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The Influence of Curiosity and Spatial Ability on Preservice Middle and Secondary Mathematics Teachers' Understanding of Geometry

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THE INFLUENCE OF CURIOSITY AND SPATIAL ABILITY ON PRESERVICE MIDDLE AND SECONDARY MATHEMATICS TEACHERS' UNDERSTANDING OF GEOMETRY

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To my mother and father

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

The objectives of this study were to investigate and characterize the geometric thinking of preservice middle and secondary mathematics teachers while taking account into their curiosity types and spatial ability levels, and to examine the relationship between the students' curiosity types (perceptual and epistemic) and their motivation, and to examine the relationship between students' spatial ability and motivation. The study used the van Hiele model of the development of geometric thought to examine description of geometric thought in preservice middle and secondary teachers and the ARCS model of motivation to examine students' motivation.

Both quantitative and qualitative methods were employed. The van Hiele levels of students were identified by using, clinical interview protocol, designed by Mayberry (1981). Four preservice teachers were interviewed. To investigate the difference, if any, exist between preservice middle and secondary teachers with different spatial ability levels and understanding geometry, pre- and post-test design were employed by using Mayberry's (1981) protocol. Pre-interview results showed three groups of levels of understanding were identified with the preservice middle and secondary mathematics teachers. One teacher whom very low in spatial ability indicated a level II, one teacher low in spatial ability indicating levels II/III, one teacher with medium spatial ability indicating level III/IV and one teacher very high in spatial ability a Level III understanding. Post interview results showed a gain among all three preservice teachers, only one teacher who were very low in spatial ability did not demonstrated such gain in geometric understanding.

Correlation design were employed examine relationships among motivation, curiosity and spatial ability. This study looked at following relationships having motivation as a dependent variable, curiosity types (epistemic and perceptual) and spatial ability as independent variables. The Pearson product-moment correlation was utilized to investigate these relationships. There was a significant correlation between perceptual curiosity and motivation.

CHAPTER 1

"*The fuels of enthusiasm and growth are curiosity and challenge*"

John M. Keller

INTRODUCTION

For many years now the International Association for the Evaluation of Educational Achievement (IEA) has conducted several international comparative studies of the mathematics and science performance of students around the world. The latest of such study is, TIMSS-R (Third International Mathematics and Science Study-Repeat) which was conducted in 1999, continued to show that United States (US) students' mathematical achievement lagged behind those of several other countries. More specifically, in the geometry content area, United States student achievement was in the bottom third of all countries tested. When geometry scores are examined, Japanese students performed at the top with a score of 575. The international average score was 487 and the US scored 473 in geometry content area. Among the 38 countries, 26 countries outperformed the US in the geometry content area (Mullis et al., 2000). One question to ask is why is US student performance in geometry so low when compared to their peers in other countries?

To understand the low performance of US students in geometry content area, one should find out which factors play main role in students' learning geometry. When it comes to learning geometry, spatial ability plays a significant role (Bishop, 1983). Learners with good spatial ability did not have difficulties grasping concepts in geometry whereas learners who had not developed spatial ability had difficulties learning and as a result showed low performance in achievement settings (Battista, 1994,1998,1999; Battista & Clements, 1990,1991,1996,2002; Bishop, 1983). To emphasize this importance, National Council of Teachers of Mathematics (NCTM, 2000) geometry standards recognize the development of student spatial ability. The difficulties students' have with learning geometry is an interest of mathematics educators. In 1957 Dina van Hiele-Geldof and Pierre M. van Hiele conducted a study to investigate why students were having such difficulties in learning geometry. The van Hieles proposed a model based on their work, in the model Hieles explained development of geometric thought in students and stages of instruction for geometry. The model will be discussed in detail later. NCTM (2000) standards also emphasize the van Hiele model for learning and teaching geometry. For now, in teaching and learning geometry, we have spatial ability to consider, and a model that explains development of geometric thought and instructional sequence. When one wants to design instruction, one needs to consider both cognitive domain and motivational domain. Researcher believes that any instructional design without a motivational design cannot be a complete design. Van Hiele was aware of the motivational dimension, in his seminal book, *Structure and Insight*, he included a section (1986,pp.187-193) on motivation. Personally I believe that motivation is a prerequisite for learning. NCTM 's standards (2000) present a vision for school mathematics.

"Imagine a classroom, a school, or a school district where all students have access to high-quality, *engaging mathematics instruction.* There are ambitious expectations for all, with accommodation for those who need it. *Knowledgeable teachers* have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment. Students confidently *engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students to make, refine, and explore conjectures* [italics added] on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. *Students are flexible and resourceful problem solvers* [italics added]. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and *engage actively in learning it* [italics added]." (p.3)

Feldman (2003) cites the same section and emphatically points out that "Unfortunately, when ones goes out to schools to observe what is taught and how, the NCTM's vision is nowhere in sight " (p.148). I posed a question earlier about the sources of low performance of US students in geometry content area.

One possible answer may be that US mathematics teachers are failing to provide their students appropriate learning opportunities in geometry. Appropriate learning opportunities are included but not limited to taking care of student motivation, appropriate instruction for students those who differ in their spatial ability for the development of geometric thought. If this is the case, then it is critical that this issue be addressed. Research in Spatial Ability and van Hiele model treated as a separate domain until 1997.Such connection made by Saads and Davis (1997) with a study to investigate the relationship between spatial ability and van Hiele levels. According to Henderson (1988) how students learn largely depends on the teachers. And van Hiele (1984) states that "the art of teaching is a meeting of three elements: teacher, student and subject matter." Thus the objective of this study was to investigate undergraduate mathematics education students' namely, preservice middle and secondary mathematics teachers' geometric understanding, as described by the van Hiele levels, while taking into account their curiosity and spatial ability levels.

Mathematics educators have been studying van Hiele levels (Burger& Shaughnessy, 1986; Fuys, Geddes, & Tischer, 1985; Gutiérrez & Jaime, 1998;Gutiérrez, Jaime, & Fortuny, 1991; Jaime & Gutiérrez, 1994; Henderson, 1988; Mayberry, 1981; Saads & Davis, 1997) and Spatial Ability (Barakat, 1951; Battista, 1994,1998,1999; Battista & Clements, 1990,1991,1996,2002; Battista, Wheatley & Talsma, 1982, 1989; Bishop, 1983; Murray, 1949;Saads & Davis, 1997; Wrigley, 1958, Wheatley, 1990;1991,1992,1998; Wheatley & Reynolds, 1999) for a while now. These two areas, van Hiele and spatial ability, have long been important to mathematics educators because of the potential they have for improving both mathematical understanding and pedagogy. This is especially true, according to several researchers, in the discipline of geometry (Battista, 1997; Bishop, 1983; Crowley, 1987; Del Grande, 1987; Jaime& Gutiérrez, 1994; Henderson, 1988; Usiskin, 1982). Although a significant amount of research exists in these two domains, spatial ability and van Hiele, research on curiosity in mathematics is scarce. However, there is a presence of literature within the discipline of Psychology grounded in an educational context (see Arnone & Grabowski, 1992;Arnone, Grabowski & Rynd, 1994; Berlyne, 1954,1960, 1965; Boyle, 1983,1989; Day, 1981; Harty, Andersen & Enochs, 1984; Koran, Koran, Foster & Fire, 1989; Langevin, 1971,1976; Leherissey, 1971; Livson, 1967; Litman, 1998,2000; Malone, 1981; Maw and Maw, 1961,1966,1967,1968,1970; Menis, 1984; Naylor, 1981; Olson &Camp, 1984; Penny & McCann, 1964; Spileberger & Star, 1993; Torrance, 1969; Vidler & Rawan, 1975). While many of these researchers have attempted to define curiosity, there exists no widely accepted definition at the present time. Curiosity has been viewed as a very important part of the learning process. It is thought to be a strong motivator for learning. Several researchers have attempted to relate curiosity to various measures of academic achievement, learning performance, and understanding (Berlyne, 1960; Day, 1981;Keller, 1999; Lowenstein, 1994; Maw and Maw, 1964 & 1968; Voss& Keller, 1983). Thus, it is important to look at preservice middle and secondary teacher's curiosity and spatial ability levels when studying geometry since they will be the future educators of US children.

Purpose of the Study

The purpose of this study was to investigate and characterize the geometric thinking and understanding of four preservice middle and secondary mathematics teachers while considering their curiosity and spatial ability levels. This study conducted using members from an undergraduate geometry course for teachers at a major research university in the southeast United States. The title of the course is Elements of Geometry. This research project will follow the model of mixed methodology (Goetz and LeCompte, 1984). Questions that will be addressed are:

- What differences, if any, exist between preservice middle and secondary mathematics teachers with different spatial ability levels and their understanding of geometry?
- What differences, if any, exist between preservice middle and secondary mathematics teachers with different curiosity types (perceptual and epistemic) and their understanding of geometry?
- Is there a relationship among motivation, spatial ability and curiosity?

These questions are important because they may be a means for providing an explanation for why US students' geometry performance remains far below the performance of their international counterparts. Also, this study hopes to assist in the development of defining a role for curiosity in the teaching and learning of geometry. This study also attempted to inform the mathematics education community in these areas because no known studies of this type have addressed this issue. Therefore the study is important in that it can serve as a future resource for mathematics teachers who are unaware of the individual differences in spatial ability and curiosity of their students.

Theoretical Framework

For more than 20 years, the van Hiele model of geometric thought, has served to characterize individuals thinking in geometry (Henderson, 1988;Mayberry, 1981). The van Hiele model of geometric thought has emerged from the work of two Dutch mathematics educators, Dina van Hiele-Geldof and Pierre M. van Hiele at the University of Utrecht. The van Hiele's model consisted of five levels, that is, they suggested that geometric thought develops in sequence of those five levels. These levels ranged from the lowest to the highest - Level 0-visualization, Level 1-analysis, Level 2-informal deduction, Level 3-formal deduction and Level 4-rigor (Burger and Shaughnessy, 1986, p.31; Shaughnessy and Burger, 1985,p.420).

Shaughnessy and Burger (1986) describe the levels of thought as they apply to geometric development as follows:

- Level 0(Visualization): The student reasons about basic geometric concepts, such as simple shapes, primarily by means of visual consideration of the concept as a whole without explicit regard to properties of its components.
- Level 1(Analysis): The student reasons about geometric concepts by means of an informal analysis of component parts and attributes. Necessary properties of the concept are established.
- Level 2(Abstraction): The student logically orders the properties of concepts, forms abstract definitions, and can distinguish between the necessity and sufficiency of set of properties in determining a concept.

- Level 3(Deduction): The student reasons formally within the context of a mathematical system, complete with undefined terms, axioms, an underlying logical system, definitions and theorems.
- Level 4(Rigor) : The student can compare systems based on different axioms and can study various geometries in the absence of concrete models.(p.31)

Characteristics of the level

There are important characteristics of the theory identified by van Hiele (1984/1959). Guiterrez (1992) classified the important characteristics of van-Hiele levels under three categories as follow:

- *Levels are ordered and sequential:* A student cannot reach level n without mastering level n-1
- *Each level has its own language:* There is a relationship between language and its uses in different levels. Two students at different level may understand the same word in different level. Gutierrez (1992) has shown the differences of the word "proof" meaning at different levels like level 2 meaning verification formal deduction at level 4 and informal deduction at level 3 (p.32).
- *Levels are continues:* First it is thought that levels are separated but results obtained from the researches (see Gutierrez, 1992 for detail) suggested that passing from one level to following one is a continuous process (p.32).

Different from Gutierrez, Usiskin (1982, pp. 5-6) discussed the important characteristics of van-Hiele levels under the following five categories:

- *Fixed sequence:* A student must pass through the level in order. That is he or she has to develop an understanding of previous level to be in the next level.
- *Adjacency:* This characteristic states that objects of thought are different at different levels. Those that are understood implicitly one level is understood explicitly at the next level.
- *Distinction:* Each level has its own set of linguistic symbols and distinction is carried in the symbols and also in the relationship connecting these symbols

- *Separation:* When two students or student-teacher are reasoning at different levels and using different linguistic symbols there is a limited understanding of the concept being discussed.
- *Attainment:* A learner moves from one level to the next with complete understanding. The learner has to go through five stages described by van Hiele (1986).

The van Hiele model not only informs us about the learning of geometry but also about the teaching of it. van Hiele (1986) described five stages which, when encountered in the process of instruction, lead to a higher level of thought. The first stage is information where pupils get acquainted with a working domain. The second stage is guided by orientation; students are guided by a task (given by the teacher or made by them) with different relations of the association that has to be formed. In the third stage, which is explication, they become conscious of the relations, they try to express them in words and they learn the technical language accompanying the subject matter. In stage four, free orientation, they learn by general tasks to find their own way in the network relations. Lastly in the fifth stage, integration, students build an overview of all they have learned on the subject, of the newly formed network of relations now at their disposal (pp. 53-54).

Although van-Hiele level of geometric thought were proposed in 1957 and accepted by the mathematics education community but it was interesting to see that after more than 50 years we still do not have any other model or framework developed. What would be the possible answer to this "walking on the same path" although everything has changed compare to 50 years ago?

In order to investigate relationship of preservice teachers' curiosity (perceptual and epistemic) and their motivation, the Attention, Relevance, Confidence, and Satisfaction Model (ARCS) will be utilized. The ARCS model is based upon the macro theory of motivation and instructional design developed by Keller (1979, 1983, 1987a,b,c). When Keller (1979) started on the development of the ARCS Model, there were no macro theories or models in existence. The ARCS Model uses a problem solving approach and it has three distinctive features; first it has four conceptual categories that subsume many of the specific concepts (Curiosity, Learned helplessness, etc.) and variables that characterize human motivation. Secondly it includes sets of strategies to use to enhance the motivational dimension of instruction. And finally it incorporates a systematic design process called motivational design (Keller, 1987a,b,c). The ARCS model of motivational design (Keller, 1987a,b,c) provides a systematic, ten-step approach to designing motivational tactics into instruction. Motivational design and tactics are based on four dimensions: attention (A), relevance(R), confidence(C), and satisfaction (S). These four categories of the ARCS model encompass the various concepts, theories, strategies, and tactics that affect to the motivation to learn (Keller, 1987a, 1999). Numerous studies and reports have described and confirmed the validity of the model in different instructional format such as distance learning and computer aided instruction, and in different part of the world such as Japan, Europe, and Africa (Means, Jonasses, and Dwyer 1997; Gibson, Herbert, P., & Mayhew, 1998; L. Visser, 1998; L. Visser et al., 2002; L.Visser et al., 1999; Song, 1998; Suzuki & Keller, 1996; Maushak, Lincecum, & Martin, 2000; Vafa,1999 cited in Gabrille,2003).

Although the model is very complicated Keller turned his model into a pill of vitamins, minerals, etc… in short everything body needs. The four dimensions of the ARCS Model are explained more closely below:

- *Attention (A):* The ARCS Model starts with capturing the learners' attention. As Keller (1987a) points out, sustaining attention is the most difficult part in this dimension. Attention has sub-categories and in each subcategory the motivational/instructional designer considers a major question, which is followed by sub questions. These are: *A1.Perceptual arousal* (What can I do to capture their interest), *A2.Inquiry arousal or curiosity arousal* (How can I stimulate an attitude of inquiry?), and finally *A3.Variability* (How can I maintain their attention).

- *Relevance (R):* Relevance connects instructional content to those things that are significant to the learners. Keller (1987a) explains that in adult learning environments, relevance of the instructional material to the learner is crucial. Relevance hopes to ensure that the learner makes the connection between what they need to know and what new learning opportunities are offered. Keller (1992)

offers three categories of relevance: (1) Goal Orientation; (2) Motive Matching, and (3) Familiarity.
Goal Orientation (R1) examines if the given instruction is related to the learners' goals of learning (Keller, 1992). Motive Matching (R2) encourages the learner to visualize achieving a learning goal. Lastly, familiarity (R3) connects the given instruction to the learner's own experiences.

- *Confidence (C):* Confidence refers to the positive expectations for the success of the learners (Keller, 1979). Keller (1992) offers three methods for achieving confidence within the learner: (1) Learning Requirements; (2) Positive Consequences, and (3) Personal Responsibility.
  Learning Requirements (C1) informs the learner of the expectations. This can occur by providing clear and understandable learning objectives and expected outcomes of the given instruction (Keller, 1992). Positive Consequences (C2) occurs when the learner is appropriately challenged by the given instructional materials. Finally, Personal Responsibility (C3) wants the leaner to feel that he/she succeeded because of their ability rather than any other external elements (Keller, 1992).

- *Satisfaction (S):* The final stage of the model, Satisfaction (S), refers to a learner's positive feelings about their own learning experiences. It also affirms, to the learner, that the instructional material was relevant and that they learned the material because of their ability. Keller (1992) identifies three areas for insuring learner satisfaction: (1) Intrinsic Reinforcement; (2) Extrinsic Rewards, and (3) Equity. Intrinsic Reinforcement (S1) is an internal desire to learn and is mostly associated with self-directedness. Extrinsic Rewards (S2) recognizes the learner for his/her accomplishments. This can be done verbally or through an authentic reward system. Equity (S3) examines the learner's perceptions of their fair and equal treatment within the learning environment.

Problem Statement and Research Questions

The mathematics education community has been studying both Van Hiele levels and Spatial Ability as separate research domain for some time now (see Battista, 2001;Bishop, 1983; Henderson, 1988; Murray, 1949;Wheatley et al, 1988; Wrigley, 1958, for examples). One such discipline where several studies have been conducted is geometry. Researchers have found through these many studies that these two areas, spatial ability and van Hiele levels of geometric thought, are essential for both students and teachers because of their potential for improving both mathematical comprehension and pedagogy. This has not been true in the case of studying curiosity in mathematics. In fact, no significant research has been found in this area. Thus the main research question of this study is:

Do differences in Spatial Ability and Curiosity makes any differences the geometric understanding of a group of preservice middle and secondary mathematics teachers? If so, how? If not, why not? Other questions that will need to be addressed in order to respond to the main research question are:

1. What differences, if any, exist between preservice middle and secondary mathematics teachers with different spatial ability levels and their understanding of geometry?
2. What differences, if any, exist between preservice middle and secondary mathematics teachers with different curiosity types (perceptual and epistemic) and their understanding of geometry?
3. Is there a relationship among motivation, spatial ability and curiosity?

3.1 Is there a statistically significant relationship between preservice middle and secondary teachers' spatial ability and motivation?

Ho: There is no relationship between spatial ability and motivation

3.2 Is there a statistically significant relationship between preservice middle and secondary teachers' epistemic curiosity and motivation?

Ho: There is no relationship between epistemic curiosity and motivation

3.3 Is there a statistically significant relationship between preservice middle and secondary teachers' perceptual curiosity and motivation? .

Ho: There is no relationship between perceptual curiosity and motivation

Relationship between motivation and spatial ability is investigated because previous research reports (see Bishop, 1981) reported positive correlation between geometric performance and spatial ability. Researcher expected that motivation and spatial ability would be correlated. Geometry in nature includes more visual, perceptual curiosity also deals with visual sensory, researcher expected that perceptual curiosity would have a higher chance to correlate with motivation than the epistemic curiosity. The Pearson product-moment correlation will be utilized to investigate these relationships. Significance of the relationships will be tested at alpha=0.1 level with two-tailed test.

Key Concepts

For the purposes of this research the following definitions were used.

Curiosity: "Curiosity is broadly defined as a desire for acquiring new knowledge and new sensory experience that motivates exploratory behavior (Berlyne, 1949,1950,1954,1960; James, 1890;Lowenstein, 1994;McDougall, 1921;Spielberger & Star, 1994)" (Litman & Spielberger, 2003, p.75)

- Perceptual curiosity: "Perceptual curiosity, evoked by complex or ambiguous patterns of sensory stimulation (e.g. sights, sounds), motivated behaviors such as visual inspection in order to acquire new information (Berlyne, 1957, 1958)." (Collins, Litman,& Spielberger, 2004)
- Epistemic curiosity: "Epistemic curiosity was aroused by complex ideas or conceptual ambiguities (e.g. scientific theories, intellectual conundrums), which motivated asking questions or testing hypotheses in order to gain knowledge (Berlyne, 1954)." (Collins, Litman, & Spielberger, 2004)

Spatial ability: Two types of spatial ability considered in this study: spatial orientation and spatial visualization.

- Spatial orientation: "The spatial orientation factor has been described as a measure of the ability to remain unconfused by changes in the orientation of visual stimuli, and therefore it involves only a mental rotation of configuration." (Bodner & Guay, 1997, p.1435)

- Spatial Visualization: The spatial visualization factor measures the ability to mentally restructure of manipulate the components of the visual stimulus and involves recognizing, retaining and recalling configurations when the figure or parts of the figure are moved." (Bodner & Guay, 1997,p.1435)

Motivation:" Simply stated, motivations are reasons individuals have for behaving in a given manner in a given situation. They exists as part of one's goal structures, one's beliefs about what is important, and they determine whether or not one will engage in a given pursuit (Ames, 1992)." (Middleton& Spanias, 1999, pp.65-66)

CHAPTER 2

REVIEW OF LITERATURE

This section consists of a review of literature relevant to the investigator's questions stated in the previous chapter. The review is organized into three sections. The first section deals with the empirical research of the van Hiele model for geometric thought. The second section contains a review of empirical research on spatial ability. The final section provides literature on motivation and curiosity.

Van Hiele Model

The first research on the van Hiele model, conducted after the van Hiele's proposed their model in 1957, was done by Soviet educators in the early 1960s. Wirzup formally introduced the van Hiele model in the United States (US) in 1974. After that, numerous US researchers conducted studies using this model (see Mayberry, 1981; Burger & Shaughnessy, 1986; Fuys, Geddes, & Tischer, 1985; Henderson, 1988; Usiskin, 1982). Among the early participants were The Oregon State, Brooklyn College, and Chicago projects, which were examples of larger scale designed research studies (for detail see Henderson,1988, pp.26-31). Since this investigation will involve pre-service middle and secondary mathematics teachers, this literature review on the van Hiele Model will concentrate on the research where the participants were pre-service teachers. US researchers that will be included consist of the following: Mayberry (1981), Henderson (1988). Spanish researchers will include Gutiérrez & Jaime (1987), and Gutiérrez, Jaime, & Fortuny, (1991). Other researchers included are, Matos (1985 – Portuguese), Saads and Davis (1997 – Britain), and Wu (1994 - China). These researchers studied the van Hiele levels of the reasoning of preservice teachers on several geometric concepts.

Research related to preservice teachers and the van Hiele Model, was first conducted by Mayberry in 1981. This involved an investigation of preservice elementary teachers' geometric knowledge based on the van Hiele model. Mayberry's research was designed to investigate the fixed sequence property of the model and to look at whether an individual would demonstrate the same level of thought in all areas of geometry to which he or she had been exposed. Mayberry (1983) constructed test items covering seven geometric concepts which included squares, isosceles triangles, right triangles, circles, parallel lines, similar figures, and congruent figures. Items for each concept and level were developed. In order to establish content validity, description of levels and items were given to a panel of 14 mathematicians and mathematics educators including Pierre M. van Hiele. The panel was asked "to judge whether items were appropriate and if appropriate whether the items were consistent with design levels" (Mayberry, 1981, p.11). The final instrument contained only those tasks deemed suitable by the panel, and included the suggested revisions.

After the validation test, 19 pre-service elementary teachers were individually interviewed using the revised questions during two 1-hour audio taped interviews. During the interviews preservice teachers were provided paper, a straight edge and a pencil. Questions were printed on eleven inches by fourteen inches cards and read by each preservice teacher. Some responses suggested further questions that were orally asked by the researcher. The preservice teachers were encouraged to think aloud and give reasons as they were answered each question.

Responses were analyzed and supported Mayberry's hypothesis that the van Hiele levels are hierarchical. However, Mayberry did not find evidence that individuals demonstrated the same level of thought in all areas of geometry to which she/he had been exposed.

After Mayberry (1983) a second study was conducted by Henderson (1988) to investigate the pre-service teachers' geometric knowledge based on the van Hiele model. Her research was designed to investigate and characterize the geometric thinking of five pre-service secondary teachers using the van Hiele model of geometric thought and to examine the relationship between preservice teachers' geometric thinking and their instructional behaviors.

Henderson (1988) used Mayberry's interview-based instrument to identify five preservice secondary mathematics teachers' geometric thinking. Following the interviews, each participant was observed and videotaped while teaching five high school geometry students. She conducted stimulated-recall interviews during and following the teaching segment for each preservice teacher.

Henderson (1988) found that preservice teachers' geometric thinking levels were reflected in their instruction and as a result, the level of understanding of preservice teachers influenced students' difficulty or insight.

Researchers from other counties also interested in the van Hiele model have also shed light on our understanding of geometric thought of preservice teachers' in different cultures. Angel Gutierrez is one of the most contributing researchers outside of the US in this arena. Gutierrez et al. (1991) conducted a study to evaluate acquisition of van Hiele level of primary preservice teachers by proposing an alternative paradigm. This research differed from other researches in that instead of assigning students to one level (e.g. Mayberry, 1981; Henderson, 1988, Usiskin, 1982) they looked at degree of acquisition of a given level (e.g. low, intermediate, high acquisitions…) in three-dimensional geometry. Gutierrez et al. (1991) designed a Spatial Geometry Test to evaluate the van Hiele level of students' thinking in three-dimensional geometry. Five different versions of the test were administered to pilot groups and for each version; one of the research team members redesigned any faulty items, which the other two group members then validated. After modification and eliminations, a final version was sent to three experts to obtain outside validation. The final version of the Spatial Geometry Test was grouped into five activities.

