2019

Capital Flow Dynamics: Theory and Evidence

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This work is dedicated to my wife, Nicole Newell, whose support and willing sacrifice was necessary to make it possible. And to my children Lathan, Ellisyn, and Corrina.
ACKNOWLEDGMENTS

I have many people to thank who helped improve this work and were valuable in developing my skills as an economist. Foremost among those is Manoj Atolia who guided my research, provided frequent feedback, and endeavored to understand the technical details of my work at a deep level. Also Mikhail Dmitriev who was a constant encouragement and conveyed the tacit knowledge of how to develop a research idea from start to finish. I would also like to thank the rest of my dissertation committee for their valuable comments and criticism regarding my work. Lastly, I would like to thank all the participants of the Florida State Economics Department Macro workshop who patiently listened to me present my work on many occasions and helped me further clarify my arguments.
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ABSTRACT

My dissertation investigates the dynamics of international capital flows, distinguishing between net and gross flows. Chapter One considers both net and gross capital flows. I develop a small open economy model which endogenous sudden stops in net capital inflows. I then show how a second electronic currency can be used as a policy tool to reduce the volatility of capital flows. I also examine the empirical regularities of gross capital flows for the G7 countries and the implications for gross flows from a data set of investment decisions from a selection of large U.S. public pension funds. I document three important patterns in the aggregate data. First, gross capital flows are highly volatile. Second, there is a strong positive relationship between capital inflows and outflows. Third, gross capital flows are acyclical when accounting for the global financial cycle; global factors, rather than the domestic business cycle, account for a significantly greater proportion of the variation in gross flows. From firm-level pension fund data, I find that international investment decisions are large enough to contribute to gross flow volatility. For periphery economies which are the recipient of these firms’ equity investments, participation in those economies is variable with firms entering, exiting, and changing the mix of markets in which they invest. The cost structure of foreign investing suggests that fixed participation costs are statistically significant and quantitatively important. The stylized facts I document are at odds with the economic theory regarding capital flows, therefore, in Chapter Two, I develop a large open-economy portfolio choice model and solve it globally with a novel solution algorithm. Using this model I show that fixed participation costs for investing abroad of less than ten basis points is sufficient to reproduce both the observed volatility of gross capital flows and the correlation between inflows and outflows.
CHAPTER 1

NET AND GROSS CAPITAL FLOWS

1.1 Introduction

Gross capital flows and financial integration go hand in hand. If domestic residents invest large quantities in the rest of the world and residents of foreign countries invest large quantities in the domestic economy, then gross capital flows and financial integration will be greater. The relative lack of financial integration during the mid-twentieth century helps to explain why the economics literature largely ignored gross capital flows in favor of net flows. As a share of output, financial integration actually peaked before World War I; collapsed and did not begin to expand again until after World War II; continued to grow in the post-War era and by some measures, it did not reach the pre-1914 level until the 1990s (Bordo et al., 1998). In the decade preceding the 2008-2009 global recession, financial integration measured by gross capital flows reached a level that dwarfed all previous experience. Consequently, this period coincides with the growing interest in the nature and dynamics of gross capital flows.

However, there are other reasons why research focused on net capital flows rather than gross flows. Data availability creates a strong bias toward net capital flows. After all, a country’s balance of payments equates the current account with the negative of the capital account, which is the measure of net capital flows. Furthermore, trade data, which defines the current account, has been available across many countries for decades—in some cases centuries. The effect of net flows on real economic variables is immediately evident; net capital flows represent external savings or borrowing which can be used to shift resources from places where the marginal product of capital is low to where it is high, impacting real investment. Compared to net flows, the importance of gross capital flows is less obvious. Gross flows may influence risk-sharing and liquidity, but their impact on real variables is not immediately apparent. Finally, the theoretical tools to deal with gross capital flows have been absent until recently. Meaningful gross capital flow dynamics require incomplete markets; however, models of this type have been notoriously difficult to solve. For these
reasons, it has only been very recently which serious attention has shifted from net capital flows to gross.

Therefore, the purpose of this paper is to review the empirical and theoretical literature regarding capital flows, document the recent empirical trends in aggregate capital flows, and examine a firm-level data set related to the dynamics of gross capital flows. Section 1.2 addresses the research primarily focused on net capital flows; Section 1.3 considers a model including an electronic money policy to reduce the volatility of the capital account; Section 1.4 addresses the research on gross capital flows; Section 1.5 provides an overview of the tools that have been developed to analyze capital flows including a novel approach used in this dissertation; Section 1.6 analyzes the aggregate data on capital flows; Section 1.7 presents firm-level foreign investment data from public pension funds and connects these observations to the aggregate capital flow patterns, and finally Section 1.8 concludes by connecting this literature to the theoretical model I develop in Chapter 2.

### 1.2 Net Capital Flows

I consider three broad strands in the literature on net capital flows, namely external balance, capital market efficiency, and macroeconomic instability. Although interest in all three overlaps across time, for historical reasons research begins on each sequentially. A natural starting point for the exposition on net capital flows as they relate to external balance is the work by Mundell (1960, 1962, 1963).\(^1\) His is not the first work to consider the subject; Hume’s price-specie-flow mechanism demonstrated how price level adjustments as a result of capital flows compensate for current account imbalances.\(^2\) However, Mundell’s work is early enough to predate the enormous policy changes that occurred with the collapse of the Bretton Woods system and the rise of floating exchange rates and late enough to remain influential in contemporary literature.

Mundell (1960) considers the difference in the adjustment dynamics of fixed versus floating exchange rate regimes. He argues that a fixed exchange rate requires monetary policy to focus on the task of maintaining external balance at the cost of other concerns such as full employment. The open economy equilibrium framework that he uses relies on standard accounting equations which

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\(^1\)In his book, Mundell (1968) writes extensively on international economics and chapters 8 through 19 address the international adjustment mechanism and capital flows.

\(^2\)Although Hume’s formulation of the price-specie-flow mechanism is quite clear; this too is probably not an early enough attribution as both Richard Cantillon and Joseph Harris were writing about the same ideas prior to Hume (O’Brien, 2004).
remain familiar today, assuming reserves are constant: domestic savings minus investment must equal the trade balance; the trade balance equals negative net capital inflows. Thus, trade deficits indicate a net capital inflow and domestic investment that exceeds domestic savings. Anticipating the famous open economy policy trilemma, Mundell observes that with fully mobile capital and a fixed exchange rate, interest rate differentials between countries (monetary policy) are quickly eliminated. He contrasts the ineffectiveness of monetary policy under fixed exchange rates with that of floating exchange rate regimes which result in the startling conclusion of increasing macroeconomic instability when capital is mobile and if central banks wish to maintain any stock of foreign reserves. Because independent monetary policy is not possible with a fixed exchange rate and capital mobility, Mundell (1962) considers fiscal policy in addition to monetary policy to achieve both external balance and internal policy goals (i.e., full employment). Mundell does not find fiscal policy to be a suitable substitute for monetary policy; even the inclusion of the additional policy instrument, monetary policy is not flexible enough to address internal macroeconomic stability. Mundell argues that monetary policy is less effective for internal stability than fiscal policy. If monetary policy targets domestic issues, such as full employment, and fiscal policy focuses on external balance, this would lead to either instability or cycling but not equilibrium. Thus, monetary policy does not gain independence with a fixed exchange rate and free capital-mobility regime even when fiscal policy is available.

The shift of major economies to flexible exchange rates during the 1970s created the demand for a greater understanding of the relationship between exchange rate dynamics with internal and external balance. To this end, a considerable quantity of research was devoted: Argy and Porter (1972); Black (1972, 1976); Kouri (1975); Niehans (1975); Dornbusch (1976a,b,c); Frenkel (1976); Mussa (1976); Henderson (1977). An important innovation by Argy and Porter (1972) is incorporating expectations in the determination of exchange rates. Rather than requiring that interest rates equalize, they argue for what is now called covered interest rate parity where forward currency contracts are used to account for interest rate differentials due to expected currency appreciation or depreciation. Under the expectations framework, Dornbusch published a series of articles arguing that the high volatility of exchange rates was consistent with rational expectations.

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3See Rey (2015).
4Working independently, Fleming (1962) reaches a similar conclusion that the effectiveness of monetary policy on domestic output is more significant for a floating exchange rate than a fixed regime.
and about the effectiveness of monetary and fiscal policy under floating exchange rates\textsuperscript{5}. Dornbusch (1976b) builds on the work of Mundell and Fleming. By assuming slowly adjusting goods prices and immediately adjusting asset and exchange markets, he shows that monetary expansion will result in an immediate depreciation of the domestic currency. If real output is fixed, an immediate initial depreciation will 'overshoot' and be followed by an appreciation back to the long run exchange rate. Alternatively, if output can adjust in the short-run, then the monetary expansion will cause an increase in income and money demand; this causes an immediate appreciation of the currency followed by gradual depreciation. In this case, higher interest rates are required to compensate for the depreciation.

By and large, the earliest literature regarding net capital flows only considered capital in so far as it balanced a country’s external position. In the Mundell-Fleming framework with fixed exchange rates and freely mobile capital, net capital flows are a residual required to clear the foreign exchange market. However, after a decade of experience post-Bretton Woods, capital flows began to receive extra attention in their own right, in particular, the efficiency of international capital markets to allocate resources to their highest valued use. A conspicuous assumption of many of the preceding models is free capital mobility or at least capital markets that adjust more quickly than goods markets; Feldstein and Horioka (1980)\textsuperscript{6} questions the assumption of free capital mobility. The importance of assuming perfect capital mobility is critical for the results of early models of international economies. With fully mobile capital, rates of return should equalize across countries and capital will always be moving toward its most valued use. If this is the case, then investment opportunities do not need to be funded by domestic savings. Likewise, country level savings behavior should be unrelated to investment decisions. To test this Feldstein and Horioka estimate the regression equation \((I_i/Y_i) = \alpha + \beta(S_i/Y_i)\) and propose that \(\beta = 0\) if capital is freely mobile, thus an increase in savings at home has little effect on domestic investment. However, if \(\beta\) is closer to one, then changes in savings are being channeled into domestic investment. Considering aggregate savings and investment over 15 years, they estimate \(\beta = 0.89\), nearly one and conclude that international capital markets are not efficient because domestic investment seems to be funded by domestic savings.

\textsuperscript{5}See Dornbusch (1991) for an exhaustive collection of his most cited work on open economy macroeconomics.

\textsuperscript{6}See also Feldstein and Bacchetta (1991)
What became known as the Feldstein-Horioka puzzle is a particularly attractive research topic as the empirical regularities are very robust and surprising. The results obtained by Feldstein and Horioka were followed by many confirmatory studies (Feldstein, 1982; Fieleke, 1982; Penati and Dooley, 1984; Murphy, 1984; Caprio and Howard, 1983; Summers, 1988) and sparked a debate about the efficiency of international capital markets to allocate scarce resources. Among those who question the interpretation of positive correlations between savings and investment as evidence of imperfect capital mobility is Dooley et al. (1987). They argue that physical capital and bonds are not perfect substitutes. Thus, even if there is a great degree of mobility of debt instruments, there is no reason to believe the same is true about physical capital. Bayoumi (1990) shows that during the gold standard period—a period of presumed free capital flows—savings and investment correlations were much less than unity and statistically indistinguishable from zero. However, he also finds that disaggregating savings into private and public results in a high savings-investment correlation for public savings but not private.

There are also those who question the validity of inferring capital mobility from savings-investment correlations. Among them, Obstfeld (1986) argues that there is no a priori relationship between the savings and investment rates in a country as many things will influence decisions such as transactions costs, taxes, or official regulation. Therefore, for savings-investment correlations to be useful, the real world needs to be compared to a benchmark 'efficient' world economy that is specified first. Furthermore, the initial estimates of the savings-investment correlation are aggregated for 15 years and follow up investigations tend to consider five years or more. In contrast, Obstfeld (1986) estimates the model annually and finds the investment-savings coefficient is statistically different than one. This pattern is also true when the sample is limited only to small countries or those without significant capital controls. The coefficient falls over time following the general collapse of fixed exchange rate regimes among developed countries.

Sachs (1983) finds evidence that increased investment is correlated with current account deficits and that this is consistent with a model which takes into account current and future economic variables. Sachs then shows how the effect will be different depending on whether shocks to the economy are perceived to be temporary or permanent. Tesar (1991) demonstrates theoretically that even under complete asset markets, savings-investment correlations are not a reliable indicator of

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See, for instance, Feldstein (1982); Fieleke (1982); Penati and Dooley (1984); Murphy (1984); Caprio and Howard (1983); Summers (1988).
capital mobility. Baxter and Crucini (1993) show that for a variety of specifications, a standard two-country model with complete markets will produce saving-investment correlations in line with Feldstein-Horioka. Although capital adjustment costs are a ubiquitous feature of open economy models, Baxter and Crucini (1993) show that these costs are necessary to produce the proper saving-investment correlation, thus, in line with Dooley et al. (1987), financial capital can be freely mobile, but limits to physical capital mobility induce high saving-investment correlation. In contrast with the literature focusing on countries’ external balance or efficient capital allocation, the third and final theme I consider in the literature on net capital flows is their connection to macroeconomic instability as a result of sudden shifts in the direction of flows.

1.3 Sudden Stops and Electronic Money

A chief concern regarding net capital inflows among policymakers and researchers alike is what are dubbed “sudden stops”. A sudden stop in net capital inflows can be caused by capital flight of domestic investors, a drying up of investment inflows from foreigners, or both. The opposite of a stop is known as a capital surge and emerging market economies are frequent recipients of both surges and stops. Surges tend to coincide with large domestic currency appreciations and increases in asset prices both of which reverse when a stop occurs. These stops have impacts on the real economy and are followed by both lower output growth and real investment. The disruptions of surges and stops have even led to the IMF reversing its long-standing policy against capital controls (Ostry et al., 2010).

The literature on sudden stops is vast and growing. The mid to late 1990s were a tumultuous time with several sudden stop episodes including the Mexican Tequila crisis in 1994, the Asian crisis of 1997-1998, and both the Russian and Argentine crises in 1998. Calvo (1998) argues that these sudden stop episodes are highly destructive to the real economy, leading to bankruptcies, underutilized human capital, and disrupted local credit channels. Furthermore, these episodes are preceded by substantial current account deficits which exacerbate disruptions because it is much more difficult to finance them during a sudden stop.

Caballero and Krishnamurthy (2004) find that countries do little to prevent sudden stops and thus conclude that governments should intervene by creating conditions which limit the disrup-

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8See, for instance, Calvo (1998); Caballero and Krishnamurthy (2004); Edwards (2004); Chari et al. (2005); Mendoza and Smith (2006); Rothenberg and Warnock (2011); Calderón and Kubota (2013).
tiveness of sudden stops, specifically advocating local private insurance markets which tend to be absent from emerging market economies. Alternative proposals aimed at averting the disastrous effects of sudden stops include encouraging capital controls to either reduce borrowing or encourage foreign direct investment and other long term financial flows which are less susceptible to sudden stops (see Levchenko and Mauro, 2007; Kose et al., 2010; Tong and Wei, 2010; Farhi and Werning, 2012; Benigno et al., 2013; Costinot et al., 2014), while others propose some form of time varying Pigouvian taxes addressing the overborrowing that precedes the crisis (see Bianchi and Mendoza, 2010; Jeanne and Korinek, 2010, 2018; Korinek, 2018). Contributing to this literature, in joint work with Mikhail Dmitriev, we propose the use of an electronic currency to manage the current account, limit the overborrowing that occurs before the crisis and mitigate the costs of sudden stops. In the model developed in subsection 1.3.1, we take for granted that the small open economy is subject to sudden stops, and our contribution is to show how an electronic currency can be used as a practical policy tool to address sudden stops.

The large amount of attention paid to Bitcoin during the 2017 run-up of its price from $1,000 to nearly $20,000 and subsequent collapse in price to below $4,000 has sparked interest in cryptocurrency among both policymakers and economists. In treatment on the theory of cryptocurrency, Barrdear and Kumhof (2016) and Schilling and Uhlig (2018) assume that its only function is as a medium of exchange and substitute for traditional money. However, cryptocurrency, and blockchain technology more generally, has already been put to many alternative private uses. Although Bitcoin is designed as a money alternative, Ethereum boasts the ability to be used for smart contracts which can reduce the need for intermediaries or escrow services. The Factom cryptocurrency is meant to be a permanent record keeper which could replace public notaries and other services which vouch for the authenticity of documents. Golem cryptocurrency works as a distributed supercomputer, similarly, Sia is a cryptocurrency which allows users to share hard drive space. Many global financial institutions are using Ripple to clear international transactions which, compared to traditional options, reduces counter-party risk and delays.

Governments and international organizations are very concerned with cryptocurrency as well. Venezuela had a brief foray into state-backed electronic money when they launched their "petro" cryptocurrency in February 2018 which was supposed to be backed by barrels of oil, however, by August 2018 it was no longer active. Riksbank, the Swedish central bank, is considering the
introduction of an "e-krona" and the Swiss government is considering an "e-franc". One of the main benefits of electronic money for central banks is the possibility of using it to overcome the "zero lower bound" (Agarwal and Kimball, 2015). The zero lower bound problem arises because hard currency (traditional money) pays zero nominal interest and thus it is difficult for central banks to achieve interest rates below zero. However, a cryptocurrency based money can pay less than zero, allowing the central bank to target negative interest rates which increases the flexibility of interest rate policy during recessions. Similar to private innovations in cryptocurrency which take it beyond a means of exchange, we propose to use a government created cryptocurrency for something other than a money substitute, specifically as a means to manage the current account.

1.3.1 Model

We consider a small open economy in discrete time. A unit measure of identical domestic agents can borrow from risk-neutral investors in the rest of the world through an international debt market. Utility comes from the consumption of a composite good made up of traded and non-traded goods, $C_t = C_{T,t}^{\alpha} C_{N,t}^{(1-\alpha)}$, and is valued by households according to the function

$$U = \mathbb{E} \sum_{t=0}^{\infty} \beta^t u(C_t) \quad (1.1)$$

where $\beta < 1$ is the time discount factor. The instantaneous utility function, $u(C_t) = \frac{C_{t}^{1-\sigma} - 1}{1-\sigma}$, is CRRA. Each period agents are endowed with tradeable goods income, $Y_{T,t}$, which grows at gross rate $g_{T,t}$, such that $Y_{T,t} = g_{T,t} Y_{T,t-1}$. The growth rate is stochastic and on average the growth rate is greater than one, however, the economy is subject to large and infrequent slowdowns in growth,

$$g_{T} \sim \begin{cases} g_h & \text{with probability } (1 - p) \\ g_l & \text{with probability } p \end{cases} \quad (1.2)$$

where $g_h$ is the normal growth rate, $g_l$ is the rare disaster growth rate, and $p$ is the disaster probability. Agents are also endowed with non-tradable goods income $Y_{N,t}$ which is priced at $P_{N,t}$. For simplicity, non-traded income is assumed to be constant and normalized to one, $Y_{N,t} = C_{N,t} = 1$. The period budget constraint is

$$C_{T,t} + P_{N,t} C_{N,t} + B_{t+1} = Y_{T,t} + P_{N,t} Y_{N,t} + B_t (1 + r) \quad (1.3)$$

where prices are denominated in traded goods. Domestic agents can borrow by issuing bonds, priced in terms of traded consumption goods, from risk neutral investors at interest rate $r$. Agents
are also bound by Kiyotaki and Moore (1997) style collateral constraints so that they can only borrow up to a fraction, \( \kappa < \frac{\alpha}{1-\alpha} \), of GDP, such that

\[
B_{t+1} \geq -\kappa(Y_{T,t} + P_{N,t}Y_{N,t}) \quad (1.4)
\]

Because the economy is growing and utility is concave, agents would like to borrow from abroad against future income, however, we set \( \kappa \) sufficiently small so that equation (1.4) is always binding at the economy level. Furthermore, agents do not internalize this constraint so they must de-lever when the rare large downturn is realized.

The combination of Cobb-Douglas consumption and constant non-traded income means that the price of non-traded goods is proportional to the consumption of traded goods

\[
P_{N,t} = \frac{1 - \alpha}{\alpha} C_{T,t} \quad (1.5)
\]

A decline in traded goods output during the downturn tightens the borrowing constraint. This effect is then amplified when the price of non-traded goods responds to the fall in traded goods consumption, exacerbating the borrowing constraint. Combining equations (1.4) and (1.5) allows us to express traded goods consumption which depends only on state variables \((B_t, Y_{T,t})\) and parameters \((\kappa, \alpha)\),

\[
C_{T,t} = \frac{B_t(1 + r) + Y_{T,t}(1 + \kappa)}{1 - \kappa\frac{1 - \alpha}{\alpha}} \quad (1.6)
\]

The decentralized equilibrium in this economy is characterized by large infrequent swings in consumption coinciding with sudden stops of capital from foreign investors. The first welfare theorem does not hold due to incomplete markets arising from the borrowing constraint. Therefore, we propose the establishment of a second (electronic) currency market as a practical means addressing the pecuniary externality arising from the borrowing constraint.

### 1.3.2 Second Currency Market

The idea of the second currency is that the government will issue to any exporter \( x_c \) units of cryptocurrency for each unit of exports, and in order to import, an importing firm must also purchase one unit cryptocurrency per unit of imports which is returned to the government and removed from the market. Additionally, to prevent speculation, unused units of cryptocurrency expire at the end of each period. Therefore, \( x_c \) is a policy tool which can be implemented to cap
the current account deficit. For instance, if $x_c = 1$ then the total value of exports must equal the total value of imports. If without the policy, a country would typically export greater value than it imports, then there will be excess tokens, and their value will be zero. If on the other hand, there would be a greater value of imports relative to exports, a policy of $x_c = 1$ would cause a shortage of tokens and cause their price to rise until the value of imports equals exports. In this way, when the policy binds, it works to simultaneously subsidize exports and tax imports. For simplicity, we will assume the target policy is a balanced current account\(^9\) (i.e., $x_c = 1$), in which case the policy imposes an additional constraint on households, such that

$$P_{x,t} (Y_{T,t} - C_{T,t} + B_t r) = 0$$ (1.7)

where $P_{x,t} \geq 0$ is the market price of the cryptocurrency. From a household perspective, access to credit is already too low due to the borrowing constraint and this further constraint makes borrowing even more difficult. In the short-run, this policy will be costly for the domestic economy. However, over time there is less accumulation of debt so that when the rare disaster does occur, it is less costly because the debt to GDP level is lower which means the borrowing constraint has more slack. On the other hand, rare disasters are infrequent and if they are not realized then the policy will be costly.

### 1.3.3 Calibration

Table 1.1 summarizes the values we use to calibrate our model. Several values are common in the literature, for an annual model we use a discount rate, $\beta = 0.92$ and utility curvature parameter of $\sigma = 2$. A constant interest rate, $r = 0.083$, which is the effective dollar interest rate estimated by Korinek (2018) for a selection of emerging market countries. The borrowing constraint parameter, which limits the debt-to-GDP ratio, is set to $\kappa = 0.43$ which is the average ratio for emerging and developing countries from 2000-2018 (IMF (2018)). The parameters of the income process are from Eichengreen and Gupta (2016) who study emerging market economies experiencing sudden stops. They estimate an average non-crisis growth rate of 4.28% ($g_h = 1.0428$). Average sudden stop

\(^9\)A balanced current account is not necessary, if $x_c > 1$ ($x_c < 1$) the country will run a current account deficit (surplus). $x_c = 1$ is not meant to be an optimal target but is consistent proposals be popular writers (Buffet, 2016) and academics (Stiglitz, 2017) alike. The main purpose of considering the use of cryptocurrency as a policy tool is its practical implementability. Considering more complicated policy rules may be interesting and enhance welfare in the theoretical realm but come at the expense of policy that can be used in the real world.
Table 1.1: Baseline Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Leverage Ratio</td>
<td>$\kappa = 0.4525$</td>
<td>IMF World Economic Outlook (10/2018)</td>
</tr>
<tr>
<td>Share Traded Cons.</td>
<td>$\alpha = 0.55$</td>
<td>Corsetti et al. (2008)</td>
</tr>
<tr>
<td>Growth Rate</td>
<td>$g_h = 1.0428$</td>
<td>Eichengreen and Gupta (2016)</td>
</tr>
<tr>
<td>Disaster Growth</td>
<td>$g_l = 0.9828$</td>
<td>Eichengreen and Gupta (2016)</td>
</tr>
<tr>
<td>Disaster Probability</td>
<td>$p = 0.08$</td>
<td>Eichengreen and Gupta (2016)</td>
</tr>
<tr>
<td>Discount Rate</td>
<td>$\beta = 0.92$</td>
<td>Aguiar and Gopinath (2007)</td>
</tr>
<tr>
<td>Utility Curvature</td>
<td>$\sigma = 2$</td>
<td>Literature</td>
</tr>
<tr>
<td>World Interest Rate</td>
<td>$r = 0.083$</td>
<td>Korinek (2018)</td>
</tr>
</tbody>
</table>

Crisis episodes last one year and the subsequent fall in GDP growth is 6% ($g_l = 0.9828$). They also report the frequency of sudden stops for developing countries to be about 8% annually ($p = 0.08$).

