Algebraic Problem Solving for Middle School Students with Autism and Intellectual Disability

Jenny R. Root and Diane M. Browder

This is the accepted author manuscript and the version of record can be found at https://doi.org/10.1080/09362835.2017.1394304
Abstract

Problem solving is an important yet neglected mathematical skill for students with autism spectrum disorder and intellectual disability (ASD/ID). In addition, the terminology and vocabulary used in mathematical tasks may be unfamiliar to students with ASD/ID. The current study evaluated the effects of modified schema-based instruction (SBI) on the algebra problem solving skills of three middle school students with ASD/ID. Mathematics vocabulary terms were taught using constant time delay. Participants were then taught how to use an iPad that displayed a task analysis with embedded prompts to complete each step of solving the word problems. This study also examined participant’s ability to generalize skills when supports were faded. Results of the multiple probe across participants design showed a functional relation between modified SBI and mathematical problem solving as well as constant time delay and acquisition of mathematics vocabulary terms. Implications for practice and future research are discussed.
Algebraic Problem Solving for Middle School Students with Autism and Intellectual Disability

The National Council for Teachers of Mathematics (NCTM) has hailed problem solving as the cornerstone of mathematical learning (NCTM, 2000). Similarly, the Common Core State Standards in Mathematics (CCSSM) has identified problem solving as a mathematical standard of practice that should be emphasized across domains and grade levels (Common Core State Standards Initiative, 2015). Despite the importance of problem solving, mathematics instruction for students with autism spectrum disorder who also have intellectual disability (ASD/ID) has often been limited to computation rather than problem solving (Browder, Spooner, Ahlgrim-Delzell, Harris, & Wakeman, 2008; King, Lemons, & Davidson, 2016).

The first studies to teach problem solving found task analytic instruction, graphic organizers, and manipulatives to be effective for students with ASD/ID (Browder, Jimenez, & Trela, 2012; Browder, Trela, et al., 2012; Jimenez, Browder, & Courtade, 2008). One limitation of these early studies was they did not include conceptual knowledge induction. Participants were able to demonstrate “how” to solve the problems, but not “when” or “why” to use those methods (Saunders, 2014). These studies did not require participants to make any decisions about why they were using a particular strategy or to make mathematical decisions; rather they only focused on procedural aspects of solving the problem. In contrast, most problem solving scenarios in both school-based and authentic contexts involve reasoning about quantities and relationships and making decisions about appropriate strategies.

Teaching through problem solving helps students address a mathematical problem by recognizing what they know and need to know about a mathematical situation. Van de Walle, Karp, and Bay-Williams (2016) argue learning to solve contextual story problems, or word
problems that are based on relevant and realistic scenarios, is the basis for solving real-world problems.

Mathematical problem solving requires a variety of skills with which students with ASD/ID typically struggle, including executive functioning, metacognition, and semantic language. Executive functioning is required for planning, organizing, and switching cognitive sets (Rockwell, Griffin, & Jones, 2011). Zentall (2007) considers executive functioning a “critical factor in math performance and achievement” (p. 234). In addition to executive functioning, mathematical problem solving also requires a certain level of metacognition, the conscious monitoring and regulation of an individual’s own thought process (Van De Walle et al., 2016). It is important for students to know what they are going to do, how they will go about doing it, and the rationale behind those choices. Executive functioning and metacognition deficits can lead to an inability to discriminate relevant events from irrelevant stimuli within word problems, moving a conceptual error in understanding the type of word problem presented into a subsequent procedural error in solving the problem due to employment of the wrong strategies (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007).

An additional barrier to mathematical problem solving for individuals with ASD/ID is the semantic language involved in solving word problems (Rockwell et al., 2011). Swanson and Beebe-Frankenberger (2004) explain problem solving requires understanding how each piece of the text comes together for meaningful interpretation. A student’s ability to comprehend mathematics is linked to their understanding of mathematics as a language and the language used to teach mathematics (Miller, 1993). The semantics of word problem solving can be problematic for students with ASD/ID on numerous levels; it requires an individual to understand both the individual terms as well as to make sense of “what is happening” in the problem. Van de Walle
et al. (2016) emphasize the importance of fluency in mathematical language, as students need to use mathematical vocabulary and articulate mathematical concepts in order to learn the language of mathematics. A barrier in problem translation due to semantic language deficits may prevent further success in the problem solving process for students with ASD/ID. A poor understanding of mathematical terminology and language prevents access to language of instruction, textbooks, and word problems (Miller, 1993).

