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The Cluster Structure of Oxygen Isotopes

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THE CLUSTER STRUCTURE OF OXYGEN ISOTOPES

By

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-Eric
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ABSTRACT

The \( \alpha \)-cluster structure of two oxygen isotopes, \(^{17}\text{O}\) and \(^{18}\text{O}\), was studied using two experimental techniques. The first technique measured resonance elastic scattering of \( \alpha \) particles and the second used the direct \( \alpha \)-transfer reactions \((^6\text{Li},d)\) and \((^7\text{Li},t)\) to determine resonance properties. Motivation for this study was two-fold.

First, the \( \alpha \)-cluster structure of \( N \neq Z \) nuclei is poorly known and is a subject of intense theoretical discussion \([1,2,3,4]\). Historically, the \( \alpha \)-particle model of the atomic nucleus was the leading model of nuclear structure. As it became clear that nuclei consist of protons and neutrons this model was replaced. The Pauli principle forbids nucleons from different \( \alpha \) particles to be in the same state since the total wave function of an atomic nucleus must be antisymmetric. However, \( \alpha \) clusters have been used to explain various nuclear effects including quasi-rotational bands of states with large \( \alpha \)-particle widths which were observed in light \( 4N \) nuclei, \(^8\text{Be}\), \(^{12}\text{C}\), \(^{16}\text{O}\) and so on. Included here is a report on the observation of the \( \alpha \)-cluster structure in the \( N \neq Z \) nucleus \(^{18}\text{O}\). We measured the \( \alpha \)-cluster structure of \(^{18}\text{O}\) using the Thick Target Inverse Kinematics (TTIK) technique \([5]\). We found that \(^{18}\text{O}\) has a very elaborate \( \alpha \)-cluster structure, including two unusual states with \( \alpha \) widths larger than the single particle limit (the Wigner limit \([6]\)). A comparison of the observed \(^{18}\text{O} \) \( \alpha \)-cluster structure with the predictions of modern theoretical approaches is given. The peculiar nature of the two very broad states is discussed.

Second, the \( \alpha \)-cluster structure of near \( \alpha \)-threshold excited states in \(^{17}\text{O}\) and \(^{18}\text{O}\) plays a crucial role in the field of nuclear astrophysics as it determines the rates of the \(^{13}\text{C}(\alpha,n)\) and \(^{14}\text{C}(\alpha,\gamma)\) reactions. These reactions are thought to play important roles in stellar evolution of Asymptotic Giant Branch (AGB) stars. Unfortunately, direct measurement of these reactions is currently impractical. This has led to the development of indirect methods to determine reaction rates. One such method is the Asymptotic Normalization Coefficient (ANC) technique. We determined the \( \alpha \)-cluster structure of several near \( \alpha \)-threshold states in \(^{17}\text{O}\) and \(^{18}\text{O}\) using this ANC technique. Using this \( \alpha \)-cluster information we were able to determine the \(^{13}\text{C}(\alpha,n)\) reaction rate and reduce its uncertainty from \( \approx 300\% \) to \( 25\% \). Also we were able to calculate the \(^{14}\text{C}(\alpha,\gamma)\) reaction rate. Accurately knowing the \(^{13}\text{C}(\alpha,n)\) reaction rate is crucial to the modeling of AGB stars, and a reliable determination of the \(^{14}\text{C}(\alpha,\gamma)\) reaction rate is needed in order to help understand the unexplained abundance of \(^{19}\text{F}\) in the universe.
CHAPTER 1

INTRODUCTION

In the early 1900’s very little was known about the internal structure of the newly discovered nucleus. Early models used combinations of known radiative particles to explain the different masses and charges of the observed nuclei. These particles included protons, electrons and α’s. It was suggested in [7] that α particles and protons should be the main constituents of the atomic nucleus. The needed number of electrons was then added to the nucleus to get the proper net charge. This α-particle model was plausible at the time since α particles were known to be emitted by nuclei. It made sense then for those same α particles to exist inside the emitting nucleus. Then in 1932 the neutron was discovered by Chadwick and these theories were replaced in favor of models using combinations of protons and neutrons, for instance the shell model [8, 9].

The shell model assumes that each nucleon is moving in a mean field potential which is the total average potential due to all of the other nucleons inside the nucleus. The resulting orbits form energy shells similar to what is seen with electrons orbiting atoms [8, 9]. The nucleons occupy the different shells according to the Pauli exclusion principle which requires that each nucleon have a unique set of quantum numbers. Of course, a detailed description of the atomic nuclear structure requires the inclusion of residual interactions. These interactions should account for all of the phenomena which cannot be described by the independent motion of nucleons in a mean field. The shell model provides a powerful framework which allows the explaintion of the properties of nuclei, including binding energies, structure, transitions, and magnetic moments to name a few [10]. However, there are many features in atomic nuclear structure which cannot be reproduced, such as the α-cluster rotational bands in certain α-conjugate nuclei (nuclei with $A = 4N$ and $N_{\text{proton}} = N_{\text{neutron}}$) [11]. One may argue that this is due to the inherently single particle nature of the shell
model. The shell model does not have the optimum basis for dealing with collective, multi-nucleon excitations, such as $\alpha$-clusters. An $\alpha$-cluster is a spacial correlation of two neutrons and two protons into a spin zero $\alpha$-particle-like structure within the nucleus. Experimental evidence that such correlations exist was very strong in the early days of nuclear physics [12, 13, 14, 15]. Therefore, $\alpha$-cluster models of the atomic nucleus were introduced.

These $\alpha$-cluster models suggest that protons and neutrons inside the nucleus form into one or more $\alpha$-clusters with the leftovers either making up a core or orbiting as valence nucleons. The original $\alpha$-cluster model (ACM) was introduced by Margenau [16], with further development being performed by Brink and Bloch [17]. The principle idea of the ACM is to form quartets of particles out of pairs of protons and neutrons coupled to a total angular momentum of zero. The wave functions of these quartets were then used to reproduce the properties of nuclei [17, 18, 19, 20]. The ACM is particularly good at reproducing the properties of states within the $\alpha$-conjugate nuclei, but it is limited because the internal excitations of the clusters are ignored. Improvements can be made in the framework of the Generator Coordinate Method (GCM) [21, 22, 23, 24, 25, 26, 27, 28, 29, 30], which takes the internal cluster excitations into account and also allows for reactions between asymptotic clusters to be studied [31, 32, 33, 34, 35]. Further improvements to the $\alpha$-cluster model have been made through the Antisymmetrized Molecular Dynamics (AMD) approach [36]. The AMD approach has the advantage that there are no assumptions being made about the $\alpha$-cluster. The AMD starts from the nucleon-nucleon interactions rather than a preformed $\alpha$-cluster, yet still predicts a clustered nuclear density [37, 1].

In the past, both theoretical and experimental efforts in studies of the $\alpha$-cluster structure were concentrated on self-conjugate $4N$ nuclei ($^8\text{Be}$, $^{12}\text{C}$, $^{16}\text{O}$, ...). There is no reason, however, that the $\alpha$-cluster degree of freedom should only exist in self-conjugate nuclei. In fact, recent calculations [38, 1, 39, 40] and a few experimental studies [41, 42, 43, 44] suggest that $\alpha$-clusterization may be an important feature in the structure of $N \neq Z$ nuclei. Moreover, new and exotic phenomena such as two(and more)-center nuclear molecules may be expected with valence nucleons orbiting these centers in a vague analogy to electrons in molecules.

The present work aims at exploring the $\alpha$-cluster degree of freedom in non-self-conjugate nuclei ($N_{\text{proton}} \neq N_{\text{neutron}}$). In order to thoroughly test the $\alpha$-cluster models more experimental information is needed on the clusterization of excited states in these nuclei.
Chapter 2 is a discussion on the measurement of elastically scattered $\alpha$ particles using the Thick Target Inverse Kinematics (TTIK) technique [5] and a detailed analysis of the $\alpha$-cluster structure of $^{18}$O. This will include a section on the $R$-Matrix theory which was used extensively in the analysis [45].

Due to the abundance of helium in stars many nuclear reactions between $\alpha$ particles and other light nuclei play an important role in nucleosynthesis. In particular, the rate at which carbon isotopes capture $\alpha$ particles is known to have a significant influence on stellar evolution. For the relevant stellar energies, the $\alpha$-capture rate is dominated by the near threshold states in oxygen, but higher-lying resonances can also play a significant role. Chapter 3 discusses the use of an indirect technique to accurately extract the parameters of near threshold resonances which can then be used to calculate the cross section of nuclear reactions which are relevant for nuclear astrophysical processes. In particular the rates of the $^{13}$C($\alpha$,n) and $^{14}$C($\alpha$,\gamma) reactions are discussed, along with the influences they have on the nucleosynthesis processes occurring inside of Asymptotic Giant Branch stars.

A summary and closing remarks are given in Chapter 4.
CHAPTER 2

THE $\alpha$-CLUSTER STRUCTURE OF $^{18}$O

2.1 Motivation

The $\alpha$-cluster degree of freedom in atomic nuclei has been extensively studied over the years. The nuclei with the most well known $\alpha$-cluster structure are $^{16}$O and $^{20}$Ne. These nuclei have well known twin $\alpha$-cluster, inversion doublet, rotational bands, as shown in Figure 2.1 [46, 47, 48, 49, 2]. The similarities of the states associated with these bands cannot be fully explained by the shell model since the different parity implies that different shells are being occupied and therefore that the states should have different properties. Additionally, the members of these bands have uncharacteristically large reduced $\alpha$ widths, comparable to the Wigner limit, which is the maximum possible reduced width that a nuclear resonance can have if it has a pure $\alpha+\text{core}$ structure [6]. Much work has been done with the $4N(N = Z)$ nuclei, but recently the $\alpha$-cluster structure of $N \neq Z$ nuclei has been getting some attention.

The picture of the $\alpha$-cluster structure in $N \neq Z$ nuclei is much less complete. This is probably due to experimental difficulties, but also due to the fact that in general the level density is higher in $N \neq Z$ nuclei and it gets much harder to identify clear $\alpha$-cluster signatures in the level structure. Currently, both nuclear experimentalists and theorists are exploring the $\alpha$-cluster structure of $N \neq Z$ nuclei. The development of powerful theoretical tools, such as the Greens Function Monte Carlo (GFMC) method, bring new dimensions into the problem of $\alpha$-clustering. It is now possible to investigate how $\alpha$-clustering emerges from the bare nucleon-nucleon interaction without a need to introduce $\alpha$-clusters artificially. For example, GFMC calculations for $^{8}$Be clearly indicate that even the ground state of $^{8}$Be has a well developed $\alpha-\alpha$ structure [2]. Unfortunately, the GFMC approach is limited to only the lightest nuclei, $A < 12$. Beyond that, calculations become intractable with present computing power. Another approach is Antisymmetrized Molecular Dynamics
Figure 2.1: Inversion doublet \(\alpha\)-cluster rotational bands for both \(^{16}\text{O}\) and \(^{20}\text{Ne}\). Inversion doublets are groups of resonances in which the positive and negative parity states are of a similar nature, with a slight difference in excitation energy.

(AMD). While it is less rigorous than the GFMC (for example the 3-body force is introduced as a \(\delta\)-interaction and is treated as a parameter), it has the advantage that it can be applied for heavier nuclei. Since \(\alpha\)-clusters are not introduced by hand, but rather are a result of the calculation, these methods allow treatment of the \(\alpha\)-cluster and nucleon degrees of freedom on an equal footing, which in turn provides an insight into the interplay between \(\alpha\)-cluster states and single nucleon states. It is clear that \(N \neq Z\) nuclei, with their extra “valence” nucleons, are a valuable proving grounds for the predictions of such models.

The study of \(\alpha\)-cluster states in neutron rich nuclei has already led to several interesting discoveries and ideas. For example, the doubling of \(\alpha\)-cluster states in \(^{22}\text{Ne}\) was experimentally discovered [42] and then theoretically matched using a microscopic \(\alpha\)-cluster model [4]. Also, the \(\alpha - \alpha\) structure of Be and B is well established, in particular the low-lying states in \(^{9}\text{Be}\) have been reproduced using an \(\alpha + \alpha + n\) cluster model [50, 51]. Furthermore, the structure of \(^{10}\text{Be}\) has been investigated using \(\alpha + \alpha + n + n\) [52] and \(\alpha : 2n : \alpha\) models [53].
Another interesting nucleus which is currently the topic of much discussion is $^{18}$O.

In principle one may consider $^{18}$O as a good "shell model" nucleus. The shell model is capable of describing many of the low-lying states in $^{18}$O using the model space of two neutrons in the $sd$-orbitals above the $N = Z = 8$ shell closure. However, $\alpha$-cluster states are known to exist in the same excitation region [54, 40]. The $\alpha$-cluster structure of $^{18}$O has been heavily investigated both theoretically and experimentally [40, 39, 55, 38, 43, 44, 56, 57, 58, 59]. Predictions for strong $\alpha$-cluster inversion doublet bands in $^{18}$O have been made in [38] and [1]. Using the Generator Coordinate Method (GCM), Descouvemont predicts that there should be one positive and one negative parity $^{14}C_{g.s.}+\alpha$ rotational band. He also suggests that rotational bands built on $^{14}C$ excited states may exist [38]. Strong $^{14}C_{g.s.}+\alpha$ rotational bands are also a feature of the calculations made by Furutachi et al. [1]. Using AMD+GCM Furutachi predicts that in addition to the above mentioned strong $^{14}C_{g.s.}+\alpha$ rotational bands of both parities, the $\alpha$ strength should be distributed among several weaker states of various spin-parity. They relate this splitting to proton excitation of the core, but state that one of the states of a specific spin-parity should have a dominant $\alpha$ width. The work by Furutachi is especially interesting because the $\alpha$-clusters are formed starting from nucleon-nucleon interactions [1].

The experimental tool that has been most successful in studying $\alpha$-cluster states is the resonance elastic scattering of $\alpha$ particles. States with strong $^{14}C_{g.s.}+\alpha$ nature in $^{18}$O should be apparent in the excitation function of $^{14}C+\alpha$ elastic scattering (an excitation function is the dependence of the differential cross section on the excitation energy of a nucleus). The first attempt to measure this excitation function dates back to 1958 [59, 60]. The analysis of this data was never completed due to the low quality of the experimental data and a very high degree of difficulty. The extra nucleons (compared to $^{16}$O) result in an increase of the level density and a decrease of the nucleon decay threshold. Both of these factors are unfavorable towards the observation of $\alpha$-cluster states. Nonetheless, measurement of the $\alpha$-cluster structure of $N \neq Z$ nuclei can lead to the discovery of new and interesting features, so it should be pursued. Recently, Goldberg et al. [41] reanalyzed part of the data taken by Morgan [59]. While the quality of the data and the fit did not allow for the evaluation of the detailed properties of the states, it was shown that $^{18}$O has a very elaborate $\alpha$-cluster structure [41]. In order to test the predictions of [1], [38], and others, a measurement of the $\alpha$-cluster states in $^{18}$O was performed through the elastic scattering of $^{14}$C on $\alpha$ particles.
In this work we are interested in studying the details of the $^{18}$O $\alpha$-cluster structure.

A detailed multi-level, multi-channel $R$-Matrix analysis of the $^{14}$C+$\alpha$ excitation function was performed. Analysis is complicated by the presence of two open neutron decay channels. The next section describes the $R$-Matrix theory that was used for the analysis of the experimental data.

### 2.2 $R$-Matrix Theory

The $R$-Matrix theory, as laid out by Lane and Thomas [45], provides an effective framework for the study of resonance reactions. It allows for the extraction of parameters of the compound nucleus resonances populated in the resonance reaction $a + A \rightarrow C^* \rightarrow b + B$ from the experimental excitation functions and angular distributions. This approach is one of the oldest and most developed theoretical methods which in its simplest form reduces to the Breit-Wigner expression for a single resonance [61]

$$
\sigma_{if} = \frac{\pi(2J + 1)}{k^2(2S_i + 1)(2S_f + 1)} \frac{\Gamma_i \Gamma_f}{(E_i - E_c)^2 + \Gamma_i^2/4}
$$

(2.1)

where $\Gamma_i$ and $\Gamma_f$ are the partial resonance widths of the incoming and outgoing channels, respectively, $\Gamma$ is the total width of the resonance, $E_i$ is the c.m. energy, $E_c$ is the resonance energy (in c.m.), $k$ is the wave number, $S_i$ and $S_f$ the spins of the incoming and outgoing channels, and $J$ is the spin of the resonance. A full derivation of the $R$-Matrix theory is beyond the scope of this work, but there will be a brief description in this section. See [45] for further details.

Consider a system of $C$ nucleons which has two solutions, $\Psi_1$ and $\Psi_2$ such that

$$
H \Psi_1 = E_1 \Psi_1,
$$

(2.2)

and

$$
H \Psi_2 = E_2 \Psi_2
$$

(2.3)

where $H = H_0 + H_{Rel} + H_a + H_A$ is the Hamiltonian operator for the $a + A$ channel, with

$$
H_0 = -\frac{\hbar^2}{2(m_a + m_A)} \nabla_R^2
$$

(2.4)

describing the centroid motion of $a + A$ where $m_a$ and $m_A$ are the masses of particle $a$ and $A$,

$$
H_{Rel} = -\frac{\hbar^2}{2\mu_{aA}} \nabla_{r_{aA}}^2 + V_{Rel}(r_{aA})
$$

(2.5)
describing the relative motion of \( a \) and \( A \) with \( \mu_{aA} = \frac{m_am_A}{m_a+m_A} \), \( r_{aA} \) as the distance between \( a \) and \( A \), and \( V_{\text{Rel}} \) as the relative potential, and

\[
H_a = (T_{\text{internal}})_a + (V_{\text{internal}})_a \tag{2.6}
\]

and

\[
H_A = (T_{\text{internal}})_A + (V_{\text{internal}})_A \tag{2.7}
\]

describing the internal state of \( a \) and \( A \) nuclei, respectively. Likewise the total wave function \( \Psi \) for the same given channel can be taken as

\[
\Psi = \Phi(R)\chi(r_{aA})\psi_a(r_a)\psi_A(r_A) \tag{2.8}
\]

where the products in Eq. 2.8 describe the centroid motion (\( \Phi \)), the relative motion of \( a \) and \( A \) (\( \chi \)), and the internal states of \( a \) (\( \psi_a \)) and \( A \) (\( \psi_A \)), respectively. Here \( R \) is the radial vector of the c.m., \( r_{aA} \) is the radius vector between \( a \) and \( A \), and \( r_a(r_A) \) is the radial coordinate of \( a(A) \). If Eq. 2.2 is left multiplied by \( \Psi^*_2 \) and the complex-conjugate of Eq. 2.3 is right multiplied by \( \Psi_1 \) the difference of the resulting equations integrated over the internal region \( \tau \) will give

\[
(E_2 - E_1) \int_\tau \Psi^*_2\Psi_1 d\tau = \int_\tau [(H\Psi_2)^*\Psi_1 - \Psi^*_2H\Psi_1]d\tau. \tag{2.9}
\]

Following the definition of \( H \), and assuming that the potentials \( V_{\text{Rel}} \) and \( V_{\text{internal}} \) are self-adjoint such that

\[
\int_\tau [(V\Psi_2)^*\Psi_1 - \Psi^*_2V\Psi_1]d\tau = 0, \tag{2.10}
\]

and then using Green’s Theorem

\[
\int_\tau (\nabla \cdot \nabla \Psi_2)^*\Psi_1 d\tau = \oint_S (\nabla_n \Psi_2)^*\Psi_1 dS, \tag{2.11}
\]

to integrate the kinetic energy terms, with \( \nabla_n \) as the gradient normal to the surface, the following can be derived (from here on the channel subscript \( aA \) will be replaced by \( c \))

\[
(E_2 - E_1) \int_\tau \Psi^*_2\Psi_1 d\tau = \frac{\hbar^2}{2\mu_c} \int_{S_c} (\Psi^*_2 \nabla_n \Psi_1 - \Psi_1 \nabla_n \Psi^*_2) dS_c = \Sigma_c (\gamma^{*_c} D_{1c} - \gamma_{1c} D^{*_c}). \tag{2.12}
\]

Where \( \gamma_{\lambda c} \) is the value of the radial part of the wave function at the surface for the \( c^{th} \) channel of the \( \lambda^{th} \) resonance,

\[
\gamma_{\lambda c} = \left( \frac{\hbar^2}{2\mu_c a_c} \right)^{\frac{1}{2}} u_c(a_c) = \left( \frac{\hbar^2}{2\mu_c a_c} \right)^{\frac{1}{2}} \int_{S_c} \varphi^{*_c} X_{\lambda JM} dA \tag{2.13}
\]
and $D_{\lambda c}$ is the value of the derivative of the radial part of the wave function at the surface

$$D_{\lambda c} = \left( \frac{\hbar^2}{2\mu_c a_c} \right)^{\frac{1}{2}} \left[ \frac{du_c}{dr} \right]_{r=a_c},$$  \hspace{1cm} (2.14)

where $\varphi_c$ is the channel wave function, $X_{\lambda J M}$ is a complete set of states for channel $c$, $u_c(r)$ is the radial part of the relative motion wave function, $\chi$, and $a_c = r_0(A_{\lambda 1}^\frac{1}{2} + A_{\lambda 2}^\frac{1}{2})$ is the channel radius at the surface with $r_0 \approx 1.4$ fm (see [45] for further description).

