more sensitive to the vertical resolution of the assimilated GPS/MET data than the specific humidity and the surface pressure. Under the current model resolution,
an increase of the assimilation GPS/MET data may give a larger improvement to the temperature analysis. Changes in the moisture and surface pressure analysis are relatively small with regard to the changes of the vertical resolution of data tested.

Numerical results assessing the impacts of observational weighting on GPS/MET bending angle assimilation presented in this study rely on the NCEP system, or on the quality of $x_0$ and the specification of $\mathbf{B}$ to be more explicit. The relative amount of improvement and degradation to temperature, specific humidity, and surface pressure analysis depends not only on the observational weightings, but also on the weightings assigned in the background term. For example, for a single bending angle observation, more than three variables ($T$, $q$, and $p_s$) are to be adjusted. The analysis increments for these variables are proportional to the sensitivity of the squared bending angle observational increment to these variables. The proportional factor is determined by the estimated variance and covariances of the background errors of $T$, $q$, and $p_s$ used in the data assimilation system (see equation (2.16)).
A systematic degradation of an analysis variable, as seen in this study for the surface pressure, could result from an over- or under-estimator of the background errors, or the existence of a bias background error of these variables. Compared with radiosonde observations, the mean bias errors of the surface pressure analysis are larger using a larger bending angle observational weighting, suggesting that the estimated background errors for the surface pressure are probably too large in the system that is used in this study.
CHAPTER 3

Assimilations of GPS Refractivity — A Computationally Efficient Method

3.1 Introduction

A simple approach to assimilate GPS refractivity is to compute model refractivity via Eq. (1.9) and then directly compare it with GPS refractivity (so called “local” refractivity assimilation). Compared to GPS bending angle assimilation, its computational cost is much lower. The method was first used to assess the impact of GPS refractivity on mesoscale forecasts using a series of four dimensional variational (4D-Var) Observing System Simulation Experiments (OSSEs) by Zou et al. (1995 [47]). Later, Kuo et al. (1997 [16]) conducted another OSSE study. Kursinski et al. (2000 [20]) and Poli et al. (2002 [31]) conducted GPS local refractivity assimilation in a 1D-Var (vertical direction) framework. Since the simulated local refractivity is a point value and GPS refractivity is a weighted average of the atmospheric refractivity along the ray-path, problems arise over regions where there exist strong horizontal gradient of refractivity, such as in the vicinity of fronts, jet streams and strong convections. The analysis obtained from this approach might be less accurate than that obtained from the GPS bending angle assimilation over these regions. In order to obtain a quantitative assessment of the accuracy of local refractivity assimilation results, two parallel GPS data assimilation experiments, one which assimilates refractivity and one which assimilates bending angle, are carried out in this chapter. Results obtained by the GPS local refractivity assimilation and those obtained by the GPS bending angle assimilation are compared. Potential benefits that any non-local refractivity assimilation scheme can thus be sought for.

The local refractivity assimilation system is described in Section 3.2. Experimental design and GPS observations are given in Section 3.3. Convergence of the minimization problem
and computational cost of the GPS observation assimilations are discussed in Section 3.4. The impact radii of the GPS bending angle and refractivity observations in model grids are compared in Section 3.5. Differences in analysis increment from two sets of experiments are discussed in Section 3.6 and 3.7, respectively. Further examination of the differences between the GPS bending angle and refractivity assimilations under different cloud regimes is carried out in Section 3.8. Finally, Section 3.9 provides a summary of the chapter.

3.2 SSI/GPS Refractivity Assimilation System

The forward operator for the GPS refractivity assimilation follows that (1) computing the refractivity by Eq. (1.9) at model grids, (2) computing geopential height and convert it to geometric height at model grids and (3) interpolating the simulated refractivity as function of geometric height to the observation space. The forward model is incorporated in the NCEP SSI analysis system introduced in Section 2.2. In this chapter, a horizontal resolution of the triangular truncation at wavenumber 170 (T170) is used. The dimension of the spatial domain, where the nonlinear physics processes and the data assimilations occur, is $384 \times 192$ (equivalent to 0.9 × 0.9 degree longitude/latitude). There are 42 $\sigma$ levels extending from the surface to a height of $\sim 40$ km with a variable vertical resolution: Generally, the resolution is $< 0.2$ km below 1 km, $\leq 1$ km between 1-14 km, $\leq 2$ km between 14-20 km, $\leq 3$ km between 20-30 km and around 8 km or more above 30 km.

3.3 Experimental Design and Observations

3.3.1 Experimental Design

To access the impacts of the GPS refractivity assimilation on large-scale analysis, we carried out three sets of data assimilation experiments. The first experiment (NOGPS) is a control run, in which all observations (including radiosonde wind and temperature and satellite-derived radiances) except for the GPS data are assimilated. The second experiment (BA) assimilates all observations in NOGPS plus the GPS bending angle observations using a 2D ray-tracing procedure (Section 2.3). The third experiment (REF) assimilates all observations in NOGPS plus the CHAMP refractivity observations using the local refractivity assimilation approach. These three experiments are carried out at 06 UTC from 21 to 31 of May 2002. GPS observations in a 6-hour time window centered at 06 UTC are incorporated in the
experiments. The same NCEP background fields are used for all three experiments.

### 3.3.2 CHAMP RO Observations

The experiments BA and REF used CHAMP observations processed by the Constellation Observing System for Meteorology Ionosphere and Climate (COSMIC) Data Analysis and Archival Center (CDAAC) at the University Corporation for Atmospheric Research (UCAR). There are a total of 4836 radio occultation soundings in May 2002, extending from near the surface to a height of \( \sim 40 \) km. Among them, 434 profiles of bending angle and refractivity were within a 6-hour window centered at 06 UTC on each of the days from 21 to 31 May 2002. The global distributions and the maximum and minimum heights of the 434 profiles are shown in Fig. 3.1. To be consistent with the model vertical resolution, the assimilated CHAMP data have been vertically smoothed to a vertical resolution of 0.1 km between 0-14 km, 0.6 km between 14-20 km, 1.0 km between 20-30 km and 2.0 km above 30 km.

The vertical error covariances \( \textbf{R} \) for CHAMP bending angle and refractivity observations are estimated based on the difference between the NCEP background fields and the CHAMP observations for all soundings available in May 2002 (Fig. 3.2c-d). The large error covariances appear in the lower troposphere, which is consistent with the fact that the observational error is larger in the lower troposphere due to multi-path, super-refractive conditions, strong horizontal refractive gradient, and receiver tracking error. The largest error covariance is about \( 1.8 \times 10^{-6} \) ((radians)\(^2\)) at 2.5 km for bending angle and about 20 ((N unit)\(^2\)) for refractivity. Fractional standard deviations \( \left( \sqrt{\text{error variance} / \text{observation average}} \right) \) are shown in Fig. 3.2c-d.

It is noted that the bending angle error is as large as 8% at the surface, decreases to about 1% between 15-30 km, and then increases to 2% around 40 km again. The percentage error of refractivity is smaller than that of bending angle at almost all levels due to smoothing effects introduced by the Abel inversion integral (Healy, 2001 [11]). It is about 2% near the surface and around 40 km and less than 1% between 10-30 km.

The vertical correlations for CHAMP bending angle and refractivity observations are also estimated using the same set of data. The formula for calculating the correlations is

\[
\text{COR} = \frac{(H(x_b) - y_{\text{obs}})(H(x_b) - y_{\text{obs}})\text{T}}{(H(x_b) - y_{\text{obs}})\text{T}(H(x_b) - y_{\text{obs}})}.
\] (3.1)

The high correlation band is rather narrow below 12 km, and its width increases with height.
Figure 3.1: (a) Distributions of the 434 CHAMP occultations within a 6-hour window centered at 06 UTC from 21 to 31 May 2002 and (b) the lowermost and uppermost altitudes of these CHAMP RO observation profiles. Distributions of the 34 CHAMP occultations within a 6-hour window centered at 06 UTC May 27, 2002 are marked by closed triangles. above 12 km (Fig. 3.3). For simplicity, the observation error covariance matrix $R$ for assimilation of either bending angle or refractivity is assumed to be diagonal.

### 3.4 Convergence and Computational Cost

As mentioned in Section 2.2, there are two embedded loops in each minimization experiment: an inner loop which seeks a local minimum of the perturbed cost function along the opposite direction of its gradient using the standard conjugate gradient method, and an outer loop which updates the non-linear model solution by adding the “optimal” increment obtained by the previous inner loop to it. The number of iterations used in the outer and inner loops determines the performance of the minimization problem.

Seven experiments with different combinations of inner and outer loops (Table 3.1) are carried out to find an optimal choice of the total inner and outer loops. A total of 34 CHAMP bending angle profiles in the 6-hour window centered at 06UTC May 27, 2002 are
Table 3.1: The number of outer and inner loops during the minimization of the cost function defined in Eq. (2.1) for each of iteration setup experiments.

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assimilated (closed triangles in Fig. 3.1). The variations of the cost function ($J$) and the norm of the gradient ($\| \nabla J \|$) with the number of iterations from these seven experiments are shown in Fig. 3.4. Small jumps can be found at each outer loop when the model basic state is updated. The value of $J$ decreases fastest within the first 50 iterations, at a slower rate from 50-100 iterations, and even slower after 100 iterations. At the 100th iteration, the value of $J$ decreases to $\sim 2 \times 10^5$ from the initial value $5.5 \times 10^5$. Simultaneously, the value of $\| \nabla J \|$ decreases continuously even at the largest iteration number (400), except for those jumps between outer loops, implying potentials for further convergence of the minimization problem. A close look at the final values of $\| \nabla J \|$ at the end of each experiment can be obtained from Fig. 3.5. We find that there is an apparent difference in the final values of $\| \nabla J \|$ between the experiments with different total iteration numbers (e.g., the value of

Figure 3.2: Estimates of (a-b) vertical covariances and (c-d) fractional standard deviations of GPS RO bending angle (a,c) and refractivity (b,d) by using CHAMP RO observations from May 2002.
Figure 3.2: -continued.

Figure 3.3: Estimates of vertical correlations of GPS RO (a) bending angle and (b) refractivity by using CHAMP RO observations from May 2002.
\[ \| \nabla J \| \text{ is } 3.4 \times 10^3 \text{ for BA}_O2I25 \text{ and } 8.2 \times 10^2 \text{ for BA}_O2I100, \text{ respectively}, \text{ while the difference between two experiments with the same total iteration number (e.g., BA}_O2I100 \text{ and BA}_O4I50) \text{ is much smaller, except for the pair of experiments with 400 iterations.} \]

The combination with more inner loops and fewer outer loops achieves a slightly smaller value for \( \| \nabla J \| \) than the opposite case, e.g., the final values of \( \| \nabla J \| \) are \( 1.4 \times 10^2 \) and \( 2.9 \times 10^2 \) for BA\_O2I200 and BA\_O4I100, respectively. The computational cost of each experiment is also presented in Fig. 3.5. All experiments are conducted on an IBM SP4 parallel system with 28 processors. The result indicates that the wallclock time of the experiments increases approximately linearly with the iteration number. There are no significant differences between two experiments with the same total iteration number.

Considering both the convergence of the minimization problem and the computational cost, we choose to carry out 2 outer loops and 100 inner loops within each outer loop for each minimization in NOGPS, BA and REF. Each minimization in BA takes about 7.5 wallclock hours, producing a decrease of three orders of magnitude of the norm of the gradient of the cost function. Each minimization in NOGPS and REF with the same numbers of outer and inner loops as in BA takes about 5 wallclock hours.

### 3.5 Impact Radius of GPS Observations

The impact radii of GPS data in both BA and REF experiments are examined. Figure 3.6 shows horizontal distributions of the moisture differences between GPS data assimilation experiments (BA and REF) and no-GPS data assimilation experiments (NOGPS) at the \( \sigma = 0.8585 \) level. We find that the specific humidity adjustments due to GPS data assimilations are limited to near circular areas surrounding the GPS occultation point. Their values are largest at the center of the occultation point and decrease to nearly zero beyond a certain radius. Such an area indicates how far the information extracted from GPS data at an occultation point could be spread to its surroundings through a data assimilation process.

