

The only functions of a set of ind. variables, which are either the points line etc themselves or functions of them.

Every projective relationship must be expressible by an equation and it must be possible to put these equations in a standard form which makes it evident whether any two of them are the same or different.

From any two equations it must be possible to eliminate any one of the ind. variables, the resulting equation being unique. Indefinite variables are such that any legitimate equation between them has a projective meaning.

From any two equations it must be possible to eliminate any ind. var., leaving one (or perhaps more) ind. equations, the resulting equation being ind. of the method of elimination.

Variables of the same type or those such that if any one is substituted for another the resulting equation is legitimate.

We can take the same function of ind. variables provided the resulting expressions are legitimate.

An equation must not contain functions of numbers.

$$\begin{vmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} a_{33} + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} a_{33}$$

$$\begin{vmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} a_{33} + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} a_{33}$$

$$(ABCP) = (RBD) = (RBE) = (RBF)$$

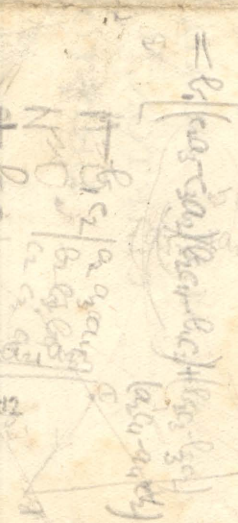
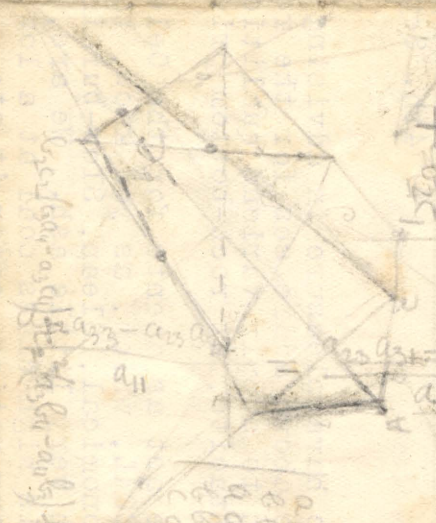
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} a_{33} + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} a_{33}$$

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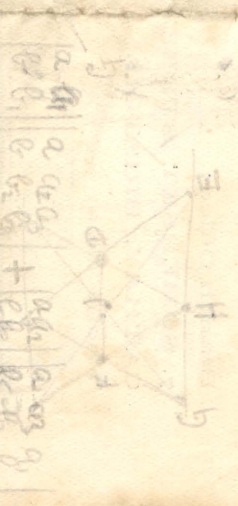
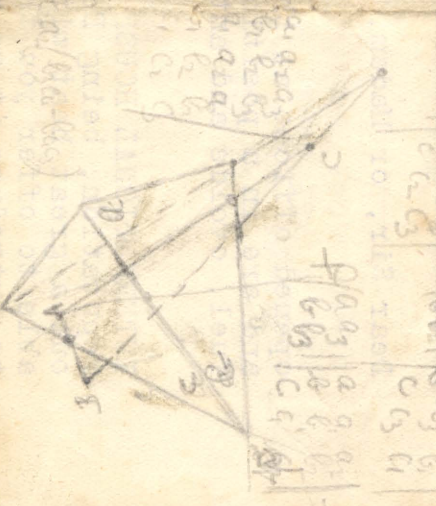
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} a_{33} + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} a_{33}$$

$$(a_{11}z_1 + a_{21}y_1 + a_{31}z_1)(a_{31}z_2 + a_{22}y_2 + a_{33}z_2) - (a_{21}z_1 + a_{11}y_1 + a_{31}z_1)(a_{31}z_1 + a_{22}y_1 + a_{33}z_1)$$

$$= x_1z_1(a_{11}a_{31} - a_{21}a_{31}) + x_1y_1(a_{21}a_{32} - a_{31}a_{22}) + x_1z_1(a_{11}a_{33} - a_{31}a_{33}) + y_1z_1(a_{21}a_{33} - a_{31}a_{33}) + y_1z_2(a_{22}a_{33} - a_{33}a_{33}) + z_1z_2(a_{33}a_{34} - a_{31}a_{33}) + z_1y_1(a_{11}a_{32} - a_{21}a_{31}) + (y_1z_2 - y_2z_1)$$



$$= \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} + \dots$$



$$= \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} + \dots$$