The next step is to specify the boundary conditions to be satisfied by the complete set of states $X_{\lambda J M}$ on the surfaces $S_c$. The boundary conditions assumed are independent of $\lambda$ and take on the form

$$\frac{D_{\lambda c}}{\gamma_{\lambda c}} = B_c.$$  \hspace{1cm} (2.15)

By applying the boundary conditions along with Eq. 2.12 to any two proper solutions $X_\lambda, X_\lambda'$, it is clear that the set of proper solutions are mutually orthogonal and normalized

$$\int_\tau X_\lambda^* X_\lambda d\tau = \delta_{\lambda\lambda'}.$$  \hspace{1cm} (2.16)

The wave function $\Psi$ can be expanded in terms of these solutions such that

$$\Psi = \sum_\lambda A_\lambda X_\lambda,$$  \hspace{1cm} (2.17)

where the coefficients $A_\lambda$ are given by

$$A_\lambda = \int_\tau X_\lambda^* \Psi d\tau.$$  \hspace{1cm} (2.18)

These coefficients may be determined by applying Eq. 2.12 to the solution $\Psi$ with energy $E$ and the solution $X_\lambda$ with energy $E_\lambda$ which gives

$$A_\lambda = (E_\lambda - E)^{-1} \sum_c D_{\lambda c}^0 \gamma_{\lambda c},$$  \hspace{1cm} (2.19)

where, $D_{\lambda c}^0 = D_{\lambda c} - B_c \gamma_{\lambda c}$. Also, $\gamma_{\lambda c}$ is the reduced width amplitude and is related to the partial decay width of the $\lambda^{th}$ resonance into the $c^{th}$ channel by $\Gamma_{\lambda c} = 2P_c(k a_c) \gamma_{\lambda c}^2$, where $P_c(k a_c) = \frac{k a_c}{F_t(k a_c) + G_t(k a_c)}$ is the penetrability factor, $F_t$ and $G_t$ are the regular and irregular Coulomb wave functions, respectively, and $k a_c$ is the product of the wave number and the channel radius. The expansion for $\Psi$ may then be written as

$$\Psi = \sum_c D_c^0 \left[ \sum_\lambda \frac{X_\lambda \gamma_{\lambda c}}{E_\lambda - E} \right].$$  \hspace{1cm} (2.20)
Next, by operating on Eq. 2.20 with Eq. 2.13, the fundamental $R$-Matrix relation is obtained

$$\gamma_{e'} = \sum_c R_{e'c} D_c^0,$$

(2.21)

or in the simpler matrix notation

$$\gamma = R D^0,$$

(2.22)

where $R \equiv R_{e'c}$ is the $R$-Matrix and

$$R_{e'c} = \sum_\lambda \gamma_{\lambda e'} \gamma_{\lambda c} / (E_\lambda - E).$$

(2.23)

In the external region the solution may be expanded in terms of incoming and outgoing waves with coefficients $x_c$ and $y_c$ such that

$$\Psi = \sum_c (x_c O_c + y_c I_c),$$

(2.24)

where $O_c$ and $I_c$ are the outgoing and incoming waves, respectively. From this, the surface and derivative values $\gamma_c$ and $D_c$ are

$$\gamma_c = \left( \frac{\hbar}{2 \rho_c} \right)^{\frac{1}{2}} (O_c x_c + I_c y_c),$$

(2.25)

$$D_c = \left( \frac{\hbar \rho_c}{2} \right)^{\frac{1}{2}} (O'_c x_c + I'_c y_c),$$

(2.26)

where $\rho_c \equiv k a_c$. The collision matrix, $U_{e'c}$, is defined in terms of the wave coefficients such that

$$x_{e'} = - \sum_c U_{e'c} y_c,$$

(2.27)

or, in matrix notation

$$x = - U y.$$

(2.28)

By combining Eqs. 2.22, 2.25, 2.26 and 2.28, a relation between the collision matrix and the $R$-Matrix can be obtained

$$U^J = \frac{I \rho^{-\frac{1}{2}} - R^J I^0 \rho^{\frac{1}{2}}}{O \rho^{-\frac{1}{2}} - R^J O^0 \rho^{\frac{1}{2}}}.$$

(2.29)

The final step is to relate the collision matrix to the differential and total cross sections. The general equation for the differential cross section for a particle of type $\alpha$, with channel spin $s$, and component $v$ is

$$d\sigma_{\alpha sv, \alpha' s' v'} = |A_{\alpha' s' v', \alpha sv}(\Omega_{\alpha'})|^2 d\Omega_{\alpha' v},$$

(2.30)
where the scattering amplitude is

\[ A_{\alpha's'v',\alpha sv}(\Omega_{\alpha'}) = \frac{\sqrt{\pi}}{\kappa_{\alpha}} \left\{ -C_{\alpha'}(\theta_{\alpha'}) \delta_{\alpha's'v',\alpha sv} + i \sum_{l'm'l} (2l + 1)^{\frac{1}{2}} \right. \]

\[ \times \left[ e^{2i\omega_{\alpha'}(\delta_{\alpha's'v',l'm'l'})} - U_{\alpha's'v',l'm'l',\alpha svl} \right] Y_{l'm'}^{l}(\Omega_{\alpha'}) \] (2.31)

with

\[ C_{\alpha}(\theta_{\alpha}) = (4\pi)^{-\frac{1}{2}} \eta_{\alpha} \csc^{2} \left( \frac{\theta_{\alpha}}{2} \right) \exp \left\{ -2i\eta_{\alpha} \log \sin \left( \frac{\theta_{\alpha}}{2} \right) \right\} , \] (2.32)

This function \( C \) represents the Coulomb amplitude. The total cross section for any reaction except charged particle elastic scattering is obtained by summing over all of the possible \( \alpha' \), and is expressed as

\[ \sigma_{\alpha}^{Total} = \frac{\pi}{k_{\alpha}^{2}} \sum_{J} \frac{2J + 1}{(2I_{1} + 1)(2I_{2} + 1)} \sum_{sl's'l'} (1 - \text{Re}(U_{\alpha svl,\alpha 'sl'}) ) \] (2.33)

with \( I_{i} \) as the spin of the \( i^{th} \) channel. For further details refer to [45].

### 2.3 Experimental Setup

#### 2.3.1 TTIK Technique

The Thick Target Inverse Kinematics (TTIK) technique was proposed by Dr. Vladilen Goldberg in the early 1990’s in order to dramatically improve the efficiency of experiments designed to measure the excitation functions of resonance elastic scattering reactions [5]. Traditionally, excitation functions are measured by accelerating a lighter beam particle into a heavier target and using many different beam energies to measure the excitation function. This process is tedious and time consuming because the number of beam energies used is equivalent to the number of data points needed in the excitation function. In this respect, the use of inverse kinematics (heavy beam with light target) can be very advantageous. In the following ‘heavy’ and ‘light’ will refer to the relative masses of the beam and target, not to their absolute masses.

The main idea of the TTIK is the following. The beam of heavy ions enters the scattering chamber through a thin entrance window. The chamber is filled with the target gas (helium). The window is used to separate the volume of the scattering chamber from the vacuum of the ion guide. As soon as the beam ions enter the chamber, they start to slow down due to energy lost on ionization of helium gas atoms. Gas pressure is adjusted so that the beam ions are stopped while still in the gas volume. If a nuclear interaction between a beam ion
Figure 2.2: Experimental setup for the elastic scattering of \(^{14}\text{C}\) on \(\alpha\) particles using the TTIK technique. The scattering chamber was filled with helium gas acting as a target and a beam absorber. An array of Si detectors measured the recoiling \(\alpha\) particles. See text for more details.

and a nucleus of the gas target occurs (elastic scattering for example) then the light recoil (\(\alpha\) particle) gains kinetic energy from the heavy projectile. This light recoil has substantially smaller specific energy loss than the beam ions, and its range in the gas target is therefore much larger. This allows the light recoils (\(\alpha\) particles) to penetrate through the target gas volume with very little energy loss. These light recoils may then be detected in the array of silicon semiconductor detectors placed directly in the volume of the gas chamber. The energy of the light recoil measured by the silicon detector is directly related to the energy of the beam ion at which this recoil was produced. If a resonance is present in the excitation function of the process of interest, it will show up in the spectrum of the light recoils as an excess of number of counts at a specific energy (peak). In this way the entire excitation function can be scanned continuously without any change in initial beam energy. This is
the main reason for the tremendous efficiency of the TTIK experimental method. Another advantage of this method is the ability to measure the reaction at $\theta_{\text{lab}} = 0^\circ$ since the beam is stopped before the detectors. This is significant due to the small potential scattering at backward angles in the center of mass (c.m.) system ($\theta_{\text{c.m.}} = 180^\circ - 2 \times \theta_{\text{lab}}$).

The success of this method is dependent upon several factors. First, the target must act simultaneously as a target, beam moderator, and beam absorber. Second, the difference in the specific energy losses of the beam and the target must be large enough so that the scattered target particles will not lose a significant portion of their energy before being detected, but the beam must lose all of its energy before reaching the detectors. Finally, there must be one reaction channel which dominates so that all other reaction channels can be neglected, which means this technique cannot be used (without further improvements) if more than one process significantly contributes to the cross section.

The inverse kinematics also has the advantage of reaching lower excitation energies in the excitation function, as will be shown. Here $E'$ and $E$ are the beam energies in the inverse and traditional geometry, respectively, $m$ is the mass of the light particle, and $M$ is the mass of the heavy particle. The c.m. energy for the two geometries is related to the beam energy by

$$E_{\text{c.m.}} = E_{\text{beam}} \frac{M}{m + M}, \quad (2.34)$$

and

$$E_{\text{c.m.}} = E'_{\text{beam}} \frac{m}{m + M}. \quad (2.35)$$

The expressions for the lab energy of the recoil are

$$E_m = E_{\text{beam}} \frac{m^2}{(m + M)^2} \left[ \cos \theta_{\text{lab}} + \sqrt{\left( \frac{M}{m} \right)^2 - \sin^2 \theta_{\text{lab}}} \right]^2 \quad (2.36)$$

and

$$E'_m = E'_{\text{beam}} \frac{4mM}{(m + M)^2} \cos^2 \theta_{\text{lab}}. \quad (2.37)$$

By letting $\theta_{\text{lab}} = 0$ and dividing Eq. 2.37 by Eq. 2.36, the following relation can be obtained
\[ E_m' = 4E_m \left( \frac{M}{m} \right)^2 \left( 1 + \frac{M}{m} \right)^2 \sim 4E_m. \quad (2.38) \]

As shown in Eq. 2.38, at the same c.m. energy, the detected energy of the recoil particle is approximately 4 times greater for inverse kinematics versus the traditional geometry. This is significant because for the same c.m. energy, the detected particle will have 4 times the energy making it easier to measure the low energy part of the excitation function (most valuable for nuclear astrophysics).

One shortcoming of this technique is the achievable resolution for the energy and angle of the detected particle. There are many factors that affect the resolution, including the energy spread, cross sectional area and angular divergence of the beam, the energy and angular resolution of the detector, and the straggling of the light particles in the target. As a result, the overall resolution of the detected particle deteriorates with lab angle. Fortunately, these factors can be corrected for by performing a Monte Carlo simulation of the experiment, the details of which are discussed in Sections 2.4.2 and 2.4.3.

### 2.3.2 Experimental Specifics

The \( \alpha \)-cluster structure of \(^{18}\text{O}\) was studied at the Florida State University, John D. Fox Superconducting Accelerator facility (see Figure 2.3) using the previously described Thick Target Inverse Kinematics (TTIK) technique through the elastic scattering of a 25 MeV \(^{14}\text{C}\) beam incident on a chamber filled with He gas [5, 62] (see Figure 2.2). The beam of negatively charged ions are produced using a unique SNICS-II cesium-sputter ion source which is capable of producing a \(^{14}\text{C}\) beam (this source is only used for \(^{14}\text{C}\) and is switched out for other beams). The ions are then deflected into the 9 MV Super FN Tandem Van de Graaf accelerator. Deflection of the ions ensures that only the ions of proper velocity (mass) will be injected into the tandem. The beam is then accelerated due to the attractive force exerted by the significant positive charge which is accumulated at the center of the tandem. When the beam reaches the center of the accelerator (charge terminal) it passes through an electron stripper (carbon foil) creating beam particles which are in a positive charge state. At this point the beam has a charge of the same polarity as the center of the tandem and is therefore accelerated again due to the repulsive force of like charges. After the tandem the beam is sent through a 90\(^\circ\) magnet which allows for accurate determination
of the beam energy and the separation of a specific charge state. The beam is then focused into the scattering chamber within Target Area II (see Figure 2.3). The entrance window of the chamber was covered with a 1.27 μm Havar (metal alloy) foil. Elastic scattering of the $^{14}$C on the Havar foil was used to measure the intensity of the incoming beam. An array of silicon detectors placed inside the gas chamber measured the recoiling α particles at forward angles from 0° to 45°. The silicon detectors measure the energy of the recoil particle through ionization. The gas chamber was filled with He gas of 99.5% purity to a pressure just high enough to stop the incoming beam before the detectors. As a result our measurement was nearly background free. This purity is unattainable when using the solid target technique.
due to the admixtures of the stable $^{12}$C and $^{13}$C isotopes in a $^{14}$C target. Using this type of inverse kinematics technique allowed us to measure a large, continuous excitation region in $^{18}$O (8-10.7 MeV) without changing the beam energy. Excitation functions covering the c.m. energy region of 1.8-4.5 MeV were measured at 20 different angles. The experimental setup is shown in Figure 2.2.

2.4 Analysis

2.4.1 Transfer to the c.m. Frame

The first step in the analysis was to ‘transfer’ the spectra from the lab frame into a c.m. excitation function, this was done using a code which takes into account the relevant experimental conditions [63]. The c.m. energy and angle of the target particle are determined by reconstructing the reaction kinematics, in particular the reaction point and the beam energy at that point, using the lab energy and angle of the target particle along with the known specific energy losses of the target and beam particles in the gas chamber. This is done by artificially dividing the target into thin layers which results in a beam energy loss of a few keV per layer. For every channel of every detector, the point of interaction is determined using the equation

$$E_m = \frac{4mM}{(m + M)^2} \left[ E_{beam} - \int_{0}^{x_1} \left( \frac{dE}{dx} \right)_M dx \right] \cos^2 \theta_{lab} - \int_{0}^{x_2} \left( \frac{dE}{dx} \right)_m dx, \quad (2.39)$$

with

$$x_2 = \frac{X - x_1}{\cos \theta_{lab}}, \quad (2.40)$$

where $x_1$ is the distance from the entrance window to the point of interaction and $X$ is the total length of the target, both parallel to the beam axis. $E_m$ and $\theta_{lab}$ are the detected energy and angle of the recoil, and $(\frac{dE}{dx})_i$ is the specific energy loss of a particle with mass $i$ in the target. The only unknown in Eq. 2.39 is $x_1$, which can be determined by solving the equation numerically. After $x_1$ is determined, the c.m. angle and energy are also known, and the last step is to convert the number of counts, $N$, in each detector channel to the c.m. cross section in $\frac{mb}{sr}$ using

$$\frac{d\sigma}{d\Omega_{c.m.}} = N \left[ 4N_t I \Delta \Omega \cos \theta_{lab} \right]^{-1} \quad (2.41)$$

where $I$ is the total number of beam particles incident on the target as measured by the elastic scattering off of the Havar foil, $\Delta \Omega = \frac{A_s}{R^2}$ is the detector solid angle with $A_s$ equal to
The surface area of the detector (or collimator) and $R$ equal to the distance from the point of interaction to the detector. $N_l$ can be determined using

$$N_l = \frac{N_A n}{M_{\text{mol}}} t,$$  \hspace{1cm} (2.42)

where $N_A$ is Avogadro’s number, $n$ is the number of target nuclei per atom ($n = 1$ for helium gas), $M_{\text{mol}}$ is the mass of one mole of the target, and $t$ is the thickness of the slab corresponding to the relative detector channel. The thickness $t$ changes with decreasing beam energy such that the amount of beam energy lost in each slab is held constant.

### 2.4.2 R-Matrix Application

The initial resonances included in the fit were mainly those observed in [41]. To confirm the existence of these resonances and as a way of looking for additional resonances, the angular
distributions were fit using
\[ \sigma = \sum A_l P_l^2, \]  
(2.43)
where \( P_l \) is a polynomial of degree \( l \), and \( A_l \) is the strength of \( P_l \). Here the degree of the polynomial, \( 0 \leq l \leq 7 \), represents the spin of the resonance. This type of approach does not take into account the interference between resonances and therefore only provided a few pieces of the \(^{18}\text{O} \) puzzle, as can be seen in Figure 2.4. Most of the spin-parity assignments were done by visually studying the behavior of resonance peaks at different angles, especially those angles for which \( P_l = 0 \).

The analysis of the excitation functions was performed using a multi-level, multi-channel \( R \)-Matrix approach. The \( R \)-Matrix was calculated using Eq. 2.23 where \( \gamma_{\lambda c} \) is the reduced width amplitude of the \( c^{\text{th}} \) channel of the \( \lambda^{\text{th}} \) resonance, and \( E_{\lambda} \) is the energy eigenvalue of the \( \lambda^{\text{th}} \) resonance. The \( R \)-Matrix parameters used in the fit are given in Section B.4.7. Both the \( \alpha \) and neutron decay channels of \(^{18}\text{O} \) were included in the fit. For the neutron channel, decay to the ground state and to the first excited state of \(^{17}\text{O} \) are open and both channels were included. The initial fitting of the data was performed by “hand,” varying the eigenvalues and the reduced widths of all the resonances in an effort to match the shape of the theoretical \( R \)-Matrix curve to the experimental excitation functions and at the same time minimize the \( \chi^2 \) value given by
\[ \chi^2 = N_{\text{free}}^{-1} \sum_i \sum_j \frac{(d\sigma_{ij} - d\sigma_{ij}^{\exp})^2}{(\Delta d\sigma_{ij}^{\exp})^2}, \]  
(2.44)
where \( N_{\text{free}} = N_{\text{data}} - N_{\text{variables}} \) is the number of free parameters, \( d\sigma_{ij}^{\exp} \) and \( \Delta d\sigma_{ij}^{\exp} \) are the experimental cross section and error in the cross section, respectively, and \( d\sigma_{ij} \) is the calculated \( R \)-Matrix cross section. This procedure was difficult due to the high density of interfering, wide \( \alpha \)-cluster states which is discussed in detail below. The initial fit was good, but it was significantly improved by implementing corrections to both the data and the calculation of the \( R \)-Matrix cross section.

During the fitting procedure it became apparent that the cross section was being increasingly underestimated in proportion to the lab angle. The underestimation was confirmed through a comparison with previous data \cite{59}. This effect was due to an unavoidable feature of our experimental setup. As shown in the inset in Figure 2.2, for a reaction occurring far from the center of the chamber the detector surface area which is
visible to the scattered $\alpha$ particle is smaller than the actual silicon surface area. This change in solid angle results in an underestimation of the cross section which increases with the angle of the scattered $\alpha$ particle. To correct this problem a Monte Carlo simulation of the experiment was performed using the code GEANT [64] in order to reproduce the cross section with the proper solid angle. The GEANT code performs an artificial $^{14}\text{C}+\alpha$ experiment. All experimental conditions and relevant physical processes (multiple scattering, straggling, etc.) were taken into account. The output of the GEANT Monte Carlo code is spectra of $\alpha$ particles as measured by the detectors in the lab frame. These spectra were then run through the previously discussed ‘transfer’ code in order to get simulated c.m. excitation functions. The ratios between the c.m. Monte Carlo cross sections at $\theta \neq 0^\circ$ and at $0^\circ$ were then used to normalize the experimental cross section. This method is valid because the normalization factor of the cross section at $0^\circ$ is always equal to 1 since there is no change in solid angle due to the blocking of $\alpha$ particles. The Monte Carlo simulation also provided the c.m. experimental energy resolution as a function of energy and angle, $\sigma_{\text{res}}(E, \theta)$. Using this resolution, a convoluted $R$-Matrix fit could be produced. Included in the procedure of implementing the experimental resolution was the development of a code to fit the excitation functions of many different angles at once through variation of the energy eigenvalues and the reduced width amplitudes of the most important resonances. Details of the code are discussed in the next section.

