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Nonparametric Change Point Detection Methods for Profile Variability

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NONPARAMETRIC CHANGE POINT DETECTION METHODS FOR PROFILE VARIABILITY

By

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I dedicate this work to my siblings. Anything is possible! I cannot wait for you to get your doctorates and explain to me what is going on...

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ABSTRACT

Due to the importance of seeing profile change in devices such as of medical apparatus, measuring the change point in variability of a different functions is important. In a sequence of functional observations (each of the same length), we wish to determine as quickly as possible when a change in the observations has occurred. Wavelet-based change point methods are proposed that determine when the variability of the noise in a sequence of functional profiles (i.e. the precision profile of medical devices) has occurred; goes out of control from a known, fixed value, or an estimated in-control value. Various methods have been proposed which focus on changes in the form of the function.

One method, the NEWMA, based on EWMA, focuses on changes in both. However, the drawback is that the form of the in-control function is known. Others methods, including the χ^2 for Phase I & Phase II make some assumption about the function. Our interest, however, is in detecting changes in the variance from one function to the next. In particular, we are interested not on differences from one profile to another (variance between), rather differences in variance (variance within). The functional portion of the profiles is allowed to come from a large class of functions and may vary from profile to profile. The estimator is evaluated on a variety of conditions, including allowing the wavelet noise subspace to be substantially contaminated by the profile's functional structure, and is compared to two competing noise monitoring methods. Nikoo and Noorossana (2013) [62] propose a nonparametric wavelet regression method that uses both change point techniques to monitor the variance: a Nonparametric Control Charts, via the mean of m median control charts, and a Parametric Control Charts, via χ^2 distribution.

We propose improvements to their method by incorporating prior data and making use of likelihood ratios. Our methods make use of the orthogonal properties of wavelet projections to accurately and efficiently monitor the level of noise from one profile to the next; detect changes in noise in Phase II setting. We show through simulation results that our proposed methods have better power and are more robust against the confounding effect between variance estimation and function estimation. The proposed methods are shown to be very efficient at detecting when the variability

has changed through an extensive simulation study. Extensions are considered that explore the usage of windowing and estimated in-control values for the MAD method; and the effect of the exact distribution under normality rather than the asymptotic distribution. These developments are implemented in the parametric, nonparametric scale, and complete nonparameric settings. The proposed methodologies are tested through simulation and applicable to various biometric and health related topics; and have the potential to improve in computational efficiency and in reducing the number of assumptions required.

CHAPTER 1

INTRODUCTION & BACKGROUND

1.1 Change Point Detection

Important questions arise from time series data when sudden variations occur, particularly; observing when the variation was occurred and/or whether more than one variation took place.

Change Point or Change Point Detection (CPD) problems are those whose focus are on identifying one or many times of change in a function or between different functions. Also of interest is the value of the statistic (mean, variance, etc.) at the time(s) of change. Change Point Detections problems are often set up as hypothesis problems. Change-point analysis (CPA) or change point detection models are designed to assess abrupt variations in a sequence of both Gaussian and non-Gaussian distributed random variables [40], [41], and [42]. CPD models are particularly robust given little to no underlying information about the either the distributional form of the sequence as well as having no prior knowledge of the distribution of parameters from different segments of observed sequences [40], [41], and [42]. Change point models were noted in the early 1920s but amplified in the late 1950s with works by Page [68], [69], Lorden [50], and Shiryayev [80].

Applications of Change Point can be found in process control, quality control, systems engineering, process engineering, industrial engineering, diagnostic analysis in many fields, and more.

1.2 Nonparametric Functions

As opposed to the Parametric distributions that depend on the family of probability distributions, nonparametric functions are not based on known distribution functions. Therefore, there are no assumption on the functions themselves.

The nonparametric statistical method was first "informally" introduced in 1710 by John Arbuthnott as means to understand higher ratios of male to female births in London between 1629 and

1710 [3]. The statistical procedure he conducted was an analogue to what is now known as the Sign Test [3]. In his paper; "Additive Partition Functions and a Class of Statistical Hypotheses" (1942), Wolfowitz formally introduced the term "non-parametric" in the context of hypothesis testing [91]. Principally, the essence of nonparametric statistical methods stems from its conventional counterpart; "parametric" statistics, for which we briefly review to better understand the principle of the nonparametric case. To examine whether a specified model accurately depicts empirical observations, distinct assumptions must be assessed depending on the family of probability distributions. Nonparametric methods are infinite-dimensional as inferences about the empirical observations are made with little to no prior assumption about the model. Knowledge about the functional forms of the distributions, particularly the assumption of the normality, of the specified model is the most important assumption in parametric statistical methods and nonparametric accounts for the lack of such assumption. Many inference problems can be assessed with nonparametric statistical models (include but not limited to): estimating the General Function, functionals, general densities, regression, normal means, wavelets, and many more. With respect to developments in theory of estimation and inference about various parameters, nonparametric methods have also been demonstrated to have high valued Asymptotic Relative Efficiency (ARE) compared to parametric statistical techniques.

1.3 Motivation

The motivation behind the use of change point problems stem from specific fields of study. For instance, its applications widely vary from economics in financial modeling with respect to correlated assets over time as detailed by Talih and Hengartner (2005) [85], to bioinformatics in genetic works in identifying changes in cell behaviors [59], or to epidemic outbreaks [72]. Other applications and motivating examples of change point detection include (but not limited to): medical condition monitoring such as heart rate and electrocardiograms (ECG) [52], [82], [92], climate change detection [43], [25], or human activity analysis [83], [46], [88].

Given the unknown underlying distribution of the sequence of a process control, developments have been made to incorporate the use of nonparametric methods to change point detection models [39]. Chakraborti (2001) [14] gives a comprehensive review of use of nonparametric in process control.

However, based on what is known from traditional statistical process control methods specially using nonparametric versions, many assumptions are still made on the function. We propose methods aiming to significantly reduce the number of assumptions made on the underlying function.

CHAPTER 2

STATISTICAL PROCESS CONTROL

Statistical process control (SPC) monitors processes for changes in mean, standard deviation, and other parameters. SPC methods are also able to compile large amounts of data, setting each observation as a noise contaminated sample of a function f measured at n points. SPC is applicable to both parametric and nonparametric methods. In this chapter we will give general review of statistical process control.

2.1 Univariate SPC

The Shewart control chart is one of the most common aspects of SPC. Let w be a statistic estimating the parameter of interest of a process. The characteristics of a Shewart control chart are the following:

$$\begin{aligned} \text{Upper Control Limit (UCL)} &= \mu_w + L\sigma_w \\ \text{Center Line} &= \mu_w \\ \text{Lower Control Limit (LCL)} &= \mu_w - L\sigma_w. \end{aligned} \tag{2.1}$$

Here μ_w denotes the mean, σ_w is the standard deviation, and L is a constant influencing the control chart's ability to detect a change as well as a false alarm rate. A process is in control (IC) if the value of w lies between the UCL and the LCL. If w lies above the UCL or below the LCL, the process is out of control (OOC). When w's probability is known, L is used to adjust the Type-I and Type-II error rates for specified values. Thus the similarity between Shewart control charts and repeated two sided hypothesis tests. See Montgomery [58] for more information.

2.1.1 $\bar{x}$ and R Charts

Let us consider a sample of size n taken from a univariate process at each of m time points. When we are interested in monitoring the mean (μ) and standard deviation (σ) of the process when these parameters are not known, we often use $\bar{x}$ and R charts. Let $\bar{\bar{x}}$ be the mean of the m observed sample means, and $\bar{R}$ be the average range of the m samples. The $\bar{x}$ chart is defined as:

$$\begin{aligned} \text{UCL} &= \bar{\bar{x}} + \frac{L}{d_2\sqrt{n}}\bar{R} \\ \text{Center Line} &= \bar{\bar{x}} \\ \text{LCL} &= \bar{\bar{x}} - \frac{L}{d_2\sqrt{n}}\bar{R} \end{aligned} \tag{2.2}$$

where d_2 is the expected value of the random variable R/σ. The process mean goes out of control for a change signaled from the $\bar{x}$ chart. The R chart is defined as:

$$\begin{aligned} \text{UCL} &= \bar{R} + Ld_3\frac{\bar{R}}{d_2} \\ \text{Center Line} &= \bar{R} \\ \text{LCL} &= \bar{R} - Ld_3\frac{\bar{R}}{d_2} \end{aligned} \tag{2.3}$$

where d_3 is the standard deviation of the random variable R/σ. The R chart is used to monitor for changes in σ. It is generally used simultaneously with the $\bar{x}$ chart.

2.1.2 Shewhart s Chart

Although using the sample range to construct $\bar{x}$ and R charts for fixed, small values of n is ideal, it is not applicable when n differs between samples. Moreover, the sample range is not an efficient estimator of σ when n is large. In these situations, we use the sample standard deviation, s, to estimate σ. We update the $\bar{x}$ chart to:

$$\begin{aligned} \text{UCL} &= \bar{\bar{x}} + L\frac{\bar{s}}{c_4\sqrt{n}} \\ \text{Center Line} &= \bar{\bar{x}} \\ \text{LCL} &= \bar{\bar{x}} - L\frac{\bar{s}}{c_4\sqrt{n}} \end{aligned} \tag{2.4}$$

where $\bar{s}$ is the average of the m observed sample standard deviations, and c_4 is the expected value of the random variable s/σ. While this modified $\bar{x}$ chart still monitors for changes in the mean, it

is now used simultaneously with the s chart. We define the latter as:

$$\begin{aligned} \text{UCL} &= \bar{s} + L\frac{\bar{s}\sqrt{1-c_4^2}}{c_4} \\ \text{Center Line} &= \bar{s} \\ \text{LCL} &= \bar{s} - L\frac{\bar{s}\sqrt{1-c_4^2}}{c_4} \end{aligned} \tag{2.5}$$

and use it to monitor for changes in σ. For cases where n varies between observations, $\bar{x}$ and s charts are still valid as long as we make sure to appropriately weight $\bar{\bar{x}}$ and $\bar{s}$.

2.1.3 Phase I and Phase II Control Charts

There are two main stages in statistical process control: Phase I and Phase II.

In Phase I, SPC analyses a set of process data to initially determine if the status remained in control for all observed samples. If the latter holds, it is then used to determine the control limits for future repetitions of this process. The Shewart control charts presented in the section above have the added benefit of being easy to obtain, and are thus commonly used in Phase I analysis.

In Phase II, SPC monitors processes that have already been identified as in-control during Phase I analysis. A defining Phase II characteristic is quickly recognizing small changes in the process. Despite its favorable aspects, Shewart control charts tend to have difficulty in detecting small changes in a process, and are thus not often used in Phase II analysis.

The remainder of this section will be devoted to introducing common methods in univariate Phase II SPC.

2.1.4 Exponentially Weighted Moving Averages (EWMA) Control Charts

We define the exponentially weighted moving average (EWMA) as:

$$z_i = \lambda x_i + (1-\lambda) z_{i-1}, \tag{2.6}$$

where λ is some constant on the unit interval and $z_0 = \mu_0$. The EWMA is an alternative to Shewart control charts that is often used in Phase II SPC. See Roberts [77]. The EWMA control chart for

monitoring a process mean is defined as:

$$\begin{aligned} \text{UCL} &= \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} \\ \text{Center Line} &= \mu_0 \\ \text{LCL} &= \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} \end{aligned} \tag{2.7}$$

for $i = 1, \cdots, n$. The main difference between EWMA and Shewart charts, besides the variable control limits, is that the EWMA chart uses all available process data to obtain the statistic being used to monitor μ.

2.1.5 Control Chart Performance via Average Run Length

Monitoring control charts is similar to repeated hypothesis testing. However, control chart performance is not typically measured in terms of the usual α and β (type-I and type-II error probability, respectively) rather quantified by the average number of observations until a change is signaled, or average run length (ARL). The ARL for an in-control process is the average number of observations until a false alarm is signaled. Ideally, this number should be large. Conversely, the ARL for an out-of-control process is the average number of observations until a change is correctly signaled. Ideally, this number should be small.

For the simplest case where points on the control chart are independent, we can model the run length performance using a geometric distribution. The ARL for an in-control process (ARL_0) is simply the mean of a geometric random variable with parameter α, where α is the constant probability of a type-I error at each point:

$$\text{ARL}_0 = \frac{1}{\alpha}. \tag{2.8}$$

Following, the ARL for an out-of-control process (ARL_1) is given by the mean of a geometric random variable with parameter $1-\beta$, where β is the probability of a type-II error at each point:

$$\text{ARL}_1 = \frac{1}{1-\beta}. \tag{2.9}$$

When α and β cannot be specified using control charts, we can obtain the in-control ARL as a function of control limits or other parameters using Monte Carlo simulations of the in-control

data. This is a common occurrence when monitoring complex processes. We then specify control limits which result in a in-control ARL. We can use the out-of-control ARL to measure the chart's performance.

2.2 Multivariate SPC

The previously mentioned control charts are useful when monitoring a univariate parameter. Although valid in the multivariate case for independent parameters, they are flawed in monitoring correlated parameters and present issues for both the type-I and type-II error rates. We look to common multivariate control charts to tackle this issue.

2.2.1 Hotelling T^2 Control Chart

Similarly to the Shewart $\bar{x}$ chart from the Shewart chart section, the Hotelling T^2 control chart is used to monitor the mean vector of a process. For a p-dimensional Normal distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$, when the listed parameters are unknown, we use the data to estimate them. We assume that m samples of size n are collected from the p-dimensional process. See Montgomery [58]. Therefore, we can estimate $\boldsymbol{\mu}$ as:

$$\begin{aligned} \bar{\bar{\boldsymbol{x}}} &= \{\bar{\bar{x}}_j;\ j = 1, \cdots, p\} \\ &= \left\{ \frac{1}{m}\frac{1}{n}\sum_{k=1}^{m}\sum_{i=1}^{n} x_{ijk};\ j = 1, \cdots, p \right\} \end{aligned} \tag{2.10}$$

and estimate $\boldsymbol{\Sigma}$ using the following $p \times p$ matrix:

$$\begin{aligned} \boldsymbol{S} &= \{\bar{s}_{jh};\ j, h = 1, \cdots, p\} \\ &= \left\{ \frac{1}{m}\frac{1}{n-1}\sum_{k=1}^{m}\sum_{i=1}^{n} \left(x_{ijk} - \bar{x}_{jk}\right)\left(x_{ijk} - \bar{x}_{hk}\right);\ j, h = 1, \cdots, p \right\}. \end{aligned} \tag{2.11}$$

Now, for an observed $p \times n$ sample with sample mean $\bar{\boldsymbol{x}}$, the Hotelling T^2 test statistic is defined as:

$$T^2 = n\left(\bar{\boldsymbol{x}} - \bar{\bar{\boldsymbol{x}}}\right)' \boldsymbol{S^{-1}} \left(\bar{\boldsymbol{x}} - \bar{\bar{\boldsymbol{x}}}\right). \tag{2.12}$$

When the process is in-control, T^2 is non-negative and should be approximately equal to 0. Although the UCL is based on the F distribution, Alt [2] advises altering it for when the chart is in

a Phase I or Phase II setting. For the Phase-I setting:

$$\text{UCL} = \frac{p(m-1)(n-1)}{mn-m-p+1} F_{\alpha,p,mn-m-p+1}, \tag{2.13}$$

for some previously specified α. For the Phase II setting:

$$\text{UCL} = \frac{p(m+1)(n-1)}{mn-m-p+1} F_{\alpha,p,mn-m-p+1}. \tag{2.14}$$

When $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are specifically known, they can be used in the T^2 equation. The UCL then becomes $\chi^2_{\alpha,p}$ for both Phase I and Phase II settings.

2.2.2 Multivariate EWMA Control Chart

As with the Shewart charts described in the earlier section, the Hotelling T^2 control chart is useful in detecting large deviations from the in-control mean. T^2 charts are not very powerful for smaller shifts due to the fact that they do not use previous sample information. To detect these small shifts, Lowry et al. [51] propose a multivariate extension of the EWMA control charts. Thus the multivariate exponentially weighted moving average (MEWMA) as the p–dimensional vector:

$$\boldsymbol{Z_i} = \lambda \boldsymbol{x_i} + (1-\lambda)\boldsymbol{Z_{i-1}}, \tag{2.15}$$

where $\boldsymbol{Z_0} = \mathbf{0}$, and the $p \times p$ covariance matrix:

$$\boldsymbol{\Sigma}_{\boldsymbol{Z_i}} = \frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]\boldsymbol{\Sigma}. \tag{2.16}$$

The test statistic for the MEWMA control chart is given by:

$$T_i^2 = \boldsymbol{Z_i'}\boldsymbol{\Sigma}_{\boldsymbol{Z_i}}^{-\mathbf{1}}\boldsymbol{Z_i}. \tag{2.17}$$

Similarly to the T^2 chart, we do not need to define a LCL for the MEWMA chart. Unlike the T^2 chart however, the UCL for MEWMA chart does not follow a parametric distribution. It is recommended selecting an UCL to achieve a desired in-control ARL. This is done via simulation for specific values of p and λ.

CHAPTER 3

WAVELETS

Wavelets are best used to represent irregular and non-linear functions. Some of their common benefits include the simultaneous analysis of time and frequency, fast computing, and precision with large amounts of data reduction. In this chapter, we will introduce the Fourier Transfrom which will lead us to the Continuous and Discrete Wavelet Transformations, including Multi-Resolution Analysis (MRA). We will cover some common wavelet bases, notably the Haar wavelet, and end by comparing some different thresholding methods. For more details on wavelets in general, see [63].

3.1 Fourier Transform

To get a better understanding of the Wavelet Transform (or Decomposition) we consider the Fourier transformation. It was discovered by Fourier that continuous periodic trigonometric functions can be used to from all square-integrable function in $L_2(R)$. We say that f is square integrable when the integral of the squared function is finite, that is:

$$\int_{-\infty}^{\infty} f^2(x)\,\mathrm{d}x < \infty \tag{3.1}$$

is satisfied. Representing f in terms of periodic functions enables us to functionally represent nonparametric functions. This is beneficial if the nonparametric functions at hand are not well modeled by other methods.

Let f be defined on the interval $[-\pi;\pi]$. For cases where f is defined on $L^2[-\pi,\pi]$, we can express f as an infinite sum of vertically scaled sines and cosine functions. This is further illustrated in the following equation:

$$f(x) = \frac{1}{2}a_0 + \sum_{j=1}^{\infty}(a_j\cos(jx) + b_j\sin(jx)). \tag{3.2}$$

For a given value of J, the above equation is well approximated:

$$S_J(x) = \frac{1}{2}a_0 + \sum_{j=1}^{J}(a_j \cos(jx) + b_j \sin(jx)). \tag{3.3}$$

The advantage of the Fourier Transform lies in its simplicity and power. The set of sine and cosine functions form a set basis functions for $L^2[\pi; -\pi]$. The latter are linearly independent groups of functions spanning the given space. The coefficients a_0, a_j, and b_j are obtained from the inner product of the function and the corresponding basis functions:

$$a_j = \frac{1}{\pi}\langle f, \cos(j\cdot)\rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(jx)\mathrm{dx}, j = 0, 1, \cdots \tag{3.4}$$

$$b_j = \frac{1}{\pi}\langle f, \sin(j\cdot)\rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(jx)\mathrm{dx}, j = 1, 2, \cdots \tag{3.5}$$

The above coefficients measure the frequency at each resolution j. As a positive, any coefficient with a small frequency can be replaced by 0 with virtually no loss of information. A negative is that the translation of sine and cosine functions only occurs in the vertical direction. The question that arises is the following: how do we account for the horizontal direction? We look to wavelets to answer; we can focus on time and frequency of a single function given wavelets' zooming and windowing abilities.

3.2 Wavelet Transform

3.2.1 Multiresolution Analysis

Wavelets analyze functions through time and frequency by projecting them into subspaces. For a function f, these subspaces, also known as resolution levels, represent its levels of precision or smoothness. For the mother and father wavelets, the smoothest to the least smooth sections appear subsequently at resolution levels j_0 and $j \geq j_0$. For any increase on level j we obtain less smooth resolution levels; correspondingly, the more our j approaches ∞, the more noise we encounter. Through its windowing feature, MRA enables us to zoom in or out of f, allowing us to analyze all (smooth or detailed) structures of f, regardless of its properties (regular or irregular).

MRA is an aggregation of nested subspaces creating the many steps of resolution levels. Let $V_j, j \in \mathbb{Z}$ be the space of a defining function, for example:

$$V_j = f \in L^2(\mathbb{R}) : f \text{ is piecewise constant on } [k2^{-j}, (k+1)2^{-j}];\ k \in \mathbb{Z}. \tag{3.6}$$

gives the Haar basis. Each V_j acts as the crosspiece of the resolution level, or is to the resolution level what rungs are to a ladder. Ogden [63] gives its properties in detail:

1. $\cdots \subset V_{-2} \subset V_{-1} \subset V_0 \subset V_1 \subset V_2 \cdots$
2. $\cap_{j\in\mathbb{Z}} V_j = 0, \overline{\cap_{j\in\mathbb{Z}} V_j} = L^2(\mathbb{R})$
3. $f \in V_j$ in and only if $f(2\cdot) \in V_{j+1}$ (Each space is a scaled version of V_0)
4. $f \in V_0$ implies $f(\cdot - k) \in V_0 \bigvee k \in \mathbb{Z}$
5. $\exists \phi \in V_0 \ni \phi_{0,k} = \phi(\cdot - k), k \in \mathbb{Z}$ forms an orthonormal basis for V_0

A MRA generated by ϕ is performed once all of $V_j = \text{span}\phi_{j,k}, k \in \mathbb{Z}$ properties are satisfied; all subspaces containing $\phi_{j,k}$ intersect at V_j. The same can be understood for a detail space as $\psi_{j,k}$ are mutually orthogonal basis with a single detailing index for the set of wavelets): $W_j = \text{span}\psi_{j,k}, k \in \mathbb{Z}$.

We define $\oplus$ to be the orthogonal sum of two subspaces. Therefore:

$$V_j = V_{j-1} \oplus W_{j-1}. \tag{3.7}$$

For $j \neq j'$, W_j and $W_{j'}$ are orthogonal. From the third property, we see:

$$V_j = (V_{j-2} \oplus W_{j-2}) \oplus W_{j-1}. \tag{3.8}$$

We can generate the above as

$$V_j = V_{j_0} \oplus (\bigoplus_{l=j_0}^{j-1} W_l);\ j \geq j_0 \tag{3.9}$$

or simply

$$V_j = \bigoplus_{l=j_0}^{\infty} W_l;\ j \in \mathbb{Z}. \tag{3.10}$$

If we are to put all spaces and limits of $\bigoplus W_{j\cdot}$ together, we obtain $L^2(\mathbb{R})$. Therefore, the space of L^2 is spanned by the closure of all $W_{j\cdot}$, or:

$$L^2(\mathbb{R}) = \overline{\bigoplus_{j\in\mathbb{Z}} W_{j\cdot}} \tag{3.11}$$

A simpler way to illustrate the ladder approach:

$$\begin{array}{ccccccccccccc} \cdots & & W_{j-2} & & W_{j-1} & & W_j & & W_{j+1} & & W_{j+2} & & \cdots \\ & \searrow & & \searrow & & \searrow & & \searrow & & \searrow & & \searrow & \\ \cdots & \to & V_{j-2} & \to & V_{j-1} & \to & V_j & \to & V_{j+1} & \to & V_{j+2} & \to & \cdots \end{array}$$

We can decompose a function f to as many or as few J resolution levels as we want. A decomposed function can be recomposed as well through Inverse Discrete Wavelet Transformation.

3.2.2 Continuous Wavelet Transform (CWT)

Wavelets are orthogonal series representation of square integrable functions. They are defined by the basis formed via translations and dilatations of the mother (ϕ) and father (ψ) wavelet functions:

$$\phi_{j,k}(x) = 2^{j/2}\phi(2^j x - k), \tag{3.12}$$

$$\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k), \tag{3.13}$$

where $j, k \in \mathbb{Z}$. Let j_0 be a fixed integer. Our orthonormal basis for L_2 is defined by

$$\phi_{j_0,k}, \psi_{j,k}, \quad j \geq j_0, k \in \mathbb{Z}. \tag{3.14}$$

A function in L_2 can be expressed by an infinite series:

$$f(x) = \sum_k \xi_{j_0,k}\phi_{j_0,k}(x) + \sum_{j=j_0}^{\infty}\sum_k \theta_{j,k}\psi_{j,k}(x), \tag{3.15}$$

where $\xi_{j,k} = \langle f, \phi_{j,k}\rangle$ and $\theta_{j,k} = \langle f, \psi_{j,k}\rangle$. The coefficients ξ and θ are estimated using the Discrete Wavelet Transform (DWT).

3.2.3 Estimating Wavelet Coefficients

Discrete Wavelet Transform (DWT). In Nonparametric Regression, the Discrete Wavelet Transform returns n coefficients estimated for all values of k and $j = j_0, j_0 + 1, \cdots, J-1$, when f is an equally spaced apart vector of length $n = 2^J$ where $J \in \mathbb{Z}^+$. Using the observed values y_i, the bivariate nonparametric function takes the form: $y_i = f(x_i) + \epsilon_i$, for the i^{th} pair of data. It follows that $i = 1, 2, \cdots, n$. Also, ϵ_i is independent and normally distributed with a standard deviation of

σ. We write the estimated coefficients as:

$$\begin{aligned}\tilde{\theta} = \{&\tilde{\xi}_{j_o,1}, \tilde{\xi}_{j_o,2}, \cdots, \tilde{\xi}_{j_o,2^{j_o}},\\ &\tilde{\theta}_{j_o,1}, \tilde{\theta}_{j_o,2}, \cdots, \tilde{\theta}_{j_o,2^{j_o}},\\ &\tilde{\theta}_{j_o+1,1}, \tilde{\theta}_{j_o+1,2}, \cdots, \tilde{\theta}_{j_o+1,2^{j_o+1}},\\ &\tilde{\theta}_{j_o+2,1}, \tilde{\theta}_{j_o+2,2}, \cdots, \tilde{\theta}_{j_o+2,2^{j_o+2}},\\ &\vdots\\ &\tilde{\theta}_{J-2,1}, \tilde{\theta}_{J-2,2}, \cdots, \tilde{\theta}_{J-2,2^{J-2}},\\ &\tilde{\theta}_{J-1,1}, \tilde{\theta}_{J-1,2}, \cdots, \tilde{\theta}_{J-1,2^{J-1}}\}\end{aligned} \tag{3.16}$$

or simply

$$\tilde{\theta} = \{\{\tilde{\xi}_{j_0,k}\}_{k=1}^{2_0^j}\}; \{\tilde{\theta}_{j,k}\}_{k=1}^{2^j}; j = j_0, \ldots, J-1\} \tag{3.17}$$

for convenience.

Let $W = \tilde{\theta}$ represent our matrix of estimated coefficients from DWT for our observed values $y = (y_1, y_2, \cdots, y_n)'$. Given the orthogonal properties of W, the equality $W^{-1} = W'$ holds. Therefore the Inverse Discrete Wavelet Transform (IDWT) follows:

$$\begin{aligned} y &= y\\ &= Iy\\ &= W'Wy \end{aligned} \tag{3.18}$$

Through IDWT we are able to return an the original sampled data y_i. Thresholding will change the reconstruction as discussed below. See Mallat's work [53, 54] for more on DWT.

Density Estimation. With the lowest and largest possible values for j_0 being 0 and $J-1$ respectively, and j and 2^j for k, the following is used to approximate f:

$$\hat{f}_J(x) = \sum_k \hat{c}_{j_0,k}\phi_{j_0,k}(x) + \sum_{j=j_0}^{J-1}\sum_k \hat{d}_{j,k}\psi_{j,k}(x). \tag{3.19}$$

In this case, the coefficients are found using a random sample $X_1, X_2 \cdots, X_n$. We can obtain $\hat{c}$ and $\hat{d}$ via density estimation:

$$\hat{c}_{j,k} = \frac{1}{n}\sum_{i=1}^{n}\phi_{j,k}(X_i) \tag{3.20}$$

and

$$\hat{d}_{j,k} = \frac{1}{n}\sum_{i=1}^{n} \psi_{j,k}(X_i), \tag{3.21}$$

id est,

$$c_{j,k} = E[\phi_{j,k}(X)] = \int \phi_{j,k}(X) f(x)\,\mathrm{d}x \tag{3.22}$$

and

$$d_{j,k} = E[\psi_{j,k}(X)] = \int \psi_{j,k}(X) f(x)\,\mathrm{d}x. \tag{3.23}$$

If density estimation is not applicable, we find $\hat{f}_J$ via DWT. The latter therefore takes f from simple function to wavelet domain. Cencov gives more detail in his 1962 paper [13].

3.3 Common Wavelet Bases

There are several well documented popualar wavelet families. In this section we will mainly focus on three: the Haar Wavelet, Daubechies' Compactly Supported Wavelets, and Daubechies' Least Asymmetric Wavelets.

3.3.1 The Haar Wavelet

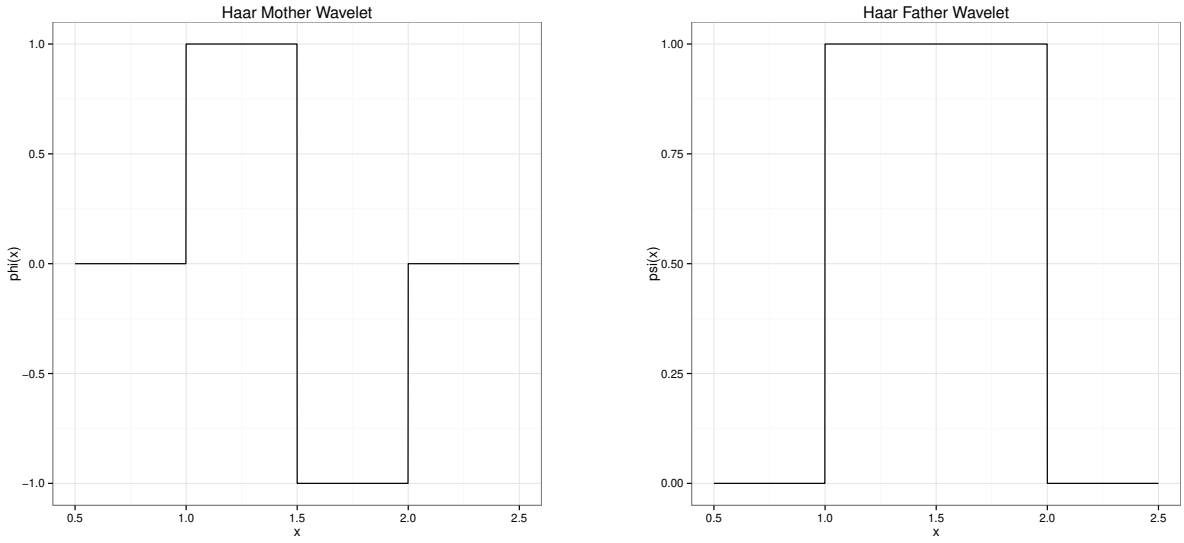
One of the most common types of wavelets is the Haar wavelet. Developed in 1910 by Alfred Haar [33], this wavelet has the ability to well approximate flat lines. One of the Haar wavelet's defining properties is that domains between coefficients do not overlap. The Haar function's mother and father wavelets are as follow: the mother wavelet for translation and dilation:

$$\psi(x) = \begin{cases} 1, & 0 \le x < \frac{1}{2} \\ -1, & \frac{1}{2} \le x < 1 \\ 0, & \text{otherwise,} \end{cases} \tag{3.24}$$

the father wavelet for scaling:

$$\phi(x) = \begin{cases} 1, & 0 \le x < 1 \\ 0, & \text{otherwise.} \end{cases} \tag{3.25}$$

Given that the Haar function is piecewise constant on intervals of $\frac{1}{2}$, it is not easily capable of well approaching curves.



(a) ϕ (b) ψ

Figure 3.1: Haar Mother and Father Wavelet Functions

3.3.2 Daubechies' Compactly Supported Wavelets

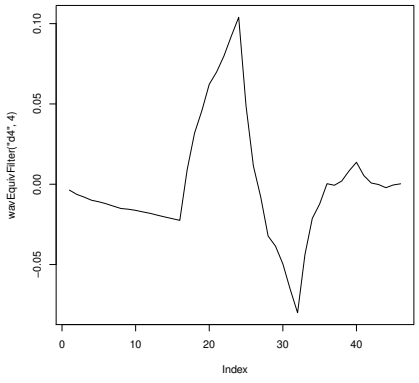
Another commonly used family of wavelet funtions are Daubechies' Compactly Supported Wavelets, of which Haar wavelets are a special case. These wavelets, for which we have compact support, are categorized by the number (N) of vanishing moments of ψ. Daubechies defines vanihing moments [21]. Suppose that ψ has $N \geq 2$) vanishing moments, then:

$$\int x^n \psi x \mathrm{d}x = 0; n = 0, 1, \cdots, N-1. \tag{3.26}$$

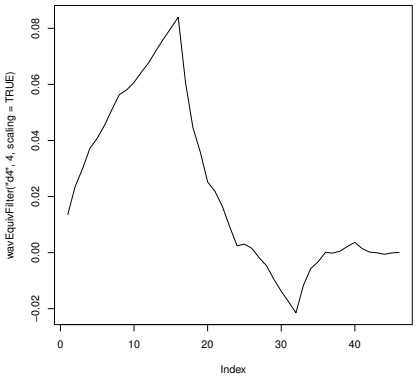
3.3.3 Daubechies' Least Asymmetric Wavelets

If we consider the family of wavelet functions in the previous subsection but whose scaling functions are closer to symmetric, we would be in the presence of Daubechies' Least Asymmetric Wavelets. Similarly, [20] shows that these wavelets, for which we have compact support, are categorized by the number (N) of vanishing moments of ϕ.

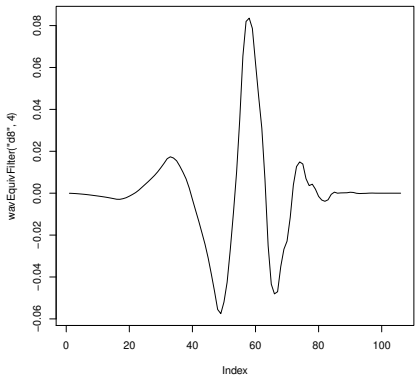
We refer to [6] for more on different types of wavelets.