After data collection and analysis, they identified that acquisition of the lower level is more complete than acquisition of higher level and they also observed that not all students used a single level of reasoning, but some of them showed several levels at the same time. The researchers did not reject the hierarchical structure of the levels, but they pointed out the complexity of the human reasoning process when teachers adapt the van Hiele theory to their own instruction. Another interesting result was that students showed better acquisition of Level 3 than of level 2.

Other research from the international perspective, Saads and Davis (1997), emerged from Britain. Although previous research in van Hiele theory was applied in 2-dimensional geometry (Mayberry, 1983; Henderson, 1988) and 3-dimensional geometry (Gutierrez et al, 1991), none of them considered the spatial abilities in geometric understanding. Saads and Davis (1997) examined the importance of spatial abilities in academic development in geometry. They designed a study to investigate 25 preservice secondary teachers' van Hiele levels and spatial abilities and to relate students' questioning and use of language to both levels. Saads and Davis (1997) constructed test items to determine participants van Hiele levels and spatial abilities. Test consists of seven items and contained various question. They used this same test to determine both van Hiele levels and spatial abilities of the participants.

Although valid and reliable instruments related to van Hiele theory (Mayberry, 1981; Gutierrez, 1991) and spatial ability (Guay, 1971; Embretson, 1987a,b, 1992, 1994, 1997) did exist in the literature, these researchers decided to construct a new test. It was very questionable to claim that a 7-item test could identify and measure both spatial ability and van Hiele levels. However, Saads and Davis (1997) claims that van Hiele levels and spatial ability are correlated without given any statistics.

The current literature related to van Hiele that concentrates on the determination of van Hiele levels of preservice teacher (Henderson, 1988; Gutierrez et al., 1991; Mayberry, 1981; Saads and Davis, 1997) is in relation to their instructional behaviors (Henderson, 1988) to their spatial ability (Saads and Davis, 1997). In this study one of the purposes is to move beyond the current literature by adding motivational (curiosity) dimensions of complex human learning phenomena in geometry, in examining geometric understanding.

Spatial Ability

Another important concept in learning geometry is spatial ability. The first studies in the field of spatial ability dates back to the 1940s and 1950s in the mathematics education literature. However, the concept was also of interest to psychologists (Spearman, 1927; Thurstone, 1938) a decade earlier. In those initial studies, numerous mathematics educators (Murray, 1949; Wrigley, 1958; Barakat, 1951) investigated the relationship between spatial abilities and mathematical abilities in different context such as algebra and geometry. They found that spatial ability was correlated more highly with ability in geometry than in algebra (cited in Bishop, 1983, p.181). Since then, spatial abilities have been accepted as crucial for high mathematical abilities (Battista, 1994,1998,1999; Battista & Clements, 1990,1991,1996,2002; Battista, Wheatley &

Talsma, 1982,1989; Bishop, 1983; Clements & Battista, 1992; Del Grande, 1987; Guay & McDaniel, 1977; Wheatley, 1990,1992). The National Council of Mathematics (2000) has identified the development of spatial abilities as a central goal in school mathematics especially in geometry. Thus, mathematics educators, and psychologists, remain interested in spatial ability today. Gardner (1983) specifies Spatial Intelligence as one type of intelligences in his Theory of Multiple Intelligences. Gardner (1993) defines spatial intelligence as the "ability to form a mental model of a spatial world and the ability to operate using that model" (p.9). Spatial abilities and their role in learning is also recognized in chemistry (Bodner& Guay, 1996; Carter, LaRussa , & Bodner, 1987; Lord,1987), engineering (Hsi, Lyn, &Bell,1997) geosciences(Kali &Orion, 1996), physics (Pallrand, &Seeber,1984).

Battista, Wheatley and Talsma (1982) conducted a study to investigate the importance of cognitive development and spatial visualization for achievement in geometry course, and effect of the instruction type on improvement of spatial ability. The study participants were 82 preservice elementary teachers enrolled in four geometry courses. Researcher, at the beginning of the semester, administered the only Purdue Spatial Visualization Test and at the end of the semester Longeot test of cognitive development and the same version of the Purdue Spatial Visualization Test. Students' grade was based on the scores on the three course exams. Battista at al. (1982) found the spatial visualization scores at the end of the semester is significantly higher than at the beginning of the scores. Among the participants, 36 students in one of the four courses, were not received any spatial activities, while other participants in other three courses received spatial activities designed for specifically for improvement of spatial visualization. The increase on the spatial visualization scores those who received spatial activities were significantly higher than who did not. As a result researcher eliminated to test-retest effect. Researcher concluded that type of activities like paper folding, tracing, symmetry etc… used in their study might improve students' spatial ability. Achievement in geometry measured by students' course grades. Both cognitive development and spatial ability positively correlated with grades, Researcher concluded "Both cognitive development and spatial visualization are important in geometry"(p.338). Although researcher found improvement on students' spatial ability due to specifically designed activities, they recommended future research stating, “More research is needed to determine if geometry instruction that emphasizes spatial activities benefits one group of students more than another” (p.338). For instance, we do not know how each students benefits when they differ in curiosity level and types (low vs. high, perceptual vs. epistemic).

Brown and Wheatley (1989) conducted a study to assess students’ meaningful mathematical knowledge when they differed in spatial ability (high and low). The study was interview-based study. Researchers designed various mathematical tasks and interviewed six students, three with high spatial ability and three with low spatial ability. Brown and Wheatley (1989) found, performance of students with high spatial ability on standardized test were average or below, they showed advanced grasp of mathematical ideas, and were able to solve non-routine problems creatively. On the other hand, students with low spatial ability performed at average or above on standardized tests but they were unable to solve non-routine problems and reasoning on mathematical tasks. Researchers’ finding opens another window in the importance of spatial ability. Spatial ability might be important factor for conceptual understanding and giving meaning to a mathematical activity.

Battista (1990) investigated the role of spatial visualization in performance and gender differences in geometry. Participants were 145 high school students from five intact classes, two of them taught one teacher and other three with another teacher. Researcher administered paper-and pencil tests in four areas: spatial visualization (Purdue Spatial Visualization Test), logical reasoning, knowledge of geometry and geometry problem solving/strategies. Battista (1990) found that male and female students differed in spatial visualization and their performance in high school geometry but not in their use of geometric problem solving strategies. More specifically, spatial visualization was only factor when geometry achievement predicted for females, whereas logical reasoning was the critical factor for males. More interesting, spatial visualization was more highly correlated with geometry learning for students in second teacher’s classes than the first teacher’s who taught three classes. Battista (1990) interviewed both teachers and second teacher required students to draw diagrams for geometric problem and first teacher strongly encouraged. Although researcher did not observe the classrooms, based on the interviews with a precaution, researcher concluded that teacher effect and instructional emphasis might play a role in developing spatial visualization skills.

Battista and Clements (1996) conducted a study to investigate students' understanding of three-dimensional rectangular arrays of cubes. Researchers' choice of topic based on previous studies demonstrated that students' having difficulties with finding the volume of rectangular solids or the number of unit cubes in it. Furthermore, students' errors were related to spatial visualization ability motivated researchers to investigate this topic (for details see, Battista and Clements, 1996, p.259). Researcher collected two types of data, quantitative, which was based on interviews and observational. Battista and Clements (1996) administered two interviews, first interviews conducted with 78 students from fifth graders, 45 students from third graders and second interviews conducted 15 fifth graders after the treatment. Researcher found that students' spatial structuring of the array was main factor determining their enumeration of it. Battista and Clements (1996) concluded that students' mental construction of arrays involves a complex interaction between spatial and numerical structuring.

Along with the spatial orientation and spatial visualization, spatial ability research produced new constructs such spatial reasoning (Clements & Battista, 1992) , spatial perception (Del Grande, 1987), spatial structuring(Battista, 1999), spatial sense (Wheatley, 1990) are name to few. These spatial ability related concepts are interrelated and there is no clear distinctions made in these relationships.

Curiosity

Historically, the fields of psychology and educational psychology have contributed to many other areas of educational studies including mathematics education and science. When they do, they have given educational researchers two main elements - a map that shows the direction of house and the keys to the doors. Metaphorically the map represents a theoretical framework and keys are definitions of concepts and instruments.

Although human salvation, development, advancement, and/or improvement of life depend heavily on human curiosity, it is hard to find literature in an educational context, especially in mathematics education. For instance there is no meta-analysis on curiosity in educational settings. Voss and Keller (1993) published a book on curiosity and explorations by reviewing theories, empirical studies, and their results. They concluded that, “At present time, no comprehensive framework exists that meets the standards of contemporary research and that can be used to classify the phenomena of curiosity and exploration” (Voss and Keller, 1993, p.18).

While this researcher is not ready to accept this conclusion (they may be wrong in this generalization), I will, for a moment, assume it to be true. Thus, if it were true then, then the first question that comes to mind might be, has there been theory proposed after their conclusion? The answer is yes: George Loewenstein published his theory of curiosity, which is called “information gap theory” in Psychological Bulletin in 1994. Loewenstein states “I propose a new theoretical account of curiosity that integrates insights from Gestalt psychology, behavioral decision theory and social psychology. The new account views curiosity as a form of cognitively induced deprivation that result from the perception of a gap in one’s knowledge” (Loewenstein, 1994, p 76). Loewenstein (1994) sees one’s awareness of the information gap as necessary precondition for experiencing curiosity. He states “failure to appreciate one does not…constitute an absolute barrier to curiosity” (p.91). Berlyne (1960), known as the father of modern curiosity research, had given some thoughts on the information gap theory 34 years ago. Berlyne (1960) stated that:

“Gestalt psychology has inspired the idea that thinking intervenes when a conceptual configuration has something missing that is needed to give it closure and this makes it stable good. Bartlett (1958) speaks of gaps in information…Description of this kind very apt in many ways. But they remain to some extent no more than metaphors. We still want to know exactly what constitutes a gap, how a subject recognize one, and what determines which gaps will have precedence over others in commanding thought”(pp.280-281).

In 1954 Daniel E. Berlyne proposed a theory for human curiosity. He differentiated two types of curiosity: perceptual and epistemic curiosity along with two type of exploratory behavior: specific and diverse (Berlyne, 1954 and 1960). Berlyne (1960) separated epistemic curiosity into two parts those being extrinsic and intrinsic epistemic curiosity. After Berlyne’s (1954) work many concepts have been introduced that are related to curiosity. For instance, Penney and McCann's introduced "reactive curiosity" referring to stimulus seeking in different situations. Day (1968), follower of Berlyne, introduced specific and diverse curiosity. After that Livson (1967) added new terms such as "productive curiosity" along with noticing, seeking and examining curiosity. Langevin (1971) investigated if the curiosity is a unitary construct. His study included several curiosity measures such as Test of Specific Curiosity, Test of Reactive Curiosity, etc..(see Langevin, 1971 for details p.363) and based on factor analysis Langevin(1971) concluded that "two weak curiosity factor derived, breadth curiosity may reflect on personality dimension …and depth curiosity may reflect the intensity of a motivational state" (p.369)Similar to Langevin(1971) study, Kreitler and Kreitler(1976) studied curiosity phenomena with first graders and their factor analysis resulted in adding more concepts; manipulatory curiosity, curiosity for complexity and adjustive-reactive curiosity, while keeping some of previously introduced ones; perceptual curiosity, conceptual curiosity.

In 1981 Malone proposed a new theory for curiosity and re-produced two concepts related to curiosity - sensory curiosity and cognitive curiosity. I said, "Re-produced" because they are no different than perceptual and epistemic curiosity. Although state and trait dimension of curiosity recognized in early studies or theoretical papers (Berlyne, 1960, Day, 1971) George J. Boyle brought the attention of curiosity research on these two categories of curiosity in 1983. Boyle (1989) further claimed that "atomistic/specific" (referring to breadth/depth, specific/diverse, perceptual epistemic and many others listed in previous section) curiosity concepts could be incorporated in the global state-trait model.

Table2.1

1954-1994 Curiosity related concepts

1	Perceptual curiosity, Sensory Curiosity	Berlyne(1954) Malone(1981)
2	Epistemic curiosity, Conceptual Curiosity, Cognitive Curiosity	Berlyne(1954) Kreitler and Kreitler (1976) Malone(1981)
3	Specific Curiosity	Day(1971)
4	Diverse Curiosity	Day(1971)
5	Reactive Curiosity, Adjustive-reactive Curiosity	Penney and McCann (1964) Kreitler and Kreitler (1976)
6	Productive Curiosity	Livson (1967)
7	Manipulatory Curiosity	Kreitler and Kreitler (1976)
8	Breadth Curiosity	Langevin(1971)
9	Depth Curiosity	Langevin(1971)
10	State Curiosity	Boyle (1989)
11	Trait Curiosity	Boyle (1989)

Until now a definition of curiosity concept, nor explained any curiosity theory in detail. But Table 1 shows the faces/dimensions of curiosity and warns the researcher about difficulty level studying about curiosity concept. Table 1 makes it clear that curiosity concept is not a unitary construct.

Maw and Maw studies

When we ask anyone if curiosity effects learning the answer will be a definite "yes". Because curiosity is part of human development from birth to death and in one way or another curiosity leads to learning something in someone's life. Maw (1967), Maw and Maw (1961a, b, c, 1964, 1965, 1967, 1968, 1970a, b, c, 1972,1977) and Maw and Magoon (1971) studied curiosity concept with school children. There are few studies that followed Maw and Maw's line of curiosity research. Torroance(1969), one of the main contributors in creativity research, investigated the creativity of gifted low and high curious children and performance on timed and untimed test of creativity. Torrance(1969) observed that highly curious children who stick with a problem and produce many original ideas but they do not produce a large number of ideas per unit of a time. He tested the hypothesis that "an untimed test of creative thinking will work more to the advantage of highly curious gifted children than it will gifted children low in curiosity"(p.156). Torrance (1969) found that high curiosity in gifted preadolescent caused him to work on a problem much more persistently and deeply than with the low curios one. Torrance (1969) put a prerequisite or precondition to his findings "If we accept the teacher's nominations of High curious and Low curious student" (p.258).

Now we can look at the Maw and Maw studies more closely. Before detailed discussion of each study done by Maw and Maw (1961a,b,c, 1964, 1965, 1967, 1968, 1970a,b,c, 1972,1977) first I will review their conceptions of curiosity construct, such as the definition of curiosity, instruments developed and used in such studies.

In their early study, Maw and Maw (1961) searched for a definition of curiosity with multiple methods including: formal and informal interviews, review of dictionary definitions, and an examination of the historical development of the word such as analyzing ancient and modern philosophical and theoretical statements regarding curiosity, and a review of literature on experiments in exploratory behavior conducted with animals and humans they developed a definition of curiosity. The definition they came up with is:

> An elementary school child was said to demonstrate curiosity when he:
> a-reacts positively to new, strange, incongruous, or mysterious elements in his environment by moving toward them, exploring them, or manipulating them
> b-exhibits a need or a desire to know more about himself and/or his surroundings
> c- scan his surroundings seeking new experiences and/or
> d-persists in examining and/or exploring stimuli in order to know more about them(Maw and Maw ,1961).

In Maw and Maw studies, three instruments were developed and used:

1-Teacher ratings: Each teacher was given the definition of curiosity and specific example for each part (a, b, c, d). Teacher was cautioned that a child may not show all the behaviors, but the more a child showed those behaviors the more curious was the child.

The teacher was also told that the child who showed the most curiosity may not be making best classroom adjustment. Then, each teacher was asked to rate his or her students in the following order:

1-Write the name of the child you consider to have most curiosity on the first line.

2-Write the name of the child you consider to have the least curiosity at the last line.

3-Write the name of the child you would rank second in curiosity on line-two.

4-Then write the name of the child you would rank next to the least curiosity child on the line above the name of the child having least curiosity.

5-Continue this manner until you have ranked all of the children in your class. (Maw and Maw, 1964, pp.31-32)

2-Peer ratings (Who-Should-Play-the-Part): The children in each classroom were asked to write the names of classmates whose behavior most nearly resembled that of a persons described in eights paragraphs. They were also told that they could use their own names. The children were told that not to use any name, including their own, more than once. Four of the paragraphs described the behavior of persons who would be thought as above average in curiosity; four of persons below the average. A child's score was a weighted sum of the number of times his name was listed. (Maw and Maw, 1964, pp.32-35)

3. *Self ratings (About Myself):* The child's estimate of his own curiosity was obtained by administering a self-rating scale entitled "About Myself". The children were told there were no "right" or "wrong" answers, but that, the best answer was what they thought was true about them. In the instrument students rated themselves on a lickert scale (0-Never, 1-Sometimes, 2-Often, 3-Always). An example from their instrument is " 26.I have a lot of curiosity. (0-1-2-3)" (Maw and Maw, 1964, pp.35-36).

The relationship among the instruments are listed below;

- Peer and teacher ratings were found to be positively correlated, r =. 54 (Maw and Maw, 1964)
- Self-ratings and peer ratings were found be positively correlated, r=. 11(Maw and Maw, 1964)

- Self-ratings and teacher ratings were found to be positively correlated, r =. 15(Maw and Maw, 1964)
- Peer, teacher and self-ratings were found to be uninfluenced by age, race, popularity and sex (Maw and Maw, 1970, p.263).
- Moderate correlations found between teacher-ratings and intelligence. (Maw and Maw, 1970, p.263).
- Moderate correlations found between peer-ratings and intelligence (Maw and Maw, 1970, p.263).
- Low, but significant relationship found between ratings and intelligence (Maw and Maw, 1961,p.298)

Nature of Sample and Rationale:

Maw and Maw studies were conducted with fifth-graders. This grade was selected to avoid, as far as possible, children with development readings and children whose interests had not crystallized compare to junior and senior high school students.

Maw and Maw (1962) reported two studies one conducted in December 1960 with 191 children, and the second in March 1961 with 749 children. The purpose of these studies was to investigate the effect of curiosity on reading more specifically comprehending the meaning of sentences. First students were divided into high and low curiosity groups based on their composite scores derived from three instruments (teacher, peer, and self ratings). The researchers developed a 52 items test to measure the ability of the children to sense the important aspects of sentences. The test items consisted of straightforward statements and some absurd statements. The children were told to mark foolish items with "X" and those not foolish with a "C". The score on the "Foolish Saying Test" was calculated by subtracting the number of incorrect answers from the number of correct answers. In the first study with 191 students the reliability of the "Foolish Saying Test" was .43. Since the reliability was low, prior to second study, the items were analyzed and the test was revised. The revised version test consisted of 22 items and reliability was found to be .91 with 749 students. The researchers compared the test scores of high and low curiosity students and they found the differences between means of both high and low curiosity groups significant at the .05 level or better. Maw and Maw concluded that high-curiosity children do comprehend more from what they read than do low curious children. In mathematics, especially in problem solving, the importance of understanding the problem has been recognized (Charles & Lester,1982; Polya, 1957).Problem solving start with reading the problem and analyzing it, determining what is given and what is being asked, separating useful information from non-useful information, interpreting important concepts, activating previous knowledge, all heavily depend on the process of comprehension during reading the problem. Maw and Maw's (1962) findings open up another dimension of the role curiosity on mathematics, since high curious students can comprehend more than low curious students, and comprehending the problem has effects on success on problem solving. Unfortunately there is no empirical study to say high curious students are better problem solvers than low curious students in mathematics.

Maw and Maw (1961) reported two studies conducted to investigate the information recognition by children with high and low curiosity. More specifically, the studies examined whether children who have high curiosity retain more information from the same lesson than those with low curiosity, even when their I.Q. scores are similar. The children were placed in high and low curiosity groups based on their composite scores on three instruments (teacher, peer and self-ratings). The first sample consisted of 145 fifth graders while the second sample consisted of 749 children. A three page single-spaced written story, that was a collection of strange but true facts mostly about animals was read to first sample in December, 1960 and in March, 1961 to second sample. At the end of the story, the pupils were asked whether they liked it and what one thing they liked best about it. Seven days later a forty-item true-false test was administered to the children. Researchers compared mean scores on the test of both high and low curiosity groups. Results were found to be significant in favor of the high curiosity group. Maw and Maw(1961) concluded that children with high curiosity learn more from a given period of exposure than do children of low curiosity, that is, they remember what they learn longer. When it comes to a comparison of individual differences in I.Q. and curiosity findings were quite interesting. Maw and Maw (1961) stated that "Child A with an I.Q. of 90 recognized more of the story material one week later than did any child in the lower-curiosity group where the mean I.Q. was over 100 and where the I.Q. of some children were 120" (p.201)

Maw and Maw (1970a) investigated the nature of creativity in 254 fifth grade boys classified as high- and low-curiosity. The 254 boys participating in the study divided into high and low-curiosity groups based on their composite scores on two instruments (teacher ratings and peer ratings). After establishing the high and low curiosity subgroups, their sample consisted of 110 high-curiosity boys and 107 low curiosity boys. Both groups were administered 38 instruments including the self-rating of curiosity and the Word Association Test of Creativity. Their factorial analysis revealed that high curiosity boys are on the positive end of continuum for Restrained Creativity (.443) and Impulsive Creativity (.459). Researchers listed that the boy high in Restrained Creativity is adaptive, confirming and helpful in dealing with others, he also lacks, some certainty about life, tending to question his own abilities and to be less independent than he would like to be. He may show emotional conflicts and feel insecure with his family. His uncertainty may be exhibited in disobedience. Researchers concluded that the boy high in Restrained Creativity is creative and sensitive and despite the recognition by his peers to be effective, reliable, accountable and loyal, he is lonely (Maw & Maw, 1970a, p.328). A boy high in Impulsive Creativity is intelligent and consistent in thinking, but unrealistic about his own abilities. Their factor analysis also revealed that low-curiosity boys are on the negative end of the continuum for Restrained Creativity (-.370) and positive side for Concrete Creativity (.251). Researchers concluded that a boy high in Concrete Creativity, does not recognize his creativity and is unaware of his own curiosity about things and about himself. Maw and Maw (1972) stated, " …boys who differ in curiosity also differ in creativity"(p.329). Maw and Maw (1970a) show the importance of curiosity in mathematics. If teacher educators do not do something to promote teacher candidates' curiosity, how can we expect those teacher candidates who are low in curiosity when they become a practicing teacher after graduation to awaken mathematical curiosity in their students and promote mathematical creativity, since they are unaware of their own curiosity and do not recognize the creativity in themselves? Unfortunately there are few empirical studies concerning promoting mathematical curiosity in students.

Maw and Maw's (1970b) study investigated the differences in preference for investigatory activities by school children that differ in curiosity level. Specific purposes were to determine whether high-curiosity children, especially boys, prefer outgoing, investigatory activities more than do low-curiosity children especially girls. First the researchers established high- and low- curiosity groups based on children's composite scores on three instruments (teacher, peer and self ratings). The researchers reported on three studies, where sample size and high and low curiosity subgroups differed in each study. For the pilot study the sample consisted of 35 children, for study two, the sample was 96 children, and study three, 442 children. Except for the pilot study, numbers of low and high curious children were equal in study two and three. Maw and Maw (1970b) developed paper-and-pencil instruments to measure children preference for different types of activities. The instruments called "What-would-you-do" tests consisted of fifty, fifty-six, and twenty-six items. These instruments were multiple-choice items in which children were allowed to choose one from among four options. The fifty-item instrument was administered in the pilot study, and item analysis revealed that some items did not discriminate and that some of them were poorly written for fifth-grade children. The researchers either eliminated items from the instrument or re-wrote and few new items were constructed. The newly formed fifty-six-item instrument was administered in the second study, and further item analysis resulted in twenty-six-item instrument. The three instruments' reliability were .65, .69, and.77 respectively. Researchers compared the means of high and low curiosity students on "What-would-you-do" tests. The results of three studies supported the hypothesis that high-curiosity children prefer investigatory activities more than do low curious children at .05 significance level or better. Based on data and analyses the researchers concluded, "High curiosity children and boys, in particular, did select outgoing, investigatory activities significantly more frequently than did low-curiosity children and girls, in particular."(Maw & Maw, 1970,p.266)

Maw and Maw's definition of curiosity set of observable behaviors. It does not include the situations or context basically they neglected the state dimension of curiosity. Berlyne 's (1960) work on curiosity was on the state dimension taking account into individual differences in curiosity. Maw and Maw preferred to study curiosity as a personality trait in educational context. I think we are in need of both type of studies since a teacher job is creating learning environments with accommodating individual differences.

Maw and Maw (1968) studies being criticized for not being reliable and valid in terms of methodology. Especially, teacher ratings were being ratings of IQ instead of curiosity (Coie, 1974). Another critics to Maw and Maw's (1968) the line of curiosity research is lack of theoretical background (for details see, Voss and Keller, 1983, p104).