For a closer look, a \( \sigma \)-longitude cross-section passing through the occultation point at (19.36\(^{\circ}\)W, 52.64\(^{\circ}\)N) is shown in Fig. 3.7. Most adjustments due to the GPS data assimilations are found in the lower troposphere. In the vertical direction, the sign of these adjustments changes alternatively, e.g., there is a water vapor decrease near \( \sigma = 0.8585 \) while there is a water vapor increase near \( \sigma = 0.6267 \). The adjustments from BA are larger in magnitude and have a wider horizontal span than those from REF, especially in the lower
troposphere (e.g., at $\sigma = 0.8585$ shown in Fig. 3.7c). Moreover, the centers of the large moisture adjustments from BA and REF are not located at similar heights but different longitudes. The positive and negative centers of the moisture adjustment resulting from refractivity assimilation are more aligned in the vertical (i.e., at the same longitude) than those from bending angle assimilation. This is an combined effect of an exact account of the vertical shifting of perigee point and inclusion of the integrated effects of the atmosphere to the total refraction in bending angle data assimilation.
Figure 3.5: Final values of the norm of the cost function gradient (square point) from each of the experiments in Fig. 3.4 and the total wallclock time that experiment has taken (circle point) running on an IBM SP4 parallel computer system with 28 processors. An exponential fitting to the gradient norm (solid line) and a linear fitting to the wallclock time (dashed line) are also given.

The mean profile of the moisture adjustments at $\sigma = 0.8585$ averaged over all 434 CHAMP occultations is shown in Fig. 3.8. It is consistent with the single sounding result in Fig. 3.7c. For example, the BA analysis has a moisture adjustment of $-0.1 \text{ g kg}^{-1}$ at the fifth model grid ($\sim 500 \text{ km in the tropics}$) away from the GPS occultation point while that value in REF is at the fourth grid ($\sim 400 \text{ km in the tropics}$). In summary, assimilations of bending angle produce larger analysis increments and larger impact radii than those of refractivity assimilation.

### 3.6 Observation and Analysis Increments of Refractivity

Refractivity increments are analyzed in the observation space. Because the tangent point heights for each sounding profile are different, all sounding results have been interpolated to fixed altitude levels from 2.5 to 38 km before their averaged increments are computed.

As mentioned before, $N^{LOC}$ is different from $N^{GPS}$. Since $N^{GPS}$ is more consistent
Figure 3.6: Distributions of the specific humidity analysis differences (a) between BA and NOGPS and (b) between REF and NOGPS at $\sigma = 0.8585$ level at 06UTC May 21, 2002. Unit: g kg$^{-1}$. The contour interval is 0.1 g kg$^{-1}$. (c) Distributions of the CHAMP occultations within the time window.
Figure 3.7: $\sigma$-longitude cross-sections of the specific humidity analysis differences (a) between BA and NOGPS and (b) between REF and NOGPS, passing through one occultation point at (19.36°W, 52.64°N) at 06UTC May 21, 2002. (c) Zonal variations of the specific humidity differences at $\sigma = 0.8585$ level between BA and NOGPS (solid line) and between REF and NOGPS (dotted line). Unit: g kg$^{-1}$. 
with the GPS observations, it is expected that after assimilating GPS observations, the observation increments of $N^{GPS}$ from BA is smaller than those from REF and NOGPS. This is confirmed in Fig. 3.9, which shows the observation increments of $N^{GPS}$ after BA, REF and NOGPS experiments. The mean profile of $N^{GPS} - N_{obs}$ from the three experiments have similar vertical profiles: the increments are positive below 4 km, negative between 4-14 km, and very small above 18 km. Although many other observations have been included in the experiment, NOGPS shows the largest bias compared with BA and REF, especially in the tropics ($30^\circ S - 30^\circ N$) where a negative bias as large as 1.5 N units is found in the middle troposphere. A positive bias as large as 3 N units is found in the lower troposphere, which is consistent with the known negative N-bias in the observation (i.e., observations are biased low relative to the models) (Sokolovskiy, 2003 [40]; Ao et al., 2003 [1]). This observational bias is mainly due to tracking errors and will be substantially diminished once open loop tracking is implemented on GPS occultation receivers anticipated in the near future (Sokolovskiy, 2001 [39]). Differences among the three experiments are also seen in the root mean square (RMS) values of the observation increments of $N^{GPS}$. Generally, the RMS values increase downward to the surface. NOGPS has the largest RMS, and BA has the smallest RMS value of the three data assimilation experiments.

The mean analysis increments of $N^{GPS}$ and their RMS values are shown in Fig.
Figure 3.9: Vertical profiles of (a-c) the mean observation increments (analysis minus observation) of $N^{GPS}$ and (d-f) their root mean square values (RMS) for BA (solid line), REF (dotted line) and NOGPS (star). Data are computed and plotted in three latitudinal bands: (a,d) $90^\circ S - 30^\circ S$, (b,e) $30^\circ S - 30^\circ N$ and (c,f) $30^\circ N - 90^\circ S$. Unit: N unit.

3.10. NOGPS produces very little changes of refractivity, while both BA and REF show noticeable analysis increments even in middle and upper troposphere. These results indicate that CHAMP GPS RO observations contain useful information in the middle and upper troposphere that is not included in observations currently incorporated in the NCEP SSI operational system, especially in the tropics.
Figure 3.10: Vertical profiles of (a-c) the mean analysis increments (analysis minus background) of $N^{GPS}$ and (d-f) their root mean square values (RMS) for BA (solid line), REF (dotted line) and NOGPS (star). Data are computed and plotted in three latitudinal bands: (a,d) $90^\circ S - 30^\circ S$, (b,e) $30^\circ S - 30^\circ N$ and (c,f) $30^\circ N - 90^\circ S$. Unit: N unit.

### 3.7 Analysis Increments of Temperature, Moisture and Surface Pressure

The atmospheric temperature, specific humidity and surface pressure are variables which determine the bending of the radio signals transmitting through the atmosphere. The fit to GPS RO observations are generated by adjusting these variables. The mean analysis increments and their RMS values for these model state variables are computed in the model space at $\sigma$ levels and shown in Figs. 3.11-3.13, respectively. The model atmosphere becomes
colder in the Southern Hemisphere, while it becomes warmer in the tropics and the Northern Hemisphere, and the analysis of REF is warmer than those of BA and NOGPS at most of these levels (Fig. 3.11). Also, the difference between NOGPS and GPS data assimilation experiments (BA or REF) is smaller than that between NOGPS and the background at these levels. Therefore, at these levels, most of the temperature increments come from other observations rather than GPS data themselves. Above $\sigma = 0.4$, NOGPS makes smaller adjustments than in the lower levels, while both BA and REF produce more temperature adjustments with significant vertical oscillations. The RMS values of these analysis increments also show that most of the differences among the experiments appear at upper levels, especially above $\sigma = 0.4$. Such results are to be expected since there is very little moisture content in the upper levels, and most contributions from GPS observations in these levels go to the temperature field. Differences between BA and REF analyses may come from the errors of the CHAMP refractivity observations introduced by the initialization of the Abel’s inversion, or by the simplification of the local refractivity assimilation approach itself.

The mean analysis increments of specific humidity and their RMS values are shown in Fig. 3.12. As expected, most differences between NOGPS and GPS data assimilation experiments (BA and REF) are found at low levels, where the moisture content is high. NOGPS produces the smallest changes in the mean specific humidity profiles. The specific humidity in the lower troposphere is reduced below $\sigma = 0.7$ and increased above this level by both BA and REF. Differences between BA and REF are also smaller than those between BA (or REF) and NOGPS.

The zonal mean of the analysis increments of the surface pressure and their RMS are computed and presented in Fig. 3.13. The number of CHAMP RO profiles within each 5° latitudinal band is also shown in Fig. 3.13. On average, the surface pressure increments from all three experiments are positive in middle-to-high latitudes in the Southern Hemisphere, near zero in the tropics and negative in middle-to-high latitudes in the Northern Hemisphere. Unlike the analysis results for temperature and specific humidity, the surface pressure of NOGPS is closer to that of BA than to REF at most latitudes, especially in the middle-to-high latitudes in the Southern Hemisphere. REF results in a larger positive adjustment of surface pressure than BA does. This is found to be related to a colder temperature analysis from the REF experiments near the model top (Fig. 3.11a).
3.8 Comparison of GPS Bending Angle and Refractivity Assimilations within Cloud Regimes

As mentioned in the introduction, the horizontal gradients of the atmosphere refractivity may be large in the vicinity of strong convections, which are often accompanied by cloud systems. Therefore, instead of examining the entire statistics of the analysis increments between BA and REF experiments, BA and REF analyses are compared within different cloud regimes in the tropics and the middle latitudes. The following five categories of cloud systems are considered: convective and cumulus clouds in the tropics, as well as convective, cumulus and stratus clouds in the mid-latitudes. By using satellite images from GOES-8
and METEOSAT-5 within a 3-hour time window, the cloud types over each occultation point have been identified and are shown in Fig. 3.14. The RO soundings that are either out of the coverage of GOES-8 and METEOSAT-5, pass through multiple types of clouds, or are ambiguous for placement in any of the above five categories are discarded. There are a total of 9 (30) GPS RO soundings within tropical (mid-latitude) convective clouds, 37 (17) soundings within tropical (mid-latitude) cumulus clouds and 23 soundings within mid-latitude stratus clouds.

Figure 3.15 shows spaghetti plots of the vertical profiles of the analysis differences of temperature between BA and REF. In the tropics, most of the profiles differ at the higher troposphere or lower stratosphere (above $\sigma = 0.3$). In the mid-latitudes, differences between
Figure 3.13: (a) Zonal averaged analysis increments (analysis minus background) of surface pressure and (b) their root mean square values (RMS) for BA (solid line), REF (dotted line) and NOGPS (star). Unit: hPa. (c) The number of GPS occultations within each of 5° latitudinal bands used in the average.
Figure 3.14: Distributions of occultations within (a) convective clouds, (b) cumulus clouds and (c) stratus clouds. The soundings in mid-latitudes (60°S-30°S and 30°N-60°N) are marked by closed circles and the soundings in tropics (30°S-30°N) are marked by closed triangles. There are 9 (30) GPS RO soundings within tropical (mid-latitude) convective clouds, 37 (17) soundings within tropical (mid-latitude) cumulus clouds and 6 (23) soundings within tropical (mid-latitude) stratus clouds.
BA and REF appear in the middle tropospheric levels, especially around $\sigma = 0.4 - 0.6$. The mean profile of temperature differences and the corresponding RMS are shown in Fig. 3.16. On average, the temperature analysis differences between BA and REF in different latitudes are similar in the upper troposphere and the stratosphere, which may be due to the lack of horizontal structure at higher altitudes. The temperature differences for those occultations passing through both convective and cumulus clouds have peak value of about -0.2 K around $\sigma = 0.3$ in the tropics, and are within 0.1 K in the mid-latitudes. The RMS values show that the occultations in the tropics are quite consistent with each other below $\sigma = 0.5$, while those in the mid-latitudes differ more significantly.

The spaghetti plots of the specific humidity differences between BA and REF are shown in Fig. 3.17, and the mean differences and RMS are shown in Fig. 3.18. It is noted that the differences for individual profiles between BA and REF are largest for those occultations which are located within convection clouds in the mid-latitudes and cumulus clouds in the tropics.

