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A Longitudinal Study on Predictors of Early Calculation Development among Young Children At-Risk for Learning Difficulties

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Abstract

The purpose of this study was to explore domain-general cognitive skills, domain-specific academic skills, and demographic characteristics that are associated with calculation development from first through third grade among young children with learning difficulties. Participants were 176 children identified with reading and mathematics difficulties at the beginning of first grade. Data were collected on working memory, language, nonverbal reasoning, processing speed, decoding, numerical competence, incoming calculations, socioeconomic status, and gender at the beginning of first grade and on calculation performance at 4 time points: the beginning of first grade, the end of first grade, the end of second grade, and the end of third grade. Latent growth modelling analysis showed that numerical competence, incoming calculation, processing speed, and decoding skills significantly explained the variance of calculation performance at the beginning of first grade. Numerical competence and processing speed significantly explained the variance of calculation performance at the end of third grade. However, numerical competence was the only significant predictor of calculation development from the beginning of first grade to the end of third grade. Implications of these findings for early calculation instructions among young at-risk children are discussed.

Keywords

calculations; processing speed; numerical competence; learning difficulties

Calculation competence is a mathematical skill children learn and develop in early elementary grades, and yet it represents a major challenge for many young children (National Council of Teachers of Mathematics, 2006). Because of the hierarchical nature of

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mathematics skills, the development of calculations serves as a foundation for developing increasingly advanced mathematical skills, such as algebra (Ashcraft, 1992; Jensen & Whang, 1994; National Council of Teachers of Mathematics, 2006). Therefore, weak calculation skills pose substantial problems for children's mathematics development in addition to their daily activities. Given the importance of calculations in early childhood, it is critical to understand factors that contribute to the early development of calculation competence. Such knowledge can guide curricular development and instructional interventions aimed at remediating poor calculation performance early on.

Prior studies examining factors that influence calculation competence have shed some light on underlying mechanisms of calculation development (e.g., Berg, 2008; Cowan & Powell, 2014; Cowan et al., 2011; Fuchs et al., 2005; Fuchs et al., 2010). These studies suggest that various domain-general cognitive skills, domain-specific academic skills, and demographic factors contribute to calculation competence. However, a majority of the studies involve typically developing children. Only a few target children with learning difficulties. We identified only two such studies that specifically addressed calculation development among children with learning difficulties (i.e., Alloway, 2009; Namkung & Fuchs, 2016). Specifically, Alloway (2009) investigated whether domain-general cognitive skills, including working memory and IQ, in children with learning difficulties between the ages of 7 and 11 predicted comprehensive mathematics skills, which included calculations and mathematics reasoning, two years later. Results indicated that only working memory was a significant predictor. However, Alloway (2009) included only two cognitive predictors, and the mathematics skills in her study were indexed by calculations and mathematics reasoning, which were not likely to represent a complete picture of the components involved in early calculation development.

Namkung and Fuchs (2016) extended Alloway (2009)'s findings by using a broader set of domain-general cognitive skills and domain-specific academic skills — working memory, processing speed, language, attention, nonverbal reasoning, and incoming calculations (i.e., calculation knowledge acquired prior to the study) — at the beginning of fourth grade to predict calculation performance at the end of fourth grade. They found that processing speed, attentive behavior, and incoming calculations uniquely predicted whole-number calculation competence. Although Namkung and Fuchs (2016) included a more comprehensive set of skills, their samples were children with learning difficulties in the intermediate grades, which could not reveal the importance of each skill in early calculation development (i.e., first grade - third grade).

Moreover, all prior studies on calculation development focused on what skills explain calculation performance concurrently (e.g., Berg, 2008; Cowan & Powell, 2014), or focused on how these skills predict calculation performance later in the intermediate grades (e.g., Alloway, 2009; Cowan et al., 2011; Fuchs et al., 2005; Fuchs et al., 2010, Namkung & Fuchs, 2016). We are unaware of any longitudinal study that has relied on domain-general cognitive skills, domain-specific academic skills, and demographic factors to predict early development (i.e., change rate/slope) of calculations, especially among young children with learning difficulties. In the present study, we used latent growth modeling, to explore early childhood cognitive, academic, and demographic factors that predict calculation development from the beginning of first grade to the end of third grade among children identified with learning difficulties at the beginning of first grade. Such analysis may help to establish a link between those children's cognitive/academic/demographic factors and development in calculations, and identify foci of school readiness programs for at-risk children. In the following sections, we briefly explain the rationale for the cognitive, academic, and demographic variables considered in the study, and how each variable may affect calculation development.

Domain-General Cognitive Skills

Language

Most prior studies have found language to be significantly associated with word problem-solving, but not calculations (e.g., Fuchs et al., 2005, 2006, 2008 e.g., Fuchs et al., 2010a, 2010b, 2013). Nevertheless, it may still be an important predictor to consider because language is the principal medium of mathematics instruction for children in early elementary grades, such as learning to count and mastering the number word sequence. Many classroom calculation activities also require articulatory processes (e.g., verbalizing the answer to the problem, "3 + 2 = ?"). In fact, some studies demonstrated that the vocabulary level of children is related to basic calculation proficiency (Durand et al., 2005; Hecht et al., 2001; Geary, 1993), and language ability has sometimes accounted for more variation in basic calculation proficiency than other cognitive skills, such as nonverbal reasoning, working memory, and quantitative skills (Cowan et al., 2005; LeFevre et al., 2010). Such evidence suggests that students with strong language ability may gain deeper understanding of calculation concepts compared to those with weak language ability, and a better understanding of calculation concepts may in turn facilitate procedural calculations as conceptual and procedural understandings have been found to influence each other (e.g., Rittle-Johnson & Siegler, 1998; Rittle-Johnson, Siegler, & Alibali, 2001).

Nonverbal reasoning

Nonverbal reasoning may also be an important predictor of calculations. Nonverbal reasoning refers to the cognitive processes of identifying, categorizing, and determining rules and concepts to solve a novel problem. Theoretically, calculation processes may involve nonverbal reasoning by allowing students to recognize and analyze quantitative relations. For example, to solve 2+3 = 6 - _, young children need to understand the two sides of the equal sign represents the same quantity before choosing or comparing strategies to solve the equation (e.g., 6-3−2 or 6-(3+2)). Moreover, pedagogy in general classroom links nonverbal reasoning to calculation development. That is, calculation problems in today's curricula often involve real-life situations, allowing students to generate their mathematical knowledge to novel situations and to engage in discussion on problem solutions and multiple strategies to solve the problem (e.g., Bottge, Heinrichs, Chan, & Serlin, 2001; Clements & Battista, 1990; McCaffrey, Hamilton, Stecher, Klein, Bugliari, & Robyn, 2001). This would draw heavily on children's nonverbal reasoning abilities.

Although the relation between nonverbal reasoning and calculation is both theoretically and pedagogically meaningful, only a few studies have directly investigated how nonverbal

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reasoning affects calculations, and mixed findings exist. For example, Steeves (1983) found nonverbal reasoning correlated moderately with incoming calculations among elementary school children ($r = .58$). In contrast, in two Fuchs et al. studies (2005, 2006), nonverbal reasoning significantly predicted word problem-solving, but not calculations among first and third graders, after controlling for other cognitive skills. Although this may suggest that nonverbal reasoning may be more important for advanced mathematics skills (e.g., word problems) than calculations, further study is warranted given the small and inconsistent findings in the literature.

