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The Energetics of Centrifugal Instability

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COLLEGE OF ARTS AND SCIENCES

THE ENERGETICS OF CENTRIFUGAL INSTABILITY

By

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ABSTRACT

Centrifugal instability (CI) has been suggested by recent numerical studies to be a boundary mechanism promoting mixing. As opposed to Kelvin-Helmholtz instability, however, little is known about its detailed energetics. The goal of this study is to address this issue. Through the use of MITgcm, a two-dimensional meridional jet with regions of negative PV simulating California Undercurrent (CUC) was studied dynamically and energetically. We observe that the system is dominated by CI and the mixing efficiency is relatively high compared to classic Kelvin-Helmholtz instability. Also, with short time scales of instability, CI is able to generate gravity wave effectively. Estimates of regional and global impact of CI are performed, suggesting while its global impact is limited, it can contribute importantly to the local mixing climatology.

CHAPTER 1

INTRODUCTION

An important question in modern physical oceanography is how the ocean is mixed. It has been recognized since Munk (1966) that the ocean is turbulent as opposed to laminar, which implies that fluid instabilities are central to the mixing and that it occurs at rates that cannot be sustained by molecular processes alone. The accepted view of mixing is that it is mainly due to Kelvin-Helmholtz instability and occurs throughout the open ocean. A separate view is that ocean boundaries are preferred locations of enhanced mixing whose products are subsequently advected into the ocean interior. The former mechanism is undoubtedly present in the ocean but some boundary processes have been recently discovered that look as if they might participate importantly in ocean mixing. The objective of this study is to examine the energetics of one of those processes as a step towards evaluating its local and global importance.

1.1 General Background

Interest in mixing comes from a desire to understand the Meridional Overturning Circulation (MOC). The connection between these two very different phenomena (mixing on the scale of a few centimeters and the MOC whose scale is global) is in the existence of a stably stratified abyssal ocean. Buoyancy is more likely added at lower latitudes and at the air-sea interface. Mixing is responsible for conducting this buoyancy downward into the deeper ocean, again in the low latitudes. Along geopotentials, then, densities gradients occur that through gravitational mechanics tends to drive an equatorial upwelling and high latitude downwelling. The accompanying horizontal advection drives a buoyant flux that balances the ocean global heat budget. Fluctuations in the MOC are thought to be central to multi-decadal to centennial climate variability. Small scale mixing is thus of interest to global climate.

As suggested by the above, it will be convenient to quantitatively discuss mixing in terms of energetics. Buoyant fluids are mixed downwards and heavy fluids upwards in the scenario, and this requires energy. The current state of knowledge of ocean energetics has recently been reviewed by Ferrari and Wunsch (2010) that is in turn an update of the earlier reviews of Ferrari and Wunsch (2008) and Wunsch and Ferrari (2004).

Two relevant facts emerge in those papers about the energetics requirements of mixing. First, an estimate can be made primarily from observations about how much energy is needed to maintain the observed stratification. There are a couple of independent ways to arrive at this number, and interestingly they both yield comparable values of 0.4TW (Munk and Wunsch, 1998; St. Laurent and Simmons, 2004). Second, this requirement can be approximately accounted for from the known mechanical drivers of the ocean.

According to the budget provided in Ferrari and Wunsch (2010), the primary power sources driving the oceans are the winds and the tides. Solar radiation is also important to oceanic structure, but as argued in the above articles, its ability to directly drive the ocean is limited. Admittedly this point is in debate (see Hughes, et al, 2006), but for our purposes, we assume radiative forcing is of secondary importance.

1.1.1 Tides

The net input from tidal forcing, particularly lunar tidal forcing, is relatively well known from a variety of astronomical measurements. Approximately 3Tera Watts ($3TW=3 \times 10^{12}W$) enters the ocean in the form of the barotropic tide, i.e. an essentially depth independent flow occurring at astronomically determined frequencies. Much of this energy (perhaps 2/3) is dissipated in the shallow, coastal oceans and does not play a role in determining the open ocean abyssal stratification. Approximately 1TW is thought to exit the barotropic tide through interaction with bottom topography. Some of these interactions result in mixing local to the topography (about 30%), and the remainder is thought to scatter into the baroclinic tide, i.e. into internal gravity waves occurring at the tidal frequencies. Bell (1976) is an early examination of this process; more recent assessments can be found in Llywellen-Smith and Young (2008). Observational evidence in support of this scenario was provided by Egbert and Ray (2001).

1.1.2 Winds

Net wind input to the ocean is much more in question. Winds over the ocean drive a lot of phenomena, from transient surface gravity waves to Langmuir circulations to Ekman flows to wind driven turbulence. None of these phenomena are well understood and hence the power provided by the winds to the ocean is poorly constrained. Several 10's of TW are no doubt involved, most of which is dissipated in the upper few hundred meters, and hence is irrelevant to abyssal stratification. Some of the energy in the higher frequencies, however, is known to penetrate into the deeper ocean in the form of near inertial oscillations. There is considerable observational verification of this.

Details aside, it is thought the downward near inertial energy flux is something like 1TW, but this is a very uncertain number. The low frequency winds manifest dominantly in Ekman flows that generate the large-scale sea surface pressure heads associated with the oceanic general circulation. The energy stored in the general circulation is enormous (on the order of $20 \times 10^{24}W$), but the power provided to it by the winds is relatively modest, at 1TW. Most of this 1TW enters in the southern ocean in the region of the Antarctic Circumpolar Current.

The net mechanical forcing of the ocean thus reduces to approximately 3TW, with 2TW directly generating the internal gravity wave field through baroclinic tidal generation and near inertial wave radiation and 1TW entering the large scale circulation. These inputs, to internal waves and large-scale flows, follow very different subsequent histories.

1.1.3 Internal Waves

Once in the internal wave field, the accepted idea is that a variety of mechanisms (e.g., induced diffusion, resonant triad interactions) transfer the energy to smaller scales and higher frequencies. Stratification allows for the existence of internal gravity waves, but the accumulation of energy at small scales eventually develops shears sufficiently strong that the flow can become unstable. The criterion governing this event is the Richardson number, $Ri = (N^2/u_z^2)$, and from idealized calculations, if Ri drops below 1/4, the flow becomes unstable. The form of the instability is growing undulations in density surfaces. At finite amplitudes, these undulations roll up and the resulting gravitationally unstable configuration can drive irreversible mixing and turbulence. The mixed product at the end of the overturning is a hybrid of the deeper denser waters and the shallower lighter waters on either side of the unstable density interface. From an energetics perspective, this is equivalent to an increase in the potential energy of the fluid through an elevation of the center of mass. Thus of the total energy cascading down to small scales and generating the mixing, some is dissipated into heat via viscous processes and the remainder into potential energy. The ratio of these two energetic fates, potential energy creation divided by viscous dissipation, is referred to as the efficiency of the mixing.

For Kelvin-Helmholtz overturning, there is now solid evidence that the efficiency is approximately 0.2 (Peltier and Caulfield, 2003). If all the energy entering the internal wave field from the winds and tides were to develop Kelvin-Helmholtz overturnings, potential energy would be created at the rate of approximately

$$Dissipation + Potential Energy = 6 \times (Potential Energy) = 2TW$$

0.3TW. This number is quite similar to that inferred as that needed to explain the abyssal stratification, although the errors on both the net source and net requirement are large. There is reasonable observational support for this chain of events, even if the global numbers are still uncertain.

1.1.4 Large Scale Flow

A less complete understanding is present for the 1TW entering the large-scale circulation. It is believed the mean circulations have been relatively stable in amplitude for at least a few hundreds of years, so roughly 1TW must be exiting the mean flow. The primary means by which this occurs is via geostrophic, baroclinic instabilities, the end result of which is the global mesoscale eddy field. This step in the global energetic budget is well studied and supported by several decades of observation, numerical study and theoretical study.

Assuming all of the 1TW mentioned above enters the mesoscale, the fate of this part is somewhat unclear. Perhaps the most defensible statement is that the mesoscale does not appear to be accelerating, so it must be drained of roughly 1TW. A major problem here is that mesoscale phenomena, characterized by spatial scales of the deformation radius (several 10s of km) and temporal scales long compared to a day, conserve both energy and potential vorticity. These constraints lead the turbulent evolution of the mesoscale to a so-called inverse cascade, namely the energy moves back to large scales rather than small scales. Direct dissipation of energy at these scales is completely negligible, implying the mesoscale must find another path to energy loss.

