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FLORIDA STATE UNIVERSITY

THE LANGUAGE OF MATHEMATICS

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INTRODUCTION

In the catalog issue of the Bulletin of Florida State University, to be published sometime during 1950, will appear a challenging innovation: the General Education course in mathematics, titled Mathematics 105, will be listed under the general area of Communication through Language. This, so far as the writer is able to ascertain, will be the first time a course in mathematics has been so listed in any university catalog.

It will be the purpose of this paper to examine some aspects of the historical development of mathematics to justify such a classification and to explore some of the implications of such an approach for the teaching of mathematics.

CHAPTER I

PRE-RENAISSANCE

The need for quantitative expression and communununication would seem to be nearly as ancient as the need for organized communication of any kind. Recent studies indicate that "... the rudiments of mathematics appear first in close connection with language and language forms; and sometimes it is difficult to discern them clearly."[1] Be that as it may, mathematics has not usually been thought of as a language but rather as a structure having a twofold nature: on the one hand practical, in its role as a tool for dealing with quantitative and spatial relationships, and on the other speculative, in its role as a pure, creative art of deduction. This dichotomy was born with mathematics, and is thriving today as evidenced by the division of the subject into "pure" and "applied" mathematics.[2]

In the beginning, the speculative side of mathematics appeared as magic.[3] As long ago as the Stone Age, religion had magical numbers such as three, four, and seven, and magical figures such as the Pantalpha and the Swastika.[4]

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[1] W.H. Werkmeister, _A Philosophy of Science_, (New York: Harper and Brothers, 1940), p. 140.

[2] E.T. Bell, _The Magic of Numbers_, (New York: Whittlesey House, 1946), p. 30.

[3] Vera Sanford, _A Short History of Mathematics_, (Boston: Houghton Mifflin Company, 1930), p.1.

[4] Dirk J. Struik, _A Concise History of Mathematics_, (New York: Dover Publications, 1948), I, 9.

But practical mathematics has always been prosaically practical, its oldest example being a tally stick for counting which dates back to paleolithic times, and which was found in 1937 in Moravia.[5]

Apparently the earliest organized mathematics was the work of the Babylonians and the Egyptians.[6] It is not clear just which was first, if either. Until recently it was generally thought that the Moscow Papyrus (ca. 1850 B.C.) and the Rhind Papyrus (ca. 1650 B.C.) were the oldest examples of systematized mathematics extant.[7] But Professor Archibald makes mention of the Babylonian Tablet, Plimpton 322 (ca. 1900 - 1600 B.C.) recently translated by Professor Neugebauer.[8] The interesting thing is that all of these, while largely practical, also indicated an interest in speculative mathematics in the form of recreational problems.[9]

In China where philosophic speculation was early venerated, speculative mathematics flowered with much interest centering on magic squares.[10] Legend has it that the magic square was discovered by the Emperor Yu (ca.2200 B.C.), who found it on the back of a tortoise near the Yellow River.[11]

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5 Ibid.

6 The term "Babylonian" is here used to include all the various peoples of the Mesopotamian Valley.

7 Florian Cajori, A History of Mathematics, (New York: The Macmillan Company, 1901), p.11.

8 Raymond Clare Archibald, "Outline of the History of Mathematics," The American Mathematical Monthly, LVI (1949), 9.

9 Sanford, op. cit., p.2.

10 Ibid. p. 4.

11 Ibid. p. 74.

In the utilitarian West, however, the bulk of mathematics remained a practical means of calculating building and commercial needs until the advent of the Greeks. As we have seen, there was some interest in recreational problems prior to the Greeks, but the change in emphasis wrought by the Hellenics was so marked that one scholar exclaims: "Both logic and an interest in pure knowledge were introductions of the utterly novel genius of Classical Greece."[12] It behooves us, then, to look next at these remarkable Greeks.