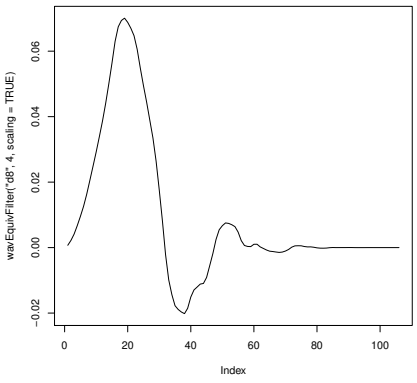
(a) $\phi(N = 2)$



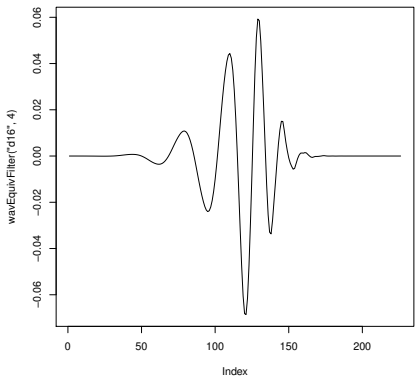
(b) $\psi(N = 2)$



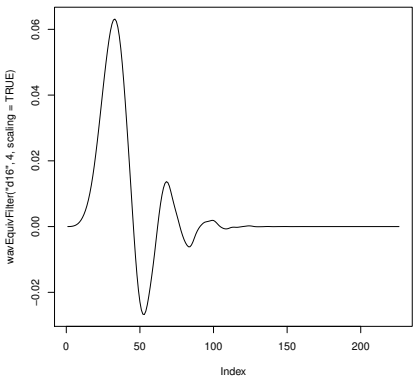
(c) $\phi(N = 4)$



(d) $\psi(N = 4)$

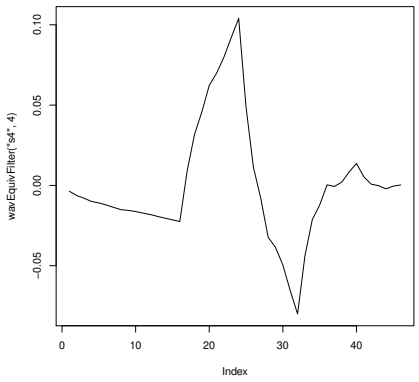


(e) $\phi(N = 8)$

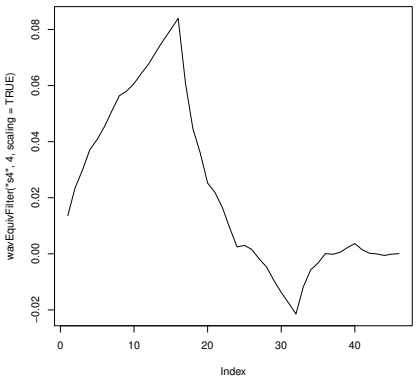


(f) $\psi(N = 8)$

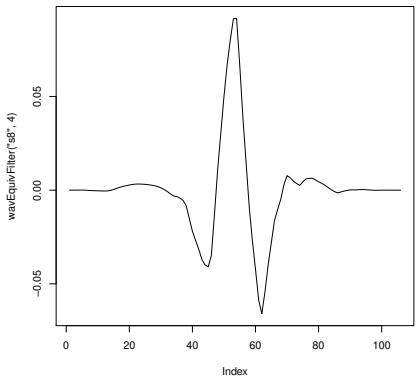
Figure 3.2: Daubechies' compactly supported wavelets and scaling functions for various N.



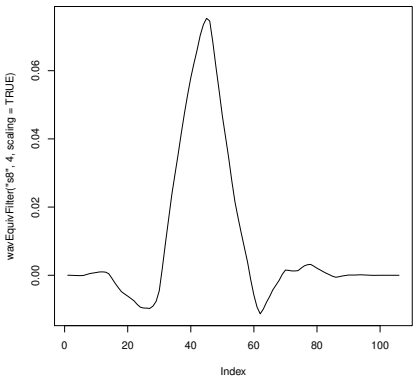
(a) $\phi(N = 2)$



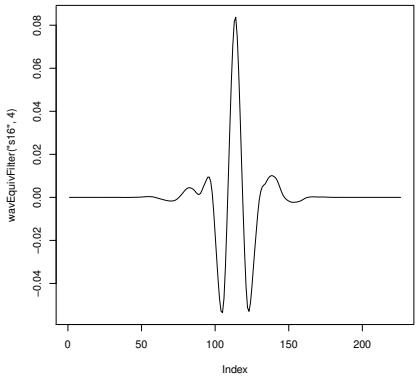
(b) $\psi(N = 2)$



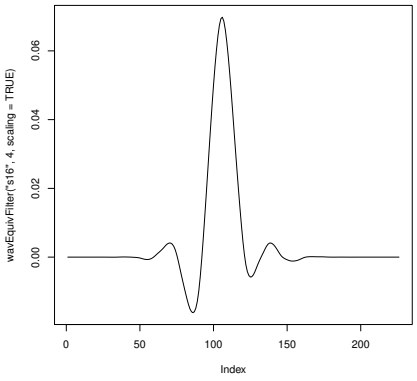
(c) $\phi(N = 4)$



(d) $\psi(N = 4)$



(e) $\phi(N = 8)$



(f) $\psi(N = 8)$

Figure 3.3: Daubechies' least asymmetric, compactly supported wavelets and scaling functions for various N.

3.4 Wavelet Thresholding

We introduce thresholding to account for noise removal when applying wavelet analysis for data reduction. Other popular methods of thresholding are Global thresholding (VisuShrink), a hybrid of Global thresholding and Stein's unbiased risk estimator (hybridShrink), and a Level Dependent thresholding (SUREShrink). Properties of each of these methods are further explained in [9, 11, 17, 18, 22, 24, 70].

3.4.1 Hard and Soft Wavelet Thresholding

As seen in [57], given a threshold value λ, different thresholding rules will apply. In the case of hard thresholding, coefficients are reduced to zero. Let $\hat{\theta}$ be the matrix of wavelet coefficients:

$$\delta^H(\tilde{\theta}_i, \lambda) = \tilde{\theta}_i \mathbf{I}[|\tilde{\theta}_i| > \lambda]. \tag{3.27}$$

In the case of soft thresholding, coefficients are shrunk toward zero. Similarly,

$$\delta^S(\tilde{\theta}_i, \lambda) = (\tilde{\theta}_i - \text{sgn}[\tilde{\theta}_i]\lambda)\mathbf{I}[|\tilde{\theta}_i| > \lambda]. \tag{3.28}$$

The following figure shows the different between both aforementioned thresholds.

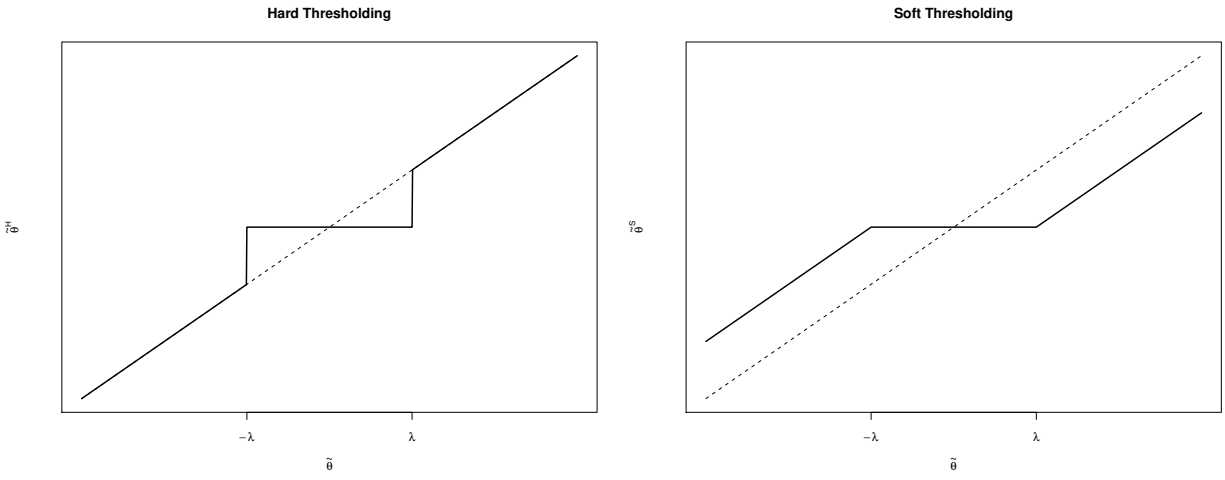


Figure 3.4: Hard and Soft Thresholding

3.4.2 Top $p\%$ Coefficient and Cross Validation Thresholding

In the Top $p\%$ Coefficient method, we identify the top $p\%$ of coefficients in magnitude, then set the remaining $(100-p)\%$ to zero. Here, the largest coefficients will contain the structure of f and the smallest, the noise. The Top $p\%$ of Coefficient Thresholding method is highly dependent on p. When p is too small, large coefficients may be unfortunately set to zero, resulting in over-smoothing of $\hat{f}$. When p is too large, small coefficients may be unnecessarily kept, resulting in under-smoothing of $\hat{f}$.

The Cross Validated method eliminates the subjective process of selecting the value of p. It is more accurate than the previous method but more computationally time consuming. We start by splitting our data into two groups. Using Nason's notation [61], the functions of the even and odd sets are:

$$f^e = \{f_2, f_4, \cdots, f_n\} \tag{3.29}$$

$$f^o = \{f_1, f_3, \cdots, f_{n-1}\} \tag{3.30}$$

Their estimates $\hat{f}^e$ and $\hat{f}^o$ are obtained through DWT. We combined the interpolated estimates to obtain the threshold:

$$\hat{M}(t) = \sum_{j=1}^{n/2} (\bar{f}^e_{t,j} - y_{2j+1})^2 + (\bar{f}^o_{t,j} - g_{2j})^2. \tag{3.31}$$

This threshold can easily be corrected to closer approximate the "true" threshold.

These two types of thresholding are also highlighted in [6]

3.4.3 (Unattainable) Optimal Thresholding

We recall the risk function and apply it to our true (f) and estimated $(\hat{f})$ functions:

$$R(f, \hat{f}) = E\|\hat{f} - f\|_2^2. \tag{3.32}$$

When f is sampled through n equally spaced points, we can apply the mean-squared-error (MSE):

$$\begin{aligned} \text{MSE}(\hat{f}, f) &= \frac{1}{n} R_n(\hat{f}, f) \\ &= \frac{1}{n} E\|\hat{f} - f\|_{l_2}^2 \\ &= \frac{1}{n} \sum_{i=1}^{n} E[\hat{f}(x_i) - f(x_i)]^2 \end{aligned} \tag{3.33}$$

The Parseval relation implies the following:

$$\text{MSE}(\hat{f}, f) = \frac{1}{n} R_n(\hat{\theta}, \theta). \tag{3.34}$$

For $\hat{\theta} = \delta(\tilde{\theta}_i, \lambda), i = 1, 2, \cdots, n$ where δ is some thresholding rule (hard or soft thresholding), the optimal threshold value, λ^* will minimize

$$\text{MSE}(\hat{f}, f) = \frac{1}{n} R_n(\hat{\theta}, \theta) = \frac{1}{n} E\|\hat{f} - f\|_{l_2}^2. \tag{3.35}$$

Donoho and Johnstone [22] also show that:

$$\begin{aligned} R_n^*(\hat{\theta}, \theta) &= \min R_n(\hat{\theta}, \theta) \\ &= R_n(\hat{\theta}|_{\lambda=\lambda^*}, \theta) \\ &= \sum_{i=1}^{n} \min(|\theta_i|^2, \sigma^2) \end{aligned} \tag{3.36}$$

This is the ideal risk. It cannot be estimated for all θ, rather help measure the performance of other thresholds. See [7, 27, 24, 44] for more details.

3.4.4 Global and Level Dependent Thresholding

A global threshold can be defined as a threshold constant for all wavelet coefficients of the observed function f. Although global thresholding λ^* can easily be implemented, the difficulty lies in setting an optimal value for λ.

From [24] we can derive the global threshold:

$$\lambda_n^* \leq \sigma\sqrt{2\log n}. \tag{3.37}$$

The above leads to the relative soft-thresholded estimator:

$$\hat{\theta}_i^* = \delta^S(\tilde{\delta}_i, \lambda_n^*), i = 1, \cdots, n \tag{3.38}$$

and the minimal risk:

$$\begin{aligned} R_n^*(\hat{\theta}^*, \theta) &= E\|\hat{\theta}^* - \theta\|_{l_2}^2 \\ &\leq \Lambda_n^* \leq 2\log n + 1. \end{aligned} \tag{3.39}$$

Asymptotically:

$$\begin{aligned} \Lambda_n^* &\sim 2\log n \qquad \text{and} \\ \lambda_n^* &\sim \sigma\sqrt{2\log n} \quad \text{as } n \to \infty \end{aligned} \tag{3.40}$$

We obtain approximately similar results when applying hard-thresholding. It should be noted that in [22], only thresholding coefficients that lie outside of the lowest resolution level are supported. In other words, only coefficients corresponding to $d_{j,k} : j_0 \leq j \leq J-1, 1 \leq k \leq 2^j$ should be thresholded. When using this strategy with soft thresholding and λ_n^*, the resulting estimator (RiskShrink) always outperforms the minimax estimator and works better on functions with non-zero means.

We refer to [22] for estimates of λ_n^* when the sample size n changes. There are multiple benefits in keeping λ fixed for all n, notably simplicity and computational effectiveness. The concept of "universal" threshold is also proposed. This concept keeps the asymptotic optimality of λ_n^* for $\lambda^u = \sigma\sqrt{2\log n}$. When combined with the RiskShrink estimator we obtain the VisuShrink summarized as:

$$\hat{\theta}^{VS} = [\tilde{\theta}_1, \cdots, \tilde{\theta}_{2^{j_0}}, \{\delta^S(\tilde{\theta}_i, \sigma\sqrt{2\log n}) : (2^{j_0}+1 \leq i \leq n)\}]'. \tag{3.41}$$

VisuShrink reconstructions are smoother and contain less noise than their RiskShrink counterparts, an additional benefit to their ease of application. However, this method comes at a cost of an inferior MSE functionality for small values of n.

An alternative to selecting a global threshold for all resolution levels is to use different thresholds at each resolution. This is the level dependent thresholding.

Donoho and Johnstone [23] developed a popular level dependent threshold using Stein's Unbiased Risk Estimator, or SURE:

$$\text{SURE}(\lambda; x) = d - 2 \cdot \#\{i : |x_i| \leq \lambda\} + \sum_{i=1}^{d} (|x_i| \wedge \lambda)^2, \tag{3.42}$$

Here, x_i are multivariate normal observarions, d is the vector of x_i's length, and $a \wedge b = min(a, b)$.

This estimate of risk is used to select a threshold λ^S:

$$\lambda^S = \underset{0 \leq \lambda \leq \sqrt{2\log d}}{\text{argmin}} \text{SURE}(\lambda; x). \tag{3.43}$$

We obtain a threshold λ^S via the SURE estimate. When d is large enough, the Law of Large Numbers shows that SURE approaches the true value of the risk and λ^S the threshold optimality. Thus method is commonly referred as SureShrink. Although the latter returns an optimal MSE, it stresses when detail coefficients have sparse resolution levels. SureShrink will therefore use the universal threshold when a level is too sparse or use the SURE based methods in other cases. We say that j is too sparse if:

$$2^{-j}\sum_{k=1}^{2^j}(d_{j,k}^2/\sigma_{j,k}^2) \leq 2^{-j/2}(\log_2 2^j)^{3/2}. \tag{3.44}$$

SureShrink is best used with soft thresholding and is more computationally expensive than VisuShrink.

3.4.5 Term-By-Term and Block Thresholding

All above thresholding methods described fall under term-by-term thresholding, that is, coefficients are compared to the threshold value individually and changed accordingly. Block thresholding however divides the data into blocks of surrounding coefficients. This method will change entire blocks simultaneously, depending if the sum of the squared coefficients in that block is less than the threshold value or not. Block thresholding methods will employ variants of the block projection estimator found in [10]. Let $B_i, i = 1, \cdots, N$ be blocks of equal size $L(\sim \log n)$ evenly dividing observed wavelet coefficients $\tilde{\theta}$, $\mathcal{H}$ be a subset of block indices $\{1, \cdots, N\}$, and $\tilde{\theta}_{B_i}$ be the L coefficients in block B_i. We are interested in having $\mathcal{H}$ consist only of blocks where the noise is less than the signal, that is:

$$\begin{aligned} \mathcal{H} &= \mathcal{H}(\tilde{\theta}) \\ &= \{i : \|\tilde{\theta}_{B_i}\|_2^2 > L\sigma^2\}, \end{aligned} \tag{3.45}$$

for $\|\tilde{\theta}_{B_i}\|_2^2 = \sum_{k\in B_i}\tilde{\theta}_k^2$. We write the final block projection estimator as:

$$\hat{\theta}_{B_i} = \begin{cases} \tilde{\theta}_{B_i} & \text{if} \quad i \in \mathcal{H} \\ 0 & \text{if} \quad i \notin \mathcal{H}, \end{cases} \tag{3.46}$$

where any value less than the threshold is set to zero. Therefore, for any specific block, the original coefficients are either kept or all coefficients in that block are set to zero. We reference the following for more on block thresholding: [9, 11, 17, 18, 35, 34, 36, 70].

CHAPTER 4

NOISE PROFILING

4.1 Profile

We recall that statistical process control methods monitor changes in mean and in standard deviation amongst other parameters. Additionally, multivariate control charts can be used to monitor changes in mean vectors and covariance matrices. We observe a profile when a characteristic is dependent on explanatory and/or independent variables. We therefore define a profile as the relationship between a response variable (y) and one or more explanatory variables ($x_1, x_2, \cdots, x_k$). For multiple pairs of data $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$, the in-control process takes the form of $y = f(x) + \epsilon$ (for a single profile). Here the function f is known and the error (or noise) ϵ is random with a mean value of zero and a standard deviation of σ. In profile monitoring, our goal is to accurately and quickly determine when a change has occurred in the profile that places it in an out-of-control state.

4.1.1 Profile Structure & Characteristics

Various methods have been proposed which focus on changes in the form of the function. Our interest, however, is in detecting changes in the standard deviation from one function to the next. In particular, we are interested not in the differences from one profile function to another, but rather the differences in the standard deviations of the profiles. Winistorfer, Young, and Walker (1996) [90] provide a prefect example of different profiles of a process in their plot of 11 VDP profiles of a plywood like material.

Let y_i^t be the observed value of the t^{th} profile for the i^{th} pair of data. From our earlier definition, the process takes the following form:

$$y_i^t = f^t(x_i) + \epsilon_i^t, \tag{4.1}$$

where $i = 1, 2, \cdots, n$ and $t = 1, 2, \cdots, T$. Here ϵ_i^t are independent and normally distributed with a standard deviation of σ_t. If the functional part f of a profile is known, the noise can be obtained by simply taking the difference between the observed value at the t^{th} profile for the i^{th} pair and the function:

$$\epsilon_i^t = y_i^t - f^t(x_i). \tag{4.2}$$

One can then monitor these errors for changes.

However, it is not generally the case that we know f. Instead, if the functional part f of a profile is unknown, but of a parametric form, it can be estimated using least squares methods (linear or non-linear). In this case, we can estimate the errors using $\hat{f}$, the estimate of f:

$$\hat{\epsilon}_i^t = y_i^t - \hat{f}^t(x_i). \tag{4.3}$$

In the situation where the functional part f of the profile is unknown and not parameterizable, we look to wavelets analysis. Our proposed change point method is based on wavelets and takes advantage of their ability to create multiple subspaces from the function (f). At the same time, they make no assumptions on the form of f other than square integrability. Thus, they are ideally suited for this third, most general case.

We recall our case of a sequence of profiles with independent and identically distributed normal errors. The DWT estimates the wavelet coefficients. This is represented notationally with the orthogonal DWT transform matrix W:

$$\tilde{\theta} = Wy^t = Wf^t + W\epsilon^t. \tag{4.4}$$

4.1.2 Coefficient Resolution Level

There are two possible cases to be considered for our noise estimation. In the first case, we consider functions f whose wavelet coefficients at the highest resolution level ($j = J - 1$) are all 0. There is therefore no structure in f that requires the highest level of coefficients. The coefficients at this highest resolution level are then from a normal distribution with mean zero and standard deviation σ due to the orthogonality of W. In the second case, we consider functions f that do require wavelet coefficients at highest resolution level. If we assume that a small number of these highest

resolution coefficients represent f, then most of these coefficients will be normal with a mean of zero and a standard deviation of σ but the remaining ones will have the same standard deviation with an unknown non-zero mean. Notice that in the second case the functions are less smooth than in the first case. This is because the functions in the second case require an additional higher resolution (less smooth) level.

The noise in an observed profile is estimated using the highest resolution level of coefficients $\tilde{\theta}_{J-1,1}, \tilde{\theta}_{J-1,2}, \cdots, \tilde{\theta}_{J-1,2^{J-1}}$. As discussed above, these coefficients are normal due to the orthogonality of the wavelet transform W. In the first case, they all have mean 0. In the second case, most have mean 0.

4.1.3 Noise Estimation Hypotheses

In the setting of this noise estimation problem, our null and alternative hypotheses can be defined as:

$$\begin{aligned} H_0 &: \sigma_0 = \sigma_1 = \cdots = \sigma_\tau = \sigma_{\tau+1} = \cdots = \sigma_T, \\ H_1 &: \sigma_0 = \sigma_1 = \cdots = \sigma_\tau \neq \sigma_{\tau+1},\ \tau + 1 > \tau. \end{aligned} \tag{4.5}$$

σ_0 is the in-control standard deviation and $\sigma_{\tau+1}$ represents the the first out-of-control noise at unknown time τ, in the alternative case. Here, $t = 1, \cdots, T$.

CHAPTER 5

EXISTING METHODS

5.1 Nonparametric Regression

To implement this procedure for noise monitoring with wavelets, set $x_i = |\tilde{\theta}_{J-1,i}|, k = 1, 2, \cdots, 2^{J-1} = n/2$, the absolute value of the estimated coefficients at the highest resolution level of detail for profile t. The value of α is taken to be 1/200 in order to have an in-control ARL of 200.

Nikoo & Noorossana [62] proposed nonparametric and parametric control charts to monitor the noise. The nonparametric method is based on Walsh averages, while the parametric methods uses an approximately χ^2 distributed statistic.

5.1.1 Nonparametric Control Chart

For a sample of data $x_i, i = 1, 2, \cdots, n/2$, from a profile, the Walsh [1] averages are given by:

$$W_r = \frac{x_l + x_h}{2}. \tag{5.1}$$

The length $n/2$ corresponds to the length of the highest resolution level of estimated wavelet coefficients. The index l runs from 1 to $n/2$, and h runs from l to $n/2$. The index r then runs from 1 to $M = (n/2)(n/2 + 1)/2$.

The median m of the Walsh averages W_r is estimated by:

$$m = \begin{cases} W_{((M+1)/2)} & \text{if } M \text{ is odd,} \\ \frac{1}{2}\left(W_{(M/2)} + W_{((M+1)/2)}\right) & \text{if } M \text{ is even,} \end{cases} \tag{5.2}$$

where $W_{(1)}, W_{(2)}, \cdots, W_{(M)}$ are the M ordered Walsh averages for the profile. A $100(1-\alpha)\%$ distribution-free confidence interval for m is obtained from the ordered Walsh averages using the Wilcoxon [89] signed-ranked test distribution.

To monitor the median m_t from profile t a distribution-free Shewhart-type chart is used, see [14]. Using a set number N of assumed in-control profiles, upper and lower control limits are found by taking the median of the N individually estimated upper and lower confidence limits as described above. For further detail see [1].

5.1.2 Parametric Control Chart

Define

$$U_t = \sum_{k=1}^{n/2} \left(\frac{\tilde{\theta}^t_{J-1,k} - \bar{\tilde{\theta}}^t_{J-1}}{\sigma} \right)^2 . \tag{5.3}$$

This statistic is used to monitor the noise at the t^{th} profile. Similarly to the nonparametric method, $\tilde{\theta}^t_{J-1,k}$ represents the estimated coefficients at the highest level of resolution and $\bar{\tilde{\theta}}^t_{J-1}$ the average of these coefficients. For m assumed in-control profiles, the value of σ^2 is estimated using the following pooled variance:

$$\hat{\sigma}^2 = \frac{\sum_{t=1}^{m} \sum_{k=1}^{n/2} (\tilde{\theta}^t_{J-1,k} - \bar{\tilde{\theta}}^t_{J-1})^2}{N - m}, \tag{5.4}$$

where $N = m(n/2)$ is the total number of wavelet coefficients at the highest level of resolution for all m assumed in-control profiles. The value of α is taken to be $1/200$ in order to have an in-control ARL of 200.

5.1.3 Simulations

For both the Parametric (Nikoo Variance) and the Nonparametric Control Chart (Nikoo Walsh) methods we considered 10 in-control profiles in our simulations. Our structural component $b = 3$ and three levels of structural contamination are used ($p = 0\%, 1\%, \text{and } 5\%$). To obtain the proper UCL to be used for the ARL estimation, we assumed an out-of-control noise $\sigma_{oc} = 1$. This is measured across different scaled values of σ_{oc}. Each simulation is done using 100 repetitions for sample sizes of $n = 2^7, 2^8, 2^9, \text{and } 2^{10}$.

5.1.4 Discussion

Rather than using coefficients with certain levels of structure, we look to implementing all of our available data. The Parametric Control Chart method calls to explore the use of the exact distribution under normality (F distribution) rather than the asymptotic distribution (χ^2 distribution). Using the estimated variance U_t can be re-written as:

$$\begin{aligned} U_t &= \frac{\sum_{k=1}^{n/2}(\tilde{\theta}_{t,J-1,k} - \bar{\tilde{\theta}}_{t,J-1})^2}{\sum_{t=1}^{m}\sum_{k=1}^{n/2}(\tilde{\theta}_{t,J-1,k} - \bar{\tilde{\theta}}_{t,J-1})^2/(N-m)} \\ &= (N-m)\frac{\sum_{k=1}^{n/2}(\tilde{\theta}_{t,J-1,k} - \bar{\tilde{\theta}}_{t,J-1})^2/\sigma^2}{\sum_{t=1}^{m}\sum_{k=1}^{n/2}(\tilde{\theta}_{t,J-1,k} - \bar{\tilde{\theta}}_{t,J-1})^2/\sigma^2}. \end{aligned} \tag{5.5}$$

Let $V_t = U_t/(N-m)$. We notice that U_t has a common distribution:

$$\begin{aligned} V_t &= \frac{\sum_{k=1}^{n/2}(\tilde{\theta}_{t,J-1,k} - \bar{\tilde{\theta}}_{t,J-1})^2/\sigma^2}{\sum_{t=1}^{m}\sum_{k=1}^{n/2}(\tilde{\theta}_{t,J-1,k} - \bar{\tilde{\theta}}_{t,J-1})^2/\sigma^2} \\ &\sim \frac{\chi^2_{(n/2)-1}/((n/2)-1)}{\chi^2_{m((n/2)-1)}/(m((n/2)-1))} \\ &\sim F_{\frac{n}{2}-1,m(\frac{n}{2}-1)}. \end{aligned} \tag{5.6}$$

The PCC method assumes that the value of σ is known. The numerator of the statistic is estimated, whereas the denominator is considered to be known. We account for variability of the latter. The upper control for the V_t statistic is thus $UCL = (N-m)F_{\frac{n}{2}-1,m(\frac{n}{2}-1),\alpha}$. In practice one can use UCL $= \chi^2_{n/2-1,\alpha}$ in lieu of the F distribution if we assume $\hat{\sigma}$ to be equivalent to σ. The sensitivity of the χ^2 is that seems to only hold when the number of in-control profiles m is small. It will fail when m is large.

We will compare the two Nikoo Nonparametric Regression methods to the MAD method in the following section.

5.2 Median Absolute Deviation

5.2.1 Methodology

A method to monitor the noise based on the median absolute deviation (MAD) and a likelihood ratio is proposed. This MAD method will estimate the change time τ and the value of the out-of-control noise σ_τ. As previously mentioned, wavelet coefficients $\tilde{\theta}_{J-1,k}$ are assumed to be normally distributed with a mean of 0 and a standard deviation of σ in the absence of any structural component at the highest level. The following equation, also detailed in Geneus' 2015 paper [31], obtains the MAD of the coefficient:

$$\begin{aligned} s_{M,t} &= \text{MAD}(\tilde{\theta}^t_{J-1}) \\ &= \frac{\text{median}(|\tilde{\theta}^t_{J-1,k} - \text{median}(\tilde{\theta}^t_{J-1})|)}{c} \\ &= \frac{\text{median }(|\tilde{\theta}^t_{J-1,k}|)}{c} \\ &= \frac{|\tilde{\theta}^t_{J-1}|_{(n/4)} + |\tilde{\theta}^t_{J-1}|_{(n/4+1)}}{2c}. \end{aligned} \tag{5.7}$$

Here $c = \Phi^{-1}(3/4)$ and Φ is CDF of a Standard Normal Distribution. The marginal density of $s_{M,t}$ is obtained by integrating the joint density of $f_{M,\sigma}(s) = \int_Y f_{S,Y}(s,y)\,\mathrm{d}y$ for $S = s_{M,t}$ and $Y = |\tilde{\theta}^t_{J-1}|_{(n/4)}$. The likelihood under the null and alternative hypotheses respectively are:

$$L_0 = \prod_{t=1}^{T} f_{M,\sigma_0}(s_{M,t}), \tag{5.8}$$

$$L_1 = \prod_{t=1}^{\tau} f_{M,\sigma_0}(s_{M,t}) \prod_{t=\tau+1}^{T} f_{M,\sigma}(s_{M,t}). \tag{5.9}$$

Thus the likelihood ratio is

$$\frac{L_1}{L_0} = \prod_{t=\tau+1}^{T} \frac{f_{M,\sigma}(s_{M,t})}{f_{M,\sigma_0}(s_{M,t})}. \tag{5.10}$$

We then obtain $\hat{\sigma}_M$ and $\hat{\tau}$, the latter by expressing the likelihood ratio as a function of τ

$$\hat{\sigma}_M(\tau) = \sigma_0 \frac{\dfrac{1}{T-\tau}\displaystyle\sum_{t=\tau+1}^{T} \text{MAD}(|\tilde{\theta}^t_{J-1}|)}{\dfrac{1}{\tau}\displaystyle\sum_{t=1}^{\tau} \text{MAD}(|\tilde{\theta}^t_{J-1}|)}, \tag{5.11}$$

$$h_M(\tau) = \prod_{t=\tau+1}^{T} \frac{f_{M,\hat{\sigma}_M(\tau)}(s_{M,t})}{f_{M,\sigma_0}(s_{M,t})}, \tag{5.12}$$

$$\hat{\tau} = \arg \max_{0 \le \tau < T} h_M(\tau). \tag{5.13}$$

The distribution of the likelihood is not known so the UCL is found via simulation. The UCL is calibrated to have an in-control ARL of 200. See [20] for additional details.

5.2.2 Simulations

All three methods described assume that the highest resolution level of coefficients, $\theta_{J-1,k}$, represent only the noise in the observed profile. This is the first case as described in Section 4.1.2. However, it is very likely that some of the wavelet coefficients at this highest level are modeling not just noise, but some function structure as well; as described in the second case. This is especially true for functions that are display some irregularity. Therefore, in addition to running simulations where the highest level of wavelet coefficients is just noise ($p = 0$), we also simulate functions where 1% ($p = 1$) and 5% ($p = 5$) of the highest level of wavelet coefficients represents noise *and* function structure, not just noise. The structure is added to randomly selected coefficients at the highest level.

The size of structural components is $3\sigma\sqrt{2\log n}$. This ensures the top structural components of our functions are big enough to be differentiated from the noise with regards to standard wavelet estimation methods. A number of true values for σ were chosen, ranging from 0.5 to 2. The in-control noise, σ_0 is set at 1. Two possibilities for the placement of out-of-control noise were considered: at the start for $\tau = 0$ and after 20 observations ($\tau = 20$). The upper control limit (UCL) is set so that the average run length (ARL) for the in-control profile is 200.

Tables A.3 and A.4, for values of $\tau = 0$ and 20 respectively, summarize the ARLs for all three methods across all percentages of structure considered and multiple cases for the out-of-control noise, σ_{oc}. In all tables ARL-1 represents the ARL excluding all false alarms and PFA is the proportion of false alarms. We use $*$ to indicate the method in question's inability to identify an ARL, or that only false alarms are reported. "Nikoo Walsh", "Nikoo Variance", and "MAD" refers to the Nonparametric Control Chart method, the Parametric Control Chart method, and the Median Absolute Deviation method, respectively. The tables are placed in the appendix section.

5.2.3 Discussion

When $\tau = 0$. As we would expect, across all percentages of structure for all methods, as the σ_{oc} noise gets closer to 1, the ARL increases. There are no false alarms when $\tau = 0$. The most significant difference between methods lies at the out-of-control noise values of $\sigma_{oc} = 0.90$ and $\sigma_{oc} = 1.10$. At the 0% of structure, the ARL value for the MAD method (17.87) is about half of those of the Nikoo Walsh (36.72) and Nikoo Variance (33.10) methods. Then same applies for $\sigma_{oc} = 1.10$ with ARLs for the MAD, Nikoo Walsh and Nikoo Variance methods of 18.18, 31.02 and 27.5 respectively. At the 0.01 proportion of structure, the Nikoo Variance method fails. The structure introduced into the detail coefficients inflates the Nikoo variance estimator so much that it rejects the null immediately. This is more obvious when $\tau = 20$, see below. The MAD method's ARL (23.72) is less than three times that of the Nikoo Walsh method (84.31) for $\sigma_{oc} = 0.90$. When the out-of-control noise is 1.10, the difference between these two techniques is of fewer runs: 15.56 and 22.56, respectively. At 5% structure at the highest detail level, the Nikoo Variance method fails again. The MAD method's ARL (46.99) is close to five times less than that of the Nikoo Walsh method (237.36) for $\sigma_{oc} = 0.90$. There doesn't seem to be any significant difference the two at $\sigma_{oc} = 1.10$.

When $\tau = 20$. As discussed above, the Nikoo Variance fails drastically with contamination ($p > 0$). Every observation, in-control or not, signals an out-of-control condition. The structure introduced into the wavelet coefficients inflates the Nikoo variance estimator so much that it rejects the null immediately. For the most part, across all proportions of structure and out-of-control noise values, the percentage of false alarm observations identified is the lowest at the MAD method with the Nikoo Walsh method being the worst. Across all proportions of structure, when the out-of-control noise is 0.90 or 1.10, the MAD method usually returns a smaller ARL than the Nikoo Walsh with values being half or fewer in most cases. The Nikoo Walsh method's ARL is 84.93 whereas the MAD method's ARL is 37.35 when $p = 1$ and $\sigma_{oc} = 0.90$. The Nikoo Walsh method's ARL is 144.55 whereas the MAD method's ARL is 56.06 when $p = 5$ with some noise. The Nikoo Walsh method has more false alarms than the MAD likelihood estimator. For example, at $\sigma_{oc} = 1.1$, there is a significant difference: not in ARL but in false alarms.

