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Feasibility Study of the Standing Accretion Shock Instability Experiment at the National Ignition Facility

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THE FLORIDA STATE UNIVERSITY
COLLEGE OF ARTS AND SCIENCES

FEASIBILITY STUDY OF THE STANDING ACCRETION SHOCK
INSTABILITY EXPERIMENT AT THE NATIONAL IGNITION FACILITY

By

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ABSTRACT

The primary hydrodynamic flow feature of early explosion phases of a core-collapse supernova is a spherical shock. This shock is born deep in the central regions of the collapsing stellar core, stalls shortly afterward, and in case of a successful explosion is revived and becomes the supernova shock. The revival process involves a standing accretion shock instability, SASI. This shock instability is considered the key processes aiding the core-collapse supernova (ccSN) explosion.

The aim of our study is to identify feasible conditions and parameters for an experimental system that is able to capture the essential characteristics of SASI. We use analytic methods and high-resolution hydrodynamic simulations in multidimensions to investigate a possible experimental design on the National Ignition Facility. The experimental configuration involves a steady, spherical shock. We explore a viable region of parameters and obtain limits on the shocked flow geometry. We study the stability properties of the shock and its post-shock region.

We discuss key differences between the experimental setup and astrophysical environment. The obtained flowfield closely resembles converging nozzle flow. The post-shock region, in contrast to the supernova setting, is found to be stably stratified and insensitive to perturbations upstream of the shock. We conclude that it is not possible to capture the characteristics of the supernova SASI for the converging shocked flow configuration considered here. However, such configuration offers a very stable setting for precision studies of shocked, dense, high temperature plasmas requiring finely-controlled conditions.

CHAPTER 1

INTRODUCTION

Core-collapse supernovae (ccSNe) are one class of stellar explosions. These events are famous for their extreme intrinsic brightness, strong asymmetries, and significant variations between occurrences. It has long been suspected their progenitors are massive stars. These explosions are radiation hydrodynamic phenomena powered by neutrinos emitted in the process of the progenitor star's core collapsing under intense gravitational force [20]. These theories have been bolstered by observations of neutrinos from SN 1987A (the most studied and documented supernova case since the rise of modern astronomy [14]). More recently it has been proposed that a process known as the standing accretion shock instability (SASI) may play a role in successful supernovae explosions.

The physics processes participating in ccSNe explosions, such as SASI, are extremely complex, and their interactions are highly nonlinear. These factors, combined with significant uncertainties in basic physics such as neutrino matter interactions [23] and nuclear equation of state [25], hampered progress of theory and development of successful computer models of supernova explosions. Therefore, it would advantageous if supernova physics could be studied under controlled conditions such as laboratory experiments.

1.1 In the Beginning...

Many science fiction writings depict the universe as a sparse group of molecular islands dotting a sea of nothing. This model is simply not true. Between each of these islands (e.g. solar systems, galaxies) is a dilute fog of matter known as the intergalactic medium (IGM). Composed primarily of hydrogen and helium (roughly 90% and 9%, respectively), this tenuous material was originally formed 400,000 years after the Big Bang.

The universe shortly after the Big Bang is believed to be a hot, thick, dense soup of subatomic particles (a photon-baryon fluid). In this fluid, rays of light (photons) bounced around erratically, constantly interacting with matter (protons and electrons) and, as a result, not being able to move very far. The primary factor in these interactions was temperature; higher temperatures cause increased interactions. Likewise, lower temperatures correspond to decreased interactions.

As the universe continued to grow, the soup became thinner (transparent) and cooler. Eventually, this soup cooled enough for photon-matter interactions to become rare. It was at this moment photons decoupled from regular matter, allowing light to stream freely without bouncing around. This allowed protons to recombine with electrons, resulting in hydrogen and helium, without being ripped apart by electromagnetic radiation. This moment is appropriately named the recombination epoch.