Most studies in the history of philosophy begin with one of the "Seven Wise Men," Thales of Miletus.[13] So also does Greek Mathematics begin with Thales. He it was who introduced the study of geometry into Greece.[14] A successful merchant, whose wealth gave him leisure for travel and study, Thales observed with great interest the geometry of Egypt. This geometry was largely a result of building needs[15] and Thales, too, applied his geometry in practical ways at first. We are told that he was able to calculate the height of a pyramid by measuring its shadow.[16] But his philosophical nature soon asserted itself , and Thales became concerned with the nature of logical proof. He interested himself in the

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[12] F. Sherwood Taylor, The March of Mind, (New York: The Macmillan Company, 1939), p. 14.

[13] Cf., Milton C. Nahm (ed.), Selections from Early Greek Philosophy, (New York: F.S. Crofts and Company, 1947), or any standard history of Philosophy.

[14] Cajori, op. cit., p. 17.

[15] Alfred Hooper, Makers of Mathematics, (New York: Random House, 1948), p. 28.

[16] Ibid., p. 37.

abstract and more general, declaring that they were worthier of deep study than the intuitive or sensible. [17] Thales it was who began the study of demonstrative geometry[18] and who was the first known individual with whom definite mathematical discoveries are associated.[19]

Once initiated, the study of deductive, or "pure," mathematics moved rapidly to the forefront of Greek Mathematics. Practical, "applied," mathematics was relegated to the counting house and a distinction was made between Arithmetic, which dealt with absolute, abstract numbers, and Logistic, which dealt with ordinary arithmetical operations.[20] This distinction reached full bloom with Pythagoras. One fellow Greek, Aristoxenus, exclaimed about the arithmetic which Pythagoras "... advanced and took out of the region of commercial utility."[21] The appraisal was indeed just, for with Pythagoras the study of numbers became no less than a religion.[22] It is conjectured that the discovery by Pythagoras that musical harmonies depended on numerical ratios was the source of his mysticism regarding numbers.[23] Whatever the source, the fact is beyond dispute.[24] Pythagoras and his followers taught that all things were

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17 H.W. Turnbull, The Great Mathematicians, (London: Methuen and Company, 1929), p. 5.

18 Sanford, op. cit., p. 6.

19 Archibald, op. cit., p. 18.

20 Ibid., p. 19.

21 Cited by Sir Thomas Heath, A Manual of Greek Mathematics, (Oxford: The Claredon Press, 1931), p. 37.

22 For a complete account of the Pythagorean religion see Bell, op. cit., p. 37.

23 Heath, op. cit., p. 37.

24 Cf., any history of philosophy.

numbers, even the soul of a man.[25] Do we not today, when we think we have discovered the nature of an individual, say that we "have his number?" This mystical emphasis initiated by Pythagoras has never been laid to rest. Only recently a renowned scientist, Sir James Jeans, exclaimed: "The Great Architect of the Universe now begins to appear as a pure mathematician."[26] Before leaving the Pythagoreans, it is well to note that in spite of their complete absorption with the mystical aspect of numbers, they inadvertently made many contributions to practical mathematics such as the famous Pythagorean Theorem.

Plato, we are told, was much influenced by the Pythagorean conception of number,[27] but with less religious emphasis. Instead of worshiping numbers as such, he sought to make use of the abstract concepts of perfect numbers and perfect geometrical forms to prove the existence of abstract ideals in general. Thus Plato has Socrates say to Glaucon:

> You are aware that students of geometry, arithmetic, and the kindred sciences assume the odd and the even and the figures and three kinds of angles and the like in their several branches of science; these are their hypotheses, which they and everybody are supposed to know, and therefore they do not deign to give any account of them either to themselves or others; but they begin with them, and go on until they arrive at last, and in a consistent manner, at their conclusion? ... and do you not know also that

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25 Paul J. Glenn, <u>The History of Philosophy</u>, (St. Louis: B. Herder Book Company, 1945), p. 45.