Working memory

Working memory is the ability to simultaneously process and store information to support ongoing cognitive tasks (Baddeley et al., 1986). It plays an important role in performing calculations because executing calculation procedures often requires a combination of storing temporal information while simultaneously performing other mental operations. For instance, to solve the problem: 15 + 6, young children must concurrently retain two or more pieces of information in short-term memory (phonological codes representing the numbers 15 and 6) and employ one or more procedures (e.g., counting) to combine the numbers while attending to place value to derive a correct answer.

Working memory may be especially important for developing calculation fluency among young children. Specifically, greater working memory capacity facilitates execution of computation strategies, making accurate solutions more likely. It also enhances the likelihood of storage of the problem and its solution, thus strengthening the associations between problems and their answers in long-term memory. This in turn enables accurate and fast retrieval of calculation facts (e.g., automatic retrieval of 6 + 5 = 11) (e.g., Geary, Brown, & Samaranayake, 1991; Geary et al., 2004; Hitch & McAuley, 1991; Siegel & Linder, 1984; Webster, 1979; Wilson & Swanson, 2001). Moreover, in a recent consideration of dual process theory in reasoning and decision-making, Evans and Stanovich (2013) distinguished between rapid autonomous processing that produces “default” responses and higher order reasoning processes that require working memory resources. For young children, very little about mathematical symbols and operations is likely to be automated. Therefore, mathematical performance on even simple calculation tasks may require working memory. If so, working memory may be especially important for calculation development among young children with learning difficulties, because those children usually do not have sufficient calculation knowledge and thus are more likely to rely on working memory in calculation tasks. This thinking is partially supported by Alloway (2009), who found that working memory was the only significant cognitive factor associated with mathematics skills that tap calculations and mathematics reasoning among children with learning difficulties. However, as mentioned, Alloway (2009) studied only two cognitive skills (IQ and working memory) and her mathematics measure was a mix of calculations and mathematics reasoning. Thus, the importance of working memory in the calculation development is still unclear among young children with learning difficulties.

Processing speed

Processing speed is another important cognitive factor that may influence calculation development. Processing speed refers to the efficiency with which information is processed. According to Processing Speed Theory, processing speed is a fundamental mechanism for higher-level cognition because it greatly influences the availability of information for advanced cognitive processing (Salthouse, 1996). For calculations, processing speed may facilitate fluent counting for figuring out answers, thereby permitting associations between problems and their answers to be held in working memory and then committed to long-term memory (Geary, 1993). With slower processing, the interval increases for deriving counted answers and for pairing a problem stem with its answer in working memory; this creates the possibility that “decay” sets in before completing the computational sequence.

In fact, Bull and Johnston (1997) found that processing speed was the best predictor of calculation competence among seven-year-olds, when they controlled for long- and short-term memory and reading performance. More recently, Fuchs et al. (2006) found that after accounting for language, working memory, phonological skills, and nonverbal reasoning, processing speed still contributed to calculation fluency for third graders. Moreover, research shows that processing speed deficit is the most prominent cognitive feature among children with mathematics difficulties (e.g., Cirino, Fuchs, Elias, Power, & Schumacher, 2015; Fuchs et al., 2006); that children with mathematics difficulties are slower at performing calculation tasks compared to their typically developing peers (Geary & Brown, 1991; Jordan & Montani, 1997; Fuchs et al., 2008); and that processing speed predicts calculation performance among children with learning difficulties in the intermediate grades (Namkung & Fuchs, 2016). However, other studies have indicated that children with weak calculation performance do not show deficits on timed measures (e.g., Andersson & Lyxell, 2007; Jordan et al., 2003; Moll, Gobel, Gooch, Landerl, & Snowling, 2014). Thus, further study of the relation between processing speed and calculation development is warranted, especially among children with learning difficulties.

Domain-Specific Academic Skills

Numerical competence

Besides the cognitive factors addressed above that apply across academic domains, children may have specialized early numerical competence that relates to calculation development (e.g., Baroody, Bajwa, & Eiland, 2009; Fuchs et al., 2010). Numerical competence, here, refers to a fundamental understanding of numbers, which many children acquire before formal schooling. Such competence includes abilities related to counting, number patterns, magnitude comparisons, estimating, and number transformation (Berch, 2005; Namkung & Fuchs, 2012). Prior studies have demonstrated that early numerical competence predicts later mathematics achievement (e.g., Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Koponen, Aunola, Ahonen, & Nurmi, 2007; Krajewski & Schneider, 2009; Passolunghi, Vercelloni, & Schadee, 2007), even after controlling for individual differences in domain-general cognitive abilities (e.g., Duncan et al., 2007; Halberda et al., 2008). However, these studies investigated the effects of numerical competence on mathematics achievement in general, and further study on how numerical competence affects early calculations is needed.

Decoding

Another potentially important academic skill for calculations is decoding. Based on the phonological representation hypothesis, completing simple calculation problems requires the retrieval of phonological codes as well as encoding and maintaining phonological representations in short-term memory (Geary, 1993; Simmons & Singleton, 2008; Vukovic & Lesaux, 2013). This hypothesis also suggests that weaknesses in decoding limit the development of skills that rely on the manipulation and storage of numerical codes, such as counting and solving simple calculations. Thus, decoding may have both direct and indirect effects (through numerical competence) on calculations (LeFevre et al., 2010). Some research supports direct effects of decoding on calculations. For example, Fuchs et al. (2005) found that decoding emerged as the most important predictor of calculation development from the fall to spring of first grade after controlling for a host of competing cognitive variables. Vukovic and Lesaux (2013) found among third graders, only decoding had a direct relation with calculation performance (numerical competence did not mediate decoding effects on calculations). Hecht et al. (2001) found that reading skills in second grade indexed by a composite of word reading, nonword reading, and reading comprehension, which is substantially related with decoding skills, make a unique prediction of calculation skills in later grades after controlling for phonological processing and vocabulary. On the other hand, research also documents that some children with calculation difficulties are nonetheless good word readers and vice versa (e.g., Fuchs, Fuchs, & Prentice, 2004; Landerl, Fussenegger, Moll, & Willburger, 2009), suggesting decoding may not be an influential factor in calculation development. Thus, mixed findings exist regarding the phonological representation hypothesis. In the present study, we included decoding to further examine its influence on calculation development, and we explored whether early numerical competence mediates the effect of early decoding on calculation performance and development.

Socioeconomic Status and Gender

We also included two demographic variables as possible predictors of calculation development: socioeconomic status (SES) and gender. SES generally affects children’s achievement such that there is a gap in overall academic performance between children with low and moderate/high SES (e.g., White, 1982). In addition, research suggests that many children from low SES develop academic skills more slowly compared to their peers from higher SES (Morgan, Farkas, Hillemeier, & Maczuga, 2009). However, the relation between SES and specific domains of academic performance remains unclear. While some studies suggest that SES is more closely related to performance on language-related tasks, such that children with low SES tend to show poorer performance on word problem-solving than calculations (e.g., Fuchs et al., 2008; Jordan, Huttenlocher, and Levine, 1992), some suggest that low SES is also associated with slow development in early numerical competence, which is related to early calculations (Jordan, Kaplan, Locuniak, & Ramineni, 2007).