In order to illustrate what may have contributed to the different degrees of spread in Fig. 3.17, Figs. 3.19-3.20 show cross sections of the background local refractivity (Figs. 3.19a and 3.20a), the analysis increments of the local refractivity for BA and REF experiments (Figs. 3.19b-c and 3.20b-c), as well as the difference between BA and REF (Figs. 3.19d and 3.20d) for two arbitrarily chosen RO soundings (Table 3.2). One is located in a middle latitude convective cloud (Fig. 3.19) and the other in a mid-latitude cumulus cloud (Fig. 3.20). These two soundings are marked by closed circles in Fig. 3.15 and 3.17. The CHAMP sounding located in a mid-latitude convective cloud measures a vertical refractivity profile at the edge of a low center of refractivity (the minimum value is about 280 N unit located near 83.5°W). A relatively large horizontal gradient of refractivity is found near this RO (Fig. 3.19a). In the case of the mid-latitude cumulus cloud, the CHAMP sounding went through a vertical column with weak horizontal refractivity gradients (Fig. 3.20a). It is found that analysis increments from BA and REF experiments are very different in a convective cloud (Fig. 3.19b and c) and quite similar in a cumulus cloud (Fig. 3.20b and c), resulting in larger refractivity differences between BA and REF in convective clouds than cumulus clouds (see Figs. 3.17, 3.19d, and 3.20d).

In order to further diagnose results from BA and REF experiments, the vertical profiles of $N^{\text{LOC}} - N_{\text{obs}}$, $N_{\text{GPS}} - N_{\text{obs}}$, and $N_{\text{GPS}} - N^{\text{LOC}}$ are plotted in Fig. 3.21. Differences between
Figure 3.15: Spaghetti plots of vertical profiles of temperature analysis differences between BA and REF (BA minus REF) under five categories of cloud systems: (a) tropical and (b) mid-latitude convective clouds, (c) tropical and (d) mid-latitude cumulus clouds and (e) mid-latitude stratus clouds. The individual profiles marked by closed circles are from the soundings listed in Table 3.2. Unit: Kelvin.

N^{GPS} and N^{LOC} are much larger for the RO in a convective cloud than in a cumulus cloud. This is reflected in the background field, the BA analysis and the REF analysis (Fig. 3.21g-i). Although the difference $N^{LOC} - N_{obs}$ is reduced in the REF experiment as expected (e.g., $N^{LOC}_{REF} - N_{obs}$ is smaller than $N^{LOC}_{background} - N_{obs}$, see Fig. 3.21a-c), the difference $N^{GPS}_{REF} - N_{obs}$ is increased after the local refractivity assimilation for the RO in a convective cloud (Fig. 3.21d-f). On the contrary, the bending angle assimilation reduces the difference $N^{GPS} - N_{obs}$. The performance of REF is only slightly different from BA for the RO in a cumulus cloud.
Figure 3.16: Vertical profiles of (a) the mean temperature analysis differences and (b) their root mean square values (RMS) between BA and REF (BA minus REF) under five categories of cloud systems in Fig. 3.15: (a) tropical (solid line) and (b) mid-latitude convective clouds (open circle), (c) tropical (dashed line) and (d) mid-latitude cumulus clouds (open square) and (e) mid-latitude stratus clouds (star). Unit: Kelvin.

Table 3.2: The CHAMP soundings chosen for the single sounding analysis (see text in Section 3.8).

<table>
<thead>
<tr>
<th>Index</th>
<th>occultation point</th>
<th>observation date</th>
<th>cloud category</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(86.07W, 34.65N)</td>
<td>29 May 2002</td>
<td>convection cloud in mid-lat.</td>
</tr>
<tr>
<td>2</td>
<td>(86.55W, 33.69N)</td>
<td>31 May 2002</td>
<td>cumulus cloud in mid-lat.</td>
</tr>
</tbody>
</table>

These results further confirm that the absence of an integrated effect of the refractivity gradients along the ray-path in REF results in large errors for assimilating RO soundings located in areas which contains large refractivity gradients.

It is found that the above single-sounding analysis is quite representative of a general performance of BA and REF experiments. Figures 3.22 and 3.23 show the vertical profiles of the mean and the RMS of $N^{LOC} - N_{obs}$ and $N^{GPS} - N_{obs}$, respectively. Also shown in Fig. 3.23 is the RMS values of $N^{GPS} - N^{LOC}$ for the two cloud types discussed above. It is found that the NCEP background has a negative bias above the middle troposphere and a positive bias in the stratosphere. The bias within cumulus clouds maximizes in the middle troposphere and the tropopause while that for convective clouds is more homogeneous. It is also seen that the horizontal gradient of refractivity, reflected by the quantity $N^{GPS} - N^{LOC}$ in Fig.
Figure 3.17: Same as Fig. 3.15, except for specific humidity. Unit: g kg\(^{-1}\).

3.23e, is on average much larger within convective clouds than cumulus clouds, resulting a poor performance of REF for assimilation of RO observations in convective clouds (Fig. 3.23a-d).

3.9 Summary and Conclusions

By incorporating CHAMP GPS RO data into the NCEP SSI data assimilation system, analysis differences between bending angle assimilation and local refractivity assimilation are examined. While both BA and REF have reduced the bias of the background fields, the BA analyses fit the GPS refractivity more closely than the REF analyses. The analysis increments of the simulated GPS refractivity from NOGPS are very small and those from
both BA and REF are much larger, implying that CHAMP GPS RO data are an important observational source which is not a redundancy with respect to existing conventional and satellite remote sensing observations. Differences between BA and REF analyses of temperature and specific humidity are smaller than the differences between BA (or REF) and NOGPS analyses. However, the REF surface pressure analysis increments are larger than those of both BA and NOGPS in high latitudes in the Southern Hemisphere.

It is also shown that the analyses obtained by assimilating GPS refractivity and bending angle are more sensitive to thick-layered cloud systems (e.g., convective clouds in the mid-latitudes and cumulus clouds in the tropics). The differences for the temperature analyses between bending angle assimilation and refractivity assimilation are also large when GPS RO profiles pass through mid-latitude clouds. It is found that larger horizontal gradients of atmosphere refractivity within (or near) cloud systems contribute to larger differences of refractivity and specific humidity analyses between BA and REF.

By comparing CHAMP GPS bending angle and refractivity data assimilation, this study provides a benchmark for quantifying potential differences between local and non-local GPS refractivity assimilation approaches. A non-local refractivity assimilation scheme, which incorporates a weighted average of refractivity along an approximated ray path, as well as differences between the non-local and local refractivity assimilation results, is described in the next chapter.
Figure 3.19: (a) For GPS sounding 1 in Table 3.2, the horizontal distribution of the local refractivity analyses from the background (contour) at $\sigma = 0.7$ (Unit: N unit) and the averaged occultation point location (triangle), the longitude-$\sigma$ cross-sections of the analysis increments (analysis minus background) of $N^{LOC}$ for (b) BA and (c) REF and (d) the difference of $N^{LOC}$ analyses between BA and REF. Unit: N unit. The cross-sections cut across the model grid closest to the corresponding occultation point.
Figure 3.20: Same as Fig. 3.19, except for GPS sounding 2 in Table 3.2.
Figure 3.21: The differences between (a-c) $N^{LOC}$ and $N_{obs}$, (d-f) $N^{GPS}$ and $N_{obs}$ and (g-i) $N^{GPS}$ and $N^{LOC}$ for the background (left panels), BA (middle panels) and REF (right panels) analyses of two GPS soundings in Table 3.2. Solid lines indicate the results from GPS sounding 1 and dotted lines are from GPS sounding 2. Unit: N unit.
Figure 3.22: Vertical profiles of the mean differences (a-b) between $N^{LOC}$ and $N_{obs}$ and (c-d) between $N^{GPS}$ and $N_{obs}$ for the background (star), BA (solid line) and REF (dotted line). Unit: N unit. Panels (a) and (c) are averaged over the soundings within mid-latitude convective clouds and panels (b) and (d) are averaged over the soundings within mid-latitude cumulus clouds.
Figure 3.23: Vertical profiles of the root mean squares (RMS) of the differences (a-b) between $N^{LOC}$ and $N_{obs}$, (c-d) between $N^{GPS}$ and $N_{obs}$ for the background (star), BA (solid line) and REF (dotted line) and (e) between $N^{GPS}$ and $N^{LOC}$ for the background. Unit: N unit. Panels (a), (c) and the solid line in panel (e) are averaged over the soundings within mid-latitude convective clouds and panels (b), (d) and the dotted line in panel (e) are averaged over the soundings within mid-latitude cumulus clouds.
CHAPTER 4

Assimilation of GPS Excess Phase Delay — An Accurate and Computationally Efficient Method

4.1 Introduction

In a local refractivity assimilation, a simulated refractivity profile is interpreted as a vertical profile at the averaged position of all tangent points of a RO. In the bending angle assimilation, the along-track refractivity and its gradient determines the bending of each individual ray-path through the integration of the ray-trajectory equation. The bending angle calculated from such an operator is physically consistent to the way the GPS observations are obtained and processed. Such a consistency renders the bending angle assimilation more desirable when accuracy is a priority. However, the integration of the ray-trajectory equation is expensive, especially for its applications in operational weather forecasts.

This chapter aims at developing a new forward observation operator which has a lower computational cost than the GPS bending angle assimilation and yet is more accurate than the local refractivity assimilation. To achieve such a purpose, the GPS excess phase delay is chosen as the assimilation variable. Section 4.2 introduces the basic concepts and the calculations of the GPS excess phase delay. Some features of the observation operator in idealized and realistic model configurations are examined in Section 4.3. Section 4.4 describes the numerical experiments carried out to test the observation operator in a global 3D-Var data assimilation system. Sections 4.5 and 4.6 discuss results from the forward simulation and data assimilation of GPS excess phase delay, respectively. Sections 4.7 proposes an alternative form of the excess phase delay forward operator, a spherically symmetric excess phase delay, which will have optimal performance in a parallel computing environment. Finally, Section 4.8 provides a summary of the chapter.
4.2 Basic Concepts and Observation Operator

4.2.1 Mathematical Expressions of the Excess Phase Delay

Along each ray path going from an occulted GPS satellite to a LEO satellite, if the numerical model used to calculate the refractivity is accurate, the following relationship holds true:

\[ \int_{r} N_{\text{obs}} \, dr = \int_{r} N^{\text{LOC}} \, dr, \]  
(4.1)

where \( \int_{r} \) represents an integration along the ray-path, \( N_{\text{obs}} \) is the GPS refractivity observation at the tangent points of rays and \( N^{\text{LOC}} \) is the local refractivity along the ray path (calculated from model variables via Eq. (1.9)). Since the expression of the excess phase delay \( (L) \) is defined as

\[ L = \int_{r} (n - 1) \, ds = 10^6 \int_{r} N^{\text{LOC}} \, dr, \]
(4.2)

where \( n \) is the refractivity index, the integrals in Eq. (4.1) are called the excess phase delay.

The actual ray paths for each RO are unknown. However, the azimuth angle (measured between the North and the tangent direction at the tangent point of a ray pointing toward the GPS satellite) is known and can be used to derive the tangent direction at the tangent point of the ray. Therefore, it is possible to define a tangent link for RO, which is a straight line satisfying the following conditions (Fig. 4.1):

1. It passes the tangent point of the observed ray;
2. It is tangent to the local curvature of the Earth at the tangent point; and
3. It is coplanar to the occultation plane which contains the tangent point and the GPS and LEO satellite positions.

Given a typical value of bending angle 0.02 rad (\( \sim 1.14 \) degree) at 1 km altitude, the point at a distance 500 km away from the tangent point along the tangent link drifts about 5 km from the ray path. Compared with the Earth radius (\( \sim 6400 \) km), such a short distance is negligible. Therefore, the integrations along the ray-path in the Eq. (4.1) can be approximated by integrations along the tangent link

\[ \int_{s} N_{\text{obs}} \, ds = \int_{s} N^{\text{LOC}} \, ds, \]  
(4.3)
where $s$ represents the tangent link of the excess phase delay. Thus, the integral of the left-hand side (LHS) equation will be referred to as the excess phase delay observation ($L_{\text{obs}}$) and the integral in the right-hand side (RHS) will be referred to as the simulated excess phase delay $L$ throughout the text.