2.4.3 MinRMatrix

The program MinRMatrix minimizes the $\chi^2$ value between an experimentally measured excitation function and a theoretical $R$-Matrix calculation of the same excitation region.

After the parameters of the most influential resonances were run through the fitting procedure, the $\chi^2$ value was reduced to approximately 1.5. In all figures of excitation functions $\theta_{\text{c.m.}}$ is the angular range covered. This notation is necessary since the excitation function is not measured at one specific angle, but rather a range of angles (see the discussion of the experimental technique in Section 2.4.1). The details of the input and output for the minimization procedure are given in Appendix B.

Details of the fitting procedure

MinRMatrix consists of 4 major processes, some of which are not always used. The first
process re-bins the experimental data with a user defined bin size in order to maximize performance. The second process is the calculation of the $R$-Matrix cross section. This is done using a loop over the user defined energy range, and an embedded loop over all of the angles. Therefore, the cross section can be calculated at any combination of energy and angle. For more details on the $R$-Matrix calculation see Sections 2.2 and [45].

The next process convolutes the $R$-Matrix cross section with a user defined experimental energy resolution. If the convolution procedure is turned on then a virtual energy data array, $E_i^v$, is created. This array is filled according to the convolution integration range and step provided in the input file (see Sections B.1 and B.4.3). A two dimensional array, $d\sigma_{ij}^v$, is then filled with the $R$-Matrix cross section using the energies from $E_i^v$ and the experimental
angles. Next, a Gaussian integral over the convolution energy range is performed numerically using

\[
d\sigma_{kj} = A_{kj}dE\sum_{i} d\sigma_{ij}^v \exp \left[ -\frac{(E_k - E_i^v)^2}{2\sigma_{res}^2} \right],
\]

(2.45)

where \( k \) is the index for the loop over the experimental energies, \( j \) is the index for the loop over the experimental angles, \( d\sigma_{kj} \) is the convoluted \( R \)-Matrix cross section, \( A_{kj} = \left[\sqrt{2\pi}\sigma_{res}^r\right]^{-1} \) is the Gaussian integration constant, \( \sigma_{res}^r \) is the Monte Carlo simulated experimental energy resolution as a function of energy and angle taken from the resolution file, \( dE \) is the user defined integration step size, and \( E_k \) is the array of experimental energies. Shown in Figure 2.5 is a comparison between the non-convoluted and convoluted \( R \)-Matrix curves.

The fourth major process is the fitting of the calculated \( R \)-Matrix curve to the experimental excitation function. There are two different methods which can be used to fit the calculated curve to the experimental data.

One method randomly chooses the number of parameters to be varied between 0 and \( \frac{1}{9} \) of the total number of parameters. After that the parameters to be varied are randomly chosen, and then the chosen parameters are varied between a user defined range. The defined range is set as a percentage of the parameter being varied (see Appendix B for details). Any parameter can be fixed by the user (see Section B.4.10). After the chosen parameters have been randomized the \( R \)-Matrix curve is calculated from those parameters. Next, the \( \chi^2 \) value is determined using Eq. 2.44, if the \( \chi^2 \) of the randomized parameters is less than that of the original parameters then the ‘best’ parameters are set to the current set of randomized parameters and the iteration index is reset to zero, if not, the process continues with the next iteration. Also at that point the parameters are saved to a file which is defined by the user according to Section B.4.1. The fitting procedure will finish when a lower \( \chi^2 \) value is not found in the user defined maximum iterations.

The other method is the FORTRAN version of the MINUIT minimization function which can use one of two minimization methods, SIMPLEX (default), or MIGRAD [65]. MINUIT takes user defined parameters and varies them inside of a user defined function until that function is a minimum. The maximum number of variable parameters which MINUIT can handle is 50, if this number is exceeded, MinNMMATRIX will give a warning but it will not abort. Any parameter above 50 will not be varied. The function which is given to MINUIT by MINNMMATRIX is Eq. 2.44. The step size implemented by MINUIT is calculated
Figure 2.6: Excitation function for elastic scattering of $\alpha$ particles on $^{14}$C at 180° with the best $R$-Matrix fit (solid curve). Observed resonances are placed at their respective excitation energies (see Table 2.1).

internally using the derivatives of the user defined function. For details on the MINUIT fitting procedure see [65].

2.5 Results

2.5.1 $\alpha$-Cluster Resonances

There are several broad overlapping $\alpha$-cluster resonances in the measured region of the $^{18}$O excitation function. The broad structures (peaks at $\approx$3.1 and 3.5 MeV) observed in Figure 2.6 are the result of interference between six strongly $\alpha$-clustered resonances. All excitation function spectra are plotted using the c.m. energy, to obtain the excitation energy from the c.m. energy add the $\alpha$-decay threshold of 6.227 MeV. See Table 2.1 for the resonance parameters and Section B.4.12 for the $R$-Matrix parameters. It will be shown that the parameters of all the resonances are well grounded.
Table 2.1: Summary of the parameters of resonances observed in the studied region of the $\alpha+^{14}\text{C}$ elastic scattering excitation function. $E_{\text{exc}}$ is the excitation energy, $J^p$ is the spin-parity, $\Gamma_{\text{tot}}$ is the total width of the state, $\Gamma_{\alpha}$ is the partial alpha width, $\Gamma_{n_i}$ is the partial neutron width for the $i^{th}$ neutron decay channel (there are two open channels each with two spin projections), and $\theta_i$ is the dimensionless reduced width of the $i^{th}$ channel.

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Figure 2.7: Excitation functions for elastic scattering of $^{14}$C on $\alpha$ particles. The solid curve is the best $R$-Matrix fit, the dash dotted curve represents the best $R$-Matrix fit without the inclusion of the $0^+$ resonance at 3.88 MeV.

2.5.2 $0^+$ Resonance

The existence of a very broad ($\Gamma_{tot} \approx 3$ MeV) $0^+$ state at 3.88 ($E_{exc}=10.11$) MeV is needed to fit the excitation function at $90^\circ$. Due to the nature of the experiment an excitation function measured purely at $90^\circ$ was unattainable, due to poor energy resolution for high angles. Fortunately, this was previously measured by G.L. Morgan et al. [59]. Through comparison with the elastic scattering data of [59], it was found that the inclusion of a broad $0^+$ resonance is necessary to fit the $90^\circ$ excitation function shown in Figure 2.7. The inclusion of this state allowed for a fit to be achieved at all angles. Also, for the final fit this $0^+$ state is influential at many angles, as shown in Figure 2.7. Discussion of the unusual nature of this broad state is done in Section 2.6.2.
2.5.3 \(1^-\) Resonances

Two broad, strongly \(\alpha\)-clustered \(1^-\) states at energies 2.94 and 3.54 (9.17 and 9.76) MeV contribute significantly to the cross section over a large angular interval. For many angles there are two broad structures which dominate the spectra. At 90° the first of these broad peak disappears, and the second is greatly reduced implying that these peaks are dominated by negative parity states, see Figure 2.7, and for the measured excitation energy only low spin (\(l \leq 4\)) states should be considered. This is because for the energy region considered, high spin states will be very narrow and won’t contribute to the observed broad structures.
Figure 2.9: Excitation functions for elastic scattering of $^{14}$C on $\alpha$ particles. The solid curve is the best $R$-Matrix fit, the dash-dotted and the dash-double-dotted curves represent the best $R$-Matrix fit without the inclusion of the $2^+$ resonances at 3.56 MeV and 4.12 MeV, respectively.

This is shown with the narrow $4^+$ peak at 4.08 MeV at 180° in Figure 2.8. On the other hand both peaks are present at $\approx 141^\circ$, which is approximately where $3^-$ states will disappear. Therefore, a $1^-$ state must exist in the region of each peak. As can be seen in Figure 2.8, it is impossible to fit the data at any angle, except 180°, without two broad $1^-$ resonances. It should be noted that a $1^-$ state at 4.5 MeV is included in the fit, but because this is at the edge of the spectra, the properties of this state are uncertain.
2.5.4 2+ Resonances

Inclusion of two broad 2+ states was also found to be necessary. The energies of the states are 3.56 and 4.12 (9.79 and 10.34) MeV. The broad peak at 90° can only be reproduced by including a 2+ state in that region. This becomes obvious when the fit is produced without including this 2+ resonance as shown in Figure 2.9. Inclusion of the second 2+ state is done for two reasons. First, it is needed to reproduce the near zero cross section at $E_{c.m.} \approx 3.9$ MeV at both 90° and 180°. Second, it is a necessary ingredient for fitting the data at $E_{c.m.} > 4.0$ MeV as seen in Figure 2.9. There are also several narrow 2+ states in this region which strongly influence the $^{14}$C($\alpha$,n) cross section (see Section 2.5.7). These states are at 2.75 and 3.15 (8.98 and 9.36) MeV, and can be seen as a shoulder of the first broad peak at 180°, and a dip in the cross section at 90°, respectively. It should be noted that a very broad 2+ state is included at 5.78 (12.14) MeV. The properties of this state are very uncertain, but they influence our energy region. After a brief analysis of the higher energy data, inclusion of this state was determined to be acceptable. The properties of this state can be confirmed through a detailed analysis of the next region in the excitation function (data is already measured).

2.5.5 3− Resonances

The 3− resonances are very influential throughout the entire angular distribution of the excitation function. In total there are five 3− resonances in the energy region studied, each of them strongly interfering with the neighboring 3− states. Of the five there is one state at 3.16 (9.39) MeV which is the dominant $\alpha$-cluster resonance, as seen in Figure 2.10. The previously observed [66] 3− resonances at 3.46, 3.88, and 4.18 (9.68, 10.10, and 10.40) MeV are narrower and therefore have less influence on the cross section. The states at 3.88 and 4.18 (10.10 and 10.40) MeV are fairly obvious from the higher angle data. The state at 3.46 (9.68) MeV has a strong influence on the neutron spectra (see Figure 2.11), and also effects the shape of the elastic scattering excitation functions. It should be noted that a 3− state is included at 4.5 (10.73) MeV but the parameters of this state are uncertain since it is at the edge of the experimental data.
Figure 2.10: Excitation functions for elastic scattering of $^{14}$C on $\alpha$ particles. The solid curve is the best $R$-Matrix fit, the dashed curve represents the best $R$-Matrix fit without the inclusion of the $3^-$ resonance at 3.16 MeV.

### 2.5.6 $4^+$ Resonance

There is an obvious $4^+$ state at 4.08 (10.30) MeV which has previously been observed [67, 66, 68]. This state is narrow and strongly $\alpha$-clustered and has an influence at many different angles. The peak of this resonance is obvious at 180° and 90°.

### 2.5.7 Neutron Excitation Function

A non-normalized total cross section for the reaction $^{14}$C($\alpha$,n) [69] was also taken into account during the fitting of the elastic scattering data. Since this neutron data was not normalized we only worried about the relative strength of the resonance structures. The total neutron
cross section is reasonably well reproduced by our $R$-Matrix parameters as can be seen in Figure 2.11. The discrepancies seen at 2.9 and 3.5 MeV can be removed with minor changes to the $R$-Matrix parameters. These discrepancies may be the result of uncertainties in the normalization of both the cross section and energy of the elastic scattering and neutron data, or they may be due to unaccounted for resonances which are influential in the neutron channel but not in the elastic scattering $\alpha$ channel.

### 2.5.8 Comparison to Previously Observed States in $^{18}$O

As previously mentioned, the $\alpha$-cluster states in $^{18}$O have been heavily studied over the last 30 years. In Table 2.2, results from various types of experimental measurements are compared to the results of this work. The work by Buchmann et al. measured the $\beta$-delayed $\alpha$ emission of $^{18}$N which allowed them to measure the excitation function for broad 1$^-$ states in $^{18}$O [70]. Goldberg et al. reanalyzed the elastic scattering data taken by [59] in order to obtain the properties of the main structures [41]. Curtis et al. measured the breakup reaction $^{14}$C($^{18}$O,$^{14}$C$\alpha$)$^{14}$C in order to observe the $\alpha$-cluster states [68]. A compilation of states is put together in [66]. Each of these works found a small piece of the $^{18}$O $\alpha$-cluster puzzle. As
shown in Table 2.2 the states found in these works match well with the states we observe. However, for all works, excluding [41], the number of states observed is only a fraction of the total number of states observed in this work. The analysis of the previously measured elastic scattering data [39] performed by Goldberg et al. [41] was made using the simplified one-channel, one-level approach. This approach does account for the interference between the resonances. It was found in this work that such interference plays a decisive role in the shape of the $^{14}$C$+\alpha$ excitation function. Therefore, while major features of the $^{18}$O $\alpha$-cluster structure were identified correctly in [41], specific values of the resonance parameters have to be treated with caution. It is not surprising that the two broadest states, the $0^+$ at 10.18 MeV and the $2^+$ at 12.01 MeV were not identified in [41]; influence of these states was emulated by a wrong interference pattern (interference phase is a free parameter in the approach used by [41]).

2.6 Discussion

2.6.1 Comparison between Observed and Predicted States

Predictions for $\alpha$-cluster rotational bands in $^{18}$O have been made [38, 4]. Descouvemont used the Generator Coordinate Method (GCM) to investigate the molecular ($\alpha$-cluster) properties of $^{18}$O [38]. This is done by calculating the quadrupole moments, rms radii, and reduced $\alpha$ widths of the resonances using the antisymmetric $\alpha+^{14}$C$_{g.s.}$ and $\alpha+^{14}$C($2^+$, 7.01 MeV) wave functions. For positive parity states, Descouvemont predicted two $0^+$ $\alpha$-cluster rotational bands relating to ground state configuration along with a $1^+$ band of the $^{14}$C$_{2^+}$ configuration. The $0^+$ rotational bands are at excitation energies which were not measured in this work, so comparisons can only be made with the $1^+$ band. Unnatural parity states could not be observed, but we do see a $2^+$ and a $4^+$ which match fairly well with the predictions made in [38] (see Table 2.3). However, he predicts these states to be of the $\alpha+^{14}$C$_{2^+}$ configuration (large $\theta$ in Table 2.3); this is not what we experimentally observe for these states. For the negative parity states, Descouvemont again predicts three $\alpha$-cluster rotational bands, the first two of which are of the $^{14}$C$_{g.s.}$ configuration, and the last of the $^{14}$C$_{2^+}$. We believe the $1^-$ and $3^-$ states predicted to be at 9.6 and 9.8 MeV excitation energy, respectively, are associated with the strongly $\alpha$-clustered states we observe at 9.16 and 9.39 MeV (see Table 2.3). This assignment of states is different than what was proposed by Descouvemont, but
Table 2.2: States observed in $^{18}$O and corresponding states from Refs. [70, 41, 68, 66, 67].

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</table>
Table 2.3: Comparison of experimental and calculated $\alpha$-cluster states [1, 38]. $\theta^2_\alpha$ is the dimensionless reduced width, which represents the percentage of $\alpha$-clusterization of the state, i.e. for a state which is entirely $\alpha + ^{14}\text{C}$, $\theta^2_\alpha = 100\%$. Here (g.s.) and ($2^+$) represent the ground and 7.01 MeV excited states of $^{14}\text{C}$, respectively (see text).

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>Present work</th>
<th>Furutachi [1]</th>
<th>Desoucheumont [38]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_{exc}$</td>
<td>$\theta^2_\alpha$ (g.s.)</td>
<td>$E_{exc}$ $\theta^2_\alpha$ (g.s.)</td>
</tr>
<tr>
<td></td>
<td>(MeV)</td>
<td>(%)</td>
<td>(MeV)</td>
</tr>
<tr>
<td>1$^-$</td>
<td>9.17</td>
<td>24</td>
<td>12.8 clustered</td>
</tr>
<tr>
<td>3$^-$</td>
<td>9.30</td>
<td>47</td>
<td>13.2 clustered</td>
</tr>
<tr>
<td>2$^+$</td>
<td>9.79</td>
<td>19</td>
<td>15.8 clustered</td>
</tr>
<tr>
<td>1$^-$</td>
<td>9.76</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>0$^+$</td>
<td>10.18</td>
<td>280</td>
<td></td>
</tr>
<tr>
<td>4$^+$</td>
<td>10.30</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>2$^+$</td>
<td>12.01</td>
<td>227</td>
<td></td>
</tr>
</tbody>
</table>

we believe the 1$^-$ state observed at 9.16 MeV is a better candidate for the band head of the second 0$^-$ $\alpha$-cluster rotational band [38]. This state fits very well with the predicted values of $E_{exc}$ and $\theta^2_{\alpha,s}$, including having good agreement with the 200 keV separation of the 1$^-$ and 3$^-$ states of the band. However, the situation is complicated by the existence of a strongly $\alpha$-clustered 1$^-$ state at 9.76 MeV. Due to energy restrictions we were unable to observe the 5$^-$ member of this band, but through observation of the data taken by [39] and our own higher energy data, it is clear that this state exists at approximately the correct excitation energy.

The $\alpha$-cluster structure of $^{18}\text{O}$ has very recently been studied by Furutachi et al. using AMD+GCM (antisymmetrized molecular dynamics plus the generator coordinate method) [1]. The wave function used for this calculation was a combination of the single particle packets for each of the nucleons in the system. This allows all of the individual nucleons to interact with one another. This is very different from the calculations performed in [38], where the $\alpha$-cluster is assumed to exist in the total wave function, since Furutachi develops $\alpha$-cluster structures starting from the nucleon-nucleon interaction [1]. Furutachi also predicts three 0$^+$ $\alpha$-cluster rotational bands. As seen in Table 2.3, the excitation energy predicted by Furutachi for the 2$^+$ state [1] in the third rotational band is significantly higher than what we observe experimentally. Also, for the negative parity rotational band there is a large difference in the excitation energy of the 1$^-$ and 3$^-$ states. It appears that there is a systematic shift in the location of the states in the calculations made in [1].

The states in Table 2.3 are those which appear to be very strongly $\alpha$-clustered. Comparing
Figure 2.12: Comparison of the $\alpha$-cluster states in $^{16}\text{O}$ and $^{18}\text{O}$. The length of the line represents the relative width of the state.

the number of states in Table 2.3 to the number in Table 2.1 shows that there are many states in the energy region which, while not strongly clustered, still have a measurable $\alpha$-reduced width. This splitting of $\alpha$-cluster strength is predicted by Furutachi but not by Descouvemont [1, 38]. Furutachi relates this splitting to proton excitation of the $^{14}\text{C}$ core, but suggests that one of the states for each spin-parity should have a dominant $\alpha$ width, saying,

"As in the case of the positive parity states, there is mixing between the $^{14}\text{C}+\alpha$ structure and the $1h\omega$ configuration. This leads to the fragmentation of the
$^{14}$C+α cluster structure into many states. In particular, the $1^{-}$, $2^{-}$, $3^{-}$, $5^{-}$ and $1^{-}$ states have considerable $^{14}$C+α cluster state components, though they are much smaller than those in the $K^\pi = 0^-$ band members [1].”

This is only partially correct. We observe the splitting of the α-cluster states, but an obviously dominant state doesn’t exist for the $1^{-}$, rather there are two very strongly α-clustered states separated by only 600 keV, as shown in Table 2.3.

Figure 2.12 shows that the α-cluster picture of $^{18}$O is very different from what has been observed in $^{16}$O. The density of states is much higher, plus there is parity mixing and splitting of the α-cluster strength. While some of the features predicted in [38, 1] agree with what is experimentally observed here, there are several aspects of the α-cluster structure which differ from the predictions (two strongly α-clustered $1^{-}$ states), and some aspects which are not predicted at all. The α-cluster states which are unpredicted are the very broad $0^+$ and $2^+$ states at 10.1 and 12 MeV, respectively.