26 Cited by E.T. Bell, <u>The Queen of the Sciences</u>, (Baltimore: The Williams and Wilkins Company, 1931), p. 20.

27 Bell, <u>The Magic of Numbers</u>, p. 164.

> although they make use of the visible forms and reason about them, they are thinking not of these, but of the ideals which they resemble; not of the figures which they draw, but of the absolute square and the absolute diameter, and so on - the forms which they draw or make, ... are converted by them into images, but they are really seeking to behold the things themselves, which can only be seen with the eye of the mind?[28]

Because mathematics illustrated the ideals so well, Plato considered training in mathematics essential to the study of philosophy. It is reported that the door of his school bore the admonition: "Let no one destitute of geometry enter."[29] Conversely, Plato had little or no use for practical mathematics. Plutarch said that Plato blamed those who tried to reduce the duplication of the cube to constructions by means of mechanical instruments, "on the ground that the good of geometry is thereby lost and destroyed, as it is thus made to revert to things of sense instead of being directed upward and grasping at eternal and incorporeal images."[30]

Not all Greeks felt so strongly about the degrading influence of applied mathematics as did Plato, however. Although Euclid's most famous treatise was his one on pure geometry called the Elements, it is well to remember that of the five complete Euclidean texts available, three are on applied mathematics, namely on phaenomena, optics, and music.[31] And Archimedes, one of the greatest of all mathematicians, worked equally in pure and applied mathematics, using them to reinforce each other.[32]

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28 Plato, The Republic, trans. Benjamin Jowett (New York: Charles Scribner's Sons, 1928), p. 270.

29 Heath, op. cit., p. 171.

30 Cited by Heath, op. cit., p. 72.

31 Archibald, op. cit., p. 21.

32 Heath, op. cit, pp. 277-288.

The Romans were as practical minded in their mathematics as they were in everything else and had little use for the spedulative side of Greek Mathematics. "In fact," Professor Sanford observes, "Cicero spoke disparagingly of the Greek interest in geometry, congratulating his countrymen because they were concerned only with the mathematics that is needed in measuring and in reckoning." [33] Apparently some interest in pure mathematics is necessary for the advance of that subject, for the Roman added nothing to the theory of mathematics.[34] The only Roman one finds in the various histories of mathematics is Boethius, and he only because he summarized the mathematics then extant.[35]

With the fall of the Roman Empire all of Europe entered an intellectual decline. The recorded advances in mathematics were wholly the contributions of the Hindu, and later the Arab and Persian, mathematicians. They seemed to have some access to the work of the Greeks[36] and continued the emphasis on pure mathematics. Their most notable contribution was, of course, the indispensible number notation which we call Hindu-Arabic numerals.[37] With the decline of the Moslem Empire, mathematics stagnated to such an extent that the "state of mathematics in Europe in 1490 was not substantially better than in Islam 500 years before."[38]

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33 Sanford, op. cit., p. 13.
34 Ibid., p. 13
35 Cajori, op. cit, pp. 81-83.
36 Sanford, op. cit., p. 14
37 H.C. Trimble, F.C. Bolser, and T.L. Wade, Basic Mathematics for General Education, (New York: Prentice-Hall, 1950), pp. 12-17.
38 Taylor, op. cit., p. 103.

CHAPTER II

THE RENAISSANCE AND AFTER

The Renaissance was felt in mathematics as in every other phase of intellectual life in the sixteenth century. A ground swell of mathematical creative activity arose that reached the proportions of a tidal wave during the two succeeding centuries. In 1533 was published the first European systematic exposition of plane and spherical trigonometry, which introduced the sine and cosine functions.[1] Trigonometry was here divorced from astronomy and thus given a "pure" aspect. "Applied" mathematics was not to be slighted, however, and made its bid with the publication in 1491 of the first account of double-entry bookkeeping.[2] A detailed account of the multitudinous advances of the sixteenth century is irrelevant to our purpose. We may note that Francois Vieta contributed a great advance to algebraic symbolism,[3] and remark that the pure and applied fields of endeavor continued to advance approximately equal claims upon men's attention.