1.3.4 Policy Experiment

To consider the welfare implications of such a policy, we simulate two economies, one without (baseline) and one with the cryptocurrency (crypto) policy. Both economies begin with the same initial steady state level of debt-to-GDP and experience the same series of exogenous shocks to traded income. Figure 1.1 shows an example of simulated paths for the baseline economy (blue) and an economy with the same endowment stream and constraints but also with the cryptocurrency policy targeting the current account (red "x"). The policy is costly in terms of consumption, because of the growing economy consumers would like to borrow to smooth consumption but are constrained by the borrowing limit, the cryptocurrency policy constrains them further and there is an immediate drop in consumption. However, when the first sudden stop arrives, the fall in GDP is about 2% (6% lower than trend growth), but the fall in aggregate consumption as a share of GDP is another 4.5%. Some of this fall is being mediated by non-traded goods, the fall in traded consumption as a share of traded income is even more extreme at 8%. The cryptocurrency economy experiences the same series of traded goods shocks, however, consumption as a share of GDP is much smoother. Over time, the accumulated debt level in the cryptocurrency economy is lower which means that even though there is lower consumption in the short run, the consumption share will eventually exceed the baseline economy, as payments on the debt will be smaller. However, we quantify the welfare gains of the cryptocurrency policy as the change in the percentage of consumption for the baseline economy which delivers the same utility as the cryptocurrency economy. The initial
impact of the policy is a welfare loss of 3.10% as a percentage of initial consumption in the baseline model. However, the benefits accrue over time and in order to achieve the same utility as agents in the policy economy, agents in the baseline economy require a permanent increase in the flow of consumption of 2.34%.\(^{10}\)

![Figure 1.1: Simulated Consumption Paths](image)

To get a sense of the implications of enacting the proposed policy, we simulate the economy 10,000 times for 200 periods and calculate the model’s moments. For the each economy total output is \(Y_{T,t} + P_{N,t}\) which is growing. To calculate the standard deviation of total output, we take the log of the series and either apply the HP-filter\(^{11}\) or first-difference the series. Consumption is logged and HP-filtered. Net exports are normalized by total output (unfiltered) and then \(NX/GDP\) is HP-filtered.

For the cryptocurrency model, implementation of the policy caps the debt level, thus as a share of GDP, debt is shrinking so that in the long run there is zero debt-to-GDP. Therefore, in the long run, the cryptocurrency economy is effectively in autarky and so the statistics reported are for the autarky economy normalized by traded-output. The data moments are from Table 1 of Aguiar and Gopinath (2007) for emerging market economies who, consistent with our model, argue that trend growth shocks are the distinguishing characteristic that differentiates emerging and developed

\(^{10}\)To compare the relative welfare gains of the cryptocurrency policy over time horizon \(T = \{1, \infty\}\), we first calculate the present discounted value of the consumption flow with the cryptocurrency policy \(W_{T,x}(C_t) = \sum_{t=1}^{T} \beta^t \frac{C_t^{1-\sigma}-1}{1-\sigma} \). The baseline economy in the absence of the policy has present discounted value of consumption of \(W_{T,0}(C_{t,0}) = \sum_{t=1}^{T} \beta^t \frac{C_t^{1-\sigma}-1}{1-\sigma} \). Thus, we calculate the welfare gain of the policy to be \(\zeta\), such that \(W_{T,x}(C_t) = W_{T,0}(\zeta C_{t,0})\). \(\zeta - 1\) is interpreted as the permanent percentage change in consumption required for the baseline economy to be as well-off as the cryptocurrency economy. See Atolia et al. (2010) for details.

\(^{11}\)The model is annual, therefore whenever the HP-filter is used, the smoothing parameter is 100.
economies. The baseline economy is consistent with the characteristic emerging market economy, output is very volatile, consumption is more volatile than output, net exports are counter-cyclical, and there is a strong positive correlation between consumption and GDP.

Table 1.2: Simulated Moments

<table>
<thead>
<tr>
<th></th>
<th>Data EM</th>
<th>Baseline</th>
<th>Crypto</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(Y)$</td>
<td>2.74</td>
<td>2.22</td>
<td>1.51</td>
</tr>
<tr>
<td>$\sigma(\Delta Y)$</td>
<td>2.87</td>
<td>3.35</td>
<td>1.61</td>
</tr>
<tr>
<td>$\sigma(C)/\sigma(Y)$</td>
<td>1.45</td>
<td>1.63</td>
<td>1.00</td>
</tr>
<tr>
<td>$\sigma(NX/Y)$</td>
<td>3.22</td>
<td>1.61</td>
<td>0.00</td>
</tr>
<tr>
<td>$\rho(NX/Y, Y)$</td>
<td>−.51</td>
<td>−.76</td>
<td>—</td>
</tr>
<tr>
<td>$\rho(C, Y)$</td>
<td>0.72</td>
<td>0.96</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Data are from Aguiar and Gopinath (2007) for emerging markets (EM) and advanced economies (AE).

The long-run effect of implementing the cryptocurrency based policy is autarky, resulting in a reduction in consumption volatility because $C_t = Y_t$ and zero net export volatility. Because consumption volatility is lower in autarky, the price volatility of non-traded goods is also lower resulting in less output volatility.

1.3.5 Extensions and Concluding Remarks

Our model is admittedly very simple and in spite of representing the stylized facts rather well, there are many margins on which it could be improved. However, most of those margins will likely only increase the cost of implementing such a policy and make it even less appealing. For instance, in a production economy where imports are intermediate goods, restrictions on imports which arise from constraining the current account will necessarily reduce output even more. Furthermore, because there is only one type of traded good, we only account for net exports, when the policy proposal is for something more nuanced which could simultaneously subsidize exports and tax imports.

1.4 Gross Capital Flows

The assumption, that differences in rates of return drive capital flows, was questioned from a very early point. The assumption that investors myopically consider the highest expected return on assets and any deviation in those expectations leads to persistent capital flows is in contrast
with ideas in the field of portfolio choice. The portfolio choice perspective emphasizes imperfect asset substitutability which requires other considerations besides expected returns when making asset acquisition decisions. The models of Markowitz (1952) and Tobin (1958) advocated that the desirability of assets depends on both the expected return and the effect of the covariance of returns on the entire portfolio. Thus, an interest rate increase (decrease) would not result in a sustained increase (decrease) in net capital inflows but instead should result in a one-time change in the allocation of assets held by domestic and foreign investors which get measured as flows. The analogy of capital flows being an open faucet or drain begins to be replaced by the new view of capital flows as the result of bailing buckets—adjustment happens suddenly, in a discrete volume, and not endlessly.

Focusing on net capital flows obscures what may be important dynamics driving gross capital flows. Grubel (1968) argues that international assets should be an important opportunity for constructing efficient mean-variance portfolios.\(^\text{12}\) Grubel shows in a two-country setting that optimal holdings of assets will be a function of both interest rate differentials and the relative growth rate of total asset holdings in the two countries. Although contemporary models presumed that higher than normal interest rates would result in net capital inflows, Grubel showed that if the portfolio growth is asymmetric, then net capital inflows can be positive even when the interest rate differential is zero or negative. Likewise, net capital outflows can occur even if the home country has a positive interest rate differential.

Following a similar line of thinking, Willett and Forte (1969) argue that a given level of interest rates will determine the stocks of assets and only changes in those interest rates will lead to transitional flows of capital. Branson (1970) highlights the importance of the portfolio choice view by showing empirically how shifts in asset and liability positions overwhelm net flows such that the effect of the shift dominates the dynamics of net capital flows and not the persistent effect of changes in interest rate differentials. Although these authors are considering the effects of the stock of foreign assets and liabilities—of which the dynamics are gross capital flows—quite early on, it took several decades for the study of gross capital flows to take a more prominent role.

In early work considering gross capital flows by evaluating the changes in stocks of foreign assets and liabilities, Stein (1965) concludes that foreign assets and liabilities have little explanatory power

\(^{12}\text{Miller and Whitman (1970) estimate mean-variance efficient portfolios for the United States which consider international assets.}\)
for short term capital movements. However, Lee (1969) provides evidence from the U.S. and Canada that taking stocks of assets and liabilities into account is necessary to explain the flow of capital between the two countries. Furthermore, by taking portfolio asset positions into account, he can address the issue of interest rate differentials between countries if there is relatively mobile capital. In a similar spirit, Golub (1990) argues that gross flows and stocks, rather than net flows, are the correct measure of capital mobility. Contrary to the claim of Feldstein and Horioka (1980) that the high savings-investment correlation implies limited capital mobility, Golub shows that by taking gross capital flows into account, there is evidence of substantial capital mobility. This was a prescient observation as global gross capital flows were about to explode in size and volatility following the publication of his essay.

The dramatic increase in gross capital flows resulted in residents around the world holding large stocks of foreign assets and liabilities and was observed in the empirical work of Lane and Milesi-Ferretti (2001) who compile a panel dataset of stocks of foreign assets and liabilities which cover 67 countries from 1970-1998. Lane and Milesi-Ferretti (2007) further revise and extend their estimates to include 145 countries over the span of 1970-2004. One influential contribution of Lane and Milesi-Ferretti (2001, 2007) which led to a flourishing of new ways of thinking about financial integration was to disaggregate net foreign assets into its asset and liability components, demonstrating the immense growth of assets and liabilities that were otherwise concealed by net positions. Assets and liabilities grew large enough, relative to GDP, that by the late 1990s fluctuations in exchange rates and asset prices posed significant risks to the wealth of countries’ residents.

Furthermore, this disaggregation allows Lane and Milesi-Ferretti to propose an intuitive measure of financial integration that is missing from the net flows paradigm, namely total foreign assets and liabilities. When considering the current account or net foreign asset position, there are several states of the world in which these measures obscure the underlying degree of financial integration. For instance, the net foreign asset position is zero when a country has no foreign asset and liabilities, but it is also zero when there are substantial and exactly matching assets and liabilities. The former represents an autarkic nation while the latter represents a financially integrated one even though

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13 Earlier work (Sinn, 1990; Rider, 1994) aimed to construct estimates of external assets and liabilities but was less comprehensive and did not include the explosion of gross capital flows occurring in the 1990’s.

14 Alternative measures of financial integration, such as Chinn and Ito (2008), measure financial openness. High levels of financial openness may result in large cross-border asset holding, however this is not necessary. The measure of financial integration of Lane and Milesi-Ferretti presupposes some degree of financial openness and then measures countries’ financial connectedness through the size of cross-border assets and liabilities.
both have zero net foreign assets. The situation is not improved by looking at flows rather than stocks. A country can have a balanced current account and zero net capital flows because the change in assets and liabilities is zero or because changes in assets are coupled with similar changes in liabilities. Again, the former country has balanced goods trade without asset trade, while the later has balanced goods trade with extensive asset trade. In the case of either stocks or flows, net positions are insufficient to measure financial integration. Lane and Milesi-Ferretti report that the correlation between the current account and changes to net foreign assets has fallen from 0.71 to 0.33 for developed countries and 0.70 to 0.46 for emerging markets. Further, the net foreign asset position is more volatile, as a percentage of GDP, than the current account.

The measure of international financial integration Lane and Milesi-Ferretti propose is defined as

$$FI_t = \frac{(FA_t + FL_t)}{GDP_t},$$

where $FA$ and $FL$ are the stocks of external assets and liabilities, respectively. Using this measure, they make a few important observations about global trends in financial integration. First, global financial integration has been increasing for industrial and developing countries since the 1970s. In 1970 foreign assets and liabilities total about 50% of GDP for both industrial and developing countries; by 1990 this number had risen to just over 100% of GDP. Developing countries have continued further along this path of financial integration so that by 2004 foreign assets and liabilities make up about 150% of GDP. The industrialized countries, however, beginning in the mid-1990s have experienced an even more spectacular increase in financial integration with their foreign assets and liabilities growing to over 300% of GDP by 2004 on a trend that continues until the Global Financial Crisis (GFC) beginning in late 2007.

Rather than consider stocks of assets and liabilities Broner et al. (2013) study the flows of assets and liabilities. Examining changes in assets and liabilities allows them to analyze not only long-term trends but business cycle fluctuations. Broner et al. characterize gross capital flows as being pro-cyclical and large in both magnitude and volatility for a large cross-section of countries annually from the 1970s up until 2007. They also analyze various crises, both local and global, and find that sudden stops in net capital flows are due both to reductions in inflows from foreigners and retrenchment of capital (negative outflows) by domestic agents.

Forbes and Warnock (2012)\textsuperscript{15} investigate gross capital flows across a number of countries during the Global Financial Crisis. They find that net capital flows obscure what is happening to

\textsuperscript{15}Rothenberg and Warnock (2011) consider periods of gross capital flow retrenchment in countries with crises from 1987-2005 and reach similar conclusions.
gross capital flows. Most countries who experienced a surge in net capital flows were Advanced Economies. Researchers have concluded from this that during the crisis there was a flight to safety. However, for most countries receiving a net capital surge, they were also experiencing a gross capital inflow stop. The net flow surge was actually a result of gross capital outflows reversing faster than gross capital inflows were collapsing. For this reason, Milesi-Ferretti and Tille (2011) characterize the Global Financial Crisis as the "Great Retrenchment".

Contessi et al. (2013) go a step further and disaggregate gross capital flows into their component parts and analyze the dynamics of those component flows. Though their analysis excludes the Global Financial Crisis, they establish some stylized facts regarding disaggregated capital flows and find that only FDI fails to behave in a pro-cyclical manner. Aizenman et al. (2013) find that the link between capital flows is tenuous in all but gross FDI flows and that after the GFC gross debt flows are negatively related with economic growth. This line of research has considered the risks and benefits of the composition of capital flows, typically favoring foreign direct investment flows and disfavoring portfolio flows (debt and equity). Levchenko and Mauro (2007) find, for a broad cross-section of countries, that during downturns inflows of FDI are the most stable, while portfolio equity is mixed and portfolio debt falls but recovers quickly. The true culprit they identify are "other" flows which are the residual component covering trade and bank credit among other things. Other flows fall during the crises and do not recover. Tong and Wei (2010) find that during the GFC, greater dependence on portfolio capital flows worsened the credit crunch, while greater reliance on FDI dampened the downturn. Calderón and Kubota (2013) analyze sudden stops in gross flows and document how the differing determinants of gross inflow stops compare to outflow stops.

Much of the early research is empirical, theoretical considerations of gross capital flows present a particularly difficult challenge. Researchers have made great progress but much of it has been only recently. In section 1.5, I describe some of the tools that have been developed and applied to consider theories of gross capital flows. Anticipating chapter 2 of this dissertation, I also briefly describe my own contribution to the theoretical tools appropriate for gross capital flow analysis.
1.5 Theoretical Tools

Tools used to develop and test theories of capital flows have evolved over the decades. Only recently have significant innovations in the analysis of economic theory and increased computing power have created opportunities to understand gross capital flow dynamics. The primary tool of Hume was argumentation by example, and this method remained dominant for centuries. Although written arguments remain an essential tool, Mundell (1960) augments this with phase diagrams to describe the dynamics of capital flows and their relation to other macroeconomic variables. However, as these models are based on accounting identities and ad-hoc behavioral response rules, Grubel (1968) suggests applying the mean-variance portfolio choice methods of Markowitz (1952) and Tobin (1958) to international portfolios to better understand the decisions of economic agents which are driving capital flows. By describing optimizing behavior of investors in the mean-variance framework, this is an improvement over ad-hoc explanations, however, it has two drawbacks. First, mean-variance models still require exogenous asset returns, and second, they are static. Mean-variance models will be unable to explain asset prices and allocations simultaneously and cannot speak to the dynamics of asset allocation choices. Furthermore, there is no forward-looking component which is very important in financial markets.

Lucas (1978) formulates an elegant way of endogenizing asset returns in a dynamic endowment model. However, because Lucas (1978) includes only one asset there is no meaningful portfolio allocation, a critical shortcoming when considering capital flows. Building on Lucas’s asset pricing model and the real business cycle model of Kydland and Prescott (1982), Backus et al. (1992, 1994) build an international real business cycle model with multiple assets and endogenous returns. However, to simplify the analysis, Backus et al. (1992, 1994) rely on the assumption that financial markets are complete. The assumption of internationally complete financial markets is emphatically rejected by the data based on an absence of risk sharing, and it introduces a strong negative correlation between gross capital inflows and outflows, a fact which is also rejected by the data.

Optimal portfolio allocations are particularly difficult to solve for in a model with incomplete markets. In a static setting, Judd (1998) uses a bifurcation theorem related to the work of Samuelson (1975) to solve for allocations. One difficulty with portfolio choice in models with incomplete markets. In a static setting, Judd (1998) uses a bifurcation theorem related to the work of Samuelson (1975) to solve for allocations. One difficulty with portfolio choice in models with incomplete markets. In a static setting, Judd (1998) uses a bifurcation theorem related to the work of Samuelson (1975) to solve for allocations. One difficulty with portfolio choice in models with incomplete markets.

\footnote{One of the implications of complete markets is perfect risk sharing, such that agents are only subject to aggregate risk and, therefore, consumption shares between agents should be constant. See Backus and Smith (1993); Kollmann (1995) for origin of the Backus-Smith-Kollmann puzzle which clearly show that international markets are not complete.}
financial markets is first-order indeterminacy in asset allocations and prices. Even the robust method of value function iteration fails if this indeterminacy is not taken into account. To address this Heaton and Lucas (1996, 1997, 2000a,b) combine value function iteration with an “auctioneer” algorithm. This algorithm has two steps, first they guess the asset pricing functions and solve for optimal asset allocations given those prices. However, if the guess of the asset pricing function is not optimal then the allocations will not clear the market. Step two, given the allocations from step one, they solve for the optimal asset prices which clear the market. Next, they repeat the process using the new pricing functions. Although the algorithm was used widely for these type of models, it faces two problems. The first drawback is the computational time required to find the optimal allocation and pricing functions, even with advances in computational power over the previous decades it still requires enough computing time to make model calibration tedious and structural estimation of parameters impractical. Second, and related to the first, the accuracy of the solution is poor, accuracy can be improved, but it comes at the expense of extra computing time.

Devereux and Sutherland (2010, 2011) and Tille and van Wincoop (2010), working concurrently, developed very similar techniques which can be easily applied to incomplete market portfolio choice models using tools that are already familiar to economists working with dynamic stochastic general equilibrium models. This provides an enormous improvement over previous methods because the computational time required is trivial which makes calibration and estimation feasible. However, the standard tools to solve DSGE models involves a linear approximation around a non-stochastic steady-state, and applying this approach to a portfolio choice problem results in two challenges. First, in a non-stochastic world, portfolios will be indeterminate or trivial because all asset returns are certain; the only thing that differentiates them will be their rate of return. If returns are equal between assets, then any allocation of assets will be equally appealing and portfolios are indeterminate. If, on the other hand, in the steady-state, some asset has higher returns than another, the optimal portfolio will only contain the high-return asset, and the portfolio allocation choice is trivial. Of course, when returns are endogenous to the model, generally any optimal steady state will be characterized by assets whose prices adjust to equate returns and the indeterminacy of portfolios is the primary issue.
Devereux and Sutherland instead emphasize the zero-order component of the optimal portfolio. The zero-order component is the constant part of a Taylor series approximation of the true portfolio allocation\textsuperscript{17}. If you consider a state of the world where the exogenous variables are at their mean, then the zero order component of the portfolio allocation is the optimal set of assets that solve the household’s problem where expectations are formed taking into account the stochastic properties of the model. The problem with portfolio allocation is that the standard first-order optimal conditions of a portfolio choice model are necessary but not sufficient to solve the allocation. The problem of asset allocation is primarily one of risk, which is a second-order problem. From this observation, Devereux and Sutherland show that in order to solve the zero-order portfolio problem, a second-order approximation of the portfolio equations is required. More generally, to solve for the $n$-th order component of optimal portfolios requires an $n+2$ order approximation of the portfolio equations. The other equations of the model only need to be approximated to the $n+1$ order.

Deriving higher order approximations to models takes effort; nevertheless, Devereux and Sutherland’s method is straightforward, can be applied to a broad variety of models, and can generally be used to obtain approximate analytic results. However, one important caveat is that it relies on local approximation methods. When considering open economies, it is often the case that the data and models exhibit non-stationary in wealth (Schmitt-Grohé and Uribe, 2003), in which case local approximation methods may be inappropriate if the endogenous state variables do not remain close enough to their zero-order levels. Rabitsch et al. (2015) compare the local methods developed by Devereux and Sutherland (2010) to global solutions methods and find the solutions produce very similar results in all respects except for the dynamics of assets. For questions related to the dynamics of gross capital flows, solution accuracy with regard to asset allocation dynamics will be important. Furthermore, and because it is a local solution method, the Devereux and Sutherland method cannot handle models with occasionally binding constraints or non-differentiability of policy functions. The problems are exacerbated when models are asymmetric as there is not always an obvious point around which to approximate the model.

The model I develop in chapter 2 is asymmetric and has non-differentiable policy functions due to a discrete choice investors make whether or not to participate in foreign assets markets, therefore, local approximation methods are not appropriate to solve the model. The solution method I use is

\textsuperscript{17}Coeurdacier et al. (2011) use a similar concept they call the risky steady state.
based on Judd et al. (2014) who solve a multi-country model with complete markets; however, to
solve the incomplete markets problem I also combine insights from Heaton and Lucas (1996) and
Devereux and Sutherland (2010). The solution method is described in further detail in Appendix B,
but here I will briefly describe the novel components of the method which can be applied generally
to solve portfolio choice problems with non-differentiable policy functions.

How policy functions get updated in the solution algorithm is critical. Judd et al. (2014) use
Euler equations of the model as fixed points to update the policy function coefficients, a method
I adopt. Consider the generic Euler equations from the representative agents of two countries, \( h \) and \( f \), investing in an equity \( s \) which pays dividends \( d \) and is ex-dividend priced at \( q \):

\[
1 = \mathbb{E}_t \left[ M_{t+1}^h \frac{(q_{t+1} + d_{t+1})}{q_t} \right] \quad \text{(1.8)}
\]
\[
1 = \mathbb{E}_t \left[ M_{t+1}^f \frac{(q_{t+1} + d_{t+1})}{q_t} \right] \quad \text{(1.9)}
\]

where \( M_{t+1}^i = \beta M_{t+1}^i / M_{t}^i \ (i \in \{ h, f \}) \) is the stochastic discount factor.

Suppose, given an initial guess for the price \( \hat{q} \) and allocation policy functions \( \hat{s} \), where hats
(\(^\wedge\)) indicate approximations which may not be optimal, that evaluating the model using these
approximations yields Euler equation residuals \( \hat{R}_t^h = \hat{M}_{t+1}^h \frac{(\hat{q}_{t+1} + \hat{d}_{t+1})}{\hat{q}_t} \) and \( \hat{R}_t^f = \hat{M}_{t+1}^f \frac{(\hat{q}_{t+1} + \hat{d}_{t+1})}{\hat{q}_t} \), in
which case equations (1.8) and (1.9) may not hold such that \( \hat{R}_t^h \neq 1 \) and \( \hat{R}_t^f \neq 1 \). The first thing
to note is that \( \hat{R}_t^i \ (i \in \{ h, f \}) \) are fixed points of the model for the optimal solution. Second, given
the price and allocation policy functions, if \( \hat{R}_t^i > 1 \ (< 1) \) then the marginal benefit of holding a
greater quantity of the asset is greater (less) than the marginal cost for agent \( i \). In which case,
the situation may be improved by increasing the quantity held or price of the asset. Numerically,
this can be accomplished by multiplying the Euler residuals by either price or quantity to obtain
a better guess for the policy function. Simplifying, if \( \hat{s}(0) \) and \( \hat{q}(0) \) are the initial guesses for the
allocation and price of an asset and produce Euler residuals \( \hat{R}^h(0) \) and \( \hat{R}^f(0) \), then subsequent
guesses \( \hat{s}(1) = \hat{R}^h(0) \hat{s}(0) \) and \( \hat{q}(1) = \hat{R}^f(0) \hat{q}(0) \) can improve the approximation. The home agent’s
share of the asset increases because they value it above its current price, and the price is raised
because the foreign agent values the asset more than its current price. Because the method does
not require the calculation of the Jacobian, it is computationally quick, however, because price and
quantity are being changed simultaneously, a sufficiently small dampening parameter is needed for
stability which increases the required number iterations. As Judd et al. (2014) show this method
can be applied generally to a large number of countries and, as I show, can also be expanded to accommodate an arbitrary number of assets.