### 4.2.2 Discrete Form in a Gridded Model

Assume a numerical forecast model has a 3D dimensions as $n\text{lon} \times n\text{lat} \times n\text{lev}$, where $n\text{lon}$, $n\text{lat}$ and $n\text{lev}$ are the total number of longitudes, latitudes and vertical levels, respectively, and that an observed GPS refractivity profile has $M$ vertical levels. Assume the GPS refractivity is observed at half levels of the model, i.e., $N_{\text{obs}}(1)$ is located in the first model layer (between model level 1 and 2), $N_{\text{obs}}(2)$ is in the second model layer, $\cdots$, and $N_{\text{obs}}(M)$ is in the $M$th model layer (therefore $n\text{lev} = M + 1$). Given the $m$th tangent link through
the tangent point in the \( m \)th layer, the discrete form of the Eq. (4.3) is written as

\[
\sum_{l=1}^{M} b_{m,l} N_{\text{obs}}(l) = \sum_{n=1}^{N} a_{m,n} N_{\text{LOC}}(n),
\]

where \( b_{m,l} \) is the length of the link \( m \) inside the layer \( l \) bounded by levels \( l \) and \( l+1 \) (Fig. 4.2), \( N_{\text{obs}}(l) \) is the value of the GPS refractivity observation at the tangent point in the layer \( l \), \( N \) is the total number of all model grid boxes and equals to \((n\text{lon}) \times (n\text{lat} - 1) \times (n\text{lev} - 1)\), \( a_{m,n} \) is the length of the tangent link intersected by the \( n \)th model grid box (Fig. 4.3), and \( N_{\text{LOC}}(n) \) represents the local refractivity inside the \( n \)th grid box. Notice that the LHS of the equation (4.4) is obtained based on the fact that \( N_{\text{obs}}(l) \) is spherically symmetric. The equation can be written in a matrix form

\[
BN_{\text{obs}} = AN_{\text{LOC}},
\]

where \( N_{\text{obs}} = (N_{\text{obs}}(1), \ldots, N_{\text{obs}}(l), \ldots, N_{\text{obs}}(M))^T \) is the vector of the GPS observed refractivity profile, \( N_{\text{LOC}} = (N_{\text{LOC}}(1), \ldots, N_{\text{LOC}}(n), \ldots, N_{\text{LOC}}(N))^T \) contains the calculated local refractivity in whole model grids, and \( B \) and \( A \) are two matrices whose elements consist of coefficients within the two integral sums in Eq. (4.3). Specifically, \( B \) is an \( M \times M \) upper triangular square matrix of the form

\[
B = \begin{pmatrix}
     b_{1,1} & b_{1,2} & \cdots & b_{1,l} & \cdots & b_{1,M-1} & b_{1,M} \\
     0 & b_{2,2} & \cdots & b_{2,l} & \cdots & b_{2,M-1} & b_{2,M} \\
     \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
     0 & 0 & \cdots & b_{m,l} & \cdots & b_{m,M} \\
     \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
     0 & 0 & \cdots & 0 & \cdots & 0 & b_{M,M}
\end{pmatrix},
\]

where \( b_{m,l} = 0 \) when \( m > l \), reflecting the fact that the link \( m \) does not go through any layer below the layer containing the tangent point. The matrix \( A \) is an \( M \times N \) matrix of the form

\[
A = \begin{pmatrix}
     a_{1,1} & a_{1,2} & \cdots & a_{1,l} & \cdots & a_{1,M-1} & a_{1,N} \\
     \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
     a_{m,1} & a_{m,2} & \cdots & a_{m,n} & \cdots & a_{m,M} & a_{m,N} \\
     \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
     a_{M,1} & a_{M,2} & \cdots & a_{M,n} & \cdots & a_{M,M} & a_{M,N}
\end{pmatrix},
\]

where \( a_{m,n} \) equals to zero if the tangent link \( m \) has no interception with the specific grid box \( n \).
Figure 4.2: Schematic illustration of the definition of the elements in the matrix $B$. Given a vertical profile of a GPS refractivity measurement, a tangent link $m$ is constructed through the tangent point ($P_t$) at layer $m$ bounded by levels $m$ and $m + 1$. The element $b_{m,l}$ in the matrix $B$ is defined as the total interception length of the link $m$ inside the layer $l$ bounded by levels $l$ and $l + 1$.

4.2.3 Forward Observation Operator

The integration of $N^{LOC}$ along the tangent link enables the inclusion of the along-track refractivity gradients in the forward simulations. Therefore, the excess phase delay is selected here as the assimilation variable. Combined with the calculation of the refractivity from the local model state variables (Eq. (1.9)), the RHS of Eq. (4.4) consists of the forward observation operator for the excess phase delay. Specifically, the simulated value of the excess phase delay ($L$) in the $m$th tangent link is calculated from the model temperature, specific humidity and pressure through the following expression:

$$L = \sum_{n=1}^{N} a_{m,n} N^{LOC}(n)$$
Figure 4.3: Schematic illustration of the definition of the elements in the matrix $A$. The element $a_{m,n}$ in the matrix is defined as the total interception length of the link $m$ inside the $n$th model grid box. (a) shows an example with the tangent link $m$ in a longitudinal plane. The grid box is indexed by counting the box number in the order of latitude, longitude and vertical levels. Therefore, the grid box $n + n_{lat}$ is the one next to the grid box $n$ as shown. (b) is a generalized 3D view of the interception of the tangent link $m$ with the $n$th grid box. The local refractivity value in the grid box $n$, required by the excess phase delay observation operator, is given either by the value of the refractivity at the left-lower corner grid of the box (point A), or by interpolating the refractivities onto the middle point of the intercepted link by the model grids (point B).
\[
\begin{align*}
\sum_{n=1}^{N} a_{m,n} \left[ c_1 \frac{p(n)}{T(n)} + c_2 \frac{p(n)q(n)}{[T(n)]^2(0.622 + 0.378q(n))} \right] \\
= H_{PHA}(T, p, q),
\end{align*}
\]

(4.6)

where \( T, p, q, c_1 \) and \( c_2 \) are defined in Eq. (1.9), and \( H_{PHA} \) represents the forward observation operator. The observed value of the excess phase delay is calculated from the GPS refractivity via a similar integration along the tangent link. In a discrete form, the observation of the excess phase delay \( (L_{obs}) \) is obtained by

\[
L_{obs} = \sum_{l=1}^{l=M} b_{m,l} N_{obs}(l)
\]

(4.7)

4.3 Features of the Observation Operator

4.3.1 Dependence of the Kernel Function to Model Grids

From Eq. (4.5), \( N_{obs} \) and \( N^{LOC} \) are related through the following equation

\[
N_{obs} = \mathbf{B}^{-1} \mathbf{A} \mathbf{N}^{LOC} = \mathbf{K} \mathbf{N}^{LOC},
\]

(4.8)

i.e., the GPS refractivity is expressed as a weighted sum of the local refractivity. The weighted matrix \( \mathbf{K} \) is also called a kernel matrix. The excess phase delay assimilation is thus equivalent to a non-local refractivity assimilation. The reason for choosing the excess phase delay observation operator over this form is to avoid any errors which may be introduced and enlarged by the calculation of \( \mathbf{B}^{-1} \). However, features of the kernel matrix can be examined in both the idealized and actual model configurations, in order to provide a better understanding of the excess phase delay operator.

Assume (1) a numerical model has 1° horizontal resolution and 1 km vertical resolution from 0 to 30 km; (2) a vertical profile of the refractivity is measured at 0° longitude, 30°N latitude and half levels of the model; (3) the tangent directions at each of the tangent points are parallel to the equator (i.e., the azimuth angles for each of the levels are either 90° or 180°); and (4) the tangent links are truncated by the uppermost model level at 30 km. Under these assumptions, the matrices \( \mathbf{B}, \mathbf{A} \) and \( \mathbf{K} \) are computed and their features are shown in Figs 4.4-4.7. Figure 4.4 shows the values of \( \mathbf{B} \) and its three selected row elements. The largest values of elements in \( \mathbf{B} \) are are on the diagonal (Fig. 4.4a), because the interception
lengths of the links are largest in the layers which contain the corresponding tangent points. For a fixed \( m \), the values of \( b_{ml} \) decreases with increasing \( l \) as the tangent links are intercepted by model layers above the tangent points (upper right triangular of the matrix). For those layers below the tangent points (\( l < m \)), the \( b_{ml} \) values are zero (lower left triangular of the matrix \( B \)). The values for the first, 11th and 21st rows of \( B \) (\( \vec{b}_1 \), \( \vec{b}_{11} \) and \( \vec{b}_{21} \)) are more clearly shown in Fig. 4.4b. They correspond to the tangent links at the tangent point altitudes of 1 km, 10 km and 20 km, respectively. For example, \( \vec{b}_{11} \) reaches its maximum value (\( \sim 230 \) km) at 10 km altitude \( (l = 11) \) and decreases to 40 km at 30 km altitude \( (l = 31) \). Below 10 km altitude \( (l < 11) \), \( b_{11} \) equals to zero.

Including elements in all grid boxes \( (= 359 \times 179 \times 30 = 1927839) \), the dimension of \( A \) is extremely large. As an example, only non-zero elements in the second row of the matrix \( A \) are shown in Fig. 4.5. They are the lengths of the intercepted tangent link defined at 1 km tangent point altitude. The x-axis of the plot indicates the distance between the tangent point and each of the intercepted grid boxes. The total length of the link is \( \sim 1200 \) km. Similar to those of \( B \), the elements in \( A \) are largest for the grid box at the tangent point and their values decrease with the increasing distances of the grid boxes to the tangent point.

Figure 4.6 shows values of the elements in \( K \) (solid line) at 1 km tangent point altitude as a function of the distance from the tangent point. Gaussian fittings to these kernel functions are also shown in Figs. 4.6 and 4.7. The kernel functions are largest at the tangent point (distance equals to zero) and decrease to zero with the distances increasing to about 600 km. The kernel functions are sensitive to the latitude of the tangent point. They are sharpest at the tangent points in the tropics, implying that the local refractivities at the tangent points will be more heavily weighted in the tropics than in the middle and high latitudes. The fitted Gaussian functions vary with latitudes. Their standard deviation (\( \sigma \)) values range between 61.4 (km) in tropics (10°N) and 333.1 (km) in high latitudes (80°N).

The sensitivity of the \( K \) distribution to the model resolution is also studied. Figure 4.8 shows the kernel functions under four different situations with the model grids along two tangent links. The two tangent links are located at the same altitude and longitude, but different latitudes, i.e., at 1 km tangent altitude, 0° longitude and 10°N (left panels) and 50°N (right panels) latitude. The grid setups in Fig. 4.8a and e are the same as in Figs. 4.4-4.7 with both latitude and longitude resolutions at 1° and vertical resolution at 1 km. Keeping the same 1° latitudinal resolution, the models in Fig. 4.8b and f increase the horizontal
Figure 4.4: The values of (a) the matrix $\mathbf{B}$ and (b) the first ($\vec{b}_1$), 11st ($\vec{b}_{11}$) and 21st ($\vec{b}_{21}$) rows of $\mathbf{B}$. The index $m$ (x-axis in (a)) represents the index of the layer where the tangent point for a tangent link is located. The index $l$ (y-axis in (a) and x-axis in (b)) is the index of the layer which a tangent link goes through. The grid setup has 1° horizontal resolution and 1 km vertical resolution. The altitudes are from 0 km to 30 km. The tangent points of the GPS observation profile are assumed at 0° longitude and 30° latitude without vertical shifts. The tangent directions are parallel to the equator.
Figure 4.5: The values of the second row elements of the matrix $\mathbf{A}$ ($a_2$). Only non-zero values are shown in the figure. X-axis indicates the distance between the tangent point and the middle point of the tangent link within each of the interception grid boxes along the tangent link. The grid and observation setup are the same as in Fig. 4.4.

resolution to $0.5^\circ$ longitude resolution, the models in Fig. 4.8c and g increase the vertical resolution to $0.5$ km vertical resolution, and the models in Fig. 4.8d and h increase both the horizontal and vertical resolution by taking $0.5^\circ$ longitude resolution and $0.5$ km vertical resolution. It shows that the increase of the horizontal resolution smoothes out the kernel function while the increase of the vertical resolution sharpens the kernel function along the tangent link. The increase of both horizontal and vertical resolutions complicated the effects of the resolution changes. Also, the kernel functions obtained under the same model grid configuration still differ in latitudes, e.g., the $\sigma$ value of the fitted Gaussian functions at $10^\circ$N is 80 (km), while it is 237.7 (km) at $50^\circ$N.