Mixed findings also exist with respect to gender. Whereas there are data to indicate that boys retrieve numerical facts faster than girls during calculation processes and develop faster on numerical skills (e.g., Jordan et al., 2007; Royer, Tronsky, Chan, Jackson, & Marchant, 1999), there are also findings that girls have stronger calculation skills than boys in the elementary grades (Hyde, Fennema, & Lamon, 1990). Yet, some research suggests that gender may not differentially affect mathematics skills in early elementary grades (e.g., Friedman, 1989; Hyde, Lindberg, Linn, Ellis, & Williams, 2008). However, virtually all these findings are based on typically developing children. It remains unclear whether gender affects calculation performance and development among children with learning difficulties.

Children with Mathematics and Reading Difficulties

In the present study, we focused on children with mathematics and reading difficulties (MDRD), because their cognitive and academic profile associated with calculation development differs from that of typically achieving children and children with other types of learning difficulties. Specifically, the learning difficulty group has subgroups including children with only reading difficulties (RD), children with only mathematics difficulties (MD), and children with MDRD. The MDRD is the most common subgroup (Badian, 1999; Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005) and has a unique cognitive and academic profile (Cirino et al., 2015).. Specifically, the MDRD group often demonstrates much lower performance on mathematics and reading than the RD or MD group (e.g., Andersson & Lyxell, 2007; Cirino et al., 2015). It is suggested that because of their poor domain-specific knowledge in mathematics and reading, children with MDRD often lack effective strategies or the ability to directly retrieve facts from long-term memory to help accomplish mathematics tasks, and thus they would, instead, rely more on their domain-general cognitive skills, such as WM and nonverbal reasoning, to help them solve mathematics problems (e.g., Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Peng, Namkung, Barnes, & Sun, 2016). Based on this view, domain-general cognitive skills, such as working memory, nonverbal reasoning, and processing speed may play a compensatory (more important than domain-specific numerical knowledge and decoding) role in mathematics tasks among children with MDRD, which may be more salient in early childhood when basic mathematics knowledge is being built up. Thus, with the MDRD group, it is expected that their cognitive skills may be significant predictors for their early calculation performance and development.

However, there are also studies showing that compared to the MD or RD group, the MDRD group demonstrates a different cognitive profile such that the MDRD group tends to show more severe deficits in working memory, processing speed, language, and nonverbal reasoning (e.g., Cirino et al., 2015; Peng, Sun, Li, & Tao, 2012). Thus, an alternative reasonable hypothesis is that the insufficient cognitive skills of children with MDRD may not be able support their early calculation performance and development. In this study, by specifically focusing on the MDRD group and by including a wide range of domain-general cognitive skills and domain-specific academic skills, we further examined these two competing hypotheses. That is, whether domain-general cognitive skills (e.g., working memory, language, IQ, and processing speed) or domain-specific skills (e.g., numerical competence and decoding) predict early calculation performance and development among children with MDRD.

Present Study

In sum, to address limitations in previous studies and to extend the body of literature on early calculation development, we included a relatively complete set of cognitive, academic, and demographic variables that have been previously identified as important predictors of calculations. More importantly, we examined the developmental trajectory of calculation competence among young children with MDRD longitudinally, rather than concurrently. Specifically, at the beginning of first grade, we collected data on working memory, nonverbal reasoning, processing speed, numerical competence, incoming calculations, decoding, language, SES, and gender. We followed these children through third grade and collected data on their calculation skills at four time points: the beginning of first grade, the end of first grade, the end of second grade, and the end of third grade. We modelled their calculation development using latent growth modeling, which allows us to understand how these variables interrelate in complex patterns and their unique roles in predicting the development of calculations among these children. Moreover, we examined whether early numerical competence mediates the effect of early decoding on calculation performance and development.

Methods

Participants

Participants were 176 children from 114 classrooms of 21 elementary schools (the number of students from each school ranging 1–18, with a median of 7) in a mid-sized city in the Southeastern United States. They were originally from the control group (i.e., students receiving regular classroom instructions) of a larger study investigating the efficacy of a reading intervention. As part of this larger study, children had been identified by their teachers in the fall of first grade as with MDRD. We individually tested this larger group of teacher-nominated children with a battery of reading measures that included timed and untimed tests of rapid letter naming, phonemic decoding, and word recognition. A factor score was derived for each child based on their performance on these measures, and these children were then rank ordered by their factor scores. The top 50% of the factor score in this sample were eliminated from study participation. Children who performed below a T-score of 37 (i.e., corresponding with a percentile rank of 10, or corresponding with the lowest standard score – 80 – within the “low average” range of IQ) on both the Vocabulary and Matrix Reasoning subtests of the Wechsler Abbreviated Scale of Intelligence were also excluded from this study due to the possible presence of intellectual disabilities.

Moreover, we used Wide Range Achievement Test 4 – Math Computation (WRAT-4) to identify children with mathematics difficulties (e.g., Fuchs, Fuchs, Compton, Bryant, Hamlett, & Seethaler, 2007) for two reasons. First, calculation is one of the most important mathematics skills young children learn in early elementary grades and yet, it represents one of the most common deficits in children with MD. Second, calculation involves little linguistic comprehension (unlike problem-solving). As suggested by most previous studies (e.g., Fuchs et al., 2004; Geary, 2004; McLean & Hitch, 1999; Wilson & Swanson, 2001), we used the 25th percentile as the cutoff criterion on WRAT-4 for identifying children with MD. Thus, the children in this study were identified with learning difficulties in both reading and mathematics based on the judgment of their classroom teachers and our screening measures on reading and calculations at the beginning of first grade. Their mean age at the beginning of first grade was 6.64 (*SD* = 0.42). Of these 176 students, 79 (44.9%) were male, 14 (8%) were English language learners, 135 (76.7%) received a subsidized lunch, and 24 (13.6%) had a school-identified disability. Race was distributed as 80 (45.5%) African American, 59 (33.5%) White, 17 (9.7%) Hispanic, and 20 of (11.4%) other races. Our research received appropriate Institutional Review Board (IRB) approval from all appropriate agencies and participants.

Measures

At the beginning of first grade, we tested these children on measures tapping working memory, language, nonverbal reasoning, decoding, processing speed, numerical competence, and incoming calculations. We also tested these children on calculations at the beginning of first grade, the end of first grade, the end of second grade, and the end of third grade. Moreover, demographics information including free/reduced lunch (indicating SES) and gender were collected at the beginning of first grade. Table 1 provides the descriptive statistics of all measures/variables included in this study.