The possibilities are that the mesoscale is unstable to gravity wave perturbations or loses energy via dissipative interactions with the boundaries. Direct loss to gravity waves has been studied extensively, with the overall result that the growth rates of these instabilities do not appear to be sufficiently strong to be the primary sink of mesoscale energy (but see Molemaker, et al., 2010). While spontaneous generation of gravity waves by the mesoscale is a possible means by which energy exits the mesoscale, observational and theoretical evidence supporting it is weak.

Dissipative interactions with the boundaries, which include the ocean surface, are a separate possibility for draining the mesoscale that holds some promise. Turbulent, and thus dissipative, boundary layers naturally develop against lateral and bottom boundaries when perturbed by flow. A recent attempt to assess this loss using bulk formulae and current meter data has been conducted by Arbic et al. (2010). The result was that perhaps 20% of the needed dissipation (0.2TW) could occur by this mechanism. Also, D'Asaro, et al., (2011) reported some very interesting observations in the Kuroshio in which it appeared some of the energy for mixing was provided by the submesoscale. A catalyst for this was downfront winds that forced the surface flow into an unstable profile.

Of direct relevance to this study, Molemaker, et al. (2013) argued that eastern boundary currents appeared capable of directly driving mixing by a geostrophic instability. The sequence of events leading to mixing were that poleward flowing currents, like the California Undercurrent (CUC, see Fig. 1.1), develop strips of strong negative relative vorticity in their bottom boundary layers. Often, the vertical component of the vorticity was well in excess of the local Coriolis parameter so that the absolute vorticity, $v_x - u_y + f$, where notation is standard, became negative. It is expected such flows should be unstable, but as was demonstrated numerically, as long as these currents run along continental shelves, they are stable. This is almost certainly due to a stabilizing influence of the boundary.

Figure 1.1: The topography of the west coast of California in the vicinity of Monterey Bay. The CUC flows poleward in this area at depths of 100-400m. Point Sur is a location of frequent separation of the CUC from the continental slope. The blue box denotes the location of the downscaling study conducted by Molemaker, et al. (2013).

However, at sharp promontories and headlands along the coast, it was observed numerically that the CUC frequently separates from the topography and enters the open ocean. As it does so, it preserves its potential vorticity, which via its negative absolute vorticity, is negative (Fig. 1.2). Free of the boundary, the flow goes unstable. It was seen numerically that the evolving flow could directly drive overturnings of potential density surfaces (Fig. 1.3).

Figure 1.2: A scatterplot of PV versus buoyancy frequency. The data comes from the downscaled model situated in the blue box of the previous figure. Note that most of the fluid (99.7%) is characterized by positive, and thus buoyantly stable stratification, but about 20% of the fluid has negative PV. Of this, further examination, not shown here, demonstrated that most of this fluid had negative absolute vorticity.

Analysis of the results argued that the instability responsible for this behavior was centrifugal instability (CI), driven largely by the strong horizontal shear. Estimates of the potential energy creation in this setting were as large as those observed in active mixing regions in the open ocean. Attempts to estimate the global significance of this mechanism suggested it was important locally, but perhaps not globally dominant. These estimates were very crude however, due to the complexity of analyzing the numerical results, which were obtained from an embedded regional general circulation model. This is nonetheless an interesting study because the source of the mixing was not the usual Kelvin-Helmholtz instability.

So to summarize, there are indications that ocean boundaries may have a role to play in the energetic decrease of the mesoscale and any subsequent mixing associated with it. One of the mechanisms by which this may occur is CI. This differs from the usual K-H mechanism thought

Figure 1.3: A timeseries of potential density versus depth at a point in the computational domain where negative PV was frequently found. Something like 4 or 5 overturning events are apparent at this point between the depths of 50 and 200m.

to be primarily responsible for ocean mixing and about which much is known. In contrast, little is known about the general energetic development of CI.

1.2 This Study

The purpose of this study is to characterize the energetics of a centrifugally unstable flow. Specifically we wish to know how much of the energy initially in the balanced, but unstable, flow is dissipated in the finite amplitude development of the instability, and how that compares to the increase in background potential energy driven by the mixing. Equivalently, we wish to estimate the efficiency of CI for comparison with that estimated for K-H instability. We also study the overall energetics development of CI.

We examine an idealized setting of a jet meeting the conditions for CI, compute the solution using the MITgcm and analyze the results. We have only examined one jet profile, although we have varied parameters in order to examine the sensitivity of our results to numerical issues. So, the specifics of the results probably reflect the profile to an extent, but qualitatively the behavior is

similar to that computed for the California Undercurrent. We view this as an illustrative exercise that demonstrates generic behavior for CI and instructs us how to examine the more complex, and realistic setting of the California Undercurrent. The generality of these results however awaits further study.

Accordingly, we argue the efficiency of CI appears to be relatively high compared to that thought to hold for Kelvin-Helmholtz instability. The implication of this is this pathway is an effective means for directly transferring balanced energy to local mixing. The local effective diffusivity can grow to $2 \times 10^{-3} m^2/s$, or values approximately 100 times those typically seen in the open ocean, for a few hours. Last, we find the unstable flow is a very effective internal wave maker. In the example case considered here, we estimate approximately 40% of the energy available to the initial balanced, but unstable, flow scatters into the internal waves bands. We do not follow the long term evolution of these waves in the calculation here, but do expect that once the energy is in the internal wave field, it is likely to follow a forward cascade with eventual mixing by classical mechanisms. Hence, CI promises to be an effective source of both local and non-local mixing via internal gravity wave generation.

Chapter 2 sets some theoretical background and introduces several concepts that are used in later analysis. Our numerical model is introduced in Chapter 3 and the solutions are described in some detail. We present the results of the energetics analysis in Chapter 4, and conclude with a Summary.

CHAPTER 2

THEORETICAL BACKGROUND

2.1 Centrifugal and Symmetric Instability

An insightful contribution to the literature on flow instability was provided by Hoskins (1974), who considered the linear stability of a unidirectional jet with constant vertical shear, constant horizontal shear and uniform stratification. The equations governing the system are

$$\begin{aligned}
 u_t - fv &= -p_x \\
 v_t - u\alpha f + w\beta f + fu &= 0 \\
 w_t &= -p_z + b \\
 u_x + w_z &= 0 \\
 b_t + uB_x + wB_z &= 0
 \end{aligned}$$

where an f -plane is used, the mean flow $V = -\alpha fx + \beta fz$ with constant α, β is balanced geostrophically, such that

$$fV_z = f^2\beta = B_x$$

the quantity B_z represents the uniform mean stratification, often called the buoyancy frequency N^2 and it is assumed all evolution is independent of the meridional coordinate, y . Note that the equations are non-hydrostatic. The continuity equation permits the introduction of a streamfunction, ψ , and straightforward manipulation of the above eventually yields the single equation in ψ

$$\nabla^2\psi_{tt} + N^2\psi_{xx} - 2f\beta\psi_{xz} + (f - \alpha)f\psi_{zz} = 0$$

This equation admits normal mode solutions of the form

$$\psi = \eta e^{i\sigma t} e^{iK(\sin\theta x + \cos\theta z)}$$

satisfying the dispersion relation

$$\sigma^2 = \cos^2\theta (N^2 \tan^2\theta - 2f\beta \tan\theta + f(f - \alpha))$$

Note that the angle θ is related to the tilt of the phase lines. The right hand side of the above becomes negative for a range of θ provided that

$$q = (f + V_x)N^2 - fV_z^2 = (1 - \alpha)fN^2 - f\beta^2 < 0$$

i.e. if the Ertel potential vorticity (PV) of the basic state is negative.

The form of the dispersion relation in this case guarantees exponentially growing solutions. Ooyama (1966) examined the problem for spatially variable shears and showed regions of negative Ertel potential vorticity exhibited exponential growth at rates consistent with the above dispersion relation.

As pointed out by Hoskins, there are several ramifications of this result. Perhaps most important for the present study is that regions of negative PV do not naturally occur in the interior of a fluid; introduction of negative PV can only occur at boundaries by frictional or diabatic mechanisms.

There are three ways PV can be negative. The most familiar is if the buoyancy frequency is negative, resulting in gravitational instability. Such settings will not be of primary interest to us. Rather, we will continue assuming stable stratification. Second, for positive horizontal shear, $(1 - \alpha)f > 0$, a strong enough vertical shear can drive the PV negative. Third, negative horizontal shear, $(1 - \alpha)f < 0$, guarantees negative potential vorticity. The prototype examination of this case is in an unstratified fluid, where it is referred to as centrifugal instability. However, as is clearly true from the dispersion relation, stratified flows admit exponential growth in this case as well.