To attend to the executive functioning, metacognitive, and semantic language challenges faced by students with ASD/ID in solving mathematical problems, researchers in the field of special education have looked to schema-based instruction (SBI), an evidence-based practice for students with high incidence disabilities (Jitendra et al., 2015) that directly addresses executive functioning, metacognitive, and semantic language demands of problem solving. According to Jitendra et al. (2013), the primary focus of SBI is to teach students the underlying mathematical structure of word problems. In SBI, students choose a schema that matches the problem type and organize information from the problem onto the schema, thereby reducing the working memory load of students by concretely group informational units. There is a general four-step process students are explicitly taught in traditional SBI, which includes (a) visual diagrams known as schemas to show the relationships between quantities in the word problem; (b) a heuristic to remember the problem solving process; (c) the use of explicit instruction to teach the problem solving process; and (d) metacognitive strategy instruction (Jitendra et al., 2013; Powell, 2011). Spooner, Saunders, Root, and Brosh (2017) proposed modified schema-based instruction (MSBI) as a viable strategy for teaching conceptual and procedural knowledge and problem solving to students with extensive support needs, such as those with ASD/ID.
MSBI supplements the essential features of SBI with (a) a task analysis and chant with hand motions to serve as a heuristic, (b) enhanced visual supports on graphic organizers, and (c) incorporation of systematic instruction along with explicit instruction (Spooner et al., 2017). Saunders (2014) was the first to investigate the effects of MSBI on problem solving of elementary students with ASD/ID. Using a multiple probe across participants design, Saunders found MSBI presented through computer-based video instruction to be effective in teaching students to solve and discriminate between two types of arithmetic problems (group and change). Root, Browder, Saunders, and Lo (2017) extended the work of Saunders by teaching elementary students to solve a different type of problem (compare). In a multiple probe across participants with embedded alternating treatment design, Root et al. found a functional relation between teacher-delivered MSBI and mathematical problem solving.

Although both Saunders (2014) and Root et al. (2017) directly taught problem solving and had positive findings, the studies set two critical boundaries on generalization. First, participants were provided several supports that were never faded, such as a visual support to fill in for the number sentence and color-coded graphic organizers. In addition, word problems contained pictures above key information in the problem. Second, the students were only taught to solve for an unknown in the final position (i.e., $4 + 3 = ?$). While these two studies provide important findings on the effectiveness of MSBI, they leave questions as to the degree of instructional supports students with ASD/ID need to independently solve word problems. In addition, it is currently unknown whether MSBI is an effective strategy for teaching students to solve word problems with unknowns in other positions, a pre-skill to algebra.

Two studies have taught students with ASD and ID to solve word problems with the unknown in the medial (i.e., $4 + ? = 7$), initial (i.e., $? + 3 = 7$) and final positions. Neef, Nelles,
Iwata, and Page (2003) taught the precurrent behaviors of identifying the component parts of a change word problem (i.e., initial set, change set, key words to identify the operation, and resulting set) to fill out a number sentence to two adults, one of who had moderate ID. Neef et al. gave visual supports in the form a diagram to participants throughout all phases of the study that was similar to those used by Root, Browder, et al. and Saunders. Rockwell et al. (2011) taught a 10-year-old student with ASD who did not have comorbid ID to use schematic diagrams to solve word problems with the unknown in the final position without permanent visual supports, as the participant drew her own diagrams. The participant was able to generalize newly acquired skills to solve problems with unknowns in other positions after only one training session.

The participants in Rockwell et al. (2011) and Neef et al. (2003) had mathematical skills that were not characteristic of all students with ASD/ID (Browder & Spooner, 2011), including the ability to independently read word problems and memorization of basic mathematic facts. The mathematical pre-skills of participants should be taken into consideration when making generalizations about the implications of their findings on problem solving instruction and learning. Rockwell et al. demonstrated students with ASD who did not have ID required minimal (one lesson) instruction in generalization to unknowns in the initial and medial positions. The findings of Neef et al. suggest individuals with moderate ID, even those with computational fluency, require systematic instruction in solving word problems with the unknowns in multiple positions and visual supports such as diagrams. Existing research on MSBI has supported the findings of Neef et al. that systematic instruction and visual supports are effective in teaching mathematical problem solving. However, the need exists to investigate a method for teaching students with ASD/ID a way to solve mathematical problems with unknowns in multiple
positions, an algebraic pre-skill which demonstrates generalization, and that is maintained after visual supports are faded.