2.6.2 Nature of the extremely broad $0^+$ and $2^+$ states

We observe an unpredicted $0^+$ state at an excitation energy of $\approx$10.1 MeV with a width of $\approx$3.0 MeV. Due to this very large width this state does not show up as a narrow peak, instead it affects the entire observed excitation function. While this broad $0^+$ state has a rather small influence on the $R$-Matrix fit of the excitation functions at scattering angles close to 180° in the c.m., it plays a major role at many angles especially in the vicinity of 90°, as can be seen in Figure 2.7. The width of this state is so large that it exceeds the Wigner limit, $\Gamma_{Wigner} \approx \frac{\hbar^2}{\mu c^2}$, at any radii chosen to calculate the penetrability factors. We made a preliminary analysis of our data at higher excitation energies, and found a similar broad $2^+$ level at $\approx$12.0 MeV with a width also exceeding the Wigner limit. These states are unusual in the sense that they appear to be pure α-particle states in which the α particle is completely separate from the core. These very broad states represent the maximum α-clusterization that a nucleus can have.

Similar pure α-particle states should exist in a large variety of nuclei since the specific structure of the core ($^{14}$C) is unimportant. Over 50 years ago, the existence of a broad $0^+$ at $\approx$11.2 MeV in $^{16}$O was tentatively suggested as a result of the analysis of the $\alpha+^{12}$C excitation function [71, 72], on top of that a similar $0^+$ state at $\approx$8.7 MeV in $^{20}$Ne was found in the
Figure 2.13: Systematic of the unusual $0^+$ states discussed in the text. The solid, dashed, dotted and dash-dotted lines represent states in $^{18}\text{O}$, $^{16}\text{O}$, $^{12}\text{C}$ and $^{20}\text{Ne}$, respectively. The peak of the triangle corresponds to the excitation energy of the state and the width of the triangle represents the width of the state in such a way that the full width at half maximum of the triangle equals the width of the state. Note that experimental information on the states in $^{12}\text{C}$, $^{16}\text{O}$ and $^{20}\text{Ne}$ is incomplete. The smaller width of the state in $^{20}\text{Ne}$ is due to the higher Coulomb barrier and as a result a smaller penetrability factor.

The $\alpha + ^{16}\text{O}$ excitation function (only an upper limit on its width is available) [73]. Furthermore, this type of $0^+$ state has been observed at $\approx 10.3 \text{ MeV}$ in $^{12}\text{C}$ [74]. The energy of the broad $0^+$ states in $^{12}\text{C}$, $^{16}\text{O}$, $^{18}\text{O}$ and $^{20}\text{Ne}$ relative to their respective $\alpha$-particle decay thresholds is shown in Figure 2.13. It is evident that the energy above the $\alpha$-decay threshold for these unusual states is consistent in different nuclei, which shows that the interaction between the $\alpha$-cluster and the core is independent of the core involved. Theoretical investigation into this interaction would be very interesting.


2.7 Conclusions

In conclusion, there are several strongly $\alpha$-clustered resonances within the excitation region which was analyzed in this work as shown in Table 2.3. The less clustered states which are observed are most likely a result of the fragmentation of the states with a large $\alpha$ reduced width caused by an overlap between the wave functions of the $\alpha$-cluster states and those of the single particle states (see Table 2.1). The sharing of the $\alpha$-cluster strength results in a more complex $\alpha$-cluster structure compared to the $N = Z$ nuclei. There is some evidence for $\alpha$-cluster rotational bands in $^{18}$O, however, due to the more complex nature of the $\alpha$-cluster structure the existence of rotational bands is not as clear as what is predicted in [38] and [1]. The splitting of the $\alpha$-cluster strength has been predicted to occur within $^{18}$O in [1], however we do not see one dominant $\alpha$-cluster state for all spin-parities as they suggest.

Also, an unusually broad $0^+$ state at $\approx$10.1 MeV was observed in $^{18}$O. Its width corresponds to at least 1.9, with a best fit of 2.8, times the Wigner limit. This is a very unusual state and comparing this with similar states in other strongly $\alpha$-clustered nuclei it appears that these states may be a common feature of the nucleus. Further theoretical and experimental studies are necessary in order to determine the nature of these pure $\alpha$-particle states.
CHAPTER 3

DETERMINATION OF ASTROPHYSICAL
REACTION RATES USING SUB-COULOMB
\(\alpha\)-TRANSFER REACTIONS

3.1 Background

The evolution of the universe is one of the driving factors behind the study of nuclear physics. There are many models which attempt to explain the formation, energy generation, and nucleosynthesis of stars. These models depend upon many factors, including the rates of nuclear reactions which occur in stellar processes. Unfortunately, many astrophysically important reactions cannot be measured directly at the relevant stellar energies. This is due to the fact that the probability for a charged particle of stellar energy to penetrate the Coulomb barrier is very low (see schematic in Figure 3.1). This makes the cross section for the corresponding nuclear reaction immeasurably small. However, in a stellar environment the density of matter is so high that even this marginal cross section allows the process to go forward. This problem necessitates the use of indirect techniques which allow the measurement of certain properties in specific nuclei. These measured quantities can then be used to extrapolate/calculate the corresponding reaction cross sections at very low energy. Typically the nuclear astrophysics capture reaction cross section is the result of two processes, direct and resonance capture. The two components have to be treated differently when extrapolating/calculating the cross section from indirect measurements.

Direct capture reactions are the result of a particle being captured by the tail of the ground and bound state wave functions. The cross section for this type of reaction at low energy can be reliably extrapolated from a higher energy measurement. On the other hand, resonance capture reactions proceed by populating a near threshold resonance in the compound nucleus. The resonance capture cross section is dependent upon the spin, partial
Figure 3.1: For low energy charged particles penetration of the Coulomb barrier prohibits the fusion process making the probability for nuclear reactions very low. The magnitude of the Coulomb barrier is exaggerated.

widths, and excitation energy of the state being populated. Therefore, determination of these resonance parameters is crucial for determining the cross section at stellar energies. For the astrophysical reactions being considered here, the resonance capture process dominates the cross section in the stellar energy region and will therefore be discussed in detail.

Unfortunately, the resonance parameters extracted from the indirect methods depend upon the nuclear model parameters used to make the extraction. The main point of this work is to develop a technique which avoids this model dependence. It will be shown that by combining the Asymptotic Normalization Coefficient (ANC) technique with a sub-Coulomb transfer reaction, it is possible to remove the dependence of the resonance parameters on the model parameters. The ANC technique and several sub-Coulomb transfer reactions will be discussed in detail. The astrophysically important reactions which will be covered here are \(^{13}\text{C}(\alpha,n)\) and \(^{14}\text{C}(\alpha,\gamma)\). The motivation, experimental design, analysis, and results will be discussed for each reaction.
3.1.1 Asymptotic Normalization Coefficient Technique

As mentioned above, direct measurement of many astrophysically important reactions cannot be done due to prohibitively small cross sections. This problem is avoided through the use of indirect methods. Consider a reaction of the type \(a + A \rightarrow C^* \rightarrow b + B\). Often the astrophysical reaction rate (defined in Eq. 3.27) is dominated by one or two near threshold resonances. In these cases, knowing the energies, spin-parities, widths, and partial widths of the resonances allows for a reliable calculation of the corresponding reaction rate. At energies close to the resonance energy, the cross section is given by Eq. 2.1. Often the partial widths, \(\Gamma_i\) and \(\Gamma_f\), are the only unknown parameters and can be measured indirectly using transfer reactions.

In principle, the cross section of the direct transfer reaction is directly proportional to the so-called spectroscopic factor which is defined as the overlap integral between the wave function of a corresponding state and the model wave function which represents the relative motion of the cluster (a) and the core (A). For example,

\[
{^{14}\text{C}}(^{6}\text{Li},d){^{18}\text{O}(3^-)}
\]

and

\[
\alpha + {^{14}\text{C}} \rightarrow {^{18}\text{O}^*(3^-)}.
\]

The spectroscopic factor is defined as

\[
S_{aA} = \int \Psi_{C^*}^* \varphi_{nlj} d\tau.
\]  

(3.1)

where, \(\Psi_{C^*}\) is the wave function of the populated resonance in the nucleus \(C\), \(\varphi_{nlj}\) describes the relative motion of the cluster and the core, and \(\tau\) is again the internal coordinates. \(S_{aA}\) holds the information on the clusterization of a state, however, the extraction of the spectroscopic factor of a resonance is dependent upon model parameters (this dependence will be considered in more detail in Sections 3.3.3 and 3.4.3). As an alternative one can determine the Asymptotic Normalization Coefficient (ANC).

Following the results of [75], we define the ANC in the following way (it is assumed here that the reaction proceeds through a sub-threshold resonance in the compound nucleus). The radial overlap wave function can be approximated as:

\[
I_{aA}(r) = \sqrt{S_{aA}\varphi_{nlj}(r)},
\]

(3.2)
where $\varphi_{nlj}$ is the normalized bound state wave function describing the relative motion of $a$ and $A$. If the radius, $r$, is taken to be larger than the nuclear interaction radius, $R_N$, then the $\text{ANC}_{aA}$ defines the amplitude of the tail of $I_{aA}$, such that

$$I_{aA}(r) \rightarrow \text{ANC}_{aA} \frac{W(2\kappa r)}{r}, \text{ for } r > R_N,$$

where $W(2\kappa r)$ is a Whittaker function that describes the asymptotic behavior of the bound state wave function such that

$$\varphi_{nlj}(r) \rightarrow b_{lj} \frac{W(2\kappa r)}{r}, \text{ for } r > R_N.$$

Here $b_{lj}$ is the single particle ANC which defines the amplitude of the tail of the bound state wave function for large $r$. Using Eqs. (3.2), (3.3), and (3.4), we can derive a relation for $\text{ANC}_{aA}$,

$$\text{ANC}_{aA}^2 = S_{aA} b_{lj}^2.$$

Using the $R$-matrix approach, it can be shown that the amplitude of the $A(a, b)$ reaction, and therefore the astrophysical $S$-factor (see Section 3.3.3), is proportional to $\text{ANC}_{aA}$,

$$M = \frac{1}{2\sqrt{k_{aA}k_{bB}}} \frac{P_l(k_{aA}, r_0)}{\mu_{aA}r_0} \tilde{W}_{-\eta,l+\frac{1}{2}}(2\kappa_{aA}r_0) \frac{\tilde{A}\tilde{\text{ANC}}_{aA}}{E_{aA} + \varepsilon + \frac{i\Gamma_{C^*}(E_{bB}, r_0)}{2}} \times e^{i\delta},$$

where $P_l(k_{aA}, r_0)$ is the Coulomb-centrifugal barrier penetration factor in the entrance channel, $\tilde{W}_{-\eta,l+\frac{1}{2}}(2\kappa_{aA}r_0) = W_{-\eta,l+\frac{1}{2}}(2\kappa_{aA}r_0) \Gamma(l + 1 + \eta)$ is the Coulomb-modified Whittaker function, $r_0$ is the channel radius, $\tilde{A}\tilde{\text{ANC}}_{aA} = \text{ANC}_{aA}^* / \Gamma(l + 1 + \eta)$ represents the Coulomb-modified ANC for the virtual decay (synthesis) $C^* \leftrightarrow a + A$, $\eta$ and $l$ are the Coulomb parameter and relative orbital angular momentum of the sub-threshold bound state $(aA)$, respectively, and $\Gamma_{bB}(E_{bB}, r_0)$ is the resonance partial width for the decay to the final channel $b + B$. The total width of the resonance $C^*$ is equal to $\Gamma_{C^*}$, $E_{aA} = \frac{\hbar^2 k_{aA}^2}{2\mu_{aA}}$ is the relative kinetic energy of particles $a$ and $A$, $k_{aA}(\varepsilon > 0) = i\kappa_{aA}(\varepsilon < 0) = \sqrt{2\mu_{aA}\varepsilon}/\hbar^2$, and $\varepsilon$ is the binding energy for the virtual decay $C^* \rightarrow a + A$. All equations in this section were taken from [75].

So far the ANC technique has been discussed in its most general form, however in the next section the benefits of using the ANC technique in parallel with a sub-Coulomb $\alpha$-transfer reaction will be made clear.
3.1.2 Distorted Wave Born Approximation

The Distorted Wave Born Approximation is vital to the extraction of a resonance ANC. As such, a brief derivation of the scattering amplitude using the Distorted Wave Born Approximation will be discussed here, for further details see [76].

The Schrödinger equation for the scattering of a particle of mass \( m \) by a central force is

\[
(\nabla^2 + k^2)\chi(r) = U(r)\chi(r),
\]

where \( k = \sqrt{2mE/\hbar} \) and \( U(r) = 2mV/\hbar^2 \). The solution of this equation, \( \chi(r) \), is the wave function which describes the motion of a particle with mass \( m \) in the field \( U(r) \). If \( V = 0 \) the particle feels no potential and this becomes the equation for a free particle of energy \( E \)

\[
(\nabla^2 + k^2)\chi_0(r) = 0,
\]

the solution of which is a plane wave, \( \chi_0(r) = e^{i k r} \). Imagine that the potential \( U(r) \) consists of two terms such that \( U = U_1 + U_2 \) and that the solution to scattering on \( U_1 \) can be determined by solving

\[
[\nabla^2 + k^2 - U_1(r)]\chi_1(k, r) = 0.
\]

The solutions can be separated into two different types, \( \chi_1^{(+)}(k, r) \) and \( \chi_1^{(-)}(k, r) \), with the former consisting of a plane wave plus an outgoing scattered wave, and the latter consisting of a plane wave plus an incoming scattered wave. The full solution to Eq. 3.7 can be written in terms of \( \chi_1^{(±)} \) as

\[
\chi(k, r) \rightarrow \chi_1^{(+)}(k, r) - \frac{e^{i k r}}{4 \pi r} \int \chi_1^{(-)*}(k', r')U_2(r')\chi(k, r')dr', \text{ for large } r.
\]

This leads to a scattering amplitude which is the sum of the contribution from the interaction \( f_1(\theta, \phi) = \langle \chi_1 | U_1 | \chi_1 \rangle \) and the contribution from the second term in Eq. 3.10,

\[
f(\theta, \phi) = f_1(\theta, \phi) - \frac{1}{4 \pi} \int \chi_1^{(-)*}(k', r')U_2(r')\chi(k, r')dr'.
\]

This amplitude is not very useful because it contains the exact solution \( \chi(k, r') \). However if the exact solution is approximated as a distorted wave, \( \chi_1 \), in the manner of the Born Approximation, then the amplitude can be written as

\[
f_{DWBA}(\theta, \phi) = f_1(\theta, \phi) - \frac{1}{4 \pi} \int \chi_1^{(-)*}(k', r')U_2(r')\chi_1^{(+)}(k, r')dr'.
\]
If $U_1$ is chosen to be an optical potential, $U_{\alpha(\beta)}$, which describes elastic scattering, and $U_2$ is a potential which produces the inelastic transition, then the scattering amplitude takes on the form

$$f_{DWBA}(\theta, \phi) = -\frac{1}{4\pi} \int \chi_{\beta}^{(-)*}(k_\beta, r_\beta) \langle b, B | U_2(r') | a, A \rangle \chi_{\alpha}^{(+)}(k_\alpha, r_\alpha) dr_\alpha dr_\beta \quad (3.13)$$

where $\chi_{\alpha}$ describes the elastic scattering in the $\alpha = a + A$ entrance channel due to the optical potential $U_{\alpha}$, and $\chi_{\beta}$ describes the elastic scattering in the $\beta = b + B$ exit channel due to the optical potential $U_{\beta}$. The term $\langle b, B | U_2 | a, A \rangle$ represents the form factor potential which depends upon the type of non-elastic interaction being considered. The importance of DWBA analysis to the technique discussed here is shown in Section 3.2.1.

### 3.2 Experimental Details

#### 3.2.1 Significance of Using a Sub-Coulomb Energy with the ANC Technique

It is possible to extract the ANC from a direct $\alpha$-transfer reaction in which an $\alpha$-cluster is transferred to the nucleus, $A$, populating the compound nucleus sub-threshold state $C^*$, i.e. $\alpha c + A \rightarrow c + C^*$, where $C^*$ is an $\alpha + A$ system. The cross section for this reaction can be calculated using the DWBA approach and is proportional to the product of single particle ANCs for the $\alpha c$ and $\alpha A$ pairs (see Eq. 3.4),

$$\frac{d\sigma}{d\Omega_{DWBA}} \sim b_{\alpha c}^2 b_{\alpha A}^2 X. \quad (3.14)$$

Here, $X$ is a function which depends on energy, angle, spin, and the parameters of the optical model potentials which describe the interaction in the entrance and exit channels. However, $X$ is independent of the cluster interaction between the $\alpha c$ and $\alpha A$ pairs, which is taken into account through $b_{\alpha c}$ and $b_{\alpha A}$. The cross section observed experimentally is related to the product of the spectroscopic factors for the $\alpha c$ and $\alpha A$ pairs and the DWBA cross section,

$$\frac{d\sigma}{d\Omega_{exp}} = S_{\alpha c} S_{\alpha A} \frac{d\sigma}{d\Omega_{DWBA}}. \quad (3.15)$$

Using this definition along with Eqs. (3.5) and (3.14), we arrive at a relation between the ANCs and the experimental cross section,

$$ANC_{\alpha c}^2 ANC_{\alpha A}^2 \sim b_{\alpha c}^2 b_{\alpha A}^2 \frac{d\sigma}{d\Omega_{exp}} b_{\alpha c}^2 b_{\alpha A}^2 X. \quad (3.16)$$
Figure 3.2: When performing a direct transfer experiment with inverse kinematics the spectating particle \((d,t)\) which is scattered to \(\theta_{\text{c.m.}} = 180^\circ\) will be measured at \(\theta_{\text{lab}} = 0^\circ\).

The single particle ANCs, \(b_{ac}^2\) and \(b_{\alpha A}^2\), are canceled out, hence the dependence of the ANC on the cluster-core interaction in both the entrance and exit channels is removed. In general, \(X\) is a function of the interaction between the projectile and the target, and also between the recoiling particles. However, if the transfer reaction is performed at sub-Coulomb energy in both the entrance and exit channels, then the Coulomb interaction is dominant and the parameters of the nuclear optical potential play only a minor role. If the ANC in the entrance channel, \(\text{ANC}_{ac}\), is known, then the exit channel ANC can be extracted from

\[
\text{ANC}_{\alpha A}^2 \sim \frac{d\sigma}{d\Omega_{\text{exp}}} X \times \text{ANC}_{ac}^2 \quad \text{for } E_{\text{c.m.}} < E_{\text{Coulomb}}
\]

Eq. (3.17) shows that if the \(\alpha\)-transfer reaction is performed at a sub-Coulomb energy, then the ANC can be extracted from the experimental cross section without any dependence upon the parameters of the nuclear cluster-core interaction between \(\alpha c\) and \(\alpha A\). Also, the dependence of the ANC on the parameters of the optical potentials in the entrance and exit channels is removed. Therefore, the value of the ANC extracted from a sub-Coulomb \(\alpha\)-transfer reaction is independent of all model parameters.

3.2.2 Setup

The sub-Coulomb \(\alpha\)-transfer reactions \(^{13}\text{C}(^6\text{Li},d)\) and \(^{14}\text{C}(^7\text{Li},t)\) were performed at the John D. Fox Superconducting Accelerator Laboratory on the campus of Florida State University. The experiments were performed using inverse kinematics, i.e. \(m_{\text{beam}} > m_{\text{target}}\), for several
reasons. First, the use of a carbon beam allowed for the removal of the background associated with the admixture of $^{12}$C in a solid $^{14,13}$C target. Second, the $\alpha$-transfer reaction needed to be performed at a sub-Coulomb c.m. energy of approximately 3 MeV. For our laboratory, this c.m. energy is experimentally easier to attain when using a carbon beam rather than a lithium beam. For energies below the Coulomb barrier, the angular distribution for a direct transfer reaction has a peak at back angles, $180^\circ$ in the c.m. frame. As can be seen in Figure 3.2 this corresponds to a peak at $0^\circ$ in the lab frame. Therefore, in order to measure the reaction cross section in the region of its maximum, the detectors were placed near $0^\circ$ in the lab frame. The detector nearest to $0^\circ$ was shielded from Rutherford scattering of the carbon beam on the lithium target by placing 5 $\mu$m of Havar foil in front of the telescope. This foil was thick enough to stop all of the carbon beam, but thin enough to allow the deuterons

Figure 3.3: Birdseye view of the scattering chamber used in the sub-Coulomb $\alpha$-transfer experiments.
Figure 3.4: $\Delta E$-vs-$E$ plot of the reaction $^{14}\text{C}+^{7}\text{Li}$ with a $^{14}\text{C}$ beam energy of 11.5 MeV. Due to the difference in specific energy losses of the reaction products, the triton spectra can be selected to produce the excitation function of $^{18}\text{O}$.

and tritons to continue on to the detectors.