It is instructive to examine the linearized energetics of the Hoskins instability problem for the latter two cases. The energy equation

$$(K + P)_t = -\nabla \cdot (p\mathbf{u}) - uvV_x - wvV_z - ub\frac{B_x}{B_z}$$

where P is the potential energy, $P = \frac{b^2}{2N^2}$ and $K = \frac{u^2 + v^2 + w^2}{2}$ is the kinetic energy, obtained from the linearized equations shows that the sources of perturbation energy are horizontal shear (HS), vertical shear (VS) and available potential energy (APE), respectively. Each of these quantities can be evaluated using the Hoskins solution and become

$$HS = -\Gamma uvV_x = \cos\theta(\beta\sin\theta - \cos\theta(1 - \alpha))\alpha$$

$$VS = -\Gamma wvV_z = \sin\theta(\beta\sin\theta - \cos\theta(1 - \alpha))\beta$$

$$APE = -\Gamma ub\frac{B_x}{B_z} = \beta\cos\theta\left(\frac{\beta f^2}{N^2}\cos\theta - \sin\theta\right)$$

where

$$\Gamma = \frac{|\sigma|}{f^2 k^2 |\psi_i|^2 e^{2|\sigma|t}}$$

is a normalizing factor.

The above formulae allow us to compare the roles of the various perturbation energy sources as a function of the mean flow parameters α , β and N^2 . Stratifications in the CUC region are $10^{-5} s^{-2} < N^2 < 10^{-4} s^{-2}$ and for mid-latitudes, typical values for β run between 0 (no shear) and 10 (strongly sheared). The California Undercurrent is described roughly by β between 5 and 10. If we consider as well lateral shears of roughly $-2f$, the linearly unstable time scales computed from the dispersion relation are typically ~ 6 hours.

We show in Fig. 2.1 comparisons of the various energy sources for unstable flows as a function of the parameters in the problem. The ratio of the vertical shear strength to the horizontal shear strength is plotted in a PV, vertical shear frame. Note, in all cases, PV is negative and contours are shown only for negative absolute vorticities. The ratio is everywhere less than one, and often much less than one, indicating that horizontal shear is the dominant energy source for flows with negative absolute vorticity.

In a recent paper, Thomas et al., (2012) considered flows with negative potential vorticity set by the vertical shear. They referred to those instabilities as symmetric instabilities to differentiate them from situations where horizontal shear drove the absolute vorticity negative. The latter case they referred to as centrifugal instability, thus extending the term to settings beyond uniform density. We shall adopt this terminology, implying an interest in strong negative horizontal shears. Then a signal of the presence of centrifugal instability (CI) is the dominance of horizontal shear as a linear instability source. We will use this later diagnostically to argue the presence of CI in our numerical results.

For better comparison, the conditions for the instabilities mentioned above and baroclinic instability on a flat bottom are listed in Table 2.1. Note that baroclinic instability is different from the others in that it is caused by gradient in potential vorticity.

Figure 2.1: The ratio of vertical shear to horizontal shear as a source of energy for instabilities. The main result is over the range of negative PV and for all vertical shears, the ratio is small, indicating horizontal shear is the primary instability driver.

2.2 Energy in Boussinesq Systems

Above we worked with linearized Boussinesq equations. The full equations are

$$u_t + uu_x + vu_y + wu_z - fv = -p_x + \nu_h \nabla_h^2 u + \nu u_{zz}$$

$$v_t + uv_x + vv_y + wv_z + fu = -p_y + \nu_h \nabla_h^2 v + \nu v_{zz}$$

$$w_t + uw_x + vw_y + ww_z = -p_z + b + \nu_h \nabla_h^2 w + \nu w_{zz}$$

$$u_x + v_y + w_z = 0$$

$$b_t + ub_x + vb_y + wb_z = \kappa_h \nabla_h^2 b + \kappa b_{zz}$$

where ν , κ represent coefficients of vertical viscosity and diffusivity, respectively. Those with subscript h are their horizontal counterparts. We also assume a fluid with a linear equation of state whose thermodynamics can be represented by a single buoyancy equation.

Table 2.1: Different instabilities and their criterions

Name of instability	Criterion
Gravitational instability	$N^2 < 0$ with no shear.
Symmetric instability	$N^2 > 0$, $(1 - \alpha)f > 0$, a strong enough vertical shear driving the PV negative and being the dominant source of perturbation energy.
Centrifugal instability	$N^2 > 0$, horizontal shear $(1 - \alpha)f < 0$ and being the dominant source of perturbation energy.
Baroclinic instability	Meridional gradient of mean potential vorticity $\frac{\partial Q_0}{\partial y}$ has the same sign for all y and for some y , $\left[\left(\frac{\partial z}{\partial y} \right)_\theta - \frac{\partial h_B}{\partial y} \right]_{z=0} \frac{\partial Q_0}{\partial y} > 0$, where θ is the potential temperature and h_B is the variation of the lower boundary. (Pedlosky, 1987)

A kinetic energy equation is obtained by multiplying the momentum equations by the appropriate velocities. After some algebra and use of the continuity equation, one obtains

$$K_t + \nabla \cdot [\mathbf{u}(K + p)] = wb + \nu_h \mathbf{u} \cdot \nabla_h^2 \mathbf{u} + \nu \mathbf{u} \cdot \mathbf{u}_{zz}$$

where K is the usual kinetic energy per unit mass and conversion to potential energy appears as wb . The latter quantity can be obtained from the buoyancy equation by multiplying it by $-z$, yielding

$$P_t + \nabla \cdot (\mathbf{u}P) + wb = -z\kappa_h \nabla_h^2 b - z\kappa b_{zz}$$

where $P = -zb$ is the potential energy per unit mass. Summing the last two equations gives

$$(K + P)_t + \nabla \cdot [\mathbf{u}(K + P + p)] = \nu_h \mathbf{u} \cdot \nabla_h^2 \mathbf{u} + \nu \mathbf{u} \cdot \mathbf{u}_{zz} - z\kappa_h \nabla_h^2 b - z\kappa b_{zz}$$

We will primarily be interested in integrated forms of this equation. Relatively common boundary conditions result in the equation

$$\iint_A (K + P)_t dA = -\epsilon + \kappa \int_{-H}^0 (b(z=0) - b(z=-H)) dx = -\epsilon + D$$

where

$$-\epsilon = - \iint_A (\nu_h (\nabla_h \mathbf{u})^2 + \nu (\mathbf{u}_z)^2) dA$$

represents the dissipation due to viscous processes. Examples of the conditions yielding this result include independence on the meridional coordinate and rigid, stress free, insulating boundaries. These will be employed in the numerical calculations done later.

Clearly dissipation is a negative definite process. In contrast, surface buoyancies are greater than bottom buoyancies in a stably stratified fluid, implying the effect of diffusion is to increase the energy. This can be understood as due to the energy implicitly associated with the coefficient κ . The final state of an initially stratified but motionless fluid is that of uniform buoyancy. This is a state of greater potential energy than the stratified state in that it is achieved by warming cold deep water and cooling light surface water, which elevates the center of mass. The energy needed to do this is provided by the processes represented by κ .

2.3 Available Potential Energy

The potential energy of a fluid parcel at height z is given by $-bz$, but due to incompressibility, not all of that can be exploited by the fluid, even aside from the usual ambiguity about the ground state of the potential energy. This has led to the useful concept of 'available potential energy', which is a more practical measure of energetic behavior.

In words, available potential energy (APE) is the potential energy relative to a state where the fluid is adiabatically levelled. The key here is the energy stored in sloping buoyancy surfaces is recognized as the important dynamical component; once all slopes relative to geopotentials are gone, the fluid has no more potential energy to give up.

In an incompressible, Boussinesq fluid, volume is conserved. In an adiabatic fluid, buoyancy is also conserved, and the combination requires that in a closed domain, the volume of fluid beneath a surface of constant buoyancy is invariant. If this amount of fluid is levelled out, it achieves the depth z^* defined by

$$z^*(b) + H = \frac{1}{L} \int_{x_w}^{x_e} \int_{-H}^{z(b)} dz dx = \frac{1}{L} \int_{x_w}^{x_e} (z + H) dx$$

where H is the (assumed constant) depth of the fluid and the meridional boundaries of the domain are given by x_w, x_e . We have also for simplicity assumed no dependence of the fluid on y . The potential energy of this levelled fluid is given by $-bz^*$. Note that the concept of available potential energy implicitly assumes the existence of some predefined volume over which levelling can take place.

