March 71

An Invariant Integral

Let $k_0^2 = k_1^2 + k_2^2 + k_3^2$ $\quad k_0 > 0$

Let $F(k)$ be a function of the k's.

$$I = \int_{\text{given region}} F(k) \frac{dk_1\, dk_2\, dk_3}{k_0} \quad \text{is Lorentz invariant.}$$

Take $F(k)$ to be homogeneous of degree -2 in the k's

so $F(k) = \frac{1}{k_0^2} \mathcal{F}(l) \qquad l_r = \frac{k_r}{k_0}$

Take the given region to be $\quad f(l) < k_0 < r f(l)$

for some function f of the l's, with r indep. of the l's.

Then $I = \int \frac{1}{k_0^3} \mathcal{F}(l)\, d^3k = \int \frac{1}{k_0} dk_0\, \mathcal{F}(l)\, d\Omega = \int \mathcal{F}(l)\, d\Omega \int_{k_0=f}^{k_0=rf} \frac{dk_0}{k_0} = \log r \int \mathcal{F}(l)\, d\Omega$

$\frac{1}{\log r} I = \int \mathcal{F}(l)\, d\Omega$. This is a Lorentz invariant integral. Write it as $\int F(k)$ symbolically

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Consider two wave fns ψ and ψ' that are homog. in the k's of degree λ.

Define $\langle \psi | \psi' \rangle = \iint \frac{\bar{\psi}(k)\, \psi'(k')}{(k \cdot k')^{\lambda+2}}$