Beyond applying the method of Judd et al. (2014) to portfolio choice models, I also make an innovative improvement, observing that the important information in the Euler residuals is the relative values between agents. For instance, it could be the case that $\hat{R}_f = \hat{R}_h > 1$. Following the algorithm described above, the next iteration would raise the price of the asset because the foreign agent values the future returns of the asset more than the cost, but the algorithm would also increase the share of the asset going to the home agent because they value the future returns more than the current price. In fact, because the Euler residuals are equal, it does not make sense to change the allocation, both agents value future returns more than the costs of the asset, so only the asset price should be increased. Alternatively, one Euler residual could be above 1 and the other below 1, in which case, the price is currently appropriate but the allocation should be shifted to the agent with the larger Euler residual.

To adjust the algorithm for this observation, I construct allocation-share and price residuals which are still fixed points of the optimal model but take relative demands into account and are more suitable to portfolio choice problems. More specifically, I construct:

\begin{align}
\hat{R}_t^s &= \hat{R}_t^h / \hat{R}_t^f \\
\hat{R}_t^q &= \hat{R}_t^h \hat{R}_t^f
\end{align}

This formulation takes account of the relative demands for assets from each agent and is better suited to asset allocation problems.

Next, in Section 1.6, I turn attention to the data on gross capital flows and document their stylized facts.

1.6 Country-Level Capital Flows

To document recent trends in capital flows, I assemble a data set for the Group of Seven (G7) countries: Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States. The data come from the IMF’s Balance of Payments Statistics, are quarterly covering 1970Q1 through 2017Q4 and reported in nominal U.S. dollars. I consider several types of capital flows—both
inflows and outflows of foreign direct investment (FDI),\textsuperscript{18} portfolio investment in equities, portfolio investment in debt, and other capital flows.\textsuperscript{19} Data on GDP, exchange rates, and prices are from the IMF’s International Financial Statistics database and supplemented with the OECD.Stat database for missing observations.

In addition to the aggregate country-level data, I compile a set of firm-level data which describe international investment positions. This type of information tends to be proprietary and closely guarded. Therefore, the data is limited to a subset of large U.S. public pension funds with foreign asset holdings. The data are annual, and coverage varies depending on the fund with the earliest beginning in 2003 and going through 2017. The data and analysis of pension funds are discussed further in Section 1.7.

\subsection*{1.6.1 Capital Flow Summary Statistics}

To make cross-country comparisons, I follow Broner et al. (2013) and normalize the data by trend nominal GDP.\textsuperscript{20} I denote gross capital inflows from foreigners as CIF and gross capital outflows from domestic agents as COD. Figure 1.2 shows the series of total gross capital flows (CIF + COD) and net capital flows (CIF - COD). Several characteristics are consistent across the group of countries. It tends to be the case that total flows are substantially larger than net flows, this is an indication of increasing financial integration as a result of domestic agents purchasing foreign assets and foreigners purchasing domestic assets simultaneously. Table 1.3 shows the average and standard deviation of net, total, gross inflows, and gross outflows of capital for the G7 countries. It shows that the level of net capital flows has remained fairly stable over time between -3\% and 3\% of GDP. The level of net flows decreased for Canada, France, and Germany while the standard deviation remained constant. For the United Kingdom and the United States, the level of net flows increased over the period. The standard deviation of net flows increased for the United Kingdom, which was the only country to experience an increase in volatility of net flows.

However, the level and standard deviation of gross capital flows have increased over the period for all countries. The UK being a financial center is the most extreme case, the level of total capital

\textsuperscript{18}In addition to direct control or ownership, FDI also includes equity purchases which amount to 10\% of the market capitalization of a firm.
\textsuperscript{19}Other capital flows are a residual with major categories including privately held foreign currency, loans, deposits, and trade credit.
\textsuperscript{20}Calculating trend Nominal GDP is done with the Hodrick-Prescott filter, using a smoothing parameter of 1600. The capital flow data are reported in nominal U.S. dollars so that normalizing by trend nominal GDP the measures of capital flows are \%GDP.
flows over the 1970Q1-2017Q4 (1995Q1-2017Q4) period is 27% (37%) of GDP with a standard
deviation of over 50% (70%) of GDP. The UK also recorded the largest positive total capital flow
just before the global financial crisis in 2007Q1, exceeding 250% of GDP. Conversely, amid the
crisis, the UK also recorded the largest negative total capital flow amounting to -230% of GDP
for 2008Q4. At the other extreme, the largest and most diversified economy, the US had more
modest but still quite significant average total capital flows over the 1973Q1-2017Q1 (1995Q1-
2017Q4) period of 8.6% (12%) of GDP and a standard deviation of 7.5% (8.9%) of GDP. Among
the G7 countries, gross capital flows have been very large as a fraction of GDP and highly volatile,
especially when compared to net capital flows.

Table 1.3: Capital Flows: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Net</th>
<th>Total</th>
<th>Inflows</th>
<th>Outflows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>2.15</td>
<td>0.99</td>
<td>12.11</td>
<td>15.68</td>
</tr>
<tr>
<td>s.d.</td>
<td>(2.62)</td>
<td>(2.64)</td>
<td>(7.97)</td>
<td>(9.05)</td>
</tr>
<tr>
<td>France</td>
<td>0.08</td>
<td>-0.10</td>
<td>16.21</td>
<td>22.38</td>
</tr>
<tr>
<td>Germany</td>
<td>-2.49</td>
<td>-3.14</td>
<td>18.61</td>
<td>20.13</td>
</tr>
<tr>
<td>s.d.</td>
<td>(4.29)</td>
<td>(4.14)</td>
<td>(20.23)</td>
<td>(21.38)</td>
</tr>
<tr>
<td>Italy</td>
<td>-</td>
<td>0.23</td>
<td>-</td>
<td>12.55</td>
</tr>
<tr>
<td>s.d.</td>
<td>(3.18)</td>
<td>(13.91)</td>
<td>(7.45)</td>
<td>(6.80)</td>
</tr>
<tr>
<td>Japan</td>
<td>-</td>
<td>-1.77</td>
<td>-</td>
<td>7.91</td>
</tr>
<tr>
<td>s.d.</td>
<td>(2.62)</td>
<td>(10.22)</td>
<td>(4.97)</td>
<td>(5.57)</td>
</tr>
<tr>
<td>U.K.</td>
<td>1.68</td>
<td>3.05</td>
<td>27.33</td>
<td>36.51</td>
</tr>
<tr>
<td>s.d.</td>
<td>(2.18)</td>
<td>(5.33)</td>
<td>(50.79)</td>
<td>(70.41)</td>
</tr>
<tr>
<td>U.S.</td>
<td>1.97</td>
<td>3.12</td>
<td>8.56</td>
<td>11.95</td>
</tr>
<tr>
<td>s.d.</td>
<td>(2.18)</td>
<td>(2.14)</td>
<td>(7.50)</td>
<td>(8.94)</td>
</tr>
<tr>
<td>G7 (median)</td>
<td>-</td>
<td>0.23</td>
<td>20.13</td>
<td>-</td>
</tr>
<tr>
<td>s.d.</td>
<td>(2.85)</td>
<td>(13.91)</td>
<td>(7.45)</td>
<td>(6.80)</td>
</tr>
</tbody>
</table>

Table 1.3 shows the average and standard deviation of net (inflows-outflows), total (inflows+outflows), gross
inflows, and gross outflows of capital for the G7 countries as a percent of gross domestic product. The full
sample begins 1970Q1 for Canada and the United Kingdom, however, because of data limitations the starting
dates are: France-1975Q1; Germany-1991Q1; Italy-1995Q1; Japan-1996Q1; U.S.-1973Q1.

21 It is important to note that gross inflows and outflows are actually net changes in foreign assets and liabilities,
when countries are experiencing negative total capital flows it is typically because domestic agents are, on net, selling
off foreign assets and foreigners are selling domestic assets.
Figure 1.2: Total and Net Capital Flows

Figure 1.2 shows total gross capital flows (Inflows + Outflows) and net capital flows (Inflows-Outflows) for the G7 countries as a share of domestic GDP.
Figure 1.3: G7 Gross Inflows and Outflows

Figure 1.3 shows gross capital inflows (CIF) and gross capital outflows (COD) for the G7 countries as a share of domestic GDP.
1.6.2 Structural Breaks

Prior to the 2008 Global Financial Crisis, it appeared as though total capital flows might not have been stationary despite net capital flows being stable. Following the crisis, the level of capital flows diminishes; however, the variance remains large. To better understand the patterns and structural changes in the underlying parameters of the data, in particular, the mean and variance of capital flows, I analyze structural breaks in the data series. Econometricians have been considering parameter instability for quite some time.\textsuperscript{22} Testing a single known breakpoint in the mean or variance of a series is straightforward with a Student-t test or Bartlett’s test respectively. However, when there are multiple breaks with potentially unknown locations, the problem increases in complexity. For example, consider the function

\[
X_t = \begin{cases} 
-1 + \epsilon_t & \text{for } 0 < t \leq n \\
1 + \epsilon_t & \text{for } n < t \leq 2n \\
0 + \epsilon_t & \text{for } 2n < t \leq 3n 
\end{cases} \\
\epsilon_t \sim iidN(0, \sigma). 
\] (1.12)

Simulated $X_t$ according to Equation 1.12, $n = 33$, and $\sigma = .75$

Figure 1.4: Series With Two Mean Breaks and Constant Variance

It is clear from the construction of $X_t$ that there are two breaks in the mean of the series and, conditional on those breaks, constant variance. However, in stochastic processes, it is not always obvious when or how many break points there are, as Figure 1.4 demonstrates. A single F-test for the hypothesis of a change in variance at either $t = n$ or $t = 2n$ will reject the null of constant variance, even though, conditional on the change in mean, the process has constant variance. Similarly, a single Student-t test of a change in mean will be significant if conducted at around the point $t = n$ but not around $t = 2n$, whereas a test of two mean changes will identify both $t = n$

\textsuperscript{22}See Hackl and Westlund (1991) and Stock (1994) for the overview of the foundational concepts and Hackl and Westlund (1989) for an annotated bibliography covering the literature.
and $t = 2n$. If we do not know \textit{a priori} the location and number of breaks, test statistics will not give a valid measure of the likelihood of changing parameter values.

There are two ways of identifying unknown breakpoints, either batch analysis which analyzes the entire series at once, or sequential analysis which analyzes the series beginning with the first period and continuing through to the end. Batch analysis is reasonable if the break points are known or there are only a few unknown breaks. When the timing of breaks is unknown, algorithms searching for breakpoints must test every possible combination of points for structural breaks and report the most likely positions for structural parameter changes. However, this suffers from the curse of dimensionality, as the number of potential breakpoints increases, the number of possible combinations of points increases exponentially. To reduce the required number of comparisons, sequential analysis considers a series sequentially rather than the entire series at once.\textsuperscript{23} Algorithms process observations sequentially over time, as new observations are introduced, a decision about whether or not a change has occurred is based only on a comparison with previous observations. When a break is detected at time-$t$ the algorithm begins again from $t$ so that the observations prior to $t$ do not effect the estimates moving forward. Sequential analysis dramatically reduces the computational challenge of identifying structural breaks.

Considering the G7 series of net and total capital flows in Figure 1.2, the mean and variance of the net capital flow series appear to be reasonably stable, while the same parameters for total flows do not appear to be stable over time. During the 1990s there is an increase in both the level and variance of total capital flows, however, following the 2008 GFC, mean total flow levels drop for several countries, but the variance remains large. The pattern described indicates that the rate of financial integration has slowed since the GFC. Prior to the GFC total flows were large and positive meaning that investors were expanding external assets and liabilities, but following the GFC the accumulation of foreign assets and liabilities is lower. Similar to Davis and van Wincoop (2018) who argue that the increasing correlation between inflows and outflows is only possible due to substantial foreign assets and liabilities, I suspect that the large and increasing variance of capital flows is only possible when stocks of foreign assets and liabilities are large. If stocks of foreign assets and liabilities are small, it is less likely that sales and purchases will be large, so that volatility will

\textsuperscript{23}Banerjee et al. (1992) use a sequential method to identify a single unknown breakpoint, Lumsdaine and Papell (1997) consider two unknown breakpoints, and Kapetanios (2005) uses a sequential method to identify an arbitrary number, $m$, breakpoints.
also be low. Therefore, because stocks of foreign assets and liabilities are large, even though the level of flows diminishes after the GFC, the variance remains high. The cause of the slow down in the rate of financial integration coincident with persistently high volatility of gross capital flows following the GFC is an open question which is beyond the scope of this chapter.

To identify structural breaks in the parameters, I use the sequential algorithm of Ross (2015). The G7 median series for total capital flows has many structural breaks for both the level and variance. Figure 1.5 plots the series for total capital flows as a share of GDP with vertical lines marking the likely changes in the mean and variance of the series. Shifts in the mean level of total capital flows occur at 1978-Q3, 1982-Q1, 1996-Q3, and 2008-Q2. Shifts in the variance occur at 1978-Q4, 1993-Q4, and 2004-Q4. Thus, even though the mean level of total flows had a significant decline following the Global Financial Crisis in 2008, the variance remains high. By contrast, analyzing the G7 series of net capital flows reveals no shifts in the mean or variance, indicating stability in those parameters.

Table 1.4 identifies likely shifts in the mean and variance for net, total, inflows, and outflows for each of the G7 countries. At the country level, as opposed to the median country, changes to the mean of net flows occur frequently. On the other hand, the country level data and the median country have relatively similar breaks for gross capital flows (total, inflows, and outflows). This is an indication of a global financial cycle at the gross capital flow level. Net flow parameters are

\[24\text{See Rey (2015)}.\]
stable for the median G7 country because the countries are switching relative rank order over time, indicating a lack of positive correlation between net flows in each country.\textsuperscript{25} However, considering gross flows, countries are moving together with little change in rank order, so that country dynamics are similar to the median country’s dynamics. Except for Italy and Japan, each country has a change in mean level and variance of gross flows in the late 1990s. France, the U.K., and the U.S. have a change in the mean level of gross capital flows during early 2008, and only the U.S. has a shift in variance after the year 2000.

Table 1.4: Capital Flows: Dating Structural Breaks

<table>
<thead>
<tr>
<th></th>
<th>Net Mean</th>
<th>Net Std Dev</th>
<th>Total Mean</th>
<th>Total Std Dev</th>
<th>Inflows Mean</th>
<th>Inflows Std Dev</th>
<th>Outflows Mean</th>
<th>Outflows Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2008Q3</td>
<td></td>
<td>1981Q4</td>
<td>1999Q2</td>
<td>2005Q4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2001Q3</td>
<td></td>
<td>2008Q2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2008Q2</td>
</tr>
<tr>
<td>Italy</td>
<td>2013Q1</td>
<td>2007Q4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2008Q2</td>
</tr>
<tr>
<td>Japan</td>
<td>2003Q1</td>
<td>2003Q1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>2004Q1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2011Q1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2007Q2</td>
<td></td>
<td>2008Q1</td>
<td>2008Q1</td>
<td>2008Q1</td>
<td>2009Q2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2000Q1</td>
<td></td>
<td>2008Q1</td>
<td>2005Q4</td>
<td>2005Q4</td>
<td>2001Q2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2008Q4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G7 (median)</td>
<td>-</td>
<td>-</td>
<td>1978Q3</td>
<td>1978Q4</td>
<td>1979Q2</td>
<td>1978Q3</td>
<td>1978Q3</td>
<td>1978Q3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1982Q1</td>
<td>1993Q4</td>
<td>1982Q1</td>
<td>2000Q3</td>
<td>1996Q3</td>
<td>1993Q4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1996Q3</td>
<td>2004Q4</td>
<td>1996Q3</td>
<td>2008Q2</td>
<td>2004Q4</td>
<td>2008Q2</td>
</tr>
</tbody>
</table>

Table 1.4 shows the estimated dates for structural breaks in the mean and standard deviation for net, total, inflows, and outflows of capital for the G7 countries.

\textsuperscript{25} Aside from measurement error, the median or average net flow should be zero globally, so it is only marginally surprising that the median net flow for the G7 is near zero and stable over time.
1.6.3 Capital Inflows and Outflows

The downward shift in the level of gross capital flows since 2008 indicates a reduction in the rate of global financial integration as investors are, on average, accumulating foreign assets and liabilities at a much slower pace. Broner et al. (2013) and Davis and van Wincoop (2018) suggest that many of the gross capital flow patterns observed in the data are a result of the rate of accumulation of foreign assets and liabilities. To test whether these patterns are the result of the rate of accumulation or the level of foreign assets and liabilities, I estimate several parameters to test the relationship between various measures of capital flows before and after 2008. If the rate of external asset and liability accumulation is important for gross capital flow properties, then the period prior to 2008 should show stronger patterns and post-2008 should be weaker. However, if the level of foreign assets and liabilities is more important, then the two periods should be similar.

The strong positive relationship between gross capital inflows and outflows is clear from inspecting Figure 1.3. Table 1.5 presents the simple contemporaneous correlation between gross capital inflows and outflows for each of the G7 countries for different time periods. The data for each country were filtered using the Hodrick-Prescott (HP) filter with smoothing parameter 1600. For each country, the correlation between inflows and outflows was weakest, albeit still strong and positive, during the earliest period of observation from 1970-1994. The relationship between inflows and outflows appears to strengthen in the subsequent years to an average of 0.94. To evaluate the apparent relationship more formally, I estimate the following regressions:

\[
CIF_{c,t} = \alpha_c + \gamma_t + \beta COD_{c,t} + \epsilon_{c,t} \tag{1.13}
\]

\[
COD_{c,t} = \alpha_c + \gamma_t + \beta CIF_{c,t} + \epsilon_{c,t} \tag{1.14}
\]

where each country is allowed to have different intercepts, capturing unobserved differences between countries, and time-specific global effects which capture unobserved factors affect all countries at each point in time.

The omission of individual fixed effects in equations (1.13) or (1.14) leads to biased and inconsistent estimates of the relationship between inflows and outflows. If the time-effects are statistically significant, then ignoring them will also lead to biased estimates (Baltagi (2008) p. 34). For each regression, I test the significance of including either the time fixed effects or both individual and time fixed effects using the Lagrange Multiplier test of Breusch and Pagan (1980). In all cases...
the individual and time fixed effects are statistically significant at $p < 0.0001$. Therefore, in the panel data set, there are important unobserved individual country influences and time specific effects which need to be taken into account to estimate the relationship between capital inflows and outflows.

Table 1.5: Capital Inflow and Outflow Correlation

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.70</td>
<td>0.91</td>
<td>0.96</td>
<td>0.93</td>
</tr>
<tr>
<td>France</td>
<td>0.94</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>Germany</td>
<td>0.68</td>
<td>0.93</td>
<td>0.98</td>
<td>0.96</td>
</tr>
<tr>
<td>Italy</td>
<td>-</td>
<td>0.96</td>
<td>0.91</td>
<td>0.93</td>
</tr>
<tr>
<td>Japan</td>
<td>-</td>
<td>0.89</td>
<td>0.88</td>
<td>0.89</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.93</td>
<td>0.99</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>United States</td>
<td>0.79</td>
<td>0.89</td>
<td>0.93</td>
<td>0.92</td>
</tr>
<tr>
<td>G7</td>
<td>0.80</td>
<td>0.94</td>
<td>0.95</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Table 1.5 reports simple correlations of Hodrick-Prescott filtered ($\lambda = 1600$) gross capital inflows and outflows for the individual G7 countries and their average at quarterly frequency for different time periods.

Table 1.6 shows the results of the estimated effect of capital inflows (CIF) on outflows (COD) and vice-versa even taking into account global unobserved effects which may be related to the global financial cycle. There is a strong positive link between gross capital inflows and outflows, and this is evident in Figure 1.3 and the reason why volatility of total capital flows exceeds net flows. When inflows and outflows are strongly positively correlated, the sum of inflows and outflows (total) will exceed the difference (net) of inflows and outflows. In light of the previous section documenting the structural shifts in the mean and variance of capital flows, I estimate the relationship between inflows and outflows for four different periods 1970Q1-1994Q4, 1995Q1-2007Q4, 2008Q1-2017Q4, and 1995Q1-2017Q4. The earliest period shows the weakest relationship between inflows and outflows; a 1% increase in capital outflows is associated with a 0.89% increase in capital inflows. The R-squared for this regression is also the smallest, only 0.78. The relationship between inflows and outflows is stronger for the more recent period. From 1995Q1-2017Q4, a 1% increase in capital outflows is associated with a 0.97% increase in capital inflows. This regression has a remarkably large R-squared of 0.936. The size of this effect is similar before and after the Global Financial Crisis, even though the rate of financial integration fell considerably following the GFC. The continuing strong positive correlation between inflows and outflows is evidence that the strong connection is
due more to the level of foreign assets and liabilities, and not the rate of accumulation of those assets and liabilities.

Table 1.6: Capital Inflow and Outflow Relationship

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CIF = $\beta$COD(a)</td>
<td>0.892***</td>
<td>0.988***</td>
<td>0.984***</td>
<td>0.970***</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.014)</td>
<td>(0.019)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>COD = $\beta$CIF(b)</td>
<td>0.872***</td>
<td>0.953***</td>
<td>0.935***</td>
<td>0.966***</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.014)</td>
<td>(0.018)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>Country FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Time FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>No. Countries</td>
<td>5</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>No. Obs.</td>
<td>384</td>
<td>360</td>
<td>280</td>
<td>1024</td>
</tr>
<tr>
<td>R-squared (a)</td>
<td>0.778</td>
<td>0.942</td>
<td>0.920</td>
<td>0.936</td>
</tr>
<tr>
<td>R-squared (b)</td>
<td>0.778</td>
<td>0.942</td>
<td>0.920</td>
<td>0.936</td>
</tr>
</tbody>
</table>

Table 1.6 shows panel regressions for the relationship between gross capital inflows and outflows for G7 countries during different time periods. Each regression includes country and time fixed effects. Robust standard errors are clustered at the country level and reported in parentheses. "∗∗∗", "∗∗", "∗", "◦" represents statistical significance at the $p < \{0.001, 0.01, 0.05, 0.1\}$ level respectively.

I next consider the disaggregated gross capital flows. The International Monetary Fund (IMF) classifies purchases of foreign assets (or sales of domestic assets) several ways.\textsuperscript{26} One form of capital flow is foreign direct investment (FDI) in which case the purchaser tends to have a controlling interest in the venture. FDI could result when a multinational corporation sets up a factory abroad, or even when an entity makes a large purchase of debt or equity. FDI is assumed to have a lasting or long-term relationship and so not only are the initial investment resources counted as a capital flow, but the data also count subsequent transactions between the investor and enterprise. For instance, home country claims on retained earnings that remain with the foreign-located enterprise are considered a gross capital outflow from the original investor’s country. Likewise, any repatriated profits from the foreign enterprise will be regarded as a negative capital outflow from the perspective of the origin (investing) country, as they are effectively selling off their foreign assets. Furthermore, to be counted as FDI, investments do not need to be full ownership or control. In addition to setting up or outright purchase of foreign enterprises, the IMF assumes that purchasing substantial equity (>10%) also counts as FDI as they expect that at this level of ownership, the investor will have some degree of long-term interest and some control.

\textsuperscript{26}See IMF (2005) for full details of the classification of capital flows.
In contrast to FDI, the IMF assumes portfolio investment is more short-term and without a controlling interest. Portfolio investment is further disaggregated into equity securities and debt securities. Equity investment covers assets to which the owner is entitled to residual claims to an enterprise after the claims of creditors have been satisfied. Mutual fund and investment trust ownership are also counted as an equity investment capital flow. Portfolio debt investment includes bonds, money market or negotiable debt instruments, or derivatives used for trading, hedging, or investment. Although this includes asset backed securities which may contain mortgages, mortgages held outright are not considered portfolio debt but credit and are put in the final category of capital flows.

The final\textsuperscript{27} component of disaggregated capital flows is the residual, simply called "other" flows. Even though this category captures all the other flows which are not represented by FDI and portfolio investment, it is a very significant source of gross capital flows. The largest components of the other investment classification are trade credit, loans, and currency and deposits. Trade credit is a typical part of doing business when importing and exporting and allows firms to bypass traditional financial intermediaries when gaining and extending credit. Trade credit accounts for between 5-20\% of export sales (Rajan and Zingales, 1995; Petersen and Rajan, 1997). Loans are created through direct lending and may finance trade or other commercial activity, create mortgages, financial leases, or repurchase agreements. Currency consists of any coins or notes that are in common circulation. Finally, there are deposits which may be anything from retail bank deposits, time deposits, or shares in credit unions, savings and loan associations, and building societies.