As the levelled fluid is stably stratified, z^* is a monotonic function of b . Therefore we are assured that the inverse of the function $b = b(z^*)$ exists. Equivalently, either z^* or b could be used as vertical coordinates.

Introducing z^* in the integrated energy equation yields

$$\left(\iint_A (K - b(z - z^*)) dA + \iint_A (-bz^*) dA \right)_t = -\epsilon + D$$

showing that dissipation and buoyancy can effect both the total 'dynamic' energy (the first integral) and the background stratification represented by the potential energy of the leveled state.

Taking a time derivative of z^*

$$\frac{d}{dt} z^* = \frac{1}{L} \int_{x_w}^{x_e} z(b)_t dx$$

The formula governing the motion of a buoyancy interface $z(b)$ is

$$z_t + uz_x - w = -e$$

where e is the diabatic flux of mass across the interface. Substituting and employing the definition of the normal to the surface $z(b)$

$$\hat{n} = \frac{\hat{k} - z_x \hat{i}}{\sqrt{1 + z_x^2}}$$

yields

$$\frac{d}{dt} z^* = \int_{x_w}^{x_e} (w - uz_x) dx + \int_{x_w}^{x_e} e dx = \iint_{A(b)} \nabla \cdot \mathbf{u} da + \int_{x_w}^{x_e} e dx = \int_{x_w}^{x_e} e dx$$

If the fluid is adiabatic, $e = 0$, and z^* is time independent. In a diabatically active fluid, the basic state of the APE will change.

The integral appearing in the energy equation is

$$\iint_A bz^* dA = \int_{x_w}^{x_e} \int_{-H}^0 bz^* dz dx$$

This can formally be converted from an integral in (x, z) to an integral in (x, b) where we choose the leveled b to define the vertical coordinate. The result is

$$\int_{x_w}^{x_e} \int_{-H}^0 bz^* dz dx = \int_{x_w}^{x_e} \int_{b_b}^{b_s} bz^* z_b^* db dx$$

where b_s, b_b denote the surface and bottom leveled buoyancies. Note that z^* and hence z_b^* are both functions of b only, and thus are independent of x . The above simplifies to

$$\int_{x_w}^{x_e} \int_{b_b}^{b_s} bz^* z_b^* db dx = L \int_{b_b}^{b_s} bz^* z_b^* db = L \int_{-H}^0 bz^* dz$$

Winters, et al., (1995) proposed a convenient way to compute the basic potential energy state from numerical output. Given a distribution in a spatial domain of buoyancy, it can be sort from highest to lowest. Assigning to each sorted buoyancy a horizontal length equivalent to the domain scale and a vertical extent such that grid volume is conserved, the sorted profile plotted against the stacked volumes corresponds to the (z^*, b) connection alluded to above. Calculation of the potential energy of the leveled system is then straightforward. The Winters et al., (1995) procedure is equivalent to the above formula.

2.4 Potential Vorticity in a Buoyancy Coordinate System

The hydrostatic momentum equations expressed in a coordinate system whose vertical coordinate is the buoyancy are

$$\begin{aligned} u_t + uu_x + vu_y + Hu_b - fv &= -M_x + X \\ v_t + uv_x + vv_y + Hv_b + fu &= -M_y + Y \\ M_b &= -z \end{aligned}$$

We have used the shorthand notation H to denote the diabatic effects acting on buoyancy

$$\frac{d}{dt}b = H$$

and (X, Y) to denote frictional effects in the horizontal. The quantity M is the so-called Montgomery potential, $M = p - bz$, and all horizontal derivatives are taken on surfaces of constant buoyancy. From the horizontal momentum equations alone, the equation for Ertel's potential vorticity can be formed

$$(z_b q)_t + \nabla \cdot (\mathbf{u} z_b q) = -\nabla \cdot \mathbf{N}$$

where $q = (v_x - u_y + f)/z_b$ is potential vorticity and $\mathbf{N}$ involves only the non-conservative effects H, X and Y . Integrating this equation on a surface of constant buoyancy relates the change of net PV to the fluxes through the boundaries

$$\frac{d}{dt} \int_{x(b)} z_b q dx = -N(x_e) + N(x_w)$$

where x_e and x_w are the locations where the interface strikes the boundary. If we assume the fluxes at those locations can be ignored, as we will do in this paper, the net PV on a buoyancy surface is conserved

$$\int_x z_b q(t) dx = \int_x z_b q(0) dx = \Pi$$

It then follows for fixed lateral boundaries that the time averaged net PV, when spatially integrated, also conserves this value

$$\int_x \overline{z_b q(t)} dx = \int_x \frac{1}{T} \int_0^T z_b q(t) dt dx = \Pi$$

Although PV is always related diagnostically to vorticity and stratification, its instantaneous form is not terribly useful. However, as in Rossby adjustment problems where it is normally assumed the system eventually settles to a steady, geostrophic state, the PV equation becomes a simple relation between PV and the Montgomery, i.e.

$$-fq = \frac{(M_{xx} + f^2)}{M_{bb}}$$

The reason for the steady state assumption is that any wave activity generated by the adjustment radiates to the far field and can be ignored. We shall see later that internal gravity wave activity is a natural result of the evolution of our initial value problem. Since we work in a finite computational domain, internal waves do not decay except by slow viscous and diffusive processes. Such processes will also degrade any underlying balanced flow as well, so we will assume that time averaging will give us a reliable estimate of the steady PV distribution.

In general

$$\zeta + f = z_b q$$

where $\zeta = v_x - u_y$, so

$$\bar{\zeta} + f = \overline{z_b q} = \frac{\overline{M_{xx}}}{f} + f$$

again assuming time averaging yields the underlying PV distribution accurately.

Defining

$$\hat{q} = \frac{\overline{z_b q}}{\overline{z_\rho}}; \quad q = \hat{q} + q'$$

it is simple to show $\overline{z_\rho q'} = 0$. The above procedure is often called thickness weighted averaging and is a standard method of averaging on density surfaces. Thus

$$\frac{\overline{M_{xx}}/f + f}{\overline{M_{bb}}} = -\hat{q}$$

This equation can be rearranged to

$$\overline{M_{xx}} + f\hat{q}\overline{M_{bb}} = -f^2$$

The above is classified as a linear elliptic equation as long as $\hat{q}$ is non-negative, which can be solved for the mean Montgomery potential. The dynamic fields corresponding to the balanced flow can then be deduced according to

$$f\bar{v} = \overline{M_x}$$

$$\bar{z} = -\overline{M_b}$$

This will be used later to estimate the balanced state which should reside in the domain, on top of which is an active internal wave field. The point of the above discussion is that the proper way to average for the mean PV is to use a thickness weighting.

CHAPTER 3

MODEL DESCRIPTION AND RESULTS

3.1 Model Description

The linear analysis in (2.1) leading to centrifugal instability was conducted in a non-hydrostatic setting. To examine the fully non-linear evolution of an unstable flow then, we have adopted the MITgcm, a well known and documented ocean general circulation model with a non-hydrostatic capacity (Marshall, et al, 1997). We have adopted a channel domain 24km in the zonal direction and 1250m deep. The grid sizes we have employed in most experiments are 15m in the horizontal and 5m in the vertical. We performed a small number of convergence tests and parameter sensitivity runs using grids of 5m in the horizontal and 2.5m in the vertical. We have also used two representations of the meridional direction. We initially conducted experiments with a down stream extent of 300m to check for possible three dimensional evolution of the initial condition. We saw none, supporting the applicability of the linear analysis which assumes $\frac{\partial}{\partial y} = 0$, and so conducted the remainder of the experiments as two dimensional in the zonal-vertical plane.

The lateral boundary conditions imposed on the model were free-slip and no-flux on all (x - z) boundaries, no-normal flow on the zonal and bottom boundaries, a rigid lid on the top and periodicity in the meridional direction. The non-hydrostatic option was turned on. We have employed viscous and diffusive coefficients of $0.5m^2/s$ in the horizontal and $0.5 \times 10^{-4}m^2/s$ in the vertical for most of our experiments. A smaller number of runs were conducted using weaker horizontal values; they will be discussed later as appropriate. The result of those experiments was that our conclusions are largely insensitive to parameter variations and grid sizes. The Coriolis parameter was set to $f = 10^{-4}s^{-1}$ and the runs were performed on a Cartesian f -plane.