The detectors used to measure the deuterons (tritons) produced by the $\alpha$-transfer reaction were silicon $\Delta E-E$ telescopes, see Figure 3.3. These telescopes reliably identify the particles of interest from the other charged particles produced in the reaction by taking advantage of the difference in specific energy losses of the products, see Figure 3.4. The $\Delta E$ detectors ranged in thickness from 15 to 25 $\mu m$, while the $E$ detectors were at least 500 $\mu m$ thick. The lithium targets used must be prepared under vacuum in order to prevent oxidation. This makes it difficult to get an accurate measure of the absolute value of the cross section due to the fact that the target thickness, which is needed to calculate the cross section cannot
be measured in a standard way. The usual way to measure the cross section is

$$\frac{d\sigma}{d\Omega_{c.m.}} = N[jkt\Delta\Omega]^{-1},$$

(3.18)

where, $N$ is the number of particles detected, $I$ is the intensity of the beam, $k$ is the kinematic factor, $t$ is the target thickness, and $\Delta\Omega$ is the detector solid angle. The method used to determine the target thickness was to measure a known elastic scattering cross section and extract the product $t\Delta\Omega$ using the relation between the known cross section and the cross section which we measured. Elastic scattering of protons on lithium targets has a cross section at 95° which is known within 3% accuracy [77]. Protons with $E_{beam} = 6.868$ MeV were scattered off of the lithium targets. The target angle relative to the beam was 45° and the telescopes were sequentially placed at 95° producing a value of $t_i\Delta\Omega_j$ for all possible combinations of $i$ and $j$, where $i$ and $j$ are the target and telescope number, respectively (see Figure 3.5).
In Sections 3.3 and 3.4, application of the previously described sub-Coulomb ANC technique to evaluate the reaction rates for $^{13}\text{C}(\alpha,n)$ and $^{14}\text{C}(\alpha,\gamma)$ are discussed.

### 3.3 Determination of the $^{13}\text{C}(\alpha,n)$ Reaction Rate Using the $^{13}\text{C} (^6\text{Li},d)$ Sub-Coulomb $\alpha$-Transfer Reaction

#### 3.3.1 Motivation

The slow neutron capture process (s-process) is thought to be responsible for producing approximately half of all the elements in the universe which are heavier than iron [78]. It was suggested in [79] that low-mass Asymptotic Giant Branch (AGB) stars may be the main site for the s-process.

The Asymptotic Giant Branch is a period which is undertaken by all low mass stars.
When the hydrogen burning phase (main sequence in Figure 3.6) of a star is completed the core contracts and begins to heat while the surface cools down, this process causes the outer layers of the star to expand and follow the Giant branch of the Hertzsprung-Russell diagram in Figure 3.6 to become a Red Giant. The cooling of the surface is stopped by a helium burning phase which moves the star backwards along the Giant branch. Once the He burning is completed, the star will asymptotically follow its previous path down the giant branch to become an AGB star [80].

AGB stars consist of four regions. The innermost is a carbon-oxygen core, next is a helium rich layer, after that is a hydrogen burning layer, and the outermost layer is a convective shell.
Figure 3.8: Diagram of a thermal pulse dredging carbon from the core up to the top of the He layer [80]. After carbon is brought to the surface through a thermal pulse, the convective envelope (mostly hydrogen) expands into the He-shell allowing protons and $^{12}\text{C}$ to mix and create $^{13}\text{C}$ (see text).

(see Figure 3.7). The helium rich layer is where the s-process is thought to occur. The helium shell goes through a burning stage called a thermal pulse approximately every $10^5$ years [80]. During these thermal pulses the temperature can be high enough to ignite the endothermic (energy absorbing) reaction $^{22}\text{Ne}(\alpha,\text{n})^{25}\text{Mg}$, but during the inter-pulse periods, the main source of neutrons for the s-process is the $^{13}\text{C}(\alpha,\text{n})$ reaction because it is exothermic (energy releasing) and can be activated at low temperatures [79]. The long inter-pulse periods are thought to be when the bulk of the s-process occurs. Therefore the $^{13}\text{C}(\alpha,\text{n})$ reaction is vitally important to modeling the s-process within AGB stars.
The two factors that determine the efficiency of the reaction are the abundance of $^{13}\text{C}$ within an AGB star and the rate of the $^{13}\text{C}(\alpha,n)$ reaction. The amount of $^{13}\text{C}$ present in the remnants of hydrogen burning stars is not sufficient to explain the observed abundance of heavy elements. The existence of $^{13}\text{C}$ rich regions in the He layer has been proposed as a possible solution [79]. Recent models show that $^{13}\text{C}$ pockets can form at the top of the He layer through the following process: as the core of the star cools, the helium burning region of the helium layer moves outward and approaches the hydrogen shell causing helium to accumulate on the inner surface of the hydrogen shell. Eventually the density of this helium becomes high enough to ignite, which causes a thermonuclear runaway within the helium layer. This results in the formation of a convective zone which reaches from the outer region of the helium shell down into the carbon-oxygen core, see Figure 3.8. This convective zone, along with an expansion of the outer convective shell, allows protons from the hydrogen shell to mix with the upper layers of the helium layer allowing for the formation of $^{13}\text{C}$ through proton capture by $^{12}\text{C}$, and the subsequent $\beta$-decay of $^{13}\text{N}$ [79].

The $^{13}\text{C}(\alpha,n)$ reaction rate determines how much of the $^{13}\text{C}$ in the pocket is burned during the inter-pulse period. If the reaction rate is high enough to burn all of the $^{13}\text{C}$ in the pockets, then the amount of carbon in the pockets becomes the limiting factor in the overall $s$-process neutron production, and if the rate is so low that not all of the $^{13}\text{C}$ is burned, then the $^{13}\text{C}(\alpha,n)$ reaction rate limits the neutron production.

Accurately determining this rate at AGB nucleosynthesis temperatures (0.08-0.1 GK) would eliminate a significant uncertainty regarding the overall neutron balance and will allow for better tests of modern stellar models of AGB stars. Before our measurement, the rate of the $^{13}\text{C}(\alpha,n)$ reaction at stellar temperatures had an uncertainty of approximately 300%. This uncertainty is due to the large uncertainty in the partial $\alpha$-width of a near threshold resonance. Direct measurement of the $^{13}\text{C}(\alpha,n)$ cross section is only available for c.m. energies above 279 keV. Below this energy the cross section must be extrapolated, which is not possible without knowing the properties of the above mentioned near threshold resonance. This problem can be avoided if the ANC technique discussed above is applied.

The $\frac{1}{2}^+$ resonance in $^{17}\text{O}$ at 6.36 MeV excitation energy is just 3 keV below the $\alpha$ decay threshold (see inset of Figure 3.11). Therefore, the partial $\alpha$ width of this state, $\Gamma_\alpha$, can
Figure 3.9: The large dependence of $S_\alpha$ on the DWBA parameters is shown for a calculation made at 60 MeV of $^6$Li. The black curve is from the optical potentials from [78], the red curve is from the deuteron optical potential from [81], the blue curve is from the [78] optical potentials but the radius of the $^{13}\text{C}+\alpha$ form factor has been decreased by 25%, and the yellow curve again is from the [78] optical potentials but the number of nodes in the $^{13}\text{C}+\alpha$ wave function has been increased by 1.

have a very strong influence on the extrapolation of the $^{13}\text{C}(\alpha,\text{n})$ cross section as shown here

$$
\sigma_{\alpha n} \sim \frac{\Gamma_\alpha \Gamma_n}{(E + \varepsilon)^2 + \left(\frac{\Gamma_{\text{tot}}}{2}\right)^2},
$$

where $\sigma_{\alpha n}$ is the reaction cross section, $\Gamma_n$ is the partial neutron width, $\Gamma_{\text{tot}}$ is the total resonance width, and $\varepsilon$ is the resonance binding energy. In fact, $\Gamma_\alpha$ is the main source of uncertainty in the $^{13}\text{C}(\alpha,\text{n})$ reaction rate.

It was assumed in the recent nuclear astrophysics compilation of reaction rates (NACRE) that the $\frac{1}{2}^+$ resonance has a well developed $\alpha$-cluster structure which would significantly enhance the $^{13}\text{C}(\alpha,\text{n})$ reaction cross section at low temperatures [82]. Recently an attempt was made by Kubono \textit{et al.} [78] to determine the spectroscopic factor, $S_\alpha$, for this state.
Figure 3.10: If the reaction is performed at sub-Coulomb energy then variations of the optical potential parameters produce variations in the DWBA cross section of only \( \approx 20\% \).

The measurement by Kubono et al. [78] was done using the \( \alpha \)-transfer reaction \(^{13}\text{C}(^6\text{Li},d)\) with a 60 MeV \(^6\text{Li}\) beam. The factor extracted from this reaction was surprisingly small, \( S_\alpha \approx 0.011 \), making the influence of this sub-threshold resonance on the astrophysical \( S \)-factor negligible [78]. This probably implies that the \(^{13}\text{C}(\alpha,n)\) reaction rate is not high enough to remove all of the \(^{13}\text{C}\) from pockets formed within AGB stars. However, it has been shown that the same experimental data is consistent with \( 0.15 \leq S_\alpha \leq 0.5 \) for the state in question [83]. This difference can be due to the heavy dependence of \( S_\alpha \) on the DWBA calculations (see Figure 3.9) and can have a significant effect on the \(^{13}\text{C}(\alpha,n)\) reaction rate. All of these uncertainties can be avoided if the \( \alpha \) transfer reaction is performed at sub-Coulomb energy, which removes the dependence of the calculated cross section on the DWBA optical potentials, and the ANC is extracted from experimental data which eliminates the dependence of the final result on the shape of the form-factor binding potentials and the
number of nodes in the wave function. This model independence can be seen in Figure 3.10. In hopes of removing the large uncertainty associated with the $S_\alpha$ value for the $\frac{1}{2}^+, 6.36$ MeV state in $^{17}$O, an experiment was performed at the Florida State University, John D. Fox Superconducting Accelerator Laboratory.

The ANC technique has previously been used to determine the properties of a near threshold state in $^{16}$O via the transfer reactions $^{12}$C($^6$Li,d) and $^{12}$C($^7$Li,t) in order to determine the effects these properties have on the $^{12}$C($\alpha,\gamma$) reaction rate [84]. However, our experiment marked the first application of the ANC technique to determine the astrophysical factor of the $^{13}$C($\alpha,n$) reaction [85].

### 3.3.2 Experimental Specifics

The measurement of the ANC of the virtual synthesis $\alpha+^{13}$C$\rightarrow^{17}$O(6.36 MeV, $\frac{1}{2}^+$) was performed using the $\alpha$-transfer reaction $^{13}$C($^6$Li,d). The energies for the $^{13}$C beam were 8.0 and 8.5 MeV, which correspond to c.m. energies of 2.53 and 2.68 MeV, respectively. These energies were chosen to be just below the Coulomb barrier in the exit channel. The exit channel Coulomb barrier of 2.81 MeV is the crucial energy due to the fact that it is smaller than that of the entrance channel, 5.81 MeV. The lithium targets used were $\approx 50 \mu g/cm^2$ and 98% enriched with $^6$Li. Deuterons from the reaction were measured using 4 $\Delta E - E$ telescopes placed 7.5° apart and ranging in angle from 6° to 27.5° (see Figure 3.3).

### 3.3.3 Analysis and Results

Deuteron spectra are shown in Figures 3.11 and A.14. These spectra were obtained using $\Delta E - E$ telescopes at the labeled laboratory angles, with a $^{13}$C beam energy of 8.5 MeV. All of the peaks in Figure 3.11 correspond to well known states (or group of states) in $^{17}$O and are labeled accordingly. The c.m. experimental resolution was approximately 250 keV (FWHM) and was determined mainly by the energy loss of the beam due to the target thickness. This resolution was adequate to separate the $\frac{1}{2}^+, 6.36$ MeV state from the neighboring states in $^{17}$O (see inset in Figure 3.11). The cross section of the 6.36 MeV state was determined by fitting the region of interest in the spectrum with 5 summed Gaussians. The parameters of the Gaussian corresponding to the 6.36 MeV state were then used to calculate the number of counts in the peak, and from that the cross section was calculated using Eq. 3.18.
Figure 3.11: Spectrum of deuterons measured from the $^6$Li($^{13}$C,d) transfer reaction with the detector placed at 6° and a $^{13}$C beam energy of 8.5 MeV. The inset shows the level scheme of $^{17}$O [72]. The solid line is the Gaussian fit described in the text.

**DWBA**

Angular distributions of the reaction $^{13}$C($^6$Li,d)$^{17}$O($\frac{1}{2}^+$, 6.36 MeV) measured at beam energies of 8.0 and 8.5 MeV are shown in Figure 3.12. The shapes of these angular distributions are typical for a sub-Coulomb transfer reaction and it was found that the normalization factor of the DWBA cross section to the experimental cross section is the same for both energies. Absolute normalization of the experimental cross section was performed by measuring elastic scattering of protons at 6.868 MeV as previously discussed is Section 3.2. The angular distribution of the $^{13}$C($^6$Li,d) reaction was calculated using the DWBA
Figure 3.12: Angular distribution of the $^6$Li($^{13}$C,d)$^{17}$O(6.36 MeV) reaction. Data taken at 8.5 MeV of $^{13}$C are shown as diamonds and 8.0 MeV data as boxes. The dashed and solid lines are DWBA calculations at $^{13}$C energies of 8.31 and 7.81 MeV, respectively (see text).

approach via the code FRESCO (version FRXY.3h) [86]. The DWBA calculations were performed for beam energies at the center of the target of 7.81 and 8.31 MeV, taking into account the 380 keV energy loss of $^{13}$C in the target.

The extracted ANC, unlike the spectroscopic factor, does not depend on the number of nodes in the $\alpha$-$^{13}$C bound state wave function or the geometrical parameters of the Woods-Saxon potential. Parameters of the optical model potentials of the usual Woods-Saxon form which were used in the DWBA calculations are given in Table 3.1 [87, 88, 89]. The calculations were performed in the finite-range transfer approach with a full complex remnant term in the interaction potential such that $V_{int} = V(^{13}$C+$\alpha$)+VDC1-VLC1. The LC1 potential was used for the $^6$Li+$^{13}$C channel. This potential reproduces experimental data on the elastic scattering of $^6$Li by $^{13}$C at c.m. energies ranging from 3 to 23 MeV [87]. Experimental data on the elastic scattering of deuterons on $^{17}$O at low energies is
Table 3.1: Parameters of optical potentials used in DWBA calculations.

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<tr>
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<td>1.25</td>
<td>6.0</td>
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Optical potential was of Woods-Saxon form $V = V_c - V_0 f(r, R_V, a_V) - iW f(r, R_W, a_W) + iW_s \left(4\alpha_W \frac{df(r, R_W, a_W)}{dR}\right)$; $f(r, R, a) = \left(1 + \exp\left(-\frac{r-r_0}{a}\right)\right)^{-1}$; $R = r_0 A_2^{1/3}$; $r_0 = 1.25$ fm and $a = 0.68$ fm were used for $\alpha + d$ and $\alpha + ^{13}C$ form factor potentials with $V$ fitted to reproduce binding energy.

not available. Thus, several potentials for the $d + ^{17}O$ channel were used [89, 81]. The angular distributions shown in Figure 3.12 were calculated with the DO1 potential, however it was verified that other potentials produce essentially identical results, with variations in the normalization factor of less than 7% [89, 81]. This demonstrates that the transfer reaction cross section at sub-Coulomb energy only weakly depends upon the parameters of the optical potentials, which was the main point of making this measurement. In fact, a DWBA calculation with the nuclear part of the optical potentials removed results in only a 40% difference in the absolute value of the cross section at large angles. Further investigation into the sensitivity of the cross section to the parameters of the optical potentials was performed. This investigation showed that the cross section varies by less than 20% if the model parameters are kept within reasonable limits. It was also found that the extracted ANC was insensitive to the parameters of the core-core DC1 interaction potential in the full DWBA transition operator.

We determined the square of the Coulomb-modified ANC for the reaction $^{13}C + \alpha \rightarrow ^{17}O(\frac{1}{2}^+, 6.36$ MeV) to be

$$\left(C_{\alpha^{13}C}^{17}O^{\frac{1}{2}^+}\right)^2 = 0.89 \pm 0.23 \text{ fm}^{-1}.$$

The contribution of the $\frac{1}{2}^+$ state to the astrophysical $S$ factor calculated using Eq. 3.6 is shown as a dashed curve in Figure 3.14. It was verified that this result is insensitive to variations of the channel radius.

**Compound Nucleus Mechanism** The DWBA approach can be used only if the contribution of the compound nucleus (CN) mechanism to the resonance cross section is small.
The contribution of the CN mechanism to the 6.36 MeV state can be estimated using the measured cross section for the first $\frac{1}{2}^+$ state at 0.87 MeV in $^{17}$O (see Figure 3.11). The cross section of the 0.87 MeV state is 24 $\mu^b_{sr}$ at 8.31 MeV at 169° in c.m. If it is assumed that all of the cross section is due to CN, then the maximum contribution of the CN mechanism to the 6.36 MeV state can be estimated. According to the Hauser-Feshbach statistical model [90], the cross section can be defined as

$$\langle \frac{d\sigma}{d\Omega} \rangle = \pi \lambda^2 \sum \omega_f(U, J) \frac{2J + 1}{2I + 1} \frac{T_f T_i}{\sum_c [T_c + \int T_c \omega_c(U_c, J)]}$$

where $\omega$ is the level density of the continuum, $\lambda \approx (\mu_i E)^{-\frac{1}{\pi}}$, and

$$T_n = \sum_{\lambda} \frac{\langle \Gamma_n \rangle \langle \Gamma \rangle}{(E - E_{\lambda})^2 + \frac{1}{4} (\Gamma_{\lambda} + \Gamma'_{\lambda})^2}$$

with $\Gamma_n \sim 2P_n(kr)$ and $\Gamma$ equal to the total width of the level. Equations 3.20 and 3.21 show that for states of the same spin parity, the cross section scales according to the penetrability factor for the decay into the $^{17}$O+d channel. Therefore, the probability to populate the $\frac{1}{2}^+$ state at 6.36 MeV is about 6 times less than that of the state at 0.87 MeV. The maximum possible cross section due to the CN mechanism for the 6.36 MeV state is 6% of the measured 70 $\mu^b_{sr}$ (calculated as the ratio of the penetrability factors of the 6.36 and 0.87 MeV states, $\frac{1}{6} \times 24 \mu^b_{sr} = 4 \mu^b_{sr}$). This value of the CN cross section was confirmed using the code EMPIRE [91]. Therefore, the DWBA approach is applicable.

**Continuum Discretized Coupled Channel (CDCC) Calculations** The DWBA calculation used to extract the ANC does not take two-step processes into account. One such process is the excitation of the $^6$Li target into its continuum. This type of excitation may effect the $^{13}$C($^6$Li,d)$^{17}$O($\frac{1}{2}^+$, 6.36 MeV) cross section and therefore the extraction of the ANC for this state. In order to determine if the continuum excitation of $^6$Li effects our result for the ANC, a continuum discretized coupled-channels (CDCC) calculation was performed.

In the early stages of coupled channels calculations, the continuum breakup states were approximated by a single channel with an average internal energy [92]. Through this method the effect of the breakup process was shown to be significant for reactions involving deuterons [93, 94, 95] and $^6$Li ions [96]. Eventually this idea was modified, and the averaged continuum state was replaced by a finite number of discrete channels [97]. This CDCC theory has been established as an accurate method of solving the three-body reaction [98].
Figure 3.13: Angular distribution of the $^6$Li($^{13}$C,d)$^{17}$O(6.36 MeV) reaction, data were taken at 8.5 MeV of $^{13}$C. The lines are calculations of the angular distribution normalized to the data points. The red line is from the DWBA approach, the green line is calculated using the CDCC approach, and the blue line is from the CDCC approach including elastic transitions for the continuum states.