To better understand the relationship between inflows and outflows I want to know the relationship between different types of gross capital inflows (outflows) and aggregate gross inflows (outflows). For instance, when a country is receiving FDI, debt, or equity inflows, does this still produces large outflows? Are some types of capital flows more important than others for the gross inflow-outflow relationship documented in Table 1.5? I evaluate the relationship between gross capital flows and their disaggregated counterparts with the panel data set controlling for individual

\textsuperscript{27}Official international reserves also are counted in gross capital flows. However, they are not privately held and for advanced economies contribute little to the stylized properties of gross capital flows.
Table 1.7: Capital Inflow and Disaggregated Outflows

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>FDI&lt;sub&gt;out&lt;/sub&gt;</td>
<td>0.881***</td>
<td>1.01***</td>
<td>1.11***</td>
<td>0.969***</td>
</tr>
<tr>
<td></td>
<td>(0.195)</td>
<td>(0.045)</td>
<td>(0.087)</td>
<td>(0.042)</td>
</tr>
<tr>
<td>Eq&lt;sub&gt;out&lt;/sub&gt;</td>
<td>0.522**</td>
<td>1.11***</td>
<td>0.857***</td>
<td>0.932***</td>
</tr>
<tr>
<td></td>
<td>(0.177)</td>
<td>(0.062)</td>
<td>(0.091)</td>
<td>(0.055)</td>
</tr>
<tr>
<td>Debt&lt;sub&gt;out&lt;/sub&gt;</td>
<td>0.906***</td>
<td>0.933***</td>
<td>0.791***</td>
<td>0.792***</td>
</tr>
<tr>
<td></td>
<td>(0.081)</td>
<td>(0.049)</td>
<td>(0.061)</td>
<td>(0.039)</td>
</tr>
<tr>
<td>Oth&lt;sub&gt;out&lt;/sub&gt;</td>
<td>0.898***</td>
<td>0.984***</td>
<td>1.03***</td>
<td>1.00***</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.017)</td>
<td>(0.021)</td>
<td>(0.014)</td>
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<td>Yes</td>
<td>Yes</td>
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<td>Time FE</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>No. Countries</td>
<td>5</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>No. Obs.</td>
<td>384</td>
<td>360</td>
<td>280</td>
<td>1024</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.782</td>
<td>0.942</td>
<td>0.927</td>
<td>0.939</td>
</tr>
</tbody>
</table>

Table 1.7 shows panel regressions for the relationship between gross capital inflows and the four disaggregated components of gross capital outflows, foreign direct investment (FDI), portfolio equity (Eq), portfolio debt (Debt), and other (Oth), for G7 countries during different time periods. Each regression includes country and time fixed effects. Robust standard errors are clustered at the country level and reported in parentheses. "***", "**", "*", "◦" represents statistical significance at the p < {0.001, 0.01, 0.05, 0.1} level respectively.

and time effects by estimating:

\[
CIF_{c,t} = \alpha_c + \gamma_t + \beta_1 FDI_{c,t}^{out} + \beta_2 EQ_{c,t}^{out} + \beta_3 Debt_{c,t}^{out} + \beta_4 Oth_{c,t}^{out} + \epsilon_{c,t} \tag{1.15}
\]

\[
COD_{c,t} = \alpha_c + \gamma_t + \beta_1 FDI_{c,t}^{in} + \beta_2 EQ_{c,t}^{in} + \beta_3 Debt_{c,t}^{in} + \beta_4 Oth_{c,t}^{in} + \epsilon_{c,t} \tag{1.16}
\]

The estimation results of equation (1.15), the relationship between gross capital inflows and disaggregated capital outflows, are reported in Table 1.7. All of the coefficients in every period are positive and statistically significant, suggesting that increases in each class of foreign asset type are associated with an increase in sales of domestic assets. A 1% of GDP increase in FDI investment abroad is associated with about a 1% of GDP increase in gross capital inflows; this was slightly lower from 1970-1994. However, based on the standard error of the estimates, it is unlikely statistically that the estimates are different. For external equity investment during the 1970-1994 period, a 1% of GDP increase in equity outflows was matched with only a 0.52% increase in gross capital inflows. The link was much stronger during the period up until the GFC, 1995-2007, with an increase in equity outflows being associated with a 1.11% increase in gross capital inflows. A 1% of GDP increase in debt outflows is associated with about a 0.9% of GDP increase in capital inflows.
up until the GFC. After the GFC there is only a 0.79% increase in capital inflows associated with increased foreign debt investment. Finally, the link between increases in other capital outflows and gross capital inflows is fairly strong. A 1% of GDP increase in other capital outflows has about a 0.9% increase in gross capital inflows associated with it from 1970-1994, this is higher at around 1% in the post-1994 period.

Table 1.8: Capital Outflow and Disaggregated Inflows

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>FDI&lt;sub&gt;in&lt;/sub&gt;</td>
<td>0.746&lt;sup&gt;***&lt;/sup&gt;</td>
<td>0.822&lt;sup&gt;***&lt;/sup&gt;</td>
<td>0.435&lt;sup&gt;***&lt;/sup&gt;</td>
<td>0.727&lt;sup&gt;***&lt;/sup&gt;</td>
</tr>
<tr>
<td>Eq&lt;sub&gt;in&lt;/sub&gt;</td>
<td>0.864&lt;sup&gt;*&lt;/sup&gt;</td>
<td>0.935&lt;sup&gt;***&lt;/sup&gt;</td>
<td>0.612&lt;sup&gt;***&lt;/sup&gt;</td>
<td>0.861&lt;sup&gt;***&lt;/sup&gt;</td>
</tr>
<tr>
<td>Debt&lt;sub&gt;in&lt;/sub&gt;</td>
<td>0.603&lt;sup&gt;***&lt;/sup&gt;</td>
<td>0.914&lt;sup&gt;***&lt;/sup&gt;</td>
<td>0.831&lt;sup&gt;***&lt;/sup&gt;</td>
<td>0.812&lt;sup&gt;***&lt;/sup&gt;</td>
</tr>
<tr>
<td>Oth&lt;sub&gt;in&lt;/sub&gt;</td>
<td>0.910&lt;sup&gt;***&lt;/sup&gt;</td>
<td>0.971&lt;sup&gt;***&lt;/sup&gt;</td>
<td>0.947&lt;sup&gt;***&lt;/sup&gt;</td>
<td>1.00&lt;sup&gt;***&lt;/sup&gt;</td>
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<tr>
<td>Country FE</td>
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<td>Yes</td>
</tr>
<tr>
<td>Time FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<td>No. Countries</td>
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<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>No. Obs.</td>
<td>384</td>
<td>360</td>
<td>280</td>
<td>1024</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.793</td>
<td>0.943</td>
<td>0.931</td>
<td>0.941</td>
</tr>
</tbody>
</table>

Table 1.8 shows panel regressions for the relationship between gross capital outflows and the four disaggregated components of gross capital inflows, foreign direct investment (FDI), portfolio equity (Eq), portfolio debt (Debt), and other (Oth), for G7 countries during different time periods. Each regression includes country and time fixed effects. Robust standard errors are clustered at the country level and reported in parentheses. "**", "*" represents statistical significance at the p < {0.001, 0.01, 0.05, 0.1} level respectively.

The estimation results of equation (1.16), the relationship between gross capital outflows and component inflows, are reported in Table 1.8. The overall connection between capital inflow components and gross capital outflows is similar to the reverse direction of flows reported in Table 1.7. Each component inflow is positively associated with an increase in gross capital outflows, and these results are statistically significant. A 1% of GDP increase in FDI inflows is associated with 0.75% of GDP increase in capital outflows during the 1970-1994 period, but only a 0.44% of GDP increase in since the GFC. Similarly, equity inflows had a stronger association with gross capital outflows prior to the GFC. A 1% of GDP increase in equity inflows as matched with a 0.86% increase in capital outflows from 1970-1994 and only a 0.61% of GDP increase since the GFC. By contrast, the effect of debt inflows has a stronger effect on capital outflows since the GFC. A 1% of GDP
increase in debt inflows increased capital outflows by only 0.6% of GDP from 1970-1994; however, since the GFC the effect has been 0.83% of GDP. The effect size for other inflows on gross capital outflows is large and stable over time. Before 1995, a 1% of GDP increase in other capital inflows is met with a 0.91% of GDP increase in gross capital outflows. Following the GFC, a 1% of GDP increase in other inflows is associated with a 0.94% of GDP gross capital outflow.

The link between gross capital outflows and inflows is undeniable despite controlling for individual differences, different time spans, and the global cycle captured by time fixed effects. Furthermore, regardless of the component gross capital flow, an increase in a component inflow (outflow) is associated with an increase in gross capital outflows (inflows). As a final check, I consider whether there is a link between inflows and outflows of the individual components of gross capital flows. From the G7 panel data, I estimate the following equations

\[ X_{in} = \alpha_c + \gamma_t + \beta X_{out} + \epsilon_{c,t} \]  
\[ X_{out} = \alpha_c + \gamma_t + \beta X_{in} + \epsilon_{c,t} \]  
\[ X \in \{FDI, Equity, Debt, Other\} \]

As shown throughout this paper, there is a clear connection between capital inflows and outflows and this is not unique to any type of capital flow, be it FDI, equity, debt, or other flows. This is evidence pointing to a global financial cycle, at the same point in time that a country is buying (selling) foreign assets, foreigners are buying (selling) domestic assets. To further shed light on this phenomenon, I test to see if there is a connection between a specific type of capital inflow (outflow) and its counterpart outflow (inflow). That is if a country invests more in FDI, does it receive more FDI, does more foreign equity investment mean more domestic equity sales to foreigners, et cetera.

The results of the estimation are presented in Table 1.9. There does appear to be a weak reciprocal relationship between FDI inflows and outflows. Regardless of the period, a country which invests 1% of GDP more in FDI is expected to simultaneously receive about 0.2% of GDP more in FDI inflows. Nevertheless, modeling FDI inflows and outflows as the only drivers of FDI explains very little of the overall variance in FDI flows, with R-squared values of less than 0.05 there are likely other important determinants of FDI.\(^{28}\) The fact that there is a statistically significant

\(^{28}\)There is an extensive literature on the factors influencing FDI, see Blonigen (2005). Although the explanations highlighted in this literature may be consistent with the FDI inflow-outflow correlation, most ignore the dynamics of FDI.
relationship at all is surprising, of all the categories of flows it is assumed that FDI is the most sluggish. Further, because FDI often involves very long-term investments, it seems least likely to be influenced by short-term fluctuations in a way that would simultaneously influence the decisions of domestic firms investing abroad and foreign firms investing in the home country.

Portfolio investments, whether equity or debt follow a different pattern. Increases in investment in foreign equity markets are associated with a decline in sales of domestic equities. Presumably, if opportunities are good enough in some country ‘B’ that country ‘A’ wants to buy more of B’s equities, then other countries may also want to hold more of B’s equities and relatively fewer of A’s equities. This pattern does not hold in the period following the GFC where there is no statistically significant relationship between equity inflows and outflows. Debt flows exhibit the same negative relationship as equity flows during the 1970-1994 period. However, the pattern reverses for the subsequent years 1995-2017, so that more recently it has also been the case that an increase in foreign debt investment coincides with more debt sales to foreigners. Like FDI, the debt and equity inflow-outflow relationship is small in magnitude and explains very little of the overall variance in debt and equity flows.

The results for other flows stand in stark contrast to those for FDI, equity, and debt. It is unfortunate that other flows are the residual of gross capital flows because the relationship between other inflows and outflows is strong and large in magnitude. In the period 1995-2017, a 1% of GDP increase in other outflows (inflows) is associated with a 0.98% (0.79%) of GDP increase in other inflows (outflows). Other flows are primarily made up of trade-credit and banking assets/liabilities. Although trade credit is an essential tool for international trade (Petersen and Rajan, 1997; Maksimovic, 2001; Fisman and Love, 2003), it likely lacks the size and variation to account for the strong relationship between other inflows and outflows.

On the other hand, banks may be the main contributor to the phenomenon. The proliferation and expansion of large banks across the world and the nature of bank balance sheet management create a nexus between international assets and liabilities creation. This explanation is consistent with the evidence Davis (2015) finds when examining data on bilateral bank flows, namely that inflows and outflows of bank capital flows are highly positively correlated even when considering country pair relationships. Furthermore, Brunnermeier et al. (2012) and Brunnermeier and Sannikov (2014) show how banks’ international assets and liabilities adjust with a nearly one-to-one
relationship and that these changes are particularly pro-cyclical as, during boom times, when the available supply of deposits become scarce, banks will turn to other types of borrowing in order to continue expanding their balance sheets.

Table 1.9: Disaggregated Capital Inflows and Outflows

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$FDI_{in} = \beta FDI_{out}$ (a1)</td>
<td>0.202***</td>
<td>0.145**</td>
<td>0.163**</td>
<td>0.171***</td>
</tr>
<tr>
<td></td>
<td>(0.066)</td>
<td>(0.045)</td>
<td>(0.057)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>$FDI_{out} = \beta FDI_{in}$ (a2)</td>
<td>0.161**</td>
<td>0.233**</td>
<td>0.211**</td>
<td>0.283***</td>
</tr>
<tr>
<td></td>
<td>(0.052)</td>
<td>(0.072)</td>
<td>(0.073)</td>
<td>(0.054)</td>
</tr>
<tr>
<td>$Eq_{in} = \beta Eq_{out}$ (b1)</td>
<td>-0.111***</td>
<td>-0.358***</td>
<td>-0.005</td>
<td>-0.180***</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.063)</td>
<td>(0.047)</td>
<td>(0.041)</td>
</tr>
<tr>
<td>$Eq_{out} = \beta Eq_{in}$ (b2)</td>
<td>-0.422***</td>
<td>-0.268***</td>
<td>-0.011</td>
<td>-0.198***</td>
</tr>
<tr>
<td></td>
<td>(0.114)</td>
<td>(0.047)</td>
<td>(0.091)</td>
<td>(0.043)</td>
</tr>
<tr>
<td>$Debt_{in} = \beta Debt_{out}$ (c1)</td>
<td>-0.242***</td>
<td>0.208**</td>
<td>0.239**</td>
<td>0.229***</td>
</tr>
<tr>
<td></td>
<td>(0.067)</td>
<td>(0.067)</td>
<td>(0.073)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>$Debt_{out} = \beta Debt_{in}$ (c2)</td>
<td>-0.183***</td>
<td>0.149**</td>
<td>0.185**</td>
<td>0.172***</td>
</tr>
<tr>
<td></td>
<td>(0.051)</td>
<td>(0.048)</td>
<td>(0.056)</td>
<td>(0.037)</td>
</tr>
<tr>
<td>$Oth_{in} = \beta Oth_{out}$ (d1)</td>
<td>0.809***</td>
<td>0.871***</td>
<td>0.974***</td>
<td>0.976***</td>
</tr>
<tr>
<td></td>
<td>(0.042)</td>
<td>(0.030)</td>
<td>(0.036)</td>
<td>(0.023)</td>
</tr>
<tr>
<td>$Oth_{out} = \beta Oth_{in}$ (d2)</td>
<td>0.708***</td>
<td>0.839***</td>
<td>0.779***</td>
<td>0.794***</td>
</tr>
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<td></td>
<td>(0.037)</td>
<td>(0.029)</td>
<td>(0.029)</td>
<td>(0.018)</td>
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<td>Yes</td>
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<tr>
<td>Time FE</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>No. Countries</td>
<td>5</td>
<td>7</td>
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</tr>
<tr>
<td>No. Obs.</td>
<td>384</td>
<td>360</td>
<td>280</td>
<td>1024</td>
</tr>
<tr>
<td>R-squared (a1, a2)</td>
<td>0.033</td>
<td>0.034</td>
<td>0.034</td>
<td>0.048</td>
</tr>
<tr>
<td>R-squared (b1, b2)</td>
<td>0.047</td>
<td>0.096</td>
<td>0.000</td>
<td>0.037</td>
</tr>
<tr>
<td>R-squared (c1, c2)</td>
<td>0.044</td>
<td>0.031</td>
<td>0.044</td>
<td>0.039</td>
</tr>
<tr>
<td>R-squared (d1, d2)</td>
<td>0.572</td>
<td>0.731</td>
<td>0.759</td>
<td>0.775</td>
</tr>
</tbody>
</table>

Table 1.9 shows panel regressions for the relationship between gross capital inflows and the four disaggregated components of gross capital outflows, foreign direct investment (FDI), portfolio equity (Eq), portfolio debt (Debt), and other (Oth), for G7 countries during different time periods. Each regression includes country and time fixed effects. Robust standard errors are clustered at the country level and reported in parentheses. "***", "**", "]", and "◦" represent statistical significance at the $p < \{0.001, 0.01, 0.05, 0.1\}$ level respectively.

1.6.4 Cyclical Properties of Capital Flows

Finally, to better understand the dynamics of capital flows, I document their cyclical properties. Several others also investigate the cyclical properties of gross capital flows and find that gross capital
flows tend to be pro-cyclical (Yeyati et al., 2007; Broner et al., 2013; Contessi et al., 2013; Cerutti et al., 2017). However, most of these studies end their analysis before the GFC and are often use annual data to cover a wider cross-section of countries. Therefore, the purpose of this section is to see if the patterns remain the same following the GFC and to see if higher frequency data impact the estimates.

To establish the stylized facts, I first calculate the simple contemporaneous correlations between domestic GDP and gross capital flows for each of the G7 countries and compute the average for all countries. I consider two measures of the business cycle, the HP-filtered ($\lambda = 1600$) real-GDP series and log-difference of real-GDP, both measured in national currency units. For capital flows, I consider gross capital inflows from foreigners (CIF), gross capital outflows from domestic agents (COD), and total gross flows (CIF+COD). I restrict the time-period to 1995Q1-2017Q4 both because of data availability and due to the shift in capital flow properties documented in subsection 1.6.2. The results are reported in Table 1.10 and for each country and both business cycle measures, capital inflows and outflows are procyclical. This correlation suggests that when the domestic country experiences an expansion, foreigners invest more in the domestic economy and domestic agents invest more abroad.

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Canada</th>
<th>France</th>
<th>Germany</th>
<th>Italy</th>
<th>Japan</th>
<th>U.K.</th>
<th>U.S.</th>
<th>G7</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIF-Y</td>
<td>0.11</td>
<td>0.10</td>
<td>0.26*</td>
<td>0.10</td>
<td>0.35***</td>
<td>0.22*</td>
<td>0.26*</td>
<td>0.20</td>
</tr>
<tr>
<td>CIF-ΔY</td>
<td>0.05</td>
<td>0.27**</td>
<td>0.29**</td>
<td>0.07</td>
<td>0.35***</td>
<td>0.39***</td>
<td>0.18°</td>
<td>0.23</td>
</tr>
<tr>
<td>COD-Y</td>
<td>0.20°</td>
<td>0.12</td>
<td>0.25*</td>
<td>0.08</td>
<td>0.30**</td>
<td>0.18°</td>
<td>0.17°</td>
<td>0.19</td>
</tr>
<tr>
<td>COD-ΔY</td>
<td>0.05</td>
<td>0.31**</td>
<td>0.33**</td>
<td>0.12</td>
<td>0.24*</td>
<td>0.42***</td>
<td>0.22°</td>
<td>0.24</td>
</tr>
<tr>
<td>Total-Y</td>
<td>0.16</td>
<td>0.11</td>
<td>0.26°</td>
<td>0.10</td>
<td>0.33**</td>
<td>0.20°</td>
<td>0.22*</td>
<td>0.20</td>
</tr>
<tr>
<td>Total-ΔY</td>
<td>0.05</td>
<td>0.29**</td>
<td>0.31**</td>
<td>0.10</td>
<td>0.30**</td>
<td>0.41***</td>
<td>0.21*</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Table 1.10 shows contemporaneous correlation between real GDP fluctuations measured and gross capital flows for the G7 countries. Real GDP fluctuations are measured in national currency and are either HP-filtered ($\lambda = 1600$) or the log-differenced. Gross capital flows are measured as either gross inflows (CIF), gross outflows (COD), or Total flows (CIF+COD). "***", "**", "*", "°" represents statistical significance at the $p < \{0.001, 0.01, 0.05, 0.1\}$ level respectively.

To test this result, I estimate a panel model to evaluate the effect of the state of the domestic economy on inflows, outflows, or total flows of capital measured as a percentage of trend-GDP.

\(^{29}\)Davis (2015) shows using bilateral capital flows from banks that the cyclicality of flows is actually due to crises and not domestic GDP fluctuations. However, the data are limited and a proxy for capital flows, with only a moderate positive correlation with gross capital flows.
I control for country differences and time effects and use robust standard errors clustered at the country level. I use the same two measures of GDP, namely HP-filtered real GDP and the log-difference of real GDP, both in national currency units. The formal model I estimate is

\[ X_{c,t} = \alpha_c + \gamma_t + \beta Y_{c,t} + \epsilon_{c,t} \]  

(1.19)

\[ X \in \{CIF, COD, Total\} \]

\[ Y \in \{rGDP_{HP}, \Delta \ln rGDP\} \]

The results of the cyclical properties of capital flows is presented in Table 1.11. Regardless of the measure of capital flows, business cycle or period, capital flows are acyclical. For the G7 countries the conditions of the local economy, whether considering levels or growth rates, appears not to affect either the percentage of GDP that is used to purchase foreign assets or the purchase of domestic assets by foreigners. The main reason for this result is that unobserved global factors are driving gross capital flows. Gross capital flows in all countries are moving together in a much stronger way than economic output is, so the time effects, \( \gamma_t \), in equation (1.19) are large in magnitude and highly statistically significant.

To show the reason for this null result when others have documented a strong statistical link between the domestic economy and gross capital flows, I plot the time-effects from (1.19) with 95% confidence interval presented in Figure 1.6. The dependent variable is Total gross capital flows from 1995-2017 and the independent variable is HP-filtered real GDP. This specification corresponds to the penultimate column of Table 1.11, however the pattern of time-effect estimates are similar for each specification in Table 1.11. The plot shows that the unobserved time effects are large in magnitude and highly statistically significant especially in the late 1990s through the global financial crisis. Controlling for these global effects, the variation in real GDP of domestic economies has no statistically significant effect on the size or direction of gross capital flows.

I also consider a model that includes country-specific time-trends but not time-dummies. The model in equation (1.19) includes time-dummies which account for shifts in the level of capital flows across all countries, and these parameters capture global factors which are important for the dynamics of capital flows. An alternative specification uses country-specific time trends\(^{30}\)

\(^{30}\)Using country time-trends is the specification estimated by Broner et al. (2013) shows procyclical gross capital flows with annual data up until the global financial crisis. The period before the GFC did appear to have country-specific time trends in capital flows as gross flows were rising exponentially and the retrenchment that occurred at
### Table 1.11: Gross Capital Flows Cyclical Estimation

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Y_{HP}$</td>
<td>$\Delta \ln Y$</td>
<td>$Y_{HP}$</td>
<td>$\Delta \ln Y$</td>
<td>$Y_{HP}$</td>
<td>$\Delta \ln Y$</td>
<td>$Y_{HP}$</td>
<td>$\Delta \ln Y$</td>
</tr>
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<td>CIF</td>
<td>0.25</td>
<td>0.11</td>
<td>0.63</td>
<td>−0.24</td>
<td>0.92</td>
<td>1.76</td>
<td>0.83</td>
<td>2.48</td>
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<tr>
<td>s.d. (a)</td>
<td>(0.25)</td>
<td>(0.88)</td>
<td>(0.24)</td>
<td>(3.28)</td>
<td>(1.31)</td>
<td>(3.65)</td>
<td>(0.87)</td>
<td>(2.71)</td>
</tr>
<tr>
<td>COD</td>
<td>−0.34</td>
<td>−0.27</td>
<td>0.24</td>
<td>0.46</td>
<td>0.15</td>
<td>2.15</td>
<td>0.31</td>
<td>3.55</td>
</tr>
<tr>
<td>s.d. (b)</td>
<td>(0.25)</td>
<td>(0.87)</td>
<td>(0.92)</td>
<td>(3.22)</td>
<td>(1.28)</td>
<td>(3.56)</td>
<td>(0.87)</td>
<td>(2.70)</td>
</tr>
<tr>
<td>Total</td>
<td>−0.09</td>
<td>−0.16</td>
<td>0.87</td>
<td>0.22</td>
<td>1.07</td>
<td>3.91</td>
<td>1.14</td>
<td>6.03</td>
</tr>
<tr>
<td>s.d. (c)</td>
<td>(0.49)</td>
<td>(1.70)</td>
<td>(1.85)</td>
<td>(6.45)</td>
<td>(2.57)</td>
<td>(7.14)</td>
<td>(1.72)</td>
<td>(5.36)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>CIF</td>
<td>0.003</td>
<td>0.000</td>
<td>0.001</td>
<td>0.000</td>
<td>0.002</td>
<td>0.000</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>s.d. (a)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>COD</td>
<td>0.007</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.002</td>
<td>0.000</td>
<td>0.003</td>
</tr>
<tr>
<td>s.d. (b)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Total</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>s.d. (c)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
</tbody>
</table>

Table 1.11 reports the estimated relationship between real GDP fluctuations and gross capital flows for the G7 countries. Real GDP fluctuations are measured as either HP-filtered ($\lambda = 1600$) or the log-difference. Gross capital flows are measured as either gross inflows (CIF), gross outflows (COD), or Total flows (CIF+COD). Each regression includes country and time fixed effects. Robust standard errors are clustered at the country level and reported in parentheses. "∗∗∗", "∗∗", "∗", "◦" represents statistical significance at the $p < \{0.001, 0.01, 0.05, 0.1\}$ level respectively.