The MITgcm requires initial conditions in zonal and meridional flow and temperature. We assumed an initially geostrophically balanced state independent of the meridional direction. Such an initial condition constitutes a potentially exact steady solution were the model purely inviscid and non-diffusive. Of course, friction and diffusion are present in these runs, but the initial condition if stable would simply spin down to a state of rest.

We are interested in a potentially centrifugally unstable initial condition, or that the horizontal shear somewhere in the fluid be negative and stronger than the Coriolis parameter. The zonal

velocity was initially set to zero and the meridional velocity was given the form

$$\begin{aligned} v(x, z) &= 0; & x < 15.01km \\ v(x, z) &= -[(x - 15010)/3990]e^{-(z-625m)^2/(150m)^2} m/s & 15.01km < x < 18km \\ v(x, z) &= -e^{-(z-18000m)^2/(900m)^2} e^{-(z-625m)^2/(150m)^2} m/s & x > 18km \end{aligned}$$

Temperature was computed by solving the geostrophic shear equation

$$T_x = fv_z/(g\alpha)$$

where $g = 9.81m/s^2$ is gravity and $\alpha = -2 \times 10^{-4}/^\circ C$ is the temperature expansion coefficient of our linear equation of state. The western edge boundary condition for the above was

$$T(0, z) = 20^\circ C + 0.015^\circ C/m(z + 2.5m)$$

Plots of both v and T appear in Fig. 3.1.

Figure 3.1: Initial fields. The upper panel is meridional velocity, v in m/s and the lower is temperature T in $^\circ C$.

The initial fields have been offset to the east from center because first tests showed a strong tendency for the initial profiles to shift west. We show in Fig. 3.2 the Ertel PV associated with these conditions. The region of negative PV is indicated by the arrow. The anticyclonic zone

on the left side of the jet was assigned a vorticity of $-2.5f$. Such vorticities and larger were frequently computed in the CUC regional model and reported in Molemaker, et al (2013). The maximum velocity here of $-1m/s$ is larger than that in the CUC, but was adopted in order to fit a representative area of the initial condition with negative PV, have far field vanishing velocities and construct a model with fine resolution that we were able to explore. The experiments were conducted for 72 hours of model time with time steps of 3 seconds. Outputs were archived at intervals of 10 minutes.

Figure 3.2: Potential vorticity of the initial condition. The region of negative potential vorticity is indicated by the arrow. The red areas are regions of strong cyclonic vorticity. Weaker positive PV zones extend to the eastern boundary.

It is likely that the results we discuss do depend on this initial condition, so this study must be considered as an exploratory examination of centrifugal instability energetics. We nonetheless feel the results are illuminating with regards to the generic behavior of such flows.

3.2 Numerical Results

The very early evolution (4hrs) appears in Fig. 3.3. The tendency is for the jet structure to shift west, particularly near mid depth. The initial spreading of isotherms seen to the east of the initial jet collapses as the shift proceeds.

Figure 3.3: State at 4hrs. The primary change in structure is a shift to the west. The minimum velocities have dropped from an initial value of $-1m/s$ to approximately $-0.5m/s$. The initial bulge of temperatures at the eastern mid depth have become two separate bulges centered on the velocity extrema.

Soon after this state, the isotherms steepen and overturn. An example appears at 8 hours in Fig. 3.4. Note the evidence of developing convection seen primarily in the velocity field in the regions of temperature inversion. Closeups of the mid-depth jet (Fig. 3.5) show that the convection is well resolved, consisting of approximately 90 grid points per overturning cell.

We also show in Fig. 3.6 a plot of the vertical velocity in the overturning cells at 8 hours. The vertical velocities at this time have achieved speeds of $0.05m/s$. Approximately 40minutes later maximum vertical velocities are in excess of $0.1m/s$.

The evolution of the system involves one primary overturning event, as suggested by the time series of w for the first 24 hours taken from a point of maximum vertical convection (Fig. 3.7). The maximum, occurring between 8 and 9 hours, is followed by a series of somewhat smaller, chaotic vertical velocities associated with secondary overturning events caused by remnant unstable regions. Very late in the time series, evidence of internal waves appears in the slower more organized oscillations at a period of 3 hours. Once the primary overturning is completed, the system is very slow to change, as suggested by the comparison of the velocity fields from 12 and 24 hours in Fig. 3.8.

Figure 3.4: State at 8hrs. Inversions in temperature appear in the lower panel, and small scale structures, more evident in the upper panel of velocity, indicate active convection.

Figure 3.5: Closeups of the region of convection. The cells are well resolved by the grid, consisting of roughly 90 gridpoints per cell.

Figure 3.6: Vertical velocities in the overturning cells at 8hrs. The flow structure consists of a series of roll cells approximately 400m wide and 100m tall. Maximum velocities here are roughly $.05m/s$ and later are in excess of $0.1m/s$.

Figure 3.7: Vertical velocity time series. The primary overturning event is associated with the spike between 8 and 9 hrs and is followed by a chaotic period. As time progresses, the time series exhibits evidence of internal waves at a period of roughly 3 hrs, which is close to the buoyancy frequency.

The next 24 hours (Fig. 3.9) involve a few minor overturning events caused by the oscillations of the field, but by the end of two day, there are only a small number of unstable density profiles and those are very weak. There is no further evidence of overturning from 48 to 72 hours, and the

Figure 3.8: Comparison between 12 and 24hrs of velocity. The primary structure after 12 hrs is very slow to change (contour intervals $.045m/s$ for both plots). Internal waves are present and are responsible for much of the difference in the two fields.

unstable profiles associated with anomalies are very weak.

Figure 3.9: Temperature at 36 (upper) and 48 (lower) hrs. Evidence of a weak overturning event can be seen in the upper diagram. Steep interfaces can be seen at about 17km in the lower, but by 48 hrs the overturning is essentially finished.

CHAPTER 4

DATA ANALYSIS

We now proceed to the analysis of the model data. The first question to address is what is the nature of the instability occurring early in the evolution. It is clear that the initial conditions meet the necessary constraint for the fluid to be unstable, i.e. regions of negative PV and stable stratification. Further the horizontal shear is sufficiently strong in the anticyclonic side of the jet that absolute vorticity is negative. We expect some form of baroclinic instability but want to differentiate between symmetric and centrifugal instability.

4.1 What Type of Instability Dominates?

We argued in Chapter 2 that a feature of centrifugal instability is the dominance of horizontal shear as an energy source for the growing instability. In Fig. 4.1, we compare the zonal integration of the horizontal ($-uvV_x$) and vertical ($-wvV_z$) sources during the early stage of the model. Note that both processes are working to release energy from the mean flow (i.e. both are positive), but that the horizontal source is about an order of magnitude larger than the vertical.

We have also computed the linear unstable modes for this initial condition using the breeding method, i.e. the linearized equations were seeded with small scale noise of maximum amplitude 10^{-5} and allowed to evolve for a short period of time. The calculation was stopped and the anomalies were extracted from the solution. The extreme value of the anomalies was located and the entire field was multiplied by the value needed to reduce the extreme anomaly back to a value of 10^{-5} . This sequence was repeated until the norm of the difference between the anomalies in successive iterations decreased beyond a stopping criterion. The structure of the unstable normal mode compares very well with that computed using the MITgcm during the very early stages of the model evolution, as argued in Fig. 4.2. It is also possible to extract a growth rate from the time series of the bred anomalies by fitting an exponential to those curves. We compute a growth time scale of 5 hours, which agrees qualitatively with the numerically observed growth.

Finally, in Fig. 4.3, we show a plot of the Richardson number after 1 hour, demonstrating that nowhere is the necessary condition for K-H instability of $Ri < .25$ met. The minimum for the field is $Ri = 0.84$. Later in the evolution, the criterion is exceeded, but that is well after the finite

Figure 4.1: Time averaged power from zonally integrated horizontal ($-uvV_x$) and vertical ($-wvV_z$) shear in the first hour.

amplitude effects develop. The lack of a subcritical Ri allows us to eliminate the possibility of K-H instability during the linear stages of the growth.

From these bits of evidence, we conclude that the flow is evolving primarily from centrifugal instability in its early stages.