In summary, more intensive supports and instruction may be needed for students with ASD/ID to overcome the barriers to solving mathematical word problems that require algebraic reasoning (i.e., with unknowns in multiple positions). To address executive functioning and metacognitive demands of problem solving, MSBI provides a student-friendly task analysis to students to promote pictorial self-instruction as a heuristic instead of a mnemonic (Spooner et al., 2017). The academic vocabulary used in mathematical problem solving may be unfamiliar to students with ASD/ID and contribute to the semantic language barrier to problem solving, and therefore may need to be explicitly taught. Constant time delay has been effective for teaching academic vocabulary to students with ASD (e.g., Browder, Root, Wood, & Allison, 2015; Knight, Spooner, Browder, Smith, & Wood, 2013; Riggs, Collins, Kleinert, & Knight, 2013).

The purpose of this study was to evaluate the effects of MSBI on algebra problem solving and explicit vocabulary instruction on identification of mathematics vocabulary related to problem solving. In addition, this study evaluated whether or not instructional supports could be faded. The following research questions were addressed:

1. What is the effect of modified schema-based instruction on the number of steps performed independently correct to solve a word problem by students with ASD/ID?

2. What is the effect of modified schema-based instruction on the number of problems solved by students with ASD/ID?

3. Are students with ASD/ID able to generalize problem solving skills, both steps completed independently correct and problems solved, when visual supports are faded?
4. What is the effect of constant time delay on the identification of mathematics vocabulary definitions by students with ASD/ID?

Method

Participants

Three middle school students with ASD/ID were recruited through teacher nomination. Participants were selected based on the following criteria: (a) educational or medical diagnosis of autism, (b) IQ at least three standard deviations below the mean, (c) participation in alternate assessment aligned with alternate achievement standards (AA-AAS), and (d) satisfactory performance on mathematical prescreening measure. The prescreening measure assessed their ability to (a) receptively and expressively identify numerals up to 10, (b) make sets of numbers 1 to 10, (c) count with one-to-one correspondence, and (d) solve one-step word problems. A participant achieved satisfactory performance on the prescreening measure if he or she completed items (a) - (c) with 100% accuracy and item (d) with no more than 25% accuracy.

Anna was a 14 year-old Caucasian female in the sixth grade with ASD and moderate ID. According to her most recent evaluation data, Anna had a cognitive scale of 53 on the Developmental Ability Scales, 2nd edition (DAS-2; Elliot, 2007), an adaptive behavior composite score of 68 (Vineland Adaptive Behavior Scales, 2nd edition; Sparrow, Cicchetti, & Balla, 2005) and a score of 74 on the Gilliam Autism Rating Scale, 2nd edition (GARS-2; Gilliam, 2005). Anna received a level four (proficient) in both language arts and math on her most recent alternate assessment. Her mathematics IEP goals included solving one-step word problems by correctly identifying the operation and ignoring excess information from the problem. Anna’s social and language skills were one of her strengths. She was able to communicate vocally in complete sentences with the interventionist.
Amanda was a 12-year-old Caucasian female in the sixth grade with ASD and moderate ID. Amanda recently transferred from another school out-of-state. According to her evaluation data, Amanda had a full scale IQ of 58 on the Wechsler Intelligence Scale for children – 4th edition (WISC-4; Wechsler, 2003), and a standard score of 66 on the Vineland-II. She had a diagnosis of Pervasive Developmental Disorder-Not Otherwise Specified (PDD-NOS) from a psychiatrist. Her current mathematics IEP goals related to solving computation and word problems with all four digits. Amanda’s transfer information did not include her alternate assessment scores. Amanda spoke in complete sentences, although she had a speech impairment.

Stephanie was a 14-year-old Caucasian female in the seventh grade with ASD and moderate ID. According to her evaluation data, Stephanie had a full scale IQ of 50 on the DAS-2, an adaptive behavior composite score of 70 according to the Vineland-2 and a diagnosis of ASD according to the GARS-2. At the end of the previous school year, she scored a level two (not proficient) on her alternate assessment in mathematics but a level four (proficient) on her alternate assessment in language arts. Her current IEP goals were related to comparing part-to-part relationships and solving one-step word problems by writing an equation and using a graphic organizer. Stephanie was able to speak in sentences composed of two to four words, although these vocalizations were often very quiet and difficult to understand.