The above studies show that the excess phase delay observation operator is independent of model fields. However it is sensitive to the grid configuration of the model. The weighting given to the local refractivities are latitude and altitude dependent. These indicate that an optimal combination of the model horizontal and vertical resolution may exist for the best representation of the kernel operator and thus the excess phase delay observation operator.
Figure 4.6: Kernel functions (solid lines) and their Gaussian function fittings (dotted) along the tangent links at 1 km tangent altitude as functions of distance. The grid and observation setup are the same as in Fig. 4.4, except that the latitudes of the tangent points are located at (a) 10°N, (b) 20°N, (c) 30°N, (d) 40°N, (e) 50°N, (f) 60°N, (g) 70°N and (h) 80°N. The x-axis is the same as in Fig. 4.5.
4.3.2 An Implementation Detail of the Observation Operator

The above formulation uses no interpolation in setting up the excess phase delay observation operators, namely, the refractivity in each grid box is assumed constant. In a realistic numerical model, it is required to use a grid point value for the refractivity within a grid box. Two ways of obtaining the refractivity values are examined: one is to use the refractivity value from a corner point (e.g. the left-lower corner point – Point A in Fig. 4.3) (corner scheme) and another is to obtain the value at the middle point of the intercepted box (Point B in Fig. 4.3) (middle-point scheme). The advantage of the former is that no interpolation is required. Using the NCEP background and its T170L42 global model (triangular truncation at wavenumber 170 with 42 vertical sigma levels), the excess phase delay values are calculated from Eq. (4.6) at two GPS occultation points using the above two schemes for refractivity calculation. The first GPS RO point is at (62.39°W, 54.29°S) for a CHAMP sounding observed at 0400UTC May 21, 2002 and the second RO is at (158.98°E, 3.92°S) for a CHAMP sounding observed at 0327UTC May 21, 2002. Figure 4.9 shows the points on

![Figure 4.7: Gaussian function fittings in Fig. 4.6 as functions of latitude.](image)
Figure 4.8: Kernel functions under four model grid setups with: (a) and (e) 1° longitude resolution and 1 km vertical resolution; (b) and (f) 0.5° longitude resolution and 1 km vertical resolution; (c) and (g) 1° longitude resolution and 0.5 km vertical resolution; (d) and (h) 0.5° longitude resolution and 0.5 km vertical resolution. The tangent links are at 1 km tangent altitude, 0° longitude and 10°N latitude (left panels) or 50°N latitude (right panels). The x-axis is the same as in Fig. 4.5.
which refractivity values are calculated in the two schemes along a tangent link at a tangent point height of 0.4 km. Notice that the use of the cartesian coordinate makes a straight line in a sphere coordinate look like a curve. The points in the middle-point scheme forms a smooth line (solid line in the figure) with those in the corner scheme (dotted line in the figure) oscillating around it. Figure 4.10 shows the vertical profiles of the differences between the observed and simulated excess phase delays at these two GPS occultation locations. The percentage differences area also shown. The solid and dotted lines represent the results using the corner scheme and the dashed lines and the lines with plus marks represent the results using the middle point scheme. Percentage differences are also shown in Fig. 4.10. It indicates that the middle point scheme produces less simulation bias at most levels in both cases. So the refractivity values are obtained using this middle-point scheme in the following assimilation experiments.

In the formulation of the excess phase delay forward observation operator, the observation
Figure 4.10: Differences and percentage differences between the simulated excess phase delays and their observation values at two GPS occultation locations: The occultation in (a) was observed at 0400UTC May 21, 2002 at the occultation point (62.39°W, 54.29°S) and the one in (b) was observed at 0327UTC May 21, 2002 located at (158.98°E, 3.92°S). Solid (plus) lines represent the difference (percentage difference) obtained from the corner scheme and dashed (dotted) lines represent the results obtained from the middle-point scheme. Difference values are obtained by subtracting the observations from the simulations ($L - L_{obs}$) and percentage values are obtained by dividing the differences over the observations ($\frac{(L - L_{obs})}{L_{obs}}$).

In brief summary, three steps are required to convert the model variables ($T$, $p$ and $q$) to the excess phase delay:

1. calculate $N^{LOC}$ at model grids from $T$, $p$ and $q$;
2. interpolate $N^{LOC}$ to the specific grid box or layer intercepted by the tangent link; and
3. calculate the value of the excess phase delay.

levels are at the half levels of the model ones. When this condition is not satisfied, which is the case for most of the realistic models, the model levels are reconstructed through interpolations at each GPS occultation point without changing the observation levels. This avoids introducing extra errors into the GPS observations.
4.4 Experimental Design and Data Sets

4.4.1 Experimental Design

The observation operator of excess phase delay (Eq. 4.6) is incorporated into the NCEP SSI analysis system. Forward simulations and data assimilations of the GPS RO excess phase delay are carried out. Results of the simulation are examined to verify when and where using excess phase delay might be beneficial for GPS data assimilation. The assimilation results are analyzed to further explore the different impacts of GPS observations on the large-scale analysis and provide the potential insights on applications of the GPS observation assimilation to the operational numerical weather forecasts.

For comparison purpose, the forward simulations include two sets of experiments: PHA and REF. PHA simulates the excess phase delay $L$ using the observation operator $H_{PHA}$ defined in Eq. (4.6), and REF simulates the local refractivity via the observation operator defined by Eq. (1.9).

Following the forward simulations, three data assimilation experiments are carried out: PHA assimilates GPS excess phase delay observations plus other conventional field and satellite observations; REF assimilates GPS refractivity observations plus other observations; and NOGPS is a control experiment, which assimilates all observations in PHA and REF except GPS data. Each minimization of the cost function employs two outer loops which each contain 100 inner loops.

In the last section of this chapter, an alternative form of the excess phase delay calculation is proposed, called symmetric excess phase delay. The forward simulation and data assimilation results using the symmetric excess phase delay, denoted by PHA-sym, are compared with PHA and REF.

4.4.2 Background and GPS Observations

For all numerical experiments, the background (first guess) fields are taken from NCEP 6 hour model forecasts at 00UTC, 06UTC, 12UTC and 18UTC during May 24-31 2002. The NCEP SSI system with resolution of T170L42 (triangle truncation at the wavenumber 170 with 42 $\sigma$ levels) is used. The post process package converts the spectral coefficients of the model outputs to grid values, including temperature, geopotential height, relative humidity, surface pressure, cloud water, wind and others, at 26 pressure levels from 1000 hPa up to 10
The CHAMP GPS refractivities and excess phase delays observed at times less than 3 hours from a background time (i.e., 6 hour interval) are compared with the simulated values from the background. For PHA, CHAMP refractivities are transformed to the “observed” excess phase delays through the operator in Eq. (4.7). There are a total of 1158 occultations within this 7 day time period. Figure 4.11 shows their global distribution. CHAMP data have been smoothed vertically to a resolution similar to that of the NCEP SSI system. Specifically, the vertical resolution is 0.2 km below 4 km model height, 0.2 km between 4 and 6 km height, 0.4 km between 6 and 8 km height, 0.6 km between 8 and 14 km height, 1.0 km between 14 and 26 km height, 2.0 km between 26 and 30 km height and 5.0 km above 30 km. As an example Fig. 4.12 shows the vertical resolution of a smoothed CHAMP vertical profile and the model levels at a sample grid. The number of vertical levels of the smoothed GPS observations are comparable to that of the model (L42).

The vertical error correlation matrices for CHAMP refractivity and its derived excess
phase delay are shown in Figs. 4.13 and 4.14. They are calculated using the same method described in Chapter 3. Due to an updated quality control procedure implemented in 2005 in COSMIC/CDAAC, the vertical correlation of the CHAMP refractivity observations becomes much sharper (Figs. 4.13). The integration along the ray-path involved in the observation operator for the excess phase delay broadens the vertical correlation of excess phase delay (Figs. 4.14). For simplicity, we still take the inverse diagonal values (variance) for the weighting matrix in the cost function for both the GPS refractivity and excess phase delay data.

4.5 Numerical Results from Forward Simulation

Since different variables are calculated in the two forward simulation experiments (PHA and REF), results are compared in terms of observational increments in percentage. Specifically, the following two quantities, observational increments in percentage, are calculated:

$$
\Delta_{per}L = \frac{L - L_{obs}}{L_{obs}} \times 100(\%),
$$

$$
\Delta_{per}N^{LOC} = \frac{N^{LOC} - N_{obs}}{N_{obs}} \times 100(\%)
$$

(4.9)
Since $L$ is derived from $N^{LOC}$ using $L = AN^{LOC}$, we obtain

$$
\triangle_{per}L = \frac{AN^{LOC} - BN_{obs}}{BN_{obs}} = B^{-1}AN^{LOC} - N_{obs} = \left( B^{-1}A - I \right)N^{LOC} + \frac{N^{LOC} - N_{obs}}{N_{obs}}.
$$

(4.10)

Therefore, there is a relationship between $\triangle_{per}L$ and $\triangle_{per}N$:

- If the atmosphere is spherically symmetric, $B^{-1}A = I$, $N^{LOC} = N^{GPS}$, $\triangle_{per}L = \triangle_{per}N^{LOC}$;

Figure 4.13: Vertical error correlation of the GPS refractivity observations calculated by using CHAMP data during May 2002.
Figure 4.14: Vertical error correlation of GPS excess phase delay observations calculated by using CHAMP data during May 2002.

• If $B^{-1}A \neq I$, $N^{LOC} \neq N^{GPS}$, $\triangle_{per}L \neq \triangle_{per}N^{LOC}$.

The forward simulations provide values of both quantities ($L$ and $N^{LOC}$) at GPS occultation locations.

4.5.1 Comparison of Observational Increments of $L$ and $N^{LOC}$

Figure 4.15 shows the spaghetti plots of observational increments (in percentage) from PHA and REF, including a total of 1158 simulated soundings during May 24-31 2002. The altitudes shown are only up to 14 km below which significant differences between GPS refractivity assimilation (REF) and bending angle assimilation (BA) were observed (Chapter 3). The
values of $\Delta_{per} N^{LOC}$ are more sporadic at all levels reaching a value as large as about 20%, while the values of $\Delta_{per} L$ are mostly confined within ±5%. These indicate that $L$ compares more favorably to $L_{obs}$ than $N^{LOC}$ to $N_{obs}$. Figure 4.16 shows the mean and standard deviations (STD) of the increments. The mean values of the observational increments from both PHA (left panel) and REF (right panel) are nearly zero. For REF, the mean profile has slightly positive bias ($N^{LOC} - N_{obs}$) ($< 0.4\%$) below 4 km altitude. The deviations from the mean profile can be as large as $\sim \pm 2\%$ of the observation values. The mean value of the increment from PHA (left panel) is slightly positive below 4 km ($< 0.2\%$) and negative above 4 km altitude ($< 0.4\%$). The deviation from the mean for PHA are smaller than 1% at all levels.
Figure 4.16: Vertical profiles of mean and standard deviation (error bars) of $\Delta_{\text{per}} L$ (left panel) and $\Delta_{\text{per}} N^{\text{LOC}}$ (right panel). The quantities are calculated from 1158 simulated soundings during May 24-31 2002.

Taking 700hPa surface as an example, Fig. 4.17 shows a scatter plot with values of $\Delta_{\text{per}} L - \Delta_{\text{per}} N^{\text{LOC}}$ and $\Delta_{\text{per}} N^{\text{LOC}}$ indicated in x-axis and y-axis, respectively. It is seen that the value of $\Delta_{\text{per}} L - \Delta_{\text{per}} N^{\text{LOC}}$ increases linearly with the value of $\Delta_{\text{per}} N^{\text{LOC}}$, with $\Delta_{\text{per}} L \simeq \Delta_{\text{per}} N^{\text{LOC}}$ when $\Delta_{\text{per}} N^{\text{LOC}} \simeq 0$. This is a desirable result, consistent to the theoretical expectation.