Maw and Maw (1964) and Maw (1967) defended themselves about the reasons why they did not use any of the curiosity theories developed. They classified the theories under two category either being "tedium" or "titillation". Titillation label goes to Berylne. (Maw, 1967, p.1)They further critique Berlyne and his works especially his seminal book, *Conflict, Arousal, and Curiosity* (Berylne, 1960). Maw and Maw (1964) complains that although there are more than 450 reference in his book but not a single of them refer to classroom behavior.(p.2)

Maw and Maw (1961a,b, c, 1964,1965,1967a,b, 1968,1970a,b, c, 1972,1977) efforts made a significant contribution in curiosity research domain in school settings along with nice suggestions for further research. Although their research was "trait" dimension of curiosity their further research suggestion was other face of the coin "state" dimension of curiosity concept. For instance, Maw and Maw (1967) found that high curiosity children comprehend the meaning of sentence more than low-curiosity children. They raised a question "What can the classroom teacher do?" Since the classroom teacher cannot change the genetic structure of students he/she need to create an environment to stimulate curiosity in low-curiosity children.

Reactive Curiosity

Individual differences and measurement of these differences gained attention during the 1960s.Similar to Maw and Maw(1964) Penney and McCann(1964) constructed, a Reactive Curiosity Scale to measure the curiosity of children in grades 4,5, and 6. Penney and McCann (1964) define reactive curiosity as:

1-a tendency to approach and explore relatively new stimulus situations
2-a tendency to approach and explore incongruous, complex stimuli
3-a tendency to vary stimulation in the presence of frequently experienced stimulation.

According to Penney and McCann (1964) their definition make a difference between curiosity and reactive curiosity because a child may be curious but may not react to his curiosity. Maw and Maw(1964) give the description of process on definition of curiosity concept described earlier, Penney and McCann(1964) did not support any of such description. We do not know how they did come up with their definition. What is the rationale or theoretical disposition behind it? Their conception of process was very similar to Livson (1967) observation and classification of noticing, seeking and examining curiosity. A child may notice curiosity but may not examine it.

Penney (1970) examined the relationship between reactive curiosity and manifest anxiety. Total 178 children,63 in grade 4, 59 children in grade5 and 56 children in grade 6 participated in the study. Reactive curiosity measured by the Reactive Curiosity Scale (RCS, Penney and McCann, 1964) and manifest anxiety measured by Children's Manifest Anxiety Scale. High- and low-reactive curiosity subgroups established based on their scores on the RCS. He found that children's reactive curiosity is negatively related to their manifest anxiety. Penney (1970) concluded, "Children who are reactively curious exhibit less anxiety than children who are not reactively curious"(p.701). And also his findings supported to Penney and McCann (1964) study, which IQ was not related to reactive curiosity.

Harty, Andersen and Enochs(1984) investigated the relationship among students' interest in science, attitude toward science and curiosity. In their research, the study sample consists of 91 fifth graders. There instruments;

1-Children's interest in Science Measurement

2-Children's Attitude toward Science Survey

3-Children's Reactive Curiosity Scale, were administered to study sample. First two instruments were directly related to science and third instrument was more general in context. They found significant positive relationship among the three variables. Researcher also further examined the students' ratings on the instruments. For instance in interest measure (1) personal activity-oriented items like using telescope rated high than other items in the instrument. Harty et al.(1984) concluded that "items rated higher were items that would involve them more actively. Items rated lower by students involved passive actions" (p.312). Researcher could not find the hierarchical rating choice of phenomenon in other two instruments (2 and 3) used in the study.

I reported studies under two categories: Maw and Maw studies and reactive curiosity. There was a connection between studies in terms of methodology or instruments they used. Now I will report the other studies that I could not make the connections under independent studies category.

Independent Studies

Day (1968) investigated the role of specific curiosity in school achievement. He pointed our emphatically "Educational systems are continually being charged with neglecting the development of curiosity in their pupils, and with rewarding students for rote learning and a display of intelligence rather than for extending their interests in the world about them" (p.37). Day (1968) found that school grades are correlated with IQ not with curiosity measured by "Test of Specific Curiosity" using a series of 28 figures generated by Berlyne (1968). Day(1968) hypothesized that if curiosity was promoted in students, curiosity and school grades must be correlated. Day(1968) concluded based on his findings that promoting sense of curiosity in students is nowhere in the sight in the education system because curiosity and achievement did not correlate. According to Day(1968) inspiring and rewarding curiosity is a necessary conditions for the development of creative individuals.

Menis (1984) examined the effect of computers on the development of student attitudes toward sciences by encouraging the students' level of curiosity. He used to measure the curiosity the revised version of Curiosity Inventory which the original questionnaire found to be reliable in the U.S.(0.89) and the revised one in Israel(0.90) developed by Champbell (1971, 1972)(as cited in Menis,1983,p.31). To measure the scientific attitudes the Menis (1983) questionnaire was used. Sample size consisted of 65 students.35 of them assigned to control group and 30 of them experimental group. Both instruments were administered to control and experimental group as pre- and post-test. Experimental group studied computer two hours a week consisted of BASIC programming language plus computer games were supported. He found significant differences between experimental group, whom exposed to computers and control group in their attitude toward science and curiosity level. Researcher did not observe any differences between experimental and control group in both their curiosity levels and attitudes toward science. Menis(1984) also reported that although computer games were not compulsory all students engaged in playing. Menis(1984) concluded that the use of computers might have potential to development of curiosity and to improve attitudes toward sciences.

Leherissey (1971) conducted a study to investigate effect of stimulating state epistemic curiosity on reducing anxiety and improvement of performance with in a computer assisted instruction. Total sample consisted of 151 female undergraduate students, and based on their scores on curiosity measured by OTIM (Ontario Test of Intrinsic Motivation; Day, 1968) and STAI (State-Trait Anxiety Inventory, Spielberger, Gorsuch,&Lushene, 1970)participants assigned equally to Curiosity-Stimulating Instruction(CSI) or No Instruction(NI) conditions with in a Reading(R) or Constructed Response(CR) program version. Curiosity-Stimulating Instructions presented to CSI group prior to the learning to increase desire to (a) know more about a learning task;(b) approach to novel or unfamiliar learning task;(c) approach a complex or ambiguous learning task; and(d) persist in information seeking behavior in learning task. On the other hand the NI group did not get any pre-instructions. She found that high state curious students had lower levels of state anxiety and performed better than low state curious students. And also she found that high trait curious students had higher state curiosity scores than low trait curious students. (Leherissey, 1971). Leherissey (1971b) developed a 20-item State Epistemic Curiosity Scale (SECS) and tested further reliability and validity of SECS and confirmed the high reliability and validity of the instrument.

Koran, Koran, Foster and Fire (1989) investigated effect of curiosity and verbal ability on learning concepts in Biology with 7^{th} and 8^{th} graders. In their study two measures of curiosity were used; ratings of observed curiosity behavior and written test. Psychomotor curiosity measures was based on observations and rating of students over a period of 10 minutes by two graduate students. Curiosity behaviors that were observed and rated were: the student’s approach to an object, touching or moving an object, subsequent observation and manipulation of the object, and finally returning the object to its original position.(Koran et al. 1989, Koran et al, 1986).Researcher used State Epistemic Curiosity Scale(SECS; Leherissey, 1971b) as a written test. Koran(1989) et al. found a significant positive correlation between ratings of curiosity based on observations and written test of curiosity. They also found verbal ability functioned positively to facilitate learning of Biology.(Koran et al. 1989, p.409)On the other hand, researcher did not find the same relationship with curiosity and learning science. According to Koran et al.(1989) their result did not confirm that curiosity do not play roles on facilitating learning because researcher believed that " instruction format was now sufficiently complex or novel to elicit a high level of curiosity…the context in which lesson was presented(class vs. lab vs. field) probably influenced behavior."(p.409)

Vidler and Rawan(1975) conducted a study to investigate the relationship between academic curiosity and performance of college students. Curiosity measured by Academic Curiosity Scale (Vidler and Rawan, 1974). Academic Performances were students' Reading and English tests scores along with students' final exam scores, final grades and grade point average on introductory Biology course. They found positive moderate relationship between academic curiosity scale and academic performance. As summarized above curiosity concept has been investigated in trait and state dimensions in educational context.

Keller (1987a) reviewed theoretical dispositions and empirical studies and synthesizes curiosity-awakening strategies under Attention in his ARCS Model. Keller (1987a) considers the multidimensionality of curiosity concepts such as perceptual curiosity and epistemic curiosity and state and trait dimensions. Berlyne (1960) separated curiosity into two types epistemic curiosity and perceptual curiosity. Berylne (1960) stated, "It must be left to the research of data of the future to clarify the exact relation that holds between them"(p.274). In mathematics education research, in geometry content area investigation of perceptual and epistemic curiosity does not exist. Present study try to find how the learners differed in curiosity types namely perceptual and epistemic develop geometric understanding in the same environment. Changing the environment should do further study to explore the relationship between epistemic and perceptual curiosity.

CHAPTER 3

METHOD

The main goal of this study was to investigate the development of geometric thought of preservice teachers while considering their spatial ability and curiosity types. Furthermore, another goal was to explore the relationship between spatial ability, curiosity types and motivation. This study was designed to seek answers to the research questions by employing both quantitative and qualitative techniques. To investigate the first two questions of this study - *What differences, if any, exist between preservice middle and secondary mathematics teachers with different spatial ability levels and their understanding of geometry? What differences, if any, exist between preservice middle and secondary mathematics teachers with different curiosity types (perceptual and epistemic) and their understanding of geometry?* - several ethnographic techniques were used. Strategies in ethnographic research have been viewed as phenomenological, naturalistic, holistic and multimodal.

A phenomenological perspective focuses on the participants' worldview. A naturalistic perspective involves accounts of phenomena as they occur in realistic contexts. A holistic perspective focuses on descriptions of total phenomena in their context, which are used to generate variables that affect human behavior and beliefs (Goetz and LeCompte, 1984). A multimodal perspective focuses on the variety of research techniques that can be used. Each of these perspectives will be considered in this study.

To answer the third question - *What are relationships among spatial ability and curiosity and motivation?* -Quantitative methods were employed. The Pearson product-moment correlation was utilized to investigate these relationships. Three hypotheses were tested at alpha=. 05-significance level.

Is there a statistically significant relationship between preservice middle and secondary teachers' spatial ability and motivation?

Ho: There is no relationship between spatial ability and motivation

Is there a statistically significant relationship between preservice middle and secondary teachers' epistemic curiosity and motivation?

Ho: There is no relationship between epistemic curiosity and motivation

Is there a statistically significant relationship between preservice middle and secondary teachers' perceptual curiosity and motivation? .

Ho: There is no relationship between perceptual curiosity and motivation

Settings and Informants. The informants were member of an undergraduate geometry course. The course is offered at a major Research University in the southeast United States. The title for this college geometry course was MAE 4816 Elements of Geometry. This course designed for preservice middle and secondary mathematics education majors and it's a required course.

There were twenty-eight, six male and twenty-two female, students in this geometry class. In terms of ethnicity three of twenty-eight students were African American; one male and two females, an Asian and a Hispanic. The physical layout of the classroom was, just like any other classrooms. There were rows of desk, chalk, blackboard and one computer and projector. Students sit side-by-side in pairs, and these pairs are lined up in rows that face the chalkboard and stretch to the back of the room. This was the main class, however starting from the fourth week; the class also met in a computerized classroom.

For the quantitative part of study survey data were collected all class members, however, two students did not included in this part of the study; one student did not want to participate (the study was completely voluntarily) and the second student was dropped from the study based on the recommendation of instructor. The instructor felt that the student was not taking the spatial ability test seriously as he "Christmas-treed" the answers. A total of 26 students, 5 male and 21 female, participated in the study. Each member of MAE 4816 was asked to complete three inventories: Spatial ability test, Perceptual Curiosity, and Epistemic Curiosity at the beginning of semester. The questionnaires allowed researcher to identify potential participants for qualitative part of the study. Researcher also received recommendation from the advisor. The criteria used by the professor were the ability to communicate and diversity with respect to knowledge of preservice teachers. Six preservice teachers were volunteered to participate in qualitative part of the study. Among the six, four preservice teachers completed both pre- and post-interviews. Of the four informants, two were female and two were male. For future reference and discussion, the respective informants were given the names Allen, David, Barbara and Cathy. In addition, the informants called preservice middle and secondary teachers because none of them had teaching experience prior to the period of data collection.

Data Collection

The use of a variety of data collection techniques permitted the researcher to establish general principles about geometric thought from the preservice of these middle and secondary mathematics preservice teachers. Taking their spatial ability and curiosity into account, the following 5 sources were utilized to gather the data:

1-Clinical interviews between the investigator and the preservice teachers: Clinical interviews were conducted with preservice teachers twice; before and after the college geometry course. These interviews concentrated on having the preservice teachers respond to the protocol designed by Mayberry (1981). Other researchers have used the same protocol with preservice teachers (e.g.Henderson, 1988). Responses to a series of 62 questions, each printed on one fourth of letter size paper, were audio taped. The interviewer also recorded responses on a check sheet. Participants provided with blank papers, a pencil and a straight edge. Instructions to participants encouraged the draw any necessary diagrams. Informants also told that the researcher was interested in investigating their thinking during the obtaining a result, therefore, the reason an answer was given of more interest than then whether correct or incorrect answer was given. Informants were encouraged to "think aloud" and to give reasons as they went through the process of getting an answer. No probing questions were asked to lead them to correct answer, however, when the clarification needed researcher asked some question needed clarification.

Interviews were conducted in a classroom on the university campus. The preservice teacher and investigator were the only people present during each interview. Each question was read a loud by the preservice teacher and then answered. Responses analyzed using criteria established by Mayberry (1981; See Appendix A, B). The analyses of structured interviews are discussed in analyses section of this chapter.

2-Collect student artifacts, i.e., test, assignments, etc.

Artifacts collected from the students will include daily work, homework, and presentations, drawings during the interviews.

3-ECI-PECI curiosity inventories: Two 10-item scales that were very recently developed assess positive reactions to different types of stimuli:

The Epistemic Curiosity scale (Litman & Spielberger, 2003) assesses feelings about stimuli that activate cognitive processes (e.g., "When I am given a new kind of arithmetic problem, I enjoy imagining solutions", "I enjoy discussing abstract concepts"). The alpha coefficient for Epistemic Curiosity scale was .85 for women and .84 for men.

The Perceptual Curiosity scale (Collins, Litman, & Spielberger, 2004) evaluates feelings about sensory stimuli (e.g., "When I smell something new, I try to find out what the odor is coming from", "I like to listen to new and unusual kinds of music"). The alpha coefficient for Perceptual Curiosity scale was .85 for men and .87 for women.

In responding to each curiosity item, the participants were instructed to report how they generally feel on a 4-point scale ranging from 1(almost never), 2(sometimes), 3(often), to 4(almost always). Total perceptual and epistemic curiosity scores were calculated summing up 10 items for each scale. Possible maximum score was 40 and minimum score was 10.

4-The Purdue Visualization of Rotations (ROT): The Purdue Visualization of Rotations Test (ROT) is one elements of the Purdue Spatial Visualization Test Battery (Guay, 1977). The Purdue Visualization of Rotations test consists of 20 items that have students to:

1. study how the object in the top of the line of the question is rotated
2. picture in your mind what the object shown in the middle of the line of the question looks like when rotated in exactly the same manner, and

3. select from among the five drawings(A, B, C, D, and E) given in the bottom line of the question the one that looks like the object rotated in the correct position.(Bodner & Guay, 1997, Pribyl at al. 1987).

As instructed in the scale (Bodner & Guay, 1997) students given 10 minutes. Researcher explained that this activity is not going to affect their grades in any way. At the beginning of semester each student took the test. For correct response a “1” point given. Total score was the sum of correct scores on the test. Possible the highest score was 20.

5-CIS (Course interest survey): Keller’s CIS (Course Interest Survey; Keller& Subbiyah, 1993) will be used to identify student motivation at the end of semester. In the instrument participants are asked to think about each statement in relation to the course itself, and to indicate how true each statement is. The response scale ranges from 1 (Not True) to 5 (Very True). Therefore, the minimum score on the 34-item survey is 34, and the maximum is 170 with a midpoint of 102. The minimums, maximums, and midpoints for each subscale vary because they do not all have the same number of items. There are 4 subscales: one for each of the ARCS components (Attention, Relevance, Confidence, Satisfaction) and one for the ARCS total score. Nine of the 34 items are reversed. The alpha coefficient for each subscale were .84(Attention), .81(Confidence),.84(Relevance), and .88(satisfaction). For the total scale alpha coefficient was .95. Total scales were calculated summing up for each subscale. Total sum of the subscale gave the motivation scores.

6-Classroom observations: In order to gain insight about the individual behaviors in the same classroom environment the classroom was observed for each class session on regular basis through the semester each seventy-five minutes. The researcher made field notes that focus on the students, the instruction and the classroom environment. Student observations were guided by mainly ARCS model of motivation (Keller, 1987) and also Maw and Maw’s definition of curiosity (e.g. Persist in examining and/or exploring mathematical ideas, concepts), Penny and McCann’s definition of reactive curiosity (e.g. tendency to vary stimulation in the presence of frequently experienced stimulation), and other curiosity related constructs such as Livson’s noticing, seeking and examining curiosity, as discussed in the second chapter. Observation of instruction will focus on effect of the instructional activities on promoting curiosity. In addition to follow up what was observed researcher also conducted short informal interviews with students about the activities right after the class. While most of the data related to curiosity and motivation in this study will be self-reports, these observations served to help triangulate the self-reported data. Delamont and Hamilton (1983) discuss that "even with small sample observations, important insights can be obtained, relationships between pupils and teachers clarified, common phenomena identified, and generalizations attempted"(as cited in Clark &Leat, 1998, p.76). Initial observation focused on the learning environment. Through the initial observations a confirmation of students curiosity as determined by PECI and ECI also made. This allowed for more focused observations as the semester progresses.

Data Analyses

In order to answer the first two questions of this study, the investigator interviewed each preservice teacher. Through the interviews and observations, informal interviews qualitative data were collected. For the first two questions analyses of data made use of triangulation and analytical induction (Goetz & LeCompte, 1984). The use of several sources of data for pinpointing the accuracy of conclusions was a form of triangulation. Triangulation assists in correcting biases that could occur when the researcher was the only observer of the phenomena under investigation. (Goetz & LeCompte, 1984). Through the use of analytical induction, the data were scanned for categories of relationships between spatial ability, curiosity types and levels of geometric thought. One purpose of this study was to characterize each preservice teachers' geometric thinking. The interviews were analyzed to categorize the responses made to the questions by each preservice teacher. Each question for a level and a concept strand was given a "1" if the response was adequate and a "0" if not.If the reasons given for the responses were also not valid, the response rated "0". For example, for the following questions(question, 17);

Does a right triangle always have a longest side?(if yes was the response, the student then asked which one), and does a right triangle always have a largest angle?(if yes was the response, the students was asked which one?)

An adequate response would have been “Yes, the hypotenuse” and “yes, the right angle.” An inadequate response to this questions would have been, “yes, one of the legs” or “yes, but I don’t know.”

Responses were independently analyzed by the researcher and by a graduate student in applied mathematics. An inter-rater reliability was determined based on percentage of agreement of adequate responses. In Henderson (1988) study reliability would be judged satisfactorily if the investigator and graduate assistant had at least a 97 % agreement on assignment of scores. Same criteria applied to this study.

A second goal of this study was to investigate the relationship between spatial ability and motivation, and also curiosity types and motivation. Surveys constituted quantitative data. Since third questions investigate correlation among three variables the Pearson product-moment correlation utilized to investigate these relationships. Three hypotheses were tested at $\alpha = .10$ significance level with two-tailed test.

Table3.1

Types of Data and Research Question

Sources of data and Instruments	Research Question - 1	Research Question -2	Research Question-3
Van Hiele Interviews	*	*	
Perceptual Curiosity (PECI)		*	*
Epistemic Curiosity (ECI)		*	*
Purdue Spatial Ability (ROT)	*		*
Course Interest Survey (CIS)		*	*
Field notes	*	*	*
Artifacts	*	*	*
Observation	*	*	*

Descriptions of learning environment and preservice middle and secondary mathematics teachers’ understanding of geometry are presented in the following chapter.

CHAPTER 4

DATA ANALYSES AND RESULTS

One purpose of this study was to characterize the development of geometric thought of middle and secondary preservice teachers while considering their spatial ability levels and curiosity types. The three research questions that were addressed:

1-What differences, if any, exist between preservice middle and secondary mathematics teachers with different spatial ability levels and their understanding of geometry?

2-What differences, if any, exist between preservice middle and secondary mathematics teachers with different curiosity types (perceptual and epistemic) and their understanding of geometry?

3-Is there a relationship among motivation, spatial ability and curiosity?

This chapter will first provide the class results for spatial ability, motivation and curiosity characteristics measured by surveys and a test. After that the contextual elements of learning environment, instructor, instruction, and activities are described. The class context is provided in order for the reader to be able to situate each informant in the overall context. Then a rich description of characterization of geometric thought for each of the four informants is described. Four informants' van Hiele levels were measured before and after the completion of informal geometry course.

Spatial Abilities, Motivation & Curiosity

The Purdue Visualization of Rotations Test (ROT) was administered to determine the spatial ability levels of students. The test is comprised of 20 questions . Table4.1 provides the results. Students were classified as having high spatial ability if they were one standard deviation above the mean and low spatial ability if they were one standard deviation below. Seven were in the high category and five in the low category. The rest of class members were between one standard deviation below and above the mean (medium category). 11 students spatial ability scores were below the mean and 15 students were above the mean.

Motivation level of students' as measured by Course Interest Survey (CIS) yielded as a mean score of 135.58 with minimum score of 101 and maximum score of 160. Eleven students scored above the mean and sixteen students scored below the mean on the survey. Low spatial ability students' mean score on motivation was 138.90 and high spatial ability students' mean score on motivation was 133.13.

Perceptual and Epistemic Curiosity inventories were used to identify trait curiosity characteristics of students. Students' mean scores did not differentiate on curiosity types. Epistemic Curiosity mean was 25.42 and perceptual curiosity mean was 25.42. Table 4.1 provides a summary of the statistical results across spatial ability, motivation, epistemic, and perceptual curiosity concepts.

Table 4.1

Descriptive Statistics: Minimal and Maximal Score, Mean and Standard Deviation for Spatial Ability, Motivation, Perceptual and Epistemic Curiosity (n=26)

	Min	Max	Mean	SD
Spatial Ability	3	19	12.77	4.23
Motivation	101	160	135.58	13.54
Perceptual C.	18	34	26.31	4.57
Epistemic C.	16	35	25.42	4.82

Learning Environment

The purpose of this section is to provide insights to the reader about the course and elements of the learning environment such as example of activities from classroom, classroom materials and student interactions.

Instructor

The course used in the study, MAE 4816 Elements of Geometry, is designed for preservice middle and secondary mathematics education majors and it is a required course. The instructor of this geometry class was a mathematics educator with over ten years classroom experience. He has a Ph.D. in mathematics education.

The instructor described the "objectives" of the course in the syllabus as follows:

“Generally, students emerging from MAE4816 should appreciate geometry as a means of analyzing, describing, and understanding the world and seeing beauty in its structures. Specifically, the course will be aligned with standards for middle and high school grades from the National Council of Teachers of Mathematics and Sunshine State Standards. Students will be expected to know and understand the objectives outlined by these national standards. In this spirit, MAE4816 will focus on proficiencies related to geometric reasoning so that students in the course will be able to develop effective strategies for understanding geometry from conceptual, representational, and problem solving perspectives. Students should be able to:

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- Specify locations and describe spatial relationships using coordinate geometric and other representational systems;
- Apply transformations and use symmetry to analyze mathematical situations;
- Use visualization, spatial reasoning, and geometric modeling to solve problems”(p.1)

Similarly the instructor also outlined the teaching strategies in the syllabus

“The participant and professor will create a learning community for sharing of reflections on reading and presentations. The emphasis will be on sharing, support in the group environment, and personal development of meaning. Although students learn individually, they profit from working together; thus MAE4816 will ask students to work cooperatively with fellow students. This means pulling your desks together and getting to know one another. When working in cooperative groups, students should be willing to listen to other students, to be an active participant, to ask one another questions when you do not understand, and to help one another.”(p.2)

The class met twice a week for 1.25 hours. The course used, *Discovering Geometry: An Inductive Approach (2nd edition*) as a textbook and the software *Sketchpad: Student Edition* as a supplementary material. Part of the class sessions were allocated to time in a computer lab.

There were twenty-eight (six male and twenty-two female) students in this geometry class. In terms of ethnicity three (one male and two females) of twenty-eight students were African American, an Asian female and a Hispanic female. The physical layout of the classroom was, just like any other classrooms. There were rows of desk, chalk, blackboard and one computer and projector. Students sit side-by-side in pairs. This was the main class, however starting from the fourth week; the class also met in a computerized classroom. The researcher observed each class and took field notes. The role of the researcher was non-participant observer.

Class sessions

The class sessions in general consisted of small lecture, problem solving activities, small group work, and class discussions. There was weekly homework from each chapter of the book. Dr. A. did not collect the homework, however, in the week following the assignments the students and instructor discussed the homework problems. Dr. A started to the topics with basic concepts such as the definition of a "point". He did not assume that students had a certain level of geometric knowledge for each concept. This was true throughout the semester for all concepts. For instance, for the concept of triangle, class investigated the sum of angles in a triangle. Dr. A handed out a paper with a triangle drawn on it. Dr. A asked the class "to cut out the triangle and label each angle with a letter or number" and then "rearrange them on a straight line, which we know is 180°". Figure 4.1 shows the steps of activity done by students.