The intervention aligned with the IEP goals and grade-level mathematics standards of all three participants. The sixth grade CCSSM standard addressed by the intervention was 6.EE.B.6 “use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number”. The seventh grade CCSSM standard addressed by the intervention was 7.EE.B.4 “Use variables to
represent quantities in real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.”

**Setting**

The study took place in a public middle school in an urban school district in the southeast United States. Each participant received all academic instruction in core content areas from a special education teacher. Intervention sessions were conducted one-on-one with each participant daily in an alcove at the end of a hallway near the participant’s classrooms approximately four days per week. A doctoral candidate (first author) in special education implemented all sessions. The interventionist was a board certified behavior analyst (BCBA) and held a valid teaching license in both special education and general education mathematics.

**Materials**

Materials included vocabulary cards, worksheets, a pencil, and an iPad displaying a task analysis. Authors chose five vocabulary words that were necessary for both conceptual understanding of problem solving as well as understanding instruction. Each term was presented on index cards and paired with a symbol. The participants were orally given a definition and asked to identify the symbol/term. The following terms and definitions were used in each vocabulary session: add (combine), subtract (take away), equal (same as), equation (statement says things are equal) and label (name of group).

Participants were given four worksheets in each session that had word problems related to a theme (e.g., school dance, working in a restaurant, video game store). The themes of the word problems represented high-interest topics and scenarios students may encounter in current and future environments. The word problems were written to be free of cultural or gender bias and were evaluated by an elementary mathematics expert for content validity. Each page of the
worksheet displayed one group word problem (see Figure 1). In group problems, two small
groups of different things are combined to make one large group, demonstrating a part/whole
relationship (Carpenter & Moser, 1984). Of the four problems presented to the participants in
each session, two of the problems required the student to solve for the whole when given two
parts (missing-whole or MW), and two problems required the student to solve for one of the
parts when given the whole and one of the parts (missing-part or MP). An example of a MW
problem is “Ethan put dishes on the table. Ethan put 2 plates on the table. Ethan put 2 cups on the
table. How many dishes did Ethan put on the table?” An example of a MP problem is “Ethan
helped set the table. Ethan put 8 dishes on the table. He put 3 plates and some cups on the table.
How many cups did he put on the table?” The order of the problems was randomized and a novel
worksheet was used in each session. Generalization worksheets contained similar word problems
at the top of each page but did not have the visual supports (equation and answer sentence).

An iPad with the SMARTnotebook application was used to display a task analysis of 10
steps for solving the problems hereto referred to as the self-instruction checklist. The self-
instruction checklist displayed the text each of the ten steps as well as a corresponding picture as
a visual support. There was space on the self-instruction checklist for participants to check off
each step as it was completed. See Table 1 for a list of the steps with corresponding expected
student response. The self-instruction checklist had embedded verbal and specific verbal
prompts. If a participant touched the number for a step, the verbal prompt was activated (e.g.,
“Step one says read the problem). If a participant touched the “?” to the right of a step, the
specific verbal prompt was activated (e.g., “Ask for help reading the problem”).

**Dependent Variables**
Four dependent variables were measured throughout the course of this study. The primary dependent variable was steps of problem solving, measured by the total number of steps of a task analysis completed independently correct (see Table 1) across four problems. The second dependent variable was the number of problems solved, measured by the number of word problems that received points for critical steps (3, 5, 6, and 10) in each session. All four critical steps had to be completed independently correct in order for the problem to be considered “solved correctly” (Test & Spooner, 1996; Weng & Bouck, 2014). The result of performing these critical steps was an accurate equation and answer sentence. The third dependent variable was generalization of problem solving when visual supports were faded, measured by the number of steps completed independently correct and number of problems solved correctly. Generalization probes were administered once during baseline and every four sessions in intervention by removing visual supports (equation and answer sentence). Finally, the fourth dependent variable was identification of mathematics vocabulary, measured by the number of correct identifications of mathematics vocabulary words when given definitions (e.g., equal/same as). Responses were scored as independent correct if the participant touched or said the correct answer with 4 s without a prompt from the interventionist. Each of the five vocabulary words were presented twice in random order for a total of 10 points available in each session.