In a private communication with Dr. Kazuyuki Ogata, the CDCC method was used to describe the $^6$Li+$^{13}$C wave function by treating it as an ($\alpha$+d)+$^{13}$C three-body system. Using this CDCC result, Dr. Ogata determined the breakup effects on the spectroscopic factor $S_\alpha$ to be between 5% and 10% (see Figure 3.13). This difference is well within the quoted uncertainty of 25%; therefore the breakup effects of the continuum can be excluded from our result.
Determination of the Contribution of Near Threshold States in $^{17}$O to the Astrophysical $S$-factor using the $R$-Matrix Approach

In order to determine the $^{13}$C($\alpha$,n) reaction rate, we need to know the cross section of this reaction at the relevant stellar energy. This energy is called the Gamow window and represents the energy range in which the cross section for this reaction will have a maximum impact on the final reaction rate at the stellar temperatures of an AGB star. For this reaction the Gamow window is 143 to 231 keV. The total cross section in this region is a combination of the resonance and direct (non-resonant) reaction cross sections such that

$$\sigma = \sigma_{\text{resonant}}(E) + \sigma_{\text{non-resonant}}(E) \quad (3.22)$$

The resonance contribution can be determined by doing an $R$-Matrix calculation including the parameters of all influential states from $^{17}$O. All resonances from 4.14 MeV (neutron decay threshold) to 8.2 MeV excitation energy in $^{17}$O were included. The parameters of the resonances were taken from [72]. Of particular importance is the partial $\alpha$ width, $\Gamma_\alpha$, of the $\frac{1}{2}^+$ state at 6.36 MeV. This partial width is related to the measured ANC by

$$\Gamma_\alpha = \frac{2P_c(ka_c)\gamma_\alpha}{1 - \gamma_\alpha \left. \frac{dS}{dE} \right|_{r=a_c}}, \quad (3.23)$$

where $P_c$ is the penetrability factor for the $\alpha$ decay channel, $k$ is the wave number, $a_c$ is the $\alpha$ decay channel radius, $\frac{dS}{dE}$ is the derivative of the shift function, and $\gamma_\alpha$ is the reduced $\alpha$ width of the resonance and is equal to

$$\gamma_\alpha = \frac{\hbar^2}{2\mu a_c} ANC^2 W^2(a_c), \quad (3.24)$$

where $\mu$ is the reduced mass and $W(a_c)$ is the Whittaker function calculated at the channel radius. The non-resonant cross section is the result of a $^{13}$C($\alpha$,n) reaction in which the compound nucleus, $^{17}$O, is never formed. For example, $^{12}$C can be transferred from $^{13}$C to the $\alpha$ particle leaving the neutron as a spectator. This non-resonant contribution to the $S$-factor was estimated to be on the order of $10^6$ MeV·b using the code FRESCO [86].

In order to remove the strong dependence of the reaction cross section on energy the astrophysical $S$-factor was introduced. This is done by defining the $S$-factor as

$$S(E_{\text{c.m.}}) = \sigma E_{\text{c.m.}} \exp[-2\pi\eta], \quad (3.25)$$
Figure 3.14: $S$-factor of the $^{13}$C($\alpha$, $n$) reaction. The experimental data, corrected for electron screening, are from Refs. [99, 100]. The best fit obtained with a $0.4 \times 10^6$ MeV·b non-resonance contribution is shown as the solid curve. The contribution of the $\frac{1}{2}^+$ state is shown as the dashed curve. The dashed-dotted curves represent a 28% uncertainty band due to the uncertainty in the $\frac{1}{2}^+$ $S(0)$ factor. The dotted curves represent the overall uncertainty band and were calculated using a $(0.2$ and 0.6$) \times 10^6$ MeV·b non-resonance contribution with a $\frac{1}{2}^+$ $S(0)$ factor of $(1.8$ and 3.2$) \times 10^6$ MeV·b, respectively (see text).

where $\eta$, defined in Eq. 3.28, is the Coulomb parameter. This removes almost entirely the energy dependence of the direct reaction cross section since this dependence is due mainly to the Coulomb barrier. It was determined that for the energy region considered, $\sigma_{non-resonant}$ is nearly a constant with respect to energy and that the non-resonant reaction is dominated by a $\frac{3}{2}^-$ partial wave which has $l=0$ in the $\alpha+^{13}$C channel. Thus possible interference of the direct process with the $\frac{1}{2}^+$ resonance should be small and was not taken into account. As
such, we determined the $S$-factor using

$$S(E_{c.m.}) = [\sigma_{\text{resonant}}(E_{c.m.}) + \sigma_{\text{non-resonant}}] E_{c.m.} \exp[-2\pi\eta],$$

(3.26)

where $\sigma_{\text{non-resonant}}$ is used to represent the contribution of the non-resonant cross section to the total cross section, and $\sigma_{\text{resonant}}$ was determined using a two channel $R$-Matrix approach as discussed above. The $\alpha$-reduced widths of resonances were fitted to reproduce the $^{13}$C($\alpha$,n) experimental data [99, 100]. The calculated $S$-factor curve along with the experimental data are shown in Figure 3.14. The best fit curve uses $\sigma_{\text{non-resonant}} E_{c.m.} \exp[-2\pi\eta] = 0.4 \times 10^6$ MeV·b in order to fit the experimental $S$-factor data in the lowest energy region. Also it was confirmed that reasonable fits can be obtained if the non-resonance contribution is varied from $(0.2$ to $0.6) \times 10^6$ MeV·b.

There are six sources of uncertainty associated with the $S(E_{c.m.} = 0)$ factor of the 6.36 MeV, $\frac{1}{2}^+$ state. They are: a statistical error of 7%, a combined systematic error of 7% originating from the determination of the product of the target thickness times the solid angle, a 20% uncertainty in the theoretical analysis, a 10% uncertainty in the total resonance width, a 10% uncertainty in the known value of the $\alpha$–d ANC, $(C_{ad}^d)^2 = 5.3 \pm 0.5$ fm$^{-1}$[101], and 4% due to the 8 keV uncertainty in the excitation energy of the $\frac{1}{2}^+$ resonance. The determined $S(0)$ value of the 6.36 MeV $\frac{1}{2}^+$ state in $^{17}$O is $(2.5 \pm 0.7) \times 10^6$ MeV·b, which is 10 times smaller than that adopted in the NACRE compilation [82] and a factor of 5 larger than the value reported in [78].

**Reaction Rate**

The reaction rate can now be calculated from the adopted $S$-factor curve using the following relation:

$$R_{\text{reaction}}(T) = A(T) \int_0^\infty S(E) e^{-2\pi\eta} e^{\frac{E}{k_B T}} dE,$$

(3.27)

where,

$$\eta = \alpha Z_1 Z_2 \sqrt{\frac{\mu}{2E}},$$

(3.28)

$\alpha$ is the fine structure constant and,

$$A(T) = N_A \sqrt{\frac{8}{\mu \pi} (k_B T)^{\frac{3}{2}}}.$$
Figure 3.15: Rate of the $^{13}$C($\alpha$, $n$) reaction. The dashed-dotted curve is from [82], the solid curve is the rate obtained here. The yellow band in the inset represents the uncertainties from [82], new uncertainties are shown in red.

The calculated reaction rates are shown in Figure 3.15. The best fit rate is shown as a solid curve in comparison with the adopted rate from NACRE (dash-dotted curve) [82]. As expected, the best fit rate is identical to the adopted curve in NACRE for temperatures greater than 0.3 GK since the contribution of the $\frac{1}{2}^+$ resonance is small in this region. However, at the temperatures which are most significant for the s-process in AGB stars, 0.08–0.1 GK, the reaction rate is smaller by a factor of 3 than that adopted in the NACRE compilation, but it is still inside the NACRE uncertainty band. Also, the uncertainty in this
Table 3.2: The rate of the $^{13}$C($\alpha$,n) reaction at temperatures from 0.08 to 0.1 GK. The rate obtained in this work is compared with the rate published in the NACRE [82] compilation. Units are [cm$^3$ s$^{-1}$ mol$^{-1}$], exp stands for $10^{\text{exp}}$. High and low values were calculated assuming a 26% uncertainty in the $\frac{1}{2}^+$, 6.36 MeV resonance contribution.

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astrophysically important reaction rate has been reduced from $\sim$300% to 25%. The inset in Figure 3.15 shows the NACRE compilation and the reaction rate obtained in this work with their uncertainty bands. Numerical values of the reaction rate for 0.08–0.1 GK are given in Table 3.2.

3.3.4 Summary

In summary, I have described an indirect technique which allows for the measurement of the astrophysical $S(0)$-factor of sub-threshold, particle unbound resonances and then applied this technique to measure the contribution of the $\frac{1}{2}^+$, 6.36 MeV resonance in $^{17}$O to the $^{13}$C($\alpha$,n) reaction rate at stellar temperatures. The combination of a sub-Coulomb $\alpha$-transfer reaction with the application of the ANC technique in the analysis eliminates nearly all dependence of the results on model parameters. This model independence makes this approach a very valuable tool for future studies of astrophysically important reaction rates with both stable and radioactive beams. The $^{13}$C($\alpha$,n) reaction rate at the relevant stellar temperatures was found to be a factor of 3 lower than the previously adopted rate [82]; and the uncertainty in this reaction rate was greatly reduced. The fact that this rate is so small may imply that not all of the $^{13}$C inside of the $^{13}$C pockets within AGB stars will be burned. This may change the neutron balance and affect the nucleosynthesis process occurring inside of AGB stars and therefore it would be of great interest to incorporate the new reaction rate into models of AGB stars.
3.4 Determination of the $^{14}$C($\alpha, \gamma$) Reaction Rate Using the $^{14}$C($^7$Li,t) Sub-Coulomb $\alpha$-Transfer Reaction

3.4.1 Motivation

Another ingredient needed for improving the models of AGB stars is an understanding of the abundance of fluorine. As discussed in Section 3.3.1, carbon rich regions may be formed within the He shell of AGB stars during thermal pulses. Observations indicate a positive correlation between the formation of carbon rich regions and an enhanced abundance of fluorine within AGB stars [102]. This correlation can be explained if $^{19}$F is produced and dredged to the surface alongside $^{12}$C and the s-process elements within the He shell [102].

The production of $^{19}$F can occur through two nucleosynthesis paths as proposed in [103]. Both paths begin with capture reactions by $^{14}$N which is produced through the CNO cycle

$$^{12}\text{C} + p \rightarrow ^{13}\text{N} + \gamma \Rightarrow ^{13}\text{N} \rightarrow ^{13}\text{C} + e^+ + \nu_e \Rightarrow ^{13}\text{C} + p \rightarrow ^{14}\text{N} + \gamma.$$ 

One path starts with the capturing of neutrons produced in the reaction $^{13}$C($\alpha$,$n$)$^{16}$O. This reaction, $^{14}$N(n,p)$^{14}$C, has a high cross section and produces both free protons and $^{14}$C which can then capture an $\alpha$ particle and produce $^{18}$O through the reaction $^{14}$C($\alpha, \gamma$)$^{16}$O. The other path begins with an $\alpha$ capture reaction by $^{14}$N, $^{14}$N($\alpha, \gamma$)$^{18}$F, followed by the $\beta$-decay of $^{18}$F to again produce $^{18}$O. The $^{18}$O produced from both paths can then capture a proton through one of two reactions, $^{18}$O(p,$\alpha$)$^{15}$N or $^{18}$O(p,$\gamma$)$^{19}$F. The $^{15}$N can capture an $\alpha$ particle through $^{15}$N($\alpha, \gamma$)$^{19}$F to also produce $^{19}$F. Here is a summary of these paths:

$$^{13}\text{C}(\alpha, n) \rightarrow ^{14}\text{N}(n, p)^{14}\text{C}(\alpha, \gamma)^{18}\text{O}(p, \alpha)^{15}\text{N}(\alpha, \gamma)^{19}\text{F}$$

or

$$^{14}\text{N}(\alpha, \gamma)^{18}\text{F}(\beta^+)^{18}\text{O}(p, \gamma)^{19}\text{F}.$$ 

Current theoretical models are only able to predict the lowest observed abundances of $^{19}$F [104, 105], as can be seen in Figure 3.16. The abundance of $^{19}$F is controlled by the production paths above along with the destruction reaction $^{19}$F($\alpha$,$p$)$^{22}$Ne, but the main uncertainty in the $^{19}$F nucleosynthesis paths is the $^{14}$C($\alpha, \gamma$)$^{18}$O reaction rate [102]. Also, $^{14}$C($\alpha, \gamma$) is a possible escape from the hot CNO cycle in the environment of white dwarf
stars which are accreting mass from a partner in a binary system [106]. The uncertainty in the $^{14}$C($\alpha,\gamma$) reaction rate is due to a prohibitively small cross section at the relevant stellar energies which cannot be measured directly. To avoid this problem we applied the sub-Coulomb ANC technique to indirectly determine the contribution of the 3$^-$ state at $E_{\text{exc}} = 6.40$ MeV in $^{18}$O to the $^{14}$C($\alpha,\gamma$) reaction rate (see Figure 3.17). The spectroscopic factor, $S_\alpha$, of this state is unknown but has been estimated by [107] (based on systematics) to be $S_\alpha = 0.02$. Using the sub-Coulomb $\alpha$-transfer reaction $^{14}$C($^7$Li,$t$) we were able to experimentally measure the ANC of the above mentioned 3$^-$ state and therefore determine the $^{14}$C($\alpha,\gamma$) reaction rate.

Figure 3.16: Relative abundance of $^{19}$F. The observed data (various points) are unexplained by the theoretical curves (points with lines) [102].
3.4.2 Experimental Specifics

The measurement of the ANC of the virtual synthesis $\alpha + ^{14}C \rightarrow ^{18}O^*$ for the contributing state was performed using the $\alpha$-transfer reaction $^{14}C(^{7(6)}Li, t(d))$ at the Florida State University, John D. Fox Superconducting Accelerator Laboratory. The energies for the $^{14}C$ beam were 8.8 MeV for the $^6Li$ target and 11.5 MeV for the $^7Li$, which correspond to c.m. energies of 2.64 and 3.83 MeV, respectively. These energies were chosen to be just below the Coulomb barrier in the exit channel. The exit channel Coulomb barrier was again the crucial energy due to the fact that it is smaller than that of the entrance channel. The lithium targets used were $\approx 20 \mu g/cm^2$. Products from the reaction were measured using $\Delta E - E$ telescopes placed 7.5° apart and ranging in angle from 8° to 50.5° (see Figure 3.3).
Figure 3.18: Spectrum of tritons measured from the $^7$Li($^{14}$C,t) transfer reaction with a $^{14}$C beam energy of 11.5 MeV.

### 3.4.3 Analysis and Results

Two different targets, and therefore two different reactions, were used during this experiment, $^{14}$C($^6$Li,d) and $^{14}$C($^7$Li,t). This was necessary due to the very low cross section for the state of interest, $3^-$ at 6.40 MeV, when using the ($^6$Li,d) reaction. Sample spectra of tritons and deuterons are shown in Figures 3.18 and 3.19. All of the peaks in the spectra correspond to well known states (or group of states) in $^{18}$O and are labeled accordingly (see Figure 3.17). All available spectra are shown in Sections A.3 and A.4. As can be seen in Figure 3.19, the number of counts in the 6.40 MeV peak is statistically insignificant. Therefore, we changed to a $^7$Li target and repeated the experiment. It appeared to be the solution we were looking for since the 6.40 MeV state then had a significant number of counts in the peak (see Figure 3.18). However, the $2^-$ state at 6.35 MeV may in principle contribute to the cross section in the region of the $3^-$ state since our energy resolution was not good enough to separate them. Fortunately, the cross section for this $2^-$ state should be small since it can only be populated through the compound nucleus mechanism, rather than the transfer reaction, due to its unnatural parity. However, since the $3^-$ is a weak state, the possible contribution of the $2^-$ state has to be carefully evaluated.
Figure 3.19: Spectrum of deuterons measured from the $^6$Li($^{14}$C,d) transfer reaction with a $^{14}$C beam energy of 8.8 MeV.

We were able to determine the ANC of the $3^-$ state at 6.40 MeV in $^{18}$O and its influence on the rate of the $^{14}$C($\alpha,\gamma$) reaction. This analysis was done using a 3 step procedure which included fitting several peaks using a Gaussian curve calculated using

$$N_{\text{counts}}(E) = \sum_i A_i \exp \left[ -\frac{(E - E_i)^2}{2\sigma_{\exp}^2} \right],$$  \hspace{1cm} (3.30)

where the sum is over the number of resonances in the group, $E$ is the channel energy, $E_i$ and $A_i$ are the excitation energy and Gaussian multiplication constant of the $i^{th}$ resonance, respectively, and $\sigma_{\exp}$ is the experimental energy resolution. First, the $1^-$ state at 4.46 MeV was fit with one Gaussian in order to extract the experimental resolution of one state so that it could be set to a fixed value when fitting groups of states. Next, the 5.3 MeV group was fit using 4 summed Gaussians with the width fixed to the value obtained in the previous fit, and including the $2^-$ peak at 5.53 MeV (see Figure 3.20). From this fit we were able to extract a value for $A_{2^-}$. According to the Hauser-Feshbach statistical model (see Eq. 3.20), for a $2^-$ state at higher excitation energy, like the one at 6.35 MeV, the cross section can be no larger than the one calculated from the Gaussian parameters of the $2^-$ at 5.53 MeV, properly scaled according to Eq. 3.20. Therefore, we can now use the extracted values of
Figure 3.20: Fit of the 5.3 MeV group measured from the $^7$Li($^{14}$C,t) transfer reaction with a $^{14}$C beam energy of 11.5 MeV. The black line is a fit of a sum of Gaussians for each state in the group. The colored lines represent the contributions of each individual state to the fit.

Figure 3.21: Fit of the 3$^-$ state at 6.40 MeV measured from the $^7$Li($^{14}$C,t) transfer reaction with a $^{14}$C beam energy of 11.5 MeV. The black line is a fit of a sum of three Gaussians, one for each state. The colored lines represent the contributions of each individual state to the fit.
Figure 3.22: Angular distribution for the $3^-$ state at 6.40 MeV measured from the \(^7\text{Li}(^{14}\text{C},t)^{18}\text{O}(3^-, 6.40 \text{ MeV})\) transfer reaction. The red line is the DWBA curve of the reaction.

\(A_2\) and \(\sigma_{exp}\) to fix the contribution of the $2^-$ state at 6.35 MeV to the group of peaks at \(\sim 6.3\) MeV. We then fit this group with 3 summed Gaussians including the fixed $2^-$ state and were able to extract the cross section of the $3^-$ state at 6.40 MeV (see Figure 3.21).

**DWBA**

The angular distribution of the reaction \(^{14}\text{C}(^{7}\text{Li},t)^{18}\text{O}(3^-, 6.40 \text{ MeV})\) measured at a beam energy of 11.5 MeV for \(^{14}\text{C}\) is shown in Figure 3.22. The shape of this angular distribution is typical for a sub-Coulomb transfer reaction. However, it is starting to show some small effects of the nuclear interaction due to the higher incident energy, as shown by the small bump at \(\approx 70^\circ\). The absolute normalization of the cross section was performed by measuring elastic scattering of protons at 6.868 MeV as previously discussed in Section 3.2. The angular distribution curve was calculated using the DWBA approach via the code \texttt{FRESCO} (version
Table 3.3: Parameters of optical potentials used in DWBA calculations.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Potential</th>
<th>$V_0^a$ [MeV]</th>
<th>$a_V$ (fm)</th>
<th>$r_V$ (fm)</th>
<th>$W$ [MeV]</th>
<th>$W^b_0$ [MeV]</th>
<th>$a_W$ (fm)</th>
<th>$r_W$ (fm)</th>
<th>$r_c$</th>
<th>$V^c_0$ [MeV]</th>
<th>$a_{so}$ (fm)</th>
<th>$r_{so}$ (fm)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^7$Li + $^{14}$C</td>
<td>LC2</td>
<td>250.0</td>
<td>0.66</td>
<td>1.35$^c$</td>
<td>-</td>
<td>30.0</td>
<td>0.66</td>
<td>1.35</td>
<td>2.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>[69]</td>
</tr>
<tr>
<td>$^7$Li + $^{14}$C</td>
<td>LC3</td>
<td>33.1</td>
<td>0.85</td>
<td>1.50$^c$</td>
<td>-</td>
<td>31.2</td>
<td>0.72</td>
<td>1.72</td>
<td>1.50</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>[69]</td>
</tr>
<tr>
<td>d + $^{18}$O</td>
<td>DO5</td>
<td>92.92</td>
<td>0.814</td>
<td>1.04$^c$</td>
<td>-</td>
<td>10.1</td>
<td>0.71</td>
<td>1.40</td>
<td>1.30</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>[67]</td>
</tr>
<tr>
<td>t + $^{18}$O</td>
<td>TO</td>
<td>130.0</td>
<td>0.72</td>
<td>1.31$^c$</td>
<td>8.0</td>
<td>-</td>
<td>0.80</td>
<td>1.60</td>
<td>1.40</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>[108]</td>
</tr>
<tr>
<td>$\alpha$ + d</td>
<td>AD</td>
<td>varied$^d$</td>
<td>0.65</td>
<td>1.55$^c$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>[67]</td>
</tr>
<tr>
<td>$\alpha$ + t</td>
<td>AT</td>
<td>varied$^d$</td>
<td>0.65</td>
<td>1.15$^c$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>[108]</td>
</tr>
<tr>
<td>$\alpha$ + $^{14}$C</td>
<td>AC</td>
<td>varied$^d$</td>
<td>0.65</td>
<td>1.25$^c$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>[67]</td>
</tr>
</tbody>
</table>

Optical potential was of Woods-Saxon form $V = V_0 - V_0 f(r, R, a_V) - iW f(r, R, a_W) + iW^c (4a_W \frac{d(r, R, a_W)}{dr})$: $f(r, R, a) = (1 + \exp \frac{r-R}{a})^{-1}$. $^a$Form Factor: Woods-Saxon. $^b$Form Factor: Woods-Saxon derivative. $^c R = r_0 A^\frac{1}{3}$. $^d$Varied to reproduce binding energies. $^e R = r_0 (A^\frac{1}{3} + a^\frac{1}{3})$. $^f$Spin-orbit potential.