The most evident conclusion is that the Nikoo Variance method fails for any proportion p value greater than 0, regardless of τ and out-of-control noise values. Although the Walsh method is preferable to the Nikoo Variance method, it is still outperformed by the MAD method. In spite of the fact that the Walsh method appears generally reasonable, the MAD method has fewer false alarms. If we discard the false latter, our ARLs are better or comparable. An added benefit to the MAD method is that it returns estimated values for both the observation at which the change in noise takes place ($\hat{\tau}$) and the change in noise itself ($\hat{\sigma}$ vs $\hat{\sigma}_0$).

CHAPTER 6

MAD METHOD EXTENSIONS

In this chapter we cover extensions to the MAD method. These expansion are estimating the in-control variance, assessing the level of added structure, and the use windowing for both the UCL and ARL steps.

Estimated In-Control and Added Structure

This step leaves the assumption that the in-control variance is known. Furthermore, adding a structural component to the $d1$ wavelet coefficients leaves the assumption that real data do not include contamination.

Windowing

The aim is to reduce computation time. The focus is on a fixed window of profiles rather than going through the entire process at once.

6.1 Estimated In-Control Variance with Added Structure

6.1.1 Method

Estimated In-Control Variance. Rather than implementing the MAD method (Geneus Et Al [31], 2015) using a known in-control value for the noise (σ_0), the latter is estimated for a predetermined number of in-control profiles (m). When $m = \infty$, the in-control noise is the same as that of the profiles error. The coefficients are obtained using a known value for our functions' error.

Estimating in-control variance is done two ways.

- *Via MAD of the wavelet coefficients of individual in-control profiles.*
  We took the absolute value of our updated list of wavelet coefficients at the highest resolution level, $\tilde{\theta}_{J-1,k}$, with added structure for an individual in-control profile. In an exercise similar

to that performed in the MAD method, we then calculated d, the median of the absolute values divided by the quantile function for the normal distribution evaluated at .75, c. This process is repeated the same number of times as the total number of in-control profiles (m) considered. From the median of the absolute values of the coefficients we obtain an m number of potential in-control variances that are then averaged out; the in-control variance is thus the mean of the m number of ds. Thus

$$\text{MAD}_0(|\tilde{\theta}^v_{J-1}|) = d_v = \frac{\text{median}(|\tilde{\theta}^v_{J-1,k}|)}{c}, \tag{6.1}$$

and

$$\begin{aligned} \hat{\sigma}_0 &= \frac{d_1 + d_2 + \cdots + d_m}{m} \\ &= \frac{1}{m}\sum_{v=1}^{m} \text{MAD}_0(|\tilde{\theta}^v_{J-1}|), \end{aligned} \tag{6.2}$$

where $v = 1, 2, \cdots, m$.

- *Via matrix of the wavelet coefficients of all in-control profiles.*
  Once the list of updated wavelet coefficients with added structure is obtained, we create a $n/2 \times m$ matrix composed of the $n/2$ coefficients for each of the m in-control profiles. Let $o = 1, 2, \cdots, n/2$. The o^{th} coefficient for the v^{th} profile is $\tilde{\theta}^{o,v}_{J-1,k}$. Our in-control matrix is of the following format:

$$\tilde{\boldsymbol{\theta}}_{J-1,k,0} = \begin{bmatrix} \tilde{\theta}^{1,1}_{J-1,k} & \tilde{\theta}^{1,2}_{J-1,k} & \cdots & \tilde{\theta}^{1,m}_{J-1,k} \\ \tilde{\theta}^{2,1}_{J-1,k} & \tilde{\theta}^{2,2}_{J-1,k} & \cdots & \tilde{\theta}^{2,m}_{J-1,k} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\theta}^{n/2,1}_{J-1,k} & \tilde{\theta}^{n/2,2}_{J-1,k} & \cdots & \tilde{\theta}^{n/2,m}_{J-1,k} \end{bmatrix} \tag{6.3}$$

  In a similar process, we take the median of absolute value of the elements of this matrix and divide it by the quantile function for the normal distribution evaluated at .75, c. Thus returning the estimated in-control variance to be used.

$$\hat{\sigma}_0 = \frac{\text{median}(|\tilde{\boldsymbol{\theta}}_{J-1,k,0}|)}{c} \tag{6.4}$$

The addition of structure at the in-control noise estimation will account for irregularity prior to the run length determining step.

In-Control & Process Coefficients Structure Level Comparison. Following the estimation of wavelet coefficients at the highest resolution level, $\tilde{\theta}_{J-1,k}$, we can introduce contamination/structure to said enumeration of coefficients. Let num be the floor of the product of number of coefficients in our list and of the proportion of contamination we want to introduce. The number of coefficients will be half of the sample size since we are considering the $d1$ level. In cases where the product is zero, that is, when the percent of contamination we wish to add is zero, we can simply set num to 1.

We create an index for which we sample num. This sampling is done without replacement and the index is matched to the initial list of coefficients, creating a new vector of coefficient values, $\tilde{\vartheta}_{J-1,k}$.

The new list of coefficient values $\tilde{\theta}^*_{J-1,k}$ is added to the product of our replication, the selected level of structural component b, the out-of-control variance σ_{oc}, and the square root of two times the log of the sample size. Thus our updated list of $d1$ wavelet coefficients with added structure:

$$\tilde{\theta}_{J-1,k} = \tilde{\theta}^*_{J-1,k} \pm b\sigma_{oc}\sqrt{2\log(n)} \tag{6.5}$$

This procedure is done twice: once we have obtained the wavelet coefficients for the in-control profiles, and once we have obtained the wavelet coefficients for the entire process. We will continue to refer to coefficients with contamination as $\tilde{\theta}_{J-1,k}$ in the remainder of this dissertation.

The added structure at the control limit and noise estimation level for the Nikoo Walsh and Variance methods [62] respectively allows for testing their limitations and a balanced analysis between the methods. Similarly to the MAD method, it is done using a set number ($m = 40$) of in-control profiles. The statistic or statistics of interest (estimated noise or estimated control limits) are then used to develop run lengths. The MAD method however calls for a new upper control limit estimation with a change of number of in-control profiles, level of structure (p) added, and/or sample size (n). All methods contain added structure ($p = 0\%$, 1% or 5%) at the in-control variance estimation, and the UCL or ARL determination step. Using sample sizes ranging from $n = 2^7$ to $n = 2^{10}$ permit comparison of their run lengths and their percentages of false alarms.

6.2 Windowing

6.2.1 Method

Nikoo's methods are not reliable when the in-control variance is estimated. Our focus is now on extending the MAD method. The goal is to increase computational efficiency. Since simulations were computationally expensive, an alternative is needed regardless of the formable results. Rather than run the simulation through all profiles, we establish a window of arbitrary value at the tail end of these functional processes.

Let "winup" be the calculated upper bound of the considered window, It is the difference between the profile at time T, the last profile, and 1. Let "winlow" be the lower end of said window, the maximum of either 0, or the difference between T and w. The latter is a set value, in other words, a fixed window size. This procedure is similar to taking a fixed sampling interval in the control charts for $\bar{x}$. See Reynolds, Amin, Arnold, & Nachlas (1988) [76], Rodrigues Dias, Infante (2008) [78], Sun, Shen, Wang, & Jin (2016) [84], and Harvey (1993) [38] for more.

We define our upper and lower interval bounds as:

$$\text{winlow} = \max(0, T - w), \tag{6.6}$$

$$\text{winup} = T - 1. \tag{6.7}$$

Rather than calculating a $\hat{\sigma}_\tau$ value for the total list of processes for taw in 0 to $T-1$, we estimate $\hat{\sigma}_\tau$ for an interval or window of processes for τ in winlow to winup. We recall from equation 5.11:

$$\hat{\sigma}_M(\tau) = \hat{\sigma}_\tau = \hat{\sigma}_0 \cdot \frac{\frac{1}{T-\tau}\sum_{t=\tau+1}^{T} \text{MAD}(|\tilde{\theta}^t_{J-1}|)}{\frac{1}{\tau}\sum_{t=1}^{\tau} \text{MAD}(|\tilde{\theta}^t_{J-1}|)}, \tag{6.8}$$

With windowing, $\hat{\sigma}_\tau$ is evaluated as:

$$\hat{\sigma}_M(\tau) = \hat{\sigma}_\tau = \frac{1}{T-\tau}\sum_{t=\tau+1}^{T} \text{MAD}(|\tilde{\theta}^t_{J-1}|). \tag{6.9}$$

An alternative method of calculating $\hat{\sigma}_\tau$ can be found in the following section.

6.3 Simulations

Simulations are ran for the same values of τ as before ($\tau = 0$ and 20) and the same percentages of contamination ($p = 0, 1, 5$). Although not explicitly specified, the Nikoo Variance method requires a lower control limit for values of the out-of-control noise ($\sigma_{oc} < 1$) when no structure is added. The study design as a whole can be repeated when the structural components' coefficient is $b = 1$ in lieu of 3 used this paper.

All tables summarize the ARLs for the presented methods across all percentages of structure considered and multiple cases for the out-of-control noise, σ_{oc}. In all tables ARL-1 represents the ARL excluding all false alarms and PFA is the proportion of false alarms. We use $*$ to indicated that only false alarms were reported. "Nikoo Walsh", "Nikoo Variance", and "MAD" refer to the Nonparametric Control Chart method, the Parametric Control Chart method, and the Median Absolute Deviation method, repectively. For tables A.11, A.12, A.13, A.14, A.15, A.16, A.23, and A.24 of the appendix, structure is added in the following order: column (p_1, structure applied to in-control profiles), then row (p_2, structure applied to process profiles). This manuscript reports results for a sample size of $n = 2^7$. Tables A.23 and A.24, along with figure 6.1, compare the methods when $p_1 = p_2 = p$.

A window of size 10 is considered. It is applied to both the UCL and ARL estimations of the proposed MAD simulations, and remaining simulations. The upper control limit and average run length values are obtained as before.

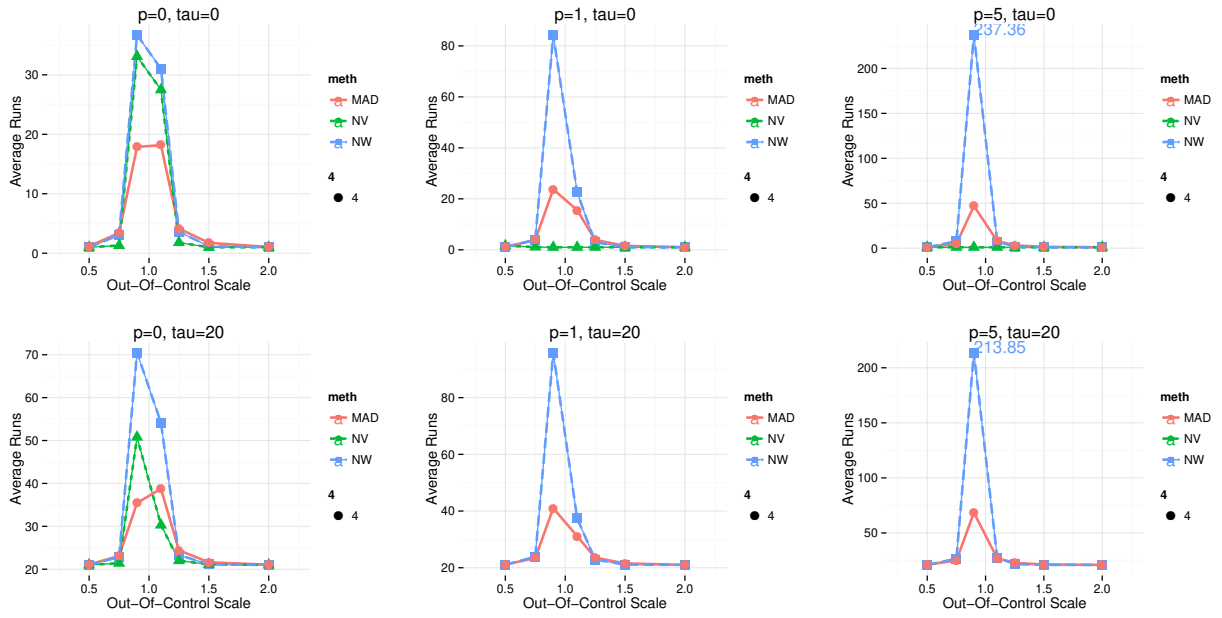


Figure 6.1: Method Comparison for $n = 2^7$. Size of Structural Component $3\sigma\sqrt{2\log(n)}$, 100 replications, $\tau = 0, 20$.

6.4 Discussion

We obtained similar results under the known in-control variance. In the absence of structure, results showed that the runs for the MAD method attained with the estimated in-control noise outperform those of both the Nikoo Walsh and Variance methods. The ARLs for $\sigma_{oc} = 1$ are closer to 200 for the MAD method (ARL-1=203.99) compared to other methods (Nikoo Walsh ARL-1=253.90, Nikoo Variance ARL-1=185.24). Furthermore, the overall proportion of false alarms recorded are lower. This significant change is due to the increase of number of in-control profiles. For the MAD, 3 UCLs based on the different levels of contamination are used throughout each average run length determination step.

Nonparametric Control Chart Method (Nikoo Walsh)

This method is somewhat resistant to added levels of structure but works best when neither p_1 nor p_2 is .05. When $\tau = 0$, the Nikoo Walsh method is conservative where structure for the in-control profiles is greater of equal to the structure added to the process profile coefficients. When $\tau = 20$,

the Walsh method is simply very conservative. The proportion of false alarm can be very high, with values of up to .34.

Parametric Control Chart Method (Nikoo Variance)

Although very favorable when $p_2 = 0$, the Variance method does not perform well when structure is at to the process profile coefficients. This holds when τ is both 0 and 20.

Median Absolute Deviation

The MAD method is not robust when contamination is introduced. This method performs best when the level of added structure is less than 1%. This is consistent when $\tau = 0$ and $\tau = 20$. In the latter case, the PFA are fairly low and never more than .15.

Comparison

Once structure is introduced beyond the in-control estimation step, all methods return unfavorable average run lengths. Therefore, we consider ARLs where the level of structure is the same for the in-control and process profiles. That is, where $p_1 = p_2 = p$.

When $\tau = 0$ we do not expect any false alarms. At $p = 0$, the Nikoo Variance method outperforms both remaining methods. However, this method fails if any structure is added. In this case, the MAD method will outperform the Walsh method (for $p = 1 and 5$). This is seen by the overall fewer runs returned by the MAD method, both when $\sigma_{oc} \neq 1$ and $\sigma_{oc} = 1$.

In the case where $\tau = 20$, the Nikoo Variance and Walsh methods' false alarm rates are comparable and relatively acceptable. However, they are both outperformed by the MAD method, with the latter resulting in fewer overall percent of false alarms. Following added structure, only false alarms were reported for the Nikoo Walsh and MAD methods. the MAD method outperforms that of the Walsh, as seen by the overall fewer runs returned, both when $\sigma_{oc} \neq 1$ and $\sigma_{oc} = 1$.

The most evident conclusion is that the Nikoo Variance method fails for any proportion p greater than 0, regardless of τ and out-of-control noise values. Although the Nikoo Walsh method is preferable to the Nikoo Variance method, it is still outperformed by the MAD. The MAD method has overall fewer false alarms, and ARL-1s either are more fit or comparable.

CHAPTER 7

ALTERNATIVE LIKELIHOOD RATIO TECHNIQUES

This is the first improvement on the previously introduced MAD method, as it incorporates the aforementioned methods. The underlying function is known.

7.1 Theory

In the traditional sense of the MAD method, the marginal distribution had a normal (known density). We therefore used a parametric function (Gaussian density) in the likelihood estimation. It is easy to get a likelihood function, and to an extent a maximum likelihood estimation and likelihood ratios, when the density is known. Let us consider a known distribution f_θ where $\theta \in \Theta \subseteq \mathbb{R}^p$ is unknown.

$$f(x_1, x_2, \cdots, x_n; \theta_n) = P(X_1 = x_1, X_2 = x_2, \cdots, X_n = x_n) \tag{7.1}$$

The likelihood of the unknown set of θs given the data x is the probability of getting the latter given the parameters. This is given by

$$\begin{aligned} L(\theta) &= L(\theta; x_1, x_2, \cdots, x_n) \\ &= f(x_1, x_2, \cdots, x_n; \theta) \\ &= P(X_1 = x_1, X_2 = x_2, \cdots, X_n = x_n), \end{aligned} \tag{7.2}$$

or

$$L(\theta|x) = f(x|\theta). \tag{7.3}$$

In our parametric case, the density is Normal,

$$f_X(x_j) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right) e^{\left(-\frac{(x_j-\mu)^2}{2\sigma^2}\right)}, \tag{7.4}$$

and the likelihood is expressed as following:

$$\begin{aligned} L(\mu, \sigma^2; x_1, \cdots, x_n) &= \prod_{j=1}^{n} f_X(x_j, \mu, \sigma^2) \\ &= \prod_{j=1}^{n} \left(\frac{1}{\sigma\sqrt{2\pi}} \right) e^{\left(-\frac{(x_j-\mu)^2}{2\sigma^2} \right)} \\ &= \left(\frac{1}{\sigma\sqrt{2\pi}} \right)^n \exp \left(\begin{array}{c} \displaystyle -\frac{\sum_{j=1}^{n} (x_j - \mu)^2}{2\sigma^2} \\ \\ \end{array} \right). \end{aligned} \tag{7.5}$$

By rule, $\hat{\theta}$ is a point estimator for θ when $L(\hat{\theta}|x) = \sup_\theta L(\theta|x)$. In other words, $\hat{\theta}$ maximizes the likelihood. If the maximum is unique, we define the maximum likelihood estimator (MLE) as:

$$\hat{\theta} = \hat{\theta}(x) = \arg\max_\theta L(\theta|x). \tag{7.6}$$

Let the null and alternative hypotheses be $\theta \in \Theta_0$ and $\theta \in \Theta_1$, respectively (or $H_0 : \ \theta = \theta_0$, $H_1 : \ \theta = \theta_1$ in the simplest form). The likelihood ratio (LR) ide defined as:

$$LR(x) = \frac{\{x|\theta_1\}}{\{x|\theta_0\}} = \frac{L(\theta_1)}{L(\theta_0)}. \tag{7.7}$$

This gives the rejection region $\mathcal{R} = \{x : LR > k\}$. [12] provides more on parametric likelihood inference.

7.2 Single In-Control Variance

Prior to obtaining a likelihood ratio, we need to estimate the variance at time τ. There are 2 ways of getting $\hat{\sigma}_M(\tau)$, each procedure comparable to how the in-control variance $\hat{\sigma}_0$ is obtained.

- *Via MAD of the wavelet coefficients of individual profiles.*
  Just as before, we took the absolute value of our updated list of $d1$ wavelet coefficients with added structure for an individual general profile. Similarly, we then calculated $s_{M,t}$, the median of the absolute values divided by the quantile function for the normal distribution evaluated at .75, c.

$$\text{MAD}(|\tilde{\theta}^t_{J-1}|) = s_{M,t} = \frac{\text{median}(|\tilde{\theta}^t_{J-1,k}|)}{c} \tag{7.8}$$

When using windowing, the variance at time τ is the mean of the number of $s_{M,t}$ in the interval profiles.

$$\begin{aligned}\hat{\sigma}_M(\tau) = \hat{\sigma}_\tau &= \frac{s_{M,\tau+1} + s_{M,\tau+2} + \cdots + s_{M,T}}{T-\tau} \\ &= \frac{1}{T-\tau}\sum_{t=\tau+1}^{T} \text{MAD}(|\tilde{\theta}^t_{J-1}|)\end{aligned} \tag{7.9}$$

- *Via matrix of the wavelet coefficients of all profiles.*
  Just as in the in-control case, one the list of updated $d1$ wavelet coefficients with added structure is obtained, we create a $n/2 \times T$ matrix composed of the $n/2$ coefficients for each of the T profiles.

  Let $o = 1, 2, \cdots, n/2$. The o^{th} coefficient at the t^{th} profile is $\tilde{\theta}^{o,t}_{J-1,k}$. The coefficient-profile matrix is:

$$\tilde{\boldsymbol{\theta}}_{J-1,k,\tau} = \begin{bmatrix} \tilde{\theta}^{1,1}_{J-1,k} & \tilde{\theta}^{1,2}_{J-1,k} & \cdots & \tilde{\theta}^{1,\tau}_{J-1,k} & \tilde{\theta}^{1,\tau+1}_{J-1,k} & \cdots & \tilde{\theta}^{1,T}_{J-1,k} \\ \tilde{\theta}^{2,1}_{J-1,k} & \tilde{\theta}^{2,2}_{J-1,k} & \cdots & \tilde{\theta}^{2,\tau}_{J-1,k} & \tilde{\theta}^{2,\tau+1}_{J-1,k} & \cdots & \tilde{\theta}^{2,T}_{J-1,k} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \tilde{\theta}^{n/2,1}_{J-1,k} & \tilde{\theta}^{n/2,2}_{J-1,k} & \cdots & \tilde{\theta}^{n/2,\tau}_{J-1,k} & \tilde{\theta}^{n/2,\tau+1}_{J-1,k} & \cdots & \tilde{\theta}^{n/2,T}_{J-1,k} \end{bmatrix} \tag{7.10}$$

  The estimated variance $\hat{\sigma}_\tau$ is the median of absolute value of the elements of this matrix, from the $\tau^{th}+1$ to T^{th} set, for the windowed case, divided by the quantile function for the normal distribution evaluated at .75, c.

$$\hat{\sigma}_M(\tau) = \hat{\sigma}_\tau = \frac{\text{median}(|\tilde{\boldsymbol{\theta}}_{J-1,k,\tau}|^T_{t=\tau+1})}{c} \tag{7.11}$$

Each procedure is repeated the same number of times as it takes the complete process to go out of control. Multiple UCLs are considered. The estimated upper control limit is then used in the ARL estimating step.

The next step is the calculate the likelihood ratio using the estimated in-control variance, and the variance at time τ. There are two ways of obtaining the likelihood ratio.

7.2.1 Scale-Out Technique

This is the MAD method with extensions. The likelihoods under the null and alternative hypotheses respectively are:

$$L_0 = \prod_{t=1}^{T} f_{M,\hat{\sigma}_0}(s_{M,t}) \tag{7.12}$$

and

$$L_1 = \prod_{t=1}^{\tau} f_{M,\hat{\sigma}_0}(s_{M,t}) \prod_{t=\tau+1}^{T} f_{M,\hat{\sigma}_\tau}(s_{M,t}). \tag{7.13}$$

The null likelihood is the product of the densities evaluated at the estimated IC noise. The alternative is the product of the densities evaluated at both the inverse estimated IC and process noises when the process is IC and OOC, respectively. f_M is the density of s when the data are Standard Normal ($Z \sim N(0,1)$). Therefore:

$$\begin{aligned} H_0 : X \sim N(0, \hat{\sigma}_0^2) &\longrightarrow \hat{\sigma}_0 Z \sim \frac{1}{\hat{\sigma}_0} f_M\left(\frac{1}{\hat{\sigma}_0}\right) \\ H_1 : Y \sim N(0, \hat{\sigma}_\tau^2) &\longrightarrow \hat{\sigma}_\tau Z \sim \frac{1}{\hat{\sigma}_\tau} f_M\left(\frac{1}{\hat{\sigma}_\tau}\right) \end{aligned} \tag{7.14}$$

Whether using all profiles or an interval of profiles, the Scale-Out Likelihood Ratio is a ratio of parametric functions evaluated at $s_{M,t}$ and the variance of the τ^{th} profile (for the numerator), and at $s_{M,t}$ and the estimated in-control variance (for the denominator). Thus the likelihood ratio with respective scale coefficients:

$$\begin{aligned} \frac{L_1}{L_0} &= \frac{\prod_{t=1}^{\tau} f_{M,\hat{\sigma}_0}(s_{M,t}) \prod_{t=\tau+1}^{T} f_{M,\hat{\sigma}_\tau}(s_{M,t})}{\prod_{t=1}^{T} f_{M,\hat{\sigma}_0}(s_{M,t})} \\ &= \prod_{t=\tau+1}^{T} \frac{f_{M,\hat{\sigma}_\tau}(s_{M,t})}{f_{M,\hat{\sigma}_0}(s_{M,t})} \\ &= \prod_{t=\tau+1}^{T} \frac{(1/\hat{\sigma}_\tau) f_M(s_{M,t}/\hat{\sigma}_\tau)}{(1/\hat{\sigma}_0) f_M(s_{M,t}/\hat{\sigma}_0)}. \end{aligned} \tag{7.15}$$

This process is repeated for values of between $\tau + 1$ and T, for which we take a product of.

7.2.2 Scale-In Technique

Let f_0 be any known distribution. For two variables, the null and the alternative hypotheses are defined as:

$$\begin{aligned} H_0 &: X \sim N(0, \hat{\sigma}_0^2) \\ H_1 &: Y \sim N(0, \hat{\sigma}_\tau^2) \end{aligned} \tag{7.16}$$

Expressing one variable in terms of the other involves using the scale coefficient with the function.

$$\begin{aligned} Y &\sim \frac{\hat{\sigma}_\tau^2}{\hat{\sigma}_0^2} X \longrightarrow P(Y \leq y) = F_X\left(X \leq \frac{\hat{\sigma}_0}{\hat{\sigma}_\tau} y\right) \\ Y &\sim \left(\frac{\hat{\sigma}_0}{\hat{\sigma}_\tau}\right) f_X\left(\frac{\hat{\sigma}_0}{\hat{\sigma}_\tau} y\right) \end{aligned} \tag{7.17}$$

The respective likelihoods are:

$$L_0 = \prod_{t=1}^{T} f_0(s_{0,t}) \tag{7.18}$$

and

$$L_1 = \prod_{t=1}^{\tau} f_0(s_{0,t}) \prod_{t=\tau+1}^{T} f_{0,(\hat{\sigma}_0/\hat{\sigma}_\tau)}(s_{0,t}). \tag{7.19}$$

The Scale-In Likelihood Ratio is a ratio of parametric functions evaluated at $s_{0,t}$ and the estimated in-control variance, over the variance of the τ^{th} profile for the numerator. It is simply evaluated at $s_{0,t}$ for the denominator. It also includes a ratio of the in-control variance and that at σ_τ.

$$\begin{aligned} \frac{L_1}{L_0} &= \frac{\prod_{t=1}^{\tau} f_0(s_{0,t}) \prod_{t=\tau+1}^{T} f_{0,(\hat{\sigma}_0/\hat{\sigma}_\tau)}(s_{0,t})}{\prod_{t=1}^{T} f_0(s_{0,t})} \\ &= \prod_{t=\tau+1}^{T} \frac{f_{0,(\hat{\sigma}_0/\hat{\sigma}_\tau)}(s_{0,t})}{f_0(s_{0,t})} \\ &= \prod_{t=\tau+1}^{T} \frac{(\hat{\sigma}_0/\hat{\sigma}_\tau) f_0(s_{0,t}\hat{\sigma}_0/\hat{\sigma}_\tau)}{f_0(s_{0,t})}. \end{aligned} \tag{7.20}$$

This is done for all values between the $\tau^{th} + 1$ and the T^{th} profile, for which we take a product of.

7.3 Multiple In-Control Variances

An alternative method of calculating the likelihood ratio is to include in-control profiles from both the entire process and the in-control process. The previous Single In-Control Likelihood Ratio

technique only includes the information from the in-control process for the $\hat{\sigma}_0$. Estimating $\hat{\sigma}_\tau$ is identical in both the Single and Multiple In-Control techniques.

- *Via MAD of the wavelet coefficients of respective profiles.*
  When $\tau = 0$, the first in control variance, $\hat{\sigma}_{0,a}$ is the mean of previously calculated d for all in control profiles. See equation 6.2. When $\tau > 0$, $\hat{\sigma}_{0,a}$ is the mean of previously calculated d and $s_{M,t}$ for all initial in-control profiles, and in-control profiles from the process, that is, profiles 1 to τ. See equation 5.7.
  At $\tau = 0$,

$$\hat{\sigma}_{0,a} = \frac{1}{m}\sum_{v=1}^{m} \mathrm{MAD}_0(|\tilde{\theta}^v_{J-1}|). \tag{7.21}$$

  At $\tau > 0$,

$$\begin{aligned}\hat{\sigma}_{0,a} &= \frac{d_1 + d_2 + \cdots + d_m + s_{M,1} + s_{M,2} + \cdots + s_{M,\tau}}{m + (\tau - 1)} \\ &= \frac{1}{m + (\tau - 1)}\left[\sum_{v=1}^{m} \mathrm{MAD}_0(|\tilde{\theta}^v_{J-1}|) + \sum_{t=1}^{\tau} \mathrm{MAD}(|\tilde{\theta}^t_{J-1}|)\right].\end{aligned} \tag{7.22}$$

  The second in-control variance, $\hat{\sigma}_{0,b}$ is the mean of previously calculated d and $s_{M,t}$ for all in-control profiles, and all profiles used throughout the process.

$$\begin{aligned}\hat{\sigma}_{0,b} &= \frac{d_1 + d_2 + \cdots + d_m + s_{M,1} + s_{M,2} + \cdots + s_{M,T}}{m + (T - 1)} \\ &= \frac{1}{m + (T - 1)}\left[\sum_{v=1}^{m} \mathrm{MAD}_0(|\tilde{\theta}^v_{J-1}|) + \sum_{t=1}^{T} \mathrm{MAD}(|\tilde{\theta}^t_{J-1}|)\right].\end{aligned} \tag{7.23}$$

- *Via matrix of the wavelet coefficients of respective profiles.*
  If $\tau = 0$, the first in-control variance, $\hat{\sigma}_{0,a}$, is the median of the absolute value of the elements of the matrix used to find the in-control variance, divided by the quantile function for the normal distribution evaluated at .75, c. See equation 6.4. When $\tau > 0$, the previous matrix is combined with the in-control profiles of the entire process, that is, the profiles for values between 1 and τ. This creates a $n/2 \times (m + \tau)$ matrix of wavelet coefficients.
  $\tau = 0$,

$$\hat{\sigma}_{0,a} = \frac{\mathrm{median}(|\tilde{\boldsymbol{\theta}}_{J-1,k,0}|)}{c} \tag{7.24}$$

$\tau > 0$,

$$\tilde{\boldsymbol{\theta}}_{J-1,k,0a} = \begin{bmatrix} \tilde{\theta}^{1,1}_{J-1,k} & \tilde{\theta}^{1,2}_{J-1,k} & \cdots & \tilde{\theta}^{1,m}_{J-1,k} & \tilde{\theta}^{1,1}_{J-1,k} & \tilde{\theta}^{1,2}_{J-1,k} & \cdots & \tilde{\theta}^{1,\tau}_{J-1,k} \\ \tilde{\theta}^{2,1}_{J-1,k} & \tilde{\theta}^{2,2}_{J-1,k} & \cdots & \tilde{\theta}^{2,m}_{J-1,k} & \tilde{\theta}^{2,1}_{J-1,k} & \tilde{\theta}^{2,2}_{J-1,k} & \cdots & \tilde{\theta}^{2,\tau}_{J-1,k} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{\theta}^{n/2,1}_{J-1,k} & \tilde{\theta}^{n/2,2}_{J-1,k} & \cdots & \tilde{\theta}^{n/2,m}_{J-1,k} & \tilde{\theta}^{n/2,1}_{J-1,k} & \tilde{\theta}^{n/2,2}_{J-1,k} & \cdots & \tilde{\theta}^{n/2,\tau}_{J-1,k} \end{bmatrix} \tag{7.25}$$

$$\hat{\sigma}_{0,a} = \frac{\text{median}(|\tilde{\boldsymbol{\theta}}_{J-1,k,0a}|)}{c} \tag{7.26}$$

The second in-control variance, $\hat{\sigma}_{0,b}$ is the median of the absolute value of the elements of the matrix used to find $\hat{\sigma}_0$, combined with all profiles of the entire process, that is, the profiles for values between 1 and T. This median is then divided by the quantile function for the normal distribution evaluated at .75, c. This creates a $n/2 \times (m + K - 1)$ matrix of wavelet coefficients.

$$\tilde{\boldsymbol{\theta}}_{J-1,k,0b} = \begin{bmatrix} \tilde{\theta}^{1,1}_{J-1,k} & \tilde{\theta}^{1,2}_{J-1,k} & \cdots & \tilde{\theta}^{1,m}_{J-1,k} & \tilde{\theta}^{1,1}_{J-1,k} & \tilde{\theta}^{1,2}_{J-1,k} & \cdots & \tilde{\theta}^{1,T}_{J-1,k} \\ \tilde{\theta}^{2,1}_{J-1,k} & \tilde{\theta}^{2,2}_{J-1,k} & \cdots & \tilde{\theta}^{2,m}_{J-1,k} & \tilde{\theta}^{2,1}_{J-1,k} & \tilde{\theta}^{2,2}_{J-1,k} & \cdots & \tilde{\theta}^{2,T}_{J-1,k} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{\theta}^{n/2,1}_{J-1,k} & \tilde{\theta}^{n/2,2}_{J-1,k} & \cdots & \tilde{\theta}^{n/2,m}_{J-1,k} & \tilde{\theta}^{n/2,1}_{J-1,k} & \tilde{\theta}^{n/2,2}_{J-1,k} & \cdots & \tilde{\theta}^{n/2,T}_{J-1,k} \end{bmatrix} \tag{7.27}$$

$$\hat{\sigma}_{0,b} = \frac{\text{median}(|\tilde{\boldsymbol{\theta}}_{J-1,k,0b}|)}{c} \tag{7.28}$$

Similar to the Single In-Control technique, we obtained the likelihood ratio using the Scale-Out and Scale-In techniques.

7.3.1 Scale-Out Technique

Let A_1 be the product of the in-control parametric function evaluated at the ratio of $s_{M,t}$ and the variance of the τ^{th} profile, and of its respective scale coefficient; for values of greater than τ. We take a similar route for each of the calculated in-control variances. Let A_2 be the product of the in-control parametric (here Gaussian) function evaluated at the ratio of $s_{M,t}$ and the first in-control control variance $\hat{\sigma}_{0,a}$, and of its respected scale coefficient; for values less than τ. Let A_3 be the

product of the in-control Gaussian function evaluated at the ratio of $s_{M,t}$ and the second in-control control variance $\hat{\sigma}_{0,b}$, and of its respected scale coefficient. The Scale-Out likelihood ratio is the product of A_1, A_2, and the inverse of A_3. The likelihoods under the null and alternative hypotheses respectively are:

$$L_0 = \prod_{t=1}^{T} f_{M,\hat{\sigma}_{0,b}}(s_{M,t}) = A_3, \tag{7.29}$$

and

$$L_1 = \prod_{t=1}^{\tau} f_{M,\hat{\sigma}_{0,a}}(s_{M,t}) \prod_{t=\tau+1}^{T} f_{M,\hat{\sigma}_{\tau}}(s_{M,t}) = A_1 A_2. \tag{7.30}$$

Thus the likelihood ratio:

$$\begin{aligned} \frac{L_1}{L_0} &= \frac{A_1 A_2}{A_3} \\ &= \frac{\prod_{t=1}^{\tau} f_{M,\hat{\sigma}_{0,a}}(s_{M,t}) \prod_{t=\tau+1}^{T} f_{M,\hat{\sigma}_{\tau}}(s_{M,t})}{\prod_{t=1}^{T} f_{M,\hat{\sigma}_{0,b}}(s_{M,t})} \\ &= \frac{\prod_{t=1}^{\tau} \hat{\sigma}_{0,a} f_M(s_{M,t}/\hat{\sigma}_{0,a}) \prod_{t=\tau+1}^{T} \hat{\sigma}_{\tau} f_M(s_{M,t}/\hat{\sigma}_{\tau})}{\prod_{t=1}^{T} \hat{\sigma}_{0,b} f_M(s_{M,t}/\hat{\sigma}_{0,b})}. \end{aligned} \tag{7.31}$$

This process is repeated for values between the max of winlow and 1, and the T^{th} profile.