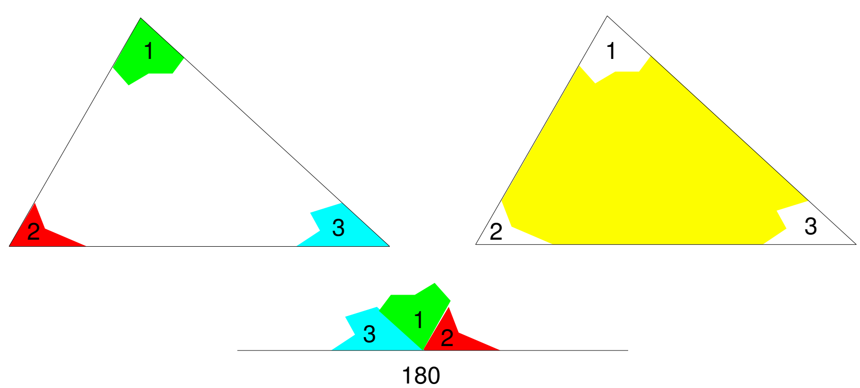


Figure 4.1

The sum of angles in a triangle activity

Students also investigated the triangle sum conjecture as a formal proof using auxiliary lines. This example shows how the instructional sequence builds geometric knowledge from the concrete to the abstract. Keller's (1987) ARCS model identifies one of the attention strategies as concreteness. More specifically Keller (1987) suggested that "Show visual representations of any important object, or set of ideas or relationships" (p.4).

Throughout the semester the class explored the course topics with a variety of activities. Some of these activities were, poolroom geometry, putt-putt golf, going to grandma's house, and discovering Pi with M&Ms. As an example, discovering Pi with M&Ms is discussed briefly here (for the full version, see Appendix D).

The activity was introduced as a way to " use a concrete model to help enrich children's understanding of the relationship between circumference and diameter of circles of various sizes" in the activity package. Dr. A hand out the activity sheets and large bag of M&Ms for each group of students. Dr. A gave a very brief explanation about the activity where " we are going to measure circumference and diameter of the circle with M&Ms and find their ratio, which is?" and the class responded, "Pi".

Dr. A asked the class " How many digits do you think we will find correctly in decimal place with this method? 3**.????**"

One student was able to guess correctly, she said, "two digits". While students engaged with the activity Dr. A made a comment to the researcher "I am surprised someone guessed correctly, usually they don't, and when they found we can find the two digit decimal place they were amazed (referring his previous classes)."

There were four different sizes of circles. Students counted the number of M&M's it took to measure the circumference and diameter of the circle and recorded these in a table.

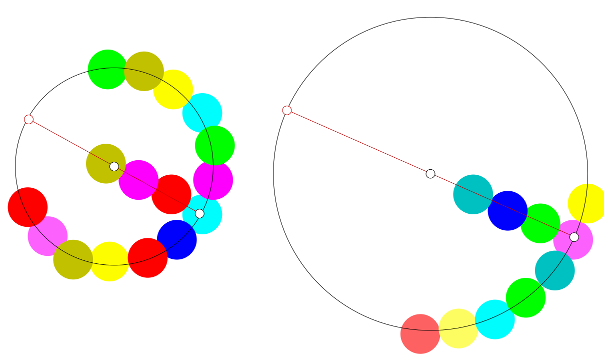


Figure 4.2

Illustration of M&M's on a circle and diameter.

Table 4.2

Data recording table for M&Ms

Circle	Number of M&Ms For circumference	Number of M&Ms for diameter	C/d (or C ÷D)
A			
B			
C			
D			

At the end a whole class average was calculated and it was 3.1419. All students enjoyed the activity, as the researcher walked around and listened to students, most of the comments were "This is cool, I think I am going to use this activity when I am teaching". These activity packages were prepared in detail, and ready for use in teaching. Students were learning activities that they could actually use in their future teaching. This exemplifies in one of the strategies in the relevance dimension of ARCS Model (Keller, 1987), which is future usefulness. Keller (1987) states, " State explicitly how the instruction relates to future activities of the learner and ask learners to relate the instruction to their own future goals" (p.4)

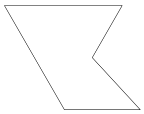
The two activities described here are examples of what students engaged in during the course. They also characterize various elements of the ARCS model.

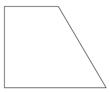
Are Learning Activities parallel to van Hiele levels?

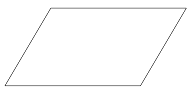
In the van Hiele model the sequence of instruction is important for the growth in understanding. This section will look at the design of instruction and show how it was consistent with the van Hiele levels. Class inclusion is an important construct as students' progress through levels one, two and three. While class inclusions are not expected for responses reflecting level one understanding they are subsequent levels.Two specific activities from the course were designed to allow students to explore relationships. The first was "Geometric Conclusions" (see figure 4.3)

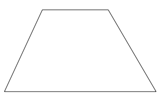
Name:…………. Date:……………

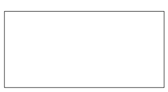
Geometric Conclusions:

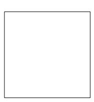
 A polygon is closed, a plane figure with sides that are line segments

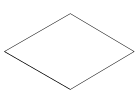
 A quadrilateral is a polygon with four sides

 A parallelogram is a quadrilateral in which each pair of opposite sides is parallel

 A trapezoid is a quadrilateral with exactly one pair of parallel sides.

 A rectangle is a quadrilateral with four right angles

 A square is a rectangle with four equal sides.

 A rhombus is a parallelogram with all sides congruent.

Complete each sentence below with the appropriate phrase:

1 = can be 2 = is always 3 = is never

1-A parallelogram …….a polygon.	16-A quadrilateral……a square.
2-A parallelogram……… a rectangle.	17-A parallelogram …….a rhombus
3-A rectangle………….a square	18-A rhombus…….a quadrilateral
4-A quadrilateral……..a rhombus.	19-A rectangle………….a rhombus.
5-A trapezoid…………a square	20-A quadrilateral……..a trapezoid
6-A square…… a rectangle.	21-A trapezoid……..a parallelogram

7-A rectangle……….a parallelogram.	22-A square……. a trapezoid
8-A parallelogram…….a square.	23-A rhombus…….a rectangle.
9-A quadrilateral……a rectangle.	24-A rectangle……….a trapezoid.
10-A square……. a parallelogram	25-A square……. a rhombus
11-A trapezoid……..a rhombus	26-A parallelogram…….a trapezoid.
12-A rhombus…….a parallelogram.	27-A rhombus…….a trapezoid.
13-A parallelogram…….a quadrilateral	28-A trapezoid…….a quadrilateral
14-A rhombus…….a square.	29-A quadrilateral……a parallelogram
15-A trapezoid…….a rectangle.	30-A polygon…..a quadrilateral.

Figure 4.3

Geometric Conclusions

During the "geometric conclusions" activity first students worked on each questions individually. And then whole class responded each question all together.

Another activity, called "Polygonal Relationships" was similar to the previous activity, but was in a "true" and "false" format.

Name:…………… Date:…………………

Polygonal Relationships:

State whether each of the following is true or false

1. Every isosceles triangle is equilateral.
2. All equilateral triangles are isosceles
3. All squares are rectangles
4. Some rectangles are rhombuses
5. All parallelograms are quadrilaterals

Figure 4.4(continue)

6. Every rhombus is a regular quadrilateral

7. Every parallelogram is a trapezoid
8. Every equilateral triangle is a scalene triangle
9. Some rectangles are squares
10. No square is a rectangle
11. No trapezoid is a parallelogram
12. Some right triangles are isosceles.

Figure 4.4

Polygonal Relationships

Similar problems were used in Mayberry's (1981) instrument for testing level III thought. For instance, question 32 (see Appendix A) asked if all isosceles triangles are right triangles or question 25 (see Appendix A) asked, if all squares are also rectangles. Thus items on the instrument used to measure geometric understanding could be mapped back onto class activities and vice versa.

The Geometer Sketchpad (Jackiw, 1995) is a dynamic geometry software program. It allows students to explore and analyze mathematical concepts and for teachers to demonstrate geometric concepts. The use of technology in the course supported NCTM's (2000) call for geometry to be learned using dynamic software along with concrete models and drawings. Students formed a group with two members. Each group did a presentation with a geometer sketchpad activity on different topic.

Discovering Geometry An Investigative Approach was used as textbook in the course. There were fourteen chapters. The content of the book was consistent with van Hiele levels and NCTM (2000) principles and standards. Detailed description of such connections with examples for each standard and van Hiele levels were given on the publisher website.

Throughout the semester students actively engaged the classroom activities. Attendance was very good. Most of the time it was hundred percent, except for few weeks; couple of students was absent due to official reasons (such as medical appointment).

Preservice Teachers' Understanding of Geometry

The following sections provide a description of the four cases, Allen, David, Barbara and Cathy. Following the four cases a cross-case discussion is provided that looks by van Hiele level at the geometric understanding of the 4 students.

The Case of Allen

Allen, a twenty-two year old male, was a junior student majoring in secondary mathematics education. Allen's high school mathematics courses were: Algebra II, Geometry, Statistics/Trigonometry and Pre-calculus. Allen indicated that the only course he "dislikes" is Geometry and he felt that the degree of difficulty for his geometry course was average, while all other high school mathematics courses were easy and he liked them. His grades for all courses were an A except for Geometry, which was a B.

Allen's college mathematics background consisted of Calculus I, Calculus II and Statistics. He liked both Calculus classes but he felt the degree of difficulty for Calculus I was easy and for Calculus II was "very hard". Although he did not like the Statistics course he rated the difficulty level as easy.

Allen mentioned that he did not care about the mathematics grades he just liked the mathematics and felt good about it. When it comes to Geometry, it was not his favorite topic or class but it was okay. He stated, "It is alright, I just really didn't like all the theorems and proofs."

Allen had the highest score on spatial ability in class.

During informal interviews with two faculty members(neither were the instructor of the course) Allen as described as a student who does what needs to be done in class and nothing more. There had been little effort with things such as meeting with professor outside of the office hours, doing extra work beyond the assignments etc…However, they both felt he was strong mathematically. He does what needs to be done in class. No sign of unusual effort such as meeting with professor outside of the office hours, doing extra work beyond the assignments, etc… However he has a strong mathematical mind.

Allen's geometric thought was at level III during the pre-interview. There was an improvement on six concepts after the finished the college geometry course. From his answers, Allen's level of geometric thought was characterized at the fourth level, except for isosceles triangle. There were no references in his answers to other axiomatic systems, which would have been indication of fifth level thinking. Table 4.4 summarizes the assigned scores for both pre- and post- interviews. A “1” indicates met the criteria for the level, a “0” was did not meet the criteria.

Table 4.4

Summary of Allen’s van Hiele Levels

Concept		Levels							
		I		**II**		**III**		**IV**	
		Pre	Post	Pre	Post	Pre	Post	Pre	Post
1	Square	1	1	1	1	1	1	0	1
2	Isosceles Triangle	1	1	1	1	1	1	0	0
3	Right Triangle	1	1	1	1	1	1	0	1
4	Circle	1	1	1	1	1	1	0	1
5	Similarity	1	1	1	1	1	1	0	1
6	Parallel Lines	1	1	1	1	1	1	0	1
7	Congruence	1	1	1	1	1	1	0	1

The Case of David

David, a twenty-one year old male, was a junior student majoring in secondary mathematics education. David’s high school mathematics courses were: Algebra I and II, Geometry, Trigonometry and Analytical Geometry. David indicated that he liked all the courses except Trigonometry, which he labeled with difficulty level of “fair”. He felt the degree of difficulty for Analytical Geometry “hard” while all other high school mathematics courses were easy. His Trigonometry grade was a “B” and Geometry was “A-” all other high school mathematics course grades were an “A”.

David’s college mathematics background consisted of Algebra, Trigonometry, Pre-Calculus, Calculus I & II and Statistics. He did not like either the Statistics or the Calculus classes. David liked Algebra and it was easy for him. All three courses: Trigonometry, Pre-Calculus and Calculus I was “hard”.

When it comes to Geometry, he stated, “I like it”. David’s spatial ability was just above the mean. His spatial ability could be classified average as compared to the other students in the class.

Through informal interviews with faculty(who did not teach the course) the researcher was given a similar picture of David by each as different from all the other students. He was described he explores the topic in details, he asks questions, and he has a passion for mathematics.

David’s geometric thinking was at level III for the concept of square and isosceles triangle during the pre-interview. David met the criteria for level IV for both concepts on the post interview. However, he did not achieve the criteria for concept of congruence both during pre- and post- interview at level III. The pattern of post-interview was non-hierarchical (1,1,0, 1) instead of (1,1,0,0) or (1,1,1,1). Table 4.5 summarizes his assigned scores for the seven concepts, a “1” indicating the criteria for the level was met and a “0” indicating the criteria for the level was not met.(Criteria for the questions and levels are in Appendix A).Since questions for the fifth level were not designed for all the concepts, the criteria was answering four of the five questions correctly. David did not achieve the fifth level on either the pre- or post-interview.

Table 4.5

Summary of David's van Hiele Levels

Concept		Levels							
		I		II		III		IV	
		Pre	Post	Pre	Post	Pre	Post	Pre	Post
1	Square	1	1	1	1	1	1	0	1
2	Isosceles Triangle	1	1	1	1	1	1	0	1
3	Right Triangle	1	1	1	1	1	1	1	1
4	Circle	1	1	1	1	1	1	1	1
5	Similarity	1	1	0	1	1	1	1	1
6	Parallel Lines	1	1	1	1	1	1	1	1
7	Congruence	1	1	1	1	0	0	1	1

The Case of Barbara

Barbara, a twenty-one year old female, was a junior student majoring in middle grades mathematics education. Barbara's high school mathematics courses were: Algebra II, Geometry (Honors), Trigonometry and Pre-Calculus. Barbara liked all the courses. For her geometry was hard, Algebra-II was easy; Trigonometry and Pre-Calculus were average in terms of difficulty. Her course grades were all A's, except for Trigonometry, which was a B.

Barbara's college mathematics background consisted of College Algebra and Calculus. She liked Algebra but she did not like the Calculus. She felt the degree of difficulty for College Algebra was "easy" and for Calculus I was "hard".
She responded to the question: How do you feel about the geometry? "I enjoy it, it just takes me more time to understand than others". Barbara scored just below the mean score on spatial ability.

Table 4.6 provides a summary of Barbara's pre and post interview results. Her geometric thinking could be classified as being mixed or between levels II and III. For the concept of square Sara had a non-hierarchical pattern, that is, the pattern was (1,1,0,1) instead of (1,1,0,0) or (1,1,1,1). Non-hierarchical pattern was consistent with Henderson (1988) findings. She explained that memorizing might be the reason for pre-service teachers being able to write a proof but not showing a depth of understanding for the relationship.

After the post-interview Barbara's geometric thinking could be classified level III for the concept of parallel lines and congruence and at level IV remaining concepts.

Table 4.6

Summary of Barbara's van Hiele Levels

Concept		Levels							
		I		**II**		**III**		**IV**	
		Pre	Post	Pre	Post	Pre	Post	Pre	Post
1	Square	1	1	1	1	0	1	1	1
2	Isosceles Triangle	1	1	1	1	1	1	0	1
3	Right Triangle	1	1	1	1	1	1	0	1
4	Circle	1	1	1	1	1	1	0	1
5	Similarity	1	1	0	1	0	1	0	1
6	Parallel Lines	1	1	1	1	1	1	0	0
7	Congruence	1	1	1	1	0	1	0	0

The Case of Cathy

Cathy, a twenty-year-old female, was a junior student majoring in middle grades mathematics education. Cathy's high school mathematics courses were; Algebra I &II

(honors), Geometry, Trigonometry. Cathy indicated that the only course she "dislikes" was Geometry and she felt the degree of difficulty for geometry average, while all other high school mathematics courses were average and she liked. Her grades for all courses were an "A", except Trigonometry, it was a "B".

Cathy's college mathematics background consisted of Trigonometry, Calculus I and College Algebra. She liked the all three classes but she felt the degree of difficulty for Trigonometry was "average" and for her Calculus-I and College Algebra were "hard".

When it comes to Geometry, it was not her favorite topic. She stated, "I do not really get into it much. The concepts are harder to get than Algebra."

Cathy had a low spatial ability. Her score was one of the lowest score in the class.

Cathy's responses during the pre-interview showed characteristics of a person whose geometric thinking could be classified at the level II and for circle concept level III. For the concept of square and similarity concepts Cathy had non-hierarchical patterns, that is, the pattern was (1,1,0,1) for square and (1,0,1,0) for similarity instead of (1,1,1,0) or (1,1,1,1). This non-hierarchical pattern was consistent with previous studies (e.g. Henderson, 1988).

Cathy's geometric thinking was at level II during the pre-interview. Cathy during the post interview gave up on some level III questions by saying "I pass". This behavior was different from the pre-interview; at least she tried to give some explanations. When it comes to level IV questions, she tried and successfully answered construction of proof type questions.

Table 4.7 summarizes the assigned scores for the concepts. Using the criteria established by Mayberry (see Appendix B), a "1" indicates the criteria for the level was met and a "0" indicates the criteria for the level was not met.

Table 4.7

Summary of Barbara's van Hiele Levels

Concept		Levels							
		I		**II**		**III**		**IV**	
		Pre	Post	Pre	Post	Pre	Post	Pre	Post
1	Square	1	1	1	1	0	0	1	0
2	Isosceles Triangle	1	1	1	1	0	0	0	0
3	Right Triangle	1	1	1	1	0	0	0	1
4	Circle	1	1	1	1	1	1	0	0
5	Similarity	1	1	0	1	1	0	0	1
6	Parallel Lines	1	1	1	1	0	0	0	1
7	Congruence	1	1	1	1	0	0	0	0

Comparison Across the Preservice Teachers

The first set of comparisons provided a view of geometric thought of the preservice middle and secondary mathematics teachers. From the interviews conducted with each of the four preservice teachers using the Mayberry protocol, a characterization of geometric thought was determined for each before and after they took the college level geometry course.Significant observations for responses for each of the five levels are discussed in the following sections. Particular attention was paid to responses at each of the levels that highlighted the differences across the preservice teachers.

Level I

The four preservice teachers correctly answered questions on recognizing and naming given figures. Naming the figures was an easy task for the preservice teachers, except during the pre-interview Barbara had difficulty with isosceles triangles by mixing them up with equilateral triangles.

Discrimination items at this level indicated that the recognition of a given shape from among examples and non-examples was a more difficult task. This is especially true for the concept of right triangle on the discrimination question for the concept of right triangles (see Figure 4.5); the preservice teachers were to select the ones that were right triangles. Three of the four preservice teachers(Allen, Barbara and Cathy) did not choose all the right triangles. The more interesting during the post interview none of the preservice teachers identified all right triangles. Table 4.8 and 4.9 shows the identified triangles by the preservice teachers. The numbers indicate the order of the choice during the pre Interview:

Three of the four preservice teachers did not identify "b" and "f". All three had chosen "a" and "e" as being right triangle in that order. David was the only preservice teacher who identified all four right triangles. He also differed on the order of selecting right triangles.

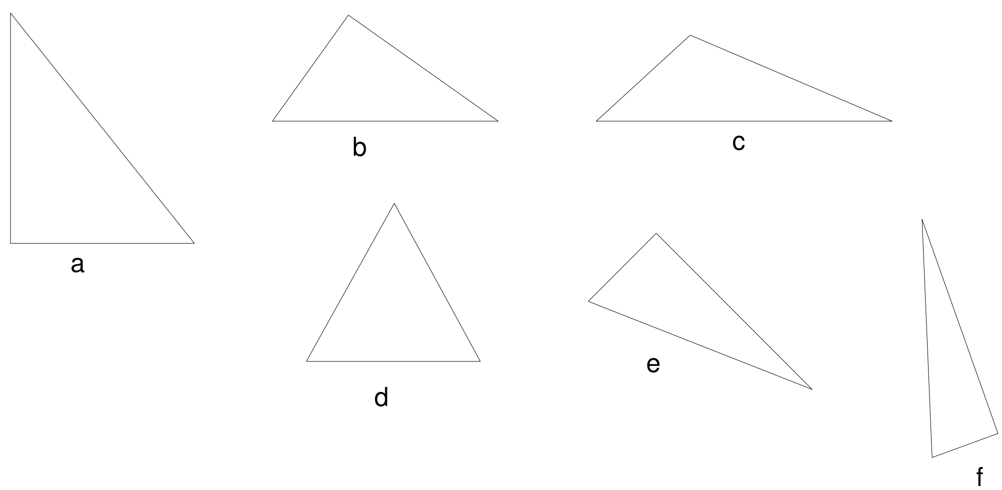


Figure 4.8

Level I Question for Right Triangles

Table4.8: Preservice Teachers' Identification of Right Triangles (Pre-Test)

Preservice Teacher	A	B	E	F
Allen	1	-	2	-
David	2	3	1	4
Barbara	1	-	2	-
Cathy	1	-	2	-

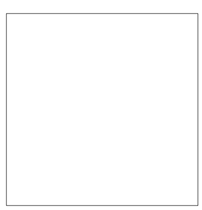
The post-interview performance(see table 4.9) of three preservice teachers' was different from the pre-interview. Allen had no difference on performance and order of his selection of right triangles. David did not identified the "f" as being right triangle during the post interview. All three chose the triangles in same order.

Allen: A→ E , David: A→B→E , Barbara: A→B→E, Cathy: A→B→E

Table4.9: Preservice Teachers' Identification of Right Triangles (Post-Test)

Preservice Teacher	A	B	E	F
Allen	1	-	2	-
David	1	2	3	-
Barbara	1	2	3	-
Cathy	1	2	3	-

The question for square (see figure 4.6) was multipurpose in that the answer given could reflect Level I thinking or Level III thinking. During the pre-interview three of the four preservice teachers named the figure as square. Only one preservice teacher, David, included more labels than square. During the post-interview Barbara included more labels than square along with David.



This figure is which of the following?

(a)triangle (b)quadrileteral
(c)square (d)parallelogram
(e)rectangle

Figure 4.6

Level I Question for Square

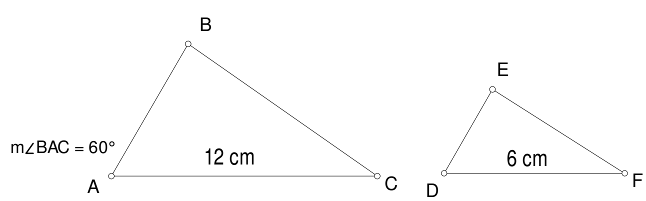
The other two preservice teachers' answer both during the pre- and post-interview were based on an "appears like" basis. Table 4.10 summarizes the responses given by each preservice teacher both during the pre- and post-interview.

Table 4.10: Labeling a square and class inclusions

Preservice Teachers	Pre-Interview	Post-Interview
David	square, quadrilateral, parallelogram, a rectangle	Square, quadrilateral, parallelogram, a rectangle
Allen	Square	Square
Barbara	Square	Square, quadrilateral, parallelogram, a rectangle
Cathy	Square	Square

Level II

At this level properties of seven concepts are tested. Preservice teachers answered all questions at this level correctly, except one question related to concept of similarity (see figure 4.7). All preservice teachers gave correct answer during the post interview, however, during the pre interview, their performance varied. Allen was the only preservice teacher who answered the questions both during pre- and post-interviews. Barbara missed the first part of the question dealing with length, while Cathy missed the second part of the question. David missed both part of the question. Table 4.11 summarizes achievement of preservice teachers on similarity problem at level II.



Triangle ABC is similar to triangle DEF

How long is ED? How do you know?

What is the size of angle EDF? How do you know?

Figure 4.7

Level II Question for Similarity

Table 4.11: Preservice Teachers Achievement on Similarity Question

Preservice Teacher	Length		Angle	
	Pre	Post	Pre	Post
Allen	1	1	1	1
David	0	1	0	1
Barbara	0	1	1	1
Cathy	1	1	0	1

During the post interview both David and Allen tried to find the actual length of DE. This might be due to their experience of conceptual conflict during the course. They had experienced different type of problems for which it looked like there was no solution or not enough information was given.

Level III

Geometric thinking at the third level was characterized by the ability to give definitions, to recognize and name the relationships, and to recognize class inclusions and implications. At this level four questions asked for necessary conditions for a square, a right triangle, an isosceles triangle and a pair of parallel lines. Table 4.12 shows the credit the preservice teachers received for these four concepts.

Table 4.12: Preservice Teachers' Achievement on Necessary Conditions Questions

Preservice Teacher	Topic							
	Square		Right Triangle		Isosceles Triangle		Parallel lines	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Allen	1	0	1	1	1	1	1	1
David	0	1	1	0	1	0	0	0
Barbara	0	0	0	0	0	0	0	1
Cathy	1	0	0	0	1	0	0	0

Three of the four preservice teachers had difficulty on this type of question both during the pre- and post- interview. The preservice teachers may have included too much information or not enough information. For instance (Questions 26 and 28; Appendix A), the preservice teachers might have failed to include that the figure was a triangle. This finding was consistent with Henderson's (1988) conclusions. Only Allen was the only preservice teacher who had answered correctly all four questions during the pre-interview.