The seventeenth century was so replete with outstanding developments in mathematics that mention must be limited to the barest outline of some of the more notable

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1 Archibald, op. cit., p. 33.
2 Ibid., p. 33.
3 Sanford, op. cit., p. 38.

contributions. Fermat laid the foundations for modern number theory, the purest of pure mathematics, containing many Pythagorean overtones. Pascal and Desargues opened new fields for pure geometry. Descartes wedded geometry and algebra. Huygens advanced the theory of probability. Newton and Leibniz gave us that standard sophomore study, the calculus, and Napier shortened computation with logarithms.[4] Pure and applied mathematics were both investigated with feverish intensity, more than ever before boosting and supplementing each other without losing their respective identities.

Especially significant for us were the works of Descartes and Newton, for these two effected what has since been called the Cartesian revolution and the Newtonian world-machine.[5] They were really the _alpha_ and _omega_ of a single stirring revolution in thought, for what Descartes initiated, Newton developed into a complete mechanical interpretation of the world in exact, mathematical, deductive terms.[6] Others contributed greatly, of course, but it was Descartes who set up the hypothesis, and Newton who, in his great _Mathematical Principles of Natural Philosophy_, formulated and completed the work.[7] Newton himself made two outstanding discoveries: he found the mathematical method that

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[4] Struik, _op. cit._, II, 125-160.

[5] Eugene G. Bewkes and others, _Experience, Reason and Faith_ (New York: Harper and Brothers, 1940), pp.450-456.

[6] John Herman Randall, Jr., _The Making of the Modern Mind_ (Boston: Houghton Mifflin Company, 1940), pp. 253-279.

[7] _Ibid_., p. 260.

would describe mechanical motion, and he applied it universally.[8]

That Newton invented the calculus is perhaps an accident; Leibniz, building on Descartes' analytic geometry, arrived at it independently;[9] while several other mathematicians, Pascal for one, seemed almost on the verge of it. Be that as it may, it was inevitable that after the Frenchman had brought algebra and geometry together, men should advance and apply algebra also to motion. Descartes had shown how to find the equation that would represent any curve,[10] and thus conveniently and accurately measure it and enable calculated prediction to be applied to all figures; but the science of mechanics, and with it any measurement of the processes of change in the world, demands a formula for the law of the growth or falling-off of a curve, that is, the direction of its movement at any point. Such a method of measuring movement and continuous growth Newton discovered. He had arrived at the most potent instrument yet found for bringing the world into subjection to man. Since any regular motion, be it of a falling body, an electric current, or the cooling of a molten mass, can be represented by a curve, he had found the means by which to attack not only the figures, but the processes of nature - the last link in the

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8 Ibid., p. 260.

9 David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1923), I, 418.

10 Ibid., P. 375.

mathematical interpretation of the world.

This mathematical approach to nature had an overwhelming success and proved to be a tremendous stimulus to both pure and applied mathematics - but something new had been added.

Hitherto, as we have seen, pure mathematics had been undertaken for faith and for fun, while applied mathematics found its raison d'etre largely in the counting-house and in building and surveying needs. Now men were explaining the universe itself in mathematical terms - and it worked! Consequently, thinkers who previously had had no interest in, or need for, the pure or functional study of mathematics, discovered that if they would understand other men's explanations of the physical universe they must understand its descriptive mathematical terms.

Nor was this "mathematizing" confined to the physical universe. Field after field of study fell under the influence of the mathematical approach with varying success, until even such a remote discipline as ethics was given mathematical form by Spinoza.[11]

Once fully under way, this quantitative approach to things was never deterred and man's communicative need for mathematics became ever more apparent. By 1904, Mr. H.G.Wells, in his Mankind in the Making, was led to declare:

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[11] Benedict de Spinoza, The Philosophy of Benedict de Spinoza, trans. R.H.M. Elwes (New York: Tudor Publishing Company, 1936), pp. 39-277.