7.3.2 Scale-In Technique

The likelihood ratio for the Multiple In-Control is the same as that of the Single In-Control. This is seen by the following. If we let A_1 be the product of the in-control parametric function evaluated at the ratio of $s_{M,t}$, the in-control variance, and the inverse of the variance of the τ^{th} profile, and the ratio of the latter two mentioned variances. This product is done for values greater than τ. We take a similar route for each of the calculated in-control variances. Let A_2 be 1 for values less than τ. Let A_3 be the in-control parametric (here Gaussian) function simply evaluated $s_{M,t}$. Whether using all profiles or an interval of profiles, the Scale-Out likelihood ratio is the product of A_1, A_2, and the inverse of A_3. We define the null and the alternative likelihoods:

$$L_0 = \prod_{t=1}^{T} f_M(s_{M,t}) = A_3, \tag{7.32}$$

$$L_1 = \prod_{t=1}^{\tau} f_M(s_{M,t}) \prod_{t=\tau+1}^{T} f_{M,(\hat{\sigma}_{0,a}/\hat{\sigma}_\tau)}(s_{M,t}) = A_1 A_2. \tag{7.33}$$

The likelihood ratio follows,

$$\begin{aligned}
\frac{L_1}{L_0} &= \frac{A_1 A_2}{A_3} \\
&= \frac{\prod_{t=1}^{\tau} f_M(s_{M,t}) \prod_{t=\tau+1}^{T} f_{M,(\hat{\sigma}_{0,a}/\hat{\sigma}_\tau)}(s_{M,t})}{\prod_{t=1}^{T} f_M(s_{M,t})} \\
&= \prod_{t=\tau+1}^{T} \frac{f_{M,(\hat{\sigma}_{0,a}/\hat{\sigma}_\tau)}(s_{M,t})}{f_M(s_{M,t})} \\
&= \prod_{t=\tau+1}^{T} \frac{(\hat{\sigma}_{0,a}/\hat{\sigma}_\tau) f_M(s_{M,t}\hat{\sigma}_{0,a}/\hat{\sigma}_\tau)}{f_M(s_{M,t})},
\end{aligned} \tag{7.34}$$

Where $\hat{\sigma}_{0,a} = \hat{\sigma}_0$. This is done for all values between the $\tau^{th} + 1$ and the T^{th} profile, for which we take a product of.

7.4 Simulations

For all the different cases of Likelihood Ratio techniques, the UCLs are obtained form the estimated in-control variance, $\hat{\sigma}_0$, whereas the process for the ARLs uses the assumed in-control variance $\sigma_0 = 1$. In addition to the simulation specifications used in the Nonparametric Regression cases, we employed a window to our process: $w = 10$. We also remain with the same number of in-control profiles used in the Estimated In-Control Variance section. For the following simulations, the level of structure added to both the in-control and the process coefficients are identical. The densities are Gaussian. Tables A.31, A.32, A.39, A.40, A.47, and A.48 of the appendix, along with figure 7.1, compare the different techniques.

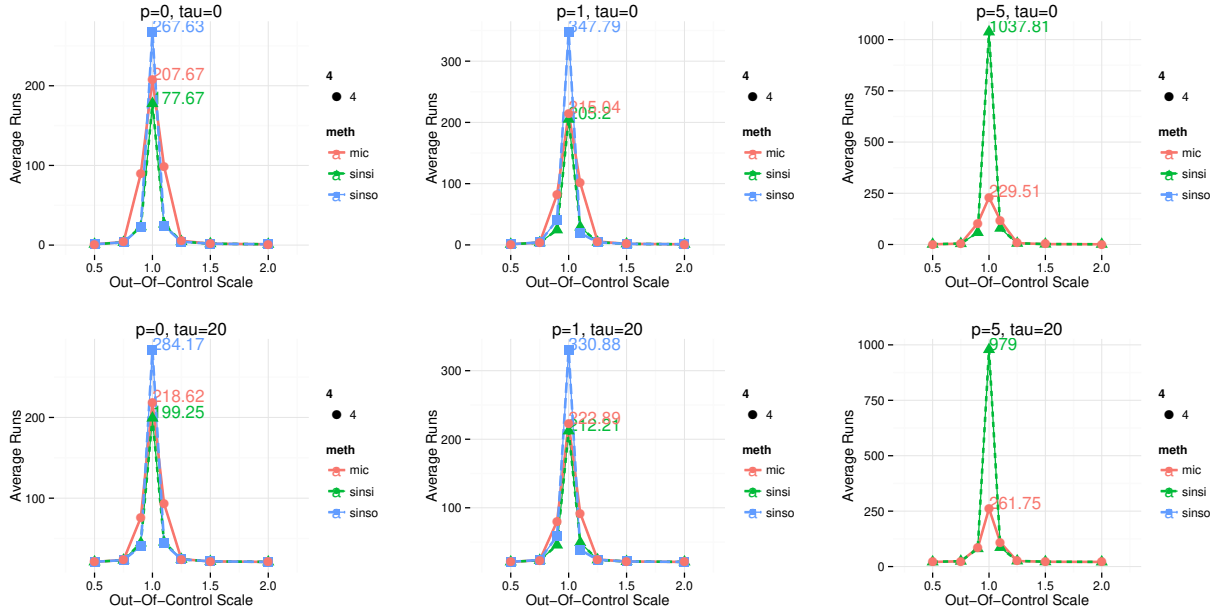


Figure 7.1: Likelihood Ratio Technique Comparison for $n = 2^7$. Size of Structural Component $3\sigma\sqrt{2\log(n)}$, 1000 replications, $\tau = 0, 20$.

7.5 Discussion

We proposed different techniques of employing the Likelihood Ratio. Each was ran with the appropriate scale option. The Single and Multiple cases are equivalent when using Scale-In. Please see appendix for tables when greater sample sizes.

7.5.1 Single In-Control, Scale-Out

This Likelihood Ratio technique shows significant progress from the original MAD method. There is now an overall improvement in the MADs ability to deal with some contamination as seen by the in-control ARLs without false alarm for $p = 0$ (ARL-1= 177.67) vs $p = 1$ (ARL-1= 205.2), when $\tau = 0$. However, just like its predecessor, this likelihood version of the MAD fails to return adequate ARLs around 200 when the percentage of noise introduced is equal to 5 (ARL-1= 1037.81). When the process goes out of control out the 20^{th} profile, the average run lengths without false alarms only slightly improve: ARL-1= 199.25, 212.21, and 979, for $p = 0, 1,$ and 2, respectively.

7.5.2 Multiple In-Control, Scale-Out

The most evident element of the Multiple In-Control Scale-Out technique for the Lilkelihood Ratio is that, as opposed its single IC counterpart, it yields realistic ARLs with 5% contamination (ARL-1= 229.51) when the process is in-control at $\sigma_{oc} = 1$. This is true for $\tau = 20$ as well (ARL-1= 261.75 at $p = 5$ and $\sigma_{oc} = 1$). When there is less contamination introduced, both the single and the multiple designs for the PLR are comparable.

7.5.3 Scale-In

The Scale-In technique is slightly more conservative than the Scale-Out technique. Similarly, it is unable to return appropriate UCLs for an ARL calculation when $p = 5$, the highest contamination percentage considered, regardless of the value of τ. The $ARL - 1$s for the smaller levels of contamination are all conservative with values above 240. In tables 17 and 18, when $\tau = 0$, the $ARL - 1$ for $\sigma_{oc} = 1$ are 267.63 ($p = 0$) and 347.79 ($p = 1$); and when $\tau = 20$, the in-control ARLs are 284.17 ($p = 0$) and 330.88 ($p = 1$), respectively. In tables 21 and 22, When $\tau = 0$, the in control $ARL - 1$ are 243.50 ($p = 0$) and 305.61 ($p = 1$); and when $\tau = 20$, the in-control ARLs are 255.75 ($p = 0$) and 322.61 ($p = 1$), respectively.

CHAPTER 8

NON-DISTRIBUTIONAL NOISE MONITORING

We propose different implementations in order to improve our nonparametric change point problem. Aspects of each of the following procedures are implemented in the next, in order to obtain the best possible method. In each section, we will explain the theory, describe the method to be used, and give the simulation specifications.

Nonparametric Scale Method

This is the second improvement on the MAD. The initial density is estimated with the process density being a scale of the in-control function.

Nonparametric Methods

Here, new methods (Log-Likelihood and KS test) are introduced. The functions are estimated and are entirely nonparametric.

8.1 Nonparametric Scale Method

8.1.1 Theory

We want to take steps away from the parametric assumption and extend the MAD method to nonparametric densities. The first step is to investigate density scale families.

We can estimate the density $\hat{h}$ using the m in-control profiles. We take a sample of size $n/2$ from the elements of a sequence of values of 1 to $mv/2$ without replacement. Each element of the unlisted matrix of in-control coefficients is matched to an index of the previous sample. We then take the MAD of a list of resampled unbounded coefficient of in-control profiles. Let e be the number of times this resampling is repeated. We use unbiased cross-validation to estimate the initial bandwidth coefficient u_0 (Turlach, & others, 1993 [87]; Chiu, 1991 [19]; Hall, Sheather, Jones, & Marron, 1991 [37]; Jones, Marron, & Sheather, 1996 [45]; Duong, & Hazelton, 2005 [26];

Scott, & Terrell, 1987 [79]; Arlot, Celisse, & others, 2010 [4]). We then create an first density $\hat{h}^*$, with u_0, based on the e above medians. This is to determine the appropriate smoothing bandwidth to use.

We now estimate the nonparametric scale family density $\hat{h}$ based on the acquired bandwidth, times a smoothing bandwidth coefficient q. We use the constant interpolation of the above density in the Scale-Out likelihood estimation. Our density is nonparametric scaled due to the fact that the densities used under the null and alternative hypotheses are from the same family of functions. Therefore, the same estimated $\hat{h}$ is used for both hypotheses.

$$\hat{h}^*(u = u^*) \to \hat{h}(u = qu^*) \tag{8.1}$$

8.1.2 Method

The method for the Nonparametric Scale Likelihood Ratio is similar to that of the Scale-In Likelihood Ratio technique, with estimated densities as the fundamental difference. We estimated these densities for the likelihood where the alternative is a scale of the null. Therefore:

$$\begin{aligned} H_0 &: \epsilon \sim h(e) \\ H_1 &: \sigma\epsilon \sim \frac{e}{\sigma} \end{aligned} \tag{8.2}$$

The wavelet coefficients of the functional process are from a scale density of the in-control profiles. The likelihoods under the null and alternative hypotheses respectively are:

$$L_0 = \prod_{t=1}^{T} \hat{h}(s_t) \tag{8.3}$$

and

$$L_1 = \prod_{t=1}^{\tau} \hat{h}(s_t) \prod_{t=\tau+1}^{T} \hat{h}_{\hat{\sigma}_0/\hat{\sigma}_\tau}(s_t). \tag{8.4}$$

Therefore, the likelihood ratio:

$$\begin{aligned}\frac{L_1}{L_0} &= \frac{\prod_{t=1}^{\tau}\hat{h}(s_t)\prod_{t=\tau+1}^{T}\hat{h}_{\hat{\sigma}_0/\hat{\sigma}_\tau}(s_t)}{\prod_{t=1}^{T}\hat{h}(s_t)} \\ &= \prod_{t=\tau+1}^{T}\frac{\hat{h}_{\hat{\sigma}_0/\hat{\sigma}_\tau}(s_t)}{\hat{h}(s_t)} \\ &= \prod_{t=\tau+1}^{T}\frac{(\hat{\sigma}_0/\hat{\sigma}_\tau)\hat{h}(s_t\hat{\sigma}_0/\hat{\sigma}_\tau)}{\hat{h}(s_t)}.\end{aligned} \tag{8.5}$$

This process is repeated for values of between $\tau + 1$ and T, for which we take a product of.

8.1.3 Simulations

For the Nonparametric Scale Likelihood Ratio simulations, we used many of the same specifications as the parametric case. In addition, the process density for the UCL step is generated from a sample of 100 in-control $d1$ level coefficients with noise. Therefore, structure is no longer needed obtain to the process coefficients in this case. The same number of in-control profiles is used for the ARL step. However, new observations are generated, just as in the Parametric method with an assumed in-control variance $\sigma_0 = 1$. When estimating the densities, sampling is done $e = 1000$ times. We also apply a bandwidth smoothing coefficient of $q = 2$. Both Gaussian and Non-Gaussian $(t(3)/\sqrt{3})$ errors for densities are considered. Refer to tables A.63, A.64, A.71, and A.72 of the appendix.

8.1.4 Discussion

The Nonparametric Scale Likelihood Ratio method works well. The ARL-1s for the in-control variances are very close to 200. When $\tau = 0$, the lowest ARL-1s are at 186 ($p = 0$) and 174 ($p = 0$) for the Gaussian and T(3) cases, respectively. When $\tau = 20$, the lowest ARL-1s are 208 ($p = 1$) and 197 ($p = 0$) for G and T densities, respectively. Additionally, the out-of-control process is immediately identified when $\sigma_{oc} \neq 1$. When $\tau = 0$, ARL-1s are never greater than 25 or 30 for G and T densities, respectively. When $\tau = 20$, the ARL-1s are always less than 45 or 50 for the Gaussian and T(3) cases, respectively. Furthermore, there is consistency across distributions with respect to the PFA, as they are never greater than .1. Please see appendix for tables when $n \geq 2^7$

8.2 Nonparametric Methods

8.2.1 Theory

In order to step away from the Nonparametric Scale assumption, we propose two methods able to operate in a completely nonparametric setting: the Log-Likelihood ratio method and the Kolmogorov-Smirnov method.

Log-Likelihood. Let us consider an unknown distribution $F(\cdot)$ for which $X_i \sim F$. Let $F(x_i) = P(X_i \leq x_i)$ and $-\infty < x_i < \infty$. If we consider $F(x_i-) = P(X_i < x_i)$ then

$$P(X_i = x_i) = F(x_i) - F(x_i-) = p_i, \tag{8.6}$$

where $p_i \geq 0$ and $\sum_{i=1}^{n} p_i = 1$. Owen (1998, 2001) [66], [67] states the empirical cumulative distribution function:

$$F_n(x) = \frac{1}{n} \sum_{i=1}^{n} I\{X_i \leq x_i\}. \tag{8.7}$$

Since $P(X_1 = x_1, X_2 = x_2, \cdots, X_n = x_n) = p_1 p_2 \cdots p_n$, the nonparametric of the cumulative distribution function F is

$$\begin{aligned} L(p_1, p_2, \cdots, p_n) &= L(p_1, p_2, \cdots, p_n | X) \\ &= \prod_{i=1}^{n} (F(X_i) - F(X_i-)) \\ &= \prod_{i=1}^{n} p_i \end{aligned} \tag{8.8}$$

Owen (2001) [67] also presents the following theorem with accompanying proof. Let $X_1, \cdots, X_n \subseteq \mathbb{R}$ be independent random variables with a common CDF F_0. Let F_n be their ECDF and let F be any CDF. If $F \neq F_n$, then $L(F) < L(F_n)$. Let us consider following: (1) distinct increasing $z_j s$ in $\{X_1, \cdots, X_n\}$ where $j = 1, \cdots, m$; (2) $n_j \geq 1$ is the number of $X_i = z_j$; (3) all $p_j =$

$F(z_j) - F(z_j-) \geq 0$; (4) for at least one j, $\hat{p}_j = \hat{F}(z_j) - \hat{F}(z_j-) = \frac{n_j}{n} \neq p$. This allows,

$$\begin{aligned} \log\left(\frac{L(F)}{L(F_n)}\right) &= \sum_{j=1}^{m} n_j \log\left(\frac{p_j}{\hat{p}_j}\right) \\ &= n\sum_{j=1}^{m} \hat{p}_j \log\left(\frac{p_j}{\hat{p}_j}\right) \\ &< n\sum_{j=1}^{m} \hat{p}_j \left(\frac{p_j}{\hat{p}_j} - 1\right) \\ &\leq 0. \end{aligned} \tag{8.9}$$

Thus the nonparametric likelihood F is maximized by the ECDF, $\hat{F} = \frac{1}{n}\sum_{i=1}^{n} \delta_{x_i}$, where δ_x is a point mass function. See Kiefer & Wolfowitz (1956) [47].

The nonparametric likelihood is described in more detail in Owen () [66] and applications of the method have been presented by Marsh & Heslington (2000) [55], Owen (1990, 1991) [64] [65], Kolaczyk (1994) [48], Qin & Lawless (1994) [74], Fan, Zhang & Zhang (2001) [30], and Fan & Jiang (2007) [29].

Kolmogorov-Smirnov Method. We also investigate an additional test. The Kolmogorov-Smirnov test, or KS test, is a Nonparametric hypothesis test that is used to determine if two samples of observations belong to the same distribution.

Let us recall the ECDF $F_n(x)$ from equation 8.7. Let F_{n1} and F_{n2} be the respective distributions of the two samples that we wish to test. We assume that the observations of each sample are continuous and independently identically distributed. The null hypothesis is that the samples are from the same distribution; the alternative is that they are not:

$$\begin{aligned} H_0 &: F_{n1}(x) = F_{n2}(x), \\ H_1 &: F_{n1}(x) \neq F_{n2}(x). \end{aligned} \tag{8.10}$$

Our test statistic D is defined as the supremum of the absolute value of the differences for F_{1n} and F_{2n} at any x. This maximum absolute difference is the greatest between the ordered observations of each sample at x. The equation is:

$$D = \sup_{-\infty<x<\infty} |F_{n1}(x) - F_{n2}(x)|. \tag{8.11}$$

More on the Kolmogorov-Smirnov test can be found in [73], [86], and in [15].

8.2.2 Log-Likelihood Method

The null and alternative hypotheses for the Log-Likelihood method are as follows:

$$\begin{aligned} H_0 &: \epsilon \sim g_0(e) \\ H_1 &: \epsilon^* \sim g_1(e^*) \end{aligned} \tag{8.12}$$

In order to obtain the UCL, we generate a function with pre-selected error (Normal, t, Exponential, etc.). This will return the in-control $d1$ coefficient estimates. The in-control density is estimated using the vector of unlisted $m \times v/2$ coefficients with added structure. We repeat this step to smooth out the in-control density by using the appropriate calculated bandwidth. We then extrapolate density values into a function $\hat{g}_0$. The process function is created by sampling $mv/2$ in-control coefficients using bootstrap. This returns the $d1$ level coefficients for the process, to which structure is added. Similar to the in-control density, the alternative density is estimated using unlisted $(T - (\tau + 1)) \times v/2$ process matrix. This step is also repeated in order to smooth out the density. The density points are extrapolated into a function $\hat{g}_1$ to be used in the likelihood steps. The likelihood ratio is the ratio of both estimated density functions evaluated at the current profile t, for all profiles from $\tau + 1$ to T.

The likelihoods under the null and alternative hypotheses respectively are:

$$L_0 = \hat{g}_0(\tilde{\theta}^{o,t}_{J-1}) \tag{8.13}$$

and

$$L_1 = \hat{g}_1(\tilde{\theta}^{o,t}_{J-1}). \tag{8.14}$$

Therefore, the log-likelihood ratio:

$$\begin{aligned} \frac{L_1}{L_0} &= \frac{\prod\limits_{t>\tau}\left[\prod\limits_{o=1}^{n/2} \hat{g}_1(\tilde{\theta}^{o,t}_{J-1})\right]}{\prod\limits_{t>\tau}\left[\prod\limits_{o=1}^{n/2} \hat{g}_0(\tilde{\theta}^{o,t}_{J-1})\right]} \\ \log\left(\frac{L_1}{L_0}\right) &= \log\left(\sum_{o=1}^{n/2} \hat{g}_1(\tilde{\theta}^{o,t}_{J-1})\right) - \log\left(\sum_{o=1}^{n/2} \hat{g}_0(\tilde{\theta}^{o,t}_{J-1})\right). \end{aligned} \tag{8.15}$$

8.2.3 Kolmogorov-Smirnov Method

Since the Kolmogorov-Smirnov method employs a technique for which an estimated density is no longer needed, statistics such as $\hat{\sigma}_\tau$ and $\hat{\sigma}_0$ are not evaluated.

To determine the UCL, we first specify the density to use in order to get the $d1$ level coefficients with noise for m in-control profiles. These densities can come from any known or unknown family of distributions (Gaussian, T, Exponential, etc). Next, the density used in the entire process is a sample of $n/2$ values of the previous in-control matrix, loosely following the initial steps of the Nonparametric Scale Likelihood Ratio.

Let $\tilde{A}$ be a vector of elements from the in-control matrix and the in-control part of the functional process. From equation 6.31:

$$\tilde{A} = \{\{\tilde{\theta}^{o,v}_{J-1,k,0}, \tilde{\theta}^{o,t}_{J-1,k}\}; o = 1, \cdots, n/2; v = 1, \cdots, m; t = 1, \cdots, \tau\}. \tag{8.16}$$

Let $\tilde{B}$ be a vector of elements from the out-of-control portion of the process. From equation 6.9:

$$\tilde{B} = \{\{\tilde{\theta}^{o,t}_{J-1,k}\}; o = 1, \cdots, n/2; t = \tau + 1, \cdots, T\}. \tag{8.17}$$

The test statistic D obtained from the KS test of vectors $\tilde{A}$ and $\tilde{B}$ is used en lieu of our likelihood ratio:

$$D = \sup_{-\infty<x<\infty} \left|\Theta_A(\tilde{\theta}^{o,v}_{J-1,k,0}, \tilde{\theta}^{o,t}_{J-1,k}) - \Theta_B(\tilde{\theta}^{o,t}_{J-1,k})\right|, \tag{8.18}$$

where Θ_A and Θ_B are CDFs of the samples $\tilde{A}$ and $\tilde{B}$, respectively. We then proceed to obtaining our UCL at before. D is obtained for every profile between values of $\tau + 1$ and the T.

8.2.4 Simulations

In the proposed Nonparamtric Likelihood Ratio methods, the UCLs are generated from a sample of 100 in-control $d1$ level coefficients with structure, just as in the Nonparameric Scale method. For the ARL, new observations are generated, just as in the Alternative Likelihood Ratio techniques and Nonparametric Scale methods, with assumed in-control variance $\sigma_0 = 1$. However the density used for the ARL step need not necessarily match that of the UCL step. Therefore, the following combinations of Gaussian and Non-Gaussian errors will be considered: Gaussian UCLs - Gaussian

ARLs, Gaussian UCLs - T(3) ARLs, T(3) UCLs - Gaussian ARLs, and T(3) UCLs - T(3) ARLs. The Kolmogorov-Smirnov method does not require any additional specifications for its simulations. In addition to the specifications previously used, we apply a bandwith smoothing coefficient of $q = 3$ for the Log-Likelihood method. However the Log-Likelihood is computationally expensive and requires a greater number of in-control profiles. See tables A.79, A.80, A.81, A.82, A.83, A.84, A.87, and A.88 of the appendix.

Table 8.1: Method Comparison for T(3) Error $n = 2^7$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, 1000 Replications, $\tau = 0$.

			Nikoo Walsh		NPSLR		NP-KS	
τ	p	σ_{oc}	ARL-1	PFA	ARL-1	PFA	ARL-1	PFA
0	0	0.50	2.00	0.00	1.14	0.00	3.11	0.00
		0.75	5.38	0.00	3.73	0.00	78.71	0.00
		0.90	57.75	0.00	23.02	0.00	184.53	0.00
		1.10	48.99	0.00	26.27	0.00	117.52	0.00
		1.25	6.65	0.00	4.65	0.00	46.25	0.00
		1.50	2.41	0.00	1.81	0.00	11.27	0.00
		2.00	2.00	0.00	1.06	0.00	2.12	0.00
		1.00	221.96	0.00	174.05	0.00	175.65	0.00
	1	0.50	2.00	0.00	1.15	0.00	3.36	0.00
		0.75	5.41	0.00	3.68	0.00	80.91	0.00
		0.90	63.18	0.00	21.01	0.00	192.79	0.00
		1.10	64.64	0.00	26.51	0.00	121.06	0.00
		1.25	7.70	0.00	5.06	0.00	46.86	0.00
		1.50	2.49	0.00	1.85	0.00	11.86	0.00
		2.00	2.00	0.00	1.06	0.00	2.31	0.00
		1.00	281.58	0.00	199.16	0.00	180.54	0.00
	5	0.50	2.01	0.00	1.18	0.00	3.95	0.00
		0.75	6.22	0.00	3.72	0.00	82.79	0.00
		0.90	78.61	0.00	25.27	0.00	195.63	0.00
		1.10	136.66	0.00	30.18	0.00	117.72	0.00
		1.25	12.82	0.00	4.93	0.00	48.56	0.00
		1.50	2.72	0.00	1.91	0.00	12.04	0.00
		2.00	2.01	0.00	1.06	0.00	2.47	0.00
		1.00	469.02	0.00	228.19	0.00	173.47	0.00

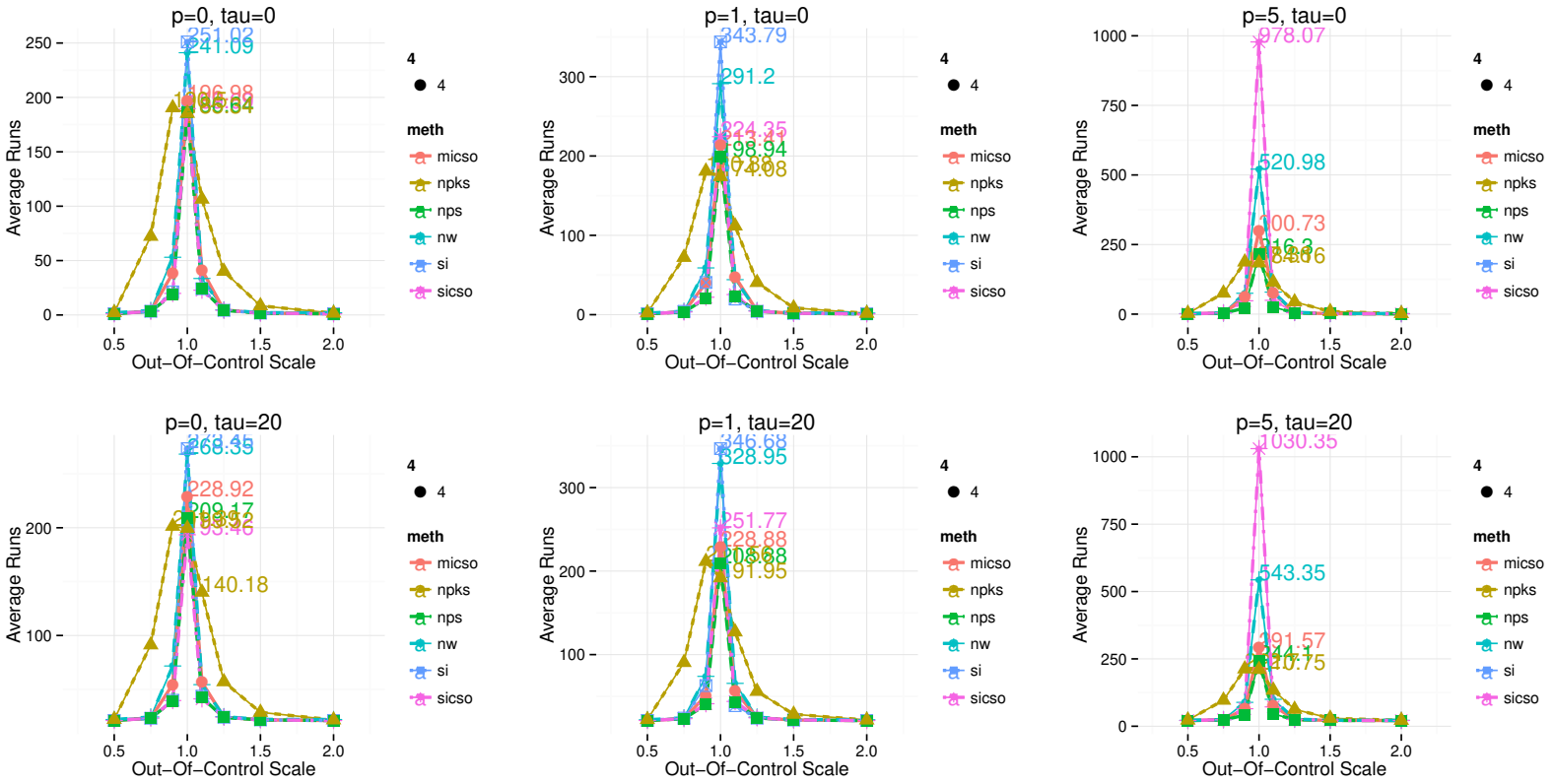


Figure 8.1: Method Comparison for Gaussian Error $n = 2^7$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $(q = 2)$, 1000 Replications, $\tau = 0, 20$.

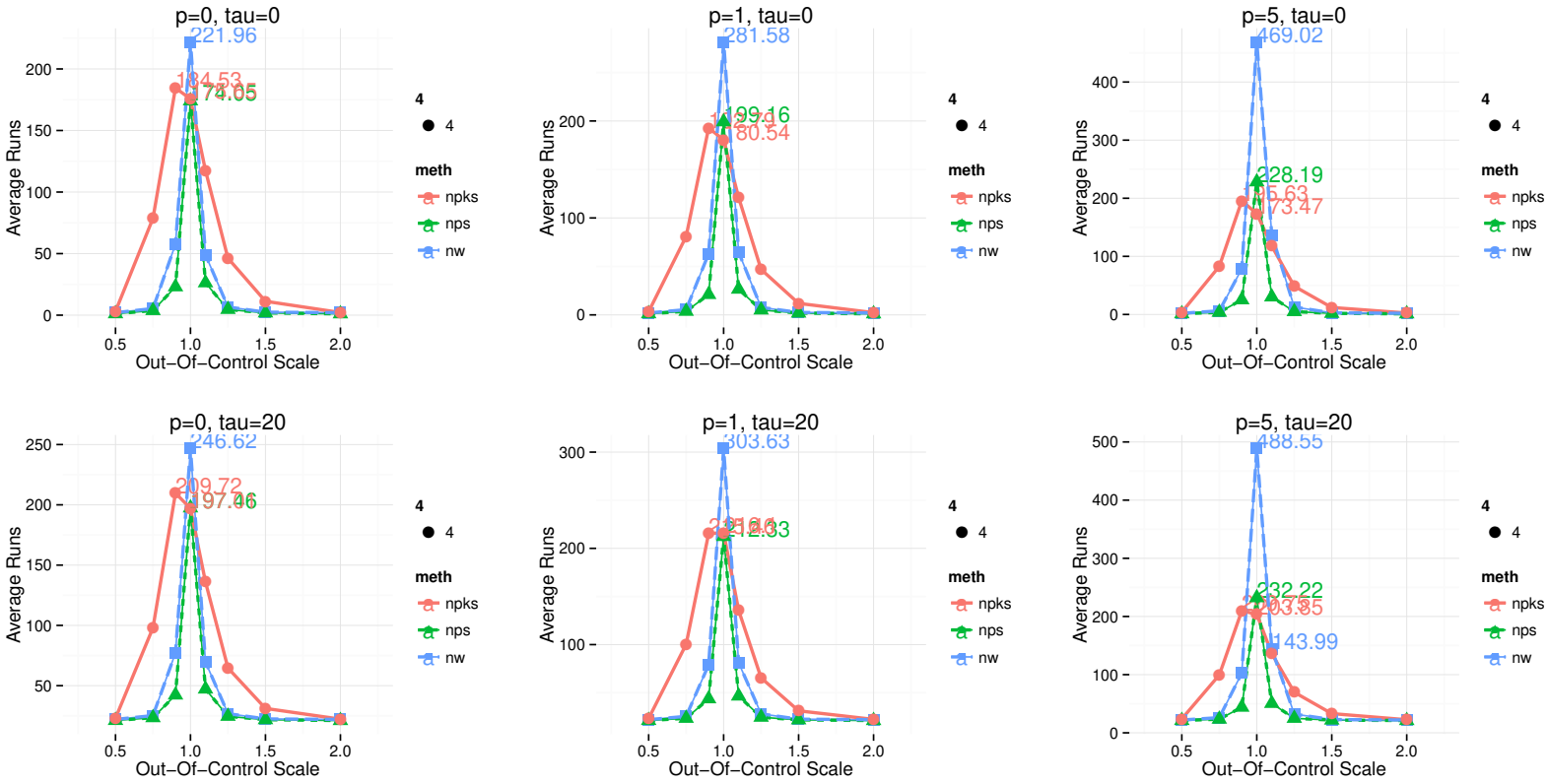


Figure 8.2: Method Comparison for Gaussian Error $n = 2^7$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $(q = 2)$, 1000 Replications, $\tau = 0, 20$.

Table 8.3: Method Comparison for Gaussian Error $n = 2^7$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $(q = 2)$, 1000 Replications, $\tau = 0$.