**Experimental Design**

A multiple probe across participants design was used (Kratochwill et al., 2010). Phase change decisions were based on student performance data on the primary dependent variable, word problem solving. Participants in baseline were probed a minimum of every eight sessions to facilitate verification and prediction, necessary components of visual analysis of single-case designs (Kratochwill et al., 2010). A generalization probe was administered once in baseline and
every four sessions during intervention to provide data on generalization during the learning process, rather than merely post-instruction as is typical in single-case designs. The criterion for mastery of word problem solving was performing nine of the 10 steps correctly, including all critical steps, for three out of four problems for two consecutive sessions. The criterion for mastery of mathematics vocabulary was correct identification of all six terms for three consecutive sessions.

**Interobserver Agreement and Procedural Fidelity**

Interobserver agreement (IOA) and procedural fidelity data was collected across all experimental conditions. IOA was evaluated using an item-by-item method and calculated by dividing the total greed times the total agreed and disagreed items and multiplied by 100. The second observer collected IOA data during baseline for 33% of baseline sessions for Anna (2 out of 6 sessions), 33% of baseline sessions for Amanda (3 out of 9 sessions), and 44% of baseline sessions for Stephanie (4 out of 9 sessions). The agreement was 100% for all three participants during baseline. The second observer collected IOA during intervention for 45% of intervention sessions for Anna (5 out of 11 sessions), 55% of intervention sessions for Amanda (5 out of 9 sessions), and 30% of intervention sessions for Stephanie (3 out of 10 sessions). For Anna, the mean agreement in intervention was 100% for mathematics vocabulary, 95.4% (range 86 to 100) and for word problem solving. For Amanda, the mean agreement in intervention was 100% for mathematics vocabulary and 98.8% (range 97-100) for word problem solving. For Stephanie, the mean agreement in intervention was 100% for mathematics vocabulary and 97% (range 96 to 100) for word problem solving.

The second observer used a procedural fidelity checklist to document the degree to which the intervention was implemented consistently as designed. To calculate the procedural fidelity,
the number of elements correctly implemented was divided by the total number of procedural elements and then multiplied by 100. The second observer collected procedural fidelity data during the same sessions that IOA was collected. The mean procedural fidelity in intervention was 99% for Anna (range 97-100), 100% for Amanda, and 98% for Stephanie (range 96-100).

Procedures

**Baseline.** Classroom mathematics instruction included small-group and individual arrangements. The teacher followed the North Carolina Extensions to the Common Core and frequently used materials from Unique Learning Systems (N2y, 2014), as prescribed by the district. Baseline and intervention sessions began with the mathematics vocabulary task, followed by solving four word problems. In baseline mathematics vocabulary sessions, the interventionist presented the instructional cue “touch the word that means _____” and provided the definition, such as “combine groups” for the word “add.” The vocabulary terms were displayed in an array of three. Each term was paired with a symbol (e.g., plus sign for the word add). Each vocabulary word was presented twice.

Next, the participant was given a worksheet, pencil with eraser, manipulatives, and an iPad 3 displaying the student self-instruction checklist. The interventionist began each trial by giving the instructional cue “Show me how to solve these word problems.” The interventionist read the word problem aloud if asked by the participant. Praise for on-task behavior was given but no error correction or reinforcement for correct answers was provided. This procedure continued until the participant attempted all four problems.

**Intervention.** Each intervention session began with the mathematics vocabulary task until students reached mastery. In the first trial, the interventionist used 0-s constant time delay to systematically teach the vocabulary definitions to the participant. To model in the 0-s trials,
the interventionist read the vocabulary word and touched the correct answer, then asked the student to do the same. Each vocabulary word definition was taught during the 0-s round one time. The definitions were shuffled between each trial and displayed in a different array of three choices. In the next two trials, 4-s will were inserted between the presentation of the math vocabulary word and definition and the model prompt. If the participant was unable to make an independent correct response, the interventionist provided a model-prompt, as was provided in the 0-s trial, and then asked the student to touch the correct definition. Two 4-s trials were conducted each session. Data were only taken during 4-s delay rounds.

To teach students to solve the word problems, the interventionist provided 2 days of strategy instruction to the participant following the steps on the student self-instruction sheet. During these training days, the interventionist modeled how to solve the problems with active student participation (e.g., “My turn. I found the label of the big group. Your turn. Can you circle the label of the big group?”). Lessons followed a model-lead-test format. The interventionist modeled use of the electronic student self-instruction checklist with embedded system of least prompts. Each session during this training period lasted approximately 10 to 15 min. The participant was taught to follow the student self-instruction sheet and check off each step as it was completed. See Table 1 for the expected participant responses for each step.