4.2 Energetics

We plot in Fig. 4.4 several measures of the energy in the flow during the first 24 hours of evolution. The units of the plot are $\frac{J}{m^3}m^2 = \frac{J}{m}$, with the appearance of the linear length scale, rather than an area, in the denominator reflecting the lack of meridional dependence. We area integrate our results, rather than volume integrate, so the resulting quantities are energy per unit distance. As is often the case, the important aspect of energy is its change, rather than its absolute value, so all the curves are relative to their initial values.

Total energy is in blue and this is quite simply

$$Total\ Energy = \iint_A \rho_0(K + P)dA$$

Figure 4.2: The upper panel shows anomalies in unstable normal mode gotten by breeding method in velocity v at 270secs. The lower panel is the difference in v between initial time and 1hr. The two profiles match each other closely in pattern. Note that contours are only plotted for positive values.

where ρ_0 is a mean density of 1030 kg/m^3 . The relation of the time derivative of this quantity to nonconservative effects is discussed in Chapter 2. There is an overall decline in total energy as time proceeds, which is expected in this unforced and viscous problem. Clearly however there is a qualitative change in the rate of decrease occurring between approximately hours 8 and 14 relative to the rest of the plot. Recall that this is the time interval over which the model experiences its primary overturning and mixing events.

The right hand side of the energy equation includes two possible sources/sinks for the energy. One of these comes from the difference in buoyancy between the top and bottom of the fluid and is proportional to the vertical diffusivity coefficient. This quantity measures the contribution to the potential energy due to the processes represented by our diffusive parameterization. The difference in the two buoyancies is positive in a stable fluid, like the present one, so this term is positive and a source of energy to the flow.

If we shift the diffusive contribution to the right hand side of the energy equation, we isolate

Figure 4.3: A close-up of the Richardson number for the field after 1 hour. Values greater than 10 have been set to 10 and are not contoured. Note that regions less than the critical K-H value of 0.25 do not appear. The field minimum is $Ri = 0.84$.

the viscous terms, which are seen to be negative definite. The result of the shift appears as the red line in Fig. 4.4. Note that dissipation, or equivalently total energy adjusted for the diffusive contribution, is more negative than total energy change. Dissipation by its form can act only on kinetic energy, and it is seen that viscosity actually reduces the energy more than the inferred total energy change in the model run. The difference between the two curves is the diffusive contribution. We should state that we also have computed the dissipation explicitly from the model output and it follows the curve appearing in Fig. 4.4.

The diffusive contributions and the changes in potential energy it drives are not of interest to this study. Note that this change would occur even if the system evolution did not involve an unstable flow. We have verified this by a control run in which the velocity and buoyancy anomalies in the initial conditions have been reduced by a factor of 10. The resulting profiles do not meet the necessary condition for instability and in fact do not change much at all over the three day interval. The net growth in potential energy in the control run matches that in the unstable run quite closely.

Background potential energy (BPE) is essentially the measure of potential energy associated with z^* , the adiabatically leveled state of the model. Equivalently, this can be determined by sorting

Figure 4.4: Energy during the first 24 hours. Total energy appears in blue, integrated dissipation in red and change of the background state stratification apart from the diffusive boundary contribution in magenta.

the buoyancy values in the model from greatest to least, multiplying by the location in the water column each piece would have and integrating the result from bottom to top. We then remove the diffusive contribution from the BPE, and the result is the magenta line. The latter adjustment insures that the change in the plot reflects the change in the BPE reference state caused primarily by the mixing from the instability driven overturning.

As with the total energy and dissipation, the period of unstable evolution stands out in the magenta curve. The leveled state potential energy grows quite quickly during this period. After about hour 15, it settles back to much lower growth rate. The reasons for this are the explicit, but weak, diffusion in the model and any other weak turbulent events. We have examined the same curve for the second day and it does not show any comparable interval of growth to that between hours 8 and 15, even though there is a secondary period of overturning around hour 36. This is due to the fact the unstable profiles are quite weak in amplitude.

The time derivative of the background curve is the power delivered to mixing by the overturning. This appears in Fig. 4.5, where we have further normalized the result by the area of the channel, thus yielding the more oceanographically familiar units of W/m^3 . The more violent phase of mixing occurs at roughly 8 hours as indicated by the sharp spike. The system then retreats to a gentler

phase which holds out to roughly hour 11.5, followed by a drop achieved by hour 12.5 to a level more consistent with background mixing. The maximum value is roughly $3.2 \times 10^{-5} W/m^3$ and the following phase is at a level of $1.5 \times 10^{-5} W/m^3$. These are sizable values, especially considering that the mixing takes place in a region that is only 10% of the total area. Locally the values are an order of magnitude larger. The values compare quite well to the elevated values seen over rough topography in the Brazil Basin (Ledwell, et al., 2000). Perhaps a fairer representation of the mixing is obtained by averaging the power over the day, which returns a value of $3.2 \times 10^{-6} W/m^3$. The relatively short, few hour period of intense overturning is sufficiently strong that it still renders the potential energy generation quite large.

Figure 4.5: The power delivered to the background stratification during the evolution of the flow. Note the sudden increase at hour 8 which is followed by a drop to a smaller, but still anomalously large value that lasts until about hour 11.5.

Standard diffusive parameterizations equate the power provided to potential energy to eddy diffusivity times the buoyancy frequency

$$\overline{-w'b'} = K_v N^2$$

Adopting this formula implies local values like $K_v = 2 \times 10^{-3} m^2/s$, and channel averaged values like $K_v = 2 \times 10^{-4} m^2/s$. The latter is anomalously large relative to typical open ocean thermocline values of $K_v = 2 \times 10^{-5} m^2/s$ (Ledwell, et al., 1992) and the former is 20 times the typically quoted

value of $K_v = 10^{-4} m^2/s$, sometimes called a 'Munk', needed for global balances of heat and salt (Munk and Wunsch, 1998).

4.3 Mixing Efficiency

An important parameter associated with Kelvin Helmholtz instability is its 'efficiency', which is defined as

$$\Gamma = \frac{\overline{-w'b'}}{\epsilon}$$

where ϵ is 'turbulent' momentum dissipation and $\overline{-w'b'}$ is the turbulent buoyancy flux, or equivalently the irreversible exchange between potential and kinetic energies. The overbar implies an appropriate averaging. Efficiency, however, does not apply uniquely to Kelvin-Helmholtz instability; the concept can be used as a descriptor for any turbulent process that mixes. The natural connection between efficiency and centrifugal instability comes from the net change in background potential energy versus the net dissipation. Defining the interval of mixing as from hour 8 to hour 13, we obtain an efficiency estimate of

$$\Gamma_{CI} = \frac{6.63 \times 10^6 J/m}{1.62 \times 10^7 J/m} = 0.4$$

This is large relative to the classical K-H value of .2, suggesting that centrifugal instability is a relatively effective way of mixing the ocean. The generality of this number for other unstable profiles remains to be examined.

4.4 Dynamical Energy

The remainder of the energy equation consists of what might be called the 'dynamical energy', consisting of the kinetic and available potential energies. This is the energy that the fluid flow can draw upon and it is interesting to examine how it evolves over the period of the instability. The dynamical energy (blue line) is compared to the total energy (red line) in Fig. 4.6; note the drop in dynamical energy is greater than that in total energy. This is due to the increase in the background potential energy.

There are a couple of possible reservoirs for the dynamic energy. Ocean flow is routinely divided up into 'balanced' flow and 'unbalanced flow'. The term balanced flow implies the evolution of that component occurs on timescales long compared to a day, while the unbalanced flow typically involves time derivatives at leading order in the momentum equations. Balanced flow is also associated with

Figure 4.6: Dynamical energy versus Total energy. The dynamical energy, consisting of kinetic and available potential energy, is in blue and total energy is in red. The drop in dynamical energy exceeds that of the total energy, consistent with the increase in the background potential energy.

potential vorticity which is a field variable that under stable conditions is close to being conserved on fluid parcels and clearly reveals the state of the system. In classical Rossby adjustment problems, initial conditions are connected to eventual steady states through PV conservation. It is common practice in such problems to compare initial and final energies, in order to comment on how effective balanced flow is in controlling the evolution of the system. A key here in our situation is that it is assumed that the system eventually relaxes to a steady state, an assumption that in some sense is inconsistent in that Rossby adjustment problems are typically inviscid and adiabatic and hence have no mechanisms by which energy can be lost. The usual explanation given to rationalize the steady assumption is that the lost energy goes into gravity waves, or equivalently unbalanced motions, which radiate to the far field.

