$\begin{vmatrix} k_1\, p_{12}\, c\, d\, e\, f \\ l\, p_{13}\, c\, d\, e\, f \end{vmatrix} = 0$ since $k_1\, p_{12}\, c\, d\, e\, f$ all meet $p_{12}$

$$\begin{vmatrix} k_1l & 0 & k_1c & k_1d & k_1e & k_1f \\ p_{12}l & p_{12}p_{13} & 0 & 0 & 0 & 0 \\ 0 & cp_{13} & 0 & cd & ce & cf \\ 0 & 0 & dc & 0 & de & df \\ 0 & 0 & ec & ed & 0 & ef \\ 0 & 0 & fc & fd & fe & 0 \end{vmatrix} = 0$$

$$k_1l \cdot p_{12}p_{13} \begin{vmatrix} cdef \\ cdef \end{vmatrix} + p_{12}l.\, cp_{13} \begin{vmatrix} k_1def \\ cdef \end{vmatrix} = 0$$

$$\frac{\begin{vmatrix} cdef \\ cdef \end{vmatrix}}{\frac{p_{12}l}{k_1l}} = -\frac{\begin{vmatrix} k_1def \\ cdef \end{vmatrix}}{\frac{p_{12}p_{13}}{cp_{13}}} = \frac{\begin{vmatrix} k_1cef \\ cdef \end{vmatrix}}{\frac{p_{12}p_{14}}{dp_{14}}} = \cdots$$

Now $\begin{vmatrix} p_{12}\, k_1\, c\, d\, e\, f \\ k_2\, l\, p_{13}\, p_{14}\, p_{15}\, p_{16} \end{vmatrix} = 0$ since $k_1\, p_{12}\, c\, d\, e\, f$ all meet $p_{12}$

$$\begin{vmatrix} 0 & p_{12}l & p_{12}p_{13} & p_{12}p_{14} & p_{12}p_{15} & p_{12}p_{16} \\ k_1k_2 & k_1l & 0 & 0 & 0 & 0 \\ ck_2 & 0 & cp_{13} & 0 & 0 & 0 \\ dk_2 & 0 & 0 & dp_{14} & 0 & 0 \\ ek_2 & 0 & 0 & 0 & ep_{15} & 0 \\ fk_2 & 0 & 0 & 0 & 0 & fp_{16} \end{vmatrix} = 0$$

$$k_1k_2 . \frac{p_{12}l}{k_1l} + ck_2 \frac{p_{12}p_{13}}{cp_{13}} + dk_2 \frac{p_{12}p_{14}}{dp_{14}} + ek_2 . \frac{p_{12}p_{15}}{ep_{15}} + fk_2 . \frac{p_{12}p_{16}}{fp_{16}} = 0$$

But $\frac{p_{12}p_{13}}{p_{12}b.\, p_{12}c} = \frac{de.\, ef.\, fd}{\begin{vmatrix} bdef \\ cdef \end{vmatrix}}$ $\quad \frac{p_{12}p_{14}}{p_{12}b.\, p_{14}d} = \frac{ce.\, ef.\, fc}{\begin{vmatrix} bcef \\ dcef \end{vmatrix}}$ etc (from proof of double 6)

$$\therefore \frac{k_1k_2.\, p_{12}l\, |cdef|}{p_{12}b.\, k_1l.\, k_2a.\, cd.ce.cf.de.df.ef} = -\left\{ \frac{|adef|}{ad.\, ae.\, af \begin{vmatrix} bdef \\ cdef \end{vmatrix}} + \frac{|acef|}{ac.\, ae.\, af \begin{vmatrix} bcef \\ dcef \end{vmatrix}} + \frac{|acdf|}{ac.\, ad.\, af \begin{vmatrix} bcdf \\ ecdf \end{vmatrix}} + \frac{|acde|}{ac.\, ad.\, ae \begin{vmatrix} bcde \\ fcde \end{vmatrix}} \right\}$$

(using formula for $ck_2$ etc)

$$-\frac{k_1k_2.\, p_{12}l.\, adef.\, acef.\, acdf.\, |cdef|.\, acde}{k_1l.\, k_2l.\, p_{12}b.\, \frac{acdef}{l}.\, cd.ce.cf.de.df.ef} = \left\{ \frac{ac.\, |adef|}{bdef} - \frac{ad\, |acef|}{bcef} + \frac{ae\, |acdf|}{bcdf} - \frac{af\, |acde|}{bcde} \right\}$$

Now $ac.\, adef - ad.\, acef + ae.\, acdf - af.\, acde = 0$ since $|acdef| = 0$

Also $ad.\, acef \left\{ \frac{adef}{bdef} - \frac{acef}{bcef} \right\} = \frac{ad.\, acef}{bdef.\, bcef} \left[ \begin{vmatrix} adef \\ bcef \end{vmatrix} - \begin{vmatrix} acef \\ bdef \end{vmatrix} \right] = -\frac{ad.\, acef.\, abef.\, cdef}{bdef.\, bcef}$