Following 2 days of explicit instruction, the interventionist used least intrusive prompting if the participant failed to make a response. The least intrusive prompting hierarchy consisted of three levels, including a verbal prompt, specific verbal prompt, and a model prompt. All verbal and specific verbal prompts were embedded within the student self-instruction sheet on the iPad as previously described. The interventionist provided the model prompt. The participant was given 5-s before each prompt. If the participant made an error, the interventionist went directly to
a model prompt and required the student to repeat the behavior. The interventionist used behavior specific praise after each correct response (prompted or unprompted). As participants demonstrated proficiency on steps of the task analysis, behavior specific praise was faded.

The interventionist took data during instruction on the number of steps the participant was able to complete independently in a type of multiple opportunity probe (Alexander, Smith, Mataras, Shepley, & Ayres, 2015), in that the participant was given the opportunity to perform a step without help for purposes of data collection and then given prompting as needed to complete the step to set up the next response. Due to the chained nature of solving a word problem, each step was dependent on the correct execution of the one before. Therefore the interventionist had to either prompt or set up each step to determine if later responses in the chain had been mastered.

**Generalization.** During the generalization probes, participants were given worksheets that were identical to those used in baseline and intervention, except they did not contain visual supports (e.g., equation and answer sentence diagrams). The interventionist did not provide prompting or feedback beyond the instructional cue “Show me how to solve these problems” and read-alouds when requested. The procedures for reinforcement of on-task behavior and task completion were continued.

**Results**

Figure 2 shows the effect of MSBI on steps of mathematical problem solving. The graph shows the number of steps of the task analysis performed independently correct across four word problems (ten steps per problem for a total of forty points). During baseline all participants had a stable pattern of responding. During intervention all three participants showed a change in level or an increasing trend, with no overlapping data with baseline performance. Visual analysis of
the graph shows a functional relation between modified SBI and the number of steps of solving a word problem performed independently correct.

Anna received an average of 4.4 points across the four problems during the five baseline sessions (range 3-7). After two sessions of modeling, she accelerated to 28 points across four problems in the first intervention session and maintained an increased level and ascending trend. Anna reached mastery after nine intervention sessions. She received an average of 34.5 points across the four problems during nine intervention sessions (range 28 to 39). Amanda received an average of 7.14 points across the four problems during seven baseline sessions (range 7-8). After two sessions of modeling, she jumped to 31 points across the four problems and maintained an ascending trend. Amanda reached mastery after seven intervention sessions. She received an average of 35.7 points across the four problems during seven intervention sessions (range 31-40). Stephanie received an average of 2.75 points across the four problems during eight baseline sessions (range 2-4). After two sessions of modeling, she jumped to 24 points across the four problems for an increase in level and continued with an overall positive trend. Stephanie reached mastery after eight intervention sessions. She received an average of 31 points across eight intervention sessions.

Figure 3 shows the effects of MSBI on the total number of word problems solved. This measure was derived from the number of problems in which the participant completed steps 3 (label), 5 (fill in equation), 6 (+ or -) and 10 (write answer). Each participant had four opportunities to solve a problem in each session. Participants were unable to solve any problems in baseline sessions. Following introduction to intervention, all participants were able to master solving problems. Stephanie has two overlapping data points with baseline, indicating it took her until the third session to independently complete the critical steps for solving the problem.
Figures 2 and 3 demonstrate the generalization performance of each participant for steps of mathematical problem solving and problems solved. Each participant was given one generalization probe during baseline and two during intervention. Figure 2 shows the number of steps of the task analysis performed independently correct across four word problems (ten steps per problem) during generalization probes, when the worksheets did not contain visual supports and no prompting or feedback was provided. During baseline, two participants demonstrated similar proficiency on the generalization problem and one participant received fewer points than other baseline probes, but no participants were able to solve any problems during baseline generalization probes. During intervention participants demonstrated an increased level of responding over baseline, however all generalization probes during intervention demonstrated a dip in performance on both steps of problem solving and problems solved.

Figure 4 shows the effects of constant time delay on the identification of mathematics vocabulary definitions. The graph shows the number of correct identifications of mathematics vocabulary terms performed by each participant. Following a stable baseline, all three participants showed a change in level or an increasing trend, with no overlapping data with baseline performance after introduction to intervention. Visual analysis of the graph indicates a functional relation between constant time delay and identification of mathematics definitions.