The radiation released during this time became known as the cosmic microwave background (CMB). This picture of the cooling of the cosmos is an important window into the early universe. The image produced at this moment was cast from the “last scattering surface” and is the farthest back in time we can visibly observe. Any information about the nature of the universe before this point must be based on inference, modeling, and experiments. After recombination, the universe chiefly consisted of fairly cool (3000 K) hydrogen and helium, transparent to electromagnetic (EM) radiation, and nearly uniformly distributed. Unfortunately for astronomers this time of the universe is difficult to see. The difficulty arises because the dominant radiation emitted at this time was the 21 cm wavelength from the neutral hydrogen

Figure 1.1: Timeline of the universe. Courtesy: NASA

atom, which occurs when the electron switches between hyperfine energy states.¹ The lack of any other types of EM radiation lends itself to naming this period, which is thought to have lasted between 150 million to 800 million years after the Big Bang, the Dark Ages. Currently, constructing detectors of the 21 cm hydrogen line is the next great push in empirically investigating the early cosmos, and may be more fruitful for knowledge than the CMB, and is known as 21 cm tomography.

Regardless of the outcome of 21 cm tomography, the CMB indicates that fluctuations in density on the order of 0.001% were present in this post-soup universe. These imprints are believed to be caused by baryon acoustic oscillations, which are sound waves in our hot soup. The ripples in density result in differing gravitational

¹In simpler terms, energy is emitted when the electron switches from spinning counter to the proton to spinning in the same direction as the proton.

accelerations affecting matter. Therefore, denser regions tended to pull on surrounding material greater than thinner regions, resulting in a congealing of our now cold soup. The resulting clouds of cooler gas begin to grow as they pull more material into them. The source of this extra matter is the IGM.

At some point during their growth, these clouds can no longer sustain their size. This occurs when the internal thermal energy of the gas drops below twice the gravitational potential energy of the cloud (via the virial theorem). At this point, the cloud of gas begins shrinking in volume in a process known as gravitational collapse. Such events are still taking place today, but are no longer triggered by primordial perturbations. The current source of perturbations are from galactic collisions, turbulence, and shocks in the IGM driven by the death of stars.

Figure 1.2: Timeline of the universe. Courtesy: NASA

During the process of collapse, gravitational energy is converted to thermal energy. For a time this thermal energy can be radiated away, allowing collapse to continue with no thermal pressure to counter the contraction. Eventually, the density increases to the point that the cloud is no longer transparent, and radiation cannot effectively remove energy (much like our soup from earlier!). This inefficiency results in a stalled collapse, at least until temperatures reach 2000 K. At that moment, H_2 breaks apart in an endothermic reaction and hydrogen and helium ionize. These events quickly draw energy from the cloud, enabling further collapse. The collapse process ends when

the now ionized plasma (an electron-proton soup) becomes hot enough to support the self-gravitating structure of what is now a newly formed protostar.

This protostar is fueled by gravitational energy from the collapse and continues accreting surrounding gas and dust. When it has “vacuumed” up the surrounding neighborhood, the protostar begins life as a pre-main sequence star. Based on the mass of the protostar there are multiple tracks it may take to becoming a main sequence star (where hydrogen is undergoing nuclear fusion). Regardless of the precise track it takes, the star exits the protostellar phase and becomes a main sequence star.

1.2 The Life of a Star

In a star’s formative years, it subsists by fusing hydrogen to helium in its core. As time proceeds, and the hydrogen is used up, the core contracts. This shrinkage increases the density and temperatures deep in the core, making conditions suitable to begin burning the new helium atoms into carbon. From here, the process repeats by fusing carbon into oxygen and heavier elements.

At this point, the remainder of the star’s life can follow one of two possible paths. First, if it is of small to modest size, the process ends here. A star in this category is referred to as a white dwarf. These carbon-oxygen stars can continue to accrete matter, and eventually result in a nova or a Type Ia supernova. A nova occurs when conditions are ripe on the surface to cause a runaway nuclear explosion. This does not tend to destroy the star, but produces intense radiation emissions and a small amount of heavy elements.