Two observations were made based on the responses to level III questions. The first observation was that the preservice teachers did not understand the role of definition, as a set of minimum conditions. This finding was consistent with previous studies (Henderson, 1988; Mayberry, 1981). The second one was that when the question did not include a drawing of the figure, the preservice teachers would think in terms of generalized figures as opposite to a specific figure that might be suggested by drawing. This finding was consistent with Henderson's (1988) conclusions.

Level IV

Characteristic of geometric thinking at this level was the ability to supply reasons for the steps in a proof and to construct a proof. . Three questions (50,51, and 53; Appendix A) were proofs of the form $a\Rightarrow b\Rightarrow c\Rightarrow d\Rightarrow e$ and the question was "What have we proved?" Question 50 (see figure 4.8) was missed by the all four preservice teachers both during the pre- and post- interview. The responses given by four included repetition of what was stated in the problem and implications of some statements, but they did not conclude that the lines were parallel.

AB is the line segment with A and B the midpoints of the equal sides of the isosceles triangle xyz. $AY \cong BY$ and Δ AYB is similar to Δ XYZ. So $\angle A \cong \angle X$ and AB is parallel to XZ. What we have proved?

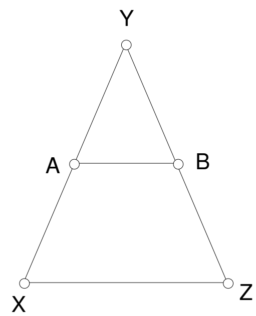


Figure 4.8

Level IV question for Isosceles Triangles

Responses of preservice teachers indicated that they had trouble deducing relevant facts from a given statement and they tend to think in terms of specific figure. This type of question right triangles, which was missed by three preservice teachers, Allen, Barbara and Cathy. Figure 4.9 shows the question given to the preservice teachers.

CD is perpendicular to AB. ∠C is a right angle. If you measure ∠ACD and ∠B, you find they have the same measure. Would this equality be true for all right triangles? Why or why not?

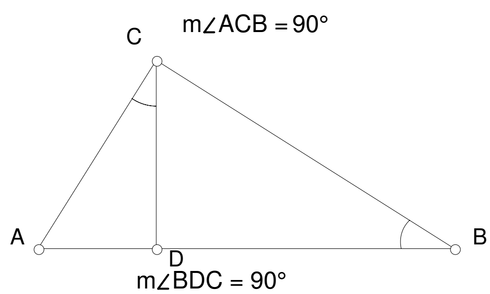


Figure 4.9

Level IV question for right triangle

Both Barbara and Allen responded the equality would be true if the triangle is an isosceles triangle. David did answer correctly and Cathy's answer was not correct.

Level V

The five questions for level were content free. They were designed to measure an understanding of the role of indirect proof and proofs by contradiction, and manipulating symbols according to laws of logic. There were a total five questions and the criterion for this level was answering four of the questions correctly. Three questions were related to axioms and their roles in geometry, one question was related to poof by contradiction and the last one was related to finite geometry. There were no signs that any of the four preservice teachers make references to other geometric systems or demonstrated the ability to work with these systems. All four preservice teachers both during the pre- and post- interview were not able to achieve the criteria at this level.

The level of geometric thought for preservice middle and secondary mathematics teachers were varied before and after the completion of the informal geometry course. Barbara started with level II –III thinking and moved to level IV thinking with Level III thinking on some concepts. Allen started with level III thinking and moved to Level IV thinking. Only for the concept of isosceles triangle there was no advancement. David started with level III-IV thinking and finished with level IV thinking for all the concepts. Although she demonstrated some performance on level IV thinking on some concepts, Cathy was the only preservice teacher who did not make more advancement in her thinking.

Table 4.13 shows a summary of the highest level reached by topic across the preservice teachers during the pre-interview and Table 4.14 summarizes the highest level reached by topic across the preservice teachers during the post –interview

Table 4.13:Highest Level Reached by Topic Across Preservice Teachers in Pre-Test

Topics	Preservice Teachers			
	Cathy	Barbara	Allen	David
Square	**II**	**II**	**III**	**III**
Right Triangle	**II**	**III**	**III**	**III**
Isosceles Triangle	**II**	**III**	**III**	**IV**
Circle	**III**	**III**	**IV**	**IV**
Congruence	**II**	**II**	**III**	**IV**
Similarity	**I**	**I**	**III**	**IV**
Parallel Lines	**II**	**III**	**III**	**IV**

Table 14: Highest Level Reached by Topic Across Preservice Teachers in Post-Test

Topics	Preservice Teachers			
	Cathy	Barbara	Allen	David
Square	**II**	**IV**	**IV**	**IV**
Right Triangle	**II**	**IV**	**IV**	**IV**
Isosceles Triangle	**II**	**IV**	**III**	**IV**
Circle	**III**	**IV**	**IV**	**IV**
Congruence	**II**	**III**	**IV**	**IV**
Similarity	**II**	**IV**	**IV**	**IV**
Parallel Lines	**II**	**III**	**IV**	**IV**

Analyses of hypotheses:

There were twenty-eight students in MAE 4816. Two students did not included in this part of the study. One student did not want to participate (the study was completely voluntarily) and other student was dropped from the study based on recommendation of instructor. A total of twenty-six, five male and twenty-one female students participated in surveys.

Three hypotheses tested. These are stated below along with the null hypotheses. Correlation test computed for each hypothesis. The first hypothesis was identified to look at what relationship exists between spatial ability and motivation.

- Is there a statistically significant relationship between preservice middle and secondary teachers' spatial ability and motivation?

Ho: There is no relationship between spatial ability and motivation

A weak negative correlation (Spearman correlation = -.291) found between spatial ability and motivation. This was surprising, because previous studies (see Bishop, 1981) found a positive relationship between spatial ability and geometric performance. The relationship was not significant, thus the null hypothesis that " there is no relationship between spatial ability and motivation" is accepted.

Hypotheses 2 and 3 were examined in order to determine if a relationship existed between other of the two types of curiosity (epistemic and perceptual) and motivation.

- Is there a statistically significant relationship between preservice middle and secondary teachers' epistemic curiosity and motivation?

  Ho: There is no relationship between epistemic curiosity and motivation
- Is there a statistically significant relationship between preservice middle and secondary teachers' perceptual curiosity and motivation?

  Ho: There is no relationship between perceptual curiosity and motivation

Both perceptual curiosity and epistemic curiosity positively correlated with motivation. This was expected, as curiosity is one element in motivation construct. The correlation between perceptual curiosity and epistemic curiosity was significant at $\alpha = 0.1$ level. Epistemic curiosity and perceptual curiosity were strongly correlated and it was significant.

Table 4.15 summarizes the statistical analyses for the constructs examined in the hypotheses.

Table 4.15 Pearson Product-Moment Correlations Between Spatial Ability, Motivation and Measures of Curiosity Subscales (Epistemic & Perceptual Curiosity) (n=26)

	Spatial Ability	Motivation	Perceptual C.	Epistemic C.
Spatial Ability	-			
Motivation	-.291	-		
Perceptual C.	.091	.331*	-	
Epistemic C.	.025	.327	.643**	-

*p<0.1 **p<0.01

In Keller's ARCS Model of motivation, curiosity concept is included in Attention dimension. Researcher also investigated if there was any relationship between Attention and curiosity types. There was a significant positive association between epistemic curiosity and attention dimension. Table 4.16 summarizes the Pearson product moment correlations between Subscales of motivation and curiosity.

Table4.16 Pearson Product-Moment Correlations Between Measures of Curiosity Subscales (Epistemic & Perceptual Curiosity) and Motivation Subscales (Attention)(n=26)

	Attention	Perceptual Curiosity	Epistemic Curiosity
Attention	-		
Perceptual Curiosity	.278	-	
Epistemic Curiosity	.494*	.643**	-

*p<.05 **p<0.01

Summary

The purpose of this study was to characterize the development of geometric thought of preservice teachers while considering their curiosity and spatial ability levels, and to investigate the relationship among motivation, spatial ability and curiosity types. This first provided the class results for spatial ability, motivation and curiosity characteristics measured by surveys and a test. After that picture of the learning environment was given. In the following sections a description of the four cases, Allen, David, Barbara and Cathy were provided. Following the four cases a cross-case discussion was provided that looks by van Hiele level at the geometric understanding of the 4 students. And finally analyses of the hypotheses were given. Next chapter will provide the conclusions and recommendations.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

This chapter provides a summary of the purpose, research questions, and summary of findings and recommendations for teacher education and for further research.

Purposes of study and research questions

Saads & Davis (1997) found a relationship between spatial ability and van Hiele levels . Since Saads and Davis 's (1997) study was a pioneering study, in terms of making connection between spatial ability and van Hiele levels of students, there appeared to be a need to explore this relationship further. This study was intended to portray the development of geometric thought of four preservice teachers with differing levels in spatial ability and curiosity types. The research questions were:

What differences, if any, exist between preservice middle and secondary mathematics teachers with different spatial ability levels and their understanding of geometry?

What differences, if any, exist between preservice middle and secondary mathematics teachers with different curiosity types (perceptual and epistemic) and their understanding of geometry?

Summary of Findings

The initial levels of geometric thought for preservice teachers were varied. Barbara's answers indicated her level of understanding was level II with some evidence of level III for the concepts of isosceles triangle, right triangle and circle and parallel lines. David's thinking was at level III and IV. For two concepts (square and isosceles triangle) he had only a level III understanding. His level of understanding was at the fourth level for other concepts. The geometric thinking of Allen was at level III understanding. For only circle concept he did have level IV understanding. Cathy was at level II geometric understanding. She demonstrated level III understanding only for the concept circle.

After the completion of the informal geometry course three preservice teachers (Barbara, David and Allen) demonstrated advancement in geometric thinking identified by van Hiele levels. Barbara moved to level IV geometric thinking for five concepts (square, isosceles triangle, right triangle circle and similarity) for parallel lines there were no advancement in her geometric thinking. Although she did not perform at level IV thinking for congruence she moved from level II to level III thinking. David moved to level IV thinking for all seven concepts. However he had non-hierarchal pattern for congruence, that was (1,1,0, 1) instead of (1,1,0,0) or (1,1,1,1) both during the pre- and post- assessment. Allen moved from level III geometric understanding to Level IV understanding on 5 concepts, only for isosceles triangle there was no advancement in his geometric thought. In Cathy's geometric thinking there was not much change. Her responses indicated little working knowledge on right triangle similarity and parallel lines level IV.

Pre-Interview

Square	**III**
Right Triangle	**III**
Isosceles Triangle	**III**
Circle	**IV**
Congruence	**III**
Similarity	**III**
Parallel Lines	**III**

⇒ Allen ⇒

Square	**IV**
Right Triangle	**IV**
Isosceles Triangle	**III**
Circle	**IV**
Congruence	**IV**
Similarity	**IV**
Parallel Lines	**IV**

Figure 5.1

Summary of Allen's Geometric Development

Pre-Interview

Square	III
Right Triangle	III
Isosceles Triangle	IV
Circle	IV
Congruence	IV
Similarity	IV
Parallel Lines	IV

→ David →

Square	IV
Right Triangle	IV
Isosceles Triangle	IV
Circle	IV
Congruence	IV
Similarity	IV
Parallel Lines	IV

Figure 5.2

Summary of David's Geometric development

Pre-Interview

Square	II
Right Triangle	III
Isosceles Triangle	III
Circle	III
Congruence	II
Similarity	I
Parallel Lines	III

→ Barbara →

Square	IV
Right Triangle	IV
Isosceles Triangle	IV
Circle	IV
Congruence	III
Similarity	IV
Parallel Lines	III

Figure 5.3

Summary of Barbara's Geometric Development

Pre-Interview

Square	II
Right Triangle	II
Isosceles Triangle	II
Circle	III
Congruence	II
Similarity	I
Parallel Lines	II

→ Cathy →

Square	II
Right Triangle	II
Isosceles Triangle	II
Circle	III
Congruence	II
Similarity	II
Parallel Lines	II

Figure 5.4

Summary of Cathy's Geometric Development

During the post- and pre-interview preservice teachers' problem solving behavior differed. David spent more time on each problem and did not give up easily. Trying to solve a problem different ways was his common behavior. Both Barbara and Allen showed confidence in their responses during the post interview. During the pre-interview both of them lack of confidence and were not sure the answer was right or wrong. Cathy had a lack of confidence both during pre- and post-interview. She gave up on more problems during the post-interview.

There was a common error across the preservice teachers when they were asked to distinguish necessary and sufficient conditions from a given list for a square, a right triangle, an isosceles triangle and a pair of parallel lines. The responses showed preservice teachers inability to define a shape using necessary and sufficient conditions and to interrelate properties with each other. The source of error seemed to lie in the understanding of the definition and the implication of properties of the concept.

Other findings showed that questions in which a person was given a series of statements (each following from the preceding one) and asked what had been proven were difficult for preservice teachers. Preservice teachers made similar comments to clarify what the question was asking. They were expecting they would have to prove a statement; they were surprised and unsure on how to proceed.

Allen was high in spatial ability, David and Barbara were medium in spatial ability and Cathy had low spatial ability levels. There were total 20 questions and possible the highest score was 20. Table 5.1 summarizes the spatial ability scores. For the first research question, mathematics education juniors enrolled in an informal geometry course were given ROT (Bodner and Guay, 1997) to determine their spatial ability levels. From this sample, four students were selected who represented various levels of spatial abilities. Two who were extremes of the class mean (see table 5.1) and two were near the mean were selected. These were also a convenience sample as students volunteered to be informants.

Table 5.1:Preservice Teachers and Spatial ability Levels (12.77)

	Allen	David	Barbara	Cathy
Spatial Ability	19	14	11	5

Pre-interview results clearly supports the findings of previous studies (see Bishop, 1981) which the higher the spatial ability the higher the geometric performance.

Previous research reports (for detail see Bishop, 1981) found a relationship between spatial ability and geometric performance. None of the studies explored motivational characteristics of learners. Table 5.2 summarizes the motivational characteristics of participants.

Table 5.2:Preservice Teachers' motivational characteristics

Preservice Teacher	Perceptual Curiosity 26.31	Epistemic Curiosity 25.42	Attention 29.85	Motivation 135.5769
Allen	26	23	33	125
Barbara	21	21	40	137
Cathy	24	24	39	141
David	26	25	40	144

Perceptual curiosity and epistemic curiosity are associated. (Spearman correlation = 643). Correlation was significant at the alpha level of .01.Preservice teachers perceptual curiosity located was between 21-25 and epistemic curiosity was located between 21-26. There was not strong or weak tendency in their curiosity levels. There was not significant relationship among students' motivation, spatial ability and epistemic curiosity. The researcher accepted the null hypotheses for two hypotheses. There was a significant correlation between perceptual curiosity and epistemic curiosity so researcher rejected the null hypothesis. However there was a strong significant correlation between epistemic curiosity and perceptual curiosity. There was also moderate and significant correlation between epistemic curiosity and Attention dimension of motivation.

The study was conducted in a semester long period. Data included structured interviews, observation, informal interviews, artifacts, surveys, which were collected from secondary mathematics education majors enrolled in a informal geometry course in Research I university in the south east of US. Four preservice volunteered to participate in the second part of the study which was to determine the development of geometric thought as reflected by van Hiele levels before and after the course.

An instrument developed by Mayberry (1981), which includes a series of 62 questions, served as the basis for the structured interviews. Questions reflecting the first four van Hiele levels focused on seven strands of the geometry curriculum and questions reflecting the fifth van Hiele level had no specific content focus. Each preservice mathematics teacher was interviewed twice using this instrument at the beginning of the course and after the completion of the course.

Keller's Course Interest Survey (CIS; Keller& Subbiyah, 1993) used to identify student motivation at the end of semester. Two instruments were used to identify student curiosity levels for each type, perceptual and epistemic. The Epistemic Curiosity scale (Litman & Spielberger, 2003) was used to assess feelings about stimuli that activate cognitive processes and The Perceptual Curiosity scale (Collins, Litman, & Spielberger, 2004) used to evaluate feelings about sensory stimuli. PECI and ECI were administered at the beginning of the course and CIS was administered at the end of the course.

The researcher was a non-participant observer for every class. Field notes were made during the observation. The researcher focused on learners (i.e. engagements with activities), instructor and instruction.

Limitations of study

The uniqueness of the sample does not allow generalizing the results to all preservice middle and secondary mathematics teachers. However, the results seem likely to characterize preservice middle and secondary teachers who possess similar mathematical backgrounds. Furthermore, the findings of this study contribute to the broader field of research in teacher education and provide a better basis for understanding how preservice teachers who differ in spatial ability (low, medium, high) have internalized mathematical content.

Second research question was investigate differences in curiosity types(perceptual and epistemic) and its results on understanding geometry. Since epistemic curiosity and perceptual curiosity correlated, the differentiation couldn't make. Furthermore, The *Perceptual Curiosity scale,* a 10-item scale that evaluates feelings about sensory stimuli such as appearance, scent and touch (Collins, Litman, & Spielberger, 2004) was used in this investigation. Some of the items in the instrument like "When I smell something new, I try to find out what the odor is coming from", "I like to listen to new and unusual kinds of music", "When I hear a strange sound, I usually try to find out what caused it", "I enjoy different kind of ethnic foods". However, both spatial orientation and spatial visualizations deals with visual stimuli. It would have been better if we had an instrument specifically designed to evaluate the feelings on visual stimuli. For example "I like solving tangram like puzzles", "I like to build new shapes from blocks" etc. Furthermore, today many specific computer games have potential for development of spatial ability (i.e. Tetris). Items related computer games can be included, for instance "I like to play… game".

Although Mayberry's instrument (Mayberry, 1981) was useful for this investigation, there was a limited number of questions at several levels for some of the concepts. For example, four concepts had only two questions at the fourth level. The criteria for meeting the levels ranged from correctly answering 50 percent of the questions (e.g., square, 1 out of 2) to correctly answering 100 percent of the questions (e.g., similarity, 1 out of 1). The concepts of right triangle and isosceles triangle had criteria for meeting the level of correctly answering 2 out of 3 questions. The number of questions for each concept should be increased to investigate the geometric understanding of students in depth. Henderson (1988) made the similar critique.

Recommendations for Teacher education

Although a preserve teacher might have completed an informal geometry course, his or her understanding of geometry is not necessarily at a level where different axiomatic systems could be compared. The findings showed that preservice teachers of middle and secondary mathematics in this study were not all at the level of deductive reasoning. Mathematics educators preparing preservice teachers should consider a means of providing preservice teachers support both in cognitive and motivational dimension.

Previous study (Saad and Davis, 1997) suggested a possible correlation between spatial ability and van Hiele levels. The findings showed that a preservice teacher might have a low spatial ability but still can develop geometric understanding equal or close to those high in spatial ability. Teachers should be aware of that when the appropriate instruction is given low spatial ability students have a chance to advance in learning geometry. Furthermore, teachers should be given opportunities to develop spatial ability as well as learning geometry.

Although a preservice mathematics teacher may not want to teach geometry, it is not acceptable to think that any mathematics teacher has a little understanding of geometric deductive reasoning. A preservice teacher may not be motivated as much as the students who desire to teach that particular course (e.g. Algebra, Geometry). Most of the case students are extrinsically motivated such as, getting a passing grade, more than for the sake of learning. Connections between geometry and other topics (i.e. algebra, trigonometry) should be made. Students should experience connections with other topics so that they do not think each topic is isolated. A parallel content and methods course could provide a natural context for helping teachers to make such connections.

The correlation between motivation and spatial ability was not significant and weak, however, it was a negative (Spearman correlation = -.291). Although previous research reports (Bishop, 1981) indicated a positive correlation between spatial ability and geometric performance, this negative relationship showed that spatial ability should be considered in motivational design.

Recommendation for future research

The complexity of the relationship between the development of geometric understanding and spatial ability was not fully addressed in this study; much more remains to be learned about the relationship. For example, studies are needed to examine high spatial ability students' differences in understanding geometry. More specifically do high spatial ability students differ in the process for advancing from one level to the next level? Furthermore, in a similar way low spatial ability students' process for advancing from one level to the next level should be examined.

Due to time limitations, the structured interview instruments (e.g. Mayberry, 1981), did not allow the researcher to interview a large number of students. However, van Hiele levels can be investigated on one concept such as right triangle. For instance in Mayberry's (1981) instrument there were a total of 10 questions for right triangle, and 8 questions for similarity for four levels. Thus a modified version of the instrument focusing on one or two central concepts could be examined with a large number of students. Further research, needed to examine the relationship between spatial ability and van Hiele levels.

Visualization plays an important role in teaching and learning geometry. In this study PECI was used and the instrument was designed for a broader perspective (like smelling, hearing). There is a need to develop an instrument related to perceptual curiosity more focused on the visual sensory.

This study looked at the van Hiele levels from learning window. Other studies should examine the instructional behaviors of teachers who differ in curiosity levels. For example, how high curious teachers differ from the low curious teachers who are on the same geometric level. How do the classroom activities and homework correspond the van Hiele levels?

Conclusion

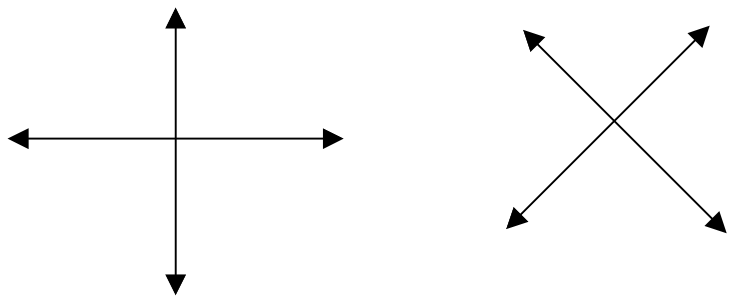
Based on the findings of this research, spatial ability is important construct for learning geometry. However, when the appropriate instruction is given low spatial ability students can develop an understanding. Furthermore, student motivation cannot be neglected in any course. Without a motivational design advantages might turn into disadvantages.

APPENDIX A

MAYBERRY INSTRUMENT

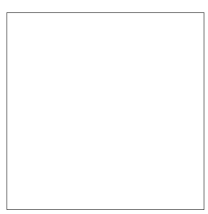
May Berry Instrument Questions

1.



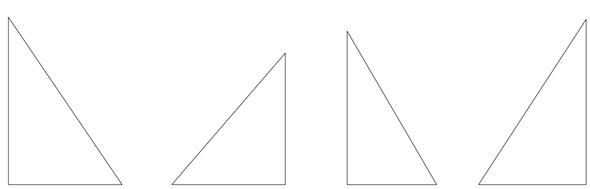
These pairs of lines appear to meet at what kind of angle? What is the word used to describe this relationship?

2. This figure is which of the following?



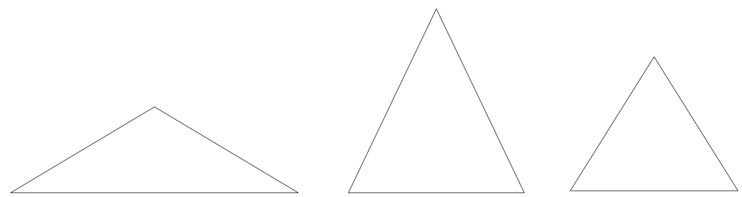
a) triangle
b) quadrilateral
c) square
d) parallelogram
e) rectangle

3.



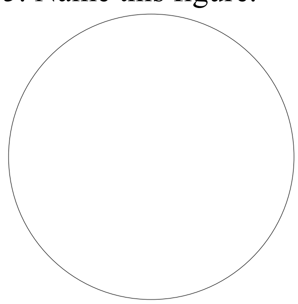
Are all of these triangles? Explain.
Do they appear to be special triangles?

4.



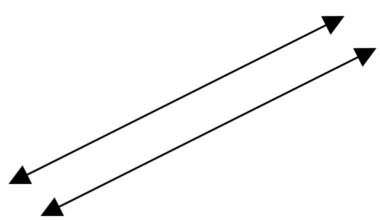
These appear to be what kind of triangles?

5. Name this figure.



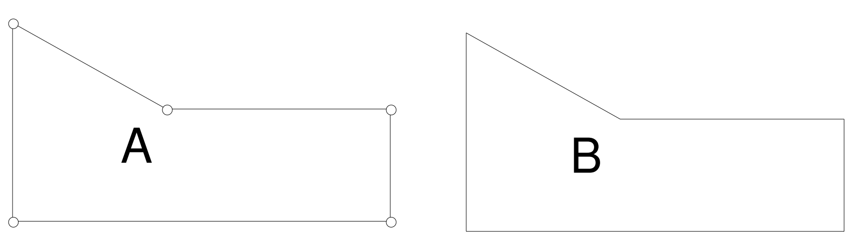
....

6.

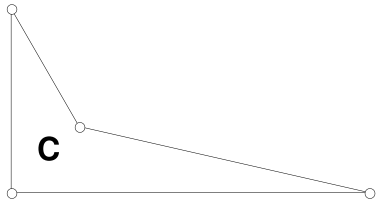


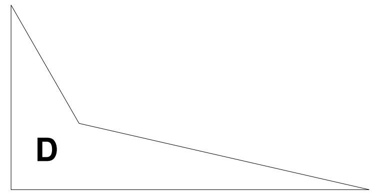
Suppose these two lines will never meet no matter how far we draw them. What word describes this?