This sequence suggests a similar decomposition is valuable in the present case, but there are some important differences. First, the initial PV distribution is unstable and so cannot immediately be linked to any eventual steady state. Instead, the PV distribution must change in time in order to disable centrifugal/symmetric instability. This is what the finite amplitude evolution does. The PV field mixes so that the initial negative regions are diluted by the much more massive positive

regions. When the PV field has become effectively non-negative, the distribution becomes stable and we can expect slower balanced evolution from then on. The tendency of the negative PV regions to mix away appears in Fig. 4.7 which is a scatter plot of PV and density at different times. The initial distribution appearing on the upper plot shows the regions of negative PV associated with the unstable profile. This same distribution is shown at hour 48, and it is seen that some of the negative PVs have retreated to 0. There are a few outliers, of course, but they are mostly weak, and dispersed in the fluid. Time evolution of area integration of negative PV is shown in Fig. 4.8, which indicates that the finite amplitude events are associated with significant drop in the amplitude of the negative PV. As a result, the negative PVs are unable to importantly influence the evolution. The effect of the instability is to move the PV distribution to a non-negative, and therefore stable distribution.

As was shown by Haynes and McIntyre (1990), PV obeys an impenetrability theorem, which states that PV does not cross lines of constant buoyancy. Thus the mixing that removes the negative PV on any buoyancy surface must ultimately represent dilution with positive PV on that same surface. Once a stable PV distribution has been achieved, we can search for the corresponding balanced field it represents.

There are also clear indications in several of the earlier plots of internal gravity waves, which are unbalanced and this relates to a separate distinction with the classic Rossby adjustment problem. There the domain is usually unbounded and gravity waves are assumed to disperse to the far field. Here, our finite domain does not allow this and hence the unbalanced and balanced fields remain mixed together. Because it is interesting to ask how much energy is transferred to the unbalanced flow, we are motivated to find a way to separate balanced and unbalanced flow, which is not part of the classic problem.

Last, in contrast to the classical Rossby adjustment problem, this is a situation in which the initial condition is itself perfectly balanced, i.e. it is in geostrophic balance and is a solution to the inviscid and adiabatic equations. However, as has been demonstrated, it is unstable to a generalized form of PV instability. The most familiar form of a PV based instability is baroclinic instability, first discussed by Eady (1949). These are well captured by quasi-geostrophic dynamics which omits unbalanced components and conserves a perfectly balanced form of energy. Baroclinic instability is able to avoid generating an unbalanced response because its timescales are quite long. In contrast, we have seen the growth rates of centrifugal instability are short, a matter of hours. This is a time scale that fits well within the internal wave frequency band, and so the natural generation of

Figure 4.7: Scatter plots of potential vorticity versus density. The upper plot is taken at initial time, the bottom one is at hour 48.

unbalanced variability is not surprising, but it does motivate us to estimate how much of the initial energy is lost to unbalanced variability.

Although it is not calculated here, presumably this energy, once in the internal wave band, follows a classical sequence leading to K-H instability and mixing at an efficiency of roughly 0.2. Thus centrifugal instability may contribute to nonlocal mixing, as well as local mixing, in a manner similar to the generation of baroclinic tides by the barotropic tide.

We here estimate the underlying balanced flow in our simulations in two ways. A reasonably obvious way to estimate it is by averaging in time. The internal waves are fast, and the balanced flow is slow, so averaging over several internal wave time scales promises to minimize their presence. This averaged flow can then be analyzed from the above perspective. While possible, it presents

Figure 4.8: Time evolution of area integration of negative potential vorticity over 3 days.

many practical issues. First, the energy in the internal waves looks sizable, so that long averaging intervals are needed to reduce their presence. But in this calculation with viscosity and diffusivity, the mean state will be degraded in a way that clouds the meaning of the result. As a compromise, we average over a 24 hour period, from hours 48 to 72.

We have also estimated the balanced flow in a second, almost independent, way by exploiting potential vorticity. By hour 48, negative PV is almost completely absent. We then declare the flow 'stable' with respect to centrifugal/symmetric instability. The potential vorticity of the underlying balanced flow can then be estimated and the elliptic equation discussed in Chapter 2 inverted to find the associated dynamical fields.

The procedure we followed to perform the PV analysis used the model data from hours 48 to 72. The density fields were inspected and found to have a small number of weak unstable density gradients dispersed throughout the domain and throughout the interval in time. These were smoothed using a running 5 point boxcar filter. Doing this twice removed all unstable gradients with very little change in the density profiles. The meridional velocity from this period was then similarly filtered to make it consistent with the density. From these smoothed fields, a PV was computed. We then projected that PV onto surfaces of constant buoyancy, also estimated from the smoothed data, and computed time series of thicknesses for those buoyancies. This data was then

thickness averaged according to the prescription in Chapter 2. The resulting averaged PV profile was found to have very weak negative PVs on .007% of the points. These were simply set to 0. With a non-negative PV now as a function of buoyancy and horizontal position, we inverted the elliptic equation

$$M_{xx} + fqM_{bb} = -f^2$$

to find the Montgomery of the underlying balanced flow.

The seemingly obvious boundary conditions to apply to this equation are fixed buoyancy at the top and bottom, and vanishing velocity at large distance. These reduce to the specification of Neumann conditions all around. However, as is often the case with Neumann conditions, an additional solvability constraint is implied on the right hand side, and this was not met. In an unbounded domain this would be worked around by specifying a Dirichlet condition at large distance. Applying this boundary condition in our finite domain resulted in Montgomery structure near the boundaries which seemed unreasonable. The conditions that we applied that yielded a solution confined to the area of the PV anomaly was fixed buoyancy at the surface, a homogenous Dirichlet condition on the Montgomery anomaly on the bottom, and Neumann conditions, equivalent to vanishing meridional flow, on the zonal boundaries. The solution of the elliptic was done using a 4th order accurate multigrid method for non-separable elliptic equations from the MUDPACK collection (Adams, 1999). Differentiating the Montgomery in x and buoyancy yielded estimates of the velocity and depths of the buoyancy interfaces where the velocities were located. These were interpolated back onto geopotential surfaces.

The velocity field from the two estimates of the balanced flow are compared in Fig. 4.9, and of the buoyancy field in Fig. 4.10. This comparison is largely successful in that the fields resemble each other closely in structure and amplitude of the anomalies. We thus accept them as reasonable estimates of the underlying balanced flow. Note that the two estimates of the dynamical energy in the balanced state, compared in Fig. 4.11 to the complete dynamical energy of the flow, are quite similar in value. Both are considerably smaller than the dynamical energy of the region over the last 24 hours of the experiment.

The energy that is unaccounted for in the balanced flow, represented by the distance between the lower and upper curves in Fig. 4.11, must be resident in the unbalanced, internal wave component. The appearance of gravity waves is apparent in the profiles shown earlier. The above is a quantitative assessment of how efficiently gravity waves are generated in this process, and the number is high. Of the total dynamical energy in the final state, approximately half is in the

Figure 4.9: Estimates of the balanced flow. The upper figure is velocity obtained by averaging the model output over the hours from 48 to 72. The lower figure is obtained by inverting the PV equation for the Bernoulli and computing the velocity according to geostrophy. The comparison between the two in structure and amplitude is quite favorable.

internal wave field ($0.9 \times 10^5 J / 1.7 \times 10^5 J$). The reason for the internal wave excitation has to do with the rapid time scales of the centrifugal instability. The initially balanced, but unstable field, changes on time scales that the fluid can react to in the form of gravity waves.

Figure 4.10: Same as 4.9, but for buoyancy. Again, the comparison is quite favorable.

Figure 4.11: Energy comparison. The dynamical energy over the three days of the experiment appears as the blue line. The two estimates of the energy of the underlying balanced flow appear as the red and green lines.

CHAPTER 5

SUMMARY

The objective of this work was to examine the energetics of centrifugal instability. This problem is motivated by some recent numerical work that argues centrifugal instability should be a common event anywhere there is a current flowing along a coast in a direction aligned with that of topographic waves (i.e. poleward on eastern boundaries, equatorward on western boundaries). An example, as argued in Molemaker, et al, (2013) is the California Undercurrent and the sequence of events is as follows.