Incoming calculations—We used the Addition and Subtraction Fact Fluency Test Battery (Fuchs, Hamlett, & Powell, 2003), which includes four tests tapping addition and subtraction skills. There are two sets of addition tests. One comprises 25 addition fact problems with answers from 0 to 8 and with addends from 0 to 8. Another comprises 25 addition fact problems with answers from 0 to 12 and with addends from 0 to 9. For subtraction, one comprises 25 subtraction fact problems with answers from 0 to 8 and with minuends/subtrahends from 0 to 8. One comprises 25 subtraction fact problems with answers from 0 to 12 and with minuends/subtrahends from 0 to 18. Problems are presented horizontally on one page. For each test, children have 1 min to write answers for each test. The score is the number of correct answers. The average Cronbach's alpha is .63 for the current sample.

Calculations—The arithmetic subtests from the Wide Range Achievement Test - 4 (WRAT-4; Wilkinson & Robertson, 2006) were administered at four time points: the beginning of first grade, the end of first grade, the end of second grade, and the end of third grade. There are two parts in WRAT-4: oral portion, consisting of 15 items measuring oral counting and number recognition, and written portion, consisting of 40 calculations items with increasing difficulty. Following the standard administration procedure of WRAT-4, we administrated the oral portion and written portion. The number of items answered correctly is the total score. The Cronbach's alpha for the first two time points are .52 and 90, respectively, on this sample. The reported Cronbach's alpha for second and third grade are above .80.

Decoding—We used Rapid Sound Naming (Fuchs et al., 2001) and Phonemic Decoding from the Test of Word Reading Efficiency (TOWRE) to measure decoding. With Rapid Sound Naming, children are shown a page with the 26 letters of the alphabet displayed in a random order and have 1 min to say the sound of each letter. The score is the number of correct sounds. If the child finishes before 1 min, the score is prorated. Cronbach's alpha is .89 for the current sample. With Phonemic Decoding Efficiency, children are required to pronounce as many phonetically regular non-words as they can from a list in 45s. Cronbach's alpha is .66 for the current sample.

Numerical competence—We used the numerical competence subtest from KeyMath-3 (Connolly, 2007), which measures children's understanding of numbers, to index early numerical competence. In this test, the tester uses a testing booklet that has both pictures and numbers on each page. Children are asked to answer 24 questions that tap skills of identifying numbers, counting objects, representing numbers on a number line, comparing magnitudes, and rounding one-, two-, and three-digit numbers as well as advanced skills, such as rational numbers, exponents, scientific notation, and square roots. However, in the beginning-of-first-grade range of performance, the KeyMath-3 numerical competence almost entirely samples basic competencies with one or two digit whole numbers. The testing is discontinued after 3 consecutive scores of 0. Children earn 1 point for each question answered correctly. Cronbach's alpha is .53 for the current sample.

Working memory—We used Listening Recall and Backward Digit Recall from the Working Memory Test Battery for Children Listening Recall (Pickering & Gathercole, 2001). For Listening Recall, the child listens to a series of short sentences, judges the veracity of each by responding "yes" or "no," and then recalls the final word of each of the sentences in sequence. There are six trials at each set size (1 to 6 sentences per set). The score is the number of trials recalled correctly. To lower the floor of this assessment for first graders, we modified its administration such that we gave feedback to the children on the first three test items. The test stopped when a child incorrectly answered three items within a set. Cronbach's alpha is .85 for the current sample. With Backward Digit Recall, the tester says a string of random numbers, and children say the series backwards. Item difficulty increases as more numbers are added to the series. We also gave feedback to the children on the first three test items to lower the floor of this assessment for first graders. The test stopped when a child incorrectly answered three items within a set. The score is the number of trials recalled correctly. Cronbach's alpha is .87 for the current sample..

Processing speed—We used the Cross Out subtest from the Woodcock-Johnson III (WJ-III; Woodcock, McGrew, & Mather, 2001). This test requires children to locate and circle five pictures that match a target picture in that row; children have 3 min to complete 30 rows and earn 1 point for each row they answer correctly. The reported test-retest reliability coefficient is .91.

Language—We used WASI Vocabulary (Wechsler, 1999) and Listening Comprehension from Woodcock Diagnostic Reading Battery (WDRB) (Woodcock, 1997). WASI Vocabulary measures expressive vocabulary, verbal knowledge, and foundation of information with 42 items. The first four items present pictures; children identify the object in the picture. For remaining items, the tester says a word that children define. Responses are awarded a score 0, 1, or 2 depending on quality. Testing is discontinued after five consecutive scores of 0. The score is the total number of points. Cronbach's alpha is .74 for the current sample.

Listening Comprehension measures the ability to understand sentences or passages that the tester reads. Children supply the word missing at the end of sentences or passages that progress from simple verbal analogies and associations to discerning implications. Cronbach's alpha is .82 for the current sample.

Nonverbal Reasoning—We used Concept Formation from Woodcock-Johnson Psycho-Educational Battery–Revised (WJ–III) (Woodcock, 1997) and Wechsler Abbreviated Intelligence Scale Matrix Reasoning (Wechsler, 1999). Concept Formation asks children to identify the rules for concepts when shown illustrations of instances and non-instances of the concept. Children earn 1 point by correctly identifying the rule that governs each concept. The score is the number of correct responses. Cronbach's alpha is .74 for the current sample. With Matrix Reasoning, children look at a matrix from which a section is missing and completes it by saying the number or pointing to one of five response options. Children earn 1 point by identifying the correct missing piece of each matrix. Testing is discontinued after four errors on five consecutive items or four consecutive errors. The score is the number of correct responses. Cronbach's alpha is .75 for the current sample.

Procedure

Trained research assistants (RAs) administered all tests at the beginning of first grade (in late August and early September) in four sessions. All cognitive and academic tests were administered to children individually in the quietest place available at their schools. The RAs administered WRAT-4 at the end of first grade, the end of second grade, and the end of third grade in one whole-class session. All children were tested by the RAs with whom they had no familiarity. For the testing at the beginning of first grade, each of the four sessions lasted about 60 min. Two project staff trained the RAs in multiple sessions during which different tests were introduced. Each training session began with project staff explaining the purpose and designs of the tests and then modeling their proper administration. The RAs next role-played as examiner and examinee and obtained immediate corrective feedback from the project staff. Following this training, the RAs were required to find partners and practice test administration for at least 10 hours prior to pretreatment testing. Two days after training, each RA "tested" the project staff on all measures. Staff recorded RA performances on detailed checklists for each test. The RAs were required to achieve at least 90% accuracy when administering and scoring every test. If they performed below 90% on one or more test, they were required to complete additional training and try again to meet administration and scoring criteria. The RAs were not permitted to test children before they did so. All test sessions were audiotaped; 20% of tapes were randomly selected, stratifying by tester, for accuracy checks by an independent scorer. Agreement on test administration and scoring exceeded 90%.

Data Analysis

Data analysis progressed in six stages. First, because we had the most missing data on numerical competence and WRAT-4 at time 4, we examined whether the data were missing at random. Our analysis showed that students with missing data on numerical competence or WRAT-4 at time 4 did not differ from those with data on all cognitive, academic, and demographic variables in this study ($ps > .13$). Thus, the missing data on numerical competence or WRAT-4 at time 4 do not depend on other variables measured in this study, which indicates those missing data were missing at random (Rubin, 1976).