In a Type Ia supernova, the star accretes mass until it approaches the Chandrasekhar mass (to be explained later) from below. Before the star reaches this critical mass, conditions become favorable in the core for nuclear burning to begin. The fusion of carbon starts, and a nuclear burning flame expands through the star. This process produces enough energy to tear apart the star, ejecting material at several percent of the speed of light.

If the star has a mass greater than roughly 8 to $9M_{\odot}$, further evolution occurs. More burning stages occur, first burning oxygen to silicon, and then silicon to iron. Other elements up to iron are produced (neon, aluminum, magnesium, etc.) in the process, but the primary products generated are the elements mentioned.

Figure 1.3: The onion-like distribution of elements in a star toward the end of its life. The shaded areas represent the interface between two layers where the conditions are right for fusion. The light areas are stable distributions of the elements in the star. The pressure, temperature, and density in these regions are outside of the range required for nuclear burning.

Eventually, the stellar interior has an onion-like structure with respect to atomic composition (see Fig. 1.3). There exists an iron core, surrounded by a silicon shell, which is itself surrounded by oxygen, carbon, helium, and hydrogen shells. At the interfaces of these shells conditions are right for nuclear burning. This only occurs near the transitions, as conditions inside the shell are not extreme enough to cause the entire shell to be burning at once. As atoms are advected from their shell toward the core, the temperature and densities increase to values making nuclear combustion feasible.

All great things must come to an end. In the case of these massive stars, the same

nuclear chemistry that has fueled their lives for thousands of millenia betrays them in the end. Fusion of elements higher than iron is an endothermic process, i.e. it requires a supply of energy to occur rather than adding energy to the surroundings. The result of this is that no new elements heavier than iron form, and the iron core continues to grow as silicon is burnt. Simultaneously, the extreme conditions in the core produce highly energetic photons. These photons can rapidly undo the results of the star's life by tearing the iron group nuclei apart. Photo-disintegration is an endothermic process that ends up removing sustaining energy (and subsequently gravity-countering pressure) from the core. The final process occurring is named electron capturing (EC) and results when free electrons and free protons interact. The result of the electron capture is a neutron and an electron neutrino. This results in a decrease in the lepton number due to the neutrinos easily streaming from the core, which decreases the mass required for the critical Chandasekhar mass. Thus, the iron core is growing while simultaneously losing size-preserving pressure and destabilizing. The relative importance of these last two affects depend on the mass of the original star, with photo-disintegration dominating when $M \gtrsim 20M_{\odot}$ and EC dominating otherwise. [26]

1.3 Collapse and Bounce

Ultimately, the iron core reaches the Chandasekhar mass ($1.4 M_{\odot}$), gravity overpowers the pressure in the core, and quantum mechanical effects come into play. Electrons are pushed inside protons resulting in neutrons, in a process aptly named neutronization, and the core rapidly contracts as it can no longer sustain its size. This shrinking happens homologously, meaning the velocity of the infalling material is proportional to the radius (farther out material falling faster). At a certain point inside the star the the flow becomes supersonic (r_{sonic}), and shortly thereafter peaks at a maximum infalling velocity (r_{vmax}). From this point outward, the magnitude of the velocity decreases nonlinearly and the shells higher than this radius appear to be suspended in “mid-air,” resulting in a “rain” of material.

The homologously infalling shells begin decelerating as they reach lower heights. This highly inertial material overshoots its equilibrium position, piling up in a dense fog and releasing strong pressure waves that collect at the sonic point mentioned earlier. Much like a spring, the material begins bouncing back, attempting to reach equilibrium. During this process the central core begins exceeding the nuclear matter density, resulting in a stiffening of the equation of state, and essentially becoming a rigid wall as the new protoneutron star (PNS). The result is an expanding density wave which travels behind the shockwave until they separate at $r_{v_{\max}}$. The shockwave continues propagating outward, shocking the infalling material. See Fig. 1.4 for a zoomed in view of the core during this process.