During baseline probes, Anna correctly identified three mathematics symbols on both trials when given the definition for a total of six points in each baseline session. She reached mastery in three trials. Amanda had similar performance to Anna during baseline, as she was able to consistently identify three symbols when given the definition in both trials in each of her seven baseline sessions. She reached mastery in four trials with an average rate of correct responding of 9.75 (range 9-10). Stephanie was able to consistently identify two symbols when
given the definition during both trials in each of her eight baseline sessions. She reached mastery in five trials, with an average rate of correct responding of 9.4 (range 8-10).

Social validity data were gathered from participants through a questionnaire with statements related to the procedures and outcomes of the intervention before and after intervention. Each participant completed the same social validity questionnaire pre and post intervention. Anna responded “Yes” to all questions both pre/post survey. Amanda increased from 5/7 to 7/7 “yes” statements and Stephanie from 4/7 to 7/7. Amanda’s new “yes” statements were that she was good at problem solving and good at math. Stephanie now agreed that she was good at problem solving and liked using manipulatives for math.

Discussion

The purpose of this investigation was to evaluate the effects of modified schema based instruction on the mathematical problem solving skills of middle school students with ASD/ID using a multiple probe across participants design. A functional relation between modified SBI and mathematical word problem solving, as well as constant time delay and identification of mathematics vocabulary, exists with three demonstrations of intervention effects at three different points in time. In addition, participants were able to correctly solve the word problems and had some success with generalizing problem solving when visual supports were faded.

Prior studies have shown that while students with ASD and ID can learn to solve mathematical word problems that require algebraic reasoning, it is likely that students with ASD/ID will need more intensive supports and instruction to be independent problem solvers (Neef et al., 2003; Rockwell et al., 2011). MSBI contains all essential components of traditional SBI as outlined by Jitendra et al. (2015) but adds modifications, or enhancements, that address
the barriers students with ASD/ID face in solving mathematics word problems, including semantic language, executive functioning, and metacognition through evidence-based practices.

Communication and language is one barrier to mathematical problem solving for students with ASD/ID that has not yet been directly addressed in empirical research. Van de Walle et al. (2016) emphasize the importance mathematical language to both receptively understanding tasks as well as expressively communicating about mathematics. In the current study, the language barrier was addressed in two ways: explicit instruction of vocabulary terms and a systematic process for modeling conceptual understanding. Vocabulary terms were chosen because they played a key role in participant understanding of strategy instruction. These results support similar findings that used constant time delay to teach academic vocabulary words that were required within tasks (e.g., Browder, et al., 2015; Knight et al., 2013; Riggs et al., 2013). This is the first application of constant time delay to teach mathematics vocabulary concepts.

The second strategy for overcoming the semantic language barrier to problem solving was providing a systematic process for modeling conceptual understanding of “what is happening” in the word problems. Key steps of the task analysis assisted students in systematically answering the question “What is happening?” (i.e., steps 2, 3, and 6). The use of visual supports to represent the structure of the problem type and to organize information from the problem is a key component of SBI (Jitendra et al., 2015). Knight and Sartini (2015) found visual supports in the form of graphic organizers, visual diagrams, picture symbols, and visuals of key phrases to be an evidence-based practice for teaching text-based comprehension to students with ASD. Although none of the studies in the Knight and Sartini review applied visual supports to a mathematical learning task, their rationale for the effectiveness of visual supports remains relevant, as mathematical word problem solving requires text-based comprehension.
The task analysis supported both executive function and metacognition, as there is cyclical relationship between the two skills in problem solving as students choose a strategy, monitor progress, and make changes as necessary. Executive functioning is required to sustain attention to the task and complete each step correctly. A written task analysis led participants through steps to choose a strategy by directing the order of choices and also ensured that the steps were carried out logically, replacing the mnemonics used in traditional SBI (Powell, 2011). The task analysis allowed for students to attend to each step in the chain and self-monitor progress. The results of this study are similar to those of previous studies that a task analysis to teach and measure mathematics problem solving (Root, Browder, et al., 2017; Saunders, 2014) and extend the field’s knowledge on how to teach problem solving that requires algebraic reasoning as well as fading instructional supports.

There is limited discussion in the literature regarding fading visual supports (Knight & Sartini, 2015). The results of this study support the use of visual supports with students with ASD/ID, however the question remains about whether they can, or should, be faded. All three participants had difficulty solving the word problems on generalization probes with no visual supports. This raises the issue of when and how to fade supports and highlights the need for explicit generalization training (Baer, Wolf, & Risley, 1968).