			Nikoo Walsh		SIC-SO		MIC-SO		Scale-In		NPSLR		NP-KS	
τ	p	σ_{oc}	ARL-1	PFA	ARL-1	PFA	ARL-1	PFA	ARL-1	PFA	ARL-1	PFA	ARL-1	PFA
0	0	0.50	2.00	0.00	1.09	0.00	1.09	0.00	1.08	0.00	1.09	0.00	2.17	0.00
		0.75	4.16	0.00	3.31	0.00	3.44	0.00	3.43	0.00	3.31	0.00	72.31	0.00
		0.90	53.05	0.00	19.82	0.00	38.46	0.00	21.25	0.00	19.08	0.00	190.50	0.00
		1.10	33.54	0.00	22.76	0.00	41.39	0.00	24.14	0.00	24.06	0.00	106.47	0.00
		1.25	4.67	0.00	4.33	0.00	4.63	0.00	4.32	0.00	4.26	0.00	40.12	0.00
		1.50	2.16	0.00	1.70	0.00	1.70	0.00	1.74	0.00	1.65	0.00	8.44	0.00
		2.00	2.00	0.00	1.03	0.00	1.03	0.00	1.04	0.00	1.04	0.00	1.64	0.00
		1.00	241.09	0.00	189.69	0.00	196.98	0.00	251.02	0.00	186.64	0.00	185.34	0.00
	1	0.50	2.00	0.00	1.10	0.00	1.07	0.00	1.19	0.00	1.09	0.00	2.24	0.00
		0.75	4.28	0.00	3.40	0.00	3.53	0.00	4.18	0.00	3.35	0.00	72.20	0.00
		0.90	58.94	0.00	21.99	0.00	39.96	0.00	40.56	0.00	20.50	0.00	180.88	0.00
		1.10	44.13	0.00	25.35	0.00	47.73	0.00	19.69	0.00	22.83	0.00	111.84	0.00
		1.25	5.14	0.00	4.40	0.00	4.56	0.00	4.21	0.00	4.25	0.00	40.57	0.00
		1.50	2.19	0.00	1.71	0.00	1.72	0.00	1.72	0.00	1.69	0.00	8.55	0.00
		2.00	2.00	0.00	1.04	0.00	1.04	0.00	1.04	0.00	1.04	0.00	1.68	0.00
		1.00	291.20	0.00	224.35	0.00	213.41	0.00	343.79	0.00	198.94	0.00	174.08	0.00
	5	0.50	2.00	0.00	1.22	0.00	1.11	0.00	*	*	1.11	0.00	2.72	0.00
		0.75	4.66	0.00	4.34	0.00	3.62	0.00	*	*	3.34	0.00	75.62	0.00
		0.90	75.00	0.00	48.00	0.00	60.84	0.00	*	*	21.65	0.00	187.36	0.00
		1.10	79.13	0.00	49.66	0.00	77.71	0.00	*	*	25.65	0.00	114.44	0.00
		1.25	6.67	0.00	5.58	0.00	4.78	0.00	*	*	4.35	0.00	44.06	0.00
		1.50	2.26	0.00	2.01	0.00	1.83	0.00	*	*	1.70	0.00	9.41	0.00
		2.00	2.00	0.00	1.09	0.00	1.04	0.00	*	*	1.04	0.00	1.88	0.00
		1.00	520.98	0.00	978.07	0.00	300.73	0.00	*	*	216.30	0.00	184.16	0.00

Table 8.4: Method Comparison for Gaussian Error $n = 2^7$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $(q = 2)$, 1000 Replications, $\tau = 20$.

			Nikoo Walsh		SIC-SO		MIC-SO		Scale-In		NPSLR		NP-KS	
τ	p	σ_{oc}	ARL-1	PFA	ARL-1	PFA	ARL-1	PFA	ARL-1	PFA	ARL-1	PFA	ARL-1	PFA
20	0	0.50	22.00	0.07	21.07	0.10	21.07	0.09	21.10	0.07	21.10	0.10	22.11	0.11
		0.75	24.35	0.10	23.18	0.10	23.26	0.08	23.18	0.07	23.25	0.10	91.28	0.12
		0.90	71.68	0.10	39.60	0.11	54.57	0.10	39.87	0.09	39.26	0.09	201.39	0.09
		1.10	54.36	0.08	41.16	0.10	56.54	0.08	43.60	0.06	42.22	0.10	140.18	0.12
		1.25	24.65	0.08	24.18	0.10	24.22	0.08	24.12	0.09	23.91	0.10	57.04	0.10
		1.50	22.16	0.09	21.69	0.08	21.61	0.09	21.67	0.08	21.64	0.10	28.65	0.12
		2.00	22.00	0.09	21.03	0.11	21.05	0.08	21.04	0.07	21.02	0.12	21.64	0.13
		1.00	268.35	0.09	193.46	0.10	228.92	0.07	273.45	0.08	209.17	0.11	199.52	0.12
	1	0.50	22.00	0.06	21.10	0.09	21.07	0.08	21.17	0.06	21.08	0.12	22.22	0.11
		0.75	24.30	0.07	23.28	0.07	23.20	0.09	23.97	0.04	23.14	0.11	90.57	0.11
		0.90	73.95	0.07	41.20	0.09	51.06	0.08	62.94	0.04	40.13	0.10	211.56	0.11
		1.10	65.56	0.06	43.93	0.09	56.29	0.09	38.84	0.06	43.17	0.10	127.18	0.11
		1.25	25.18	0.08	24.30	0.08	24.47	0.10	23.93	0.06	24.15	0.09	55.99	0.10
		1.50	22.21	0.06	21.70	0.08	21.64	0.08	21.69	0.06	21.62	0.10	28.59	0.11
		2.00	22.00	0.08	21.04	0.08	21.03	0.08	21.03	0.05	21.02	0.10	21.68	0.11
		1.00	328.95	0.07	251.77	0.09	228.88	0.08	346.68	0.05	208.88	0.10	191.95	0.12
	5	0.50	22.00	0.04	21.24	0.02	21.10	0.06	*	*	21.11	0.10	22.81	0.10
		0.75	24.82	0.04	24.22	0.02	23.45	0.06	*	*	23.26	0.09	96.58	0.08
		0.90	88.58	0.05	64.93	0.02	64.20	0.05	*	*	41.04	0.09	212.70	0.12
		1.10	99.93	0.04	71.88	0.02	77.44	0.06	*	*	45.73	0.09	133.85	0.09
		1.25	26.62	0.05	25.39	0.01	24.60	0.05	*	*	24.28	0.09	62.69	0.11
		1.50	22.28	0.04	21.92	0.02	21.74	0.07	*	*	21.64	0.10	29.48	0.10
		2.00	22.00	0.05	21.07	0.02	21.05	0.06	*	*	21.04	0.08	21.89	0.10
		1.00	543.35	0.05	1030.35	0.02	291.57	0.06	*	*	244.10	0.10	210.75	0.11

Table 8.2: Method Comparison for T(3) Error $n = 2^7$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, 1000 Replications, $\tau = 20$.

			Nikoo Walsh		NPSLR		NP-KS	
τ	p	σ_{oc}	ARL-1	PFA	ARL-1	PFA	ARL-1	PFA
20	0	0.50	22.00	0.06	21.14	0.10	23.09	0.09
		0.75	25.19	0.08	23.44	0.11	98.05	0.11
		0.90	77.26	0.09	42.26	0.12	209.72	0.11
		1.10	69.64	0.10	47.20	0.11	136.09	0.11
		1.25	26.83	0.10	24.59	0.12	64.66	0.09
		1.50	22.41	0.09	21.77	0.11	31.14	0.11
		2.00	22.00	0.09	21.06	0.09	22.20	0.10
		1.00	246.62	0.08	197.46	0.11	197.01	0.10
	1	0.50	22.00	0.08	21.15	0.11	23.34	0.09
		0.75	25.45	0.07	23.40	0.09	100.29	0.12
		0.90	79.06	0.08	43.70	0.12	215.46	0.11
		1.10	81.11	0.06	46.54	0.10	135.65	0.13
		1.25	28.05	0.06	24.52	0.10	65.40	0.11
		1.50	22.48	0.08	21.79	0.09	31.20	0.13
		2.00	22.01	0.06	21.07	0.11	22.28	0.10
		1.00	303.63	0.08	212.33	0.10	216.10	0.11
	5	0.50	22.00	0.05	21.15	0.09	23.99	0.11
		0.75	26.12	0.06	23.46	0.09	99.43	0.11
		0.90	102.62	0.04	44.01	0.10	209.75	0.12
		1.10	143.99	0.04	50.54	0.09	136.12	0.10
		1.25	32.25	0.04	24.92	0.09	69.75	0.10
		1.50	22.79	0.05	21.89	0.10	33.04	0.12
		2.00	22.01	0.04	21.07	0.10	22.50	0.10
		1.00	488.55	0.06	232.22	0.09	203.85	0.10

8.2.5 Discussion

Kolmogorov-Smirnov Method. The KS method's results are very consistent with the same distribution family, which is expected. For the Gaussian UCLs - Gaussian ARLs case, the in-control ARL-1s are no more than 266.56 and no less than 181.02. For the T(3) UCLs - T(3) ARLs case, the in-control ARLs are slightly below average when $p = 5$. In the Gaussian UCLs - T(3) ARLs and the T(3) UCLs - Gaussian ARLs cases, the ARL-1s are consistently close to 200.

Log-Likelihood Method. For the Gaussian UCLs - Gaussian ARLs case, The Log-Likelihood method is more accurate when then number of IC profiles is increased from $m = 100$ to $m = 250$. This is consistent across all values of p and τ.

8.3 Summary

See Summary tables 8.3, 8.4, 8.1, 8.2, and figures 8.1, 8.2 for method comparisons.

8.3.1 Alternative Likelihood Ratio Techniques

Of the different Alternative Likelihood Ratio Techniques, the Scale-Out cases are favorable. The addition of in-control profiles to the alternative likelihood L_1, for the Multiple In-Control (MIC) variance Scale-Out technique, has a clear advantage over its absence in the Single In-Control (SIC) variance Scale-In technique. The proportion of false alarms are less than .14 for the SIC technique and almost always less than .1 in the MIC variance, another indication of the slight advantage of the MIC over the SIC. Furthermore, the MIC variance is able to generate values slightly above an average run length of 200; a disadvantage of the SIC variance. Beyond this, the SIC and the MIC variance Scale-Out techniques are nearly identical, with both yielding favorable ARL-1s, regardless of τs value. The SIC is a slightly more conservative technique than its multiple in control variance counterpart.

Overall, the Scale-In technique is outperformed by the Scale-Out techniques, with the Scale-In case being unable to account for increased percentages in noise. The Scale-In MADs are not as robust to outliers/noise compared to the Scale-Out MADs. The latter have the added benefit of being able to handle the highest percentage of added contamination (5%), in the MIC case. Also The MIC Scale-Out techniques produced $ARL - 1s$ closer to 200 than that of the Single In-Control. There is a slight increase in proportion of false alarms form the previous Likelihood Ratio technique.

8.3.2 Nonparametric Scale Method

In the Nonparamtric Scale method, the Scale-In case is employed. Although conservative, the Nonparametric Scale method is an amelioration to the MIC Scale-Out Likelihood Ratio technique,

as it allows for a change in scale for the same family distribution. ARLs at $\sigma_{oc} = 1$ are favorable for both Gaussian and Non-Gaussian errors. However overall, the percentage of false alarms is greater in the Nonparametric Scale setting than in the MIC Scale-Out setting.

8.3.3 Nonparametric Method

Kolmogorov-Smirnov. The Nonparametric Kolmogorov-Smirnov method is not as robust as the Nonparametric Scale method. Nevertheless, it performs adequately, specifically when different densities are used for the UCL determination and to generate the ARLs. The PFAs are low but still greater than those of Nonparametric Scale method; and the ARLs are in the general area of 200 for the most part. One drawback of the KS method is its large ARL when $\sigma_{oc} \neq 1$. This Nonparametric method is not as fast as the Nonparametric Scale method, requiring as many as 223.48 runs on average before identifying the out-of-control process. We would rather this to be immediate.

Log-Likelihood. The Log-Likelihood method requires a larger number of IC profiles than the other methods; 100 IC profiles are insufficient for 1000 replications. This is a direct result of fewer assumptions needed by the method as compared to all aforementioned methods.

CHAPTER 9

CONCLUSION

The purpose of this dissertation is the pursuit of a method for profile variability, and of an understanding of the requirements necessary to go from a parametric to a nonparametric setting.

We started with the proposed MAD method and looked to improve on it. The MAD method is either comparable or outperforms the Nonparametric Regression methods described by Nikoo & Noorossana [62]. However, some issues remained with this proposed method: it is computationally expensive, and not particularly resistant to increased levels of added structure. Although the assumptions made on the function are limited, the method can be ameliorated.

We are able to estimate the in-control variance to be used in the UCL step's process, and we assess the level of structure needed for the in-control and process profiles at each step (UCL and ARL). Additionally, we incorporate a fixed window which decreases computation time significantly.

In the Parametric method, we investigated the use of two in-control variances: one for just the in-control profile, and one for both the in-control profile and the in-control portion of the process. This is the MIC case. It is weighted against using just one variance for the in-control profile, the SIC case. We also examined the differences in using the in-control variance as an inner (Scale-In) vs an outer (Scale-Out) scale coefficient, and used the appropriate fit. The proportion of false alarms reported comparable to the initial MAD method.

In the Nonparametric Scale method, we further took an additional step away from the Gaussian assumption. We were able to generate a function from the estimated in-control coefficients using any distribution error, rather than assume normality. The null and alternative likelihood functions are from the same scale family distribution. For this method, we sampled the process density from the in control coefficients UCLs step while all while using new observations in the ARL step.

In the Nonparametric method, we employed the Log-Likelihood and the Kolmogorov-Smirnov techniques to take steps away from the scale assumption. In the Log-Likelihood method, we estimate

the null likelihood function, in a procedure similar to that of the previous nonparametric scale method. The alternative likelihood function is based on the estimated coefficients of the process. We then evaluated each function at the current profile t, and take the log of the likelihood ratio. In the KS method, we performed a distribution test on estimated coefficients for the in-control profile and process. The statistic of KS test replaces the likelihood ratio of the previous methods.

The Nonparametric Scale method outperformed the Nikoo Walsh method, the Multiple In Control Scale-Out technique and The Nonparametric Kolmogorov-Smirnov method for the Gaussian UCLs to Gaussian ARLs case. The Nonparametric Scale method is also ideal for the Non-Gaussian UCLs to Non-Gaussian ARLs case when compared to the Nikoo Walsh and Nonparametric KS methods. Although low in power, the KS method can be considered for the Gaussian UCLs - Non-Gaussian ARLs, Non-Gaussian UCLs - Gaussian ARLs cases. The Log-Likelihood method is a substitute when a large number of IC profiles is available.

Future

Rather than use two monitoring schemes, we can investigate the possibility of one method accounting for both f & σ. The Pseudo Standard Error (PSE) method proposed by Lenth in 1989 [49] defines the estimate as:

$$s_{P,t} = 1.5 \cdot \text{median}|\tilde{\theta}^*_{J-1,k}|, \tag{9.1}$$

where the $\tilde{\theta}^*_{J-1,k}$ are those $\tilde{\theta}_{J-1,k}$ which have absolute values less than $2.5 \cdot s_0$ with

$$s_0 = 1.5 \cdot \text{median}|\tilde{\theta}_{J-1,k}|. \tag{9.2}$$

This estimate assumes there are few non-zero means among the normal random variables $\tilde{\theta}_{J-1,k}$, and that these means are large. The PSE method is much more robust to outliers than the MAD method.

Thr window w and the bandwidth coefficient q were given fixed values. One future possibility is to consider them as variables and estimate optimal values for both.

We can also look into Bayesian methods by placing a prior on the IC portion of the process.

Another Bayesian approach is to investigate the Change Point problem where the τ^{th} profile has a prior distribution. See Chib (1998) [16], Hartigan & Barry (1993) [5], and Raftery & Akman (1986) [75] for more information.

Additionally, we can examine Nonparametric Bayesian methods on the function, or on the appropriate level of structure to use with the help of posterior information. Additional readings: Brodsky & Darkhovsky (2013) [8], Pettitt (1979) [71], Müller & Quintana (2004) [60], and Matteson & James (2014) [56].

Lastly, conceivably using a method outside of Wavelets. Perhaps B-splines can be a suitable/more formidable replacement. Eilers & Marx (1996) [28], Silverman (1985) [81], and Green & Silverman (1993) [32] provide more on this topic.

APPENDIX A

ADDITIONAL TABLES

A.1 Existing Methods

Table A.1: Nonparametric Regression, $n = 2^7$. Structural Component $3\sigma\sqrt{2\log(n)}$, $m = 40$, 100 Replications, $\tau = 0$.

			Nikoo Walsh			Nikoo Variance		
τ	p	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00	1.00	1.00	0.00
		0.75	3.06	3.06	0.00	1.30	1.30	0.00
		0.90	36.72	36.72	0.00	33.10	33.10	0.00
		1.10	31.02	31.02	0.00	27.50	27.50	0.00
		1.25	3.60	3.60	0.00	1.80	1.80	0.00
		1.50	1.18	1.18	0.00	1.00	1.00	0.00
		2.00	1.00	1.00	0.00	1.00	1.00	0.00
	1	0.50	1.00	1.00	0.00	1.60	1.60	0.00
		0.75	3.93	3.93	0.00	1.10	1.10	0.00
		0.90	84.31	84.31	0.00	1.00	1.00	0.00
		1.10	22.56	22.56	0.00	1.00	1.00	0.00
		1.25	2.93	2.93	0.00	1.00	1.00	0.00
		1.50	1.11	1.11	0.00	1.00	1.00	0.00
		2.00	1.00	1.00	0.00	1.00	1.00	0.00
	5	0.50	1.00	1.00	0.00	1.00	1.00	0.00
		0.75	7.77	7.77	0.00	1.00	1.00	0.00
		0.90	237.36	237.36	0.00	1.00	1.00	0.00
		1.10	7.34	7.34	0.00	1.00	1.00	0.00
		1.25	1.78	1.78	0.00	1.00	1.00	0.00
		1.50	1.05	1.05	0.00	1.00	1.00	0.00
		2.00	1.00	1.00	0.00	1.00	1.00	0.00

Table A.2: Nonparametric Regression, $n = 2^7$. Structural Component $3\sigma\sqrt{2\log(n)}$, $m = 40$, 100 Replications, $\tau = 20$.

			Nikoo Walsh			Nikoo Variance		
τ	p	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA
20	0	0.50	20.20	21.00	0.08	17.10	21.00	0.30
		0.75	21.63	22.83	0.10	21.40	21.40	0.00
		0.90	63.59	70.52	0.11	50.80	50.80	0.00
		1.10	51.03	54.13	0.07	30.30	30.30	0.00
		1.25	23.00	23.32	0.04	20.00	22.00	0.20
		1.50	20.21	21.13	0.09	19.40	21.11	0.10
		2.00	19.71	21.00	0.09	21.00	21.00	0.00
	1	0.50	19.54	21.00	0.13	1.00	$*$	1.00
		0.75	22.69	23.91	0.10	1.00	$*$	1.00
		0.90	84.93	95.92	0.13	1.00	$*$	1.00
		1.10	31.81	37.41	0.20	1.00	$*$	1.00
		1.25	21.05	22.89	0.12	1.00	$*$	1.00
		1.50	19.65	21.09	0.14	1.00	$*$	1.00
		2.00	19.31	21.00	0.13	1.00	$*$	1.00
	5	0.50	16.84	21.02	0.36	1.00	$*$	1.00
		0.75	20.93	26.70	0.37	1.00	$*$	1.00
		0.90	144.55	213.85	0.34	1.00	$*$	1.00
		1.10	20.81	27.65	0.40	1.00	$*$	1.00
		1.25	18.16	21.79	0.34	1.00	$*$	1.00
		1.50	17.81	21.01	0.30	1.00	$*$	1.00
		2.00	17.38	21.00	0.33	1.00	$*$	1.00

Table A.3: Preliminary Method Comparison, $n = 2^7$. Structural Component $3\sigma\sqrt{2\log(n)}$, $m = 40$, 100 Replications, $\tau = 0$.

			Nikoo Walsh			Nikoo Variance			MAD		
τ	p	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00	1.00	1.00	0.00	1.17	1.17	0.00
		0.75	3.06	3.06	0.00	1.30	1.30	0.00	3.42	3.42	0.00
		0.90	36.72	36.72	0.00	33.10	33.10	0.00	17.87	17.87	0.00
		1.10	31.02	31.02	0.00	27.50	27.50	0.00	18.18	18.18	0.00
		1.25	3.60	3.60	0.00	1.80	1.80	0.00	4.15	4.15	0.00
		1.50	1.18	1.18	0.00	1.00	1.00	0.00	1.74	1.74	0.00
		2.00	1.00	1.00	0.00	1.00	1.00	0.00	1.06	1.06	0.00
	1	0.50	1.00	1.00	0.00	1.60	1.60	0.00	1.14	1.14	0.00
		0.75	3.93	3.93	0.00	1.10	1.10	0.00	3.88	3.88	0.00
		0.90	84.31	84.31	0.00	1.00	1.00	0.00	23.72	23.72	0.00
		1.10	22.56	22.56	0.00	1.00	1.00	0.00	15.56	15.56	0.00
		1.25	2.93	2.93	0.00	1.00	1.00	0.00	3.88	3.88	0.00
		1.50	1.11	1.11	0.00	1.00	1.00	0.00	1.57	1.57	0.00
		2.00	1.00	1.00	0.00	1.00	1.00	0.00	1.02	1.02	0.00
	5	0.50	1.00	1.00	0.00	1.00	1.00	0.00	1.12	1.12	0.00
		0.75	7.77	7.77	0.00	1.00	1.00	0.00	4.61	4.61	0.00
		0.90	237.36	237.36	0.00	1.00	1.00	0.00	46.99	46.99	0.00
		1.10	7.34	7.34	0.00	1.00	1.00	0.00	7.94	7.94	0.00
		1.25	1.78	1.78	0.00	1.00	1.00	0.00	3.20	3.20	0.00
		1.50	1.05	1.05	0.00	1.00	1.00	0.00	1.37	1.37	0.00
		2.00	1.00	1.00	0.00	1.00	1.00	0.00	1.02	1.02	0.00

Table A.4: Preliminary Method Comparison, $n = 2^7$. Structural Component $3\sigma\sqrt{2\log(n)}$, $m = 40$, 100 Replications, $\tau = 20$.

			Nikoo Walsh			Nikoo Variance			MAD		
τ	p	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
20	0	0.50	20.20	21.00	0.08	17.10	21.00	0.30	20.51	21.11	0.06
		0.75	21.63	22.83	0.10	21.40	21.40	0.00	22.51	23.08	0.04
		0.90	63.59	70.52	0.11	50.80	50.80	0.00	33.77	35.44	0.07
		1.10	51.03	54.13	0.07	30.30	30.30	0.00	37.32	38.77	0.05
		1.25	23.00	23.32	0.04	20.00	22.00	0.20	23.82	24.41	0.05
		1.50	20.21	21.13	0.09	19.40	21.11	0.10	20.79	21.61	0.07
		2.00	19.71	21.00	0.09	21.00	21.00	0.00	20.03	21.05	0.09
	1	0.50	19.54	21.00	0.13	1.00	$*$	1.00	20.11	21.15	0.09
		0.75	22.69	23.91	0.10	1.00	$*$	1.00	22.75	23.40	0.05
		0.90	84.93	95.92	0.13	1.00	$*$	1.00	37.35	41.01	0.11
		1.10	31.81	37.41	0.20	1.00	$*$	1.00	29.72	31.14	0.08
		1.25	21.05	22.89	0.12	1.00	$*$	1.00	22.56	23.55	0.09
		1.50	19.65	21.09	0.14	1.00	$*$	1.00	21.22	21.54	0.05
		2.00	19.31	21.00	0.13	1.00	$*$	1.00	20.15	21.00	0.08
	5	0.50	16.84	21.02	0.36	1.00	$*$	1.00	19.36	21.19	0.23
		0.75	20.93	26.70	0.37	1.00	$*$	1.00	21.48	24.97	0.25
		0.90	144.55	213.85	0.34	1.00	$*$	1.00	56.06	68.46	0.22
		1.10	20.81	27.65	0.40	1.00	$*$	1.00	22.88	27.00	0.27
		1.25	18.16	21.79	0.34	1.00	$*$	1.00	19.90	22.79	0.25
		1.50	17.81	21.01	0.30	1.00	$*$	1.00	19.17	21.41	0.30
		2.00	17.38	21.00	0.33	1.00	$*$	1.00	17.71	21.00	0.35

Table A.5: Nonparametric Regression, $n = 2^8$. Structural Component $3\sigma\sqrt{2\log(n)}$, $m = 40$, 100 Replications, $\tau = 0$.

			Nikoo Walsh			Nikoo Variance		
τ	p	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA
0	0	0.50	1.99	1.99	0.00	1.00	1.00	0.00
		0.75	2.37	2.37	0.00	1.04	1.04	0.00
		0.90	23.99	23.99	0.00	13.00	13.00	0.00
		1.10	17.04	17.04	0.00	7.16	7.16	0.00
		1.25	2.52	2.52	0.00	1.20	1.20	0.00
		1.50	2.00	2.00	0.00	1.00	1.00	0.00
		2.00	1.99	1.99	0.00	1.00	1.00	0.00
	1	0.50	2.00	2.00	0.00	21491.44	21491.44	0.00
		0.75	2.37	2.37	0.00	2.81	2.81	0.00
		0.90	41.70	41.70	0.00	1.17	1.17	0.00
		1.10	14.77	14.77	0.00	1.00	1.00	0.00
		1.25	2.40	2.40	0.00	1.00	1.00	0.00
		1.50	2.00	2.00	0.00	1.00	1.00	0.00
		2.00	2.00	2.00	0.00	1.00	1.00	0.00
	5	0.50	1.93	1.93	0.00	1.00	1.00	0.00
		0.75	3.47	3.47	0.00	1.00	1.00	0.00
		0.90	135.77	135.77	0.00	1.00	1.00	0.00
		1.10	4.79	4.79	0.00	1.00	1.00	0.00
		1.25	2.08	2.08	0.00	1.00	1.00	0.00
		1.50	1.97	1.97	0.00	1.00	1.00	0.00
		2.00	1.96	1.96	0.00	1.00	1.00	0.00

Table A.6: Nonparametric Regression for $n = 2^8$. Structural Component $3\sigma\sqrt{2\log(n)}$, $m = 40$, 100 Replications, $\tau = 20$.

			Nikoo Walsh			Nikoo Variance		
τ	p	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA
20	0	0.50	20.13	22.00	0.15	20.29	21.00	0.06
		0.75	20.09	22.36	0.16	19.69	21.07	0.11
		0.90	34.06	40.91	0.22	29.31	31.32	0.10
		1.10	30.90	34.62	0.16	26.66	29.21	0.13
		1.25	21.27	22.82	0.16	20.26	21.29	0.07
		1.50	19.91	21.99	0.17	19.45	21.00	0.14
		2.00	19.90	22.00	0.17	19.06	21.00	0.14
	1	0.50	20.31	21.98	0.12	1.02	*	1.00
		0.75	20.22	22.34	0.18	1.03	*	1.00
		0.90	44.03	52.38	0.20	1.02	*	1.00
		1.10	31.33	35.41	0.18	1.00	*	1.00
		1.25	20.92	22.84	0.15	1.00	*	1.00
		1.50	19.93	21.98	0.15	1.00	*	1.00
		2.00	20.35	21.99	0.15	1.00	*	1.00
	5	0.50	14.09	21.98	0.56	1.00	*	1.00
		0.75	14.98	23.19	0.58	1.00	*	1.00
		0.90	68.37	110.84	0.42	1.00	*	1.00
		1.10	15.68	24.73	0.56	1.00	*	1.00
		1.25	14.61	22.27	0.55	1.00	*	1.00
		1.50	15.39	21.98	0.47	1.00	*	1.00
		2.00	16.16	21.96	0.46	1.00	*	1.00

Table A.7: Nonparametric Regression, $n = 2^9$. Structural Component $3\sigma\sqrt{2\log(n)}$, $m = 40$, 100 Replications, $\tau = 0$.

			Nikoo Walsh			Nikoo Variance		
τ	p	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA
0	0	0.50	2.00	2.00	0.00	1.00	1.00	0.00
		0.75	1.98	1.98	0.00	1.00	1.00	0.00
		0.90	7.91	7.91	0.00	3.33	3.33	0.00
		1.10	6.67	6.67	0.00	4.16	4.16	0.00
		1.25	2.08	2.08	0.00	1.02	1.02	0.00
		1.50	1.99	1.99	0.00	1.00	1.00	0.00
		2.00	1.99	1.99	0.00	1.00	1.00	0.00
	1	0.50	1.99	1.99	0.00	111.93	111.93	0.00
		0.75	2.02	2.02	0.00	1.03	1.03	0.00
		0.90	10.65	10.65	0.00	1.00	1.00	0.00
		1.10	5.74	5.74	0.00	1.00	1.00	0.00
		1.25	2.06	2.06	0.00	1.00	1.00	0.00
		1.50	2.00	2.00	0.00	1.00	1.00	0.00
		2.00	1.99	1.99	0.00	1.00	1.00	0.00
	5	0.50	1.89	1.89	0.00	1.00	1.00	0.00
		0.75	2.05	2.05	0.00	1.00	1.00	0.00
		0.90	69.13	69.13	0.00	1.00	1.00	0.00
		1.10	2.35	2.35	0.00	1.00	1.00	0.00
		1.25	1.93	1.93	0.00	1.00	1.00	0.00
		1.50	1.88	1.88	0.00	1.00	1.00	0.00
		2.00	1.91	1.91	0.00	1.00	1.00	0.00

Table A.8: Nonparametric Regression for $n = 2^9$. Structural Component $3\sigma\sqrt{2\log(n)}$, $m = 40$, 100 Replications, $\tau = 20$.

			Nikoo Walsh			Nikoo Variance		
τ	p	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA
20	0	0.50	19.63	21.98	0.18	19.81	21.00	0.09
		0.75	20.56	21.99	0.12	19.37	21.00	0.14
		0.90	25.18	27.79	0.14	22.11	23.90	0.13
		1.10	25.97	27.81	0.12	22.35	23.72	0.12
		1.25	20.24	22.04	0.15	19.30	21.01	0.16
		1.50	20.58	22.00	0.13	20.25	21.00	0.05
		2.00	20.40	21.99	0.14	20.15	21.00	0.08
	1	0.50	19.66	21.99	0.21	1.00	*	1.00
		0.75	20.56	22.00	0.14	1.00	*	1.00
		0.90	25.58	29.76	0.21	1.00	*	1.00
		1.10	22.61	26.31	0.22	1.00	*	1.00
		1.25	21.01	22.10	0.10	1.00	*	1.00
		1.50	18.96	21.97	0.23	1.00	*	1.00
		2.00	19.56	21.98	0.20	1.00	*	1.00
	5	0.50	10.41	22.00	0.77	1.00	*	1.00
		0.75	10.12	22.05	0.81	1.00	*	1.00
		0.90	18.06	61.05	0.79	1.00	*	1.00
		1.10	9.20	22.53	0.83	1.00	*	1.00
		1.25	9.70	21.93	0.85	1.00	*	1.00
		1.50	10.50	22.00	0.80	1.00	*	1.00
		2.00	10.94	21.92	0.76	1.00	*	1.00

Table A.9: Nonparametric Regression, $n = 2^{10}$. Structural Component $3\sigma\sqrt{2\log(n)}$, $m = 40$, 100 Replications, $\tau = 0$.

			Nikoo Walsh			Nikoo Variance		
τ	p	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA
0	0	0.50	1.99	1.99	0.00	1.00	1.00	0.00
		0.75	1.99	1.99	0.00	1.00	1.00	0.00
		0.90	3.49	3.49	0.00	1.55	1.55	0.00
		1.10	3.54	3.54	0.00	1.65	1.65	0.00
		1.25	1.99	1.99	0.00	1.00	1.00	0.00
		1.50	1.99	1.99	0.00	1.00	1.00	0.00
		2.00	2.00	2.00	0.00	1.00	1.00	0.00
	1	0.50	1.99	1.99	0.00	1.00	1.00	0.00
		0.75	2.00	2.00	0.00	1.00	1.00	0.00
		0.90	4.07	4.07	0.00	1.00	1.00	0.00
		1.10	2.47	2.47	0.00	1.00	1.00	0.00
		1.25	2.00	2.00	0.00	1.00	1.00	0.00
		1.50	2.00	2.00	0.00	1.00	1.00	0.00
		2.00	1.99	1.99	0.00	1.00	1.00	0.00
	5	0.50	1.80	1.80	0.00	1.00	1.00	0.00
		0.75	1.71	1.71	0.00	1.00	1.00	0.00
		0.90	48.01	48.01	0.00	1.00	1.00	0.00
		1.10	1.80	1.80	0.00	1.00	1.00	0.00
		1.25	1.75	1.75	0.00	1.00	1.00	0.00
		1.50	1.73	1.73	0.00	1.00	1.00	0.00
		2.00	1.80	1.80	0.00	1.00	1.00	0.00

Table A.10: Nonparametric Regression for $n = 2^{10}$. Structural Component $3\sigma\sqrt{2\log(n)}$, $m = 40$, 100 Replications, $\tau = 20$.

			Nikoo Walsh			Nikoo Variance		
τ	p	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA
20	0	0.50	19.68	22.00	0.20	19.08	21.00	0.16
		0.75	20.33	22.00	0.15	19.81	21.00	0.11
		0.90	20.25	23.78	0.24	19.77	21.44	0.13
		1.10	20.46	23.70	0.23	21.05	21.56	0.07
		1.25	20.67	21.99	0.10	19.34	21.00	0.14
		1.50	20.43	21.99	0.14	19.76	21.00	0.11
		2.00	20.05	21.99	0.18	20.21	21.00	0.10
	1	0.50	18.81	22.00	0.28	1.00	*	1.00
		0.75	18.54	21.99	0.26	1.00	*	1.00
		0.90	20.76	24.42	0.23	1.00	*	1.00
		1.10	18.91	22.85	0.29	1.00	*	1.00
		1.25	19.55	21.98	0.19	1.00	*	1.00
		1.50	19.02	21.99	0.23	1.00	*	1.00
		2.00	19.72	21.97	0.24	1.00	*	1.00
	5	0.50	4.70	22.00	0.99	1.00	*	1.00
		0.75	5.50	22.20	0.95	1.00	*	1.00
		0.90	5.11	38.50	0.98	1.00	*	1.00
		1.10	5.51	22.00	0.96	1.00	*	1.00
		1.25	4.60	22.00	0.97	1.00	*	1.00
		1.50	4.89	22.00	0.97	1.00	*	1.00
		2.00	5.16	22.00	0.95	1.00	*	1.00

A.2 MAD Method Extensions

A.2.1 Structure Comparison - Parametric Control Chart Method

Table A.11: Nikoo Walsh Structure Level Comparison for $n = 2^7$. Structural Component $3\sigma\sqrt{2\log(n)}$, $m = 40$, 1000 Replications, $\tau = 0$.