### 4.5.2 Dependence of Excess Phase Delay and Refractivity Simulation Differences on Refractivity Gradient

The forward simulations from PHA and REF are carried out at the GPS occultation locations (geodetic longitude/latitude/altitude, observation space), while the refractivity gradients are calculated at model grids (longitude/latitude/pressure, model space) from the NCEP background. In order to examine the dependence of excess phase delay and refractivity simulations on the refractivity gradients, the refractivity gradients at model
grids are interpolated horizontally onto the geodetic longitudes and latitudes of the GPS occultation points, and the simulated excess phase delay/local refractivity are interpolated vertically onto the pressure levels. In other words, the refractivity gradients and the model forward simulation values are compared at GPS occultation points horizontally but at vertical pressure levels.

As an example, Fig. 4.18 shows the global distribution of the NCEP refractivity background field at the 700 hPa surface at 06UTC May 30, 2002. A latitudinal dependence is noticed. Large values of $N^{LOC}$ (> 230 N unit) are found in the tropics, and small values
of $N^{LOC}$ are found in the extratropics ($< 200$ N unit). Along a latitudinal band, refractivity values tend to be larger over land than over the oceans (e.g. America, Asia, Africa in the subtropics and Antarctica). Figures 4.19 and 4.20 show the vertical and horizontal gradients of the refractivity, respectively. Similar to the refractivity distribution, the vertical gradient distributions are largest in the tropics and larger around 60° than around 30°. Large horizontal gradients are more localized but most of the large horizontal gradients are located in the tropics and midlatitudes.

The geographic dependence of the differences between $\Delta_{\text{per}} L$ and $\Delta_{\text{per}} N^{LOC}$ ($\Delta_{\text{per}} L - \Delta_{\text{per}} N^{LOC}$) on refractivity horizontal and vertical gradients are shown in Fig. 4.21. Mostly, simulations from PHA and REF have slightly larger differences in the tropics between 30°S and 30°N, where both the horizontal and vertical gradients of the refractivity are larger than other latitudes. The distributions of $\Delta L - \Delta_{\text{per}} N^{LOC}$, the refractivity horizontal and vertical gradients are almost uniform along the longitudes. There is no significant
Figure 4.19: The global distribution of the vertical gradient of the NCEP refractivity background in Fig 4.18. 32 CHAMP occultation available in the time window are indicated by the triangles. Unit: N unit/km.

To examine a relationship between the vertical structure of the refractivity gradients and the difference between the simulated excess phase delay and refractivity, a pair of soundings are selected with one in a region of relatively small refractivity gradients and the other in a region with large refractivity gradients, respectively. The first one is at the location of the GPS occultation (258.63°E, 23.05°N) observed at 0723UCT May 30, 2002 (referred to as “RO1”), and the second one is at the location of the GPS occultation (81.87°E, 51.85°S) observed at 0643UTC May 30, 2002 (referred to as “RO2”). Their averaged occultation locations and tangent links (straight line 'A-B' for RO1 and 'C-D' for RO2) are shown in Fig. 4.22. Notice for both GPS occultations, the tangent point location changes slightly in the vertical direction. Along the directions of their averaged tangent links, both soundings can be plotted in the averaged occultation planes as shown in Figs. 4.23 and 4.24. The line
Figure 4.20: Same as Fig 4.19, except for the horizontal gradient of the refractivity. Unit: N unit/km

contours are the refractivity field and the shaded contours are the horizontal gradients (in Fig. 4.23) and the vertical gradients of the refractivities (in Fig. 4.24). The tangent points in the vertical directions for RO1 and RO2 are marked by the solid dots. The x-axis in both figures indicates the longitude of the points on the tangent links (i.e., line A-B for RO1 and C-D for RO2 in Fig. 4.22). The tangent links of RO1 pass a large refractivity gradient area. The horizontal gradient is $\sim 0.15$ N unit/km and the vertical gradient is $\sim 60$ N unit/km near the surface. The tangent links of RO2 pass an area with much smaller refractivity gradients, with the horizontal gradient is no more than 0.03 N unit/km (achieved at 600hPa) and the vertical gradient is no more than 30 N unit/km (achieved near the surface). For the purpose of clear comparison, the vertical profiles of the gradients at the two occultation points are given in Fig. 4.25. Both the horizontal and vertical refractivity gradients for RO1 are much larger than those for RO2 below 400hPa.

Percentage observational increments for PHA and REF are plotted for these two occult-
Figure 4.21: The percentage difference between PHA and REF ($\Delta_{\text{per}} L - \Delta_{\text{per}} N^{LOC}$) (a,d), the horizontal refractivity gradient (b,d) and the vertical refractivity gradient (c,f) at the 700hPa surface as functions of latitude (left panels) and longitude (right panels).
Figure 4.22: The averaged occultation points and the tangent links for two single occultations RO1 and RO2. RO1 is at the GPS occultation (258.63°E, 23.05°N) observed at 0723 UTC May 30, 2002 and RO2 is at the GPS occultation (81.87°E, 51.85°S) observed at 0643 UTC May 30, 2002.

For both occultations, the observational increments of $\Delta_{\text{per}} N^{\text{LOC}}$ are larger than those of $\Delta_{\text{per}} L$. Comparing RO1 with RO2, we find that the absolute values of the increments are larger for RO1 in a larger gradient area, especially near 5 km where $|\Delta_{\text{per}} N^{\text{LOC}}|$ is about 5%. The peak value of $|\Delta N^{\text{LOC}}|$ is about 1.4% for RO2 at 1.5 km and 2 km altitudes. In other words, the simulations of $L$ and $N^{\text{LOC}}$ are more different from their corresponding observations in an area with larger refractivity gradients. Impact of refractivity gradient on the simulation of $L$ versus $N^{\text{LOC}}$ in the low troposphere can not be assessed in this case as RO1 did not go below 3 km.
Figure 4.23: Cross-sections of the distributions of refractivity (contour) and its horizontal gradient (shaded) in the averaged occultation planes for RO1 (upper panel) and RO2 (lower panel). The tangent points in the vertical direction are marked by the solid dots.
Figure 4.24: Same as in Fig. 4.23 except that the shaded contours are the vertical gradients for each of occultations.
4.6 Data Results from Assimilation

Through the minimization of the cost function defined in Eq. (2.1), the observations could be incorporated into the numerical model and therefore the model state is adjusted. Thus, analyzing the GPS data assimilation results enables us to explore the different impacts of GPS observations on the model variables. Results from three experiments, NOGPS, PHA and REF, are discussed in this section. NOGPS is the control run assimilating observations excluding GPS data; PHA assimilates GPS excess phase delay observation and REF assimilates GPS refractivity observation.

Besides the percentage observational increments discussed in the previous section, ob-
Figure 4.26: Vertical profiles of $\triangle_{\text{per}L}$ (solid line) and $\triangle_{\text{per}N^{\text{LOC}}}$ (dashed line) for (a) RO1 and (b) RO2.

Suppose $V$ is a variable to be analyzed, three types of increments are defined

\[
\begin{align*}
\Delta V &= V_{\text{EXP}} - V_{\text{obs}} \quad \text{(Observation increment)}, \\
\Delta_{\text{ANL}} V &= V_{\text{EXP}} - V_{\text{guess}} \quad \text{(Analysis increment)}, \\
\Delta_{\text{GPS}} V &= V_{\text{EXP}} - V_{\text{NOGPS}} \quad \text{(GPS increment)},
\end{align*}
\]

(4.11)

where $V_{\text{EXP}}$ represents the analysis from a specific experiment (e.g., REF, PHA, PHA-sym and NOGPS), $V_{\text{guess}}$ is the background value, and $V_{\text{obs}}$ is the observed value. $\Delta V$ is the observational increment, indicating the differences between the analysis after data assimilation and its observed value; $\Delta_{\text{ANL}} V$ is the analysis increment, indicating the adjustments the assimilation makes to the background field and $\Delta_{\text{GPS}} V$ is the GPS increment, indicating the additional adjustments the GPS data assimilation makes to the analysis.
4.6.1 Minimization Test

To assure that the minimization in each of the assimilation experiments works, we performed three verifications: First, the correctness of the tangent linear and adjoint operators of the excess phase delay is checked. Second, the correctness of the gradient of the cost function is verified. Finally, a sufficient decrease of the cost function and the norm of the gradient of that cost function is achieved. As an example, Fig. 4.27 shows the values of $\| \nabla J \|^2$ as a function of iterations. GPS observations within the ±3 hour time window centered at 06UTC May 30, 2002 are assimilated. For all experiments, the values of $\| \nabla J \|^2$ decreased about 5 orders of magnitude after 200 iterations.

In terms of the fit to observations, Fig. 4.28 shows the mean and STD of the differences between the simulated and observed excess phase delay and refractivity for both GPS assimilation experiments (i.e., $\Delta_{\text{per}} L$ for PHA and $\Delta_{\text{per}} N^{\text{LOC}}$ for REF) before (solid lines) and after (dashed lines) the minimization. Statistics are calculated over all the 1158 GPS occultations. The mean differences between $L$ and $L_{\text{obs}}$ (i.e., $L - L_{\text{obs}}$) are reduced in most levels by the experiment PHA. The mean difference between $N^{\text{LOC}}$ and $N_{\text{obs}}$ is reduced below 6 km altitude but slightly increased above 6 km altitude in the experiment REF. The
4.6.2 Assimilation Impacts on Refractivity in Observation Space

Having obtained an appropriate fit of the model to direct observations, we examine changes in local refractivity ($N^{\text{LOC}}$) and GPS refractivity ($N^{\text{GPS}}$) after the GPS data assimilation. Here, the simulated $N^{\text{GPS}}$ is obtained through a ray-tracing and Abel inversion, which are consistent with the GPS refractivity observations. The following two observational increments are calculated at each occultation point:

$$\triangle N^{\text{LOC}} = N^{\text{LOC}} - N_{\text{obs}},$$

$$\triangle N^{\text{GPS}} = N^{\text{GPS}} - N_{\text{obs}}.$$  (4.12)

Figure 4.28: Vertical profiles of the mean (left panels) and STD (right panels) of $\Delta_{\text{per}} L$ (a,b) and $\Delta_{\text{per}} N^{\text{LOC}}$ (c,d) before (solid lines) and after the GPS data assimilations (dashed lines).

STDs of the analyses from both GPS assimilation experiments are much closer to the GPS observations after data assimilations as is expected.
Vertical profiles of these two refractivity increments for NOGPS, PHA and REF are shown in Figs. 4.29-4.31, respectively. The left and middle panels in each of the figures are the mean and STD profiles of $\Delta N^{\text{LOC}}$ and $\Delta N^{\text{GPS}}$ before and after data assimilations averaged over 1158 GPS occultation locations, and the right panels combine two profiles of $\Delta N^{\text{LOC}}$ and $\Delta N^{\text{GPS}}$ after assimilations in one figure. Since no GPS data are incorporated in NOGPS, the observational increments for both $N^{\text{LOC}}$ and $N^{\text{GPS}}$ are up to 2% both before and after assimilation (left and middle panels in Fig. 4.29). The values of the observational increments of $N^{\text{GPS}}$ are even larger than those of the $N^{\text{LOC}}$, especially between 1 and 6 km, in terms of both the mean and STD values of their analyses (right panels in Fig. 4.29). This confirms that GPS observations are not redundant to in the conventional and satellite observations and can provide much needed temperature, moisture and pressure information to large-scale analysis.