Figure 1.4: Depiction of the density during the collapse and bounce process in the iron core of the star. Black and white shading represents density, red shading signifies the outward shock. Not drawn to scale. (upper-right) The initial core at the critical mass just prior to collapse. (bottom-right) The collapsed, neutronized core (now the protoneutron star). (bottom-left) As material piles up against the PNS, a shock is generated and travels outward. (top-left) The shock overruns the lighter layers of the star, pushing them away and leaving the degenerate star behind. Eventually the shock runs out of energy and its movement stalls, leading to the focus of this work.

1.4 Standing Accretion Shock Instability

As the outward traveling shock passes through the rain of outer shells, photo-disintegration reenters the picture. Energetic photons present in the post-shock region

begin to strip apart heavy elements, extracting energy from the flow, and decreasing the pressure behind the shock. Additionally, the shock is moving outward, sweeping through the infalling matter. This expansion requires an increasing amount of energy to sustain its progress. These two factors conspire to bring the shock to a halt. Typically, the shock will overshoot its equilibrium position and reverse direction, falling back toward the star. It will then enter a period of oscillation about some nominal radius.

If the shock was to stay in one location indefinitely then this would not be much of an explosion. Therefore, shock motion must eventually revive, so that it can push through the outer layers of our onion-star, expelling them into space.

The revival of the shock depends on the introduction of new energy sources to the post-shock region. These sources must exist prior to the shock stalling and/or emerge from external perturbations after the shock has stalled. One possible source of energy is neutrino heating due to electron capture near the core. Other processes participating in the explosion may include magnetic fields and the rotation the progenitor star.

The ccSN has now reached the point we are interested in. Its evolution has taken it from a massive star, orders of magnitude larger than our own, to a compressed core of neutrons (or a black hole) roughly the size of a large US city. Outside of our city-star lies a standing shockwave, approximately 100km in radius. The thickness of this structure is the distance that two atoms can move before touching one another. Any traveler passing through this infinitesimal divider finds themselves, suddenly, in a very different world. This brand new world, spanning from cosmic lengths to atomic scales, is what we are interested in studying in the laboratory.

1.5 Connecting Astrophysics to the Lab

In order to reconcile the difference in scale between the phenomena in a star and the phenomena in a laboratory, consider one of the most famous supernovae ever

recorded, SN 1987A. The current consensus is that the progenitor star of the core-collapse supernova (ccSN) was of the red supergiant type. These stars generally have radii in the range of $200\text{--}800R_{\odot}$, or $34,775,000,000,000$ (3.48×10^{13}) cm, on average. The pellets used as the source of material in HEDLA experiments are typically on the size of a BB, with a radius of approximately a quarter centimeter (2.5×10^{-1}). This discrepancy is 14 orders of magnitude! How could experiments done in a laboratory possibly compare to these large scales?

The answer to this lies in dimensionless scaling laws that may be obtained using reasonable simplifications. One example of this is the Reynolds number, which describes the ratio of inertial forces to viscous forces, or the total momentum transfer to molecular momentum transfer. For engineers this quantity is indispensable, as it allows them to design things such as piping systems independent of physical size. Therefore, modeling a large oil pipe network in the laboratory can be done by simply scaling it down in size, as long as the Reynolds number is the same in both situations. This can work in the other direction as well, by scaling micron-sized microchannels found in rock structures up to machinable laboratory experiments, which is useful in determining other dimensionless quantities involved in porous media flows.

Ryutov et al.’s [34] seminal work on similarity criteria for HEDLA experiments provides us relations that need to be maintained in order to construct reasonable investigations. In their derivation using inviscid “fluid flow” plasma, they assume the following:

- Heat conduction and viscosity are negligible
- Accelerations caused by the explosion dominate gravitational accelerations

In the paper they present a rigorous investigation of the details of these assumptions which will not be repeated here. Their conclusion, however, is that the above are reasonable assumptions.