			Nikoo Walsh, $p_1 = 0$			Nikoo Walsh, $p_1 = 1$			Nikoo Walsh, $p_1 = 5$		
τ	p_2	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
0	0	0.50	2.00	2.00	0.00	2.00	2.00	0.00	1.98	1.98	0.00
		0.75	3.71	3.71	0.00	4.00	4.00	0.00	2.80	2.80	0.00
		0.90	52.05	52.05	0.00	34.47	34.47	0.00	16.08	16.08	0.00
		1.10	40.50	40.50	0.00	86.38	86.38	0.00	355.84	355.84	0.00
		1.25	4.68	4.68	0.00	5.85	5.85	0.00	21.63	21.63	0.00
		1.50	2.18	2.18	0.00	2.23	2.23	0.00	2.77	2.77	0.00
		2.00	2.00	2.00	0.00	1.96	1.96	0.00	1.98	1.98	0.00
		1.00	253.90	253.90	0.00	296.16	296.16	0.00	138.52	138.52	0.00
	1	0.50	1.97	1.97	0.00	2.00	2.00	0.00	1.99	1.99	0.00
		0.75	5.52	5.52	0.00	5.12	5.12	0.00	3.24	3.24	0.00
		0.90	85.81	85.81	0.00	61.44	61.44	0.00	29.76	29.76	0.00
		1.10	22.54	22.54	0.00	53.52	53.52	0.00	348.05	348.05	0.00
		1.25	3.90	3.90	0.00	4.46	4.46	0.00	16.26	16.26	0.00
		1.50	2.08	2.08	0.00	2.27	2.27	0.00	2.35	2.35	0.00
		2.00	1.97	1.97	0.00	2.00	2.00	0.00	2.00	2.00	0.00
		1.00	186.61	186.61	0.00	280.29	280.29	0.00	234.55	234.55	0.00
	5	0.50	1.98	1.98	0.00	1.99	1.99	0.00	2.00	2.00	0.00
		0.75	9.07	9.07	0.00	7.89	7.89	0.00	5.68	5.68	0.00
		0.90	225.64	225.64	0.00	189.65	189.65	0.00	75.33	75.33	0.00
		1.10	9.67	9.67	0.00	17.39	17.39	0.00	89.93	89.93	0.00
		1.25	2.82	2.82	0.00	3.41	3.41	0.00	6.94	6.94	0.00
		1.50	2.03	2.03	0.00	2.12	2.12	0.00	2.30	2.30	0.00
		2.00	1.99	1.99	0.00	2.00	2.00	0.00	1.99	1.99	0.00
		1.00	72.84	72.84	0.00	153.82	153.82	0.00	447.59	447.59	0.00

Table A.12: Nikoo Walsh Structure Level Comparison for $n = 2^7$. Structural Component $3\sigma\sqrt{2\log(n)}$, $m = 40$, 1000 Replications, $\tau = 20$.

			Nikoo Walsh, $p_1 = 0$			Nikoo Walsh, $p_1 = 1$			Nikoo Walsh, $p_1 = 5$		
τ	p_2	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
20	0	0.50	21.16	22.00	0.08	20.87	21.99	0.07	19.94	21.99	0.18
		0.75	23.04	24.79	0.11	22.39	23.58	0.08	20.58	23.01	0.16
		0.90	64.05	70.11	0.10	52.68	55.30	0.06	31.71	40.36	0.28
		1.10	44.71	49.48	0.13	65.91	72.75	0.11	304.15	392.56	0.23
		1.25	24.00	24.81	0.07	25.65	26.23	0.05	39.03	44.02	0.14
		1.50	20.60	22.20	0.11	21.07	22.29	0.11	20.57	22.73	0.16
		2.00	21.26	22.00	0.08	21.14	22.00	0.07	19.30	22.00	0.25
		1.00	243.31	260.91	0.07	277.86	304.30	0.09	142.06	169.25	0.17
	1	0.50	20.42	21.98	0.11	21.13	21.98	0.06	20.96	21.99	0.09
		0.75	23.67	25.25	0.09	23.74	24.77	0.09	22.29	23.74	0.13
		0.90	94.51	107.33	0.13	94.25	95.98	0.02	49.84	56.01	0.13
		1.10	40.60	43.67	0.09	74.68	77.94	0.05	261.61	283.29	0.08
		1.25	22.60	24.14	0.12	23.76	24.44	0.05	30.39	33.26	0.12
		1.50	20.14	22.08	0.14	20.70	22.20	0.13	21.65	22.55	0.06
		2.00	21.14	22.00	0.09	20.87	21.99	0.11	20.56	21.99	0.11
		1.00	249.64	265.28	0.06	290.27	301.91	0.04	272.82	302.21	0.10
	5	0.50	18.65	21.97	0.34	20.54	21.98	0.12	21.72	22.00	0.04
		0.75	23.82	29.73	0.27	24.78	26.81	0.14	24.16	24.61	0.05
		0.90	162.63	231.13	0.31	154.78	180.09	0.15	85.50	89.39	0.05
		1.10	24.32	30.16	0.26	31.12	34.87	0.14	100.83	102.61	0.02
		1.25	18.09	22.86	0.34	21.59	23.17	0.11	25.53	26.56	0.06
		1.50	18.50	22.07	0.30	20.48	22.10	0.13	22.07	22.13	0.01
		2.00	19.06	22.00	0.27	20.87	21.99	0.10	21.66	22.00	0.05
		1.00	78.57	96.82	0.21	171.27	195.67	0.13	428.03	454.53	0.06

A.2.2 Structure Comparison - Nonparametric Control Chart Method

Table A.13: Nikoo Variance Structure Level Comparison for $n = 2^7$. Structural Component $3\sigma\sqrt{2\log(n)}$, $m = 40$, 1000 Replications, $\tau = 0$.

			Nikoo Variance, $p_1 = 0$			Nikoo Variance, $p_1 = 1$			Nikoo Variance, $p_1 = 5$		
τ	p_2	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		0.75	1.86	1.86	0.00	1.79	1.79	0.00	1.82	1.82	0.00
		0.90	26.78	26.78	0.00	30.44	30.44	0.00	27.34	27.34	0.00
		1.10	18.93	18.93	0.00	17.74	17.74	0.00	16.17	16.17	0.00
		1.25	1.94	1.94	0.00	2.07	2.07	0.00	2.07	2.07	0.00
		1.50	1.03	1.03	0.00	1.02	1.02	0.00	1.06	1.06	0.00
		2.00	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.00	185.24	185.24	0.00	194.29	194.29	0.00	177.56	177.56	0.00
	1	0.50	1.93	1.93	0.00	1.73	1.73	0.00	1.65	1.65	0.00
		0.75	1.04	1.04	0.00	1.09	1.09	0.00	1.07	1.07	0.00
		0.90	1.01	1.01	0.00	1.01	1.01	0.00	1.02	1.02	0.00
		1.10	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.25	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.50	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		2.00	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.00	1.01	1.01	0.00	1.00	1.00	0.00	1.00	1.00	0.00
	5	0.50	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		0.75	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		0.90	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.10	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.25	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.50	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		2.00	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.00	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00

Table A.14: Nikoo Variance Structure Level Comparison for $n = 2^7$. Structural Component $3\sigma\sqrt{2\log(n)}$, $m = 40$, 1000 Replications, $\tau = 20$.

			Nikoo Variance, $p_1 = 0$			Nikoo Variance, $p_1 = 1$			Nikoo Variance, $p_1 = 5$		
τ	p_2	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
20	0	0.50	19.52	21.00	0.11	20.23	21.00	0.10	20.50	21.00	0.07
		0.75	21.16	21.87	0.06	21.34	21.78	0.04	21.26	22.10	0.07
		0.90	39.85	45.18	0.15	42.87	47.94	0.14	39.82	42.23	0.07
		1.10	33.76	36.49	0.12	34.66	36.33	0.06	35.88	39.78	0.13
		1.25	20.55	22.18	0.11	21.65	22.42	0.09	20.99	22.07	0.09
		1.50	20.46	21.02	0.08	20.21	21.02	0.08	20.30	21.04	0.09
		2.00	19.56	21.00	0.14	19.76	21.00	0.12	20.02	21.00	0.09
		1.00	172.57	190.76	0.10	201.41	213.52	0.06	189.17	200.63	0.06
	1	0.50	1.00	$*$	1.00	1.00	$*$	1.00	1.00	$*$	1.00
		0.75	1.02	$*$	1.00	1.00	$*$	1.00	1.01	$*$	1.00
		0.90	1.00	$*$	1.00	1.01	$*$	1.00	1.01	$*$	1.00
		1.10	1.00	$*$	1.00	1.00	$*$	1.00	1.01	$*$	1.00
		1.25	1.00	$*$	1.00	1.00	$*$	1.00	1.00	$*$	1.00
		1.50	1.00	$*$	1.00	1.00	$*$	1.00	1.00	$*$	1.00
		2.00	1.00	$*$	1.00	1.00	$*$	1.00	1.00	$*$	1.00
		1.00	1.00	$*$	1.00	1.01	$*$	1.00	1.01	$*$	1.00
	5	0.50	1.00	$*$	1.00	1.00	$*$	1.00	1.00	$*$	1.00
		0.75	1.00	$*$	1.00	1.00	$*$	1.00	1.00	$*$	1.00
		0.90	1.00	$*$	1.00	1.00	$*$	1.00	1.00	$*$	1.00
		1.10	1.00	$*$	1.00	1.00	$*$	1.00	1.00	$*$	1.00
		1.25	1.00	$*$	1.00	1.00	$*$	1.00	1.00	$*$	1.00
		1.50	1.00	$*$	1.00	1.00	$*$	1.00	1.00	$*$	1.00
		2.00	1.00	$*$	1.00	1.00	$*$	1.00	1.00	$*$	1.00
		1.00	1.00	$*$	1.00	1.00	$*$	1.00	1.00	$*$	1.00

A.2.3 Structure Level Comparison - Median Absolute Deviation Method

Table A.15: MAD Structure Level Comparison for $n = 2^7$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			MAD, $p_1 = 0$			MAD, $p_1 = 1$			MAD, $p_1 = 5$		
τ	p_2	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
0	0	0.50	1.05	1.05	0.00	1.04	1.04	0.00	1.06	1.06	0.00
		0.75	3.43	3.43	0.00	3.26	3.26	0.00	2.66	2.66	0.00
		0.90	25.63	25.63	0.00	18.75	18.75	0.00	10.56	10.56	0.00
		1.10	39.89	39.89	0.00	43.56	43.56	0.00	123.17	123.17	0.00
		1.25	4.52	4.52	0.00	5.47	5.47	0.00	8.47	8.47	0.00
		1.50	1.76	1.76	0.00	1.88	1.88	0.00	2.03	2.03	0.00
		2.00	1.03	1.03	0.00	1.02	1.02	0.00	1.09	1.09	0.00
		1.00	203.99	203.99	0.00	138.03	138.03	0.00	75.19	75.19	0.00
	1	0.50	1.16	1.16	0.00	1.10	1.10	0.00	1.05	1.05	0.00
		0.75	3.46	3.46	0.00	3.39	3.39	0.00	2.78	2.78	0.00
		0.90	41.40	41.40	0.00	27.96	27.96	0.00	14.44	14.44	0.00
		1.10	23.90	23.90	0.00	34.51	34.51	0.00	102.66	102.66	0.00
		1.25	3.85	3.85	0.00	4.60	4.60	0.00	7.12	7.12	0.00
		1.50	1.79	1.79	0.00	1.71	1.71	0.00	2.07	2.07	0.00
		2.00	1.02	1.02	0.00	1.03	1.03	0.00	1.08	1.08	0.00
		1.00	195.87	195.87	0.00	222.54	222.54	0.00	129.34	129.34	0.00
	5	0.50	1.39	1.39	0.00	1.39	1.39	0.00	1.21	1.21	0.00
		0.75	6.45	6.45	0.00	4.94	4.94	0.00	4.45	4.45	0.00
		0.90	276.58	276.58	0.00	169.92	169.92	0.00	78.14	78.14	0.00
		1.10	19.06	19.06	0.00	28.46	28.46	0.00	132.86	132.86	0.00
		1.25	4.19	4.19	0.00	4.58	4.58	0.00	6.05	6.05	0.00
		1.50	1.73	1.73	0.00	1.77	1.77	0.00	2.17	2.17	0.00
		2.00	1.02	1.02	0.00	1.05	1.05	0.00	1.04	1.04	0.00
		1.00	284.22	284.22	0.00	450.11	450.11	0.00	896.53	896.53	0.00

Table A.16: MAD Structure Level Comparison for $n = 2^7$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			MAD, $p_1 = 0$			MAD, $p_1 = 1$			MAD, $p_1 = 5$		
τ	p_2	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
20	0	0.50	20.16	21.07	0.08	20.28	21.03	0.06	20.14	21.03	0.10
		0.75	21.97	23.18	0.09	22.06	22.79	0.06	22.14	22.45	0.04
		0.90	41.30	42.91	0.05	35.77	36.78	0.04	26.98	27.48	0.03
		1.10	44.74	48.38	0.10	54.94	59.88	0.10	149.16	162.69	0.09
		1.25	23.05	24.61	0.11	25.14	26.01	0.06	27.31	28.47	0.07
		1.50	21.07	21.53	0.06	21.55	21.86	0.05	21.76	22.15	0.04
		2.00	20.45	21.02	0.04	20.65	21.01	0.05	20.33	21.10	0.08
		1.00	178.89	193.66	0.08	150.44	165.93	0.10	93.58	96.14	0.03
	1	0.50	20.18	21.18	0.09	20.75	21.07	0.04	20.18	21.02	0.09
		0.75	22.64	23.59	0.07	22.41	23.39	0.06	22.03	22.94	0.10
		0.90	54.54	57.05	0.06	46.15	47.78	0.04	30.62	32.02	0.07
		1.10	42.26	46.51	0.11	56.52	60.08	0.07	113.35	120.94	0.07
		1.25	23.50	24.55	0.06	24.10	24.85	0.05	25.65	26.91	0.10
		1.50	21.07	21.54	0.05	21.44	21.73	0.05	20.23	22.00	0.14
		2.00	20.68	21.07	0.04	20.69	21.06	0.04	20.20	21.03	0.12
		1.00	200.99	212.82	0.06	224.04	230.59	0.03	141.26	149.39	0.06
	5	0.50	20.21	21.35	0.12	20.91	21.29	0.06	21.03	21.21	0.02
		0.75	25.14	25.95	0.06	23.98	24.96	0.06	23.23	24.03	0.07
		0.90	300.69	316.00	0.05	140.18	159.51	0.13	55.95	61.39	0.11
		1.10	35.03	37.32	0.10	39.39	42.79	0.11	93.06	97.51	0.05
		1.25	22.16	23.66	0.12	22.54	23.67	0.08	24.55	25.82	0.10
		1.50	21.11	21.72	0.06	21.31	21.79	0.06	21.36	22.07	0.09
		2.00	20.20	21.02	0.07	19.66	21.06	0.15	20.56	21.04	0.08
		1.00	425.59	476.71	0.11	483.00	553.49	0.13	803.63	901.26	0.11

Table A.17: MAD Structure Level Comparison for $n = 2^8$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			MAD, $p_1 = 0$			MAD, $p_1 = 1$			MAD, $p_1 = 5$		
τ	p_2	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		0.75	2.01	2.01	0.00	1.90	1.90	0.00	1.47	1.47	0.00
		0.90	11.42	11.42	0.00	13.23	13.23	0.00	4.78	4.78	0.00
		1.10	14.13	14.13	0.00	22.76	22.76	0.00	102.35	102.35	0.00
		1.25	2.35	2.35	0.00	3.22	3.22	0.00	4.48	4.48	0.00
		1.50	1.16	1.16	0.00	1.30	1.30	0.00	1.41	1.41	0.00
		2.00	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.00	116.12	116.12	0.00	196.85	196.85	0.00	48.18	48.18	0.00
	1	0.50	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		0.75	2.09	2.09	0.00	2.23	2.23	0.00	1.67	1.67	0.00
		0.90	16.45	16.45	0.00	15.53	15.53	0.00	5.42	5.42	0.00
		1.10	12.25	12.25	0.00	14.84	14.84	0.00	63.12	63.12	0.00
		1.25	2.56	2.56	0.00	2.84	2.84	0.00	3.50	3.50	0.00
		1.50	1.16	1.16	0.00	1.15	1.15	0.00	1.35	1.35	0.00
		2.00	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.00	177.56	177.56	0.00	173.06	173.06	0.00	66.72	66.72	0.00
	5	0.50	1.02	1.02	0.00	1.01	1.01	0.00	1.00	1.00	0.00
		0.75	3.86	3.86	0.00	4.34	4.34	0.00	2.70	2.70	0.00
		0.90	465.23	465.23	0.00	373.93	373.93	0.00	40.03	40.03	0.00
		1.10	9.89	9.89	0.00	13.42	13.42	0.00	51.58	51.58	0.00
		1.25	2.57	2.57	0.00	2.86	2.86	0.00	3.44	3.44	0.00
		1.50	1.20	1.20	0.00	1.15	1.15	0.00	1.47	1.47	0.00
		2.00	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.00	249.96	249.96	0.00	566.23	566.23	0.00	2054.37	2054.37	0.00

Table A.18: MAD Structure Level Comparison for $n = 2^8$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			MAD, $p_1 = 0$			MAD, $p_1 = 1$			MAD, $p_1 = 5$		
τ	p_2	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
20	0	0.50	20.02	21.00	0.10	19.90	21.00	0.10	16.02	21.00	0.48
		0.75	20.37	21.92	0.15	21.40	21.91	0.06	17.39	21.53	0.43
		0.90	28.07	30.40	0.12	27.54	28.84	0.07	17.87	24.37	0.46
		1.10	28.74	31.68	0.13	34.11	36.50	0.10	58.07	102.73	0.48
		1.25	19.63	22.45	0.23	22.09	22.72	0.08	18.25	25.00	0.45
		1.50	20.20	21.17	0.10	19.96	21.19	0.12	16.44	21.35	0.49
		2.00	19.16	21.00	0.18	20.56	21.00	0.06	15.47	21.00	0.50
		1.00	147.76	166.33	0.12	169.98	189.65	0.11	60.83	82.70	0.30
	1	0.50	20.04	21.00	0.10	20.07	21.00	0.10	18.81	21.00	0.23
		0.75	20.85	22.17	0.10	21.47	22.13	0.07	17.76	21.59	0.34
		0.90	30.21	32.95	0.12	28.73	29.67	0.05	19.77	24.84	0.36
		1.10	27.75	29.53	0.08	32.82	35.53	0.11	48.17	68.38	0.35
		1.25	20.84	22.45	0.13	21.43	22.59	0.12	20.14	23.61	0.29
		1.50	20.27	21.13	0.09	20.45	21.11	0.07	18.42	21.40	0.33
		2.00	19.85	21.00	0.11	19.88	21.00	0.12	17.36	21.00	0.30
		1.00	168.49	180.23	0.07	147.90	156.78	0.06	59.60	79.46	0.29
	5	0.50	20.12	21.01	0.10	20.55	21.07	0.04	21.01	21.01	0.00
		0.75	22.31	23.81	0.12	23.42	23.81	0.04	22.60	22.60	0.00
		0.90	332.21	372.09	0.11	259.12	280.75	0.08	42.04	42.04	0.00
		1.10	27.06	29.14	0.14	27.32	28.22	0.05	53.19	53.19	0.00
		1.25	21.54	22.40	0.11	22.38	22.54	0.01	23.50	23.50	0.00
		1.50	19.41	21.22	0.17	20.77	21.27	0.05	21.16	21.27	0.01
		2.00	19.93	21.00	0.11	20.81	21.00	0.02	21.01	21.01	0.00
		1.00	378.12	414.47	0.09	596.61	621.05	0.04	1901.08	1901.08	0.00

Table A.19: MAD Structure Level Comparison for $n = 2^9$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			MAD, $p_1 = 0$			MAD, $p_1 = 1$			MAD, $p_1 = 5$		
τ	p_2	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		0.75	1.33	1.33	0.00	1.22	1.22	0.00	1.04	1.04	0.00
		0.90	5.01	5.01	0.00	5.14	5.14	0.00	2.68	2.68	0.00
		1.10	6.17	6.17	0.00	7.28	7.28	0.00	40.51	40.51	0.00
		1.25	1.60	1.60	0.00	1.60	1.60	0.00	2.47	2.47	0.00
		1.50	1.01	1.01	0.00	1.01	1.01	0.00	1.04	1.04	0.00
		2.00	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.00	170.97	170.97	0.00	178.15	178.15	0.00	23.59	23.59	0.00
	1	0.50	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		0.75	1.33	1.33	0.00	1.26	1.26	0.00	1.12	1.12	0.00
		0.90	6.85	6.85	0.00	4.91	4.91	0.00	3.14	3.14	0.00
		1.10	4.99	4.99	0.00	7.33	7.33	0.00	39.71	39.71	0.00
		1.25	1.39	1.39	0.00	1.69	1.69	0.00	2.26	2.26	0.00
		1.50	1.01	1.01	0.00	1.02	1.02	0.00	1.05	1.05	0.00
		2.00	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.00	205.67	205.67	0.00	182.68	182.68	0.00	35.49	35.49	0.00
	5	0.50	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		0.75	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		0.90	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		1.10	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		1.25	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		1.50	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		2.00	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		1.00	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00

Table A.20: MAD Structure Level Comparison for $n = 2^9$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			MAD, $p_1 = 0$			MAD, $p_1 = 1$			MAD, $p_1 = 5$		
τ	p_2	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
20	0	0.50	19.36	21.00	0.13	19.97	21.00	0.10	13.77	21.00	0.61
		0.75	20.43	21.22	0.07	20.41	21.17	0.11	13.11	21.03	0.68
		0.90	23.35	24.91	0.11	22.95	24.51	0.10	13.04	22.39	0.72
		1.10	24.89	26.21	0.09	25.26	28.20	0.16	22.97	49.11	0.65
		1.25	20.20	21.60	0.12	20.76	21.67	0.09	14.53	22.29	0.65
		1.50	20.17	21.01	0.12	19.82	21.01	0.13	13.10	21.06	0.69
		2.00	19.72	21.00	0.11	19.58	21.00	0.12	12.66	21.00	0.71
		1.00	167.34	184.23	0.10	125.42	153.61	0.20	28.01	56.61	0.62
	1	0.50	20.28	21.00	0.07	19.97	21.00	0.13	15.22	21.00	0.49
		0.75	20.75	21.29	0.07	20.61	21.22	0.08	17.41	21.08	0.39
		0.90	24.29	26.02	0.12	23.66	25.21	0.11	15.24	22.67	0.57
		1.10	23.20	25.58	0.16	24.32	25.47	0.07	26.21	41.57	0.46
		1.25	21.05	21.49	0.05	20.26	21.56	0.11	16.21	22.40	0.53
		1.50	20.02	21.00	0.11	19.64	21.00	0.11	15.10	21.09	0.55
		2.00	20.04	21.00	0.08	20.31	21.00	0.08	16.03	21.00	0.46
		1.00	192.27	208.02	0.08	177.55	193.76	0.09	34.46	62.00	0.52
	5	0.50	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		0.75	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		0.90	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		1.10	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		1.25	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		1.50	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		2.00	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		1.00	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00

Table A.21: MAD Structure Level Comparison for $n = 2^{10}$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			MAD, $p_1 = 0$			MAD, $p_1 = 1$			MAD, $p_1 = 5$		
τ	p_2	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		0.75	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		0.90	3.13	3.13	0.00	2.73	2.73	0.00	1.60	1.60	0.00
		1.10	3.73	3.73	0.00	4.05	4.05	0.00	27.37	27.37	0.00
		1.25	1.13	1.13	0.00	1.16	1.16	0.00	1.64	1.64	0.00
		1.50	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		2.00	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.00	165.39	165.39	0.00	101.01	101.01	0.00	9.53	9.53	0.00
	1	0.50	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		0.75	1.04	1.04	0.00	1.04	1.04	0.00	1.00	1.00	0.00
		0.90	4.03	4.03	0.00	3.37	3.37	0.00	1.83	1.83	0.00
		1.10	2.96	2.96	0.00	3.49	3.49	0.00	17.24	17.24	0.00
		1.25	1.07	1.07	0.00	1.16	1.16	0.00	1.42	1.42	0.00
		1.50	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		2.00	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.00	149.36	149.36	0.00	334.74	334.74	0.00	21.89	21.89	0.00
	5	0.50	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		0.75	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		0.90	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		1.10	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		1.25	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		1.50	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		2.00	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		1.00	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00

Table A.22: MAD Structure Level Comparison for $n = 2^{10}$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			MAD, $p_1 = 0$			MAD, $p_1 = 1$			MAD, $p_1 = 5$		
τ	p_2	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
20	0	0.50	19.38	21.00	0.17	18.96	21.00	0.20	8.16	21.00	0.96
		0.75	19.54	21.00	0.12	19.00	21.00	0.20	9.18	21.00	0.84
		0.90	21.39	22.58	0.10	20.83	22.52	0.14	8.16	21.63	0.92
		1.10	22.98	23.39	0.03	22.09	24.29	0.17	10.78	38.45	0.89
		1.25	20.04	21.07	0.10	19.12	21.11	0.17	9.58	21.31	0.84
		1.50	19.93	21.00	0.12	19.46	21.00	0.14	8.43	21.00	0.91
		2.00	20.01	21.00	0.11	19.54	21.00	0.17	7.72	21.00	0.90
		1.00	164.21	182.81	0.11	119.12	135.18	0.13	9.94	38.80	0.90
	1	0.50	19.74	21.00	0.14	20.40	21.00	0.06	11.82	21.00	0.73
		0.75	20.47	21.06	0.10	20.63	21.00	0.05	12.00	21.00	0.76
		0.90	22.57	23.85	0.09	22.35	23.30	0.07	12.24	21.81	0.68
		1.10	21.78	22.86	0.10	22.74	23.66	0.07	12.13	29.42	0.81
		1.25	20.41	21.08	0.07	21.14	21.15	0.01	11.41	21.75	0.80
		1.50	19.95	21.00	0.13	20.41	21.00	0.06	12.46	21.00	0.74
		2.00	19.67	21.00	0.14	20.47	21.00	0.05	12.92	21.00	0.74
		1.00	218.66	239.36	0.09	293.34	321.52	0.09	19.18	41.67	0.70
	5	0.50	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		0.75	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		0.90	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		1.10	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		1.25	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		1.50	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		2.00	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00
		1.00	1.00	*	1.00	1.00	*	1.00	1.00	*	1.00

A.2.4 Method Comparison

Table A.23: Method Comparison for $n = 2^7$. Structural Component $3\sigma\sqrt{2\log(n)}$, $m = 40$, 1000 Replications, $\tau = 0$.

			Nikoo Walsh			Nikoo Variance			MAD		
τ	p	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
0	0	0.50	2.00	2.00	0.00	1.00	1.00	0.00	1.05	1.05	0.00
		0.75	3.71	3.71	0.00	1.86	1.86	0.00	3.43	3.43	0.00
		0.90	52.05	52.05	0.00	26.78	26.78	0.00	25.63	25.63	0.00
		1.10	40.50	40.50	0.00	18.93	18.93	0.00	39.89	39.89	0.00
		1.25	4.68	4.68	0.00	1.94	1.94	0.00	4.52	4.52	0.00
		1.50	2.18	2.18	0.00	1.03	1.03	0.00	1.76	1.76	0.00
		2.00	2.00	2.00	0.00	1.00	1.00	0.00	1.03	1.03	0.00
		1.00	253.90	253.90	0.00	185.24	185.24	0.00	203.99	203.99	0.00
	1	0.50	2.00	2.00	0.00	1.73	1.73	0.00	1.10	1.10	0.00
		0.75	5.12	5.12	0.00	1.09	1.09	0.00	3.39	3.39	0.00
		0.90	61.44	61.44	0.00	1.01	1.01	0.00	27.96	27.96	0.00
		1.10	53.52	53.52	0.00	1.00	1.00	0.00	34.51	34.51	0.00
		1.25	4.46	4.46	0.00	1.00	1.00	0.00	4.60	4.60	0.00
		1.50	2.27	2.27	0.00	1.00	1.00	0.00	1.71	1.71	0.00
		2.00	2.00	2.00	0.00	1.00	1.00	0.00	1.03	1.03	0.00
		1.00	280.29	280.29	0.00	1.00	1.00	0.00	222.54	222.54	0.00
	5	0.50	2.00	2.00	0.00	1.00	1.00	0.00	1.21	1.21	0.00
		0.75	5.68	5.68	0.00	1.00	1.00	0.00	4.45	4.45	0.00
		0.90	75.33	75.33	0.00	1.00	1.00	0.00	78.14	78.14	0.00
		1.10	89.93	89.93	0.00	1.00	1.00	0.00	132.86	132.86	0.00
		1.25	6.94	6.94	0.00	1.00	1.00	0.00	6.05	6.05	0.00
		1.50	2.30	2.30	0.00	1.00	1.00	0.00	2.17	2.17	0.00
		2.00	1.99	1.99	0.00	1.00	1.00	0.00	1.04	1.04	0.00
		1.00	447.59	447.59	0.00	1.00	1.00	0.00	896.53	896.53	0.00

Table A.24: Method Comparison for $n = 2^7$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			Nikoo Walsh			Nikoo Variance			MAD		
τ	p	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
20	0	0.50	21.16	22.00	0.08	19.52	21.00	0.11	20.16	21.07	0.08
		0.75	23.04	24.79	0.11	21.16	21.87	0.06	21.97	23.18	0.09
		0.90	64.05	70.11	0.10	39.85	45.18	0.15	41.30	42.91	0.05
		1.10	44.71	49.48	0.13	33.76	36.49	0.12	44.74	48.38	0.10
		1.25	24.00	24.81	0.07	20.55	22.18	0.11	23.05	24.61	0.11
		1.50	20.60	22.20	0.11	20.46	21.02	0.08	21.07	21.53	0.06
		2.00	21.26	22.00	0.08	19.56	21.00	0.14	20.45	21.02	0.04
		1.00	243.31	260.91	0.07	172.57	190.76	0.10	178.89	193.66	0.08
	1	0.50	21.13	21.98	0.06	1.00	$*$	1.00	20.75	21.07	0.04
		0.75	23.74	24.77	0.09	1.00	$*$	1.00	22.41	23.39	0.06
		0.90	94.25	95.98	0.02	1.01	$*$	1.00	46.15	47.78	0.04
		1.10	74.68	77.94	0.05	1.00	$*$	1.00	56.52	60.08	0.07
		1.25	23.76	24.44	0.05	1.00	$*$	1.00	24.10	24.85	0.05
		1.50	20.70	22.20	0.13	1.00	$*$	1.00	21.44	21.73	0.05
		2.00	20.87	21.99	0.11	1.00	$*$	1.00	20.69	21.06	0.04
		1.00	290.27	301.91	0.04	1.01	$*$	1.00	224.04	230.59	0.03
	5	0.50	21.72	22.00	0.04	1.00	$*$	1.00	21.03	21.21	0.02
		0.75	24.16	24.61	0.05	1.00	$*$	1.00	23.23	24.03	0.07
		0.90	85.50	89.39	0.05	1.00	$*$	1.00	55.95	61.39	0.11
		1.10	100.83	102.61	0.02	1.00	$*$	1.00	93.06	97.51	0.05
		1.25	25.53	26.56	0.06	1.00	$*$	1.00	24.55	25.82	0.10
		1.50	22.07	22.13	0.01	1.00	$*$	1.00	21.36	22.07	0.09
		2.00	21.66	22.00	0.05	1.00	$*$	1.00	20.56	21.04	0.08
		1.00	428.03	454.53	0.06	1.00	$*$	1.00	803.63	901.26	0.11

Table A.25: Method Comparison for $n = 2^8$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			Nikoo Walsh			Nikoo Variance			MAD		
τ	p	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
0	0	0.50	2.00	2.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		0.75	2.21	2.21	0.00	1.04	1.04	0.00	2.01	2.01	0.00
		0.90	15.99	15.99	0.00	9.42	9.42	0.00	11.42	11.42	0.00
		1.10	15.49	15.49	0.00	8.19	8.19	0.00	14.13	14.13	0.00
		1.25	2.72	2.72	0.00	1.23	1.23	0.00	2.35	2.35	0.00
		1.50	2.02	2.02	0.00	1.00	1.00	0.00	1.16	1.16	0.00
		2.00	1.99	1.99	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.00	187.13	187.13	0.00	185.89	185.89	0.00	116.12	116.12	0.00
	1	0.50	2.00	2.00	0.00	25764.82	25764.82	0.00	1.00	1.00	0.00
		0.75	2.29	2.29	0.00	3.01	3.01	0.00	2.23	2.23	0.00
		0.90	20.75	20.75	0.00	1.18	1.18	0.00	15.53	15.53	0.00
		1.10	17.76	17.76	0.00	1.00	1.00	0.00	14.84	14.84	0.00
		1.25	2.62	2.62	0.00	1.00	1.00	0.00	2.84	2.84	0.00
		1.50	1.99	1.99	0.00	1.00	1.00	0.00	1.15	1.15	0.00
		2.00	1.99	1.99	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.00	220.62	220.62	0.00	1.03	1.03	0.00	173.06	173.06	0.00
	5	0.50	2.00	2.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		0.75	2.26	2.26	0.00	1.00	1.00	0.00	2.70	2.70	0.00
		0.90	20.85	20.85	0.00	1.00	1.00	0.00	40.03	40.03	0.00
		1.10	28.46	28.46	0.00	1.00	1.00	0.00	51.58	51.58	0.00
		1.25	3.38	3.38	0.00	1.00	1.00	0.00	3.44	3.44	0.00
		1.50	2.03	2.03	0.00	1.00	1.00	0.00	1.47	1.47	0.00
		2.00	2.00	2.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.00	483.24	483.24	0.00	1.00	1.00	0.00	2054.37	2054.37	0.00

Table A.26: Method Comparison for $n = 2^8$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			Nikoo Walsh			Nikoo Variance			MAD		
τ	p	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
20	0	0.50	21.28	21.99	0.06	19.60	21.00	0.11	20.02	21.00	0.10
		0.75	20.79	22.28	0.12	20.03	21.07	0.10	20.37	21.92	0.15
		0.90	36.53	39.20	0.09	28.02	29.82	0.09	28.07	30.40	0.12
		1.10	32.41	35.00	0.10	26.45	28.14	0.09	28.74	31.68	0.13
		1.25	21.23	22.65	0.09	19.55	21.18	0.12	19.63	22.45	0.23
		1.50	21.08	22.00	0.09	20.32	21.00	0.09	20.20	21.17	0.10
		2.00	20.82	22.00	0.10	19.97	21.00	0.07	19.16	21.00	0.18
		1.00	183.62	200.89	0.09	165.54	192.95	0.15	147.76	166.33	0.12
	1	0.50	20.59	22.00	0.12	1.03	*	1.00	20.07	21.00	0.10
		0.75	20.75	22.27	0.10	1.03	*	1.00	21.47	22.13	0.07
		0.90	35.42	37.81	0.09	1.01	*	1.00	28.73	29.67	0.05
		1.10	33.74	36.38	0.09	1.00	*	1.00	32.82	35.53	0.11
		1.25	20.98	22.56	0.10	1.00	*	1.00	21.43	22.59	0.12
		1.50	20.68	22.00	0.11	1.00	*	1.00	20.45	21.11	0.07
		2.00	20.95	22.00	0.11	1.00	*	1.00	19.88	21.00	0.12
		1.00	262.56	284.98	0.08	1.05	*	1.00	147.90	156.78	0.06
	5	0.50	21.13	22.00	0.06	1.00	*	1.00	21.01	21.01	0.00
		0.75	22.06	22.58	0.04	1.00	*	1.00	22.60	22.60	0.00
		0.90	49.77	50.81	0.03	1.00	*	1.00	42.04	42.04	0.00
		1.10	41.28	44.13	0.08	1.00	*	1.00	53.19	53.19	0.00
		1.25	22.72	23.13	0.04	1.00	*	1.00	23.50	23.50	0.00
		1.50	21.44	22.02	0.05	1.00	*	1.00	21.16	21.27	0.01
		2.00	20.60	22.00	0.11	1.00	*	1.00	21.01	21.01	0.00
		1.00	323.02	336.04	0.04	1.00	*	1.00	1901.08	1901.08	0.00