For PHA, the assimilation of the excess phase delay observations reduces the observational increments of both $N^{\text{LOC}}$ and $N^{\text{GPS}}$ (left and middle panels in Fig. 4.30). The mean and STD values of the $N^{\text{LOC}}$ and $N^{\text{GPS}}$ are comparable in magnitude at most levels (right panels in Fig. 4.30). Similar to PHA, the observation increments for both $N^{\text{LOC}}$ and $N^{\text{GPS}}$ are reduced in REF (Fig. 4.31). However, $N^{\text{LOC}}$ analysis has smaller observational increment than $N^{\text{GPS}}$ after assimilation. The reductions of the $N^{\text{LOC}}$ and $N^{\text{GPS}}$ observational increments after assimilations for PHA and REF confirm a positive impact of the GPS data on the model refractivity simulations. Smaller observational increment of $N^{\text{LOC}}$ from REF, compared with that of $N^{\text{GPS}}$, implies that REF observation operator may be over-fitting the GPS observations.

Figure 4.32 shows the mean and STD profiles of $\Delta N^{\text{GPS}}$ background and analyses from all three data assimilation experiments. PHA has the smallest $N^{\text{GPS}}$ observation increment, while NOGPS have the largest values, which are even slightly bigger than the background at the lowest levels. REF has larger $\Delta N^{\text{GPS}}$ than that of PHA and smaller $\Delta N^{\text{GPS}}$ than that of NOGPS.

### 4.6.3 Impacts on Temperature and Moisture Analyses

First, the temperature and moisture analyses at GPS occultation points are examined. The model fields are interpolated onto the occultation points horizontally but remain at pressure levels vertically. Figure 4.33 presents the spaghetti plots of the GPS increments of the temperature ($\Delta_{\text{GPS}}T$) from PHA and REF. The mean profiles have been calculated in three
different latitudinal zones: the tropics ($30^\circ S - 30^\circ N$), the mid-latitudes ($30^\circ S - 60^\circ S$ and $30^\circ N - 60^\circ N$) and the high-latitudes ($(60^\circ S - 90^\circ S$ and $60^\circ N - 90^\circ N$). The values of $\Delta_{GPS}T$ differ remarkably in three latitudinal zones, especially for PHA. Generally, their values are the smallest in the tropics and the largest in high-latitudes. Figure 4.34 shows the mean and STD by averaging all $\Delta_{GPS}T$ over 1158 GPS occultation locations. In all three latitudinal zones, the GPS experiments (PHA and REF) tend to have a warmer model atmosphere at most levels. PHA produces larger positive temperature adjustments to the models based on NOGPS results than REF does. The differences between the results from GPS data assimilation experiments and NOGPS are larger at higher latitudes. The results for specific humidity analyses ($\Delta_{GPS}q$) are shown in Figs. 4.35 and 4.36. Generally, the
largest (smallest) adjustments occur in the tropics (the high-latitudes). Both experiments tend to reduce (increase) the humidity value below (above) 600hPa. PHA produces a more moisture atmosphere than REF at all levels.

 Due to the characteristics of the observation operator, PHA modifies the model atmosphere not only at the tangent point locations but also in their surrounding areas through the adjoint of the observation operator and the background error covariance matrices. Therefore, it is of interest to examine the differences between PHA and REF analyses horizontally. As an example, Fig. 4.37 shows the global distribution of the NCEP temperature background field at the 500 hPa surface at 06UTC May 30, 2002. The temperature has a zonal distribution. The highest temperature is about \(-5^\circ C\) in the tropics. The lowest temperature \((<-45^\circ C)\) is located in the higher latitudes of the Southern Hemisphere. Figure 4.38 shows the global distribution of the analysis increment of temperature \((\Delta_{ANL}T)\) at 500hPa from PHA. The value of \(\Delta_{ANL}T\) ranges from -3 K to 3 K, with contributions from all observations.

Figure 4.30: Same as Fig. 4.29, except for the experiment PHA.
Figure 4.31: Same as Fig. 4.29, except for the experiment REF.

including GPS data. The swath pattern of the increment mostly comes from satellite radiance observations. Zooming into the subdomain (10°E – 140°E, 90°S – 10°S), Fig. 4.39 shows $\Delta ANLT$ from NOGPS, REF and PHA at the 500hPa surface. There are 5 GPS occultations observed within the assimilation time window in this domain. There is one large temperature increment around 80°E for NOGPS (panel a). The amplitude of this center wakens and splits into two small centers for REF (panel b). A similar structure is observed for PHA, with larger amplitude (panel c). PHA has larger positive increment area than NOGPS and REF, which might indicate a larger influence radius of the GPS excess phase delay than the GPS refractivity. The moisture increments for the same subdomain in lower pressure levels (figure not shown) do not have significant differences among different experiments in the area.

A case study for moisture is taken also from the analysis at 06UTC May 30, 2002. The global distribution of the specific humidity at this time is shown in Fig. 4.40. The moisture is higher in the tropics than higher latitudes. The analysis increments of the specific humidity
Figure 4.32: Vertical profiles of the mean (left) and STD (right) of $\triangle N^{GPS}$ background (square) and analyses from the experiment NOGPS (dotted line), PHA (star) and REF (solid line).

Figure 4.33: Spaghetti plots of $\triangle GPS T$ for (a) PHA and (b) REF. The profiles have been color-coded according to their corresponding occultation latitudes: blue represents soundings in the tropics ($30^\circ S - 30^\circ N$), cyan represents soundings in mid-latitudes ($30^\circ S - 60^\circ S$ and $30^\circ N - 60^\circ N$) and magenta represents soundings in high-latitudes ($60^\circ S - 90^\circ S$ and $60^\circ N - 90^\circ N$).
Figure 4.34: Vertical profiles of the mean (upper panels) and STD values (low panels) of $\Delta_{GPS}T$ for PHA (stared lines) and REF (dashed lines). (a) and (d) are for soundings in the tropics ($30^\circ S - 30^\circ N$), (b) and (e) are for mid-latitude soundings ($30^\circ S - 60^\circ S$ and $30^\circ N - 60^\circ N$) and (c) and (f) are for high-latitude soundings ($60^\circ S - 90^\circ S$ and $60^\circ N - 90^\circ N$).

$\Delta_{ANLq}$ for PHA is shown in Fig. 4.41. Large moisture analysis increments are found in the Southern Hemisphere. For comparison purpose, the GPS increments $\Delta_{GPSq}$, instead of $\Delta_{ANLq}$, are plotted in a selected subdomain ($100^\circ E - 140^\circ E, 0 - 30^\circ N$) shown in Fig. 4.42. There are two GPS occultations in this area. For REF, there is no significant moisture adjustment to NOGPS experiment around the GPS occultations at $120^\circ E$. For PHA, moisture is increased within a large area centered around the two occultation locations. The value of $\Delta_{GPSq}$ is about 0.7 g/kg.
Figure 4.35: Same as Fig. 4.33, except for specific humidity.

Figure 4.36: Same as Fig. 4.34, except for specific humidity.
Assuming that the simulated local refractivity at the tangent point is spherically symmetric, an alternative form of $L$ in Eq. (4.6) can be obtained by

$$L^{\text{sym}} = \sum_{l=1}^{l=M} b_{m,l} N^{\text{LOC}}(l)$$

$$= \sum_{l=1}^{l=M} b_{m,l} \left[ c_1 \frac{p(l)}{T(l)} + c_2 \frac{p(l)q(l)}{T(l)^2 (0.622 + 0.378q(l))} \right]$$

$$= H_{PHA}^{\text{sym}}(t, p, q), \quad (4.13)$$

where $H_{PHA}^{\text{sym}}$ represents the observation operator for $L^{\text{sym}}$. $H_{PHA}^{\text{sym}}$ are different from $H_{PHA}$ defined in Eq. 4.6. The former one integrates the local refractivities at tangent points and only involves the refractivity gradients in the vertical direction, while the latter one integrates
local refractivities along the tangent link, thus involving the refractivity gradients in the areas adjacent to the tangent link both in horizontal and vertical directions.

The advantage of $H^\text{sym}_{PHA}$ is that it adds little work to the local refractivity calculation except for an integration in the vertical direction. All computations related to the tangent links in $H_{PHA}$ is not necessary, including the requirement for the tangent directions at the tangent points, the interception of the tangent links with the vertical layers and grid boxes and the interpolations of the local refractivities onto the interception points. Actually, the calculation of $L^\text{sym}$ at one tangent point can be done by integrating the local refractivities vertically at the levels above the tangent point along a distance given a priori. The distance can be defined by the altitudes between the tangent point and the uppermost level of the model.

As an alternative form of the excess phase delay observation operator, $H^\text{sym}_{PHA}$ is incorporated into the NCEP SSI system. The forward simulation and assimilation of GPS excess
Figure 4.39: Distribution of the analysis increments of 500 hPa temperature in subdomain (10° – 140°E, 90° – 10°S) at 06UTC May 30, 2002 for (a) NOGPS, (b) REF and (c) PHA. The triangles indicate the GPS occultation points.

phase delay observations in terms of it are carried out (referred to as PHA-sym). The following discussion contain some preliminary results.

Similar to Fig. 4.15, Fig. 4.43 shows the spaghetti plots of $\Delta_{per}L$ from PHA and $\Delta_{per}L^{sym}$ from PHA-sym. Most of the points overlap, indicating that the simulation results from the two experiments are quite similar. On average (Fig. 4.44), like PHA, PHA-sym also have slight positive mean values ($< 0.2\%$) below 4 km altitude. The STD of the simulations from PHA-sym is up to 1%, slightly larger than PHA. Compared with the result from REF, the differences between PHA and PHA-sym results are very small.

Differences between PHA-sym and PHA simulations can be compared with those between PHA-sym and REF by comparing Figs. 4.15 and 4.43. Clearly, PHA-sym simulations differ from REF results much more than they differ from PHA results. Figure 4.45 quantifies such differences at the 700hPa pressure level. The differences between PHA and PHA-sym simulations are within 1% mostly, while differences between PHA-sym and REF simulations exceed 5%.

For the same soundings (RO1 and RO2) in Fig. 4.26, Fig. 4.46 shows the vertical profiles
Figure 4.40: Global distribution of the 850hPa NCEP specific humidity background at 06UTC May 30, 2002.

... of $\Delta L$ and $\Delta L^{\text{sym}}$. Compared with $\Delta N_{\text{LOC}}$ in Fig. 4.26, $\Delta L$ and $\Delta L^{\text{sym}}$ are quite close to each other for both soundings. It is noted that the difference between $\Delta L$ and $\Delta L^{\text{sym}}$ is slightly larger for RO1, which is located in an area with larger refractivity gradients.

For data assimilation experiments, a similar analysis is carried out to PHA-sym result as was done to PHA and REF results in Section 4.5. The cost function of PHA-sym (not shown) is reduced by a similar amount to other three experiments shown in Fig 4.27. The mean and STD values of $N_{\text{GPS}}$ analysis after GPS excess phase delay assimilation from PHA-sym are compared with those from background, NOGPS, PHA and REF in Fig. 4.47. PHA-sym has a slightly larger $N_{\text{GPS}}$ bias from the observation than REF, but still much smaller than those from the background field and NOGPS. Spaghetti plots and averaged profiles of temperature and moisture analysis increments at the occultation points are shown in Figs. 4.48–4.51. Results from PHA-sym are in between those from PHA and REF. In terms of the mean profiles of the temperature analysis (solid lines in Fig. 4.49a,b and c), results from PHA-sym...
Figure 4.41: Global distribution of the 850hPa specific humidity analysis increment from PHA at 06UTC May 30, 2002.

Figure 4.42: Distribution of the GPS increments of the 850hPa specific humidity subdomain (10° – 140°E, 90° – 10°S) at 06UTC May 30, 2002 for (a) REF and (b) PHA. The triangles indicate the GPS occultation points.
are closer to those of REF. For example, in the tropics, temperature analysis increments at 750 hPa from PHA-sym and REF are around 0.2 K while PHA is 2 times bigger, equal to 0.4 K. But for moisture analysis increments (solid lines in Fig. 4.51a, b and c), results from PHA-sym are much closer to PHA compared with those from REF. The reason for this is probably that moisture is a more localized variable and therefore the neglect of the horizontal gradient along the tangent link in PHA-sym has smaller influence on moisture variable. For the subdomain analysis of temperature analysis increments ($\Delta_{ANLT}$) (Fig. 4.52), PHA-sym also presents a two-center pattern of the analysis increment at 80°E, but the amplitudes of the centers are slightly weaker than those of PHA and REF. The subdomain analysis of moisture GPS increment ($\Delta_{GPSq}$) for PHA-sym is shown in Fig. 4.42b. PHA-sym produces an increase of moisture with a smaller influence area than that of PHA.