The results of the estimation for equation (1.20) are reported in Table 1.12 and tell a different story. For this specification, both gross inflows and outflows appear to be procyclical with a high degree of statistical significance. Again, this is due to misspecification as the estimation does not account for the global financial cycle. The country time-trend estimates, $\gamma_c$, are small in magnitude and, except for the United Kingdom, statistically indistinguishable from zero. Even though it is statistically different from zero, the effect size of the U.K.’s time-trend is about three orders of magnitude smaller than the effect size of GDP.

The time of the crisis was absent from the data. The data following the GFC reveal near stationary time-trends for each country, and thus country-specific trends are not necessary. However, time-dummies which account for changes in global dynamics are important and help to differentiate the effects of domestic economies and global influences.
Figure 1.6: Total Capital Flows Time Fixed-Effects

Figure 1.6 Shows the estimated time fixed-effects and 95% confidence interval for Equation (1.19), where the independent variable is Total capital flows and the dependent variable is HP-filtered (λ = 1600) real GDP for the G7 countries during the period 1995Q1-2017Q4.

The stark contrast in the estimates between equations (1.19) and (1.20) reveal the important quantitative impact of unobserved time effects which are affecting capital flows in all countries and may be due to spurious regression.\textsuperscript{31} As a final test to evaluate the cyclical properties, I estimate the first-difference (FD) model. As Wooldridge (2010)\textsuperscript{32} observes the fixed effect (FE) and first difference models are identical for \( T = 2 \) and should be similar for \( T > 2 \), however, the efficiency of the two models varies depending on the serial correlation of the residuals. The FE estimator is more efficient when the residuals are serially uncorrelated and the FD estimator is more efficient when the residuals follow a random walk. Formally, I estimate:

\[
\Delta X_{c,t} = \alpha_c + \beta \Delta Y_{c,t} + \Delta \epsilon_{c,t}
\]

\( X \in \{ \text{CIF, COD, Total} \} \)

\( Y \in \{ \text{rGDP}_{HP}, \Delta \ln \text{rGDP} \} \)

where, because of differencing, the country specific intercept term is equivalent to a time trend. The results of the FD estimation are reported in Table 1.13 and emphatically reject any relationship between capital flows and domestic GDP. This finding contributes to the growing literature considering the importance of a global financial cycle (Bruno and Shin, 2015; Miranda-Agrippino

\textsuperscript{31}See Granger and Newbold (1974).
\textsuperscript{32}See \textit{Chapter 10.7.1}.
Table 1.12: Gross Capital Flows Cyclical Estimation Without Time Fixed Effects

<table>
<thead>
<tr>
<th>Year Range</th>
<th>Y_{HP}</th>
<th>Δ ln Y</th>
<th>Y_{HP}</th>
<th>Δ ln Y</th>
<th>Y_{HP}</th>
<th>Δ ln Y</th>
<th>Y_{HP}</th>
<th>Δ ln Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970:1-1994:4</td>
<td>0.50***</td>
<td>−0.78</td>
<td>1.54**</td>
<td>3.80</td>
<td>1.12°</td>
<td>9.10***</td>
<td>2.41***</td>
<td>9.99***</td>
</tr>
<tr>
<td>s.d.</td>
<td>(0.15)</td>
<td>(0.62)</td>
<td>(0.59)</td>
<td>(2.78)</td>
<td>(0.68)</td>
<td>(2.53)</td>
<td>(0.49)</td>
<td>(1.97)</td>
</tr>
<tr>
<td>1995:1-2007:4</td>
<td>0.21</td>
<td>−1.20*</td>
<td>1.47*</td>
<td>3.56</td>
<td>0.81</td>
<td>9.62***</td>
<td>2.24***</td>
<td>10.88***</td>
</tr>
<tr>
<td>s.d.</td>
<td>(0.15)</td>
<td>(0.61)</td>
<td>(0.59)</td>
<td>(2.79)</td>
<td>(0.67)</td>
<td>(2.49)</td>
<td>(0.49)</td>
<td>(1.96)</td>
</tr>
<tr>
<td>2008:1-2017:4</td>
<td>0.70*</td>
<td>−1.98°</td>
<td>3.01*</td>
<td>7.36</td>
<td>1.93</td>
<td>18.72***</td>
<td>4.65***</td>
<td>20.87***</td>
</tr>
<tr>
<td>s.d.</td>
<td>(0.29)</td>
<td>(1.19)</td>
<td>(1.18)</td>
<td>(5.54)</td>
<td>(1.34)</td>
<td>(4.97)</td>
<td>(0.97)</td>
<td>(3.91)</td>
</tr>
<tr>
<td>1995:1-2017:4</td>
<td>0.135</td>
<td>0.121</td>
<td>0.225</td>
<td>0.214</td>
<td>0.053</td>
<td>0.094</td>
<td>0.105</td>
<td>0.112</td>
</tr>
</tbody>
</table>

Table 1.12 reports the estimated relationship between real GDP fluctuations and gross capital flows for the G7 countries. Real GDP fluctuations are measured as either HP-filtered (λ = 1600) or the log-difference. Gross capital flows are measured as either gross inflows (CIF), gross outflows (COD), or Total flows (CIF+COD). Each regression includes country fixed effects and country time-trend. Robust standard errors are clustered at the country level and reported in parentheses. "***", **", *", and °" represent statistical significance at the p < {0.001, 0.01, 0.05, 0.1} level respectively.

and Rey, 2015; Rey, 2015; Banerjee et al., 2016; Cerutti et al., 2017; Borio, 2017; Coimbra and Rey, 2017). I next consider firm-level foreign investment decisions and their impact on aggregate measures.

### 1.7 Firm-Level Data

To better understand the dynamics of capital flows, I compile data from four U.S. public pension funds including California Public Employees Retirement System (CalPERS); California State Teachers’ Retirement System (CalSTRS); Florida Retirement System Trust Fund (FRS); and Teacher Retirement System of Texas (TRS). These funds were selected because of their size and based on the availability of the data. In general, financial institutions are reluctant to reveal asset holding positions either to maintain the secrecy of successful strategies or to conceal failures which may damage the prestige and confidence in the firm’s leadership. However, public pension
Table 1.13: Gross Capital Flows Cyclical Estimation FD Estimator

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Y_{HP}$</td>
<td>$\Delta \ln Y$</td>
<td>$Y_{HP}$</td>
<td>$\Delta \ln Y$</td>
</tr>
<tr>
<td>CIF (a)</td>
<td>-0.01</td>
<td>-0.82</td>
<td>0.96</td>
<td>4.67</td>
</tr>
<tr>
<td>s.d.</td>
<td>(0.37)</td>
<td>(0.63)</td>
<td>(1.65)</td>
<td>(3.25)</td>
</tr>
<tr>
<td>COD (b)</td>
<td>-0.12</td>
<td>-0.97</td>
<td>1.12</td>
<td>5.70</td>
</tr>
<tr>
<td>s.d.</td>
<td>(0.35)</td>
<td>(0.61)</td>
<td>(1.62)</td>
<td>(3.18)</td>
</tr>
<tr>
<td>Total (c)</td>
<td>-0.13</td>
<td>-1.77</td>
<td>2.08</td>
<td>10.37</td>
</tr>
<tr>
<td>s.d.</td>
<td>(0.68)</td>
<td>(1.21)</td>
<td>(3.24)</td>
<td>(4.72)</td>
</tr>
<tr>
<td>Co.</td>
<td>5</td>
<td>5</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>N</td>
<td>384</td>
<td>381</td>
<td>360</td>
<td>359</td>
</tr>
<tr>
<td>$R^2$ (a)</td>
<td>0.000</td>
<td>0.004</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>$R^2$ (b)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$R^2$ (c)</td>
<td>0.000</td>
<td>0.006</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 1.13 reports the estimated relationship between real GDP fluctuations and gross capital flows for the G7 countries using the first difference model. Real GDP fluctuations are measured as either HP-filtered ($\lambda = 1600$) or the log-difference. Gross capital flows are measured as either gross inflows (CIF), gross outflows (COD), or Total flows (CIF+COD). Robust standard errors are clustered at the country level and reported in parentheses. "**", "*", "\#" represents statistical significance at the $p < \{0.001, 0.01, 0.05, 0.1\}$ level respectively.

Funds are often subject to more stringent reporting and "Sunshine Laws" making it possible to get a clearer picture of investment decisions at very sophisticated firms. Furthermore, these firms also tend to have relatively large holdings of foreign assets, although in aggregate U.S. investors have disproportionately large holdings of domestic assets, more sophisticated investors tend to have less home bias (Hau and Rey, 2008). Funds hold various foreign asset classes including publicly traded and private equities, corporate debt, government debt, real estate, and real estate backed assets, currency, and futures contracts.

For public pension funds, there tend to be additional constraints and broad goals which research tends to overlook when considering general models of investment decisions. For instance, in the latest round of revisions in 2013, the CalPERS Board of Administration adopted ten investment beliefs which are meant to be a guide to and commitment of the fund to certain principles which may be at odds with each other. To some degree these principles trade-off high returns and lower risk in favor of other goals; Investment Belief 3 states that "CalPERS investment decisions may reflect wider stakeholder views, provided they are consistent with its fiduciary duty to members and beneficiaries." Although this statement is vague and open to interpretation, in practice, it means
that CalPERS does not invest in certain industries, such as tobacco. It also means a reallocation of investments away from fossil fuel industries and into renewables, in spite of the impact this may have on the portfolio risk and return. These goals have an impact on international asset holdings as well; Investment Belief 6 states "Strategic asset allocation is the dominant determinant of portfolio risk and return." This sort of principle is common for large diversified funds; they typically have broad asset allocation goals for different asset classes and frequently rebalance the portfolio to maintain those goals within a few percentage points. These types of goals also tend to apply to the relative size of domestic and foreign assets. Therefore, it is possible that a fund that is pre-committed to a certain asset allocation may forgo profitable investment opportunities to maintain the overall allocation goal. However, on occasion discretion is exercised regarding portfolio allocation targets. For instance, the FRS Annual Investment Report acknowledges that "[t]he investment strategy for the Pension Plan is to implement the policy allocation within relatively narrow bands around the policy target weights... one rebalance was triggered May 7, 2013, but was not executed. The rebalance would have shifted $729.8 million from Global Equity to Fixed income, but adverse expectations for Fixed Income returns made such a move unattractive." (Pg. 28). The result of this significantly increased the share of foreign assets in the portfolio, an outcome that persists through 2016, the final year of reporting for this fund in the data set. Had the fund remained consistent with their targets and rebalanced the portfolio, this would have been recorded as a large negative gross capital flow for the United States and would have represented a staggering 0.5% of GDP. The U.S. has a standard deviation of gross outflows of 4.3% of GDP, and although FRS management did not take this action, it is easy to imagine that individual firm-level decisions like this can have a significant impact on aggregate statistics.

1.7.1 Data Description

The data available from each fund varies on many dimensions and there is more information available about each fund than is relevant for this investigation. CalPERS is the largest public pension fund in the United States with $375 billion in assets as of 2018. The data contains all domestic and foreign equity positions at fiscal year-end (June) for 2010 through 2016 including the number of shares, book value, and market value. Additional asset positions are given for domestic cash equivalents, foreign currency, asset backed securities, corporate bonds, sovereign bonds,
U.S. treasuries and agency debt, mortgage loans and securities, international debt, alternative investments, inflationary linked assets, domestic real estate, domestic real estate investment trusts (REITs), and international REITs. For 2005 through 2016, the data also contain disaggregate international holdings by currency-denomination and asset class. Asset classes of foreign holdings are equities, alternative investments, debt, real estate, forward contracts, and currency. Among the data collected, CalPERS has the greatest granularity concerning asset positions, the comparable measures of other funds are only at a more highly aggregated level.

CalSTRS is the second largest public pension fund with $229.2 billion in assets as of 2018. The data series for CalSTRS is annual at fiscal year-end (June) from 2012 through 2017. The data for CalSTRS include the overall asset allocation shares between cash, equities, debt, alternative investments, real estate, and inflation-sensitive investments. Cash holdings include domestic currency as well as other highly liquid assets, and also includes foreign currency holdings. For equities, data include the relative share of domestic and international equities as well as the currency-denomination of foreign equities. Additionally, foreign-denominated debt, spot, and forward contracts are known by currency-denomination.

FRS is the 5th largest U.S. pension fund with $163.6 billion in assets as of 2018. The series for FRS is annual and reported at fiscal year-end (June) from 2004 through 2017. The FRS does not manage their investments themselves, but rely on asset managers. From the information on asset managers, it is possible to get a measure of the proportion of equities investments that are in domestic and foreign markets, however, unlike the other funds, the data do not reveal the specific countries of investment.

TRS is the 6th largest U.S. pension fund with $153.1 billion in assets of 2018. The data for TRS is reported annually at fiscal year-end (August) from 2006 through 2017. TRS reports the overall shares of various asset classes including, domestic equities, international equities, debt, alternative investments, and short-term investments (i.e., cash). Foreign-currency denominated assets are disaggregated at the currency level and across asset classes: debt, equity, alternative investments, cash, forward contracts, and commingled funds. Data on forward contracts is only available for 2006.

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33 Alternative investments tend to be high-risk non-marketable investments sometimes called private equity, such as distressed assets and corporate restructurings.
34 Data for CalPERS foreign currency forward contracts is missing for 2007-2009.
35 Commingled funds contain a mixture of equity and fixed income assets.
1.7.2 Home Bias in Equities

Even though large public pension funds hold substantial foreign assets, they tend to maintain a relatively large share of domestic assets. French and Poterba (1991) observe that on average, investors hold a disproportionately large percentage of local assets relative to foreign ones, even though it is widely known that a properly diversified portfolio can reduce risk without reducing the expected return. Pension funds internalize this by creating specific asset allocation targets; the effect of these targets is that large pension funds tend to be less home-biased than the average investor. Many measures of home bias compare actual holdings of domestic assets to some ideal level of domestic asset holdings. Typically this involves showing that a different asset allocation which includes more foreign assets would produce portfolio returns with less volatility and with the same return. However, the way that the ideal portfolio varies depending on the assumptions of the authors. All sorts of frictions and other considerations may exist which influence the ideal portfolio. For instance, a naïve ideal allocation could be that a portfolio should contain the same fraction of domestic assets as the share of domestic asset market to the global asset market. Other factors such as transaction costs, inflation, exchange rate risk, or asymmetric information can influence the ideal portfolio allocation. It is beyond the scope of this paper to commit to an ideal asset allocation for these funds so I will adopt a non-ideal measure of home equity bias equal to the share of domestic equities in total equity holdings. By this measure French and Poterba (1991) find that the U.S. holds about 91% domestic equities. For the years 2010-2016, the average year-end value of domestic equities to total equities for CalPERS was 0.51 or about half domestic and half foreign equities. For 2012-2017 the typical share of domestic equities was 0.65 for CalSTRS, for FRS from 2004-2017 it was 0.56, and for TRS from 2006-2017, it was 0.53. These shares are all lower than current estimates of aggregate U.S. home-bias, however, they vary significantly over time.

Figure 1.7 shows the annual share of domestic equities as a proportion of overall equity holdings. For FRS and TRS before 2008, domestic equities represented over 70% of equity holding; however, the share drops precipitously beginning in 2008 so that by 2011 the share is about 45%. A similar drop occurs later with the CalSTRS portfolio going from a value of about 70% in 2012 down to about 54% by 2017. Overall these three funds hold a much greater share of foreign equities in 2017 than they did previously, furthermore, the size of equity portfolios is growing over this period, so the

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36See Coeurdacier and Rey (2013).
absolute increase in foreign equities is amplified. For CalPERS the pattern is different, home-bias
is around 60% in 2006, falls to just under 50% in 2010, and rises to just over 50% by 2017.

Figure 1.8 shows the number of foreign currencies that funds are exposed to through their
equity portfolios. This measure is a proxy for the number of countries that funds are holding
assets in. There are two reasons why this measure differs from the actual number of countries a
firm is invested in. (1) The countries in the Euro-zone have a common currency and (2) some
foreign assets are denominated in dollars. TRS is the only fund which reports the value of foreign
equities denominated in U.S. dollars and in 2006 this accounted for only about 8% of foreign equity
holdings. The fact that the Euro-zone countries are aggregated together will mean that the actual
number of countries that each fund invests in is likely substantially higher than what is reported
here. However, because the Euro-zone countries are advanced economies, it seems plausible that
the funds were invested in each one of them throughout the entire period and thus this aggregation
does not affect the variation over time.

In the mid-2000s CalPERS was increasing the number of countries they were exposed to, this
CalSTRS follows a pattern of slight increases in the number of countries they are invested in from
2012 through 2017. For CalSTRS, this steady rise masks underlying volatility, for 2013, 2014, and
2015 they simultaneously entered some markets and exited from others. TRS follows a different pattern; in 2009 there is a massive push to diversify into more international markets which increases their exposure to 13 new markets. The number of new markets moderates until 2013-2014 where they again increase their exposure to an even greater number, which is steady afterward. For TRS this jump in diversification coincides with the fall in home bias documented in Figure 1.7, which is in contrast to the steadiness of CalSTRS exposure to foreign markets event though there is a large decline in home bias. CalPERS, on the other hand, has constant exposure but increasing home bias.

![Figure 1.8: Number of Foreign Equity Markets](image)

Number of foreign equity markets in which funds’ are invested.

1.7.3 Estimating the Cost of Investing Abroad

I show in Chapter 2 that an international portfolio choice model is consistent with the stylized capital flow facts if there are fixed costs associated with participating in foreign asset markets. To evaluate this assumption empirically, I consider the cost structure of three pension funds for which I observe the foreign currency exposure of their equity positions, namely CalPERS, CalSTRS, and TRS. Large pension funds vary in the proportion of their portfolio which they actively manage, leaving a large part of equity investment to firms and managers outside the fund. For this reason, I use the fees paid to external asset managers as a proxy for the total cost of participating in foreign
asset markets. It is typical for individual external managers/firms to have compensation structures which are linked both to the size of assets being managed and incentivized based on the investment returns. To consider the relative contributions of fixed and variable costs to participating in foreign asset markets I estimate the effects on external manager costs from the number of countries a fund is invested in, the overall value of the international portfolio, and the returns to that portfolio over the previous year. More formally, I estimate

\[ \log(\text{Ext.Mgmt.Fees}_{i,t}) = \alpha_i + \beta(\text{No.Co.}_{i,t}) + \gamma \log(\text{Int'l Assets}_{i,t}) + \delta(\text{Int'l Return}_{i,t}) + \epsilon_{i,t} \]  

(1.22)

The results of the estimation are reported in Table 1.14 and show that a fund investing in an additional country is associated with a 20% increase in the cost of external investment managers. Over the sample, the average fund paid external managers about 121 million per year, or equivalent to 38 basis points on an average international portfolio of $35 billion. Although a 20% increase in external management costs associated with investing in a new country sounds like a lot, it amounts to only about 6.8 basis points when compared with the average portfolio of international assets. The size of these costs is highly statistically significant, reasonably sized, and quantitatively consistent with the calibrated fixed costs of the model developed in Chapter 2.

The association of external management costs and the size of the portfolio is negative. However, the estimate is not statistically significantly different than zero; this means that, in the sample, external management costs are not scaling with the size of the total portfolio. It is relatively common for investment management firms to charge a percentage of assets under management. One possibility for this null result is that because these pension funds are so large, they can negotiate favorable contracts with external management firms which do not charge a fee for assets under management. Lastly, a 1% higher return to the international portfolio is associated with 1.3% higher external management costs; this result is statistically significant at the \( p < 0.05 \) level and makes intuitive sense. Funds writing contracts which pay higher amounts to investment managers when returns are higher helps to align the incentives of the funds and investment management firms.

Finally, to test whether these costs are participation costs or entry-exit costs, I estimate an additional specification which includes the change in the number of markets as a measure of entry and exit. If costs are participation type costs which must be paid to be in a market, then the
Table 1.14: Pension Cost Structure

<table>
<thead>
<tr>
<th></th>
<th>log(Ext Mgmt Fees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>No. Int’l Markets</td>
<td>0.202***</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
</tr>
<tr>
<td>log(Int’l Assets)</td>
<td>−0.396</td>
</tr>
<tr>
<td></td>
<td>(0.613)</td>
</tr>
<tr>
<td>Int’l Returns</td>
<td>0.013*</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
</tr>
<tr>
<td>Δ Int’l Markets</td>
<td>−0.015</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
</tr>
<tr>
<td>Fixed Effect</td>
<td>Yes</td>
</tr>
<tr>
<td>Time Effect</td>
<td>Yes</td>
</tr>
<tr>
<td>No. Funds</td>
<td>3</td>
</tr>
<tr>
<td>No. Obs.</td>
<td>23</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.846</td>
</tr>
</tbody>
</table>

This table shows the panel regressions for the relationship between pension fund’s (log) external manager fees and the number of international markets they are invested in, the (log) value of international assets, and investment returns on international assets. Each regression has a fund specific mean. Standard errors are reported in parentheses. "∗∗∗", "∗∗", "∗", "◦" represents statistical significance at the \( p < \{0.001, 0.01, 0.05, 0.1\} \) level respectively.

number of markets a firm is invested in will have an important impact on their costs. However, if the real cost is a setup or entry-exit cost, then the change in the number of markets a fund is invested in will be an important driver of costs. The results in Table 1.14 show that including the change in international markets does not have a statistically significant relationship with external manager costs and it does not change the estimates for other variables.

Though the data are limited and represent only a small fraction of total foreign asset holdings for the U.S., nevertheless, decisions at the firm level can have an impact on aggregate gross capital flow statistics. The decision of fund managers to invest in a country is a discrete choice, and although firms are always invested in large advanced economies such as the U.K and Japan, they do choose to enter and exit asset markets in more peripheral economies. Furthermore, the increase in costs is associated with participation in foreign equity markets, not variable costs associated with portfolio size or entry and exit costs for new markets.
1.8 Conclusion

This paper provides stylized facts for gross capital flows for the G7 countries, which includes the period following the Global Financial Crisis and is measured at a quarterly frequency. Consistent with what other researchers have found with a broader set of countries at annual frequencies, gross capital inflows and outflows are strongly positively correlated with each other and highly volatile. This relationship has been strengthening over time even in the post-GFC period where the level of gross capital flows has fallen. Although gross capital inflows and outflows are positively correlated with the real domestic economy, controlling for unobserved global time fixed effects shows that the relationship between the domestic business cycle and either gross inflows or outflows is tenuous. I show that, controlling for global movements, additional variation of the domestic economy is unrelated to gross inflows and outflows. Global factors are a more important determinant of gross capital flow dynamics than local ones.

I also provide data and insights on firm-level international investment decisions for several large U.S. public pension funds. I show that firms do not just vary the size of their international portfolios, but also vary in the countries that they are invested. The discrete choice of the number of countries to invest in is positively related to firms' investment costs. I characterize these costs as participation costs and show that they are not variable costs that scale with the size of the portfolio, nor are they entry-exit costs which go away after a fund has entered a new market.

The stylized facts and evidence I present in this paper are generally quite strong and yet inconsistent with standard international portfolio choice models; this poses a puzzle for researchers. Although there may be several possible explanations, in Chapter 2, I develop a model of international portfolio investment which includes fixed participation costs for holding foreign assets. The model matches several of the key statistical moments of capital flows documented in this chapter and is also consistent with the firm-level evidence. Firm-level data is often proprietary and the micro-data presented here is very limited. Future research directions should be directed to compiling more data with longer time horizons. Public pension funds may be amenable to this effort because they are often subject to transparency laws which require them to comply with reasonable requests for information. A larger data set will be useful to confirm or conflict with the findings presented here on the structure of costs.
CHAPTER 2

FIXED COSTS AND GROSS CAPITAL FLOW DYNAMICS

2.1 Introduction

International capital flows have dramatically increased in both size and volatility in most countries since the 1970’s. Early research focused on net capital inflows, documenting their pro-cyclical tendency and demonstrating how sudden and dramatic changes in capital flows can be destabilizing (Calvo et al., 1993, 1996; Dornbusch et al., 1995; Fernandez-Arias, 1996; Calvo, 1998; Kaminsky et al., 1998, 2004). The pro-cyclicality of net capital inflows is intuitively appealing. Higher productivity attracts capital from foreign investors and allows agents in the domestic economy to finance investment without sacrificing current consumption. Conversely, during recessions, domestic and foreign investors move capital abroad. This fact is precisely captured by the workhorse international real business cycle (IRBC) model of Backus et al. (1994) who show this is the case when physical capital is mobile.1

However, while the size of net inflows has remained fairly stable for many decades, an increase in financial integration led to the explosive growth of gross capital flows. This observation has prompted additional recent research focusing specifically on gross capital flows. Gross capital inflows refer to the acquisition of domestic assets by foreign investors; gross capital outflows refer to the acquisition of foreign assets by domestic agents. Net capital inflows are defined as the difference between gross inflows and outflows. Broner et al. (2013) document the dynamics of gross capital flows showing that inflows and outflows are strongly positively correlated, pro-cyclical, and both larger and more volatile than net flows. This poses a problem for the models which successfully explain net capital flows. While the data show that during expansions foreign agents invest more in the domestic economy, which is consistent with IRBC models, the domestic agents also invest more abroad, contrary to standard model predictions.