7.



What is true for A and B

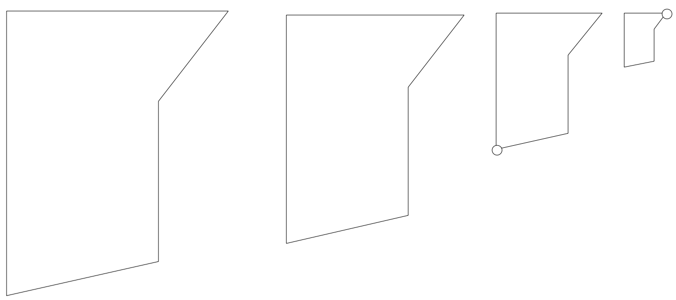




What is true C and D?

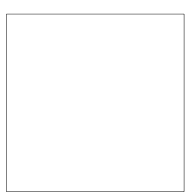
What words describes this?

Q8.

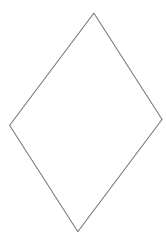


Are these figures alike in what way?
What word describes this?

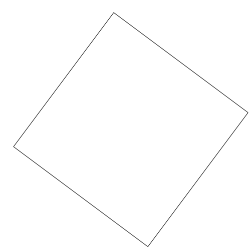
9. Which of these figures are squares? Which of these figures are rectangles?



a

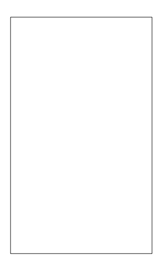


b

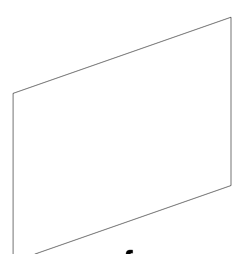


c

d



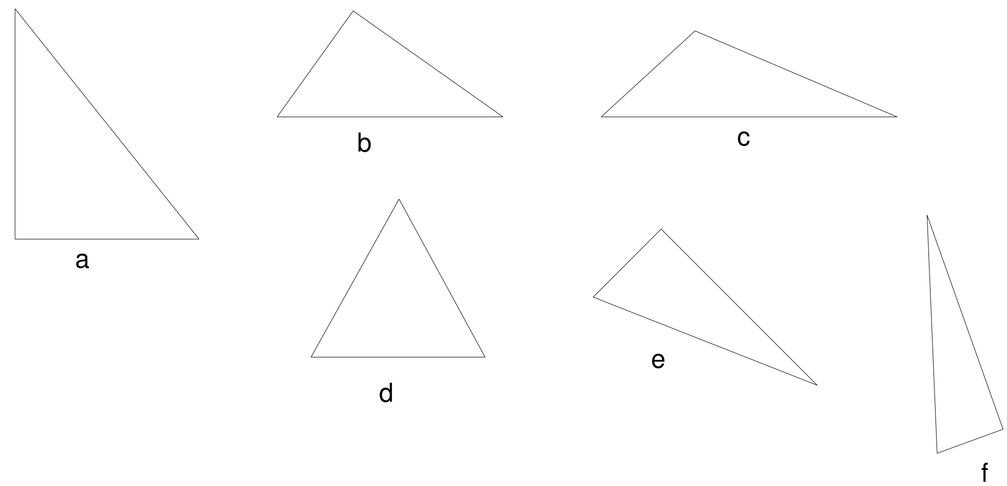
e



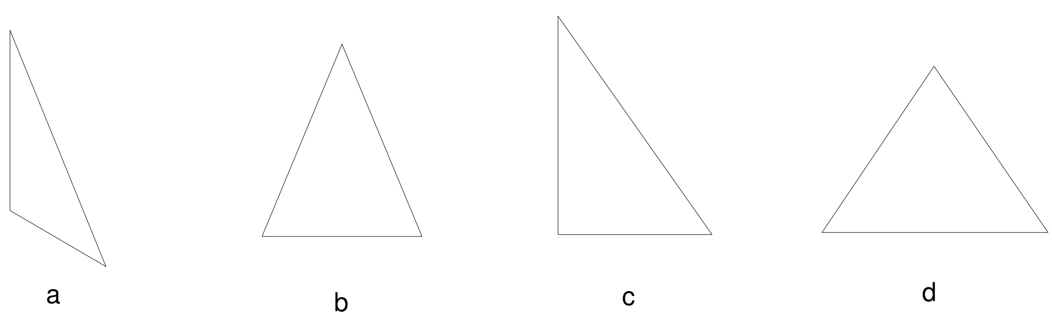
f

Rectangles:................................ Squares:...

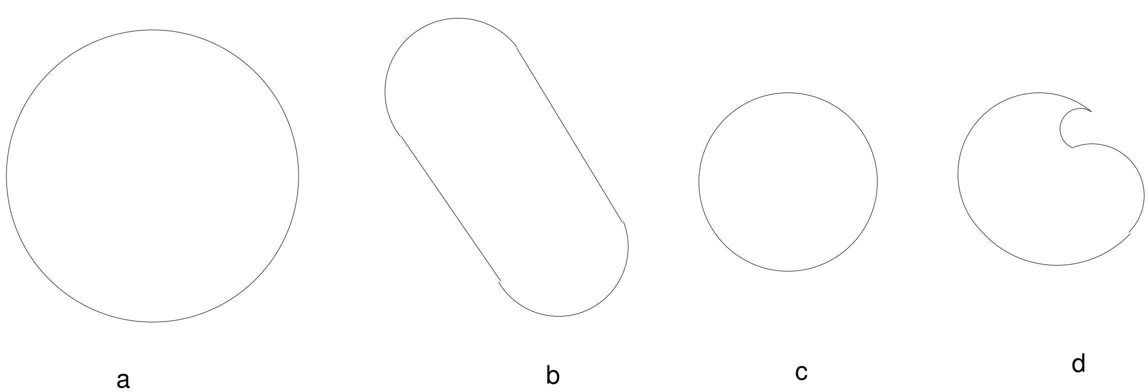
10. Which of these are right triangles?



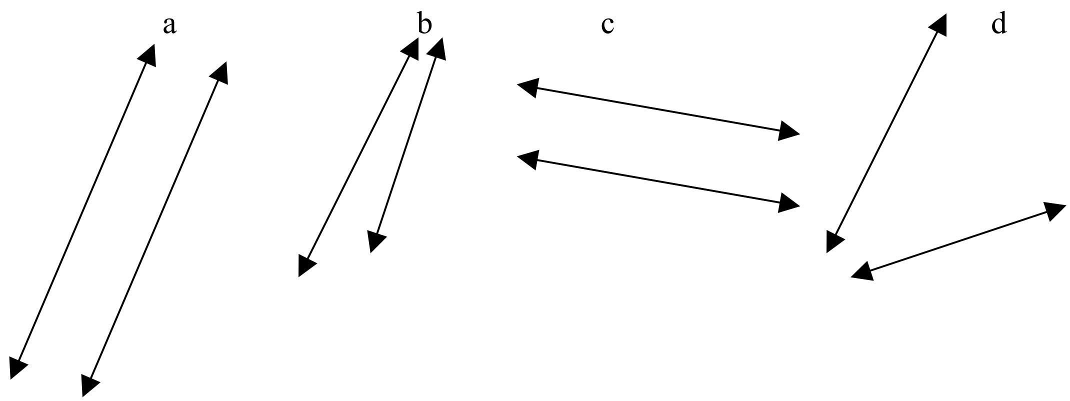
11. Which of these figures appear to be isosceles triangles?



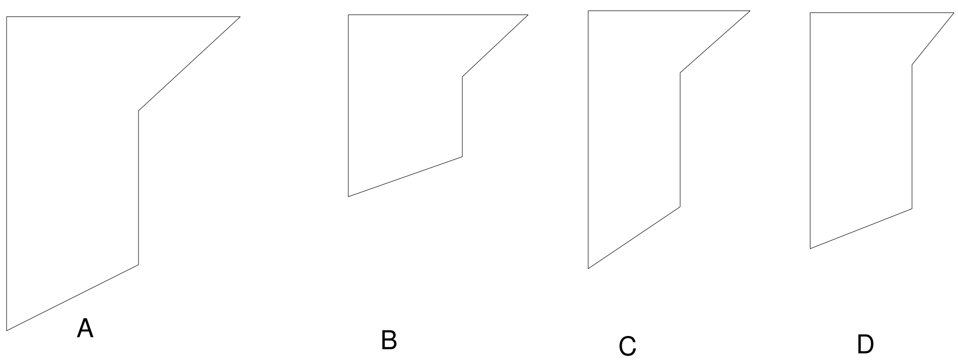
12.Which of these are circles?



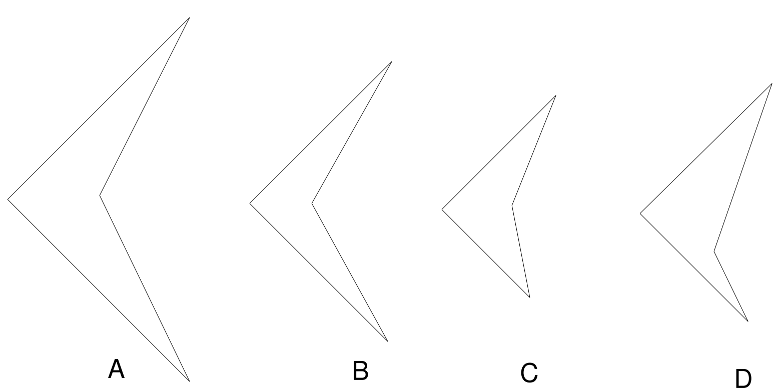
13. Which pair(s) of lines appears to be parallel?



14. Which figures appears to be similar to A?



15. Which figure is congruent to A?



16. Draw a square. What must be true about the sides? What must be true about the angles?

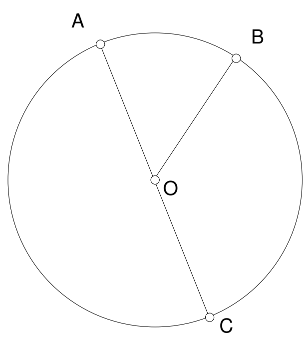
17. Does a right angle triangle always have a longest side? (If student answers yes, ask which one?)

Does a right angle triangle always have a largest angle? (If student answers yes, ask which one?)

18. What can you tell me about the sides of an isosceles triangle?

What can you tell me about the angles of an isosceles triangle?

19 .This figure is a circle. O is the center



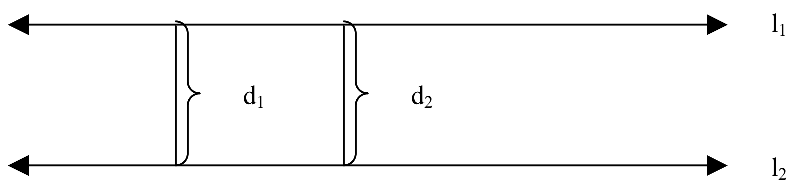
Name the following segments.

OB is a .... of the circle.

OC is a .... of the circle.

AC is a .... of the circle.

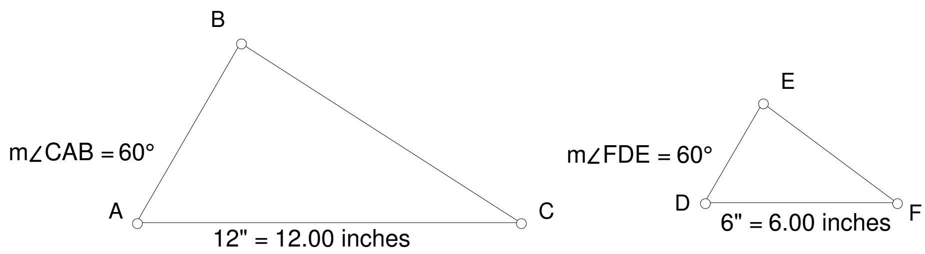
20.



If $d_1 = d_2$, what is true about l_1 and l_2

If $d_1 \neq d_2$ what is true about l_1 and l_2?

21.

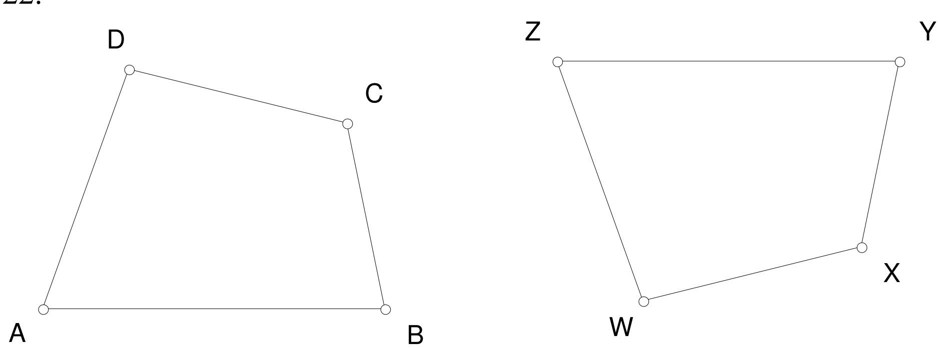


Triangle ABC is similar to triangle DEF.

How long is ED?
How do you know?

What is the size of ∠ EDF?
How do you know?

22.

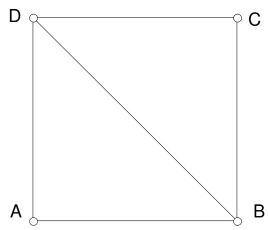


These are congruent figures.

What is true about their sides? (If student says sides are equal or congruent, ask which is equal or congruent to AD?)

What is true about their angles? (If student says angles are equal or congruent, ask which is equal or congruent to angle B?)

23.



ℹCD is a square BD is a diagonal.

me any angle concurrent to ∠ ABD

w do you know?

24. Which combination of the following guarantees a figure to be a square?
   a. It is a parallelogram
   b. It is a rectangle
   c. It has right angles
   d. Opposite sides are parallel
   e. Adjacent sides are equal in length
   f. Opposite sides are equal in length

(After response, ask "Can you use less?")
(Ask again, "Can you use less?")

25. A. Name some ways in which squares and rectangles are alike?

   B. Are all squares also rectangles? Why?
   Are all rectangles also squares? Why?

26. Which combination of the following guarantees a figure to be a right triangle?
   a. It is a triangle
   b. It has two acute angles
   c. The measures of angle add up to180.
   d. An altitude is also a side.
   e. The measures of two angles up to 90.

(After response, ask "Can you use less?")
(Ask again, "Can you use less?")

27. QAB is a triangle
   a. Suppose angle Q is a right angle. Does that tell you anything about angles A and B? If so, what?
   b. Suppose angle Q is less than 90^{o}. Could the triangle be a right triangle? Why?
   c. Suppose angle Q is more than 90^{o}. Could the triangle be a right triangle? Why?

28. Which combination of the following guarantees a figure to be an isosceles triangle?

a. It has two congruent angles
b. It is a triangle
c. It has two congruent sides.
d. An altitude bisects the opposite side.
e. The measures of angles add up to 180.

(After response, ask, “Can you use less?”)
(Ask again, “Can you use less?”)

29. Give a definition of isosceles triangle.

30. Suppose all we know about ΔMNP is that∠M is the same size as ∠N. What do you know about the size MP and NP?

Suppose ∠M is larger than ∠N. What do you know about MP and NP? Could ΔMNP be isosceles?

31. Triangle DEF has three congruent sides. Is it an isosceles triangle?

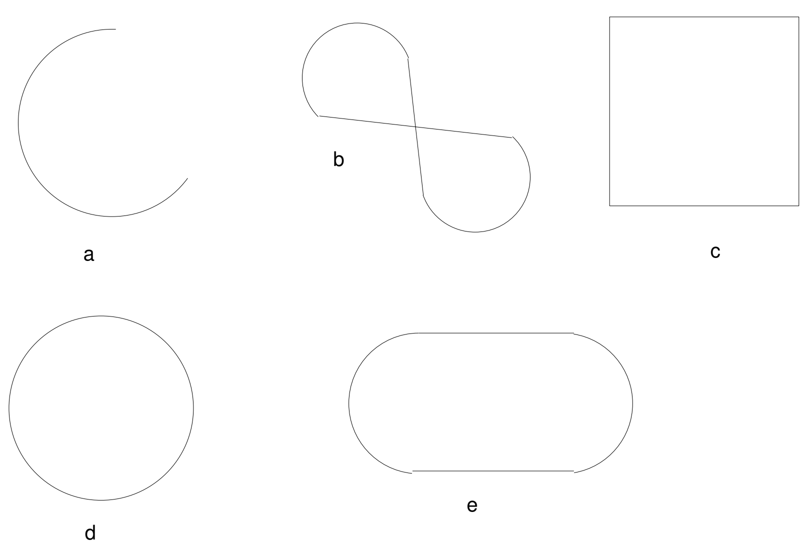
Why or why not?
Which of the following is/are true?

a. All isosceles triangles are equilateral.
b. All equilateral triangles are isosceles.

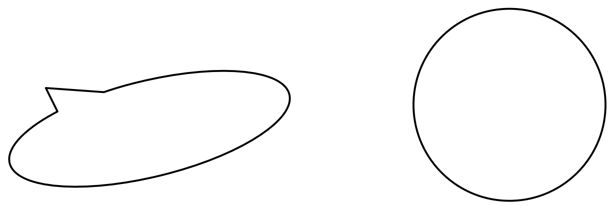
32.Which are true? Give reasons?

a. All isosceles triangles are right triangles.
b. Some right triangles are isosceles triangles.

33. Tell why each of figures is or is not a circle?



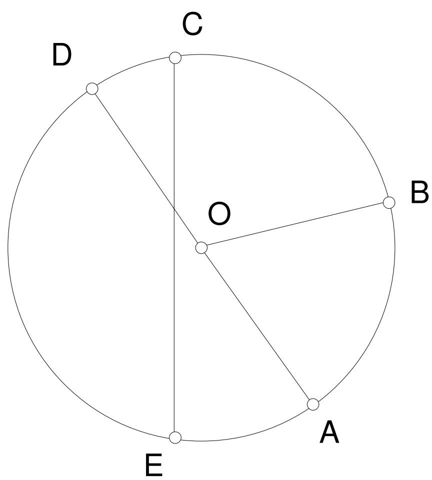
34. Figure A is a simple closed curve. Figure B is a circle.



Is figure B is a simple closed curve?
How are these figures alike? How are they different?

(T – F) All simple closed curves are circles.
(T- F) All circles are simple closed curves.

35.

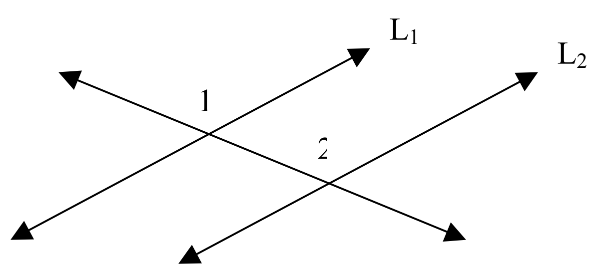


This figure is a circle with center O.
Would the following be:

a) certain
b) possible or
c) impossible

1) OB ≅ OA
2) OD ≅ OA
3) 2OB = AD
4) AD ≅ EC

36.



Suppose ∠1 and ∠2 are congruent. What does that tell you about L_1 and L_2?

Suppose ∠1 is larger than ∠2. What does that tell you about L_1 and L_2?

37. How do you recognize lines that are parallel?

38.Are these lines are line segments parallel?

a) always b) sometimes c) never

a) Two lines which do not intersect
b) Two lines which perpendicular to the same line
c) Two line segments in a square
d) Two line segments in a triangle
e) Two line segments which do not intersect.

Give reasons to your answers.

39.Which combination of the following guarantee that two lines are parallel?

a. There are everywhere the same distance apart.
b. They have no points in common.
c. They are in the same plane.
d. They never met

(After response, ask, "Can you use less?")
(Ask again, "Can you use less?")

40.What does it mean to say that two figures are similar?

41.Triangle ABC is similar to triangle DEF(in that order).Are the following a) certain , b) possible, or c) impossible?

a. AB≅ DE
b. AB> DE
c. ∠A≅∠E
d. m∠A > m∠E
e. AB≅ EF
f. m∠A > m∠D

42. Will the figures A and B be similar a) always, b) sometimes, c) never?

	A	B
a	A square	A square
b	An isosceles triangle	An isosceles triangle
c	a Δ congruent to B	a Δ congruent to A
d	A rectangle	A square
e	A rectangle	A triangle

Give your reasons to your answers.

43. Δ ABC is congruent to ΔDEF(in that order).Are the following a)certain b) possible c) impossible

a. AB≅ DE
b. ∠A≅∠E
c. m∠A < m∠D
d. AB≅ EF

Give your reasons for your answers.

44. Will the figures A and B be similar a) always, b) sometimes, c) never?

	A	B
a	A square	A triangle
b	A square with a 10 cm. side	A square with a 10 cm. side
c	a right triangle with a 10 cm. hypotenuse	a right triangle with a 10 cm. hypotenuse
d	A circle with a 10 cm chord	A circle with a 10 cm chord
e	a Δ similar to B	a Δ triangle similar to A

The subject was given these written statements after #44

If the angles of triangle A have the same measures as the angles of triangle B, the triangles are similar.

If the sides of triangle A are proportional to the sides of triangle B, the triangles are similar.

If the two triangles are similar

1) corresponding angles are similar.
2) corresponding sides are proportional.

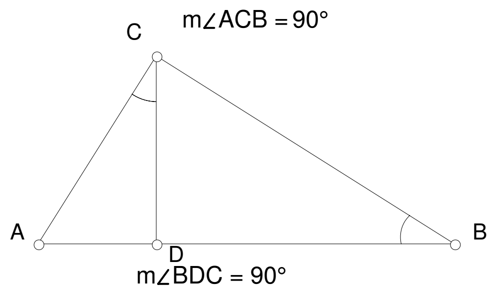
The sum of the measures of the angles of a triangle is 180 degrees.

Two triangle are congruent if
1) 3 sides of A are congruent to 3 sides of B
2) 2 sides of A and included angle are congruent respectively to 2 sides of B and included angle
3) 2 angles and included side of A are congruent to 2 angles and included side of B.

45. ABCD is a four-sided figure. Suppose we know that opposite sides are parallel. What are the fewest facts necessary to prove that ABCD is a square?

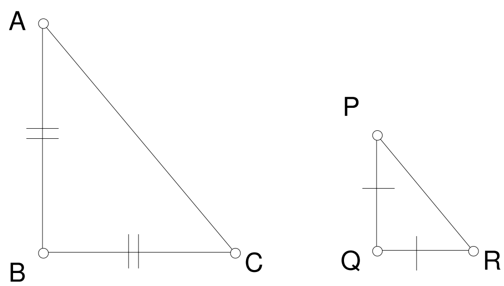
46. Figure ABCD is a parallelogram. AB ≅ BC and ∠ ABC is a right angle. Is ABCD is a square?
Prove your answer.

47. CD is perpendicular to AB. ∠C is a right angle.



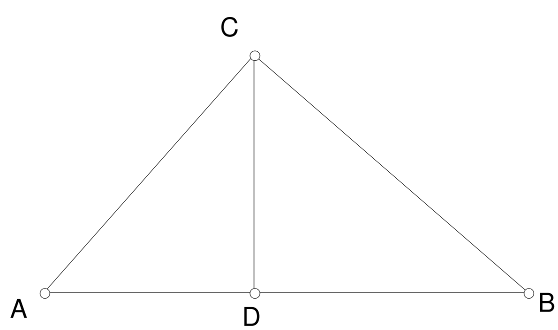
If you measure ∠ ACD and ∠B, you find they have the same measure. Would this equality be true for all right angles? Why or why not?

48.



Figures ABC and PQR are right isosceles triangles with angle B and Q being right angles. Prove that ∠ A ≅ ∠ P and ∠ C ≅ ∠ R

49. ABC is a triangle ∠ ADC≅∠ BDC

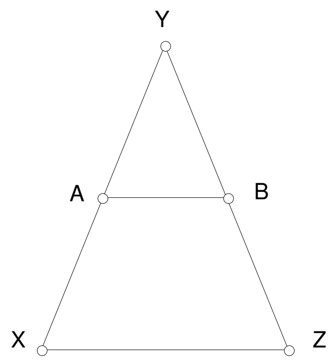


a. What kind of triangle is Δ ABC ? Why

b. AD ≅ BD . Why?

c. CD is perpendicular to AB. Why?

50.



AB is the line segment with A and B the midpoints of the equal sides of the isosceles triangle xyz. AY ≅ BY and Δ AYB is similar to Δ XYZ. So ∠ A ≅ ∠ X and AB is parallel to XZ. What we have proved?

51.