> The new mathematics is a sort of supplement to language, affording a means of thought about form and quantity and a means of expression, more exact, compact, and ready than ordinary language. The great body of physical science, a great deal of essential facts of financial science, and endless social and political problems are only accessible and only thinkable to those who have a sound training in mathematical analysis, and the time may not be very remote when it will be understood that for complete initiation as an efficient citizen of one of the new great complex world-wide states that are now developing, it is as necessary to be able to compute, to think in averages, and maxima and minima, as it is now to be able to read and write.[12]

While Mr. Wells was making this prophecy, the concept of mathematics as a language was being bolstered from another direction. As the scientists grew more successful in describing the world in which we live in terms of mathematics, the mathematicians were making new discoveries about the nature of mathematics itself. During the first half of the nineteenth century, Lobatchewsky published a startling geometry that denied one of Euclid's postulates and yet remained consistent.[13] Mathematics tottered upon its foundations, but soon recovered and began to rebuild on firmer ground. Other men trod the newly-blazed paths until by the time of the publication of such works as Hilbert's Grundlagen der Geometrie,(1899), and

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[12] H.G. Wells, Mankind in the Making, (New York: Charles Scribner's Sons, 1904), pp. 191-192.

[13] E.T. Bell, Men of Mathematics, (New York: Simon and Schuster, 1937), pp. 294-306.

the <u>Principia Mathematica</u>,(1910), of Whitehead and Russell, the definition of mathematics had undergone considerable revision. No longer was mathematics to be defined simply as the science of quantity and space. Instead, such a mathematical philosopher as Professor Katsoff lists no less than eleven definitions.[14] He introduces the problem by stating that "it [mathematics] cannot be uniquely defined by stating its structure or its field of application alone."[15] He concludes by saying, "It is almost possible to say that mathematics is a language and that its elements (groups of symbols) are the 'words' of the language. It would then appear to be possible to analyze mathematics linguistically."[16] Possible indeed, for such an analysis was published in 1932.[17]

Dr. Katsoff reservedly states that the concept of mathematics as a language is "almost possible," but others were more definite. Let us cite two, one a scientist, the other a linguist. In <u>Mathematics for the Million</u>, Lancelot Hogben writes: "Mathematics is a language in which we describe the sizes of things in contrast to the ordinary languages which we use to describe the sorts of things in the world."[18] Mr. Hogben goes so far as to devote an entire

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14 Louis O. Katsoff, <u>A Philosophy of Mathematics</u> (Ames: Iowa State College Press, 1948), pp. 9-13.

15 <u>Ibid</u>., p. 15.

16 <u>Ibid</u>., p. 16.

17 A.F. Bentley, <u>A Linguistic Analysis of Mathematics</u> (Bloomington: Principia Press, 1932).

18 Lancelot Hogben, <u>Mathematics for the Million</u> (New York: W.W. Norton and Company, 1937), p. 13.

chapter to "The Grammar of Size, Order and Number."[19] M.M. Lewis, a linguist, says: "... logic and metaphysics and even mathematics are in essence social structures fundamentally linguistic in nature."[20]

In 1867 a leading linguist, W.D. Whitney wrote: "The essential characteristic of our speech is that it is arbitrary and conventional ...."[21] Now in all of the definitions assembled by Dr. Katsoff there is one quality that finds no disagreement, namely that mathematics is arbitrary and conventional. The study of modern mathematics was thus lending philosophical support to the functional concept of mathematics as a language.

However, in 1904, when H.G. Wells postulated the need of mathematics as a necessary tool for communication and understanding, it was still possible to keep up with the deverlpment of science by other means. For by the use of models, analogies, or verbal descriptions, it was possible to "translate," as it were, these new discoveries from the mathematical language of science into the tongue that all could understand. And there were many who were thus engaged in "popularizing" the achievements of science. Something further was needed to establish a widespread need for this newly recognized language, and it was not long in

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19 Ibid., pp. 69-110.