FRXY.3h) [86]. It should be noted that this code is designed to be used with bound states, so in order to perform this calculation we treated the 3$^-$ state as such. A small artificial binding energy of 10 keV with respect to the $\alpha$-decay threshold was used.

The extracted ANC, unlike the spectroscopic factor, does not depend on the number of nodes in the $\alpha$+$^{14}$C bound state wave function or the geometrical parameters of the Woods-Saxon potential. Parameters of the optical model potentials of the usual Woods-Saxon form which were used in the DWBA calculations are given in Table 3.3. The calculations were performed in the finite-range transfer approach with a full complex remnant term in the interaction potential such that $V_{int} = V^{(14C+\alpha)} + V^{TC-VLC3}$. The LC3 potential was used for the $^7$Li+$^{14}$C channel. The angular distribution shown in Figure 3.22 was calculated with the TO and DO5 potentials.

The ANC was determined using Eq. 3.17, where the ANC for $^7$Li($\alpha$-t) is well known. From this we determined the square of the Coulomb-modified ANC, for the reaction $^{14}$C+$\alpha$ $\rightarrow$ $^{18}$O(3$^-$, 6.40 MeV) to be

$$(C^{18O}_{\alpha14C})^2 = (2.96 \pm 1.12) \times 10^{-9} \text{ fm}^{-1}.$$  

The uncertainty in the DWBA cross section was determined by performing a Monte Carlo variation of all the nuclear potential parameters given in Table 3.3. This process resulted in a Gaussian distribution of values for the ratio between the experimental and DWBA cross sections, with a standard deviation of 30%. This uncertainty is somewhat larger than hoped for relative to our result on the $^{13}$C($\alpha$,n) reaction rate, and is due to the higher
Figure 3.23: Reaction rate curves of various resonance contributions to the $^{14}\text{C}(\alpha, \gamma)$ reaction rate. Each curve consists of an adopted rate plotted over a wider band representing the uncertainty in the reaction rate [110]. The green band represents the stellar temperature range for nucleosynthesis in AGB stars.

incident energy of the beam for the $^7\text{Li}$ target. Even so, this uncertainty again demonstrates that the transfer reaction cross section at a sub-Coulomb energy only weakly depends upon the parameters of the optical potentials. There are 4 sources of uncertainty in this ANC value. They are: a statistical uncertainty of 20\%, a combined systematic uncertainty of 7\% originating from the determination of the product of the target thickness times the solid angle, a 30\% uncertainty in the theoretical analysis, and a 9\% uncertainty in the known value of the $\alpha - t$ ANC, $(C_{\alpha t}^{L i})^2 = 12.74 \pm 1.1 \text{fm}^{-1}$ [109].
Reaction Rate

The resonance strength of the 6.40 MeV, $3^-$ state in $^{18}$C is equal to

$$\omega\gamma = \frac{2J_{18O^*} + 1}{(2J_{14C} + 1)(2J_{\alpha} + 1)} \Gamma_{\alpha} \Gamma_{\gamma}. \quad (3.31)$$

Since $\Gamma_{\alpha} \ll \Gamma_{\gamma}$, $\Gamma_{\gamma} \approx \Gamma_{\text{total}}$ and $\omega\gamma$ is directly proportional to the observed partial $\alpha$ width, $\Gamma_{\alpha}$, which is related to the measured ANC of the $\frac{1}{2}^+$, 6.36 MeV state in $^{17}$O by Eqs. 3.23 and 3.24. We determined $\Gamma_{\alpha}$ and $\omega\gamma$ to be

$$\Gamma_{\alpha} = (1.03 \pm 0.39) \times 10^{-13} \text{ eV},$$
$$\omega\gamma = (7.21 \pm 2.72) \times 10^{-13} \text{ eV}.$$ 

The resonance strength can then be used to calculate the $^{14}$C($\alpha, \gamma$) reaction rate using

$$N_A \langle \sigma v \rangle = 1.54 \times 10^{11}(\mu T_9)^{-\frac{3}{2}} \omega\gamma \exp \left[\frac{-11.605E_r}{T_9}\right], \quad (3.32)$$

taken from [111], where $T_9$ is the temperature in GK, $\mu$ is the unitless reduced mass, and $E_r$ is the c.m. resonance energy. The calculated reaction rates of the $3^-$ resonance and several others are shown in Figure 3.23. The black and red lines represent the contribution of the $4^+$ at 7.11 MeV [110] and the $3^-$ at 6.40 MeV, respectively. The green shaded region in Figure 3.23 represents the temperature region for which nucleosynthesis occurs inside of AGB stars. In this region the $3^-$ state dominates the reaction rate. The blue line represents the direct capture contribution and is always of secondary influence [110]. The purple line is due to the $1^-$ state at 7.61 MeV, which never dominates the reaction rate for stellar temperatures.

3.4.4 Summary

In summary, the sub-Coulomb ANC technique described in Section 3.1.1 was applied to the transfer reaction $^{14}$C($^7$Li,$t$) in order to determine the $^{14}$C($\alpha, \gamma$) reaction rate. This was done by measuring the contribution of the $3^-$, 6.40 MeV resonance in $^{18}$O. Previously, the effects of this state on the $^{14}$C($\alpha, \gamma$) reaction rate had only been statistically estimated. As shown in Figure 3.23, this state is dominant in the temperature region of an AGB star. The valuable combination of a sub-Coulomb $\alpha$-transfer reaction with the application of the ANC technique in the analysis greatly reduces the dependence of the results on the model parameters. Implementation of this result into AGB stellar models may help to better understand the unexplained abundance of $^{19}$F in the universe.
CHAPTER 4

CONCLUSION

The TTIK technique has been shown to be an efficient method of studying the properties of \( \alpha \)-cluster states in nuclei. Having the ability to measure an excitation function with only one beam energy will allow for much more detailed study of the cluster states in non-\( \alpha \)-conjugate nuclei.

There are several strongly clustered resonances within the excitation region which was analyzed in this work as shown in Table 2.3. These states give some indications for an \( \alpha \)-cluster rotational band of negative parity, but fragmentation of states along with the inversion of the \( 1^- \) and \( 3^- \) states indicate that the \( \alpha \)-cluster structure of \( ^{18} \text{O} \) is more complicated than in neighboring \( \alpha \)-conjugate nuclei. The fragmentation of the states with a large \( \alpha \) reduced width may be caused by an overlap between the wave functions of the cluster states and those of the single particle states, and results in many states of the same spin-parity with small \( \alpha \) widths being close in excitation energy (see Table 2.1). This type of splitting has been predicted to occur within \( ^{18} \text{O} \) in [1], however we do not see a dominant \( \alpha \)-cluster state for all spin-parities as they predict. The observed mixing of \( \alpha \)-cluster states with single particle states can help to understand the connections between the \( \alpha \)-cluster and shell models.

Also, we observed a \( 0^+ \) state at \( \sim 10.1 \) MeV in \( ^{18} \text{O} \) which has the maximum amount of \( \alpha \)-clusterization possible. Similar states have also been observed in \( ^{12} \text{C}, ^{16} \text{O}, \text{and} ^{20} \text{Ne} \) which indicates that this type of extremely clustered state may be a general feature of nuclei in this mass region. A \( 2^+ \) state of the same nature was also observed at \( \sim 12.1 \) MeV in \( ^{18} \text{O} \), however, further analysis of the excitation function at higher energy (than in this work) is needed to confirm the existence of this \( 2^+ \) state.

While the common properties of \( \alpha \)-clusterization in nuclei with different ratios of protons and neutrons is vital for understanding nuclear structure it is also important for astrophysics.
Due to the abundance of helium in stars, many reactions between \( \alpha \) particles and other light nuclei play an important role in nucleosynthesis.

Two important astrophysical reaction rates were determined using an indirect technique which allows for determination of the astrophysical \( S(E_{\text{c.m.}} = 0) \) factor of near threshold resonances. The contributions of the \( \frac{1}{2}^+ \) resonance at 6.36 MeV in \( ^{17}\text{O} \) to the \( ^{13}\text{C}(\alpha,n) \) reaction rate and the \( 3^- \) resonance at 6.40 MeV in \( ^{18}\text{O} \) to the \( ^{14}\text{C}(\alpha,\gamma) \) reaction rate, both at the relevant stellar temperatures, were determined. The combination of a sub-Coulomb \( \alpha \)-transfer reaction with the application of the ANC technique in analysis practically eliminates all dependence of the results on model parameters. This model independence makes this approach a very valuable tool for future studies of astrophysically important reaction rates with both stable and radioactive beams. The \( ^{13}\text{C}(\alpha,n) \) reaction rate at the relevant stellar temperatures was found to be a factor of 3 lower than the previously adopted rate \([82]\), also uncertainty in this reaction rate was greatly reduced. The small value of this rate implies that not all of the \( ^{13}\text{C} \) inside of the \( ^{13}\text{C} \) pockets within AGB stars will be burned. The \( ^{14}\text{C}(\alpha,\gamma) \) reaction rate is dominated by the near threshold \( 3^- \) state in \( ^{18}\text{O} \) for AGB nucleosynthesis temperatures. Previously, the effects of this state on the \( ^{14}\text{C}(\alpha,\gamma) \) reaction rate had only been statistically estimated. Applying the calculated contribution of this \( 3^- \) state to the nucleosynthesis models of AGB stars may help to decrease the discrepancy between the observed and calculated abundance of \( ^{19}\text{F} \).

Fully understanding the \( \alpha \)-cluster structure of oxygen isotopes may help to gain insight into the interplay between the single nucleon and \( \alpha \)-cluster degrees of freedom in atomic nuclei. The \( \alpha \)-cluster structure of oxygen isotopes also plays an important role in nucleosynthesis. The work outlined here has shown that \( \alpha \)-clusterization can lead to interesting and new phenomena in the field of nuclear structure such as pure \( \alpha \)-particle states. Also, cluster properties of near threshold states have a heavy impact on the astrophysical \( \alpha \)-capture reactions.

The author expects that detailed study of the \( \alpha \)-cluster structure of \( N \neq Z \) nuclei coupled with theoretical analysis may produce interesting and unexpected results. However, even now it has become clear that the \( \alpha \)-cluster degree of freedom plays a significant role in the \( N \neq Z \) nuclei.
APPENDIX A

EXPERIMENTAL SPECTRA

A.1 $^{18}$O Excitation Functions

Included here are all the angles of the $^{14}$C($\alpha$, $\alpha$) elastic scattering excitation functions used in the \textit{R}-Matrix fitting procedure referenced in Chapter 2.

![Figure A.1: Excitation function of elastic scattering of $^{14}$C on $\alpha$ particles along with a convoluted \textit{R}-matrix fit(solid)](image)
Figure A.2: Excitation function of elastic scattering of $^{14}$C on $\alpha$ particles along with a convoluted $R$-matrix fit (solid)

Figure A.3: Excitation function of elastic scattering of $^{14}$C on $\alpha$ particles along with a convoluted $R$-matrix fit (solid)
Figure A.4: Excitation function of elastic scattering of $^{14}\text{C}$ on $\alpha$ particles along with a convoluted $R$-matrix fit(solid)

Figure A.5: Excitation function of elastic scattering of $^{14}\text{C}$ on $\alpha$ particles along with a convoluted $R$-matrix fit(solid)
Figure A.6: Excitation function of elastic scattering of $^{14}$C on $\alpha$ particles along with a convoluted $R$-matrix fit (solid)

Figure A.7: Excitation function of elastic scattering of $^{14}$C on $\alpha$ particles along with a convoluted $R$-matrix fit (solid)
Figure A.8: Excitation function of elastic scattering of $^{14}$C on $\alpha$ particles along with a convoluted $R$-matrix fit (solid).

Figure A.9: Excitation function of elastic scattering of $^{14}$C on $\alpha$ particles along with a convoluted $R$-matrix fit (solid).
Figure A.10: Excitation function of elastic scattering of $^{14}$C on $\alpha$ particles along with a convoluted $R$-matrix fit (solid).

Figure A.11: Excitation function of elastic scattering of $^{14}$C on $\alpha$ particles along with a convoluted $R$-matrix fit (solid).
Figure A.12: Excitation function of elastic scattering of $^{14}$C on $\alpha$ particles along with a convoluted $R$-matrix fit (solid).

Figure A.13: Excitation function of elastic scattering of $^{14}$C on $\alpha$ particles along with a convoluted $R$-matrix fit (solid).
A.2 $^{13}\text{C}(^{6}\text{Li},d)$ Deuteron Spectra

Spectra of deuterons measured from the $^{13}\text{C}(^{6}\text{Li},d)$ transfer reaction with a $^{13}\text{C}$ beam energy of 8.5 MeV. Each spectrum is measured at the given angle.

Figure A.14: Spectra of deuterons measured from the $^{13}\text{C}(^{6}\text{Li},d)$ transfer reaction, the lab angle is given in the figure.
A.3 $^{14}\text{C}(^7\text{Li},t)$ Triton Spectra

Spectra of deuterons measured from the $^{14}\text{C}(^7\text{Li},t)$ transfer reaction with a $^{14}\text{C}$ beam energy of 11.5 MeV. Each spectrum is measured at the given angle.

![Graphs showing spectra at different angles](image)

Figure A.15: Spectra of tritons measured from the $^{14}\text{C}(^7\text{Li},t)$ transfer reaction.
Figure A.16: Spectra of tritons measured from the $^{14}\text{C}(^7\text{Li},t)$ transfer reaction.
A.4  $^{14}\text{C}(^{6}\text{Li,}d)$ Deuteron Spectra

Spectra of deuterons measured from the $^{14}\text{C}(^{6}\text{Li,}d)$ transfer reaction with a $^{14}\text{C}$ beam energy of 8.8 MeV. Each spectrum is measured at the given angle.

Figure A.17: Spectra of deuterons measured from the $^{14}\text{C}(^{6}\text{Li,}d)$ transfer reaction.
APPENDIX B
DETAILS OF THE PROGRAM MINRMATRIX

B.1 Input

The MinRinput file contains the names of all input and output files and paths which are needed by MINRmatrix and should be redirected into the MINRmatrix executable. All of the input files should be in a directory named input, and all of the output files should be sent to an output directory (see Section B.4). The files and paths listed in MinRinput must remain in the order described below. The following paragraphs will describe the information contained in the input files.

FileList The first input file contains the names of the data files to be included in the calculation/fitting procedure and will be referred to as FileList. If the number of files exceed 50, the program will abort in order to avoid memory leaks. The first line within FileList should be a header which can be used to define the individual columns. The first column of all successive lines is the path and name of the files containing the experimental excitation function data. The second column contains a switch to turn the convolution procedure either ON or OFF for each individual file. The third column is a multiplication factor which can be used to scale the value of the experimental resolution, \( \sigma^{res}(E, \theta) \), obtained from the resolution file (see below) as a function of angle. The fourth column is a switch for the normalization of the cross section, ON or OFF, for individual files using the normalization file discussed below. The fifth column contains a shift in energy for all points within the file. This energy shift can be used to make small corrections to the calibration of your experimental data. Each line within FileList must contain a value in each of the columns discussed (see Section B.4.2).

General Parameter File The second input file contains several input parameters used in the calculation. Each line should contain a dummy string defining the parameter in the first column and the parameter value in the second column. The first parameter is the energy step size used in the convolution procedure and should be given in keV. The second parameter is the size of the bin to be used for re-binning the experimental data, also in keV. Note that if the number of energy bins exceeds 4096 the program will abort. The third and fourth parameters are the lower and upper energy limits of the convolution integration in MeV, respectively. This energy range is also equal to the \( \chi^2 \) energy range and should be less than the data energy range, which is defined below in the *.inp paragraph. The fifth is the type of target which was used for obtaining the experimental data, its value should be either SOLID or GAS. The sixth parameter is YES or NO, and tells whether or not there are correlated parameters (see below). The seventh line of this file should contain a header string along with two input parameters which are used for bounding all variables by \( \pm N \ast Step \). Here \( N \) is an integer given as a parameter and \( Step \) is the step size used in the fitting procedure which is set internally. The first parameter of this line is a switch for this bounding procedure, YES or NO, and the next parameter is \( N \). The eighth line should be a list of resonances for which the physical parameters are to be calculated and printed (1, 3 or 1 – 10, 14, 16 or NONE or ALL). The resonance number refers to the order in which they are placed in the *.par file. The physical parameters are the channel and total widths and excitation energy of the resonance in MeV. These same parameters
will be automatically written to a ResPars.tex file in the output directory which can be used in writing papers. The ninth line determines whether or not the Monte Carlo analysis will be used and should be either YES or NO. The tenth line determines the type of Monte Carlo analysis to perform, which is either FIT or ERR. The eleventh line contains the Value which sets the range used to randomly vary parameters in the Monte Carlo analysis and is used in the following way

\[(P_{\text{current}} - \text{Value} \times P_{\text{current}}) < P_{\text{random}} < (P_{\text{current}} + \text{Value} \times P_{\text{current}}).\]  

The twelfth line is the number of iterations which must be completed before the Monte Carlo analysis is finished. For more details on this procedure please see Section 2.4.3. The thirteenth line is a switch for the minuit fitting procedure and should be either ON or OFF. Finally, the last line is the name of the fitting subroutine which will be called by minuit, and by default is set to simplex (see Section B.4.3). Once again, for details of the fitting procedures and error calculation please refer to Section 2.4.3 and [65].

**Experimental Cross Section Normalization** The third input file contains a calculated ratio of the cross section at all lab angles to the cross section at 0° in the lab frame. The ratio should be calculated using a Monte Carlo simulation of the experimental data, including all experimental parameters. This information is used to normalize the cross section for the experimental data as a function of energy and angle. The input file should have a one line header, and each subsequent line should have four columns. The columns are c.m. energy, lab angle, Monte Carlo normalization factor, the number of counts from the Monte Carlo peak integration (see Sections 2.4.2 and B.4.4).

**Experimental Energy Resolution** The fourth input file contains information on the experimental resolution, \(\sigma_{\text{res}}\), which is calculated using the same Monte Carlo simulation as for the normalization. This file is made up of three columns, the first of which is the c.m. energy in MeV, the second column is the experimental resolution in MeV, and the last column should be the c.m. angle in degrees (see Section B.4.5).

**R-Matrix Reaction Parameters *.inp** The fifth file contains the information needed for the FORTRAN portion of the code which performs the R-Matrix calculation, usually it is of the form *.inp. The first line of this file is a header, and anything following a ‘;’ is a comment. Some of the parameters are no longer used, but because the FORTRAN code is recycled they have not been removed and therefore are still needed in the input files; this also allows for the same input files to be used for two different programs. After the header comes the parameter definitions for the different reaction channels. Each line holds information for one open channel and the last channel should start with a minus sign. The information contained in each line is as follows, ProjectileMass ProjectileCharge ProjectileSpin(GS) TargetMass TargetCharge TargetSpin(GS) ReactionQ-Value ChannelSpin ChannelRadius, where GS stands for ground state. After the last channel definition comes the maximum value of the angular momentum, \(L\), to be used in the calculation, this number is typically 8 or 12. The next line contains three parameters, \(E_{\text{start}}\), \(E_{\text{final}}\), \(E_{\text{step}}\) which make up the energy range used in creating the data arrays, \(E_{\text{step}}\) is not used. The next line contains three unused parameters, \(\theta_{\text{start}}\), \(\theta_{\text{final}}\), \(\theta_{\text{step}}\). The next line defines which of the previously defined reaction channels will be used in the calculation, the channel numbers start from 1. The next line is obsolete and should be set to zero. The following line is a switch for using experimental data, if it is '1' the data is used, and if it is '0' the data is not used. This switch allows for a calculation to be made in a region of the excitation function in which there is no data. The next line defines the threshold energy in MeV and should be properly associated with the reaction Q-Values previously defined in the channels. After that is the energy range which will be used for calculating the \(\chi^2\) value, \(E_{\chi_{\text{start}}}\), \(E_{\chi_{\text{final}}}\) but is now obsolete. The next line should have five numbers corresponding to the parameters for an imaginary phase which accounts for loss of beam flux due to absorption by non-measured processes, however this phase was not used in the current analysis. The last line defines which channel will be used to calculate physical parameters for the resonances (see Section B.4.6).