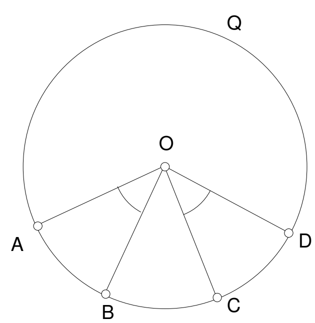
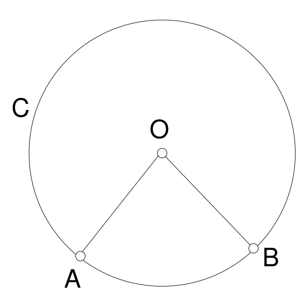


Figure Q is a circle. O is the center.

∠ AOB ≅ ∠ COD. Since AO, BO, CO, and DO are radii, ΔAOB ≅ ΔCOD so AB≅CD.

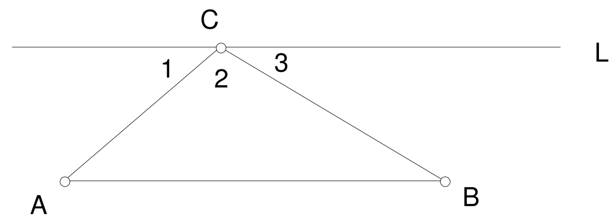
What we have proved?

52. Figure C is a circle and O is the center.



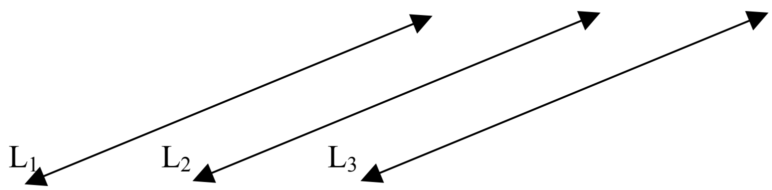
Prove that Δ AOB is isosceles.

53.



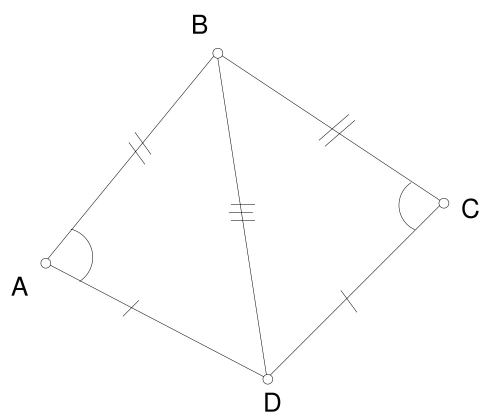
Line L is parallel to AB.
Because of the properties of parallel lines we can prove that ∠1≅∠ A and ∠3≅∠ B. Now L is a straight angle (180°). What we have proved?

54.

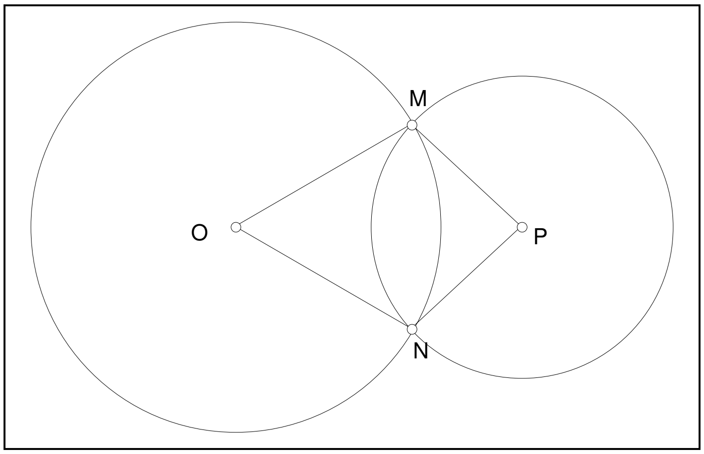


Prove: If L_1 is parallel to L_2 and L_3 is parallel to L_3, then L_1 is parallel to L_3.

55. In this figure AB and CB are the same length. AD and CD are the same length. Will ∠ A and ∠ C be the same size? Why or why not?



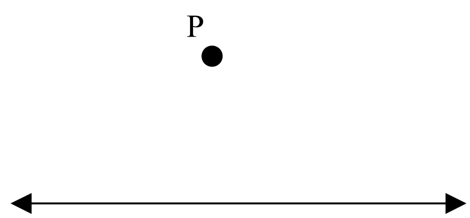
56. These circles which centers Q and P intersect at M and N. Prove: ΔOMP ≅ ΔONP.



57. Prove that the perpendicular from a point not on a line to the line is the shortest line segment that can be drawn from the point to the line.

58. What is the difference between an axiom and theorem.

59.



Given the line L and point P not on L. In order to prove that there exist exactly one line through P perpendicular to L

L_1

you might assume existence of two such lines.

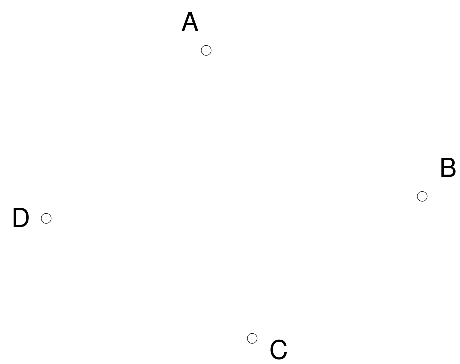
a) Why would make such an assumption?
b) How would this assumption help your proof?

60. Why does a course like geometry have axioms?

61.Are axioms true?

62. In one finite geometry A, B, C, and D, are the only points. {A, B}, {B, C}, {B, D}, {C,D} are the only lines. {A, B} intersects {A,C} because they have in common. {A,B} is parallel to {C,D} because they have no points on common.

Here is a possible picture:



True or False

a) {A,C} intersects {B,D}
b) {A,C} is parallel to {B,D}

APPENDIX B

MAYBERRY INSTRUMENT SCORING GUIDE

Mayberry scoring

Concept	Level	Question Type	Question Number	Possible Score	Criteria if for question	Criteria for level
SQUARE	I	Name	2	1		
		Discriminate	9a	2	2/2 =1	1 of 2
	II	Properties	16	2		2 of 3
			23a	1		
	III	Definitions	24	1		6 of 9
		Class Inclusions	9b	4	¾ = 1	
			25	3		
		Relationships	42a	1		
			44b	1		
			42d	1		
		Implications	23b	1		
	IV	Proof	45	1		1 of 2
			46	1		
RIGHT TRIANGLE	I	Name	3	1		1 of 2
		Discriminate	10	4	¾=1	
	II	Properties	17	4		3 of 4
	III	Definitions	26	1		5 of 7
		Class Inclusions	32	2		
		Relationships	44c	1		
		Implications	27	3		
	IV	Proof	47	1		2 of 3
			48	1		
			57			
CIRCLE	I	Name	5	1		1 of 2
		Discriminate	12	2	2/2=1	
	II	Properties	19	3	2/3=1	5 of 6
			33	5		
	III	Definitions	33	1		6 of 10
		Relationships	35	4		
			44d	1		
		Class Inclusions	34	4		
	IV	Proof	51	1		1 of 2
			52	1		

Scoring continues....

Concept	Level	Question Type	Question Number	Possible Score	Criteria if for question	Criteria for level
ISOSCELES TRIANGLE	I	Name	4	1		1 of 2
		Discriminate	11	2	2/2=1	
	II	Properties	18	2		2 of 2
	III	Definition	28	1		8 of 12
		Implications	29	1		
		Class Inclusions	30	3		
			49a	1		
			42b	1		
			31	3		
			32	2		
	IV	Proof	48	1		2 of 3
			50	1		
			52	1		
PARALLEL LINES	I	Name	6	1		1 of 2
		Discriminate	13	1		
	II	Properties	20	2		2 of 2
	III	Definitions	37	1		6 of 9
			39	1		
		Relationships	38	5		
		Implications	36	2		
	IV	Proof	53	1		1 of 2
			54	1		
SIMILARITY	I	Name	8	1		1 of 2
		Discriminate	14	1		
	II	Properties	21	4		3 of 4
	III	Definition	40	1		8 of 13
		Relationships	41	6		
			42	5		
		Class Inclusions	44e	1		
	IV	Proof	48	1		1

Scoring continues...

Concept	Level	Question Type	Question Number	Possible Score	Criteria if for question	Criteria for level
	I	Name	7	1		
		Discriminate	15	1		1 of 2
	II	Properties	22	4		3 of 4
	III	Relationships	43	4	Deduct 1 point if miss a and c	
		Implications	44	5		6 of 10
		Class Inclusions	42c	1		
	IV	Proof	55	1		
			56	1		1 of 2

APPENDIX C

EPICTEMIC CURIOSITY SCALE

ECI

A number of statements that people use to describe themselves are given below. Read each statement and then blacken the appropriate space on the answer sheet to indicate how you generally feel. There are no right or wrong answers. Do not spend too much time on any one statement but give the answer that seems to describe how you generally feel.

1 = Almost Never **2 =** Sometimes **3 =** Often **4 =** Almost Always

1	It is important to me to feel knowledgeable.	1	2	3	4
2	I enjoy exploring new ideas.	1	2	3	4
3	It troubles me when there doesn't seem to be a reasonable solution to a problem.	1	2	3	4
4	Difficult conceptual problems can keep me awake all night thinking about solutions.	1	2	3	4
5	I am interested in discovering how things work.	1	2	3	4
6	I spend time formulating my ideas as clearly as possible, so that I can be understood by others.	1	2	3	4
7	I enjoy learning about subjects that are unfamiliar to me.	1	2	3	4
8	It really gets on my nerves when I know that I'm close to solving a puzzle, but still can't figure it out.	1	2	3	4
9	I can spend hours on a single problem because I just can't rest without knowing the answer.	1	2	3	4
10	When someone asks me a riddle, I am interested in trying to solve it.	1	2	3	4
11	I don't like not knowing things, so I try to learn new information about even the most complex topics.	1	2	3	4
12	I find it fascinating to learn new information.	1	2	3	4
13	I have a hard time accepting that some mysteries just can't be solved.	1	2	3	4
14	If I read something that puzzles me at first, I keep reading until I understand it.	1	2	3	4
15	When I am given a new kind of arithmetic problem, I enjoy imagining solutions.	1	2	3	4
16	It bothers me if I come across a word that I don't know, so I will look up its meaning in a dictionary.	1	2	3	4
17	When I learn something new, I would like to find out more about it.	1	2	3	4
18	I feel frustrated if I can't figure out the solution to a problem, so I work even harder to solve it.	1	2	3	4
19	I brood for a long time in an attempt to solve some fundamental problem.	1	2	3	4

20	When I see a complicated piece of machinery, I like to ask someone how it works.	1	2	3	4
21	I am critical of current ideas and theories.	1	2	3	4
22	I enjoy discussing abstract concepts.	1	2	3	4
23	It aggravates me if I can’t remember a fact that I’ve learned before, so I think about it until the answer comes to me.	1	2	3	4
24	I work like a fiend at problems that I feel must be solved	1	2	3	4
25	If I am given an incomplete puzzle, I like to try and imagine the final solution.	1	2	3	4

APPENDIX D

PERCEPTUAL CURIOSITY SCALE

PECI

Directions: A number of statements that people use to describe themselves are given below. Read each statement and then blacken the appropriate space on the answer sheet to indicate how you *generally* feel using the scale below. There are no right or wrong answers. Do not spend too much time on any one statement but give the answer that seems to describe how you *generally* feel.

1 = Almost Never **2 =** Sometimes **3 =** Often **4 =** Almost Always

1.	I like exploring my surroundings.	1	2	3	4
2.	I enjoy exploring new ideas.	1	2	3	4
3.	When I smell something new, I try and find out what the odor is coming from.	1	2	3	4
4.	I find it fascinating to learn new information.	1	2	3	4
5.	I like to discover new places to go.	1	2	3	4
6.	I enjoy learning about subjects that are unfamiliar to me.	1	2	3	4
7.	If I hear something rustling in the grass I have to see what it is.	1	2	3	4
8.	When I learn something new, I would like to find out more about it.	1	2	3	4
9.	I like visiting art galleries and art museums.	1	2	3	4
10.	I enjoy discussing abstract concepts.	1	2	3	4
11.	When I see a new fabric, I like to touch and feel it.	1	2	3	4
12.	If I am given an incomplete puzzle, I like to try and imagine the final solution.	1	2	3	4
13.	I like to listen to new and unusual kinds of music.	1	2	3	4

14.	When I see a complicated piece of machinery, I like to ask someone how it works.	1	2	3	4
15.	When I hear a musical instrument and I am not sure what it is, I like to see it.	1	2	3	4
16.	When I am given a new kind of arithmetic problem, I enjoy imagining solutions.	1	2	3	4
17.	I enjoy trying different kinds of ethnic foods.	1	2	3	4
18.	I am interested in discovering how things work.	1	2	3	4
19.	When I hear a strange sound, I usually try to find out what caused it.	1	2	3	4
20.	When someone asks me a riddle, I am interested in trying to solve it.	1	2	3	4

APPENDIX E

COURSE INTEREST SURVEY

COURSE INTEREST SURVEY

INSTRUCTIONS

1-There are 34 statements in this questionnaire. Please think about each statement in relation to the instructional materials you have just studied, and indicate how true it is. Give the answer that **truly applies to you**, and not what you would like to be true, or what you think others want to hear.

2-Think about each statement **by itself** and indicate how true it is. Do not be influenced by your answers to other statements.

3- Record your responses on the scale.

Thank you.

Please circle each questions based on the scale.

A-Not true **B** –Slightly true **C**- Moderately true **D**- Mostly True **E**-Very True

1-The instructor knows how to make us feel enthusiastic about the subject matter of this course	A	B	C	D	E
2-The things I am learning in this course will be useful to me.	A	B	C	D	E
3-I feel confident that I will do well in this course	A	B	C	D	E
4-This class has very little in it that captures my attention	A	B	C	D	E
5- The instructor makes the subject matter of this course seem important	A	B	C	D	E
6-You have to be lucky to get good grades in this course.	A	B	C	D	E
7- I have to work too hard to succeed in this course	A	B	C	D	E
8-I do NOT see how the content of this course relates to anything I already know.	A	B	C	D	E
9-Wheter or not I succeed in this course up to me	A	B	C	D	E
10-The instructor creates suspense when building a point.	A	B	C	D	E
11- The subject matter of this course is just too difficult for me	A	B	C	D	E

12-I feel that this course gives me a lot of satisfaction.	A	B	C	D	E
13. In this class, I try to set and achieve high standard of excellence	A	B	C	D	E

A-Not true **B** –Slightly true **C**- Moderately true **D**- Mostly True **E**-Very True

14-I feel that the grades or other recognition I receive are fair compared to other students	A	B	C	D	E
15-The students in this class seem curious about the subject matter.	A	B	C	D	E
16-I enjoy working for this course.	A	B	C	D	E
17-It is difficult to predict what grade the instructor will give my assignments	A	B	C	D	E
18- I am pleased with instructor's evaluations of my work compared to how well I think I have done.	A	B	C	D	E
19-I feel satisfied with what I am getting from this course	A	B	C	D	E
20- The content of this course related to my expectations and goals	A	B	C	D	E
21-The instructor does unusual or surprising things that are interesting	A	B	C	D	E
22-The students actively participate in this class.	A	B	C	D	E
23-To accomplish my goals; it is important that I do well in this course.	A	B	C	D	E
24- The instructor uses an interesting and variety of teaching techniques	A	B	C	D	E

25-I do NOT think I will benefit much from this course	A	B	C	D	E
26-I often daydream while in this class.	A	B	C	D	E

A-Not true **B** –Slightly true **C**- Moderately true **D**- Mostly True **E**-Very True

27-As I am taking this class, I believe that I can succeed if I try hard enough	A	B	C	D	E
28-The personal benefits of this course are clear to me.	A	B	C	D	E
29- My curiosity often stimulated by the questions asked or the problems given on the subject matter in this class.	A	B	C	D	E
30. I find the challenge level in this course to be about right: neither too easy, nor too hard.	A	B	C	D	E
31- I feel rather disappointed with this course.	A	B	C	D	E
32-I feel that I get enough recognition of my work in this course by means of grades, comments, or other feedback.	A	B	C	D	E
33-The amount of work I have to do is appropriate for this type of course.	A	B	C	D	E
34. I get enough feedback to know how well I am doing.	A	B	C	D	E

APPENDIX F

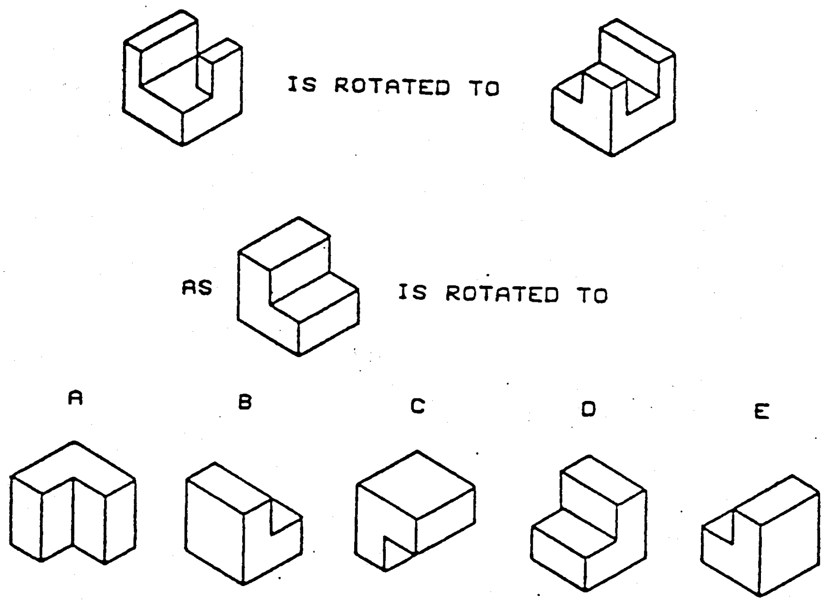
The Purdue Visualization of Rotation Test

The Purdue Visualization of Rotation Test

Do NOT make any marks on this exam.
Mark your answers on the separate answer sheet

DIRECTIONS

This test consists of 20 questions designed to see how well you can visualize the rotation of three-dimensional objects. An example of the type of question included in this test is shown below.



For each question, you should:

I. Study how the object in the top line of the question is rotated.

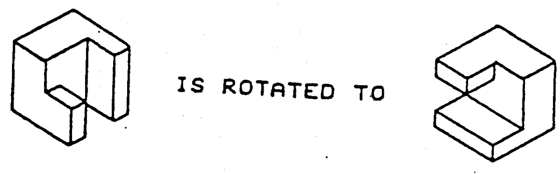
II. Picture in your mind what the object shown in the middle line of the question looks like when rotated in exactly the same manner.

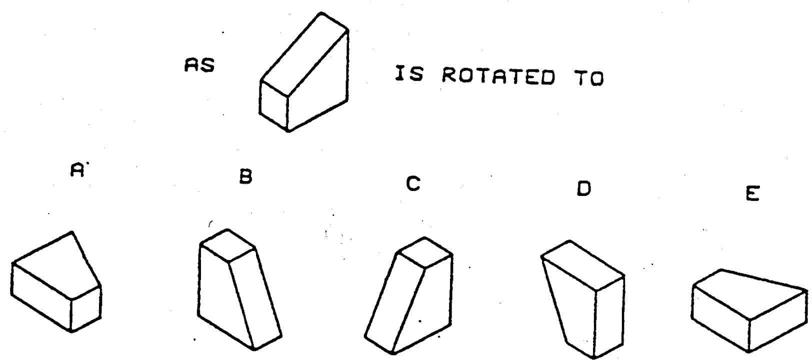
III. Select from among the five drawings (A, B, C, D, or E) given in tbe bottom line of the question the one that looks like the object rotated in the correct position.

What is the correct answer to the example shown above?

Answers A. B. C. and E are wrong. Only drawing D looks like the object after it hsa been rotated. Remember that each question has only one correct answer.

Now look at the example shown below and try to select the drawing that looks like the object in the correct position when the given rotation is applied

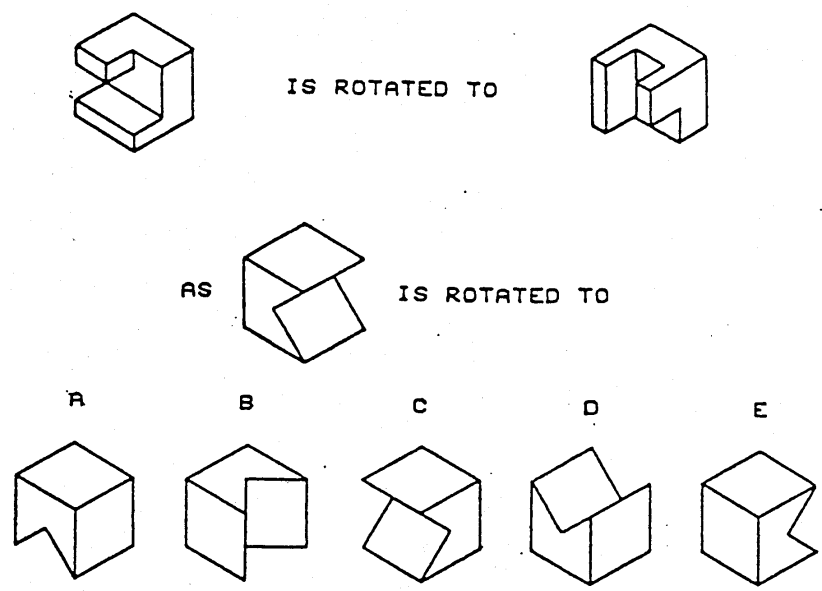




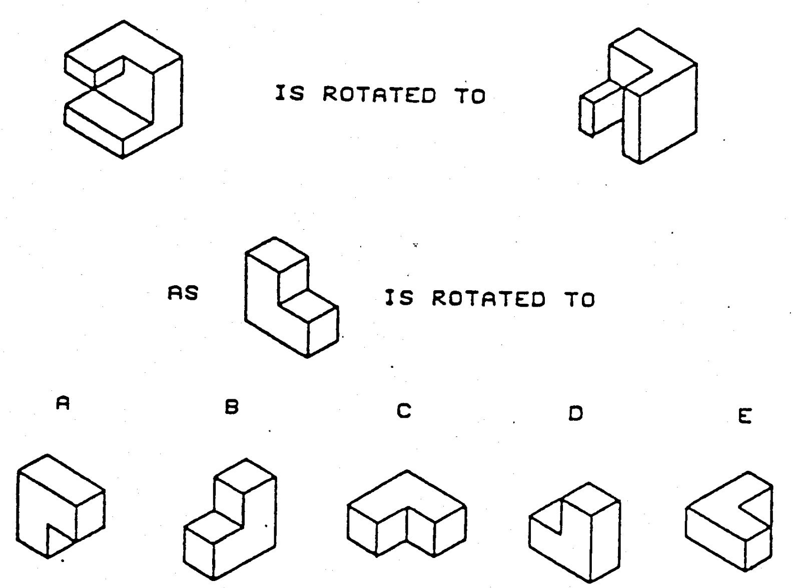
Note that the rotation in this example is more complex. The correct answer for this example is B.