Second, we log transformed the skewed measures and created a factor score using principal factor analysis for nonverbal reasoning (based on Matrix Reasoning and Concept Formation), working memory (based on Listening Recall and Backward Digit Recall), language (based on Vocabulary and Listening Comprehension), incoming calculations (based on Addition 0–12, Addition 0–8, Subtraction 0–12, Subtraction 0–8), and decoding (based on Rapid Sound Naming and Phonemic Decoding Fluency).

Third, because the data on WRAT-4 across were collected from different schools, we examined the Intra-Class Correlation Coefficients (ICCs) to evaluate school effects on WRAT-4. Results showed that the ICCs for schools were less than 1% and not significant at each time point and for the slope of WRAT-4. Thus, we did not apply multi-level analyses to these data.

Fourth, we tested the baseline latent growth model that estimated intercept (mean level) and slope (rate of change) in calculations across the four time points. In line with latent growth modeling conventions (Byrne, 2010), two latent traits — intercept and slope — were extracted from WRAT-4 variables at 4 time points. Intercept was centered at the beginning of first grade as well as at the end of third grade. The slope was defined as the change of children' calculation performance across the four time points. Intercept and slope were allowed to covary as to account for the association between the level of calculations and the rate of change. Residual covariance was estimated to represent the time-varying covariates that have not been included in the model.

Fifth, building upon the baseline latent growth model, we ran structural models to test the effects of nonverbal reasoning, working memory, language, incoming calculations, decoding, processing speed, numerical competence, free/reduced lunch (SES), and gender at the beginning of first grade on the intercepts (at the beginning of first grade and at the end of the third grade) and slope of calculations. Path analysis was used to model the covariance structure among the predictors, intercept, and slope. Because we did not expect gender to covary with SES, we did not include the covariance between gender and free/reduced lunch in the model.

Last, following the recommendation by Baron and Kenny (1986) and Preacher and Kelley (2011), we examined whether numerical competence mediated the relationship between decoding and calculation performance at the beginning of first grade and development. We described the mediation data-analytic steps in greater detail in the results section.

Model Evaluation Criteria

All analyses on the two models were carried out using the Mplus statistical software version 7.0 (Muthen & Muthen, 2012). Full information Maximum Likelihood Estimation was employed to construct models using a scaled Chi-square estimated with robust standard errors to handle missing data points (missing at random) (Arbuckle, 1996). Because of our relatively small sample, we adopted the bootstrapping method (with bootstrap values of draws n = 10000) to check the stability of the findings (Chernick, 1999). Model fit was assessed with the Chi-square Test (χ^2) of model fit, the Comparative Fit Index (CFI), the Tucker- Lewis Index (TLI), Standardized Root Mean Square Residual (SRMR), and the Root Mean Square Error of Approximation (RMSEA). The non-significant of χ^2 test of model fit indicates a good model fit. That said, χ^2 test of model fit is sample size dependent and performs optimally in a range of at least 200–400 (Kenny, 2012). CFI and TLI indicate an adequate model fit at values of .90 or above (Hu & Bentler, 1999). For SRMR, a value less than .08 is generally considered a good fit (Hu & Bentler, 1999). RMSEA values of .08 and below are considered acceptable (Browne & Cudeck, 1993). However, models with small dfs (df < 70) can have artificially large values of the RMSEA, and for this reason, RMSEA for low-df models, such as ours, is not reliable and thus is considered as an optional index for the evaluation of model fit (Kenny, Kaniskan, & McCoach, 2014).

Results

Baseline Model

The model fit indices for the baseline models are as follows, $\chi^2 = 5.67$, df = 2, $p = .06$, CFI = .98, TLI = .94, RMSEA = .10, and SRMR =.03. As mentioned, the small dfs make RMSEA unreliable (Kenny et al., 2014). Thus, despite the relatively large RMSEA value, we considered the baseline latent growth models have a good model fit based on other model fit indices. Overall, the variance of the intercepts and slope was significant, *ps* < .01, indicating that even among those young low-performing children, there were still individual differences on their calculations at the beginning of first grade and at the end of third grade as well as on their growth of calculations from the beginning of first grade to the end of third grade. Table 3 presents the path coefficients and covariance for the baseline models estimated based on Maximum Likelihood Estimation and Bootstrapping in Mplus 7.0.

Structural Model

The model fit indices for the structural models are as follows, $\chi^2 = 29.23$, df = 20, $p = .08$, CFI = .98, TLI = .95, RMSEA = .05, and SRMR = .04, indicating a good model fit. The path coefficients and covariance for the structural model are presented in Table 4. Specifically, incoming calculations, processing speed, decoding, and numerical competence have unique effects on calculations at the beginning of first grade controlling for other skills. That is, children with faster processing speed, richer prior calculation knowledge, stronger decoding and numerical competence tend to show stronger performance on calculations at the beginning of first grade. Processing speed and numerical competence also predicted unique variance of calculations at the end of third grade while other skills were controlled. That is, children with faster processing speed and richer prior calculation knowledge tended to show stronger performance on calculations at the end of third grade. More importantly, numerical competence at the beginning of first grade also significantly predicted the development of calculations across time up to third grade with other skills controlled. That is, children with learning difficulties with relatively stronger numerical competence at the beginning of first grade tend to improve faster on calculations from first grade to third grade. Although we found that processing speed predicted the calculation development on a marginally significant level, $p = .06$, this marginal significance was gone after we ran the model with bootstrapping (Bootstrapping 95%CI [−.001 .15] of unstandardized coefficient). Other variables at the beginning of first grade, including free/reduced lunch, gender, working memory, nonverbal reasoning, and language, did not exert significant impact on the beginning level of calculations, the level of calculations at the end of third grade, or the development of calculations. Regarding the R^2, all the predictors explained 45.2%, 61.6%, and 15.6% of variance in the intercept at the beginning of first grade, the intercept at the end of third grade, and the slope, respectively.

Last, we examined whether numerical competence mediated the effect of decoding on the calculation performance at the beginning of first grade and calculation development through third grade. Specifically, we first ran the full structural model excluding numerical competence in the model. Results showed that decoding significantly explained variance in calculation performance at the beginning of first grade, standardized coefficient = .16, $p <$.01, Bootstrapping 95%CI [.12, .55], but did not explain variance in in calculation performance at the end of third grade, standardized coefficient = .13, $p <$.01, Bootstrapping 95%CI [−.12, .89], or the development of calculations, standardized coefficient = −.02, $p =$ .88, Bootstrapping 95%CI [−.20, .16]. Thus, we next only examined whether numerical competence mediate the relation between decoding and calculation performance at the beginning of first grade. We ran the full structural model, having decoding predicting numerical competence. Results showed that decoding significantly explained variance in the calculation performance at the beginning of first grade, standardized coefficient = .14, $p <$ .01, Bootstrapping 95%CI [12, .54], and variance in numerical competence, standardized coefficient = .02, $p < .05$, Bootstrapping 95%CI [.01, .04]. Then, we calculated the effect size for the seeming mediation of numerical competence on the relation between decoding and calculation performance at the beginning of first grade. Based on recommendation of mediation effect size calculation by Preacher and Kelley (2011), the mediation effect size was $k^2 = .05$, $p = .40$, which was small and non-significant. To sum, numerical competence did not mediate the effects of decoding on calculation performance at the beginning of first grade, at the end of third grade, or the development of calculations among young children with learning difficulties.