1In the absence of physical capital, net capital flows become counter-cyclical as they primarily serve the role of insurance, moving current resources to countries experiencing adverse shocks for consumption smoothing purposes.
To explain the dynamics of capital flows, I build a dynamic general equilibrium model with portfolio choice and fixed costs of investing abroad. One benefit of international investment is diversification. However, if investors have to pay a fixed cost to participate in the foreign asset market (for instance, to gather information about foreign investment opportunities) then they will only do so if the expected value of diversification exceeds the value of the fixed cost. Because the marginal utility of consumption is highest during bad times and lowest in good times, agents will be the least willing to pay these costs after adverse shocks. Therefore, they will exit the foreign asset market and only re-enter after conditions improve, leading to pro-cyclical gross outflows. Coincidentally, when domestic agents liquidate their foreign assets, they sell them to foreign agents. Foreigners finance these purchases through sales of their claims to domestic assets, producing large negative gross inflows for the domestic economy. As conditions in the economy improve, the process reverses resulting in pro-cyclical gross inflows as well.

The model presented in this paper uses a two-country two-good DSGE framework with trade in equities and incomplete markets. Each economy is endowed with two stochastic sources of income, both in the form of a country-specific tradable good. One income stream can be collateralized into equity-type claims and traded internationally. The second source of income cannot be collateralized (e.g. labor income), therefore, agents in each economy have sole claim to it. Contrary to Lucas (1982), where the number of tradable assets matches the number of sources of stochastic variation in endowments, here the presence of non-financial income results in incomplete financial markets and thus uninsured consumption risk. Consumption is a composite of domestic and foreign goods with imperfect substitutability. The key contribution of this paper is the addition of a fixed cost paid each period by agents in the home country in order to participate in the foreign equity market.

Solving portfolio choice problems with incomplete markets is not trivial even in the absence of fixed participation costs. Recent advances have employed perturbation methods to solve various forms of these models. However, the introduction of fixed costs creates two additional computational difficulties. First, the objective function of the household is non-differentiable because it has a kink at the point where agents switch from investing abroad to only at home. Second, because the benefits of diversification are quantitatively small, the solution accuracy around the kink is of principal importance. I resolve these difficulties by using a Smolyak sparse-grid projection algo-
rithm with Chebyshev polynomials, this provides high accuracy and is well suited to problems with non-differentiable objective functions. To my knowledge, this is the first DSGE model to be solved with incomplete market portfolio choice and fixed participation costs.

2.1.1 Related Literature

Despite being the first dynamic model to consider portfolio choice with fixed costs, this paper relates to contemporary theoretical models investigating the properties of gross capital flows. I build on two literatures: (i) the static cost-based portfolio home-bias literature and (ii) the gross capital flows literature.

The portfolio home-bias literature is vast and focuses on the fact that across countries and time, domestic agents tend to hold a disproportionately large share of domestic assets. This observation is particularly surprising since Baxter and Jermann (1997) emphasize that domestic financial and non-financial incomes tend to be correlated. Consequently, foreign assets could provide a good hedge for domestic non-financial income and there should actually be foreign bias in financial assets.

Early explanations for equity home bias focused on transactions costs. These costs asymmetrically affect foreign assets and could be fixed or proportional costs, differences in tax treatments, or other policies that make owning assets in the domestic economy more expensive for foreign investors vis-à-vis domestic investors. However, using a mean-variance framework, French and Poterba (1991) conclude that these costs would need to be unreasonably large in order to induce the degree of home bias observed in the data. Moreover, Tesar and Werner (1995) postulate that if transactions costs are the reason for home bias then this would imply that turnover should be lower for foreign assets; however, the opposite is true.

Amadi and Bergin (2008) challenge the conclusion that high turnover of foreign assets precludes a transaction-cost explanation of home bias. They use a mean-variance framework to show that a combination of variable and fixed costs can explain both home bias and high turnover of foreign assets. Nevertheless, numerically the transaction costs still need to be unrealistically large. To generate their results, Amadi and Bergin use variable costs ranging between 100 to 300 basis points

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3See Lewis (1999) and Coeurdacier and Rey (2013) for comprehensive reviews of the home-bias literature.

4Heathcote and Perri (2013) show how real exchange rate fluctuations provide a good hedge against non-collateralizable labor income risk, justifying equity home bias.
and fixed foreign investment costs equivalent to $0.4\% - 0.7\%$ of income. Costs of this magnitude have been rejected as unreasonably large (French and Poterba, 1991; Cooper and Kaplanis, 1994).

In a static model fixed costs will need to be very large to deter agents from investing abroad because an agent who chooses not to participate in the foreign equity market will essentially be excluded forever. In a dynamic model the decision to invest abroad is made every period; agents who forgo investment in foreign equities in the current period may always re-enter in the future. The dynamic choice means that fixed costs do not need to be so large to have a significant impact on capital flows. This paper is unique in its application of fixed costs to study capital flow dynamics. It joins just a few other papers which attempt to address gross capital flow dynamics.

The empirical regularities of gross capital flows demonstrate a clear pattern (Lane and Milesi-Ferretti, 2001, 2007; Gourinchas, 2011; Milesi-Ferretti and Tille, 2011; Obstfeld, 2012; Broner et al., 2013; Davis, 2015): inflows and outflows are large, volatile, and pro-cyclical. With few exceptions this tends to be the case for high-income, low-income, large and small countries. However, the tendencies are most strongly observed for high-income countries. Three papers closely related to this one try to address these with theoretical contributions.

Tille and van Wincoop (2014) use a model of asymmetric information between countries combined with private information among investors within countries. They show how this combination causes gross capital inflows and outflows to be highly volatile and positively correlated. They do not report on the cyclicality of gross capital flows. However, if domestic agents have better information about domestic assets, then when times are good and agents receive positive signals about future asset returns they will be confident about high returns at home and uncertain about returns abroad. The response will be to move capital home during good times and the opposite during bad times. In contrast with the evidence, this would imply counter-cyclical gross capital flows.

Gourio et al. (2016) is a thorough empirical work that documents the relationship between stock market volatility and gross capital flows for 26 emerging market countries. They observe that political risk and stock market volatility are correlated and use a model of exogenous sovereign expropriation to explain capital flow patterns. However, the empirical regularities in capital flows are strongest for developed counties and though expropriation risk may be a concern for investing in emerging markets, it is unlikely to be important for developed countries. They do not report the cyclicality of gross flows, however, the expropriation is a random variable and proportional to
the portfolio size, so it is likely similar to proportional costs which reduce the expected return of foreign assets held by domestic agents.

Finally, for a subset of developing countries Walsh (2016) applies a small open economy model with partial sovereign default. Again, the mechanism is similar to Gourio et al. except that the likelihood and size of the expropriation is endogenously determined. Rather than measuring capital flows relative to trend-GDP, Walsh normalizes flows by wealth. This normalization complicates the interpretation of cyclicality because wealth is a function of assets, prices, and current-GDP. Nevertheless, the model matches the cyclicality of gross inflows and outflows for developing countries. As with the explanation found in Gourio et al., developed country default is extremely rare in the post-war period so it is probably not what is responsible for the patterns observed in the largest and wealthiest economies.

2.2 Model

This section introduces a model of optimal portfolio choice with fixed costs which must be paid each period in order to invest abroad.

2.2.1 Description

The model is a discrete-time endowment economy with two countries, home \((h)\) and foreign \((f)\). Endowments are country-specific non-storable consumption goods. A representative agent in each country maximizes utility which is time-separable with constant relative risk aversion:

\[
U(C_t) \equiv \sum_{\tau=t}^{\infty} E_t \left[ \theta(C^1_{\tau}) \frac{C^{1-\gamma}_{\tau} - 1}{1 - \gamma} \right]
\]

\[
U(C^*_t) \equiv \sum_{\tau=t}^{\infty} E_t \left[ \theta(C^*_t) \frac{C^{1-\gamma}_{\tau} - 1}{1 - \gamma} \right]
\]

It is well known that if markets are incomplete and the discount factor is constant then models of this type will have a unit root in wealth dynamics. Without stationarity in wealth, local solution methods fail because there is not a steady-state level of wealth about which to approximate the solution. Global methods use a predetermined grid and so will require bounds on wealth resulting in extrapolation errors. While it is possible to accomplish this with an ad-hoc constraint on net foreign assets, instead I follow Schmitt-Grohé and Uribe (2003) who propose several different endogenous
discount factors which induce stationarity. For simplicity I assume that this discount factor is a function of aggregate consumption, $\bar{C}$, borrowing the functional form from Devereux and Sutherland (2009, 2011) such that $\theta(\bar{C}) = \beta \bar{C}^{-\eta}$.

2.2.2 Consumption Goods and Prices

Aggregate consumption is a CES combination of home and foreign goods. Where home country aggregate consumption is:

$$C_t = \left[ a^{\frac{1}{\theta}} c_{h,t}^{\frac{\theta-1}{\theta}} + (1-a) c_{f,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{2.3}$$

c_{h,t}$ is the home agent’s consumption of home goods and $c_{f,t}$ the home consumption of foreign goods. When $a > \frac{1}{2}$, there is home-bias in consumption. Similarly, foreign aggregate consumption (foreign variables indicted by *) is given by:

$$C^*_t = \left[ (1-a)^{\frac{1}{\theta}} c^*_{h,t}^{\frac{\theta-1}{\theta}} + a c^*_{f,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{2.4}$$

Home and foreign consumption goods are priced at $p_{h,t}$ and $p_{f,t}$ respectively. Cost minimization for total consumption expenditure ($P_t C_t = p_{h,t} c_{h,t} + p_{f,t} c_{f,t}$) yields a home consumption price index:

$$P_t = \left[ a p_{h,t}^{1-\theta} + (1-a) p_{f,t}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{2.5}$$

Similarly, cost minimization for foreign consumption expenditure ($P^*_t C^*_t = p_{h,t} c^*_{h,t} + p_{f,t} c^*_{f,t}$) results in a foreign consumption price index:

$$P^*_t = \left[ (1-a) p_{h,t}^{1-\theta} + a p_{f,t}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{2.6}$$

2.2.3 Income and Assets

The total goods endowment of the home and foreign country is $Y_{h,t}$ and $Y_{f,t}$, respectively. The endowment for each country is the sum of two random components following a multivariate AR(1) process. One part is a collateralizable capital income component $Y^k_{h,t}$ ($Y^k_{f,t}$) which is paid to equity holders as dividends. These payments are priced in local goods so that the total value of dividends paid to equity holders is $p_{h,t} Y^k_{h,t}$ ($p_{f,t} Y^k_{f,t}$). Equities can be internationally traded with domestic holdings of the home-country equity represented by $S_{h,t}$ and foreign holdings of the home-country
equity are $S^*_{h,t}$. Home-country holdings of the foreign equity are represented by $S_{f,t}$, while foreign-country claims to the foreign equity are $S^*_{f,t}$. Equities are priced at $Q_{h,t}$ ($Q_{f,t}$) and the quantity of equity is normalized to one so that the asset market clearing conditions are:

\[
1 = S_{h,t} + S^*_{h,t} \quad (2.7) \\
1 = S_{f,t} + S^*_{f,t} \quad (2.8)
\]

In contrast, the second component of national income is non-collateralizable labor income $Y^\ell_{h,t}$ ($Y^\ell_{f,t}$) and is priced in local goods so that its value is $p_{h,t} Y^\ell_{h,t}$ and $p_{f,t} Y^\ell_{f,t}$ for the home and foreign countries respectively. Agents cannot commit to claims on future labor income and with only two available assets they face a risk that cannot be perfectly hedged, therefore, markets are incomplete.

The income process (in logs) is:

\[
y_t = Ay_{t-1} + \epsilon_t \quad (2.9)
\]

Where $y_t = \left[ \ln \left( \frac{Y^k_{h,t}}{\bar{Y}^k_h} \right), \ln \left( \frac{Y^k_{f,t}}{\bar{Y}^k_f} \right), \ln \left( \frac{Y^\ell_{h,t}}{\bar{Y}^\ell_h} \right), \ln \left( \frac{Y^\ell_{f,t}}{\bar{Y}^\ell_f} \right) \right]^T$, $A$ is a matrix of persistence and spill-over parameters, and $\epsilon_t = \left[ \epsilon^k_{h,t} \epsilon^k_{f,t} \epsilon^\ell_{h,t} \epsilon^\ell_{f,t} \right]^T$ is vector of multivariate-normal shocks with mean zero and variance-covariance matrix $\Sigma$.

In order to hold claims to foreign capital, agents in the home country must pay a fixed amount, $\kappa$, valued in terms of the home consumption good priced at $p_{h,t}$. This payment can be interpreted as an information acquisition cost\(^5\). In contrast to models of asymmetric information, this approach means information differences are not insurmountable; domestic investors who pay the fixed cost have the same knowledge as foreign agents. It is assumed that domestic investors who do not pay the cost cannot effectively evaluate foreign assets and therefore do not hold any. Further, this is not a one-time decision, because information is always changing, the cost must be paid each period to remain up-to-date. I also assume foreign agents do not pay any additional costs to have access to the home-country equity markets. This asymmetry represents an environment where the home country may have more open or accessible financial markets and thus participation costs for outsiders are negligible.

### 2.2.4 Budget Constraints

Here I adopt prime (‘) notation to indicate time $t+1$ variables. The budget constraints vary depending on the decision of domestic savers whether or not to hold foreign equities. In the event

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\(^5\)Merton (1987) argues for this reduced form interpretation of information asymmetries.
they choose to invest abroad, the home budget constraint becomes:

\[ PC + Q_hS'_h + Q_fS'_f + p_h\kappa = (Q_h + p_hY^k_h)S_h + (Q_f + p_fY^k_f)S_f + p_hY^\ell_h \]  \hspace{1cm} (2.10)

Alternatively, home agents may choose not to participate in the foreign equity market and avoid paying \( p_h\kappa \); the left hand side of the budget constraint changes by dropping both the fixed cost and the foreign equity holdings. Note that this does not affect the right hand side as the income from previous period foreign equity holdings remains. The budget constraint in this case becomes:

\[ PC + Q_hS'_h = (Q_h + p_hY^k_h)S_h + (Q_f + p_fY^k_f)S_f + p_hY^\ell_h \]  \hspace{1cm} (2.11)

Similar to many portfolio choice models, it is possible to restate the equilibrium of the economy such that the only endogenous state variable is the home country’s relative claim to financial wealth, \( \omega \):

\[ \omega = \frac{(Q_h + p_hY^k_h)S_h + (Q_f + p_fY^k_f)S_f + p_hY^\ell_h}{Q_h + p_hY_h + Q_f + p_fY_f} \]  \hspace{1cm} (2.12)

The right hand side of the budget constraints can thus be restated in terms of \( \omega \) so that equations (2.10) and (2.11) become:

\[ PC + Q_hS'_h + Q_fS'_f + p_h\kappa = \omega(Q_h + p_hY_h + Q_f + p_fY_f) \]  \hspace{1cm} (2.13)

\[ PC + Q_hS'_h = \omega(Q_h + p_hY_h + Q_f + p_fY_f) \]  \hspace{1cm} (2.14)

Because the budget constraints no longer depend directly on the previous allocation between the two assets, this allows the state-space to be reduced by one-dimension. Therefore, the current state of the economy, \( s \), is defined by the endogenous variable \( \omega \) and the four exogenous income streams:

\[ s \equiv \{\omega, Y^k_h, Y^k_f, Y^\ell_h, Y^\ell_f\} \]

### 2.2.5 Optimal Choices

Let \( I_\kappa = 1 \) be an indicator variable if the home country pays the fixed cost in the current period investing in the foreign asset market and zero otherwise. Given the current state of the economy, \( s \), the value of participating in the foreign equity market for home investors is the Bellman equation:

\[ v_\kappa(s) = \max_{\{C, S'_h, S'_f\}} \left[U(C) + \theta(\bar{C})\mathbb{E}[V(s')|I_\kappa]\right] \quad s.t. \quad (2.13) \]  \hspace{1cm} (2.15)

---

6Fixed costs assumed to be priced in home goods as it is the home country who is paying them, however, pricing the fixed cost in terms of the home price index, foreign price index, or foreign goods do not change the model results.
where expectations about the future state of the economy are formed conditional on the choice to
hold foreign equities. Otherwise, the value to home investors of abstaining from the foreign equity
market is given by the Bellman equation:
\[
v_A(s) = \max_{\{C, S_h^s\}} U(C) + \theta(\bar{C}) \mathbb{E} \left[ V(s') I_\kappa \right]
\tag{2.16}
\]
Again, expectations of the future state of the economy are conditional on the choice not to hold
foreign equities.

The home representative agent will choose to participate in the foreign equity market only if
the value of doing so exceeds the value of abstaining. Therefore, for the home agent the present
value of the current state of the economy is:
\[
V(s) = \max \{ v_\kappa(s), v_A(s) \}
\tag{2.17}
\]

2.2.6 Foreign Problem

The foreign agent has a similarly defined problem except that they do not face the fixed cost
of investing abroad and always participate in both equity markets. The foreign representative
agent optimizes conditional on the choice of the home agent. The value of the current state of the
economy to the foreign representative agent, conditional on the home agent participating in the
foreign asset market is the Bellman equation:
\[
v^*_\kappa(s) = \max_{\{C^*, S^*_h, S^*_f\}} U(C^*) + \theta(\bar{C}^*) \mathbb{E} V^*[s'|I_\kappa]
\tag{2.18}
\]
subject to
\[
P^* C^* + Q_h S^*_h + Q_f S^*_f = (1 - \omega)(Q_h + p_h Y_h + Q_f + p_f Y_f)
\tag{2.19}
\]
Note that expectations regarding the future state of the economy are conditional on the home
country’s current period decision to invest abroad or not.

On the other hand, conditional on the home agent abstaining from the foreign asset market,
the present value of the state of the economy for the foreign agent is the Bellman equation:
\[
v^*_A(s) = \max_{\{C^*, S^*_h^s\}} U(C^*) + \theta(\bar{C}^*) \mathbb{E} V^*[s'|I_\kappa]
\tag{2.20}
\]

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subject to

\[ P^*C^* + Q_h S_h^* + Q_f = (1 - \omega)(Q_h + p_h Y_h + Q_f + p_f Y_f) \]  \hspace{1cm} (2.21)

Combining these, the present value of the current state of the economy to the foreign agent is:

\[ V^*(s) = I_\kappa v^*_h(s) + (1 - I_\kappa)v^*_A(s) \]  \hspace{1cm} (2.22)

where \( I_\kappa \) is an indicator function which takes the value one if the home country participates in the foreign asset market \( V = \max \{ v_A, v_\kappa \} = v_\kappa \) and zero otherwise.

Normalizing the home country price index, \( P = 1 \), optimization conditions produce a set of four equilibrium Euler equations:

\[ U_C Q_h = \theta(\bar{C})E\left[ U'_C(Q'_h + p'_h Y'_h) \right] \]  \hspace{1cm} (2.23)
\[ U_C Q_f = \theta(\bar{C})E\left[ U'_C(Q'_f + p'_f Y'_f) \right] \]  \hspace{1cm} (2.24)
\[ \frac{U_C^* Q_h}{P^*} = \theta(\bar{C}^*)E\left[ \frac{U_{C^*}^*}{P^*}(Q'_h + p'_h Y'_h) \right] \]  \hspace{1cm} (2.25)
\[ \frac{U_C^* Q_f}{P^*} = \theta(\bar{C}^*)E\left[ \frac{U_{C^*}^*}{P^*}(Q'_f + p'_f Y'_f) \right] \]  \hspace{1cm} (2.26)

When home agents do not invest abroad, equation (2.24) does not hold and \( S'_f = 0 \). Finally, to close the model, the foreign goods market clearing condition is:

\[ Y^*_f + Y^*_f = c_f + c'_f \]  \hspace{1cm} (2.27)

Appendix A lists the full system of equilibrium equations.

2.2.7 Competitive Equilibrium

Given the state of the economy at time \( t, s = \{ \omega, Y^k_h, Y^k_f, Y^\ell_h, Y^\ell_f \} \), a competitive equilibrium is a set of home agent’s policy functions: \( C(s), c_h(s), c_f(s), S_h'(s), S_f'(s), I_\kappa; \) value functions: \( V(s), v_\kappa(s), v_A(s); \) foreign agent’s policy functions: \( C^*(s), c^*_h(s), c^*_f(s), S^*_h'(s), S^*_f'(s); \) foreign value function, \( V^*(s); \) and pricing functions: \( Q_h(s), Q_f(s), P^*(s), p_h(s), p_f(s) \) such that: the home agents solve equation (2.17) and the foreign agents solve equation (2.22) subject to the relevant budget constraints; equity and goods markets clear; and the exogenous income process follows equation (2.9).
2.3 Calibration and Numerical Methods

2.3.1 Calibration

A summary of parameter values is given in Table 2.1. To demonstrate the effect of fixed costs on capital flows, I choose parameters that are commonly found in the literature for similar demonstrative models. On average, the capital share of income makes up 30% of total income and the labor share is 70% with total income normalized to one. The share of expenditure on domestic consumption goods is $a = 0.75$ so that agents spend three times as much on local goods relative to imports. The variance-covariance matrix of shocks to income is symmetric such that the standard deviation of shocks to each income stream is 2% and the correlation between capital and labor income within each country is 0.5. Income shocks are uncorrelated across countries. The relative risk aversion parameter is $\gamma = 2$ implying an intertemporal elasticity of substitution of 0.5. For the endogenous discount factor, $\theta(\bar{C}) = \beta \bar{C}^{-\eta}$, $\beta = 0.95$ which is the steady-state discount value because aggregate consumption is calibrated to be one. The consumption-elasticity of the discount factor $\eta = 0.001$, which imposes mathematical stationarity on the dynamics of relative wealth of the two countries, but is small enough that relative wealth deviations persist for a very long time making it indistinguishable from a unit root.

There is considerable debate about the price elasticity of traded goods\(^7\), although micro-sectoral estimates tend to be larger, macro estimates range from about 0.5 to 2. Coeurdacier et al. (2010) note that when this elasticity is low ($\theta < 1$) and in the absence of bonds, home bias in equities will be much larger than what is observed in the data because of real exchange-rate risk. Therefore, under the baseline model the elasticity of substitution between home and foreign goods is set to $\theta = 1.1$; and $\theta = 1.5$ for an alternative specification.

To my knowledge, there is no estimate of the fixed cost parameter $\kappa$ and it is not easily measured. I calibrate $\kappa$ to match the data for capital outflow (COD) volatility. Values of $\kappa$ range from $6.5 \times 10^{-5}$ to $10 \times 10^{-5}$. Considering that GDP for the model economy is normalized to one, by comparison, for a $18$ trillion economy the fixed cost would represent just $12–18$ billion. Although this is no small sum, compared to U.S. holdings of foreign assets of approximately $27$ trillion\(^8\), the fixed

\(^7\)See Ruhl (2008), Imbs and Mejean (2015), and Feenstra et al. (2018) for detailed discussions regarding traded goods price elasticity estimates.

\(^8\)US International Investment Position 2018Q2 reported by the BEA.
Table 2.1: Baseline Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Cost</td>
<td>$\kappa = 1 \times 10^{-4}$</td>
</tr>
<tr>
<td>Share of Home goods</td>
<td>$a = 0.75$</td>
</tr>
<tr>
<td>Goods Elasticity</td>
<td>$\theta = 1.10$</td>
</tr>
<tr>
<td>Discount Factor</td>
<td>$\beta = 0.95$</td>
</tr>
<tr>
<td>Endogenous Discount</td>
<td>$\eta = 0.001$</td>
</tr>
<tr>
<td>Utility Curvature</td>
<td>$\gamma = 2$</td>
</tr>
<tr>
<td>Mean Endowment</td>
<td>$\bar{Y}^k = 0.3$</td>
</tr>
<tr>
<td>AR(1) Income Persistence</td>
<td>$A = 0.8$</td>
</tr>
<tr>
<td>% Std of Shocks</td>
<td>$\sigma = 0.02$</td>
</tr>
<tr>
<td>Corr($\epsilon^k_i$, $\epsilon^f_i$)</td>
<td>$\rho = 0.5$</td>
</tr>
</tbody>
</table>

Cost would reduce returns on foreign assets by less than ten basis points. Unlike static models or those with transactions cost applied to buying foreign assets, here a reasonably sized fixed cost is sufficient to produce highly volatile capital flows.