20 M.M. Lewis, Language in Society (New York: Social Science Publishers, 1948), p. 239.

21 W.D. Whitney, Language and the Study of Language, cited by Lewis, op. cit., p. 237.

coming.

Mr. Wells made his prophecy in 1904. In 1900 Professor Max Planck suggested that energy was atomic or granular in character,[22] and the new quantum physics was born. In 1905 Einstein formulated his special theory of relativity.[23] With the development of these new approaches to nature, physics became increasingly dependent on mathematics for description and understanding, and translations from the language of mathematics to our ordinary language became ever more difficult. It became virtually impossible by 1925, for it was then that Heisenberg, in seeking to resolve some of the difficulties of quantum physics, decided that much of the trouble so far encountered was the result of assuming too simple a model for the atom. He tried a new philosophical approach, which met with great success. What is important for us is that Heisenberg discarded all models, pictures, and parables. He limited himself strictly to observational numerical data, with the consequence that his results were inevitably mathematical in form.[24] No longer could one rely upon such analogies for an accurate description of man's observations of his universe.

Nor was science alone in its increasing dependence upon mathematical language. This same first quarter of the twentieth century saw an ever greater use of mathematics in

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22 Randall, op. cit., p. 473.
23 Ibid., p. 475.
24 Sir James Jeans, Physics and Philosophy (New York: The Macmillan Company, 1944), p. 155.

many fields. Statistics flourished, the intelligence quotient came into prominence, poll-taking became a science, and at least one notable psychologist became so aware of the psychological use of multiple factor analysis that he wrote an important book on the subject.[25] That medium of the day-by-day report upon the state of the world, the press, is relying more and more on graphs, tables, business curves, moving averages, etc.. Truly, the time when the efficient citizen must "... be able to compute, to think in averages, and maxima and minima...," in other words to read mathematics, is rapidly approaching.

That this need is gaining wide recognition is evidenced by the large number of mathematical books, both texts and popular works, which devote at least a chapter to the language of mathematics. Alfred Korzybski has labored mightily to convince the world that if we wish to understand the world and ourselves, it follows that we should use a language whose structure corresponds to physical structure, and that we possess the model for such a language in mathematics.[26] Perhaps most convincing is this glowing tribute from the pen of Stuart Chase:

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25 Louis Leon Thurstone, <u>The Vectors of Mind</u> (Chicago: University of Chicago Press, 1935).

26 Alfred Korzybski, <u>Science and Sanity</u> (new York: International Non-Aristotelian Library Publishing Company, 1941).

> There is however one language which is capable of expressing the structural relation found in the known world and in the nervous system. It is used with equal facility by a Japanese, a Russian, a Chilean, or an American. The name of this useful, well ordered language is mathematics. I dislike testimonials, but honesty seems to demand them in this subject, and here is a testimonial on mathematics. Convinced by Korzybski that an understanding of mathematics improves communication, I bought a little book, Calculus Made Easy, by S.P. Thompson, and set to work. In a few days of hard sweating I brushed up what higher mathematics I had learned in the Massachusetts Institute of Technology.... Then I returned to Korzybski's account of Einstein. For the first time in my life, and in wild excitement, I caught a genuine glimmer of the meaning of "relativity." It was not a matter of words; it was an inner meaning. I think it is safe to say that no language but mathematics could have given me this light.[27]

Clearly, the approach that mathematics is a language is a justifiable one. It is demanded by practical considerations and well supported by philosophical speculation. It is not too much to say that for many people, just as for Mr. Stuart Chase, the language function may be the most useful and interesting role that mathematics can play.

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27 Stuart Chase, The Tyranny of Words (New York: Harcourt, Brace and Company, 1938), p. 81.