Discussion

In present study, we explored how early cognitions, academic skills, and demographics predict the development of calculations from the beginning of first grade to the end of third grade among children identified with MDRD at the beginning of first grade. Latent growth model analysis showed that early numerical competence, processing speed, decoding, and incoming calculations significantly explained variance of calculations at the beginning of first grade. Numerical competence and processing speed significantly explained the variance of calculation performance at the end of third grade. Numerical competence, in particular, significantly predicted the calculation development. Numerical competence did not mediate the effects of decoding on calculation performance at the beginning of first grade, at the end of third grade, or the development of calculations.

The Influence of Academic Skills

Specifically, numerical competence was the only academic skill that exerted direct impact on calculation performance at the beginning of first grade, at the end of third grade, and calculation development from first grade through third grade. Our finding is in line with most previous work on typically children showing that after controlling for domain-general cognitive skills such as working memory, language, and nonverbal reasoning, numerical competence still made unique contributions to calculation performance (e.g., Fuchs et al., 2010; Sowinski, LeFevre, Skwarchuk, Kamawar, Bisanz, & Smith-Chant, 2015) and that numerical competence in kindergarten predicted calculation skills in grade 1 and 2 (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Desoete & Grégoire, 2006; Jordan, Kaplan, Locuniak, & Ramineni, 2007). It is also worth noting that even compared to incoming calculations, which seem more closely related to calculations, numerical competence still plays a more important role in calculation development from first grade to third grade. This may be because incoming calculations mainly tap simple math facts retrieval, whereas numerical competence also tap other numerical skills including counting and number estimation, which are the foundations for early calculation development.

We found decoding's unique relation to calculations at the beginning of first grade. This is in line with the phonological representation hypothesis (Vukovic & Lesaux, 2013), indicating that calculations may require the retrieval of phonological codes, encoding, and maintaining phonological representations (e.g., numbers) in short-term memory (Fuchs et al., 2006; Koponen, Aunola, Ahonen, & Nurmi, 2007; Simmons & Singleton, 2008). Previous research proposed numerical competence may serve as the mediator between phonological decoding and arithmetic development (e.g., LeFevre et al., 2010). The rationale is that children use phonological systems to code numerical information (e.g., counting and sequencing), which in turn are critical to the development of calculation skills (Aunola et al., 2004; Geary & Brown, 1991; Lemaire & Siegler, 1995). However, we did not find numerical competence mediated the effects of decoding on calculation performance at the beginning of first grade, at the end of third grade, or on calculation development. This finding indicates that decoding directly affects calculations among children with MDRD but this effect may be limited at an early stage. It is the numerical competence, not decoding, that exerted direct impact on early calculation development. That said, we used decoding to serve as a proxy for phonological processing. Longitudinal studies are needed to further investigate the interplay among the development of phonological processing, decoding, numerical competence, and calculations.

The Influence of Cognitive Skills

Regarding processing speed, Bull and Johnston (1997) and Fuchs et al. (2006) indicated that after controlling for language and memory, processing speed still explained variance in calculations at the beginning of first grade. Some researchers suggest that processing speed may facilitate counting speed, so that as young children gain speed in counting sets to figure sums and differences. This in turn leads to successfully pairing problems with their answers in working memory before decay sets in, thus establishing associations in long-term memory (e.g., Geary, Brown, & Samaranayake, 1991; Lemaire & Siegler, 1995). Although we found that processing speed predict calculation performance at the beginning of first grade and the end of third grade, we did not find that processing speed significantly predicted the calculation development from the beginning of first grade to the end of third grade. Thus, our findings are in line with previous research showing that processing speed predicts calculation performance among children with mathematics difficulties (e.g., Namkung & Fuchs, 2016), but also add to the literature by showing that processing speed does not predict the growth of calculation among children with MDRD. In other words, it is numerical competence not processing speed that plays a more essential role in helping children with MDRD develop competence with calculation. That said, further research is needed to investigate whether processing speed moderate the effect of numerical/calculation intervention on calculation performance among children with MDRD.

We did not find working memory, language, or nonverbal reasoning to be significant predictors of calculations at the beginning of first grade or early calculation development. This finding is consistent with the work of Fuchs et al. (2005), which also did not find these cognitive skills predicted calculations among third graders. However, our finding contradicts most previous work showing the importance of working memory and language in calculations (Geary et al., 1991; Hitch & McAuley, 1991; LeFevre et al., 2010; Siegel & Linder, 1984; Webster, 1979; Wilson & Swanson, 2001). One possible explanation is that unlike most previous studies, our study and Fuchs et al. (2005) examined the effects of working memory, language, and nonverbal reasoning on calculations with simultaneous consideration of other cognitive abilities and academic skills. Simultaneously considering a fuller set of these variables may provide different results than considering a single or fewer variables because each construct competes for variance against others included in the model. Another possible explanation may come from how we defined these cognitive skills. For example, we operationalized working memory with a particular set of measures, assessing memory span for language stimuli as well as for backward digit span. Although these measures are well accepted for indexing working memory, it is possible that different instruments tapping different domains (i.e., visual-spatial domain) of working memory may emerge as a significant predictor. Moreover, we noted that although we modified the administration of the working memory tests to reduce the floor effects, the mean and SD still suggest that our working memory tests may not fully reflect young at-risk children' working memory capacity. Future studies should use more suitable/sensitive working memory tests for low-performing children to confirm our findings.

The Influence of Demographics

It is also interesting to note that SES correlated with working memory and language, but not with numerical competence, incoming calculations, or calculation development. Because our working memory tests were language-based tests (e.g., listening span and backward digit recall), this finding is partially consistent with prior work suggesting that SES may be more likely to influence language-related task performance (e.g., Fuchs et al., 2008; Jordan, Huttenlocher, & Levine, 1992). This finding is also consistent with some work that suggests although SES is associated with mathematics performance, it is not necessarily associated with the mathematics development at an early age (Jordan, Kaplan, Olah, & Locuniak, 2006). Regarding the gender effect, previous research suggests there may be a gender difference on calculation processes, in which boys seem to have faster numerical facts retrieval than girls (e.g., Royer et al., 1999), whereas some research shows that there are no gender differences in math performance during the elementary school years (Friedman, 1989; Hyde et al., 1990; Skaalvik & Rankin, 1994). Our finding is consistent with the latter, suggesting that gender did not contribute to early calculation development among young children with learning difficulties.

Limitations and Implication for Future Studies

Our findings should be considered in the context of some limitations. First, we note that we could not examine specific numerical skills separately in our numerical competence measure because of a limited number of items on each of the skills included in KeyMath-3. Research shows that different subcomponents within early numerical competence may play differential roles in mathematics development among typically developing children. Specifically, Lyons et al. (2014) systematically examined the effects of subcomponents of early numerical competence on arithmetic development among typically developing children. They found that in grades 1–2, children's ability to judge the relative magnitude of numerical symbols was most predictive of early arithmetic skills. The unique contribution of children's ability to assess ordinality in numerical symbols steadily increased across grades, overtaking all other predictors by grade 6. Children's ability to judge the relative magnitude of approximate, non-symbolic numbers was uniquely predictive of arithmetic ability at any grade. Although Lyons et al. (2014) focused on typically developing children, and they did not control for potentially important cognitive skills (e.g., processing speed and working memory) and demographic variables, their findings shed a light on the importance of differentiating the contributions of subcomponents in early numerical to mathematics development. Thus, future studies should specifically investigate what specific skills within numerical competence serve as the active ingredient in helping young children with learning difficulties develop early calculation skills.