2.3.2 Solution Method

In many ways the model with fixed costs is no different than a standard portfolio choice model which itself is not trivial to solve. However, the discrete choice of whether or not to participate in the foreign asset market by the home country creates a kink in the value function and complicates the solution. Nevertheless, considering a simpler model is sufficient to understand the intuition and effect of fixed costs; assume there are only two lotteries and one time period. If the home and foreign lotteries are independent and identical, then a risk averse agent will invest half their wealth in each one. A fixed cost paid in order to participate in one of the lotteries will have no impact on the allocation between the two lotteries. Because the cost is paid first and then the allocation decision between lotteries is made, it is still optimal for agents to equally split their investable resources evenly. The important impact of the fixed cost is the discrete choice regarding the decision to participate at all in the lottery with an extra cost. Agents will participate in both lotteries so long as the diversification value of participating exceeds the fixed cost.

---

9These costs are similar in magnitude to the estimates of fixed participation costs for international investing in Chapter 1.
I solve the model using global methods. Although breakthroughs in solving open economy portfolio allocation models have used perturbation methods (Devereux and Sutherland, 2010, 2011; Tille and van Wincoop, 2010; Evans and Hnatkovska, 2012) and these are computationally very efficient, they rely on approximation around a stochastic steady-state. However, the model presented here has a kink in the objective function and, therefore, local methods will fail to capture this fact. Furthermore, Rabitsch et al. (2015) indicate that in portfolio choice models local solutions are comparable to global ones with respect to the non-portfolio variables of the model, such as consumption and net foreign assets; however, relatively large discrepancies occur in the dynamics of asset positions. In models with asymmetries they show the disparity is even more significant so that solving the model with global methods is imperative when portfolio allocations are of interest.

I employ a projection-collocation algorithm using Smolyak sparse-grids and Chebyshev polynomial basis functions similar to the process described in Judd et al. (2014). One of the important benefits of the Smolyak algorithm is that it is not subject to the so-called curse of dimensionality, making it is well suited to address multi-country models with incomplete markets.\textsuperscript{10} Updating the polynomial coefficients is done with a derivative-free-fixed point method which avoids using a numerical solver and thus greatly increases the iteration speed. Further details regarding the solution algorithm are given in Appendix B.

### 2.4 Results

In this section I report how the presence of fixed costs affects capital flow dynamics and compare these outcomes with the stylized facts for capital flows. Specifically, I conduct experiments varying some important parameters of the model such as the price elasticity for traded goods and the correlation of shocks to capital and labor income. I examine how variation in these parameters affects the outcome of the model to shed light on the underlying mechanism. This analysis relies both on comparisons of second moments and impulse response functions from the model with the data.

\textsuperscript{10}Other papers using Smolyak interpolation for portfolio choice problems include Gavilan-Gonzalez and Rojas (2009) and Gourio et al. (2016).
2.4.1 Simulated Moments

I simulate the model 100 times for 11,000 periods. I drop the first 1,000 observations and HP-filter\textsuperscript{11} the remaining observations from each simulation. I calculate each of the statistics using the HP-filtered observations and report the average across all simulations. I compare several different specifications, (1) and (2) are models with high and low price elasticity in the absence of fixed costs. Specifications (3) and (4) add fixed costs to the models with high and low price elasticity; The value of $\kappa$ is calibrated in models (3) and (4) to match the standard deviation of gross capital outflows, the second row of Table 2.2.

Using the same methodology as Broner et al. (2013) to measure capital flows, I normalize the simulated data by trend-GDP. Thus measured variables are percentages of trend-GDP. Normalizing country capital flows this way allows for more meaningful cross-country comparison; and because trend-GDP is slow moving compared to GDP, spurious correlation is negligible.

Gross capital outflows by domestic agents are defined as: \( COD = q_f (S_f' - S_f)/\bar{Y}_h \); where \( \bar{Y}_h \) is home trend-GDP. Gross capital inflows from foreigners are defined similarly as: \( CIF = q_h (S_h' - S_h)/\bar{Y}_h \). Total flows are just the sum of inflows and outflows: \( TOT = CIF + COD \); net flows are inflows minus outflows: \( NET = CIF - COD \). Following Coeurdacier and Rey (2013) home-bias in equities is calculated as \( 1 - \frac{q_f S_f q_h S_h}{q_f + q_h} \) and measures the degree to which the share of foreign assets in the home portfolio matches the share of foreign assets in the world portfolio. A zero value implies that the home country holds foreign equities in the same ratio as the world portfolio, a negative value indicates a foreign bias, and a positive value implies home bias.

Table 2.2 summarizes the moments of each model. Model (1) uses the same parameter values as Rabitsch et al. (2015). This specification has zero fixed costs for holding foreign equities, high elasticity of substitution \( (\theta = 1.5) \), and low positive correlation between in-country capital and labor income \( (\rho = 0.2) \). The standard model (1) vastly underestimates the variability of inflows and outflows in addition to the equity home-bias. This specification largely matches the co-movements of capital flows including: the strong positive correlation between gross inflows and outflows; and the procyclical nature of capital flows; indicating that during expansionary periods in the home economy, home agents tend to increase holdings of foreign assets and foreigners increase holdings of home assets.

\textsuperscript{11}I use a smoothing parameter of 6.25 for annual frequency data. See Ravn and Uhlig (2002) for a discussion of parameter choices for various data frequencies.
Table 2.2: Simulated Moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Fixed Cost $\kappa$</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Goods Elasticity $\theta$</td>
<td>1.5</td>
<td>1.1</td>
</tr>
<tr>
<td>Corr($\epsilon_i^k, \epsilon_i^f$) $\rho$</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>Equity home-bias</td>
<td>0.63</td>
<td>0.48</td>
</tr>
<tr>
<td>StdDev (%GDP)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>COD</td>
<td>8.05</td>
<td>0.93</td>
</tr>
<tr>
<td>CIF</td>
<td>7.81</td>
<td>0.87</td>
</tr>
<tr>
<td>NET</td>
<td>3.92</td>
<td>0.38</td>
</tr>
<tr>
<td>TOT</td>
<td>15.49</td>
<td>1.49</td>
</tr>
<tr>
<td>Correlation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(COD,CIF)</td>
<td>0.89</td>
<td>0.91</td>
</tr>
<tr>
<td>(COD,\Delta y)</td>
<td>0.25</td>
<td>0.37</td>
</tr>
<tr>
<td>(CIF,\Delta y)</td>
<td>0.30</td>
<td>0.49</td>
</tr>
<tr>
<td>(NET,\Delta y)</td>
<td>0.22</td>
<td>0.23</td>
</tr>
<tr>
<td>(TOT,\Delta y)</td>
<td>0.28</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Data, annual, are from the IMF balance of payments statistics. They are converted to local currency and divided by the trend component of HP-filtered nominal GDP. For models (3) and (4), capital outflow (COD) volatility is targeted to match the data by adjusting the fixed cost parameter. The mean equity home bias is targeted by adjusting the correlation between capital and labor income within a country, there is no correlation between income shocks across countries.

However, one shortcoming of model (1) is the low level of equity home-bias, model (2) corrects this using a stronger correlation coefficient (within country) between capital and labor income shocks. Because of the higher correlation between a country’s capital and labor income, foreign equities provide a better hedge against income risk. Additionally, model (2) uses lower elasticity of substitution between home and foreign goods ($\theta = 1.1$). With a lower elasticity of substitution between home and foreign goods, shocks to endowments produce larger price responses, effectively providing a stronger insurance mechanism through the terms of trade.\(^{12}\) Due to this effect, capital flow volatility is much smaller for this specification. Furthermore, the correlation between inflows and outflows becomes strongly negative resulting in a correlation between GDP growth and inflows which is also negative, in contrast to the data.

Models (3) and (4) include fixed costs to target the median annual volatility of capital outflows as a percentage of GDP for the G7 countries from 1970-2017. However, this model also matches the

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\(^{12}\)Cole and Obstfeld (1991) document how the terms of trade can serve as a risk-sharing mechanism even in the absence of financial markets.
standard deviation of gross inflows and total flows. Strong positive co-movement between inflows and outflows produces a correlation coefficient that is slightly higher than what is observed in the data. Finally, fixed costs have a dampening effect on the cyclicality of capital flows. In the absence of fixed costs, the cyclicality of gross flows is very sensitive to the elasticity of substitution, however, the inclusion of fixed costs results in acyclical inflows and outflows regardless of the elasticity. Although this does not match the correlation coefficient for gross flows and GDP, in Chapter 1 I present evidence which is suggestive that the coefficient is not statistically different from zero, meaning that gross flows may be acyclical.

To shed further light on the model dynamics and the effect of fixed costs, I run several experiments comparing the impulse response functions of the economy.

### 2.4.2 Impulse Response Functions

To better understand the dynamics of capital flows, I first demonstrate the differential impact of shocks to domestic capital versus labor income. Second, to highlight the effects of fixed costs I compare the models with and without fixed costs. Around the stochastic steady state the results are broadly similar regardless of the size of $\kappa$. But away from the stochastic steady-state the presence of fixed costs has an important effect. The fixed cost model has a threshold point in the state-space where the fixed cost exceeds the value of diversification and as a result home agents withdraw from the foreign equity market. It is only after a series of negative shocks that there is a retrenchment of domestic capital. Therefore, the second set of experiments I run compares the results away from the stochastic steady-state when the home country has less wealth.

Figure 2.1 shows the effect of a negative one standard deviation shock to home capital income in the absence of fixed costs. The price of home goods rises (panel c) in response to the fall in supply, while the price of foreign goods falls on account of reduced demand and larger relative supply. The change in goods prices helps to moderate the effects of the fall in home dividend payments, the home equity price falls only a little further than the foreign equity price (panel d). This means that the shock to capital income only has a very moderate effect on relative wealth (panel b) and diminishes just as quickly. Gross flows (panel e) are initially positive, meaning domestic agents invest more abroad and foreign agents invest more in the home country, however, they reverse the next period and remain negative. Net capital flows (panel f) are nearly zero.
Figure 2.1: Effect of Negative Home Capital Income Shock without Fixed Costs

A negative one standard deviation shock to home capital income. Relative wealth begins at steady-state; fixed costs are zero; the elasticity of substitution between goods is 1.5; and correlation of the shocks to capital and labor income within a country is 0.2. \( \omega_0 = \bar{\omega}; \kappa = 0; \theta = 1.5; \rho = 0.2 \)
In contrast, Figure 2.2 shows the effect of a negative one standard deviation shock to home labor income in the absence of fixed costs. Because labor income makes up a larger share of GDP, the effect on goods prices and asset prices is larger than a shock to capital. Nevertheless, like the shock to capital, the labor income shock results in a rising home good price (panel c) and falling foreign good price which combine to drive the prices of both assets down (panel d). Unlike capital, labor income is uncollateralizable so that the negative labor income shock results is a persistent decline in relative wealth, only returning to the original level after an extended period of time. This results in the opposite effect on capital flows (panel e), an initial sudden stop of capital inflows and an even larger retrenchment of capital outflow. After one period gross flows are reversed. Net capital flows are positive, resulting in a drop in net foreign assets for the home country but allowing domestic agents to smooth consumption.

Figure 2.1 and Figure 2.2 are responses of the model to income shocks around the stochastic steady-state.\textsuperscript{13} Although the introduction of fixed costs changes the steady-state, it has no discernible effect on the impulse response functions of the model. However, away from the steady-state fixed costs play a very important role in increasing the variability of capital flows.

Figure 2.3 shows the effect of a negative one standard deviation shock to capital income when there are fixed costs for holding foreign assets. However, the home country starts with relative wealth below the stochastic steady-state level, equal to 0.8\(\bar{\omega}\). Due to the endogenous discount factor, even in the absence of shocks to income, the endogenous variables are trending to their steady-state levels. Therefore, the plots compare the path of an economy that receives the shocks relative to any economy starting at the same relative wealth and experiencing no shock. Both goods and asset prices (panels c-d) respond the same as was the case around the stochastic steady-state. The effect on relative wealth (panel b) is slightly different. The initial decline is smaller than before and relative wealth actually returns to the stochastic steady-state level more quickly. The reason for this is because the foreign agent, having a relatively larger share of wealth, is more exposed to the home country capital income shock. The movement of gross capital flows (panel e) is an order of magnitude smaller than before. Gross outflows are initially positive and, despite quickly diminishing, remain positive. Gross inflows are initially positive but then reverse and turn negative.

\textsuperscript{13}The standard deterministic steady-state concept does not apply to portfolio choice models because, in the absence of risk, all assets will have the same return and thus the portfolio will be indeterminate. I adopt the concept of a stochastic steady state developed by Coeurdacier et al. (2011). The stochastic steady-state is a state of the economy which reproduces itself if the realization of current shocks is zero.
A negative one standard deviation shock to home labor income. Relative wealth begins at steady-state; fixed costs are zero; the elasticity of substitution between goods is 1.5; and correlation of the shocks to capital and labor income within a country is $0.2 \omega_0 = \bar{\omega}$; $\kappa = 0$; $\theta = 1.5$; $\rho = 0.2$.
This difference between inflows and outflows results in negative net capital flows (panel f) that are nearly as large in size as gross flows.

Figure 2.4 shows the effect of a negative one standard deviation shock to labor income when there are fixed costs for holding foreign assets. As with Figure 2.3, initial relative wealth is $0.8\bar{w}$ and the variable plots are the path of variables subject to the shocks relative to what their path would be in the absence of any shocks. The time paths of income, relative wealth, goods prices, asset prices and net capital flows are similar to that of a shock around the stochastic steady-state. The effect on relative wealth (panel b) is slightly larger. However, because home agents are exiting the foreign equity market, there is a sudden stop of capital inflows and retrenchment of domestic capital leading to negative gross capital flows of over one-hundred percent of GDP. After a few years of recovery, the domestic agents rejoin the foreign equity market leading to large positive capital inflow and outflow. Net flows follow the same basic pattern as before, they are positive and small compared to GDP. However, the positive net capital flow diminishes at a faster rate while the domestic agents are absent from the foreign equity market, when they rejoin the rate of decline in net flows eases.

2.5 Conclusion

In contrast to the data, standard two-country portfolio choice models exhibit low volatility of gross capital flows relative to that of net flows, a negative correlation between inflows and outflows, and consequently counter-cyclical gross outflows. The model presented here resolves this inconsistency by including fixed costs for investing abroad into an otherwise standard international business cycle model with portfolio choice. Fixed costs of investing abroad drive up the volatility of gross capital flows and improve the cyclicality of gross flows so that, consistently with the data, both inflows and outflows are pro-cyclical and positively correlated with each other. While the model developed in the paper explains the stylized facts regarding gross capital flows, it also provides insights for future research.

First, the model is an endowment economy framework. One possible extension is the inclusion of productive capital with physical investment. I expect that the main impact of physical capital will be on net flows as agents optimally shift capital between countries in response to productivity
A negative one standard deviation shock to home capital income. Home relative wealth begins at 80% of steady-state; fixed costs are $1 \times 10^{-4}$; elasticity of substitution between home and foreign goods is 1.5; and the correlation of the shocks to within country capital and labor income is 0.2. $\kappa = 1 \times 10^{-4}$; $\theta = 1.5$; $\rho = 0.2$
A negative one standard deviation shock to home labor income. Home relative wealth begins at 80% of steady-state; fixed costs are $1 \times 10^{-4}$; elasticity of substitution between home and foreign goods is 1.5; and the correlation of the shocks to within country capital and labor income is $0.2$. $\omega_0 = 0.8\bar{\omega}$; $\kappa = 1 \times 10^{-4}$; $\theta = 1.5$; $\rho = 0.2$.
shocks. The portfolio choice problem, fixed cost mechanism, and gross capital flows will remain practically unaffected.

Second, allowing for heterogeneous fixed costs may further improve the performance of the model. In a two-country setting with representative agents the fixed cost mechanism produces maximum capital flows which are too extreme. When agents exit the foreign market they completely liquidate their foreign holdings resulting in zero foreign assets, which is unrealistic. If agents have varying fixed costs, then those with lower costs may stay in the foreign asset market, whereas, agents with higher fixed costs may never enter. Agents in the middle will shift their portfolios in and out of foreign equities so that large gross capital flows will result as the proportion of investors participating in the foreign asset market changes. This extension is left to future research.
APPENDIX A

EQUILIBRIUM EQUATIONS

The system of equilibrium equations are listed in Table A.1. Prices are normalized by the home-country price index, $P = 1$. The resulting system totals 41 equations and 41 unknown variables. This is relatively large because it contains both on- and off-path variables; even if the home country participates in the foreign asset market, equilibrium requires optimal off-path variables as well. Thus, the variables of the model are the six exogenous ones; 17 endogenous variables conditional on the home agent holding foreign equities; 17 conditional on the home agent holding zero foreign equities; and one for the home present value $V$. 
Table A.1: System of Equilibrium Equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E1) 0 = $V - \max{v_k, v_A}$</td>
<td></td>
</tr>
<tr>
<td>(E2-5) 0 = $y' - Ay - e'$, $e' \sim N_4(0, \Sigma)$</td>
<td></td>
</tr>
<tr>
<td>(E6-7) 0 = $Y_i - Y_i^k - Y_i^f$, $i = {h, f}$</td>
<td></td>
</tr>
</tbody>
</table>

Conditional on choice $v_k$

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E8) 0 = $E \left[ \omega' - (Q_h + p_h Y_h) S_h + (Q_f + p_f Y_f) S_f + Y_h^r \right]$</td>
<td></td>
</tr>
<tr>
<td>(E9) 0 = $v_k - U(C) - \theta(C) E[V^k I_k]$</td>
<td></td>
</tr>
<tr>
<td>(E10) 0 = $U_C Q_h - \theta(\bar{C}) E[U_C'(Q_h + p_h Y_h^k)]$</td>
<td></td>
</tr>
<tr>
<td>(E11) 0 = $U_C Q_f - \theta(\bar{C}) E[U_C'(Q_f + p_f Y_f^k)]$</td>
<td></td>
</tr>
<tr>
<td>(E12) 0 = $\frac{U_C Q_h}{\bar{p} \bar{h}} - \theta(\bar{C}^*) E[U_C'(Q_h + p_h Y_h^k)]$</td>
<td></td>
</tr>
<tr>
<td>(E13) 0 = $\frac{U_C Q_f}{\bar{p} \bar{f}} - \theta(\bar{C}^*) E[U_C'(Q_f + p_f Y_f^k)]$</td>
<td></td>
</tr>
<tr>
<td>(E14) 0 = $C - \left[ \frac{1}{\bar{a}} c_h^{\frac{\theta}{1-\theta}} + (1 - \frac{1}{\bar{a}}) c_f^{\frac{\theta}{1-\theta}} \right]^{\frac{1}{\theta-1}}$</td>
<td></td>
</tr>
<tr>
<td>(E15) 0 = $C^* - \left[ (1 - \frac{1}{\bar{a}}) c_h^{\frac{\theta}{1-\theta}} + \frac{1}{\bar{a}} c_f^{\frac{\theta}{1-\theta}} \right]^{\frac{1}{\theta-1}}$</td>
<td></td>
</tr>
<tr>
<td>(E16) 0 = $1 - \left[ a p_h^{\frac{1}{\theta-1}} + (1 - a) p_f^{\frac{1}{\theta-1}} \right]^{\frac{1}{\theta-1}}$</td>
<td></td>
</tr>
<tr>
<td>(E17) 0 = $P^* - \left[ (1 - a) p_h^{\frac{1}{\theta-1}} + a p_f^{\frac{1}{\theta-1}} \right]^{\frac{1}{\theta-1}}$</td>
<td></td>
</tr>
<tr>
<td>(E18) 0 = $c_h/c_f - \frac{\bar{a}}{1-\bar{a}} (m_h/m_f)^{-\theta}$</td>
<td></td>
</tr>
<tr>
<td>(E19) 0 = $c_h/c_f - \frac{\bar{a}}{1-\bar{a}} (m_h/m_f)^{-\theta}$</td>
<td></td>
</tr>
<tr>
<td>(E20) 0 = $\omega(Q_h + Q_f + p_h Y_h + p_f Y_f) - C - Q_h S_h^\prime - Q_f S_f^\prime - p_h k$</td>
<td></td>
</tr>
<tr>
<td>(E21) 0 = $(1 - \omega)(Q_h + Q_f + p_h Y_h + p_f Y_f) - P^* C^* - Q_h S_h^\prime - Q_f S_f^\prime$</td>
<td></td>
</tr>
<tr>
<td>(E22) 0 = $1 - S_h^\prime - S_f^\prime$</td>
<td></td>
</tr>
<tr>
<td>(E23) 0 = $1 - S_h^\prime - S_f^\prime$</td>
<td></td>
</tr>
<tr>
<td>(E24) 0 = $Y_h^k + Y_f^k - c_f - c_f^*$</td>
<td></td>
</tr>
</tbody>
</table>
Table A.1: (cont.)

<table>
<thead>
<tr>
<th>Conditional on choice $v_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E25) $0 = \mathbb{E} \left[ \omega' - \frac{(Q_h'p_hY_k')S_h' + (Q_f'p_fY_f')S_f' + Y_f'}{(Q_h'Q_f'p_h'Y_k' + p_f'Y_f')} \right]$</td>
</tr>
<tr>
<td>(E26) $0 = v_A - U(C) - \theta(C)\mathbb{E}[V'</td>
</tr>
<tr>
<td>(E27) $0 = U_C Q_h - \theta(C)\mathbb{E} [U'_C (Q_h' + p_h'Y_k')]$</td>
</tr>
<tr>
<td>(E28) $0 = S_f'$</td>
</tr>
<tr>
<td>(E29) $0 = \frac{U_C^* Q_h}{p_h} - \theta(C^<em>)\mathbb{E} \left[ \frac{U'_C^</em> (Q_h' + p_h'Y_k')}{p_h} \right]$</td>
</tr>
<tr>
<td>(E30) $0 = \frac{U_C^* Q_f}{p_f} - \theta(C^<em>)\mathbb{E} \left[ \frac{U'_C^</em> (Q_f' + p_f'Y_f')}{p_f} \right]$</td>
</tr>
<tr>
<td>(E31) $0 = C - \left[ \frac{a}{2\alpha} c_h \frac{e^{-1}}{\alpha} + (1-a) \frac{1}{2\alpha} c_f \frac{e^{-1}}{\alpha} \right] \frac{\alpha}{\alpha^2}$</td>
</tr>
<tr>
<td>(E32) $0 = C^* - \left[ (1-a) \frac{1}{2\alpha} c_h \frac{e^{-1}}{\alpha} + a \frac{1}{2\alpha} c_f \frac{e^{-1}}{\alpha} \right] \frac{\alpha}{\alpha^2}$</td>
</tr>
<tr>
<td>(E33) $0 = 1 - \left[ a p_h^{1-\theta} + (1-a)p_f^{1-\theta} \right] \frac{1}{1-\theta}$</td>
</tr>
<tr>
<td>(E34) $0 = P^* - \left[ (1-a)p_h^{1-\theta} + a p_f^{1-\theta} \right] \frac{1}{1-\theta}$</td>
</tr>
<tr>
<td>(E35) $0 = \frac{c_h}{c_f} - a/(1-a) \left( p_h/p_f \right) + \theta$</td>
</tr>
<tr>
<td>(E36) $0 = \frac{c_h^<em>}{c_f^</em>} - (1-a)/a \left( p_h^<em>/p_f^</em> \right) + \theta$</td>
</tr>
<tr>
<td>(E37) $0 = \omega(Q_h + Q_f + p_hY_k + p_fY_f) - C - Q_hS_h'$</td>
</tr>
<tr>
<td>(E38) $0 = (1-\omega)(Q_h + Q_f + p_hY_k + p_fY_f) - P^<em>C^</em> - Q_hS_h' - Q_f$</td>
</tr>
<tr>
<td>(E39) $0 = 1 - S_f' - S_f''$</td>
</tr>
<tr>
<td>(E40) $0 = 1 - S_f' - S''_f$</td>
</tr>
<tr>
<td>(E41) $0 = Y_f^* + Y_f^* - c_f - c_f^*$</td>
</tr>
</tbody>
</table>
APPENDIX B

SOLUTION ALGORITHM

Portfolio choice problems are notoriously difficult to solve because the allocation between assets depends on risk and so the notion of steady-state is not defined by first order conditions. Devereux and Sutherland (2010, 2011) provide a general method of approximation for solving many portfolio choice problems,¹ but because it relies on perturbation methods it requires a point around which to approximate. The model with fixed costs has no natural point around which to approximate because of the kink in the value function. Global methods are most appropriate, however, standard value function iteration still presents a problem. In general equilibrium, simultaneously determining optimal asset allocations and prices has posed a problem. The popular algorithm of Lucas (1994) and Heaton and Lucas (1996) uses value function iteration to solve general equilibrium portfolio choice problems. However, in order to solve for asset prices and allocations simultaneously they employ an 'auctioneer' algorithm for which asset market clearing errors are too large for the current problem. Accuracy can always be improved through additional grid points but this comes at a cost of computational time and is plagued by the curse of dimensionality. I employ a fixed point algorithm inspired by Judd et al. (2014) to simultaneously solve for asset allocations and prices with high accuracy.