Table A.27: Method Comparison for $n = 2^9$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			Nikoo Walsh			Nikoo Variance			MAD		
τ	p	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
0	0	0.50	1.99	1.99	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		0.75	2.02	2.02	0.00	1.00	1.00	0.00	1.33	1.33	0.00
		0.90	7.55	7.55	0.00	3.41	3.41	0.00	5.01	5.01	0.00
		1.10	7.55	7.55	0.00	3.41	3.41	0.00	6.17	6.17	0.00
		1.25	2.10	2.10	0.00	1.02	1.02	0.00	1.60	1.60	0.00
		1.50	2.00	2.00	0.00	1.00	1.00	0.00	1.01	1.01	0.00
		2.00	1.98	1.98	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.00	160.58	160.58	0.00	177.47	177.47	0.00	170.97	170.97	0.00
	1	0.50	1.99	1.99	0.00	86.23	86.23	0.00	1.00	1.00	0.00
		0.75	2.01	2.01	0.00	1.03	1.03	0.00	1.26	1.26	0.00
		0.90	6.68	6.68	0.00	1.00	1.00	0.00	4.91	4.91	0.00
		1.10	6.50	6.50	0.00	1.00	1.00	0.00	7.33	7.33	0.00
		1.25	2.05	2.05	0.00	1.00	1.00	0.00	1.69	1.69	0.00
		1.50	1.99	1.99	0.00	1.00	1.00	0.00	1.02	1.02	0.00
		2.00	2.00	2.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.00	144.09	144.09	0.00	1.00	1.00	0.00	182.68	182.68	0.00
	5	0.50	2.00	2.00	0.00	1.00	1.00	0.00	1.00	*	1.00
		0.75	2.00	2.00	0.00	1.00	1.00	0.00	1.00	*	1.00
		0.90	8.06	8.06	0.00	1.00	1.00	0.00	1.00	*	1.00
		1.10	9.32	9.32	0.00	1.00	1.00	0.00	1.00	*	1.00
		1.25	2.17	2.17	0.00	1.00	1.00	0.00	1.00	*	1.00
		1.50	2.00	2.00	0.00	1.00	1.00	0.00	1.00	*	1.00
		2.00	2.00	2.00	0.00	1.00	1.00	0.00	1.00	*	1.00
		1.00	353.72	353.72	0.00	1.00	1.00	0.00	1.00	*	1.00

Table A.28: Method Comparison for $n = 2^9$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			Nikoo Walsh			Nikoo Variance			MAD		
τ	p	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
20	0	0.50	20.19	21.99	0.14	19.76	21.00	0.11	19.36	21.00	0.13
		0.75	20.01	21.99	0.16	19.68	21.00	0.13	20.43	21.22	0.07
		0.90	24.80	25.77	0.07	22.29	23.62	0.08	23.35	24.91	0.11
		1.10	23.92	26.17	0.14	20.65	23.59	0.20	24.89	26.21	0.09
		1.25	21.24	22.11	0.08	20.17	21.01	0.08	20.20	21.60	0.12
		1.50	21.63	21.99	0.02	20.10	21.00	0.08	20.17	21.01	0.12
		2.00	20.54	22.00	0.12	19.62	21.00	0.12	19.72	21.00	0.11
		1.00	170.02	194.03	0.13	213.31	235.96	0.10	167.34	184.23	0.10
	1	0.50	20.72	21.99	0.10	1.00	*	1.00	19.97	21.00	0.13
		0.75	20.41	22.00	0.14	1.00	*	1.00	20.61	21.22	0.08
		0.90	26.23	27.51	0.08	1.00	*	1.00	23.66	25.21	0.11
		1.10	24.71	26.94	0.13	1.00	*	1.00	24.32	25.47	0.07
		1.25	20.65	22.08	0.13	1.00	*	1.00	20.26	21.56	0.11
		1.50	21.03	21.99	0.09	1.00	*	1.00	19.64	21.00	0.11
		2.00	20.21	22.00	0.16	1.00	*	1.00	20.31	21.00	0.08
		1.00	184.70	197.80	0.07	1.00	*	1.00	177.55	193.76	0.09
	5	0.50	21.39	21.99	0.06	1.00	*	1.00	1.00	*	1.00
		0.75	21.15	22.01	0.05	1.00	*	1.00	1.00	*	1.00
		0.90	29.05	30.07	0.05	1.00	*	1.00	1.00	*	1.00
		1.10	27.75	29.80	0.11	1.00	*	1.00	1.00	*	1.00
		1.25	21.62	22.11	0.04	1.00	*	1.00	1.00	*	1.00
		1.50	21.04	22.00	0.06	1.00	*	1.00	1.00	*	1.00
		2.00	21.03	22.00	0.06	1.00	*	1.00	1.00	*	1.00
		1.00	341.01	377.57	0.10	1.00	*	1.00	1.00	*	1.00

Table A.29: Method Comparison for $n = 2^{10}$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			Nikoo Walsh			Nikoo Variance			MAD		
τ	p	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
0	0	0.50	1.99	1.99	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		0.75	1.99	1.99	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		0.90	3.32	3.32	0.00	1.36	1.36	0.00	3.13	3.13	0.00
		1.10	3.28	3.28	0.00	1.63	1.63	0.00	3.73	3.73	0.00
		1.25	1.98	1.98	0.00	1.00	1.00	0.00	1.13	1.13	0.00
		1.50	2.00	2.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		2.00	1.99	1.99	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.00	167.78	167.78	0.00	194.04	194.04	0.00	165.39	165.39	0.00
	1	0.50	2.00	2.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		0.75	2.00	2.00	0.00	1.00	1.00	0.00	1.04	1.04	0.00
		0.90	3.17	3.17	0.00	1.00	1.00	0.00	3.37	3.37	0.00
		1.10	3.57	3.57	0.00	1.00	1.00	0.00	3.49	3.49	0.00
		1.25	1.99	1.99	0.00	1.00	1.00	0.00	1.16	1.16	0.00
		1.50	2.00	2.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		2.00	2.00	2.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00
		1.00	187.39	187.39	0.00	1.00	1.00	0.00	334.74	334.74	0.00
	5	0.50	2.00	2.00	0.00	1.00	1.00	0.00	1.00	*	1.00
		0.75	2.00	2.00	0.00	1.00	1.00	0.00	1.00	*	1.00
		0.90	3.43	3.43	0.00	1.00	1.00	0.00	1.00	*	1.00
		1.10	4.06	4.06	0.00	1.00	1.00	0.00	1.00	*	1.00
		1.25	2.01	2.01	0.00	1.00	1.00	0.00	1.00	*	1.00
		1.50	2.00	2.00	0.00	1.00	1.00	0.00	1.00	*	1.00
		2.00	2.00	2.00	0.00	1.00	1.00	0.00	1.00	*	1.00
		1.00	372.26	372.26	0.00	1.00	1.00	0.00	1.00	*	1.00

Table A.30: Method Comparison for $n = 2^{10}$. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			Nikoo Walsh			Nikoo Variance			MAD		
τ	p	σ_{oc}	ARL	ARL-1	PFA	ARL	ARL-1	PFA	ARL	ARL-1	PFA
20	0	0.50	20.37	21.99	0.13	20.29	21.00	0.05	19.38	21.00	0.17
		0.75	20.73	21.99	0.11	19.54	21.00	0.14	19.54	21.00	0.12
		0.90	22.11	23.37	0.10	20.37	21.58	0.09	21.39	22.58	0.10
		1.10	22.07	23.46	0.10	20.64	21.53	0.07	22.98	23.39	0.03
		1.25	20.32	21.99	0.12	19.89	21.00	0.11	20.04	21.07	0.10
		1.50	20.32	22.00	0.14	19.73	21.00	0.13	19.93	21.00	0.12
		2.00	20.67	22.00	0.11	20.14	21.00	0.08	20.01	21.00	0.11
		1.00	172.94	206.27	0.17	182.72	213.56	0.15	164.21	182.81	0.11
	1	0.50	20.31	22.00	0.13	1.00	*	1.00	20.40	21.00	0.06
		0.75	21.15	22.00	0.08	1.00	*	1.00	20.63	21.00	0.05
		0.90	21.90	23.11	0.10	1.00	*	1.00	22.35	23.30	0.07
		1.10	22.12	23.65	0.11	1.00	*	1.00	22.74	23.66	0.07
		1.25	20.29	22.00	0.15	1.00	*	1.00	21.14	21.15	0.01
		1.50	20.41	22.00	0.13	1.00	*	1.00	20.41	21.00	0.06
		2.00	20.60	22.00	0.11	1.00	*	1.00	20.47	21.00	0.05
		1.00	168.96	186.50	0.10	1.00	*	1.00	293.34	321.52	0.09
	5	0.50	21.54	22.00	0.04	1.00	*	1.00	1.00	*	1.00
		0.75	21.05	21.99	0.06	1.00	*	1.00	1.00	*	1.00
		0.90	23.20	23.48	0.04	1.00	*	1.00	1.00	*	1.00
		1.10	23.19	24.05	0.06	1.00	*	1.00	1.00	*	1.00
		1.25	20.97	22.00	0.08	1.00	*	1.00	1.00	*	1.00
		1.50	21.08	22.00	0.07	1.00	*	1.00	1.00	*	1.00
		2.00	21.27	21.99	0.04	1.00	*	1.00	1.00	*	1.00
		1.00	294.83	326.18	0.10	1.00	*	1.00	1.00	*	1.00

A.3 Alternative Likelihood Ratio Techniques

A.3.1 Single In-Control Variance Methods

Table A.31: Likelihood Ratio with Single In-Control Variance, Scale-Out Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			LR-out, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.09	1.09	0.00
		0.75	3.40	3.40	0.00
		0.90	23.12	23.12	0.00
		1.10	26.39	26.39	0.00
		1.25	4.41	4.41	0.00
		1.50	1.76	1.76	0.00
		2.00	1.04	1.04	0.00
		1.00	177.67	177.67	0.00
	1	0.50	1.10	1.10	0.00
		0.75	3.59	3.59	0.00
		0.90	24.61	24.61	0.00
		1.10	29.18	29.18	0.00
		1.25	4.39	4.39	0.00
		1.50	1.71	1.71	0.00
		2.00	1.03	1.03	0.00
		1.00	205.20	205.20	0.00
	5	0.50	1.29	1.29	0.00
		0.75	4.48	4.48	0.00
		0.90	58.62	58.62	0.00
		1.10	79.52	79.52	0.00
		1.25	5.97	5.97	0.00
		1.50	2.07	2.07	0.00
		2.00	1.08	1.08	0.00
		1.00	1037.81	1037.81	0.00

Table A.32: Likelihood Ratio with Single In-Control Variance, Scale-Out Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			LR-out, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.07	21.08	0.10
		0.75	21.71	23.21	0.12
		0.90	41.40	44.53	0.09
		1.10	41.60	45.60	0.12
		1.25	22.70	24.22	0.11
		1.50	20.21	21.65	0.13
		2.00	19.75	21.03	0.13
		1.00	180.29	199.25	0.10
	1	0.50	19.98	21.10	0.11
		0.75	22.02	23.36	0.11
		0.90	42.00	45.68	0.11
		1.10	44.91	49.40	0.12
		1.25	22.82	24.09	0.09
		1.50	20.63	21.67	0.11
		2.00	19.93	21.03	0.11
		1.00	195.75	212.21	0.08
	5	0.50	21.13	21.26	0.02
		0.75	24.08	24.38	0.02
		0.90	79.82	81.00	0.02
		1.10	84.85	86.40	0.02
		1.25	25.42	25.70	0.02
		1.50	21.77	22.00	0.02
		2.00	20.84	21.08	0.02
		1.00	955.83	979.00	0.02

Table A.33: Likelihood Ratio with Single In-Control Variance, Scale-Out Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			LR-out, $n = 2^8$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0
		0.75	2.01	2.01	0
		0.90	9.95	9.95	0
		1.10	12.09	12.09	0
		1.25	2.52	2.52	0
		1.50	1.17	1.17	0
		2.00	1.00	1.00	0
		1.00	171.84	171.84	0
	1	0.50	1.00	1.00	0
		0.75	2.00	2.00	0
		0.90	10.70	10.70	0
		1.10	12.61	12.61	0
		1.25	2.56	2.56	0
		1.50	1.17	1.17	0
		2.00	1.00	1.00	0
		1.00	192.94	192.94	0
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.34: Likelihood Ratio with Single In-Control Variance, Scale-Out Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			LR-out, $n = 2^8$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	19.77	21.00	0.118
		0.75	20.54	21.96	0.133
		0.90	28.23	30.26	0.107
		1.10	29.54	31.51	0.099
		1.25	21.21	22.46	0.109
		1.50	20.29	21.18	0.092
		2.00	19.87	21.00	0.119
		1.00	166.27	186.23	0.114
	1	0.50	20.02	21.00	0.102
		0.75	20.88	21.92	0.098
		0.90	28.01	30.09	0.11
		1.10	30.30	32.35	0.095
		1.25	21.08	22.49	0.116
		1.50	20.03	21.20	0.111
		2.00	19.95	21.00	0.114
		1.00	178.68	196.01	0.094
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.35: Likelihood Ratio with Single In-Control Variance, Scale-Out Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			LR-out, $n = 2^9$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0
		0.75	1.26	1.26	0
		0.90	5.57	5.57	0
		1.10	6.13	6.13	0
		1.25	1.59	1.59	0
		1.50	1.01	1.01	0
		2.00	1.00	1.00	0
		1.00	169.49	169.49	0
	1	0.50	1.00	1.00	0
		0.75	1.29	1.29	0
		0.90	5.50	5.50	0
		1.10	6.36	6.36	0
		1.25	1.60	1.60	0
		1.50	1.01	1.01	0
		2.00	1.00	1.00	0
		1.00	206.61	206.61	0
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.36: Likelihood Ratio with Single In-Control Variance, Scale-Out Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			LR-out, $n = 2^9$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	19.68	21.00	0.126
		0.75	20.24	21.24	0.105
		0.90	23.21	24.85	0.124
		1.10	24.19	25.70	0.103
		1.25	20.44	21.58	0.111
		1.50	19.86	21.01	0.122
		2.00	19.82	21.00	0.109
		1.00	165.58	185.03	0.112
	1	0.50	20.18	21.00	0.09
		0.75	20.31	21.26	0.091
		0.90	24.00	25.22	0.086
		1.10	25.09	26.17	0.08
		1.25	20.72	21.60	0.082
		1.50	20.02	21.00	0.095
		2.00	20.11	21.00	0.094
		1.00	188.33	212.16	0.119
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.37: Likelihood Ratio with Single In-Control Variance, Scale-Out Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			LR-out, $n = 2^{10}$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0
		0.75	1.01	1.01	0
		0.90	3.12	3.12	0
		1.10	3.39	3.39	0
		1.25	1.10	1.10	0
		1.50	1.00	1.00	0
		2.00	1.00	1.00	0
		1.00	175.73	175.73	0
	1	0.50	1.00	1.00	0
		0.75	1.02	1.02	0
		0.90	3.42	3.42	0
		1.10	3.73	3.73	0
		1.25	1.14	1.14	0
		1.50	1.00	1.00	0
		2.00	1.00	1.00	0
		1.00	283.17	283.17	0
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.38: Likelihood Ratio with Single In-Control Variance, Scale-Out Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			LR-out, $n = 2^{10}$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	19.96	21.00	0.104
		0.75	20.01	21.01	0.107
		0.90	21.68	22.94	0.113
		1.10	22.13	23.23	0.1
		1.25	19.92	21.10	0.109
		1.50	19.96	21.00	0.11
		2.00	19.96	21.00	0.12
		1.00	174.09	191.57	0.097
	1	0.50	20.08	21.00	0.09
		0.75	20.40	21.01	0.068
		0.90	22.57	23.27	0.064
		1.10	22.89	23.59	0.062
		1.25	20.61	21.15	0.059
		1.50	20.44	21.00	0.055
		2.00	20.45	21.00	0.059
		1.00	289.83	313.18	0.077
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.39: Likelihood Ratio with Single In-Control Variance, Scale-In Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			LR-in, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.10	1.10	0.00
		0.75	3.50	3.50	0.00
		0.90	22.33	22.33	0.00
		1.10	24.03	24.03	0.00
		1.25	4.24	4.24	0.00
		1.50	1.71	1.71	0.00
		2.00	1.04	1.04	0.00
		1.00	267.63	267.63	0.00
	1	0.50	1.17	1.17	0.00
		0.75	4.09	4.09	0.00
		0.90	40.96	40.96	0.00
		1.10	19.22	19.22	0.00
		1.25	4.21	4.21	0.00
		1.50	1.68	1.68	0.00
		2.00	1.04	1.04	0.00
		1.00	347.79	347.79	0.00
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.40: Likelihood Ratio with Single In-Control Variance, Scale-In Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			LR-in, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.42	21.09	0.07
		0.75	22.24	23.33	0.09
		0.90	38.44	40.21	0.06
		1.10	41.75	43.86	0.07
		1.25	23.31	24.23	0.07
		1.50	21.06	21.72	0.06
		2.00	20.54	21.04	0.05
		1.00	267.79	284.17	0.06
	1	0.50	20.59	21.12	0.05
		0.75	23.16	23.91	0.06
		0.90	55.52	58.87	0.07
		1.10	37.26	38.67	0.05
		1.25	23.18	24.00	0.06
		1.50	21.13	21.65	0.05
		2.00	20.59	21.04	0.05
		1.00	315.53	330.88	0.05
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.41: Likelihood Ratio with Single In-Control Variance, Scale-In Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			LR-in, $n = 2^8$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00
		0.75	2.08	2.08	0.00
		0.90	10.34	10.34	0.00
		1.10	11.77	11.77	0.00
		1.25	2.63	2.63	0.00
		1.50	1.18	1.18	0.00
		2.00	1.00	1.00	0.00
		1.00	253.60	253.60	0.00
	1	0.50	1.00	1.00	0.00
		0.75	2.17	2.17	0.00
		0.90	13.51	13.51	0.00
		1.10	10.23	10.23	0.00
		1.25	2.53	2.53	0.00
		1.50	1.18	1.18	0.00
		2.00	1.00	1.00	0.00
		1.00	277.41	277.41	0.00
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.42: Likelihood Ratio with Single In-Control Variance, Scale-In Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			LR-in, $n = 2^8$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.36	21.00	0.07
		0.75	21.34	21.98	0.06
		0.90	28.84	30.16	0.07
		1.10	30.12	31.48	0.07
		1.25	21.49	22.46	0.08
		1.50	20.46	21.16	0.07
		2.00	20.45	21.00	0.05
		1.00	257.93	275.30	0.07
	1	0.50	20.29	21.00	0.07
		0.75	21.36	22.12	0.06
		0.90	31.72	33.12	0.06
		1.10	28.51	29.62	0.06
		1.25	21.81	22.40	0.06
		1.50	20.56	21.15	0.06
		2.00	20.46	21.00	0.06
		1.00	270.70	284.68	0.05
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.43: Likelihood Ratio with Single In-Control Variance, Scale-In Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			LR-in, $n = 2^9$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00
		0.75	1.30	1.30	0.00
		0.90	5.47	5.47	0.00
		1.10	6.07	6.07	0.00
		1.25	1.63	1.63	0.00
		1.50	1.01	1.01	0.00
		2.00	1.00	1.00	0.00
		1.00	269.70	269.70	0.00
	1	0.50	1.00	1.00	0.00
		0.75	1.39	1.39	0.00
		0.90	7.20	7.20	0.00
		1.10	5.55	5.55	0.00
		1.25	1.65	1.65	0.00
		1.50	1.01	1.01	0.00
		2.00	1.00	1.00	0.00
		1.00	310.21	310.21	0.00
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.44: Likelihood Ratio with Single In-Control Variance, Scale-In Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			LR-in, $n = 2^9$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.11	21.00	0.08
		0.75	20.60	21.26	0.07
		0.90	24.38	25.22	0.06
		1.10	25.08	25.85	0.05
		1.25	20.74	21.52	0.07
		1.50	20.20	21.01	0.08
		2.00	20.23	21.00	0.08
		1.00	253.17	272.31	0.07
	1	0.50	20.36	21.00	0.07
		0.75	20.71	21.40	0.07
		0.90	26.14	27.02	0.06
		1.10	24.51	25.29	0.05
		1.25	20.95	21.51	0.05
		1.50	20.49	21.01	0.06
		2.00	20.40	21.00	0.06
		1.00	318.50	337.43	0.06
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.45: Likelihood Ratio with Single In-Control Variance, Scale-In Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			LR-in, $n = 2^{10}$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00
		0.75	1.01	1.01	0.00
		0.90	3.24	3.24	0.00
		1.10	3.48	3.48	0.00
		1.25	1.10	1.10	0.00
		1.50	1.00	1.00	0.00
		2.00	1.00	1.00	0.00
		1.00	260.16	260.16	0.00
	1	0.50	1.00	1.00	0.00
		0.75	1.05	1.05	0.00
		0.90	4.64	4.64	0.00
		1.10	3.59	3.59	0.00
		1.25	1.14	1.14	0.00
		1.50	1.00	1.00	0.00
		2.00	1.00	1.00	0.00
		1.00	539.34	539.34	0.00
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.46: Likelihood Ratio with Single In-Control Variance, Scale-In Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			LR-in, $n = 2^{10}$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.26	21.00	0.08
		0.75	20.42	21.01	0.06
		0.90	22.09	22.98	0.07
		1.10	22.32	23.28	0.08
		1.25	20.41	21.10	0.06
		1.50	20.37	21.00	0.07
		2.00	20.39	21.00	0.06
		1.00	249.56	269.27	0.08
	1	0.50	20.65	21.00	0.04
		0.75	20.73	21.05	0.03
		0.90	24.13	24.54	0.04
		1.10	22.84	23.27	0.04
		1.25	20.69	21.14	0.04
		1.50	20.61	21.00	0.04
		2.00	20.67	21.00	0.04
		1.00	532.26	548.35	0.03
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

A.3.2 Multiple In-Control Variances Methods

Table A.47: Likelihood Ratio with Multiple In-Control Variances, Scale-Out Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			LR-out, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.10	1.10	0.00
		0.75	3.78	3.78	0.00
		0.90	90.10	90.10	0.00
		1.10	98.99	98.99	0.00
		1.25	5.44	5.44	0.00
		1.50	1.75	1.75	0.00
		2.00	1.04	1.04	0.00
		1.00	207.67	207.67	0.00
	1	0.50	1.10	1.10	0.00
		0.75	3.76	3.76	0.00
		0.90	82.10	82.10	0.00
		1.10	100.95	100.95	0.00
		1.25	5.57	5.57	0.00
		1.50	1.73	1.73	0.00
		2.00	1.03	1.03	0.00
		1.00	215.04	215.04	0.00
	5	0.50	1.09	1.09	0.00
		0.75	4.03	4.03	0.00
		0.90	104.16	104.16	0.00
		1.10	118.40	118.40	0.00
		1.25	6.62	6.62	0.00
		1.50	1.81	1.81	0.00
		2.00	1.05	1.05	0.00
		1.00	229.51	229.51	0.00

Table A.48: Likelihood Ratio with Multiple In-Control Variances, Scale-Out Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			LR-out, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.40	21.07	0.07
		0.75	22.32	23.39	0.09
		0.90	71.44	76.09	0.07
		1.10	87.41	93.33	0.07
		1.25	23.18	24.42	0.08
		1.50	20.82	21.66	0.08
		2.00	20.26	21.04	0.08
		1.00	204.55	218.62	0.07
	1	0.50	20.10	21.08	0.10
		0.75	22.43	23.45	0.09
		0.90	74.79	80.21	0.08
		1.10	84.56	91.97	0.09
		1.25	23.69	24.57	0.07
		1.50	20.88	21.65	0.08
		2.00	20.29	21.04	0.08
		1.00	203.99	222.89	0.09
	5	0.50	20.55	21.08	0.06
		0.75	22.58	23.53	0.08
		0.90	79.42	84.27	0.07
		1.10	103.65	108.83	0.05
		1.25	23.80	24.74	0.07
		1.50	21.09	21.72	0.06
		2.00	20.37	21.04	0.06
		1.00	243.20	261.75	0.07

Table A.49: Likelihood Ratio with Multiple In-Control Variances, Scale-Out Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			LR-out, $n = 2^8$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00
		0.75	2.10	2.10	0.00
		0.90	31.66	31.66	0.00
		1.10	40.57	40.57	0.00
		1.25	2.74	2.74	0.00
		1.50	1.18	1.18	0.00
		2.00	1.00	1.00	0.00
		1.00	189.36	189.36	0.00
	1	0.50	1.00	1.00	0.00
		0.75	2.09	2.09	0.00
		0.90	33.30	33.30	0.00
		1.10	43.31	43.31	0.00
		1.25	2.70	2.70	0.00
		1.50	1.18	1.18	0.00
		2.00	1.00	1.00	0.00
		1.00	192.35	192.35	0.00
	5	0.50	1.00	1.00	0.00
		0.75	2.23	2.23	0.00
		0.90	50.62	50.62	0.00
		1.10	63.02	63.02	0.00
		1.25	2.84	2.84	0.00
		1.50	1.21	1.21	0.00
		2.00	1.00	1.00	0.00
		1.00	283.90	283.90	0.00

Table A.50: Likelihood Ratio with Multiple In-Control Variances, Scale-Out Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			LR-out, $n = 2^8$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.23	21.00	0.09
		0.75	20.92	21.93	0.09
		0.90	35.00	37.36	0.09
		1.10	37.25	39.77	0.09
		1.25	21.67	22.47	0.07
		1.50	20.38	21.15	0.07
		2.00	20.09	21.00	0.09
		1.00	195.39	210.98	0.08
	1	0.50	20.18	21.00	0.08
		0.75	21.14	21.95	0.08
		0.90	35.39	37.16	0.07
		1.10	41.03	43.16	0.07
		1.25	21.72	22.58	0.07
		1.50	20.31	21.15	0.08
		2.00	20.25	21.00	0.09
		1.00	190.28	208.24	0.09
	5	0.50	20.44	21.00	0.06
		0.75	21.46	22.09	0.06
		0.90	45.24	47.33	0.06
		1.10	52.08	54.80	0.06
		1.25	22.13	22.68	0.05
		1.50	20.80	21.19	0.04
		2.00	20.51	21.00	0.06
		1.00	306.47	322.97	0.05

Table A.51: Likelihood Ratio with Multiple In-Control Variances, Scale-Out Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			LR-out, $n = 2^9$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00
		0.75	1.26	1.26	0.00
		0.90	7.50	7.50	0.00
		1.10	8.47	8.47	0.00
		1.25	1.61	1.61	0.00
		1.50	1.01	1.01	0.00
		2.00	1.00	1.00	0.00
		1.00	185.24	185.24	0.00
	1	0.50	1.00	1.00	0.00
		0.75	1.30	1.30	0.00
		0.90	7.45	7.45	0.00
		1.10	9.94	9.94	0.00
		1.25	1.63	1.63	0.00
		1.50	1.01	1.01	0.00
		2.00	1.00	1.00	0.00
		1.00	214.33	214.33	0.00
	5	0.50	1.00	1.00	0.00
		0.75	1.42	1.42	0.00
		0.90	15.03	15.03	0.00
		1.10	19.90	19.90	0.00
		1.25	1.78	1.78	0.00
		1.50	1.01	1.01	0.00
		2.00	1.00	1.00	0.00
		1.00	462.91	462.91	0.00

Table A.52: Likelihood Ratio with Multiple In-Control Variances, Scale-Out Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			LR-out, $n = 2^9$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.16	21.00	0.09
		0.75	20.42	21.25	0.08
		0.90	24.59	25.86	0.09
		1.10	25.47	26.93	0.09
		1.25	20.72	21.56	0.09
		1.50	20.28	21.01	0.07
		2.00	20.30	21.00	0.07
		1.00	193.83	211.05	0.09
	1	0.50	20.33	21.00	0.07
		0.75	20.59	21.28	0.07
		0.90	24.94	26.10	0.08
		1.10	26.31	27.39	0.07
		1.25	20.74	21.61	0.08
		1.50	20.36	21.00	0.07
		2.00	20.23	21.00	0.08
		1.00	207.92	224.67	0.08
	5	0.50	20.70	21.00	0.03
		0.75	20.98	21.34	0.03
		0.90	27.29	27.83	0.03
		1.10	29.41	30.09	0.04
		1.25	21.32	21.70	0.04
		1.50	20.58	21.02	0.04
		2.00	20.70	21.00	0.03
		1.00	469.91	487.55	0.04

Table A.53: Likelihood Ratio with Multiple In-Control Variances, Scale-Out Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			LR-out, $n = 2^{10}$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00
		0.75	1.01	1.01	0.00
		0.90	3.36	3.36	0.00
		1.10	3.89	3.89	0.00
		1.25	1.10	1.10	0.00
		1.50	1.00	1.00	0.00
		2.00	1.00	1.00	0.00
		1.00	195.42	195.42	0.00
	1	0.50	1.00	1.00	0.00
		0.75	1.01	1.01	0.00
		0.90	3.49	3.49	0.00
		1.10	3.77	3.77	0.00
		1.25	1.10	1.10	0.00
		1.50	1.00	1.00	0.00
		2.00	1.00	1.00	0.00
		1.00	199.66	199.66	0.00
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.54: Likelihood Ratio with Multiple In-Control Variances, Scale-Out Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			LR-out, $n = 2^{10}$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.28	21.00	0.08
		0.75	20.22	21.01	0.08
		0.90	22.27	23.08	0.07
		1.10	22.57	23.54	0.08
		1.25	20.47	21.10	0.07
		1.50	20.19	21.00	0.08
		2.00	20.20	21.00	0.08
		1.00	208.29	220.86	0.06
	1	0.50	20.34	21.00	0.07
		0.75	20.13	21.01	0.08
		0.90	22.27	23.06	0.07
		1.10	22.66	23.42	0.06
		1.25	20.57	21.10	0.06
		1.50	20.23	21.00	0.08
		2.00	20.15	21.00	0.09
		1.00	208.33	225.20	0.08
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.55: Likelihood Ratio with Multiple In-Control Variances, Scale-In Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			LR-in, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.10	1.10	0.00
		0.75	3.33	3.33	0.00
		0.90	22.79	22.79	0.00
		1.10	24.86	24.86	0.00
		1.25	4.31	4.31	0.00
		1.50	1.67	1.67	0.00
		2.00	1.04	1.04	0.00
		1.00	243.50	243.50	0.00
	1	0.50	1.15	1.15	0.00
		0.75	4.00	4.00	0.00
		0.90	44.93	44.93	0.00
		1.10	18.46	18.46	0.00
		1.25	4.14	4.14	0.00
		1.50	1.65	1.65	0.00
		2.00	1.05	1.05	0.00
		1.00	305.61	305.61	0.00
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.56: Likelihood Ratio with Multiple In-Control Variances, Scale-In Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			LR-in, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.36	21.09	0.08
		0.75	22.29	23.24	0.08
		0.90	39.17	41.01	0.06
		1.10	40.35	43.26	0.09
		1.25	23.30	24.17	0.07
		1.50	20.96	21.69	0.07
		2.00	20.10	21.04	0.09
		1.00	240.06	255.75	0.06
	1	0.50	20.59	21.16	0.06
		0.75	23.21	23.86	0.05
		0.90	58.99	61.55	0.05
		1.10	37.28	38.28	0.04
		1.25	22.98	23.77	0.06
		1.50	21.11	21.69	0.05
		2.00	20.42	21.03	0.06
		1.00	309.25	322.61	0.04
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.57: Likelihood Ratio with Multiple In-Control Variances, Scale-In Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			LR-in, $n = 2^8$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00
		0.75	2.07	2.07	0.00
		0.90	10.24	10.24	0.00
		1.10	11.91	11.91	0.00
		1.25	2.58	2.58	0.00
		1.50	1.18	1.18	0.00
		2.00	1.00	1.00	0.00
		1.00	249.40	249.40	0.00
	1	0.50	1.00	1.00	0.00
		0.75	2.22	2.22	0.00
		0.90	13.37	13.37	0.00
		1.10	10.39	10.39	0.00
		1.25	2.57	2.57	0.00
		1.50	1.17	1.17	0.00
		2.00	1.00	1.00	0.00
		1.00	293.97	293.97	0.00
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.58: Likelihood Ratio with Multiple In-Control Variances, Scale-In Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			LR-in, $n = 2^8$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.42	21.00	0.07
		0.75	21.08	21.87	0.07
		0.90	28.76	30.00	0.07
		1.10	29.56	31.06	0.08
		1.25	21.90	22.53	0.06
		1.50	20.30	21.15	0.08
		2.00	20.39	21.00	0.07
		1.00	260.00	279.54	0.07
	1	0.50	20.48	21.00	0.05
		0.75	21.49	22.16	0.06
		0.90	31.53	32.96	0.06
		1.10	28.41	29.69	0.07
		1.25	21.97	22.46	0.05
		1.50	20.58	21.17	0.06
		2.00	20.31	21.00	0.07
		1.00	282.50	299.86	0.06
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.59: Likelihood Ratio with Multiple In-Control Variances, Scale-In Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			LR-in, $n = 2^9$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00
		0.75	1.28	1.28	0.00
		0.90	5.39	5.39	0.00
		1.10	5.96	5.96	0.00
		1.25	1.60	1.60	0.00
		1.50	1.01	1.01	0.00
		2.00	1.00	1.00	0.00
		1.00	243.13	243.13	0.00
	1	0.50	1.00	1.00	0.00
		0.75	1.41	1.41	0.00
		0.90	7.06	7.06	0.00
		1.10	5.39	5.39	0.00
		1.25	1.59	1.59	0.00
		1.50	1.01	1.01	0.00
		2.00	1.00	1.00	0.00
		1.00	286.03	286.03	0.00
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.60: Likelihood Ratio with Multiple In-Control Variances, Scale-In Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			LR-in, $n = 2^9$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.49	21.00	0.06
		0.75	20.55	21.24	0.07
		0.90	24.18	25.27	0.07
		1.10	24.74	25.71	0.06
		1.25	20.85	21.59	0.07
		1.50	20.35	21.01	0.07
		2.00	20.51	21.00	0.06
		1.00	250.44	268.74	0.07
	1	0.50	20.47	21.00	0.05
		0.75	20.70	21.36	0.07
		0.90	25.70	26.53	0.06
		1.10	24.29	25.29	0.07
		1.25	20.91	21.52	0.06
		1.50	20.42	21.00	0.06
		2.00	20.44	21.00	0.06
		1.00	296.25	316.74	0.07
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.61: Likelihood Ratio with Multiple In-Control Variances, Scale-In Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 0$.