From the above discussions, we find that PHA-sym has a potential to produce an intermediate result compared with PHA and REF. Under the following situations, results from PHA-sym and PHA can be comparable:

Figure 4.43: Spaghetti plots of $\Delta_{per L}$ and $\Delta_{per L^{sym}}$. $\Delta_{per L}$ (cyan) is plotted over $\Delta_{per L^{sym}}$ (blue). There are a total of 1158 soundings during May 24-31 2002.
• Horizontal components of the refractivity gradient are small enough to be ignored or

• Horizontal gradient is not well represented, e.g., due to the model horizontal resolution.

As mentioned earlier, $L_{sym}$ is calculated from $H_{PHA}^{sym}$ at the tangent point, without involving any interpolation in the horizontal direction. This feature is especially attractive for the operational numerical models which divide the model space into sub-domains horizontally for a parallel processing algorithm. Application of $L_{sym}$ (or combined with $L$) is probably more desirable in such a computational environment.

## 4.8 Summary and Conclusions

This chapter develops and tests two new schemes for GPS data assimilation. The excess phase delay and the symmetric excess phase delay are selected as the assimilation variables because the integral involved in their formulation enables the along-track refractivity and refractivity gradient information be included in data assimilation. The observation operators
for both variables are simple and has relatively low computational cost.

The observation operator for the excess phase delay is incorporated into the NCEP SSI system (T170L42). The forward simulation and assimilation experiments using the system are conducted (PHA) for the time period from May 20 to 31, 2002. A total of 1158 CHAMP RO sounding observed in the time window during the period are used. Two more experiments, local refractivity simulations and assimilations (REF) and a control run without the assimilation of GPS data (NOGPS) are carried out. The results from PHA, REF and NOGPS are compared. Based on the same background of refractivity, the excess phase delay simulated by PHA has less bias from its observation value than the local refractivity simulated by REF from its observation. After assimilations of GPS data, PHA results in a smallest bias of $N^{GPS}$ than REF and NOGPS, indicating that the analysis from PHA is more accurate than those from the other two experiments. The impacts of the assimilation of the excess phase delay observations are studied in terms of both the general statistical mean and STD and case studies. We find that PHA tends to produce a warmer and more moist atmosphere than REF. It also introduces a finer structure with a larger radius of influence.

Figure 4.45: Comparison of the simulation difference between PHA-sym and PHA ($\Delta_{per} L^{sym} - \Delta_{per} L$) (y-axis) and the simulation difference between PHA-sym and REF ($\Delta_{per} L^{sym} - \Delta_{per} N^{LOC}$) (x-axis) at the 700hPa surface.
Figure 4.46: Vertical profiles of $\Delta_{per}L$ (solid line) and $\Delta_{per}L_{sym}$ (dashed line) for (a) RO1 and (b) RO2.

Figure 4.47: Same as Fig. 4.32, except that the vertical profiles of the mean (left) and STD (right) values of $\Delta N_{GPS}$ analysis from PHA-sym (dashed line) are added.
Figure 4.48: Same as Fig. 4.33, except for results from PHA-sym.

Figure 4.49: Same as Fig. 4.34, except that results from PHA-sym are added (solid lines).
Figure 4.50: Same as Fig. 4.35, except for results from PHA-sym.

Figure 4.51: Same as Fig. 4.36, except that results from PHA-sym are added (solid lines).
to the temperature and specific humidity analysis fields.

Under the assumption of the spherical symmetry of the local refractivity, an alternative form of the excess phase delay, symmetric excess phase delay, and its observation operator are derived. Simulation and assimilation experiments using the forward operator are carried out (PHA-sym). It shows that PHA-sym produces intermediate results between PHA and REF. Since it is simpler than PHA, and meanwhile keeps most information for the along-track refractivity gradients, it may be more desirable in a operational numerical model implemented via a localized parallel algorithm without any changes of the model structure. However, when and where this symmetric excess phase delay can substitute the excess phase delay are yet to be studies further.
In summary, the simulation and assimilation of the GPS excess phase delay using CHAMP data have shown a great potential of the application of the excess phase delay observation operator. This method combines the advantages of both GPS bending angle and GPS refractivity assimilations. It is computationally less expensive than GPS bending angle assimilation and has better accuracy than GPS refractivity assimilation. Therefore it is a desirable GPS data assimilation algorithm for operational numerical weather forecasts and simulations.
In recent years, numerical models have advanced rapidly with more detailed descriptions of the atmospheric processes and higher resolutions. Uncertainties in the initial conditions become more critical to the capabilities of the numerical weather prediction (NWP) and simulation. Having an accurate initial condition becomes particularly important in certain geographical areas, e.g. those in which small perturbations to the state of the atmosphere will amplify rapidly. Many of these are found in the vicinity of atmospheric fronts and jet-streams in which cloud cover is often extensive. Among other requirements, there is a clear demand for higher accuracy, better vertical resolution and higher stability in observations of temperature and humidity. Unlike conventional and satellite observations, the GPS radio occultation (RO) remote sensing technique provides a new way to see through the atmosphere. The RO observations are all-weather observations, requiring no calibration. They complement other observing systems very well and may thus contribute to improved global analyses and forecasts as well as climate studies. Therefore, it is important to come out with the best strategy for the assimilation of GPS RO observations into numerical models.

Based on theoretical studies, GPS bending angle and refractivity appear to be the possible choices for assimilation than other retrievals from the GPS measurements. Both GPS bending angle and refractivity assimilations have their advantages and disadvantages. For the assimilation of GPS bending angle, the observation is in a rawer form of the measurement and has simpler error characteristics. The forward observation operator for bending angle is more accurate but computationally expensive. For the assimilation of GPS refractivity, the observation may carry additional errors due to the uncertainties and error propagations in the retrieval procedure and its forward observation operator is not as accurate as that of GPS bending angle. However the operator is simple and easy to implement in a numerical model.
and is computationally inexpensive. To achieve both accuracy and computational efficiency, a new method that is not only accurate but also computationally efficient is developed and tested.

Using a 2D ray-tracing model, a set of GPS bending angle assimilation (BA) experiments was first carried out to investigate the sensitivity of the assimilation analyses to the observational weightings, which are estimates of the inverse of the error covariance of the GPS observation. Three ways to calculate the weighting functions are examined. The bending angle assimilation with any of the three weightings produced temperature and specific humidity analyses which compare favorably with independent radiosonde data. The temperature analysis was much more sensitive to the observational weighting than the specific humidity. It was also noted that the sign of the differences between the background refractivity and the GPS observations determined the sign of the analysis increments. Thus the efficiency of the data assimilation was partly determined by the quality of the background fields. Moreover, the sensitivities of the bending angle assimilation to the gravity and the vertical data resolution were tested. Differences between the experiments including and excluding the variation of the gravity in the ray-tracing operator were rather small. The temperature analysis was found to be more sensitive to the vertical resolution of the assimilated GPS data than the specific humidity and the surface pressure.

The GPS refractivity assimilation via a local refractivity forward operator (REF) was then conducted and compared with BA. For the assimilation of 34 GPS ROs, the computational cost of BA is about 1.5 times more than REF. By analyzing the GPS increments of the specific humidity, i.e., differences of specific humidity analysis with and without GPS data, we noted that BA tended to produce larger analysis increments and larger impact radii than those of REF. The accuracy of the BA and REF analyses were evaluated in terms of the observational increments: $N^{GPS} - N_{obs}$, where $N^{GPS}$ is the GPS refractivity obtained in a physically consistent manner with the GPS observations. While both BA and REF experiments have reduced the bias of the background fields of $N^{GPS}$, the BA analyses fit the GPS refractivity more closely than the REF analyses. Impacts of these differences of refractivity on the model variables were shown in the analysis increments of the model variables, i.e., temperature, specific humidity and surface pressure, in the troposphere. The analysis differences of temperature between BA and REF occurred mostly at the middle-to-high troposphere, especially for the ROs in the tropics. Those of moisture appeared mostly
at the lower troposphere. The surface pressure analyses from BA and REF were quite close in the tropics and remarkably different in the high latitudes, especially in the Southern Hemisphere. To assess the performance of these two GPS data assimilation strategies, BA and REF analyses were compared within different cloud regimes in the tropics and middle latitudes. The differences between BA and REF analyses were found larger in thick-layered cloud systems (e.g., convective clouds in the mid-latitudes and cumulus clouds in the tropics). For the specific humidity analysis in the convective clouds in the mid-latitudes and cumulus clouds in the tropics, the differences could be 3 times more than those in other cloud systems. It indicates that larger horizontal gradients of the atmosphere refraction within or near cloud systems contribute to these large differences between BA and REF.

BA and REF analyses were also compared with a control run of the NCEP analysis system without assimilating the GPS observations (NOGPS). The comparison reveals that NOGPS produced the largest refractivity bias from the GPS observations and smallest analysis increments of temperature, moisture and surface pressure among three experiments. It confirms that GPS observations are able to provide non-redundant information to the NWP models.

Based on the above studies, a new observation operator for simulating the GPS excess phase delay was finally proposed and tested for GPS RO data assimilation. Using excess phase delay, the along-track refractivity and refractivity gradient information can be included while the observation operator remains simple and requires little additional computational cost. By incorporating the excess phase delay observation operator into the NCEP analysis system, results from the forward simulation and assimilation experiments of the excess phase delay (PHA) were compared with those of REF and NOGPS. It is found that the excess phase delay simulated by PHA had less bias from its observation than the local refractivity simulated by (REF) from its observation. The latter one was 2 times more than the former one by average. After assimilations of GPS data, PHA tended to produce a smallest bias of $N_{GPS}$ than REF and NOGPS, indicating that the analysis from PHA is more accurate than those from the other two experiments. In the model space, PHA tended to produced a warmer and more moist model atmosphere than REF. The differences of temperature analyses between PHA and REF are largest in high latitudes and those of moisture analyses are largest in the tropics. For a single sounding, the difference could reach 1 to 2 degrees for temperature analyses and 1 to 2 g/kg for specific humidity analyses. A finer structure
with a larger radius of influence to the temperature and specific humidity analysis fields is also found in the PHA analysis compared with REF. Under the assumption of the spherical symmetry of the local refractivity, an alternative form of the excess phase delay and its observation operator are also derived (PHA-sym). It is a point scheme in which only a vertical profile of model refractivity is required and therefore simpler than PHA and meanwhile the information for the along-track refractivity gradients in the vertical direction is still taken into consideration. PHA-sym mostly produces intermediate numerical results between PHA and REF. It has the potential to be used in an operational numerical model without changing the existing model structure in a parallel environment.

This dissertation studied and compared several methods for the assimilation of the GPS radio occultation observations using the NCEP three dimensional data analysis system and concluded on a new scheme as the best choice for the assimilation of GPS observations. More detailed studies on the GPS data assimilation are under way, including further assessments on the impacts of the excess phase delay observation operator on model states, aiming at answering the following questions:

- where and when is the excess phase delay assimilation most needed? and
- how does the model resolution limit the current impact assessment?

Forecast impacts of the GPS observations are being conducted and the assimilation results will also be verified with independent observations, e.g., the Advanced Infrared Sounder (AIRS) temperature and moisture retrievals.
REFERENCES


146


BIOGRAPHICAL SKETCH

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Hui Shao was born in 1972 in Shengyang, China. In 1995, she completed her Bachelors degree in Atmospheric Sciences at Nanjing University, China. Under the advisement of Dr. Yongfu Qian, she obtained her Masters degree in Climatology in 1998, also from Nanjing University. She came to Florida State University in 2000 and has pursued her doctorate in Meteorology since then.