CHAPTER III

IMPLICATIONS FOR TEACHING

One of the chief difficulties confronting the teacher of mathematics is the problem of stimulating interest. Hitherto this problem has seemed to have two principal answers. There are always some students who delight in the problems and processes of mathematics for their own sake. From this group come our "pure" mathematicians. They represent but a small minority of all students, however. There is a larger group which interests itself in the practical or "applied" aspect of mathematics. Future engineers, statisticians, physical and social scientists, etc., recognize their need of mathematics.

But there remains a very great number of students, perhaps a majority, who have neither a liking for mathematics <u>per se</u>, nor any interest in the practical uses of computational skills. These people would, in the main, fall into that group primarily interested in literature, art, religion, history, etc.. With many exceptions, of course, it has been a very challenging task to interest such students in mathematics. If we can convey to this latter group the fact that mathematics is a language, "...not only the simplest and most easily understood of any, but the shortest also,"[1] and that "for some

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[1]H.L. Brougham, cited by R.E. Moritz, <u>Memorablia Mathematica</u> (New York: The Macmillan Company, 1914),p. 194.

of the more complicated aspects of nature, mathematics provides the only key; for every day activities in the Power Age, it provides a very useful aid to clear thinking,"[2] and we shall have added a very useful tool for the task of stimulating interest.

The words of ordinary language first came into use as nouns. A simple word represented some concrete object. As the language grew older, there developed words to represent actions and more abstract ideas. Philologists tell us that the antiquity of a language can usually be measured by the number of words representing high-order abstractions. This is also an indication of the complexity of the civilization which fostered the language. Paradoxically, as a language grows more abstract it gains in preciseness. There are more words to express fine and subtle differences, nuances of meaning.

Similarly mathematics has developed a large group of terms having the same two general characteristics - ever more abstract and ever less ambiguous. Why is this important to the teacher, and ultimately to the student, of mathematics? Because:

> Nothing is more impressive that the fact that as mathematics withdrew increasingly into the upper regions of ever greater extremes of abstract thought, it returned back to earth with a corresponding growth of importance for the analysis of concrete fact....The paradox is now fully established that the utmost abstractions are the true weapons to control our thought of concrete fact.[3]

It is not proposed that the teacher attempt to convince all students of the desirability of pursuing mathematics

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[2] Chase, <u>op. cit.</u>, p. 142.

[3] Alfred North Whitehead, <u>Science and the Modern World</u> (New York: The New American Library, 1948), p. 34.

into the "upper regions" mentioned here. However, armed with an understanding of this function of mathematics, the teacher should be able to convince almost any student that "...ability to handle a little algebra and geometry, to plot a few simple graphs - is worth having. It helps to solve many problems of cummunication and learning."[4] Indeed it does! As we have seen, mathematics is in fact rapidly becoming the <u>only</u> means we have of communicating ideas about the most vital and interesting facts concerning our very existence - the age-old questions of why, how, what, when, and where. Especially since the advent of the quantum and relativity, actual mathematical symbols, terms and equations have become our whole stock of words by which we can express our findings about the nature of things. We might say that the symbols and terms are our nouns, the operations our verbs, and the equations our sentences.

Still, one might ask how all that can be related to a classroom situation. The answer may be found in the fact that as our dependence upon mathematical concepts and notations for describing the nature of the physical and social world grows, our literature, our periodicals, and our daily press make increasing use of the language of mathematics. The teacher can readily find many examples with which he can create a pupil awareness of the need for an ability to read mathematics.

The writer has, for instance, successfully used <u>Human Destiny</u>, a very popular book, to introduce the study of scientific notation. Usually a majority of the class will have

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[4] Chase, <u>op. cit.</u>, p. 142.

read, attempted to read, or heard of this book. The rest can be quickly interested in a vital point made by the book, which is the chance, or probability, that life could have started by accident on this planet. Curiosity concerning this point is practically universal, and at the proper peak of interest, the writer presents the book's answer which is 2.02×10^{-321}[5]. Here is an illustration of scientific notation which reaches and stimulates a very high percentage of the students. With an explanation that a short study of scientific notation will enable one to read understandingly many such terms, the study is launched advantageously.