Second, although we tried to include a comprehensive set of skills in our model, we did not include some skills due to limited testing resources. For example, previous research suggests that attention and spatial skills may be important predictors of calculation and numerical abilities in early grades (e.g., Fuchs et al., 2006; Gunderson, Ramirez, Beilock, & Levine, 2012; Newcombe, 2010). Thus, future studies should investigate the role of attention and spatial skills in early calculation development among the MDRD group. Also, we only used WRAT-4 Math Computation as the measure for calculations. Although this test is widely accepted as a calculation test, it includes a few items that tap other domains of mathematics, such as word problem-solving and understanding of fractions. Future studies should look at whether other calculation tests, such as Woodcock Johnson – Calculation subtest (Woodcock et al., 2001), result in different findings.

Third, it is important to note that the predictors included in this study explained 45.2% ~ 61.6% of variance in the calculation performance and 15.6% of variance in the slope of calculations, respectively, leaving a fair amount of variance to be explained. It is likely that other extra-child variables may affect calculation competence and development, such as classroom instructions. In this study, we could not estimate the ICCs for the classroom accurately for two reasons. First, at the beginning of first grade, our sample came from 114 classrooms across 21 elementary schools. On average, fewer than two students came from the same class. With only 2 students on average per classroom, the student and the classroom are the same unit. Second, in our follow-up data collection from the second grade through the third grade, we found it very common that students moved from class to class, and their classroom teachers changed from time to time. Thus, we did not document the information on students' classrooms/teachers after the first grade. Moreover, we point out that even if we had students' classrooms/teacher information after the first grade, we would still be underpowered to run HLM using the classroom as the second level. This is because in this case, we need to use a cross-classified model in HLM, which requires an even larger sample size. Thus, future work may look at whether teachers' behavioral control or instructional approaches for at-risk children may help enhance the prediction of calculation development (Fuchs et al., 2005; Morgan, Farkas, & Maczuga, 2015). Lastly, we only included children with MDRD and most children in our sample came from low SES background. Future studies may include both MD alone and RD alone group and more children from middle and high SES background to investigate whether the calculation development trajectories are influenced by learning difficulty subtype and SES.

With these limitations in mind, our study provides insights on the contributions of early cognitive and academic skills to early calculation development among children with MDRD. Specifically, the non-significant effect of language on early calculation performance and development is partially consistent with Morgan et al. (2015)'s findings on effective mathematics instructions for young children with MD. Morgan et al. (2015) found that for first graders with MD, compared to student-centered language-based activities (e.g., solving a math problem with a partner, peer tutoring, and explaining math), teacher-directed activities that focused on explicit and direct number knowledge instruction and practice showed a stronger correlation with students' mathematics achievement. This finding, together with ours, suggests meaningful instruction in number with opportunities for practice (e.g., Frye, Baroody, Burchinal, Carver, Jordan, & McDowell, 2013; Lynn et al., 2013), not language-based student-centered activities, may more likely facilitate the improvement of calculations among at-risk young children. That said, this suggestion is based on our correlational findings and warrants further intervention studies to confirm the causal relationship between language (or other cognitive skills) and early calculation development among young children with MDRD.

More importantly, our findings did not support the view that cognitive skills, such as processing speed and working memory, play a compensatory role in calculation development among children with MDRD (e.g., Geary et al., 2007). Instead, our finding indicates that even with comprehensive and severe deficits in both cognitions and mathematics, young children with MDRD still primarily rely on the numerical knowledge, not general abilities, to develop their calculation competence. Thus, this finding provides evidence for the importance of numerical skills instruction among children with MDRD in early childhood. Prior research suggests that numerical competence can be remediated with targeted interventions (e.g., Bryant, 2011; Siegler, 2009; Ramani & Siegler, 2008). Early interventions in key areas of numerical competence (e.g., counting knowledge, quantity comparison) have shown to improve numerical competence of preschoolers from low-income families (Siegler, 2009) and at-risk kindergarteners (Bryant, 2011; Chard, Baker, Clarke, Jungjohann, Davis, & Smolkowski, 2008). Given that numerical competence is highly predictive of later mathematics achievement and that numerical competence is a key determinant of calculation development among MDRD students, early interventions in numerical competence hold promise for early identification and remediation purposes. However, further experimental study is warranted to determine active ingredients (e.g., intensity) for MDRD students because MDRD students may experience more pervasive deficits compared to students with a single area deficit and require more distinctive and intensive interventions.

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Table 1

Descriptive Statistics of Study Measures

	N	Mean	SD	Skewness	Kurtosis	Min	Max
Gender (Female)	174 (95)	-	-	-	-	-	-
Free/Reduced Lunch (Yes)	169 (135)	-	-	-	-	-	-
Ethnicity (Caucasian/Black/Hispanic/Others)	174 (59/80/17/20)	-	-	-	-	-	-
Age (at the beginning of first grade)	176	6.64	.42	-	-	-	-
Rapid Sound Naming	176	25.76	12.79	.39	.52	0	71
Phonemic Decoding Fluency	176	2.17	2.76	1.35	1.23	0	12
Matrix Reasoning	176	6.13	3.48	1.18	2.98	0	23
Concept Formation	176	5.74	3.78	2.04	6.64	1	27
Vocabulary	176	15.58	6.66	−.13	−.29	1	34
Listening Comprehension	176	11.91	4.93	−.19	−.37	1	24
Listening Recall	176	1.48	2.47	1.25	.20	0	11
Backward Digit Recall	176	4.06	3.75	.41	−.54	0	16
Addition 0–8	176	6.64	4.08	.46	.08	0	18
Addition 0–12	176	2.28	1.88	1.01	.88	0	9
Subtraction0–8	176	1.77	2.30	1.44	1.34	0	10
Subtraction 0–12	176	1.23	1.16	.88	.11	0	4
Processing Speed	166	7.95	3.26	−.49	.29	0	16
Numerical Competence	131	5.11	1.45	.17	.14	2	9
WRAT-Time1	176	11.66	2.15	−.08	−.08	6	17
WRAT-Time2	156	15.63	2.89	−.40	.18	6	22
WRAT-Time3	151	18.98	3.30	−.67	.39	8	26
WRAT-Time4	134	23.12	3.66	−.21	.48	13	34
WRAT-Slope	134	3.74	1.16	−.22	.33	.57	6.50

Note. N = the number of data points.