Limitations and Future Research

This study has several limitations that should be considered in future research, including the format and content of word problems and limited generalization. The word problems all followed a predictable and structured format to compensate for participants’ limited text comprehension. Despite each session using novel worksheets with new word problems, they followed the same format depending on the problem type with the big group found in the first
sentence, small groups found in second/third sentences, and no irrelevant information. Students were only exposed to problems that contained quantities of one through nine. Future research should consider methods for teaching students to solve problems that include extraneous information and quantities above ten.

A second limitation of the current study was the limited generalization in missing part problems when visual supports were removed. Although data for the two problem types are collapsed Figures 2 and 3, researcher anecdotal notes indicate performance was lower on missing part problems on generalization probes when they did not have the visual supports. This suggests supports may need to be faded more gradually or that students may need ongoing access to a visual referent when completing algebra problems. Future research should include a plan for generalization using one of the nine methods outlined by Stokes and Baer (1977). The train and hope method, which has been cited as both the most common and least effective (Stokes & Baer, 1977), did not prove successful for the participants in this study for the missing part problem type. Future research should consider the use of sequential modification when fading the visual supports use modified SBI. This planned fading of supports would allow participants to attend to one component at a time and future research may prove this a successful approach. An additional strategy to promote generalization would be teaching each problem type separately to mastery and then providing explicit discrimination training.

**Recommendations for Practice**

The findings of this study provide practical implications for mathematics instruction for overcoming barriers to problem solving faced by students with ASD/ID. First, practitioners should consider the vocabulary demands of problem solving tasks. In order for students to develop conceptual understanding and understand the procedural directions provided by their
instructor, they need fluency in mathematical vocabulary (Van de Walle et al., 2016). Second, providing students with accessible task analyses can address executive functioning and metacognitive barriers to problem solving, thereby improving self-monitoring and independence. Breaking down the steps of solving a word problem into a task analysis that is provided to the student in an accessible format, such as pictures paired with words and using technology to provide read alouds, is one way that practitioners can encourage self-instruction. Making the steps of the problem available to the student increases independence, as the student can self-manage behaviors and even self-correct when a mistake has been made.

References


Table 1.  
*Expected Responses for Each Step of Self-Instruction Sheet*

<table>
<thead>
<tr>
<th>Step</th>
<th>Expected Student Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Read the problem</td>
<td>Read problem or ask instructor to read problem</td>
</tr>
<tr>
<td>2. Circle the groups</td>
<td>Circle the whole (“big group”) and parts (“small groups”) in problem</td>
</tr>
<tr>
<td>3. Label equation</td>
<td>Write letter for whole above third box in equation and letters for parts above first and second boxes in equation</td>
</tr>
<tr>
<td>4. Circle the numbers</td>
<td>Circle numbers in word problem</td>
</tr>
<tr>
<td>5. Fill-in equation</td>
<td>Write numbers from problem in corresponding boxes in equation and an “x” in the box in the equation representing the unknown amount</td>
</tr>
<tr>
<td>6. + or -</td>
<td>Write addition symbol in the circle in equation</td>
</tr>
<tr>
<td>7. Use my rule</td>
<td>State rule for group problem type (“small group plus small group equals big group”) and/or use hand motions</td>
</tr>
<tr>
<td>8. Make sets</td>
<td>Use manipulatives to create a set under each of the small groups and (missing whole) or make a set to represent the known small group (missing part)</td>
</tr>
<tr>
<td>9. Solve</td>
<td>Combine two sets and count total (missing whole) or count up with manipulatives under unknown small group to known whole amount (missing part)</td>
</tr>
<tr>
<td>10. Write answer</td>
<td>Write number of manipulatives combined from two small groups (missing whole) or amount counted up (missing part) in the answer sentence at the bottom of the page</td>
</tr>
</tbody>
</table>
Eli cleaned up books in the classroom library.
Eli put 9 books on the shelf.
He put 4 picture books on the shelf and some chapter books.
How many chapter books did he put on the shelf?

\[ \square \bigcirc \square = \square \]

\[ x = \square \]

*Figure 1.* Example missing-part worksheet.
Figure 2. Graph of number steps of task analysis completed independently correct
Figure 3. Graph of total problems solved in each session
Figure 4. Graph of independent correct identifications of mathematics vocabulary terms when given the symbol and definition.