Do NOT make any marks in this booklet.
Mark your answers on the separate answer sheet.
You will be told when to begin

1


IS ROTATED TO
AS
IS ROTATED TO
A
B
C
D
E

2


IS ROTATED TO
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A
B
C
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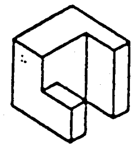
E

3



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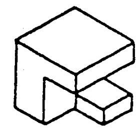

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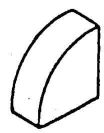
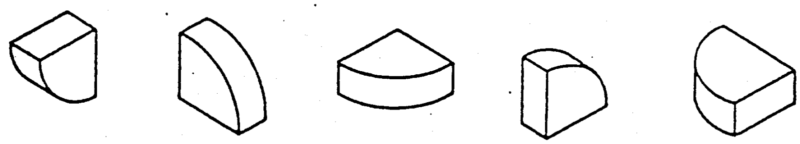


IS ROTATED TO

A B C D E



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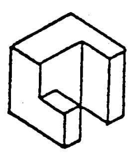

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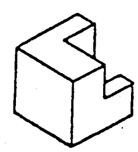

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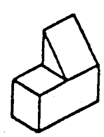
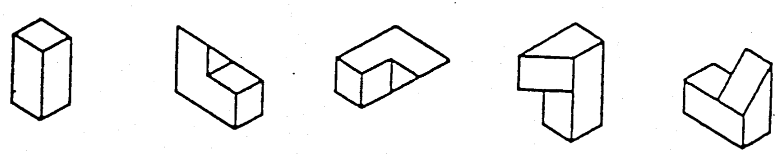
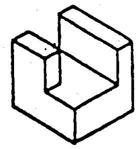
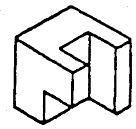
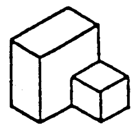


IS ROTATED TO

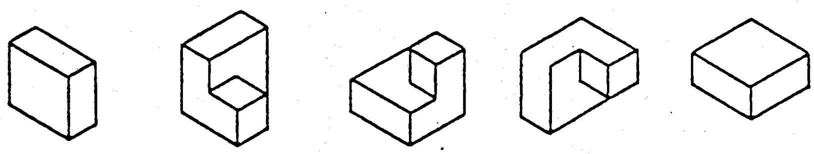
A B C D E

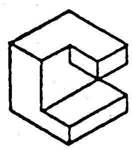
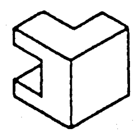


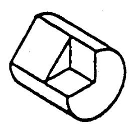
5  IS ROTATED TO 

AS  IS ROTATED TO

A B C D E



6  IS ROTATED TO 

AS  IS ROTATED TO

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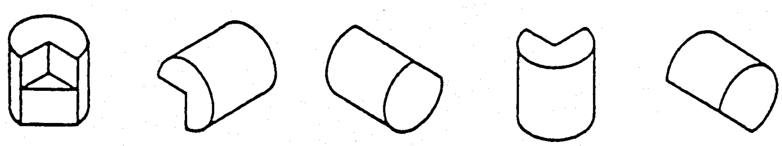
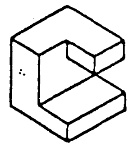
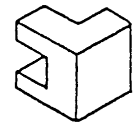
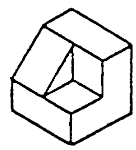


7  IS ROTATED TO 

AS  IS ROTATED TO

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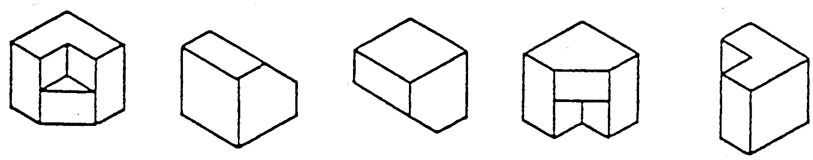
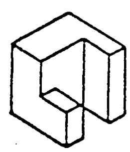
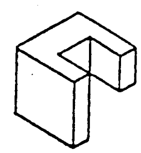
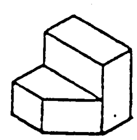


8  IS ROTATED TO 

AS  IS ROTATED TO

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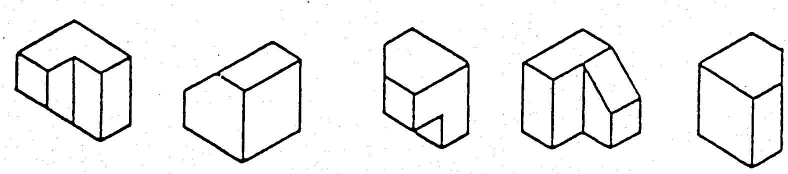
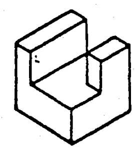
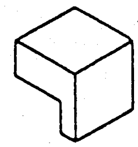
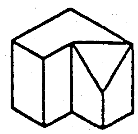


9  IS ROTATED TO 

AS  IS ROTATED TO

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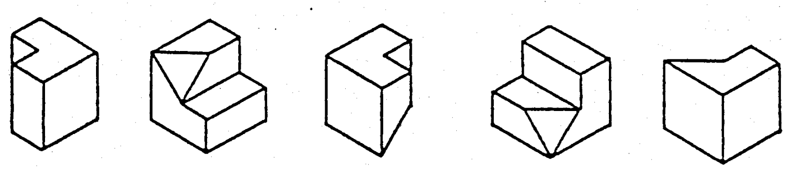
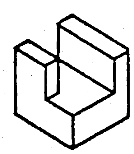
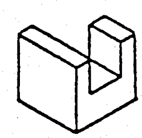
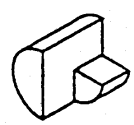


10  IS ROTATED TO 

AS  IS ROTATED TO

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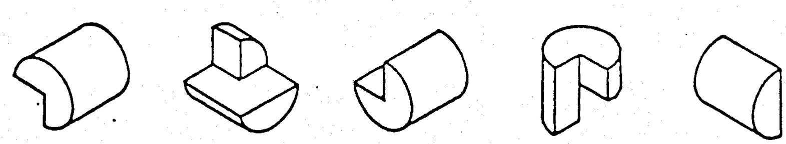

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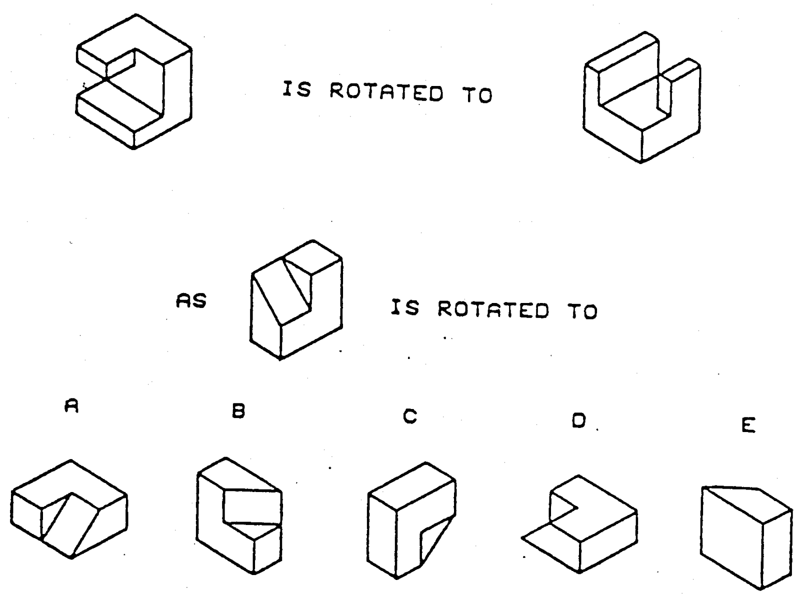
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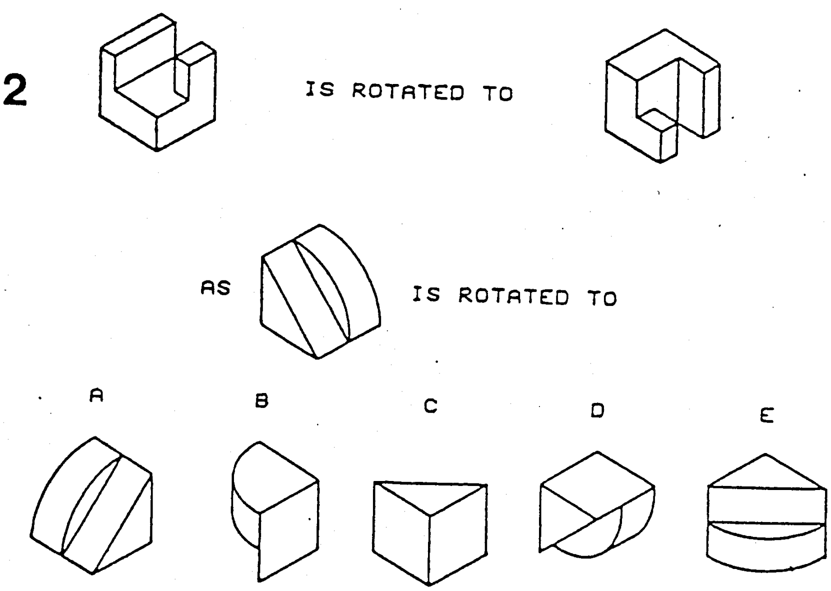
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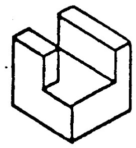
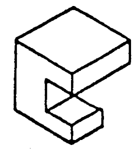
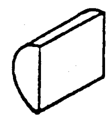
13  IS ROTATED TO 

AS  IS ROTATED TO

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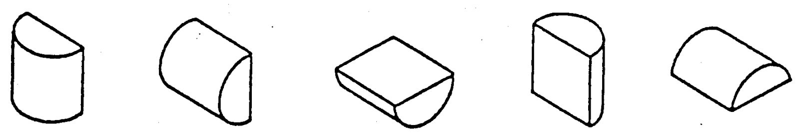
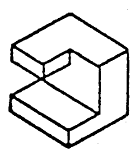
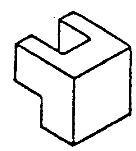
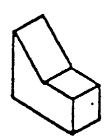


14  IS ROTATED TO 

AS  IS ROTATED TO

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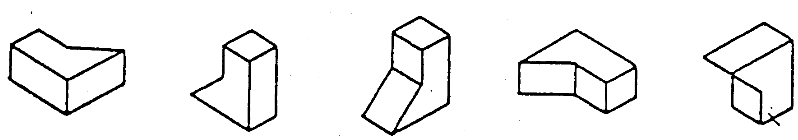
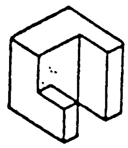
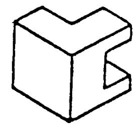
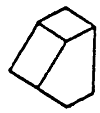
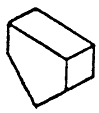
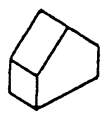
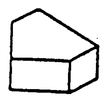


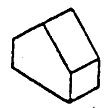
15  IS ROTATED TO 

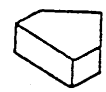
AS  IS ROTATED TO

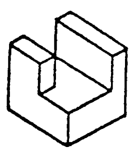
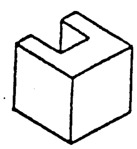
A 

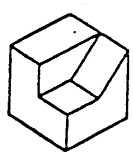
B 

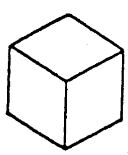
C 

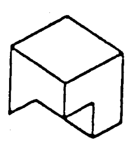
D 

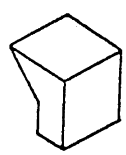
E 

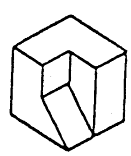
16  IS ROTATED TO 

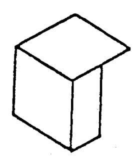
AS  IS ROTATED TO

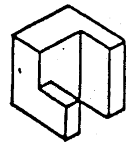
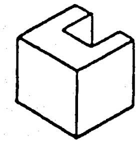
A 

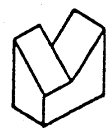
B 

C 

D 

E 

17  IS ROTATED TO 

AS  IS ROTATED TO

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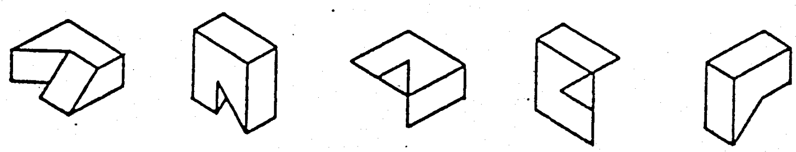
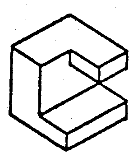
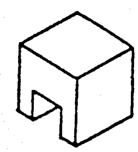
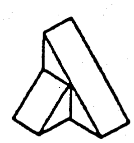


18  IS ROTATED TO 

AS  IS ROTATED TO

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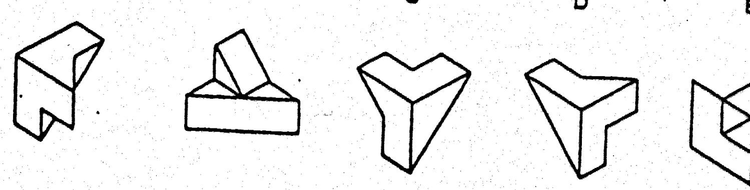
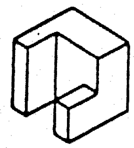
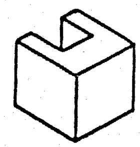
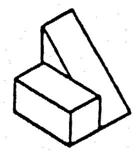


19  IS ROTATED TO 

AS  IS ROTATED TO

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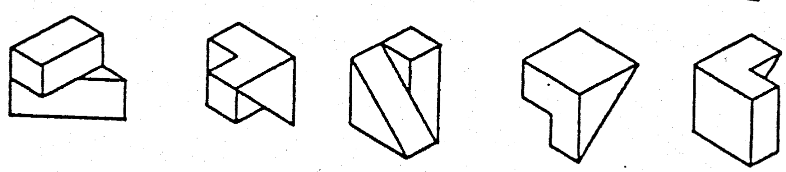
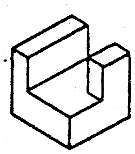
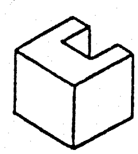
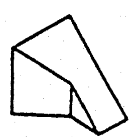


20  IS ROTATED TO 

AS  IS ROTATED TO

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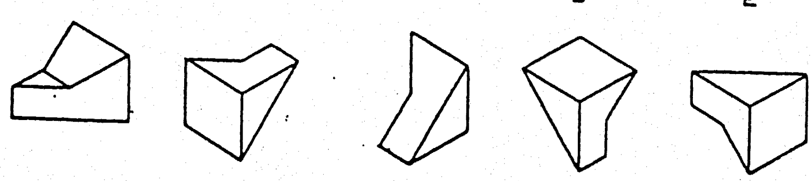

APPENDIX G

COURSE SYLLABUS

Course Syllabus

MAE 4816

Elements of Geometry

Instructor Information	Professor: Dr. Leslie Aspinwall Office: 209 Milton Carothers Hall Phone: 850-644-8427 FAX: 850-644-1880 Email: aspinwal@coe.fsu.edu Office Hours: Thursday 10:00-12:00 Other hours by appointment
Objectives	Generally, students emerging from MAE 4816 should appreciate geometry as a means of analyzing, describing, and understanding the world and seeing beauty in its structures. Specifically, the course will be aligned with standards for middle and high school grades from the National Council of Teachers of Mathematics and the Sunshine State Standards. Students will be expected to know and understand the objectives outlined by these national standards. In this spirit, MAE 4816 will focus on proficiencies related to geometric reasoning so that students in the course will be able to develop effective strategies for understanding geometry from conceptual, representational, and problem-solving perspectives. Students should be able to: • analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships; • specify locations and describe spatial relationships using coordinate geometric and other representational systems; • apply transformations and use symmetry to analyze mathematical situations; • use visualization, spatial reasoning, and geometric modeling to solve problems
Required Materials	Serra, M. (1997). *Discovering Geometry: An Inductive Approach* (2nd ed.). Berkeley: Key Curriculum Press. Key Curriculum Press. (2000). *The Geometer's Sketchpad: Student Edition*. Berkeley: Key Curriculum Press.
Topical Course Outline	• Geometric Art • Inductive Reasoning • Introducing Geometry • Using Tools of Geometry • Line and Angle Properties • Triangle Properties

	• Polygon Properties • Circles • Transformations and Tessellations • Area • Pythagorean Theorem • Volume • Similarity • Trigonometry • Deductive Reasoning • Geometric Proof • Sequences of Proofs
Teaching Strategies	The participants and professor will create a learning community for sharing of reflections on readings and presentations. The emphasis will be on sharing, support in the group environment, and personal development of meaning. Although students learn individually, they profit from working together; thus MAE 4318 will ask students to work cooperatively with fellow students. This means pulling your desks together and getting to know one another. When working in cooperative groups, students should be willing to listen to other students, to be an active participant, to ask one another questions when you don't understand, and to help one another.
Student Responsibilities	• Read assignments in preparation for class activities and discussions. • Read current literature about geometry education. • Respond to questions on geometric concepts. • Review journal articles. • Develop and solve geometry problems. • Develop a portfolio of *Geometer's Sketchpad* activities. • Sit for exams and quizzes. • Develop a portfolio of geometry proofs. • Participate in in-class group discussions.
Abstract	This is a succinct review of an article from one of these journals: • *Mathematics Teaching in the Middle School* • *Mathematics Teacher*. The abstract should relate to a current topic in geometry education, such as assessment, equity, teaching and learning, technology, classroom learning environment, national standards, or state frameworks. The abstract is due **February 10** and should be **one page, typewritten, double-spaced**. The abstracted article should be cited according to APA guidelines.
Geometer's Sketchpad Portfolio	The portfolio is a set of activities from *Geometer's Sketchpad* that are completed in class or at home. It should **not** contain other items from class, such as handouts, abstracts, journal entries, or class notes. Solutions should be placed into the portfolio so that they are organized and clear to the reader. The portfolios are due **April 13**.
Mid-term Exam	At midterm, all students will complete an in-class test of problems from the

first part of the semester.

End-of-Term Exam

At the end of the semester, all students will complete an in-class test of problems from the last part of the semester.

Evaluation

Assignment/Points		Final Grade		
Assignment	**Points**	**Grade**	**Percent**	**Points**
Mid-Term Exam	150	A	92%-100%	460-500
End-of-Term Exam	150	B	83%-91%	415-459
Abstract of Journal Article	50	C	74%-82%	370-414
Geometer's Sketchpad Portfolio	100	D	65%-73%	325-369
Participation/Attendance	50	F	0%-64%	0-324
Total	500			

Participation/ Attendance

In Class Attendance and Participation is vital for knowledge construction by each student; numerous learning experiences conducted in class cannot be duplicated in any other way. In-class activities (e.g., cooperative group work, presentations) are not formatted for completion outside of class. Timely, relevant feedback is an important component of any learning environment. Full participation in all discussions, whether instructor-facilitated, student-facilitated or small group, play an integral role in the success of this course and is required. It is expected that each student will be in attendance for every class and will participate in the whole group and small group discussions. Many of the activities involve interaction and collaboration with other students. To be effective, all participants must contribute. **Accordingly, students who miss more than three days of class will receive a five-percentage point final grade reduction for each absence. These three absences are provided for you for emergency use only; thus guard your absences, and use them only when you need them.** In addition, your grade for this assessment will be based partly on your percentage of attendance.

Academic Honor Code

Students are expected to uphold the Academic Honor Code published in The Florida State University Bulletin and the Student Handbook. The Academic Honor System of The Florida State University is based on the premise that each student has the responsibility to (1) uphold the highest standards of academic integrity in the student's own work, (2) refuse to tolerate violations of academic integrity in the university community, and (3) foster a high sense of integrity and social responsibility on the part of the university community.

Students with Disabilities

Students with disabilities needing academic accommodations should (1) register with and provide documentation to the Student Disability Resource Center (SDLC), 850-644-9566 (voice), 850-644-8504 (ADD); (2) bring a

<table>
<tr><td></td><td>letter to the instructor from SDLC indicating the need for the accommodation and what type. This should be done within the first week of class.</td></tr>
<tr><td>Important Dates</td><td>Last day to drop/add and have fees adjusted is <u>January 12</u>.
Last day to cancel enrollment and have fees removed is <u>January 13</u>.
Last day to drop a course without receiving a grade is <u>February 3</u>.</td></tr>
<tr><td>Calendar (Completed after assignments are made)</td><td>
<table>
<tr><th>Class</th><th>Activity</th></tr>
<tr><td>January 8</td><td>Introductions</td></tr>
<tr><td>January 13, 15</td><td></td></tr>
<tr><td>January 20, 22</td><td></td></tr>
<tr><td>January 27, 29</td><td>January 29 – Classroom MCH 302</td></tr>
<tr><td>February 3, 5</td><td></td></tr>
<tr><td>February 10, 12</td><td>February 12 – Classroom MCH 302</td></tr>
<tr><td>February 17, 19</td><td>February 19 – Classroom MCH 302</td></tr>
<tr><td>February 24, 26</td><td>February 26 – Journal abstract due</td></tr>
<tr><td>March 2, 4</td><td>March 4 – Classroom MCH 302</td></tr>
<tr><td>March 9, 11</td><td>Spring Break – No classes</td></tr>
<tr><td>March 16, 18</td><td>March 18 – Classroom MCH 302</td></tr>
<tr><td>March 23, 25</td><td>March 25 – Classroom MCH 302</td></tr>
<tr><td>March 30, April 1</td><td>April 1 – Classroom MCH 302</td></tr>
<tr><td>April 6, 8</td><td>April 8 – Classroom MCH 302</td></tr>
<tr><td>April 13, 15</td><td>April 15 – Sketchpad portfolios due</td></tr>
<tr><td>April 20, 22</td><td>April 22 – Last class</td></tr>
<tr><td>Finals Week</td><td></td></tr>
</table>
</td></tr>
<tr><td>SUGGESTED READING</td><td>Journals
<u>Selected Mathematics Education Journals</u>
Educational Studies in Mathematics
Focus on Learning Problems in Mathematics
Journal for Research in Mathematics Education
Journal of Mathematical Behavior
Journal of Mathematics Teacher Education
Journal of the Teaching of Mathematics and Informatics
Mathematical Teaching in the Middle School
Mathematics Teacher
Mathematical Thought (http://www.soton.ac.uk/~gary/Math_thought.html)
Mathematical Thinking and Learning
Research in Collegiate Mathematics Education
School Science and Mathematics

<u>Selected Mathematics Journals</u>
American Mathematical Monthly
The College Mathematics Journal
Mathematics Magazine
Pi Mu Epsilon Journal</td></tr>
</table>

	Selected Journals Related to Mathematics Education American Educational Research Journal Developmental Psychology Educational Leadership Educational Research Quarterly Educational Researcher Journal of Educational Psychology Journal of Instructional Psychology Journal of Educational Research Middle School Journal **Books** Grouws, D. (Ed.). (1992). Handbook of research on mathematics teaching and learning. New York: McMillan. Janvier, C. (1987). Problems of representation in the teaching and learning of mathematics. Hillsdale, NJ: Lawrence Erlbaum. National Council of Teachers of Mathematics (2000). Principles and Standards for School Mathematics. Author: Reston, VA. **Proceedings** Proceedings of the Psychology of Mathematics Education-North America (PME-NA) Proceedings of the American Mathematical Society

APPENDIX H

M&M ACTIVITY

Discovering the Relationship between Circumference and Diameter

by
Leslie Aspinwall

Overview: Use a concrete model to help enrich children's understanding of the relationship between circumference and diameter of circles of various sizes.

Materials: A large bag of M&Ms for each group of students..

Groups: Work individually or in groups of 2-4.

Introduction Students will gather and analyze data to determine the relationship between circumference and diameter of circles. The nonstandard unit of measurement will be the M&M; that is, students will measure the distance around and across circles with M&Ms.

<u>Activity One</u>

1. Place M&Ms around the circumferences of circles A, B, C, D, E, and F. Arrange them so that the line for the circumference of each circle passes through the center of each M&M and so that each M&M touches another M&M on either side. If there is space left over where a whole M&M will not fit, approximate how what part of an M&M – 1/4,1/2, or 3/4 – it would take to fill the space.

2. Place M&Ms across the diameters of circles A, B, C, D, E, and F. Again , arrange them so that the line for the diameter of each circle passes through the center of each M&M and so that each M&M touches another M&M on either side. If there is space left over where a whole M&M will not fit, approximate how what part of an M&M – 1/4,1/2, or 3/4 – it would take to fill the space.

3. Count the number of M&Ms it takes to measure the circumference and diameter of each circle. Record your data in Table 1.

4. Using a calculator, calculate the ratio of the circumference to the diameter for each circle: C/d (or C) d). Round your answers to two decimal places. Record this data in Table 1.

5. Organize the data in ascending order. Calculate the mean, median, and mode of

the ratios.

6. Construct a histogram of the ratios.

7. Construct a scatterplot of the data with diameter as the independent variable on the horizontal axis and circumference as the dependent variable on the vertical axis.

8. Try to fit a line to the data. Estimate the slope and y-intercept for this line. Write the equation of the line in slope-intercept form: y = mx +b.

9. Look at the mean and median ratios and the slope of the line. What do you see? Why do these numbers –the number that is the mean or median and the number representing the slope of the line – related this way?

10. What do we call this number?

Table for Data

Circle	Number of M&Ms for Circumference	Number of M&Ms for Diameter	C/d (or C) d)
A			
B			
C			
D			
E			
F			

Activity Two
Using lists on the TI-83

1. Press STAT, 4 (to Clear lists of previous data), 2^{nd}, 1, comma (above 7 key), 2^{nd} , 2, comma, 2^{nd}, 3, comma, 2^{nd}, 4, comma, 2^{nd}, 5, comma, 2^{nd}, 6.

2. Press STAT, 1, ENTER. Then move cursor so that it is on the first line under L_1.

3. Type in data values for diameters of circles (in M&Ms) and press ENTER after each one.
4. Move the cursor to the first line under L_2, and type in data values for circumferences (in M&Ms).

Using lists like a spreadsheet to make a new list of ratios of C/d.

1. Let the calculator form the ratio of each circumference to the diameter with the numbers in the list. Move cursor so that it is on top on L_3 (not on the line under it)

and press ENTER..

2. Type in the calculation you want to perform in terms of lists. This time, we want pi, which is circumference, or L_2, divided by diameter or L_1. Type 2^{nd}, 2,), 2^{nd}, 1, ENTER. Our ratios that approximate pi will be placed in list 3.

3. To leave the list screen, press 2^{nd}, QUIT.

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BIOGRAPHICAL SKECTH

Hasan Unal was born in Ankara, Turkey, in 1973. He graduated from Istanbul Technical University with a Bachelor of Science Degree in Civil Engineering in 1994. After graduation Hasan worked as civil engineer and continued on Master of Science at same university. After finishing his course work, in 1998 he was awarded a scholarship by the Turkish Ministry of National Education to pursue both a Masters and Ph.D. in Mathematics Education at a university of his choice. Hasan gained acceptance to Florida State University and began his graduate studies in 1999.

Hasan Unal received his Masters degrees in Mathematics Education at Florida State University in 2001.During his graduate studies he taught an undergraduate problem-solving course and helped with online classes as a teaching assistant. His research interests are student motivation, problem solving, and creativity.