Table 2

Correlations between Study Measures

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1. Gender	-																			
2. Free/Reduced Lunch	.09	-																		
3. Rapid Sound Naming	−.09	−.03	-																	
4. Phonemic Decoding Fluency	−.05	−.05	.46**	-																
5. Matrix Reasoning	.01	−.15	−.01	.16*	-															
6. Concept Formation	−.00	−.26**	.08	.18*	.47**	-														
7. Vocabulary	.07	−.24**	.13	.13	.17*	.34**	-													
8. Listening Comprehension	.03	−.22**	.21**	.20**	.30**	.45**	.62**	-												
9. Listening Recall	−.03	−.20*	.16*	.16*	.24**	.35**	.29**	.35**	-											
10. Backward Digit Recall	.01	−.15	.16*	.31**	.19*	.32**	.33**	.44**	.37**	-										
11. Addition 0–8	.03	−.13	.29**	.21**	.24**	.29**	.20**	.33**	.21**	.36**	-									
12. Addition 0–12	.06	−.13	.27**	.27**	.22**	.22**	.19*	.24**	.20**	.27**	.61**	-								
13. Subtraction0–8	−.09	−.15*	.14	.02	.10	.08	.03	.09	.17*	.10	.39**	.22**	-							
14. Subtraction 0–12	.09	−.08	.15*	.02	.30**	.14	.15*	.14	.28**	.12	.42**	.39**	.48**	-						
15. Processing Speed	−.09	−.07	.26**	.12	.38**	.30**	.22**	.25**	.23**	.30**	.44**	.36**	.17*	.37**	-					
16. Numerical Competence	.10	−.09	.18*	.15	.18*	.23**	.31**	.41**	.28**	.33**	.42**	.24**	.23**	.27**	.30**	-				
17. WRAT-Time1	−.04	−.13	.31**	.25**	.30**	.30**	.27**	.36**	.31**	.36**	.54**	.44**	.34**	.32**	.50**	.47**	-			
18. WRAT-Time2	.00	−.02	.40**	.28**	.20*	.29**	.24**	.27**	.37**	.27**	.42**	.29**	.22**	.27**	.51**	.48**	.51**	-		
19. WRAT-Time3	−.03	−.14	.27**	.15	.29**	.30**	.27**	.30**	.30**	.30**	.45**	.30**	.29**	.35**	.49**	.50**	.55**	.64**	-	
20. WRAT-Time4	.04	−.16	.27**	.07	.22*	.14	.21*	.20*	.26**	.27**	.31**	.25**	.17	.19*	.48**	.52**	.33**	.46**	.63**	-
21. WRAT-Slope+	.06	−.09	.10	−.11	.08	−.01	.09	.05	.06	.06	.04	−.01	−.02	.30	.18*	.22*	−.21*	.10	.45**	.83**

Note.

* $p<.05$;

** $p<.01$;

‡WRAT-Slope: the slope from time 1 to time 4

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Table 3

Path Coefficients and Covariance and Estimates with Bootstrapping Estimates and Confidence Interval of the Baseline Latent Growth Model

Path	Standardized Coefficient	*p*	95% CI based on Bootstrapping Estimates (N = 10000)[+]
Intercept1/Intercept4 → WRAT-Time1[#]	1.18/1.45	-	-
Intercept1/Intercept4 → WRAT-Time2[#]	.88/1.08	-	-
Intercept1/Intercept4 → WRAT-Time3[#]	.76/.94	-	-
Intercept1/Intercept4 → WRAT-Time4[#]	.69/.86	-	-
Slope → WRAT-Time2[#]	.38	-	-
Slope → WRAT-Time3[#]	.66	-	-
Slope → WRAT-Time4[#]	.89	-	-
Intercept1/Intercept4 ↔ Slope	**−.44/−.69**	**.01/.01**	**[−2.54, −.06]/[−4.02, −.64]**

Note.

[+]Non-standardized estimates (Mplus version 7 only generates unstandardized estimates and 95%CI with bootstrapping).

[#]because the loading coefficient is set as a fixed number in latent growth modelling, no *p* values or bootstrapping estimated is computed.

Intercept 1 is the WRAT performance at the beginning of first grade; Intercept 4 is the WRAT performance at the end of third grade.

Table 4

Path Coefficients Estimates with Bootstrapping Estimates and Confidence Intervals of the Latent Growth Model with Predictors

Path	**Standardized Coefficient**	***p***	**95% CI based on Bootstrapping Estimates (N = 10000)**[+]
Intercept1/Intercept4 → WRAT-Time1[#]	1.03/1.39	-	-
Intercept1/Intercept4 → WRAT-Time2[#]	.80/1.08	-	-
Intercept1/Intercept4 → WRAT-Time3[#]	.69/.93	-	-
Intercept1/Intercept4 → WRAT-Time4[#]	.60/.80	-	-
Slope → WRAT-Time1[#]	0	-	-
Slope → WRAT-Time2[#]	.34	-	-
Slope → WRAT-Time3[#]	.58	-	-
Slope → WRAT-Time4[#]	.75	-	-
Intercept1/Intercept4 ↔ **Slope**	**−.70**/−.77	**<.01**/.09	**[−1.80, −.15]**/[−2.50, .36]
Free/Reduced Lunch → Intercept1/Intercept4	.03/−.04	.61/.71	[−.46, .76]/[−1.76,1.28]
Numerical Competence → Intercept1/Intercept4	**.23/.47**	**.003/<.001**	**[1.83, 8.21]/[7.64, 19.02]**
Processing Speed→ Intercept1/Intercept4	**.23/.42**	**.002/<.001**	**[.06, .25]/[.18, .58]**
Decoding → Intercept1/Intercept4	**.14**/.13	**.01**/.13	**[.07, .56]**/[−.11, .85]
Working Memory → Intercept1/Intercept4	.09/.004	.16/.97	[−.09, .48]/[−.66, .65]
Incoming Calculation→ Intercept1/Intercept4	**.22**/.03	**.001**/.80	**[.20, .75]**/[−.50, .69]
Language → Intercept1/Intercept4	.03/−.02	.59/.82	[−.21, .40]/[−.65, .48]
Nonverbal Reasoning → Intercept1/Intercept4	.05/−.01	.47/.92	[−.21, .40]/[−.78, .56]
Gender → Intercept1/Intercept4	−.03/.06	.61/.49	[−.60, .34]/[−.63, 1.35]
Free/Reduced Lunch → Slope	−.06	.56	[−.70, .42]
Numerical Competence → Slope	**.32**	**.02**	**[.51, 5.03]**
Processing Speed→ Slope	.26	.06	[−.001, .15]
Decoding → Slope	.02	.83	[−.16, .20]
Working Memory → Slope	−.07	.58	[−.30, .17]
Incoming Calculation→ Slope	−.14	.23	[−.36, .06]
Language → Slope	−.05	.64	[−.27, .16]
Nonverbal Reasoning → Slope	−.05	.64	[−.32, .19]
Gender → Slope	.09	.39	[−.22, .55]

Note.

[+] Non-standardized estimates (Mplus version 7 only generates unstandardized estimates and 95%CI with bootstrapping).

[#] because the loading coefficient is set as a fixed number in latent growth modelling, no p values or bootstrapping estimated is computed.

Intercept 1 is the WRAT performance at the beginning of first grade; Intercept 4 is the WRAT performance at the end of third grade.