To increase the likelihood that the problem is computationally feasible I use a sparse-grid method developed by Smolyak (1963) to replace traditional tensor-product methods. Initially introduced to the economics literature by Krueger and Kubler (2004) to solve overlapping generations models with many cohorts, and subsequently applied to many other economic models. Implementations of the Smolyak sparse-grids applied to portfolio choice problems include Gavilan-Gonzalez and Rojas (2009) and Horneff et al. (2016). Gavilan-Gonzalez and Rojas solve a standard model to demonstrate the feasibility of the approach, while Horneff, Maurer, and Schober show an effective parallelization of a standard portfolio choice model with many overlapping generations.

¹Alternative approaches to solving dynamic general equilibrium portfolio choice problems include Tille and van Wincoop (2010) and Evans and Hnatkovska (2012).
Smolyak interpolation requires a good starting guess for the policy coefficients. The solution algorithm has two steps: the first to find a good starting guess, the second to get a more accurate solution. For step one, I follow the generalized stochastic simulation algorithm introduced by Judd et al. (2011) to find rough approximations to the policy functions. Step 2 follows Judd et al. (2014) and uses the Smolyak method of interpolation to find a more accurate solution. In both cases I write and debug the code in MATLAB, however, following Gibson and Henson (2016) I use the MATLAB coder application to automatically translate the MATLAB code into C++, decreasing computational time by a factor of 10 (or more).

### B.1 Stochastic Simulation

#### B.1.1 Initialization

1. Simulate the income process for \( T + 1 \) periods.

\[
y' = Ay + \epsilon' \tag{B.1}
\]

\[
y = \left[ \ln \left( \frac{Y^k_h}{\bar{Y}^k_h} \right), \ln \left( \frac{Y^k_f}{\bar{Y}^k_f} \right), \ln \left( \frac{Y^\ell_h}{\bar{Y}^\ell_h} \right), \ln \left( \frac{Y^\ell_f}{\bar{Y}^\ell_f} \right) \right]^T
\]

2. Define the projection basis as a function of the state variables, in this case it is a \( T \times 11 \) matrix:

\[
x^{(0)} \equiv \left\{ 1, \frac{\omega^{(0)} - \bar{\omega}}{\bar{\omega}}, \left( \frac{\omega^{(0)} - \bar{\omega}}{\bar{\omega}} \right)^2, y, y^2 \right\}_{t=1}^T \tag{B.2}
\]

3. Choose endogenous variables to parameterize.

   (a) \( S^\kappa_h, S^\kappa_f, Q^\kappa_h, Q^\kappa_f, \bar{E}_{\kappa} \)

   (b) \( S^A_h, S^A_f, Q^A_h, Q^A_f, \bar{E}_{v'}^A \)

Where the superscript represents the which discrete choice is being made: to invest in the foreign equity (\( \kappa \)), or not (\( A \)). And by construction \( S^A_f = 0 \).

4. Parameterize two sets of policy functions with an initial guess for the coefficients:

\[
\Phi^{\kappa(0)} \equiv \left[ \Phi^{\kappa(0)}_{S^\kappa_h}, \Phi^{\kappa(0)}_{S^\kappa_f}, \Phi^{\kappa(0)}_{Q^\kappa_h}, \Phi^{\kappa(0)}_{Q^\kappa_f}, \Phi^{\kappa(0)}_{\bar{E}_{\kappa}} \right] \tag{B.3}
\]

\[
\Phi^{A(0)} \equiv \left[ \Phi^{A(0)}_{S^A_h}, \Phi^{A(0)}_{S^A_f}, \Phi^{A(0)}_{Q^A_h}, \Phi^{A(0)}_{Q^A_f}, \Phi^{A(0)}_{\bar{E}_{v'}} \right] \tag{B.4}
\]

5. Such that,

\[
x^{(0)} \Phi^{\kappa(0)} = \left[ S^{\kappa(0)}_h, S^{\kappa(0)}_f, Q^{\kappa(0)}_h, Q^{\kappa(0)}_f, \bar{E}_{\kappa}^{(0)} \right]_{t=1}^T
\]

\[
x^{(0)} \Phi^{A(0)} = \left[ S^{A(0)}_h, S^{A(0)}_f, Q^{A(0)}_h, Q^{A(0)}_f, \bar{E}_{v'}^{(0)} \right]_{t=1}^T
\]

yields the selected endogenous variables for each time period and investment choice.
B.1.2 Sequential Evaluation

For each iteration, (i) and policy functions $\Phi^A(i)$ and $\Phi^\kappa(i)$:

Compute policies each time period: $(t = 1 \text{ to } T)$ sequentially.

at time-$t$:

1. Evaluate Option 1, as if home investors do hold foreign equity.
   
   (a) Evaluate $x_t(i)\Phi^\kappa(i) = [S_h'^\kappa(i), S_f'^\kappa(i), Q_h'^\kappa(i), Q_f'^\kappa(i), \mathbb{E}v'^\kappa(i)]$

   (b) Given $S_h'^\kappa(i), S_f'^\kappa(i), Q_h'^\kappa(i), Q_f'^\kappa(i), \mathbb{E}v'^\kappa(i)$, solve for $C'^\kappa(i)$.

   (c) Calculate the value of this choice as
      
      \[ v'^\kappa(i) = U(C'^\kappa(i)) + \theta(C'^\kappa(i))\mathbb{E}v'^\kappa(i) \]

2. Evaluate Option 2, as if home investors do not hold foreign equity

   (a) Evaluate $x_t(i)\Phi^A(i) = [S_h'^A(i), S_f'^A(i), Q_h'^A(i), Q_f'^A(i), \mathbb{E}v'^A(i)]$

   (b) Given $S_h'^A(i), S_f'^A(i), Q_h'^A(i), Q_f'^A(i), \mathbb{E}v'^A(i)$, solve for $C'^A(i)$.

   (c) Calculate the value of this choice as
      
      \[ v'^A(i) = U(C'^A(i)) + \theta(C'^A(i))\mathbb{E}v'^A(i) \]

3. Set $V(i) = \max \{ v'^A(i), v'^\kappa(i) \}$

   (a) Use the corresponding endogenous variables which were conditional on this choice. i.e.
   
   if $v'^A(i) > v'^\kappa(i)$ then
   
   \[ S_h'^A(i) = S_h'^\kappa(i), S_f'^A(i) = S_f'^A(i), Q_h'^A(i) = Q_h'^\kappa(i), Q_f'^A(i) = Q_f'^\kappa(i), V(i) = v'^A(i) \]

4. Update next period’s state $x'(i)$ for $\omega'(i)$.

   Since the basis is a function of future wealth, $x'(i)(\omega'(i), Y')$, the wealth evolution equation is a high order polynomial and satisfies:

   \[
   \omega'(i) = \frac{\left( \sum_{h} \left( x'(i) + p_h(i) x'(i) Y_h \right) S_h'^i(i) + \left( Q_f(i) x'(i) + p_f(i) x'(i) Y_f \right) S_f'^i(i) + p_h(i) x'(i) Y_h \right)}{\sum_{h} \left( x'(i) + p_h(i) x'(i) Y_h \right) + Q_f(i) x'(i) + p_f(i) x'(i) Y_f} (B.5)
   
   Note that the future endogenous variables in equation (B.5) must be optimized for the discrete choice in the same way as steps (1)-(3) above.

5. Advance to $t+1$ and repeat steps (1)-(4)
B.1.3 Updating Policy Rules

1. Construct Euler equation and value-function 'residual' time-series

\[ R_{hh}^{A(i)} = \frac{\theta(C^{A(i)} \partial U/\partial C^{A(i)}(Q_h^{A(i)} + p_h^{A(i)}Y_h^{kr}))}{\partial U/\partial C^{A(i)}(Q_h^{A(i)})} \]

\[ R_{hh}^{\kappa(i)} = \frac{\theta(C^{\kappa(i)} \partial U/\partial C^{\kappa(i)}(Q_h^{\kappa(i)} + p_h^{\kappa(i)}Y_h^{kr}))}{\partial U/\partial C^{\kappa(i)}(Q_h^{\kappa(i)})} \]

\[ R_{hf}^{A(i)} = \frac{\theta(C^{A(i)} \partial U/\partial C^{A(i)}(Q_f^{A(i)} + p_f^{A(i)}Y_f^{kr}))}{\partial U/\partial C^{A(i)}(Q_f^{A(i)})} \]

\[ R_{hf}^{\kappa(i)} = \frac{\theta(C^{\kappa(i)} \partial U/\partial C^{\kappa(i)}(Q_f^{\kappa(i)} + p_f^{\kappa(i)}Y_f^{kr}))}{\partial U/\partial C^{\kappa(i)}(Q_f^{\kappa(i)})} \]

\[ R_{fh}^{A(i)} = \frac{\theta(C^{A(i)} \partial U/\partial C^{A(i)}(Q_h^{A(i)} + p_h^{A(i)}Y_h^{kr}))}{\partial U/\partial C^{A(i)}(Q_h^{A(i)})} \]

\[ R_{fh}^{\kappa(i)} = \frac{\theta(C^{\kappa(i)} \partial U/\partial C^{\kappa(i)}(Q_h^{\kappa(i)} + p_h^{\kappa(i)}Y_h^{kr}))}{\partial U/\partial C^{\kappa(i)}(Q_h^{\kappa(i)})} \]

\[ R_{ff}^{A(i)} = \frac{\theta(C^{A(i)} \partial U/\partial C^{A(i)}(Q_f^{A(i)} + p_f^{A(i)}Y_f^{kr}))}{\partial U/\partial C^{A(i)}(Q_f^{A(i)})} \]

\[ R_{ff}^{\kappa(i)} = \frac{\theta(C^{\kappa(i)} \partial U/\partial C^{\kappa(i)}(Q_f^{\kappa(i)} + p_f^{\kappa(i)}Y_f^{kr}))}{\partial U/\partial C^{\kappa(i)}(Q_f^{\kappa(i)})} \]

\[ R_V^{A(i)} = \frac{U(C^{A(i)}) + \theta(C^{A(i)})Y^{TA(i)}}{V^{A(i)}} \]

\[ R_V^{\kappa(i)} = \frac{U(C^{\kappa(i)}) + \theta(C^{\kappa(i)})Y^{TK(i)}}{V^{\kappa(i)}} \]

2. Construct the \( \hat{\phi}^{(i)} \) time-series, where at time-\( t \):

\[ \hat{\phi}^{A(i)}_{S_h} = S_h^{A(i)} p_h^{A(i)} / R_{fh}^{A(i)} \quad \hat{\phi}^{A(i)}_{S_f} = S_f^{A(i)} p_f^{A(i)} / R_{ff}^{A(i)} \]

\[ \hat{\phi}^{\kappa(i)}_{S_h} = S_h^{\kappa(i)} p_h^{\kappa(i)} / R_{fh}^{\kappa(i)} \quad \hat{\phi}^{\kappa(i)}_{S_f} = S_f^{\kappa(i)} p_f^{\kappa(i)} / R_{ff}^{\kappa(i)} \]

\[ \hat{\phi}^{A(i)}_{Q_h} = Q_h^{A(i)} R_{hh}^{A(i)} / R_{fh}^{A(i)} \quad \hat{\phi}^{A(i)}_{Q_f} = Q_f^{A(i)} R_{hf}^{A(i)} / R_{ff}^{A(i)} \]

\[ \hat{\phi}^{\kappa(i)}_{Q_h} = Q_h^{\kappa(i)} R_{hh}^{\kappa(i)} / R_{fh}^{\kappa(i)} \quad \hat{\phi}^{\kappa(i)}_{Q_f} = Q_f^{\kappa(i)} R_{hf}^{\kappa(i)} / R_{ff}^{\kappa(i)} \]

\[ \hat{\phi}^{A(i)}_{V^{TA}} = V^{TA(i)} R_{V}^{A(i)} \quad \hat{\phi}^{\kappa(i)}_{V^{TK}} = V^{TK(i)} R_{V}^{\kappa(i)} \]

Intuitively, the numerator of each Euler equation residual is the discounted marginal value of holding each security and the denominator is the marginal cost. If the policy function
coefficients are optimal then the entire residual series should be equal to one. Thus the $\hat{\varphi}^{(i)}$ will represent a fixed point of the system.

However, consider the home Euler equation residual for the home equity $R_h^{\kappa(i)}$ away from the fixed point of the system. Given the policy functions, there may be a point in time where the discounted marginal value of holding an asset exceeds the marginal cost, the residual will exceed one. Two non-exclusive possibilities exist, the home asset price or allocation of the home asset is too low. The foreign Euler equation is used to weight the relative importance of pricing and allocation errors. If the foreign Euler residual for the home equity $R_h^{\kappa(i)}$, similar to the home residual, exceeds one, then $\hat{\varphi}^{\kappa(i)}_S \approx S^{\kappa(i)}_h$, but $\hat{\varphi}^{\kappa(i)}_Q > Q^{\kappa(i)}_h$. Thus the updated (i+1) coefficients found in Step 3 will increase the price of home assets in that state, without changing allocations.

3. Find $\Phi^{(i+1)}$ by OLS. ($x^{(i)}$ and $\hat{\varphi}^{(i)}$ are de-meaned and standardized before OLS).

$$\tilde{\Phi}^{(i)} = (x^{(i)}') (x^{(i)} - \hat{\varphi}^{(i)})^{-1} (x^{(i)}')$$

Set $\Phi^{(i+1)} = (1 - \xi) \Phi^{(i)} + \xi \tilde{\Phi}^{(i)}$. Where $\xi$ is a dampening parameter. Because the algorithm is changing allocations and asset prices simultaneously in response to Euler equations errors, it is important that the dampening parameter be small particularly when policy functions are far from optimal.

4. Check convergence, if $|x^{(i)} \Phi^{(i)} / x^{(i-1)} \Phi^{(i-1)}| > \tau$, repeat B.1.2 with the updated policy function $\Phi^{(i+1)}$.

**B.2 Smolyak Iteration**

**B.2.1 Initialization**

1. Set approximation level $\mu$, stopping criterion $\tau$, and dampening parameter $\xi$.

2. Construct the Smolyak grid $\mathcal{H}^{d,\mu} = (x_1, \ldots, x_d)$ on $[-1, 1]^d$. Where $d$ is the dimension of the state space.

3. Compute the basis function $\mathcal{P}^{d,\mu}$ for each grid point. The resulting square matrix is the initial basis function $X_0$.

4. Choose a domain for the state space $[s, \bar{s}]$ and the operator $\Omega : (s) \rightarrow (x_1, \ldots, x_d)$ which transforms the model domain to the Chebyshev domain $[-1, 1]^d$.

5. Choose integration nodes, $\{ \epsilon_{h,j}^k, \epsilon_{f,j}^k \}$, and weights, $w_j$ for $j = 1, \ldots, J$.

---

*See Judd et al. (2014) for an efficient method for constructing a sparse Smolyak grid.*
6. Compute possible future income paths, \( Y_{1,j} = \{ Y_{h,1,j}^{k}, Y_{f,1,j}^{k}, Y_{h,1,j}^{f}, Y_{f,1,j}^{f} \} \) from each grid point for each integration node according to equation (B.1).

7. Choose an initial guess for the polynomial coefficients, \( \Psi = \left[ \Psi^{A(0)}, \Psi^{\kappa(0)} \right] \). which is a change in basis from the PEA coefficients found in Appendix B.1.

B.2.2 Iteration

For iteration (i) to find \( \Psi^{(i+1)} = \left[ \Psi^{A(i+1)}, \Psi^{\kappa(i+1)} \right] \).

1. At iteration \( i \), compute policy functions for each point on the grid:
   \[
   \Psi^{A(i)} = [S_{h}^{A(i)}, S_{f}^{A(i)}, Q_{h}^{A(i)}, Q_{f}^{A(i)}, EV^{A(i)}] \quad \text{and} \quad \Psi^{\kappa(i)} = [S_{h}^{\kappa(i)}, S_{f}^{\kappa(i)}, Q_{h}^{\kappa(i)}, Q_{f}^{\kappa(i)}, EV^{\kappa(i)}]
   \]
   For each possible decision, calculate consumption \( c^{A(i)}, c^{\kappa(i)} \), goods prices \( p_{h}^{A(i)}, p_{f}^{A(i)}, p_{h}^{\kappa(i)}, p_{f}^{\kappa(i)} \) and the value \( v_{A}^{(i)}, v_{\kappa}^{(i)} \) of each potential decision.

2. For each possible future state, \( j = 1, \ldots, J \): solve for wealth, \( \{ \omega_{j}^{A(i)}, \omega_{j}^{\kappa(i)} \} \), according to equation (B.5) for each possible decision. Future goods prices are solved in the process of solving for future wealth.

3. Compute the one-period forward basis function for each choice \( \lambda_{j}^{AA(i)} \) and \( \lambda_{j}^{\kappa(i)} \) using \( \{ w_{j}^{A(i)}, Y_{j}^{A} \} \) and \( \{ w_{j}^{\kappa(i)}, Y_{j}^{\kappa(i)} \} \).

4. Compute future state dependent policies for each possible decision path:
   \[
   \lambda_{j}^{A(i)} \Psi^{A(i)} = [S_{h}^{AA(i)}, S_{f}^{AA(i)}, Q_{h}^{AA(i)}, Q_{f}^{AA(i)}, EV^{AA(i)}] \quad \text{and} \quad \lambda_{j}^{\kappa(i)} \Psi^{\kappa(i)} = [S_{h}^{\kappa(i)}, S_{f}^{\kappa(i)}, Q_{h}^{\kappa(i)}, Q_{f}^{\kappa(i)}, EV^{\kappa(i)}]
   \]
   Calculate future consumption for each possible decision path and state:
   \[
   C_{j}^{AA(i)}, C_{j}^{A\kappa(i)}, C_{j}^{\kappa A(i)}, C_{j}^{\kappa\kappa(i)}
   \]
   The superscript indicates a series of decisions, for instance, \( C_{j}^{A\kappa(i)} \) is the future consumption in state \( j \) if the home agent did not participate in the foreign equity market (A) in the initial period but does participate in the future period (\( \kappa \)).

5. Calculate \( V_{j}^{A} = \max\{ v_{A,j}^{A(i)}, v_{A,j}^{\kappa(i)} \} \) and \( V_{j}^{\kappa} = \max\{ v_{A,j}^{A(i)}, v_{A,j}^{\kappa(i)} \} \) the time-1 value functions given each choice at time-0. For each possible transition path, set the future policy variables according to the choice with higher present value. For instance, suppose the home country participates only in the home equity market at time-0, if at time-1 and state-\( j \): \( v_{A,j}^{\kappa(i)} < v_{A,j}^{\kappa(i)} \), then set the one-period forward policy for state-\( j \) equal \( \lambda_{j}^{A(i)} \Psi^{\kappa(i)} \). (e.g. \( Q_{h}^{A(i)} = Q_{h}^{A\kappa(i)} \))
6. Construct Euler equation and value-function ‘residual’ time-series

\[
R_{hh}^{A(i)} = \frac{\theta(C^{A(i)})}{\frac{\partial U}{\partial C^{A(i)}} Q_h} \sum_j \left[ w_j \frac{\partial U}{\partial C^{A(i)}} (Q_{hj}^{A(i)} + p_{hj}^{A(i)} Y_{hj}^{k_f}) \right]
\]

\[
R_{hh}^{\kappa(i)} = \frac{\theta(C^{\kappa(i)})}{\frac{\partial U}{\partial C^{\kappa(i)}} Q_h} \sum_j \left[ w_j \frac{\partial U}{\partial C^{\kappa(i)}} (Q_{hj}^{\kappa(i)} + p_{hj}^{\kappa(i)} Y_{hj}^{k_f}) \right]
\]

\[
R_{hf}^{A(i)} = \frac{\theta(C^{A(i)})}{\frac{\partial U}{\partial C^{A(i)}} Q_f} \sum_j \left[ w_j \frac{\partial U}{\partial C^{A(i)}} (Q_{fj}^{A(i)} + p_{fj}^{A(i)} Y_{fj}^{k_f}) \right]
\]

\[
R_{hf}^{\kappa(i)} = \frac{\theta(C^{\kappa(i)})}{\frac{\partial U}{\partial C^{\kappa(i)}} Q_f} \sum_j \left[ w_j \frac{\partial U}{\partial C^{\kappa(i)}} (Q_{fj}^{\kappa(i)} + p_{fj}^{\kappa(i)} Y_{fj}^{k_f}) \right]
\]

\[
R_{fh}^{A(i)} = \frac{\theta(C^{A(i)})}{\frac{\partial U}{\partial C^{A(i)}} Q_h} \sum_j \left[ w_j \frac{\partial U}{\partial C^{A(i)}} (Q_{hj}^{A(i)} + p_{hj}^{A(i)} Y_{hj}^{k_f}) \right]
\]

\[
R_{fh}^{\kappa(i)} = \frac{\theta(C^{\kappa(i)})}{\frac{\partial U}{\partial C^{\kappa(i)}} Q_h} \sum_j \left[ w_j \frac{\partial U}{\partial C^{\kappa(i)}} (Q_{hj}^{\kappa(i)} + p_{hj}^{\kappa(i)} Y_{hj}^{k_f}) \right]
\]

\[
R_{ff}^{A(i)} = \frac{\theta(C^{A(i)})}{\frac{\partial U}{\partial C^{A(i)}} Q_f} \sum_j \left[ w_j \frac{\partial U}{\partial C^{A(i)}} (Q_{fj}^{A(i)} + p_{fj}^{A(i)} Y_{fj}^{k_f}) \right]
\]

\[
R_{ff}^{\kappa(i)} = \frac{\theta(C^{\kappa(i)})}{\frac{\partial U}{\partial C^{\kappa(i)}} Q_f} \sum_j \left[ w_j \frac{\partial U}{\partial C^{\kappa(i)}} (Q_{fj}^{\kappa(i)} + p_{fj}^{\kappa(i)} Y_{fj}^{k_f}) \right]
\]

\[
R_V^{A(i)} = \frac{\theta(C^{A(i)})}{\frac{\partial U}{\partial C^{A(i)}} V^{A(i)}} \sum_j \left[ w_j V^{A(i)} \right]
\]

\[
R_V^{\kappa(i)} = \frac{\theta(C^{\kappa(i)})}{\frac{\partial U}{\partial C^{\kappa(i)}} V^{\kappa(i)}} \sum_j \left[ w_j V^{\kappa(i)} \right]
\]
7. Construct the $\phi^{(i)}$ time-series, where at time-$t$:

$$
\hat{\phi}^{A(i)}_{s_h} = S^{A(i)}_{h} R^{A(i)}_{s_h} / R^{A(i)}_{f_{h}} \\
\hat{\phi}^{A(i)}_{s_f} = S^{A(i)}_{f} R^{A(i)}_{s_f} / R^{A(i)}_{f_{f}} \\
\hat{\phi}^{A(i)}_{q_h} = Q^{A(i)}_{h} R^{A(i)}_{q_h} R^{A(i)}_{f_{h}} \\
\hat{\phi}^{A(i)}_{q_f} = Q^{A(i)}_{f} R^{A(i)}_{q_f} R^{A(i)}_{f_{f}} \\
\hat{\phi}^{A(i)}_{v_x} = E V^{A(i)} R^{A(i)}_{V}
$$

$$
\hat{\phi}^{\kappa(i)}_{s_h} = S^{\kappa(i)}_{h} R^{\kappa(i)}_{s_h} / R^{\kappa(i)}_{f_{h}} \\
\hat{\phi}^{\kappa(i)}_{s_f} = S^{\kappa(i)}_{f} R^{\kappa(i)}_{s_f} / R^{\kappa(i)}_{f_{f}} \\
\hat{\phi}^{\kappa(i)}_{q_h} = Q^{\kappa(i)}_{h} R^{\kappa(i)}_{q_h} R^{\kappa(i)}_{f_{h}} \\
\hat{\phi}^{\kappa(i)}_{q_f} = Q^{\kappa(i)}_{f} R^{\kappa(i)}_{q_f} R^{\kappa(i)}_{f_{f}} \\
\hat{\phi}^{\kappa(i)}_{v_x} = E V^{\kappa(i)} R^{\kappa(i)}_{V}
$$

8. Calculate the coefficients which reproduce $\phi^{(i)}$ for the basis $x^0$:

$$
\tilde{\Psi}^{(i)} = x^0^{-1} \phi^{(i)}
$$

9. Update the policy coefficients using dampening parameter $\xi$:

$$
\Psi^{(i+1)} = (1 - \xi) \Psi^{(i)} + \xi \tilde{\Psi}^{(i)}
$$

10. Check convergence, if $|x_0^0 \Psi^{(i)}/x_0^0 \Psi^{(i-1)}| > \tau$, repeat B.2.2 with the updated policy function $\Psi^{(i+1)}$. 

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REFERENCES


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BIOGRAPHICAL SKETCH

Graham David Newell was born May 14, 1985 in Columbia, MD. He graduated from Virginia Polytechnic Institute and State University in May, 2008, earning a Bachelor of Arts degree with a major in Economics and minor in Business. He earned his Master of Arts degree in Economics from San José State University in May 2011 and Master of Science degree in Economics from Florida State University in August, 2015.