Ryutov et al. determined that the primary similarity criterion that can be used in comparing the supernova system and laboratory experiment is the Euler number,

defined as

$$Eu = \frac{u}{\sqrt{P/\rho}} \quad (1.1)$$

where u is the flow speed, P is the thermal pressure, and ρ is the fluid density. One may note that this looks surprisingly like the Mach number ($M = u\sqrt{\rho/\gamma P}$). In some sense, the Euler number may be seen as a composition-independent Mach number.

Additionally, they conclude that effects operating on some fraction of a characteristic timescale in one system correspond to the same fraction of the timescale in the other system. An example of such a timescale is the time required for material to move from the shock to the exit of the domain, also known as the advective crossing time (τ_{adv}). Therefore, if hydrodynamic instabilities are generated within $10 \tau_{\text{adv}}^{\text{ccSN}}$ for the ccSN setting, they should be generated within $10 \tau_{\text{adv}}^{\text{exp}}$ in the experimental setting. Note that, in principle, $\tau_{\text{adv}}^{\text{ccSN}}$ and $\tau_{\text{adv}}^{\text{exp}}$ can be very different.

Therefore, if it is possible to satisfy the above similarity criteria then creating laboratory supernovae should be possible. Nothing to this point has clarified how to go about generating such events.

1.6 High Energy Density Laboratory Astrophysics & The National Ignition Facility

High Energy Density Laboratory Astrophysics (HEDLA) is the combined informal effort of several research groups study, experimentally, astrophysics. In particular, research efforts are focused on experimentally reproducing conditions and phenomena prevalent in large-scale astrophysical settings. High energy density physics (HEDP) experiments are meant to probe the physics of matter and radiation under extreme conditions, which is the particular situation matter in a star finds itself in. Therefore, HEDP experiments are a natural fit for investigating the evolution of stellar objects.

These HEDP experiments initially started as efforts to achieve nuclear fusion in a controlled environment, via a technique called Inertial Confinement Fusion (ICF).

The original approach to ICF was to focus multiple high-power lasers at a pellet of hydrogen isotopes (a “direct drive” system). The heating caused by the incident laser beams results in the outer layer of the pellet exploding, or ablating. This ablating material moving away from the pellet requires that the material not being heated move the opposite way. Therefore, the non-heated material in the pellet is rapidly driven towards the center of the pellet. The resulting shock wave and compression in the pellet heats its core to the point of ignition (theoretically). Once this point is reached, nuclear burning (fusion) becomes possible and a flame begins to consume the core. This flame overruns the compressed pellet faster than the expansion caused by this new heating source, confining the nuclear fuel. Confinement is the result of inertia of the outer layers, and is the namesake for ICF. Direct drive experiments have long been plagued by nonuniform compression of the pellet, leading to hydrodynamic instabilities (e.g. Rayleigh-Taylor). These instabilities severely limit performance of the target and ultimately result in failed fusion attempts.

The laser system at the National Ignition Facility (NIF) uses a newer method called “indirect drive”. In this method, the pellet resides inside of a cylindrical cavity called a hohlraum (German for “hollow space”). Lasers are then shone through openings or windows of the hohlraum, heating the inner walls and creating an extremely hot plasma. This plasma then radiates X-rays, uniformly bathing the pellet. From this stage on-ward, it is as though the pellet is being driven in the direct drive manner, but by radiation created by the inner walls of the hohlraum. The intent of the hohlraum is to cause the incident radiation to be uniformly distributed over the pellet. Using this technology, NIF may be the first fusion project with a positive energy yield, making the first steps toward creating a system that can sustain itself and offer commercially viable technology. The facilities at NIF are also used for nuclear stockpile research and validation of numerical codes.

Here we consider NIF a platform for conducting the investigation into the SASI of ccSN. All that remains to nail down is a promising experimental design.