			LR-in, $n = 2^{10}$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00
		0.75	1.01	1.01	0.00
		0.90	3.23	3.23	0.00
		1.10	3.39	3.39	0.00
		1.25	1.11	1.11	0.00
		1.50	1.00	1.00	0.00
		2.00	1.00	1.00	0.00
		1.00	248.96	248.96	0.00
	1	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

Table A.62: Likelihood Ratio with Multiple In-Control Variances, Scale-In Technique. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 40$, 1000 Replications, $\tau = 20$.

			LR-in, $n = 2^{10}$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.26	21.00	0.08
		0.75	20.40	21.01	0.06
		0.90	21.97	22.94	0.08
		1.10	22.70	23.40	0.06
		1.25	20.45	21.12	0.07
		1.50	20.36	21.00	0.06
		2.00	20.37	21.00	0.06
		1.00	235.25	249.62	0.06
	1	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*
	5	0.50	*	*	*
		0.75	*	*	*
		0.90	*	*	*
		1.10	*	*	*
		1.25	*	*	*
		1.50	*	*	*
		2.00	*	*	*
		1.00	*	*	*

A.4 Nonparametric Scale Likelihood Ratio Method

A.4.1 Gaussian Error

Table A.63: Nonparametric Scale Likelihood Ratio, Gaussian Method. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $e = 1000$, $q = 2$, 1000 Replications, $\tau = 0$.

			NPSLR-G, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.09	1.09	0.00
		0.75	3.31	3.31	0.00
		0.90	19.08	19.08	0.00
		1.10	24.06	24.06	0.00
		1.25	4.26	4.26	0.00
		1.50	1.65	1.65	0.00
		2.00	1.04	1.04	0.00
		1.00	186.64	186.64	0.00
	1	0.50	1.09	1.09	0.00
		0.75	3.35	3.35	0.00
		0.90	20.50	20.50	0.00
		1.10	22.83	22.83	0.00
		1.25	4.25	4.25	0.00
		1.50	1.69	1.69	0.00
		2.00	1.04	1.04	0.00
		1.00	198.94	198.94	0.00
	5	0.50	1.11	1.11	0.00
		0.75	3.34	3.34	0.00
		0.90	21.65	21.65	0.00
		1.10	25.65	25.65	0.00
		1.25	4.35	4.35	0.00
		1.50	1.70	1.70	0.00
		2.00	1.04	1.04	0.00
		1.00	216.30	216.30	0.00

Table A.64: Nonparametric Scale Likelihood Ratio, Gaussian Method. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $e = 1000$, $q = 2$, 1000 Replications, $\tau = 20$.

			NPSLR-G, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.13	21.10	0.10
		0.75	22.03	23.25	0.10
		0.90	36.67	39.26	0.09
		1.10	39.13	42.22	0.10
		1.25	22.69	23.91	0.10
		1.50	20.62	21.64	0.10
		2.00	19.75	21.02	0.12
		1.00	187.95	209.17	0.11
	1	0.50	19.96	21.08	0.12
		0.75	21.90	23.14	0.11
		0.90	37.28	40.13	0.10
		1.10	40.00	43.17	0.10
		1.25	22.97	24.15	0.09
		1.50	20.56	21.62	0.10
		2.00	20.00	21.02	0.10
		1.00	189.32	208.88	0.10
	5	0.50	20.07	21.11	0.10
		0.75	22.12	23.26	0.09
		0.90	38.34	41.04	0.09
		1.10	42.89	45.73	0.09
		1.25	23.01	24.28	0.09
		1.50	20.61	21.64	0.10
		2.00	20.19	21.04	0.08
		1.00	221.81	244.10	0.10

Table A.65: Nonparametric Scale Likelihood Ratio, Gaussian Method. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $e = 1000$, $q = 2$, 1000 Replications, $\tau = 0$.

			NPSLR-G, $n = 2^8$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00
		0.75	1.99	1.99	0.00
		0.90	9.51	9.51	0.00
		1.10	11.53	11.53	0.00
		1.25	2.46	2.46	0.00
		1.50	1.16	1.16	0.00
		2.00	1.00	1.00	0.00
		1.00	177.04	177.04	0.00
	1	0.50	1.00	1.00	0.00
		0.75	1.96	1.96	0.00
		0.90	10.07	10.07	0.00
		1.10	11.37	11.37	0.00
		1.25	2.46	2.46	0.00
		1.50	1.18	1.18	0.00
		2.00	1.00	1.00	0.00
		1.00	177.44	177.44	0.00
	5	0.50	1.00	1.00	0.00
		0.75	2.01	2.01	0.00
		0.90	9.66	9.66	0.00
		1.10	12.16	12.16	0.00
		1.25	2.55	2.55	0.00
		1.50	1.19	1.19	0.00
		2.00	1.00	1.00	0.00
		1.00	221.42	221.42	0.00

Table A.66: Nonparametric Scale Likelihood Ratio, Gaussian Method. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $e = 1000$, $q = 2$, 1000 Replications, $\tau = 20$.

			NPSLR-G, $n = 2^8$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	19.87	21.00	0.12
		0.75	20.77	21.92	0.11
		0.90	27.41	29.44	0.11
		1.10	28.02	30.35	0.12
		1.25	21.07	22.39	0.12
		1.50	20.11	21.14	0.10
		2.00	19.80	21.00	0.13
		1.00	178.07	197.21	0.10
	1	0.50	19.86	21.00	0.12
		0.75	20.76	21.94	0.11
		0.90	27.32	29.44	0.11
		1.10	28.53	30.48	0.10
		1.25	21.47	22.50	0.09
		1.50	20.13	21.14	0.10
		2.00	19.91	21.00	0.12
		1.00	182.85	201.48	0.10
	5	0.50	20.18	21.00	0.09
		0.75	21.02	21.97	0.09
		0.90	28.01	29.56	0.08
		1.10	29.63	31.70	0.10
		1.25	21.54	22.46	0.08
		1.50	20.40	21.17	0.09
		2.00	20.19	21.00	0.08
		1.00	210.82	234.03	0.10

Table A.67: Nonparametric Scale Likelihood Ratio, Gaussian Method. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $e = 1000$, $q = 2$, 1000 Replications, $\tau = 0$.

			NPSLR-G, $n = 2^9$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00
		0.75	1.28	1.28	0.00
		0.90	5.23	5.23	0.00
		1.10	6.11	6.11	0.00
		1.25	1.55	1.55	0.00
		1.50	1.01	1.01	0.00
		2.00	1.00	1.00	0.00
		1.00	169.86	169.86	0.00
	1	0.50	1.00	1.00	0.00
		0.75	1.28	1.28	0.00
		0.90	5.16	5.16	0.00
		1.10	6.04	6.04	0.00
		1.25	1.57	1.57	0.00
		1.50	1.01	1.01	0.00
		2.00	1.00	1.00	0.00
		1.00	190.24	190.24	0.00
	5	0.50	1.00	1.00	0.00
		0.75	1.28	1.28	0.00
		0.90	5.59	5.59	0.00
		1.10	6.10	6.10	0.00
		1.25	1.61	1.61	0.00
		1.50	1.01	1.01	0.00
		2.00	1.00	1.00	0.00
		1.00	211.33	211.33	0.00

Table A.68: Nonparametric Scale Likelihood Ratio, Gaussian Method. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $e = 1000$, $q = 2$, 1000 Replications, $\tau = 20$.

			NPSLR-G, $n = 2^9$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	19.91	21.00	0.12
		0.75	20.48	21.26	0.09
		0.90	23.13	24.74	0.12
		1.10	24.16	25.58	0.10
		1.25	20.43	21.49	0.11
		1.50	19.85	21.01	0.12
		2.00	20.07	21.00	0.10
		1.00	174.60	194.85	0.11
	1	0.50	19.89	21.00	0.11
		0.75	20.17	21.26	0.11
		0.90	23.63	25.12	0.11
		1.10	24.03	25.66	0.12
		1.25	20.48	21.52	0.10
		1.50	19.93	21.01	0.11
		2.00	20.04	21.00	0.10
		1.00	180.01	201.75	0.11
	5	0.50	20.25	21.00	0.08
		0.75	20.41	21.27	0.09
		0.90	23.99	25.05	0.08
		1.10	24.41	25.79	0.09
		1.25	20.69	21.55	0.09
		1.50	20.11	21.01	0.09
		2.00	20.27	21.00	0.08
		1.00	225.72	245.85	0.09

Table A.69: Nonparametric Scale Likelihood Ratio, Gaussian Method. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $e = 1000$, $q = 2$, 1000 Replications, $\tau = 0$.

			NPSLR-G, $n = 2^{10}$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00
		0.75	1.01	1.01	0.00
		0.90	3.03	3.03	0.00
		1.10	3.28	3.28	0.00
		1.25	1.09	1.09	0.00
		1.50	1.00	1.00	0.00
		2.00	1.00	1.00	0.00
		1.00	174.58	174.58	0.00
	1	0.50	1.00	1.00	0.00
		0.75	1.01	1.01	0.00
		0.90	3.05	3.05	0.00
		1.10	3.23	3.23	0.00
		1.25	1.10	1.10	0.00
		1.50	1.00	1.00	0.00
		2.00	1.00	1.00	0.00
		1.00	173.10	173.10	0.00
	5	0.50	1.00	1.00	0.00
		0.75	1.01	1.01	0.00
		0.90	3.08	3.08	0.00
		1.10	3.34	3.34	0.00
		1.25	1.12	1.12	0.00
		1.50	1.00	1.00	0.00
		2.00	1.00	1.00	0.00
		1.00	226.98	226.98	0.00

Table A.70: Nonparametric Scale Likelihood Ratio, Gaussian Method. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $e = 1000$, $q = 2$, 1000 Replications, $\tau = 20$.

			NPSLR-G, $n = 2^{10}$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	19.84	21.00	0.11
		0.75	19.91	21.01	0.10
		0.90	21.60	22.88	0.11
		1.10	21.74	23.20	0.11
		1.25	20.09	21.10	0.11
		1.50	20.10	21.00	0.10
		2.00	19.91	21.00	0.11
		1.00	180.29	198.51	0.10
	1	0.50	19.98	21.00	0.10
		0.75	19.84	21.01	0.12
		0.90	21.51	22.85	0.11
		1.10	22.00	23.16	0.10
		1.25	20.09	21.09	0.10
		1.50	19.95	21.00	0.12
		2.00	19.85	21.00	0.12
		1.00	186.50	203.57	0.09
	5	0.50	20.28	21.00	0.08
		0.75	19.89	21.00	0.12
		0.90	22.00	23.00	0.08
		1.10	22.23	23.32	0.09
		1.25	20.12	21.11	0.10
		1.50	20.04	21.00	0.10
		2.00	20.31	21.00	0.08
		1.00	229.68	248.68	0.08

A.4.2 Non-Gaussian, $T(3)$ Error

Table A.71: Nonparametric Scale Likelihood Ratio, Non-Gaussian Method. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $e = 1000$, $q = 2$, 1000 Replications, $\tau = 0$.

			NPLR-T, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.14	1.14	0.00
		0.75	3.73	3.73	0.00
		0.90	23.02	23.02	0.00
		1.10	26.27	26.27	0.00
		1.25	4.65	4.65	0.00
		1.50	1.81	1.81	0.00
		2.00	1.06	1.06	0.00
		1.00	174.05	174.05	0.00
	1	0.50	1.15	1.15	0.00
		0.75	3.68	3.68	0.00
		0.90	21.01	21.01	0.00
		1.10	26.51	26.51	0.00
		1.25	5.06	5.06	0.00
		1.50	1.85	1.85	0.00
		2.00	1.06	1.06	0.00
		1.00	199.16	199.16	0.00
	5	0.50	1.18	1.18	0.00
		0.75	3.72	3.72	0.00
		0.90	25.27	25.27	0.00
		1.10	30.18	30.18	0.00
		1.25	4.93	4.93	0.00
		1.50	1.91	1.91	0.00
		2.00	1.06	1.06	0.00
		1.00	228.19	228.19	0.00

Table A.72: Nonparametric Scale Likelihood Ratio, Non-Gaussian Method. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $e = 1000$, $q = 2$, 1000 Replications, $\tau = 20$.

			NPSLR-T, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.19	21.14	0.10
		0.75	22.16	23.44	0.11
		0.90	38.67	42.26	0.12
		1.10	43.40	47.20	0.11
		1.25	23.12	24.59	0.12
		1.50	20.65	21.77	0.11
		2.00	20.20	21.06	0.09
		1.00	177.50	197.46	0.11
	1	0.50	20.00	21.15	0.11
		0.75	22.28	23.40	0.09
		0.90	39.74	43.70	0.12
		1.10	43.05	46.54	0.10
		1.25	23.25	24.52	0.10
		1.50	20.94	21.79	0.09
		2.00	20.03	21.07	0.11
		1.00	192.42	212.33	0.10
	5	0.50	20.29	21.15	0.09
		0.75	22.41	23.46	0.09
		0.90	40.81	44.01	0.10
		1.10	46.93	50.54	0.09
		1.25	23.73	24.92	0.09
		1.50	20.82	21.89	0.10
		2.00	19.99	21.07	0.10
		1.00	211.42	232.22	0.09

Table A.73: Nonparametric Scale Likelihood Ratio, Non-Gaussian Method. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $e = 1000$, $q = 2$, 1000 Replications, $\tau = 0$.

			NPLR-T, $n = 2^8$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00
		0.75	2.21	2.21	0.00
		0.90	11.42	11.42	0.00
		1.10	12.13	12.13	0.00
		1.25	2.76	2.76	0.00
		1.50	1.20	1.20	0.00
		2.00	1.00	1.00	0.00
		1.00	173.85	173.85	0.00
	1	0.50	1.00	1.00	0.00
		0.75	2.17	2.17	0.00
		0.90	10.87	10.87	0.00
		1.10	13.04	13.04	0.00
		1.25	2.76	2.76	0.00
		1.50	1.24	1.24	0.00
		2.00	1.00	1.00	0.00
		1.00	189.04	189.04	0.00
	5	0.50	1.00	1.00	0.00
		0.75	2.25	2.25	0.00
		0.90	11.81	11.81	0.00
		1.10	15.21	15.21	0.00
		1.25	2.87	2.87	0.00
		1.50	1.27	1.27	0.00
		2.00	1.00	1.00	0.00
		1.00	203.91	203.91	0.00

Table A.74: Nonparametric Scale Likelihood Ratio, Non-Gaussian Method. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $e = 1000$, $q = 2$, 1000 Replications, $\tau = 20$.

			NPSLR-T, $n = 2^8$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.03	21.00	0.10
		0.75	20.93	22.10	0.11
		0.90	28.92	30.71	0.10
		1.10	29.83	31.88	0.09
		1.25	21.34	22.70	0.12
		1.50	20.29	21.23	0.10
		2.00	19.80	21.00	0.12
		1.00	188.20	211.34	0.12
	1	0.50	19.83	21.00	0.12
		0.75	21.07	22.10	0.10
		0.90	29.08	31.12	0.11
		1.10	30.61	32.41	0.09
		1.25	21.70	22.76	0.09
		1.50	20.12	21.21	0.11
		2.00	20.04	21.00	0.10
		1.00	189.50	209.01	0.10
	5	0.50	20.01	21.00	0.10
		0.75	21.21	22.12	0.08
		0.90	29.52	31.20	0.08
		1.10	31.29	33.18	0.09
		1.25	21.59	22.75	0.09
		1.50	20.24	21.24	0.10
		2.00	20.28	21.00	0.08
		1.00	204.15	226.01	0.10

Table A.75: Nonparametric Scale Likelihood Ratio, Non-Gaussian Method. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $e = 1000$, $q = 2$, 1000 Replications, $\tau = 0$.

			NPLR-T, $n = 2^9$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00
		0.75	1.35	1.35	0.00
		0.90	5.60	5.60	0.00
		1.10	6.39	6.39	0.00
		1.25	1.68	1.68	0.00
		1.50	1.02	1.02	0.00
		2.00	1.00	1.00	0.00
		1.00	158.27	158.27	0.00
	1	0.50	1.00	1.00	0.00
		0.75	1.36	1.36	0.00
		0.90	5.94	5.94	0.00
		1.10	6.70	6.70	0.00
		1.25	1.73	1.73	0.00
		1.50	1.01	1.01	0.00
		2.00	1.00	1.00	0.00
		1.00	204.63	204.63	0.00
	5	0.50	1.00	1.00	0.00
		0.75	1.39	1.39	0.00
		0.90	5.98	5.98	0.00
		1.10	7.02	7.02	0.00
		1.25	1.76	1.76	0.00
		1.50	1.01	1.01	0.00
		2.00	1.00	1.00	0.00
		1.00	228.03	228.03	0.00

Table A.76: Nonparametric Scale Likelihood Ratio, Non-Gaussian Method. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $e = 1000$, $q = 2$, 1000 Replications, $\tau = 20$.

			NPSLR-T, $n = 2^9$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	19.83	21.00	0.11
		0.75	20.03	21.32	0.12
		0.90	23.77	25.40	0.12
		1.10	24.39	26.00	0.11
		1.25	20.76	21.65	0.10
		1.50	20.15	21.01	0.09
		2.00	19.95	21.00	0.10
		1.00	173.01	191.39	0.10
	1	0.50	19.96	21.00	0.10
		0.75	20.47	21.34	0.09
		0.90	24.17	25.53	0.10
		1.10	24.75	26.27	0.10
		1.25	20.63	21.65	0.10
		1.50	19.97	21.02	0.10
		2.00	19.90	21.00	0.11
		1.00	196.75	211.32	0.07
	5	0.50	20.32	21.00	0.07
		0.75	20.54	21.36	0.08
		0.90	24.54	25.81	0.08
		1.10	25.07	26.37	0.09
		1.25	20.81	21.71	0.09
		1.50	20.29	21.02	0.08
		2.00	20.16	21.00	0.09
		1.00	209.74	227.26	0.08

Table A.77: Nonparametric Scale Likelihood Ratio, Non-Gaussian Method. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $e = 1000$, $q = 2$, 1000 Replications, $\tau = 0$.

			NPLR-T, $n = 2^{10}$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00
		0.75	1.02	1.02	0.00
		0.90	3.29	3.29	0.00
		1.10	3.69	3.69	0.00
		1.25	1.17	1.17	0.00
		1.50	1.00	1.00	0.00
		2.00	1.00	1.00	0.00
		1.00	188.46	188.46	0.00
	1	0.50	1.00	1.00	0.00
		0.75	1.02	1.02	0.00
		0.90	3.31	3.31	0.00
		1.10	3.70	3.70	0.00
		1.25	1.17	1.17	0.00
		1.50	1.00	1.00	0.00
		2.00	1.00	1.00	0.00
		1.00	179.85	179.85	0.00
	5	0.50	1.00	1.00	0.00
		0.75	1.03	1.03	0.00
		0.90	3.44	3.44	0.00
		1.10	3.74	3.74	0.00
		1.25	1.18	1.18	0.00
		1.50	1.00	1.00	0.00
		2.00	1.00	1.00	0.00
		1.00	205.16	205.16	0.00

Table A.78: Nonparametric Scale Likelihood Ratio, Non-Gaussian Method. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $e = 1000$, $q = 2$, 1000 Replications, $\tau = 20$.

			NPSLR-T, $n = 2^{10}$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.06	21.00	0.10
		0.75	20.02	21.01	0.11
		0.90	21.66	23.08	0.12
		1.10	22.28	23.56	0.10
		1.25	20.15	21.13	0.10
		1.50	20.15	21.00	0.09
		2.00	20.07	21.00	0.10
		1.00	172.69	191.60	0.11
	1	0.50	19.97	21.00	0.11
		0.75	19.94	21.03	0.11
		0.90	22.00	23.21	0.10
		1.10	22.21	23.52	0.11
		1.25	20.13	21.13	0.10
		1.50	20.12	21.00	0.09
		2.00	20.23	21.00	0.08
		1.00	185.47	204.34	0.10
	5	0.50	20.06	21.00	0.10
		0.75	20.04	21.03	0.10
		0.90	22.18	23.32	0.10
		1.10	22.71	23.78	0.09
		1.25	20.28	21.16	0.10
		1.50	19.99	21.00	0.11
		2.00	19.96	21.00	0.10
		1.00	218.08	240.15	0.10

A.5 Nonparametric Likelihood Ratio Method

A.5.1 Log-Likelihood Method

Table A.79: Nonparametric Log-Likelihood Ratio, Gaussian UCL, Gaussian ARLs. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $q = 2$, 1000 Replications, $\tau = 0$.

			LL-GG, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00
		0.75	1.98	1.98	0.00
		0.90	8.19	8.19	0.00
		1.10	125.31	125.31	0.00
		1.25	4.81	4.81	0.00
		1.50	1.38	1.38	0.00
		2.00	1.00	1.00	0.00
		1.00	162.53	162.53	0.00
	1	0.50	1.01	1.01	0.00
		0.75	2.40	2.40	0.00
		0.90	8.91	8.91	0.00
		1.10	82.54	82.54	0.00
		1.25	5.42	5.42	0.00
		1.50	1.63	1.63	0.00
		2.00	1.01	1.01	0.00
		1.00	101.81	101.81	0.00
	5	0.50	1.00	1.00	0.00
		0.75	2.47	2.47	0.00
		0.90	9.59	9.59	0.00
		1.10	110.38	110.38	0.00
		1.25	6.34	6.34	0.00
		1.50	1.69	1.69	0.00
		2.00	1.01	1.01	0.00
		1.00	131.01	131.01	0.00

Table A.80: Nonparametric Log-Likelihood Ratio, Gaussian UCL, Gaussian ARLs. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, $q = 2$, 1000 Replications, $\tau = 20$.

			LL-GG, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	19.83	21.00	0.13
		0.75	20.47	21.90	0.14
		0.90	25.47	27.35	0.12
		1.10	122.02	136.55	0.12
		1.25	23.17	24.69	0.12
		1.50	20.20	21.40	0.12
		2.00	19.99	21.00	0.11
		1.00	168.21	190.42	0.12
	1	0.50	19.48	21.00	0.17
		0.75	20.54	22.21	0.17
		0.90	25.41	27.78	0.15
		1.10	88.87	102.84	0.15
		1.25	23.29	25.42	0.16
		1.50	20.05	21.58	0.15
		2.00	19.72	21.00	0.14
		1.00	101.53	119.05	0.16
	5	0.50	19.66	21.00	0.15
		0.75	21.02	22.30	0.13
		0.90	26.25	28.62	0.14
		1.10	116.11	131.90	0.13
		1.25	24.05	25.86	0.13
		1.50	20.53	21.67	0.12
		2.00	19.65	21.00	0.16
		1.00	125.88	144.38	0.14

Table A.81: Nonparametric Log-Likelihood Ratio, Gaussian UCL, Gaussian ARLs. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 250$, $q = 2$ 1000 Replications, $\tau = 0$.

			LL-GG, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	1.00	1.00	0.00
		0.75	2.01	2.01	0.00
		0.90	9.40	9.40	0.00
		1.10	59.81	59.81	0.00
		1.25	3.84	3.84	0.00
		1.50	1.27	1.27	0.00
		2.00	1.00	1.00	0.00
		1.00	175.80	175.80	0.00
	1	0.50	1.00	1.00	0.00
		0.75	2.49	2.49	0.00
		0.90	10.76	10.76	0.00
		1.10	56.70	56.70	0.00
		1.25	4.68	4.68	0.00
		1.50	1.51	1.51	0.00
		2.00	1.00	1.00	0.00
		1.00	141.60	141.60	0.00
	5	0.50	1.00	1.00	0.00
		0.75	2.60	2.60	0.00
		0.90	11.41	11.41	0.00
		1.10	80.16	80.16	0.00
		1.25	5.15	5.15	0.00
		1.50	1.55	1.55	0.00
		2.00	1.00	1.00	0.00
		1.00	177.10	177.10	0.00

Table A.82: Nonparametric Log-Likelihood Ratio, Gaussian UCL, Gaussian ARLs. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 250$, $q = 2$, 1000 Replications, $\tau = 20$.

			LL-GG, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	19.75	21.00	0.12
		0.75	20.91	21.91	0.10
		0.90	26.82	28.63	0.10
		1.10	72.24	80.48	0.12
		1.25	22.41	23.72	0.11
		1.50	20.22	21.27	0.11
		2.00	19.87	21.00	0.12
		1.00	172.50	193.65	0.12
	1	0.50	19.77	21.00	0.13
		0.75	21.19	22.29	0.11
		0.90	27.33	29.10	0.11
		1.10	71.03	78.74	0.12
		1.25	23.03	24.59	0.12
		1.50	20.32	21.46	0.11
		2.00	19.74	21.00	0.13
		1.00	142.40	159.44	0.12
	5	0.50	20.07	21.00	0.10
		0.75	21.48	22.45	0.09
		0.90	28.90	31.15	0.11
		1.10	87.86	94.85	0.08
		1.25	23.84	25.04	0.09
		1.50	20.57	21.59	0.10
		2.00	19.94	21.00	0.12
		1.00	179.75	197.50	0.10

A.5.2 Kolmogorov-Smirnov Method

Table A.83: Nonparametric KS Method, Gaussian UCL, Gaussian ARLs. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, 1000 Replications, $\tau = 0$.

			KS-GG, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	2.17	2.17	0.00
		0.75	72.31	72.31	0.00
		0.90	190.50	190.50	0.00
		1.10	106.47	106.47	0.00
		1.25	40.12	40.12	0.00
		1.50	8.44	8.44	0.00
		2.00	1.64	1.64	0.00
		1.00	185.34	185.34	0.00
	1	0.50	2.24	2.24	0.00
		0.75	72.20	72.20	0.00
		0.90	180.88	180.88	0.00
		1.10	111.84	111.84	0.00
		1.25	40.57	40.57	0.00
		1.50	8.55	8.55	0.00
		2.00	1.68	1.68	0.00
		1.00	174.08	174.08	0.00
	5	0.50	2.72	2.72	0.00
		0.75	75.62	75.62	0.00
		0.90	187.36	187.36	0.00
		1.10	114.44	114.44	0.00
		1.25	44.06	44.06	0.00
		1.50	9.41	9.41	0.00
		2.00	1.88	1.88	0.00
		1.00	184.16	184.16	0.00

Table A.84: Nonparametric KS Method, Gaussian UCL, Gaussian ARLs. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, 1000 Replications, $\tau = 20$.

			KS-GG, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.86	22.11	0.11
		0.75	81.08	91.28	0.12
		0.90	185.05	201.39	0.09
		1.10	124.51	140.18	0.12
		1.25	52.52	57.04	0.10
		1.50	26.42	28.65	0.12
		2.00	20.24	21.64	0.13
		1.00	177.05	199.52	0.12
	1	0.50	20.85	22.22	0.11
		0.75	81.42	90.57	0.11
		0.90	188.99	211.56	0.11
		1.10	114.70	127.18	0.11
		1.25	51.69	55.99	0.10
		1.50	26.54	28.59	0.11
		2.00	20.40	21.68	0.11
		1.00	170.46	191.95	0.12
	5	0.50	21.47	22.81	0.10
		0.75	89.74	96.58	0.08
		0.90	187.92	212.70	0.12
		1.10	122.54	133.85	0.09
		1.25	57.00	62.69	0.11
		1.50	27.56	29.48	0.10
		2.00	20.73	21.89	0.10
		1.00	188.83	210.75	0.11

Table A.85: Nonparametric KS Method, Gaussian UCL, T ARLs. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, 1000 Replications, $\tau = 0$.

			KS-GT, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	2.19	2.19	0.00
		0.75	71.04	71.04	0.00
		0.90	183.59	183.59	0.00
		1.10	100.90	100.90	0.00
		1.25	38.27	38.27	0.00
		1.50	8.49	8.49	0.00
		2.00	1.63	1.63	0.00
		1.00	181.20	181.20	0.00
	1	0.50	2.27	2.27	0.00
		0.75	77.46	77.46	0.00
		0.90	192.22	192.22	0.00
		1.10	110.65	110.65	0.00
		1.25	42.58	42.58	0.00
		1.50	8.83	8.83	0.00
		2.00	1.66	1.66	0.00
		1.00	192.53	192.53	0.00
	5	0.50	2.64	2.64	0.00
		0.75	75.19	75.19	0.00
		0.90	189.46	189.46	0.00
		1.10	115.72	115.72	0.00
		1.25	41.98	41.98	0.00
		1.50	9.99	9.99	0.00
		2.00	1.87	1.87	0.00
		1.00	174.81	174.81	0.00

Table A.86: Nonparametric KS Method, Gaussian UCL, T ARLs. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, 1000 Replications, $\tau = 20$.

			KS-GT, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	22.03	23.38	0.12
		0.75	90.49	99.73	0.10
		0.90	200.86	218.41	0.08
		1.10	120.17	131.88	0.10
		1.25	57.54	63.66	0.12
		1.50	29.03	31.39	0.11
		2.00	20.78	22.13	0.11
		1.00	175.41	194.58	0.10
	1	0.50	21.92	23.26	0.11
		0.75	86.50	95.82	0.11
		0.90	188.43	209.13	0.10
		1.10	120.20	130.88	0.09
		1.25	62.62	68.28	0.10
		1.50	28.70	31.26	0.12
		2.00	21.04	22.15	0.09
		1.00	167.82	188.10	0.11
	5	0.50	22.35	23.94	0.12
		0.75	97.74	109.03	0.12
		0.90	188.07	208.34	0.10
		1.10	121.22	130.90	0.08
		1.25	64.34	70.45	0.10
		1.50	31.18	33.12	0.09
		2.00	21.28	22.52	0.10
		1.00	181.34	202.55	0.11

Table A.87: Nonparametric KS Method, T UCL, Gaussian ARLs. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, 1000 Replications, $\tau = 0$.

			KS-TG, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	2.19	2.19	0.00
		0.75	71.04	71.04	0.00
		0.90	183.59	183.59	0.00
		1.10	100.90	100.90	0.00
		1.25	38.27	38.27	0.00
		1.50	8.49	8.49	0.00
		2.00	1.63	1.63	0.00
		1.00	181.20	181.20	0.00
	1	0.50	2.27	2.27	0.00
		0.75	77.46	77.46	0.00
		0.90	192.22	192.22	0.00
		1.10	110.65	110.65	0.00
		1.25	42.58	42.58	0.00
		1.50	8.83	8.83	0.00
		2.00	1.66	1.66	0.00
		1.00	192.53	192.53	0.00
	5	0.50	2.64	2.64	0.00
		0.75	75.19	75.19	0.00
		0.90	189.46	189.46	0.00
		1.10	115.72	115.72	0.00
		1.25	41.98	41.98	0.00
		1.50	9.99	9.99	0.00
		2.00	1.87	1.87	0.00
		1.00	174.81	174.81	0.00

Table A.88: Nonparametric KS Method, T UCL, Gaussian ARLs. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, 1000 Replications, $\tau = 20$.

			KS-TG, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	20.86	22.07	0.10
		0.75	85.04	92.86	0.09
		0.90	189.72	209.71	0.10
		1.10	118.42	131.98	0.11
		1.25	55.13	60.65	0.11
		1.50	26.52	28.36	0.11
		2.00	20.33	21.61	0.12
		1.00	173.97	195.71	0.12
	1	0.50	21.21	22.42	0.10
		0.75	86.49	93.75	0.09
		0.90	206.11	228.35	0.10
		1.10	127.40	140.01	0.10
		1.25	53.87	59.39	0.11
		1.50	27.13	28.70	0.08
		2.00	20.70	21.73	0.09
		1.00	191.28	214.02	0.11
	5	0.50	21.20	22.60	0.12
		0.75	91.99	100.03	0.09
		0.90	188.51	207.20	0.10
		1.10	121.81	132.91	0.09
		1.25	58.72	63.50	0.09
		1.50	27.35	29.21	0.10
		2.00	20.61	21.86	0.11
		1.00	176.68	194.79	0.10

Table A.89: Nonparametric KS Method, T UCL, T ARLs. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, 1000 Replications, $\tau = 0$.

			KS-TT, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
0	0	0.50	3.11	3.11	0.00
		0.75	78.71	78.71	0.00
		0.90	184.53	184.53	0.00
		1.10	117.52	117.52	0.00
		1.25	46.25	46.25	0.00
		1.50	11.27	11.27	0.00
		2.00	2.12	2.12	0.00
		1.00	175.65	175.65	0.00
	1	0.50	3.36	3.36	0.00
		0.75	80.91	80.91	0.00
		0.90	192.79	192.79	0.00
		1.10	121.06	121.06	0.00
		1.25	46.86	46.86	0.00
		1.50	11.86	11.86	0.00
		2.00	2.31	2.31	0.00
		1.00	180.54	180.54	0.00
	5	0.50	3.95	3.95	0.00
		0.75	82.79	82.79	0.00
		0.90	195.63	195.63	0.00
		1.10	117.72	117.72	0.00
		1.25	48.56	48.56	0.00
		1.50	12.04	12.04	0.00
		2.00	2.47	2.47	0.00
		1.00	173.47	173.47	0.00

Table A.90: Nonparametric KS Method, T UCL, T ARLs. Structural Component $3\sigma\sqrt{2\log(n)}$, $w = 10$, $m = 100$, 1000 Replications, $\tau = 20$.

			KS-TT, $n = 2^7$		
τ	p	σ_{oc}	ARL	ARL-1	PFA
20	0	0.50	21.95	23.09	0.09
		0.75	88.66	98.05	0.11
		0.90	187.35	209.72	0.11
		1.10	122.92	136.09	0.11
		1.25	60.11	64.66	0.09
		1.50	28.97	31.14	0.11
		2.00	21.00	22.20	0.10
		1.00	178.04	197.01	0.10
	1	0.50	22.17	23.34	0.09
		0.75	89.50	100.29	0.12
		0.90	193.40	215.46	0.11
		1.10	120.02	135.65	0.13
		1.25	59.24	65.40	0.11
		1.50	28.67	31.20	0.13
		2.00	21.11	22.28	0.10
		1.00	193.39	216.10	0.11
	5	0.50	22.64	23.99	0.11
		0.75	90.08	99.43	0.11
		0.90	185.40	209.75	0.12
		1.10	123.92	136.12	0.10
		1.25	63.92	69.75	0.10
		1.50	30.27	33.04	0.12
		2.00	21.34	22.50	0.10
		1.00	184.33	203.85	0.10

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BIOGRAPHICAL SKETCH

Vladimir was born in Boston, MA on June 1st, 1987 to loving and stubborn physicians Chantal & Jacques Geneus. At a young age, his family moved to Haiti where he attended middle school and high school at "L'Institution Saint-Louis de Gonzague". Vladimir graduated from the University of Massachusetts, Amherst with a Bachelor of Science in Mathematics in 2009. He then graduated from Northeastern University with a Master of Science in Applied Mathematics in 2011. In 2014 Vladimir graduated from the Florida State University with a Master of Science in Statistics. He defended his dissertation in 2017. Vladimir enjoys making music with his siblings and watching his favorite Boston Sports teams win championships.