Extreme anticyclonic vorticities are generated in the bottom boundary layer. At irregular points along the boundary, such as promontories like Point Sur, separation of the current from the coast is commonplace and a jet with negative absolute vorticity is injected into the open ocean. Such fluid has regions of negative potential vorticity and thus is potentially unstable to symmetric and centrifugal instabilities. Numerical calculations show for the CUC case that centrifugal instability dominates the evolution, with the result that potential density surfaces overturn and 'mix'.

Interest in this sequence focuses on the mixing, because it is a boundary mechanism promoting mixing, as has been speculated for several years now, and the mixing occurs for reasons other than the classical paradigm of Kelvin-Helmholtz instability. In comparison to K-H instability, however, little is known about the detailed energetics of centrifugal instability, a gap in knowledge this study seeks to address.

The study is largely numerical and process oriented. A two-dimensional (x - z plane), meridional jet with regions of negative PV is studied. The maximum jet speed exceeds those observed for the CUC, but the size of the negative PV regions compare well with those in the CUC and the amplitude of the negative lateral shears is similar as well. An initial value problem is computed in a non-hydrostatic model and analyzed for its energetics evolution. Parameter studies have argued the results are robust to reasonable modifications to the problem. While an examination of jet structures has not been performed, the study is offered as an example of generic behavior for centrifugally unstable flows.

We observe that the evolving system is dominated by centrifugal instability, which at finite amplitude drives direct overturns of density surfaces. A comparison of the energy dissipation

during the overturning to the growth in mean potential energy argues this process is relatively efficient:

$$\Gamma_{CI} \approx 0.4$$

This should be compared to the classical efficiency of Kelvin Helmholtz instability, which is close to 0.2. The reason for the overturning is that the instability tries to erase the negative potential vorticity values along isopycnals by stirring laterally. In a fluid with tilted isopycnals, like any vertically sheared rotating flow, this results in a wrapping up of the isopycnals. The relatively high efficiency implies the overturning can occur with relatively little kinetic energy loss to heat. We should mention that the same parameter at finer resolution model increases a bit to 0.6.

Beyond this, the times scales of the instability are short. Analytical dispersion relations suggest growth times of several hours. This implies the adjustment away from the purely balanced initial state is rapid and on time scales that can excite gravity waves. We have estimated the separation between balanced and unbalanced energy in the final state of the system, and found that half of the final energy is 'unbalanced'.

A rough accounting of the energy budget thus is as follows. Of the total initial dynamical energy of the flow (initial kinetic plus initial available potential energy), about 10% is directly dissipated as a result of the finite amplitude instability and another 5% goes into the background potential energy. Of the remaining 85% of the energy, approximately half, or $\sim 40\%$ of the initial, scatters into the unbalanced, and predominately internal gravity wave field and the remainder ($\sim 40\%$ of the initial) remains in the balanced flow. All told, well over half of the initially balanced energy is lost from the balanced state. This constitutes a very effective means by which energy can exit the balanced flow.

The transfer to the mean state potential energy computed above represents a local contribution to mixing. One presumes the strong excitation of gravity waves as suggested by the experiments generates a classical energy trajectory through the internal gravity wave field. Namely, nonlinear mechanisms within the unbalanced field develop a forward cascade of energy and eventually lead to mixing via the classical K-H mechanism. Assuming this occurs with a classical efficiency (0.2), approximately 8% more of the initial energy eventually ends up in the mean potential energy profile. Importantly, this is not necessarily local, and probably is a non-local means by which centrifugal instability can initiate mixing. In total, something like 10-15% of the initially balanced energy ends up in the mean state stratification.

5.1 Estimation of Regional and Global Impact

Now we speculate on the interesting question of the possible regional and global impact of centrifugal instability. These are very difficult to estimate. As a metric, the estimated 0.4TW needed to sustain the stratification translates to an energy consumption per unit area of

$$\frac{4 \times 10^{11} W}{0.7\pi \times (6.4 \times 10^6)^2} = 4 \times 10^{-3} W/m^2$$

i.e. $4mW/m^2$. In Chapter 4, we estimated an average transfer to mean state stratification by a centrifugal event as $3.2 \times 10^{-6} W/m^3$. This was characteristic of the transfer over a depth of about 400m, resulting in $1.3mW/m^2$. This is almost one third of the global average requirement per unit area. Further, the time series shown in Section 1 suggests a day as a typical separation in time between events for the CUC, so this number might represent a reasonable average.

One can ask if this number obtained from an idealized calculation applies to the real world. The conditions favorable to centrifugal instability are cyclonic type flows against topography, translating into poleward flows on eastern ocean boundaries and equatorward flows on western ocean boundaries. The strongest currents in the world largely do not meet this requirement. Examples are the Gulf Stream, the Kuroshio and the Antarctic Circumpolar Current. The higher latitude western boundary currents do meet the requirement, like the Ooyashio, the Labrador Current (the subpolar western boundary current of the North Atlantic) and the Malvinas. There are several near Equatorial western boundary currents flowing equatorward, like the Mindanao current and the North Brazil Current. Perhaps more significantly, the deep current of the North Atlantic flows equatorward on the west, as do many of the deep currents in the Indian and Pacific Oceans. Many eastern ocean boundary currents are poleward, and thus cyclonically oriented. Examples include the often mentioned California Undercurrent and the Leeuwen Current off western Australia.

In summary, several boundary regions in the ocean have currents that meet the criteria of centrifugal instability. Whether absolute vorticity of opposite sign to the local Coriolis is generated in all of them is unknown. For the present purposes, we suppose they are all potentially unstable and that the associated boundaries have enough promontories and points that separation from the coast is relatively common. This last assumption is easily supported by examining topographic maps of the coastlines.

We expect something like 1TW to enter the large scale circulation and to largely exit the same by geostrophic instabilities. This powers up the mesoscale at a rate of 1TW. Of this, we might expect as much as .5TW to end up in boundary currents, the rest being dissipated by 'frictional'

topographic interactions and surface processes. Most of the boundary current energy will be in currents that do not meet the criteria for centrifugal instability, but suppose 25% of the energy makes it to those that do. This puts 0.1TW in possibly favorably structured currents. Not all parts of these currents are available for CI. For example, the California Current system includes the CUC, but also several other parts, some of which are equatorward. Suppose 10% of the currents do meet this constraint, suggesting .01TW are available to CI. Of this, our analysis suggests about 10%, or .001TW, will end up in the mean stratification.

The western coasts of the major continents (North/South America, Europe/Africa) are prime locations for this process. They are roughly 4000km long, providing 8000km along which centrifugal instability might be active. Other currents meet the necessary criteria as well; suppose the total linear length available for centrifugal instability is $5 \times 4000km = 20000km$. We might expect centrifugal instability to occur over an offshore distance of 10km along these coasts. If we now divide .001TW by the area over which the instability takes place, we obtain a power input to the stratification of

$$\frac{10^9 W}{2 \times 10^7 m \times 10^4 m} = 5 \times 10^{-3} W/m^2$$

$5mW/m^2$, which compares very favorably with the global average per unit area. This suggests over the areas where CI is active, it may contribute quite significantly to the regional mixing.

In contrast, because the areas of active CI are confined, its global effect is limited to

$$\frac{10^9 W}{4 \times 10^{11} W} = 2.5 \times 10^{-3}$$

less than 1% of the total requirement.

These are clearly very uncertain estimates with very large error bars, but it is not unreasonable to suggest CI can contribute importantly to regional mixing in eastern boundary coastal areas. We also mention some experiments with lower horizontal viscosity suggest even higher efficiencies ($\Gamma_{CI} \sim 1$), although a careful study of the convergence of these results has not been completed. Our stated estimates are nonetheless likely to be conservative. Also, the locations where much of the associated mixing should take place are relatively near surface and in the permanent ocean thermocline. This can amplify them in importance in maintaining the shallow stratification. Mixing in such areas is also important to the maintenance of near surface ecosystems through delivery of deep nutrients to the photic zone.

To summarize, CI appears to be able to drive vigorous mixing in ocean-like stratified fluids. While its global impact appears limited, it is likely an important regional contributor to the mixing

climatology. The estimates suggesting this are based on what has been learned from this highly idealized study, and it would be of interest to examine the generality of these results to other centrifugally unstable profiles.

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BIOGRAPHICAL SKETCH

I come from Beijing, China. Over there I got a BS in applied mathematics. Upon graduation, with an interest in climate science I applied FSU for a degree in physical oceanography.