1.7 Proposed Experimental Design

One particular experimental realization of the standing accretion shock (SAS) model has been proposed recently by Ohnishi et al. [31]. In Ohnishi’s design, supersonic converging flow is created by deposition of laser energy at the outer surface of the target. This flow is reminiscent of the collapse stage of core collapse, where material begins falling onto the protoneutron star (PNS).

The converging material then strikes the hard, reflecting hemisphere mimicking the PNS core. Like the bounce stage of the ccSN, a shock then propagates outward from the core, overrunning the inward-moving material. The hope now is that the shock will reach a quasi-steady state in order to reproduce the behavior of the standing accretion shock. A quasi-steady shock is an essential requirement to studying the standing accretion shock instability experimentally.

Based on this setup, it is hoped all of the hydrodynamic instabilities present in ccSN that are believed to be the predominant driver for the revival of the supernova explosion will be captured. The one hydrodynamic aspect it does not directly address is the settling of material onto the PNS surface. In ccSN, as material falls towards the core, it cools and settles, effectively removing matter from the domain. The design by Ohnishi et al. compensates for this mass removal by leaving a toroidal opening between the reflecting hemisphere and rigid walls. Once the initial inflow and reflection occurs, material should be allowed to flow out of this opening in order to remove mass from the domain. It is unknown how the rigid wall and toroidal opening will affect the development of the bounced shock.

1.8 Aim of Work

My aim is to begin preliminary studies on the feasibility of replicating fundamental characteristics of the SAS phenomena in a laboratory setting. In particular, I will focus on a design similar to Ohnishi et al.’s. At this point, the question remains

Figure 1.5: Schematic diagram of the possible experimental system outlined by Ohnishi et al.[31].

whether all the elements of the Ohnishi scenario can reproduce the proper elements of the ccSN.

This work will explore three primary areas: 1) constraints put on the system by analytic considerations, 2) the stability of the shock in one dimension, and 3) the ability to generate a relevant multidimensional scenario that would provide insight into the SAS explosion model. The analytical considerations are probed by examining the steady-state Euler equations for an ideal fluid in spherical geometry. This investigation will provide initial insight into the differences in the flow fields between the experimental and supernova settings. In addition, it will provide the foundation for constraints on parameters when moving to time and space varying simulations.

The second part will focus on the ability to maintain stationarity of the radial

position of the shock over time. This component is critical to the success of any experiments. In order to probe the validity of the SAS model, the prerequisite is to be able to generate a standing shock wave for “long enough.” The shock must not only be stable when placed upon the grid and not perturbed, but should also withstand time-dependent deviations in the upstream flow field (e.g., clumps of varying density falling through the shock). These computer simulations are constrained by the results of the analytic investigation and are performed using the FLASH hydrodynamics code.

If it is possible to maintain the radial stability of the shock in one dimension, the goal is to repeat the investigation in two dimensions. Performing similar density perturbations in two-dimensions will allow us to determine if any multidimensional effects may impact the radial stability. If the shock remains stable, it will allow investigation of multidimensional hydrodynamic instabilities that are believed to be crucial to shock revival.

As a note, this work does not focus on generating the standing accretion shock in an experimental setting. Failing to *create* a standing shock, or failing to *maintain* a standing shock with the proper characteristics, will cast doubt on the feasibility of such experiments. Thus, the choice was made to investigate the branch most readily available to solution (maintainability of the standing accretion shock). Creation of the standing shock becomes a problem of radiation-hydrodynamics, as one must simulate the deposition of energy into the material, resulting in ablation, and then resolve the induced hydrodynamic response. That problem quickly becomes overwhelmed with parameters and seems to be a viable area of research only if the benefits of creating the shock can be turned into sensible experimental results.