The everyday reading world grows daily less reticent about using mathematical language such as in the illustration offered above. Recently Harper's, a magazine supposed to be on the high school reading level,[6] carried a series of three articles entitled "The Universe and Dr. Einstein."[7] In these articles the author, Lincoln Barnett, labored hard to avoid any criticism of being too mathematical in language. Yet in a paragraph concerning the Lorentz transformation we read:

> Suppose, for example, that a system, or reference body, is moving in a certain direction, then according to the old principle of the addition of velocities, a distance or length x', measured with respect to a relatively stationary system, by the equation $X' = X \pm VT$, where V is the velocity of the moving system and T is the time....Here are the equations of the Lorentz

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[5] Lecomte du Nouy, Human Destiny (New York: Longmans, Green and Company, 1947), p.34.

[6] Harper's Magazine, CXCVI (1948), 393.

[7] Lincoln Barnett, "The Universe and Dr. Einstein," Harper's Magazine, CXCVI (1948), 303-312, 465-476, 529-539.

transformation which have supplanted the older and evidently inadequate relationship cited above:[8]

$$X' = \frac{X - VT}{\sqrt{1 - \left(\frac{V^2}{C^2}\right)}}$$

Admittedly, the average person today doesn't have to know these terms, and doesn't have to read Harper's or Human Destiny. But whenever certain terms are necessary to explain or describe something, interest in the terms used is bound to increase in direct ratio with interest in the thing described. And people are generally interested, in varying degrees, in the nature of the world and the life on it. They are, after all, part of that life. Tell them that they are composed of electrons and they will want to know more. Tell them an electron can best be described as a field of probability and they will want to understand more about probability.

When Albert Einstein predicted the curvature of light, and Lise Meitner suggested the possibility of splitting the atom, via pure mathematics, they performed feats of great public interest. This is reflected by the popularity of such books as One-Two-Three-Infinity, Mathematics for the Million, etc., and by the appearance of authors like Alfred North Whitehead in the popular paper bound pocket-books. The widely read news magazine, Time, recently carried a lead article on

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[8] Barnett, "The Universe and Dr. Einstein," op.cit., p. p. 472.

Dr. Robert Oppenheimer and his work in the Institute For Advanced Studies at Princeton with no apologies to the ordinary reader.[9]

Surely this adds up to the fact that the teacher of mathematics has been provided with another tool with which to stimulate interest. As the use of mathematics as a language becomes more widespread, it will seep through in mounting quantity to newspapers, advertisements, science fiction, and the like. Here are the aids that may, with discriminating selection, be used even on the lower levels. From an advertisement appearing in Collier's we take these words: "The Hurricane has a compression ratio of 7.4 to 1... ."[10] America's youngsters have a great deal of interest in anything pertaining to motors, so such advertisements as the one mentioned provide easily accessible aids for provoking interest in the language of mathematics.

There remains to be considered the question of whether provoking interest in mathematics as a language is kin to arousing interest in schoolbook mathematics. In a general way the answer is yes. Any language whatsoever cannot escape a certain amount of syntax which must be mastered to some extent if the language is to become a useful tool, and mathematics is no exception. Pure mathematics may be likened to pure grammar, and usually if there is an interest in a language there is some interest in its grammar. If, as seems highly possible, the capable teacher can evoke interest in

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[9] Time, LII (1948), 70-72.
[10] Collier's, (April 29, 1950), p.44.

mathematics as a language, there will be a corollary interest, to some degree, in schoolbook mathematics.

We may sum up then by saying that the teaching implications in approaching mathematics as a language are that since it can be demonstrated that mathematics is a necessary language, with a high factor of use and interest, we have here a new implement for motivating the study of mathematics and for reaching greater numbers of students.

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