**R-Matrix Resonance Parameters *.par** The sixth file contains the resonance parameters to be used in the R-Matrix calculation, and usually has the form *.par. The file should have four lines of comments
as a header, and then the parameters for as many resonances as needed. The resonance parameters are defined using two types of lines, the first is the

\[ \text{EnergyEigenvalue Spin Parity}(\pm1.0), \]

and is either the first input line or should follow a line beginning with a minus sign. The second type defines the

\[ \text{ChannelNumber AngularMomentum BoundaryValue ReducedWidth}, \]

the number of lines of this type depend upon how many open channels are being used for the particular resonance. The last line of this type for each resonance must begin with a minus sign so that the next line is read as type one. For resonances of the same spin and parity the boundary value must be the same for each channel (see Section B.4.7).

MINUIT Variables The seventh input file contains the names of the resonance parameters which should be varied by MINUIT, along with any bounds you wish to set. If you want the parameter unbounded set the bounds to zero. The first line of the file should be a header showing the format of the file. Each line after that should contain a variable name, a lower bound and an upper bound, respectively. The format of the variable name is as such, \( E_i \) is the energy eigenvalue of the \( i^{th} \) resonance contained in the \( R \)-Matrix resonance file (i.e. Section B.4.7). \( W_{ij} \) is the reduced width of the \( j^{th} \) channel of the \( i^{th} \) resonance, again in the \( R \)-Matrix resonance file (see Section B.4.8).

Correlations Between \( R \)-Matrix Parameters The eighth input file contains any correlations between non-varied parameters and variables. The format for the names of the parameters in this file should be the same as that of the variables above. There should be a one line header describing the format, which is correlated parameter,correlated variable, correlation factor (see Section B.4.9).

Non-Convoluted Cross Section Output Path The ninth line in the input file is the path for the output of the calculated cross section before the convolution and fitting is performed. For example output/csarr/csarr would print the cross section to files in the directory output/csarr with the names csarr#, where # is a number which refers to the corresponding data file from the FileList. The format of these files is

\[ \text{Energy}_{c.m.}(\text{MeV}) \text{ Angle}_{c.m.} (\text{degrees}) \text{ Non - ConvolutedRMatrixCS}, \]

where CS stands for cross section.

Fixed Variation of Monte Carlo Variables The tenth input file contains the names of the parameters which should be varied using a different range during the Monte Carlo fitting procedure. The format is a one line header followed by lines containing the parameter name and the variation factor. The parameter names have the same format as the variables file which is defined above, and the range is set by Eq. B.1. If you want to fix the parameter simply set the parameter range value to zero (see Sections 2.4.3 and 2.4.10).

Output Path for Monte Carlo Errors The eleventh line is the path for the output files which will contain the information produced during the calculation of the errors. This is only used when the Monte Carlo procedure is set to YES and the type is set to ERR. See Section B.3 for more details on the calculation of errors.

Convoluted Cross Section Output Path The twelfth line in the input file is the path for the output of the calculated cross section after the convolution has been performed. For example output/csint/csint would print the cross section to files in the directory output/csint with the names csint#, where # is a number which refers to the corresponding data file from the FileList. The format of these files is

\[ \text{Energy}_{c.m.}(\text{MeV}) \text{ Angle}_{c.m.} (\text{degrees}) \text{ ConvolutedRMatrixCS ExpCS ExpCSError}. \]
Best Fit Parameters File *.par.fit  The thirteenth line contains the name of the file which will contain the resonance parameters which correspond to the best fit, and should be different than the initial resonance parameter file to avoid overwriting.

B.2 Output

The output from MinRMatrix begins with several lines letting the user know which input files are being used, which procedures are switched on, and whether or not the experimental data is being used. If a fit is being performed there will be output which depends upon the procedure being used. The output from the Monte Carlo procedure is the $\chi^2$ value from before the fit, which is the 'best fit' value. Then the value of the $\chi^2$ for the current iteration is printed (every tenth iteration is output), if this value is less than the 'best fit' value it will tell you, and the name of the new 'best fit' parameter file will be printed.

Once the MinRMatrix fitting procedure has converged the R-Matrix cross section from before the fit along with the cross section from after the fit and the re-binned experimental cross section are placed in their respective user defined files (see Sections B.1 and B.4). The total cross section is also printed to the file output/TotalCS.dat.

Finally, MinRMatrix saves the new parameters to the user defined file, and prints out the final $\chi^2$ value along with the total run time. The current run time will also be printed every ten minutes if the program is running that long (see Section B.4.11).

B.3 Calculation of the Parameter Errors using a Monte Carlo technique

The Monte Carlo procedure can also be used to calculate the errors of the parameters used in the fit. At this time it is not efficient enough to be used, but the idea is as follows. The variable parameters are randomly varied within the user defined range just as described above, after the chosen parameters have been randomized the $R$-Matrix curve is calculated from those parameters. Next, the $\chi^2$ value is determined using Eq. 244, if the $\chi^2$ of the randomized parameters is less than the upper limit, $\chi^2_{upper} = \chi^2_{best} + \Delta \chi^2$, then the parameters are printed to file. Here $\Delta \chi^2$ is set internally to a one $\sigma$ confidence interval of a Gaussian distribution. Given enough iterations each variable should have a range of values forming a Gaussian distribution, the half-width of this distribution will give an error estimate for that variable within one standard deviation. Unfortunately, the efficiency is too low to make this work with the current experiment, or possibly due to the current implementation.
B.4 Examples of Input/Output Files for the Program MINRMATRIX

B.4.1 Example input

input/FileList
input/MinR.param
input/Enormalize.dat
input/gauss.dat
input/180.inp
input/180.par
input/variables.dat
input/Correlations.dat
input/FixedPars
output/Errors/
output/csint/csint
input/180new.par

B.4.2 Example FileList

<table>
<thead>
<tr>
<th>path/filename</th>
<th>Integration</th>
<th>Gauss Factor</th>
<th>Normalization</th>
<th>Shift</th>
<th>Ch</th>
</tr>
</thead>
<tbody>
<tr>
<td>exc/exc15.dat</td>
<td>OFF</td>
<td>0.3</td>
<td>ON</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>exc/exc21.dat</td>
<td>OFF</td>
<td>1.00</td>
<td>OFF</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>exc/exc25.dat</td>
<td>OFF</td>
<td>0.5</td>
<td>ON</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

B.4.3 Example param

Step_of_integration(keV)     2.0
Size_of_rebin(keV)           10.0
Lower_limit_of_integration&Xi~2(MeV) 2.5
Upper_limit_of_integration&Xi~2(MeV) 4.5
Target(GAS/SOLID)            GAS
Correlated_params(YES/NO)    NO
Bound_all_parameters(YES/NO,&_#Steps) NO 2
Calc_Physical_params_for_Res#(NONE,ALL,Res#) 7
MonteCarlo_Analysis(YES/NO)  YES
MonteCarlo_Type(FIT/ERR)     FIT
Fit_using_changing_interference(ON/OFF) ON
Size_of_randomization_for_MC(% of param) 0.0
Number_of_random_iterations_for_MC       8
MINUIT_fitting_procedure(ON/OFF)         OFF
Name_of_MINUIT_fitting_procedure         SIMPLEX
MINUIT_error_calculation(ON/OFF)         OFF
Frame_for_the_angle_data(LAB/CM)        LAB
### B.4.4 Examp.Enormalize

<table>
<thead>
<tr>
<th>Energy, Angle, Ratio, Integration</th>
</tr>
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<tbody>
<tr>
<td>1.5 0 1 63.83</td>
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<tr>
<td>1.5 5 0.98 62.74</td>
</tr>
<tr>
<td>1.5 10 0.94 59.76</td>
</tr>
<tr>
<td>1.5 15 0.91 57.85</td>
</tr>
<tr>
<td>1.5 20 0.84 53.44</td>
</tr>
<tr>
<td>1.5 25 0.81 51.98</td>
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<tr>
<td>1.5 35 0.69 44.16</td>
</tr>
<tr>
<td>1.5 40 0.4 28.85</td>
</tr>
<tr>
<td>1.7 0 1 58.32</td>
</tr>
<tr>
<td>1.7 5 1.04 60.42</td>
</tr>
<tr>
<td>1.7 10 0.95 55.24</td>
</tr>
<tr>
<td>1.7 15 0.93 54.19</td>
</tr>
<tr>
<td>1.7 20 0.86 50.03</td>
</tr>
<tr>
<td>1.7 25 0.83 48.66</td>
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<td>1.7 30 0.78 45.67</td>
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<td>1.7 35 0.71 41.68</td>
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<td>1.7 40 0.62 36.27</td>
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<td>1.9 0 1 58.23</td>
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<tr>
<td>1.9 5 0.92 53.48</td>
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<tr>
<td>1.9 10 0.91 52.81</td>
</tr>
<tr>
<td>1.9 15 0.85 49.3</td>
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<tr>
<td>1.9 20 0.79 46.13</td>
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<td>1.9 25 0.77 44.9</td>
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<td>1.9 30 0.75 43.4</td>
</tr>
<tr>
<td>1.9 35 0.67 38.89</td>
</tr>
<tr>
<td>1.9 40 0.64 37.17</td>
</tr>
</tbody>
</table>

### B.4.5 Examp.gauss

<table>
<thead>
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</tr>
</thead>
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</tr>
<tr>
<td>1.500 0.038142 168.115128</td>
</tr>
<tr>
<td>1.500 0.043678 157.540588</td>
</tr>
<tr>
<td>1.500 0.059416 146.577621</td>
</tr>
<tr>
<td>1.500 0.072998 136.303909</td>
</tr>
<tr>
<td>1.500 0.111409 125.488754</td>
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<tr>
<td>1.500 0.170849 115.042564</td>
</tr>
<tr>
<td>1.500 0.300215 104.884018</td>
</tr>
<tr>
<td>1.700 0.025187 175.960052</td>
</tr>
<tr>
<td>1.700 0.033835 169.283386</td>
</tr>
<tr>
<td>1.700 0.043281 159.616425</td>
</tr>
<tr>
<td>1.700 0.049522 149.601120</td>
</tr>
<tr>
<td>1.700 0.066638 139.608734</td>
</tr>
<tr>
<td>1.700 0.102015 130.083298</td>
</tr>
<tr>
<td>1.700 0.145094 120.169395</td>
</tr>
<tr>
<td>1.700 0.236157 110.441139</td>
</tr>
<tr>
<td>1.700 0.310741 100.795395</td>
</tr>
<tr>
<td>1.900 0.029376 176.428177</td>
</tr>
<tr>
<td>1.900 0.029567 170.113007</td>
</tr>
<tr>
<td>1.900 0.034433 161.471664</td>
</tr>
</tbody>
</table>

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B.4.6 Examp.inp

Channel cards: Ap, Zp, Sp, At, Zt, St, Qvalue, ChSp, ChR

4.0 2.0 0.0 14.0 6.0 0.0 0.0 0.0 0.0 5.20
1.0 0.0 0.5 17.0 8.0 2.5 -1.8173 2.0 4.60
1.0 0.0 0.5 17.0 8.0 2.5 -1.8173 3.0 4.60
1.0 0.0 0.5 17.0 8.0 0.5 -2.6880 0.0 4.60
-1.0 0.0 0.5 17.0 8.0 0.5 -2.6880 1.0 4.60

8 ; Lax - Max. angular momentum in partial wave expansion
0.90 5.00 0.005 ; Energy range and step. Estart, Efinal, Estep
90.0 90.0 20.0 ; obsolete
1 ; Exit channel. Defines the reaction to be calc.
0 ; obsolete
1 ; Data key: 1=use data, 0=not using data
0.0 ; Threshold Energy (MeV)
2.8 4.5 ; obsolete
0 -1. 0.3 3.0 0.3 ; imphase, APA, APAL, APEP, APAP
1 ; Channel used to calculate res. width

; The last channel indicated by negative value of Ap.
; If Estart=Efinal cross section calculated only for one energy Estart
; If Astart=Afinal cross section calculated only for one angle Angle

B.4.7 Examp.par

; Resonance parameters: Eres, Jtotal, Parity (3F8.3)
; ChN, L, Bc, G (2x, 2I3, 3F8.3)
; "-------""-------""-------"
; -ChN - last channel-spin group

1.811 1.000 -1.000
1 1 -2.000 0.050
2 1 -2.000 0.000
3 3 -3.000 0.000
4 1 -2.000 0.000
5 -1 -2.000 0.000
1.988 2.000 1.000
1 2 -2.500 0.050
2 0 0.000 0.000
3 2 -2.500 0.000
4 2 -2.500 0.000
5 -2 -2.500 0.000
3.140 2.000 1.000
1 2 -2.500 0.033
2 0 0.000 0.084
3 2 -2.500 0.000
4 2 -2.500 0.000
5 -2 -2.500 0.000
2.755 2.000 1.000
1 2 -2.500 0.090
2 0 0.000 0.165
3 2 -2.500 0.000
4 2 -2.500 0.000
5 -2 -2.500 0.000
B.4.8 Examp.variables

Variable name, Lower bound, Upper bound
E7  0  
W71 0  

B.4.9 Examp.correlations

Correlated Parameter Correlated Variable Correlation Factor
W13 W11 0.95

B.4.10 ExampFixedPars

Parameter Name, Percentage Error for randomization
E7  0.01

B.4.11 Examp.Output

File w/ data files: input/FileList
Integration: ON
Number of files read: 3
Total data points: 3952
MinR param file: input/MinR.param
Normalization is ON
Normalization file: input/Enormalize.dat
Resolution file: input/gauss.dat
Rmatrix input file: input/180.inp
Rmatrix param file: input/180.par
Using experimental data
Variables file: input/variables.dat
Correlated parameters file: input/Correlations.dat

MINUIT RELEASE 96.03  INITIALIZED. DIMENSIONS 100/ 50 EPSMAC= 0.89E-15

PARAMETER DEFINITIONS:

NO.  NAME VALUE  STEP SIZE LIMITS
1 'E7  3.2770  0.50000E-01 no limits
2 'W71 0.10900  0.54500E-02 no limits

************
** 1 **SET STRAT 1.000
************

NOW USING STRATEGY 1: TRY TO BALANCE SPEED AGAINST RELIABILITY

************
** 2 **SET PRI 1.000
************

************
** 3 **MIGRAD
************

FIRST CALL TO USER FUNCTION AT NEW START POINT, WITH IFLAG=4.
CSarr output file path: output/csarr/csarr
START MIGRAD MINIMIZATION. STRATEGY 1. CONVERGENCE WHEN EDM .LT. 0.10E-03

FCN= 1.729420 FROM MIGRAD STATUS=INITIATE 6 CALLS 7 TOTAL
EDM= unknown STRATEGY= 1 NO ERROR MATRIX

EXT PARAMETER CURRENT GUESS STEP FIRST
NO. NAME VALUE ERROR SIZE DERIVATIVE
 1 E7  3.2770 0.50000E-01 0.50000E-01 -2.8228
 2 W71 0.10900 0.54500E-02 0.54500E-02  1.7704

MIGRAD MINIMIZATION HAS CONVERGED.
MIGRAD WILL VERIFY CONVERGENCE AND ERROR MATRIX.
COVARIANCE MATRIX CALCULATED SUCCESSFULLY

FCN= 1.643535 FROM MIGRAD STATUS=CONVERGED 81 CALLS 82 TOTAL
EDM= 0.22E-05 STRATEGY= 1 ERROR MATRIX ACCURATE

EXT PARAMETER STEP FIRST
NO. NAME VALUE ERROR SIZE DERIVATIVE
 1 E7  3.3410 0.39912E-01 0.38586E-05 -0.48108E-01
 2 W71 0.12480 0.19035 0.14864E-03 -0.18750E-02

EXTERNAL ERROR MATRIX. NDIM= 50 NPAR= 2 ERR DEF= 1.00
0.159E-02  0.361E-02
 0.361E-02  0.362E-01

PARAMETER CORRELATION COEFFICIENTS
   NO. GLOBAL    1   2
     1  0.47554  1.000  0.476
     2  0.47554  0.476  1.000

***********
**  4 **CALI 3.000
***********

CSInt output file path: output/csint/csint
New parameter file: input/180new.par

CHI^2: 1.64354

RUNTIME: 5.38333 min
B.4.12  \( R \)-Matrix Resonance Input Parameters for the Best Fit

; Resonance parameters: Eres, Jtotal, Parity (3F8.3)
; ChN, L, Bc, G (2x, 2I3, 3F8.3)

```
1.811 1.000 -1.000
1 1 -2.000 0.090
2 1 -2.000 0.000
3 3 -3.000 0.000
4 1 -2.000 0.000
-5 1 -2.000 0.000
2.000 2.000 1.000
1 2 -2.500 -0.150
2 0 0.000 0.040
3 2 -2.500 0.000
4 2 -2.500 0.000
-5 2 -2.500 0.000
2.072 3.000 -1.000
1 3 -3.000 0.220
2 1 -2.000 -0.160
3 1 -2.000 0.000
4 3 -3.000 0.000
-5 3 -3.000 0.000
2.560 2.000 1.000
1 2 -2.500 0.045
2 0 0.000 -0.200
3 2 -2.500 0.000
4 2 -2.500 0.000
-5 2 -2.500 0.000
2.755 2.000 1.000
1 2 -2.500 0.080
2 0 0.000 0.165
3 2 -2.500 0.000
4 2 -2.500 0.000
-5 2 -2.500 0.000
4.191 0.000 1.000
1 0 -0.600 1.530
2 2 -2.500 0.000
3 2 -2.500 0.000
4 0 0.000 0.000
-5 0 0.000 0.000
3.187 1.000 -1.000
1 1 -2.000 0.371
2 1 -2.000 0.154
3 3 -3.000 -0.228
4 1 -2.000 0.231
-5 1 -2.000 0.131
3.150 2.000 1.000
1 2 -2.500 -0.025
2 0 0.000 0.100
3 2 -2.500 0.027
4 2 -2.500 0.000
```
<table>
<thead>
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<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
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</tr>
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<tr>
<td>-5 2</td>
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<td>3.323</td>
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<td>-1.000</td>
<td>1 3</td>
<td>-3.000</td>
</tr>
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<td>2 1</td>
<td>-2.000</td>
<td>-0.198</td>
</tr>
<tr>
<td>3 1</td>
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<td>-0.086</td>
<td>4 3</td>
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BIBLIOGRAPHY


99
[41] V.Z. Goldberg, K.M. Källman, T. Lömroth, P. Manngard, and B.B. Skorodumov. *Phys. of Atomic Nuclei*, 68-7:0, 2005. (document), 1, 2.1, 2.4.2, 2.5.8, 2.2

100


[64] Geant. *CERN Program Library*. 2.4.2
[65] F. James. *CERN Program Library*. 2.4.3, B.1


[76] G.R. Satchler. *Introduction to Nuclear Reactions*. 1990. 3.1.2


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BIOGRAPHICAL SKETCH

Eric D. Johnson

Eric D. Johnson was born on May 31, 1982 in West Point, NE. In the Spring of 2004, he received his Bachelor of Science in Physics with a minor in Mathematics at Nebraska Wesleyan University. Under the advisement of Dr. Grigory V. Rogachev he obtained his Master of Science in the summer of 2007 from the Department of Physics at Florida State University. He then enrolled in the doctoral program with the Department of Experimental Nuclear Physics at FSU in the Fall of 2007. Eric graduated in the Fall of 2008 with his PhD in Experimental Nuclear Physics from FSU.

Eric’s research interests include nuclear reaction rates of stellar processes, and the structure of neutron rich nuclei.

Eric lives in Tallahassee, FL with his wife Lisa and dog Arya.