CHAPTER 2

STANDING SHOCKS AND THEIR MANIFESTATION

2.1 Theoretical Background

Consider the Euler equations of compressible inviscid hydrodynamics in one spatial dimension including gravity from the central point-like mass [3],

$$\begin{aligned}
 \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) &= 0 \\
 \frac{\partial \rho \vec{u}}{\partial t} + \nabla \cdot (\vec{u} \otimes (\rho \vec{u})) + \nabla P &= \frac{\rho G M}{r^2} \\
 \frac{\partial \rho E}{\partial t} + \nabla \cdot [u (\rho E + P)] &= 0
 \end{aligned} \tag{2.1}$$

where ρ is the gas density, u is the gas velocity, $E = 1/2 u^2 + e - GM/r$ is the total gas energy, and e is the internal gas energy. We assume that pressure is related to the density and internal energy through the ideal equation of state, $P = (\gamma - 1)\rho e$, where γ is the constant ratio of specific heats. The quantities G and M are, respectively, the gravitational constant and the mass of the gravitational point source.

The instantaneous conditions across the shock are given by the Rankine-Hugoniot equations [1],

$$\begin{aligned}
 s [\rho] &= [\rho u] \\
 s [\rho u] &= [\rho u^2 + P] \\
 s [\rho E] &= [u (\rho E + P)]
 \end{aligned} \tag{2.2}$$

where $[a] = a_2 - a_1$, with a_1 and a_2 the quantity values upstream/downstream the shock, and s is the speed of the shock front. We note that Eqns. 2.2 are independent of the problem geometry, i.e. they apply equally to spherical or Cartesian geometry without regard for the surface area of the shock or its radius. This is due to their application across a volume of zero thickness (a surface) and therefore curvature effects do not come into play.

We are interested in the case of standing shocks, which arises when the shock speed goes to zero ($s \rightarrow 0$). When this occurs, the Rankine-Hugoniot relations may be simplified slightly (Eqn. 2.3).

$$\begin{aligned}\rho_1 u_1 &= \rho_2 u_2 \\ \rho_1 u_1^2 + P_1 &= \rho_2 u_2^2 + P_2 \\ u_1 (\rho_1 E_1 + P_1) &= u_2 (\rho_2 E_2 + P_2)\end{aligned}\tag{2.3}$$

These simplified equations provide a direct relation between the upstream and downstream conditions at the standing shock.

2.2 Core-Collapse Supernova Setting

The stalled core-collapse supernova shock can be considered a steady accretion shock. For the core-collapse supernova explosion, this shock must be revived. The shock revival phase has been studied by means of semi-analytic methods by Janka et al. [19].

The steady spherical shock configuration, including the necessary heating and cooling neutrino process and gravity, has been studied in detail by Houck & Chevalier [15] in the context of accretion onto compact objects. Due to the similarity of the above setting to the supernova problem, the results of Houck & Chevalier's analysis are expected to be relevant for the current application. Their results can be summarized as follows. For $\gamma = \frac{5}{3}$, the shock position is unstable due to curvature when subjected to radial and/or lateral perturbations. Also, both the shock instability growth timescale and the shock oscillation frequency are sensitive to the form of

the gas cooling function. Additionally, as γ decreases to $\frac{4}{3}$, the shock becomes only mildly unstable and its stability is progressively less sensitive to the cooling function. Finally, Houck & Chevalier found that nonradial oscillation modes with $l > 1$ are strongly stabilized and become increasingly damped as l increases.

More recently, Blondin et al. [2] investigated the stability of a standing accretion shock (SAS) in the context of core-collapse supernovae. Their one-dimensional computational model accounts for only radial perturbations, but in contrast to Houck & Chevalier [15] is limited only to hydrodynamics and gravity without heating or cooling. Blondin et al. [2] concluded that SAS systems are stable to spherically symmetric perturbations. Blondin et al. also examined two-dimensional flows with nonradial perturbations. Their findings indicate that in this case the shock evolution consists of three distinct stages. First, the isentropic and adiabatic initial state evolves to redistribute energy (primarily from thermal to turbulent kinetic energy) while the shock front roughly maintains its initial spherical shape. After the energy is redistributed, the shock behavior becomes highly oscillatory and its evolution is dominated by the $l = 1$ mode. Ultimately, the shock structure becomes dominated by the $l = 2$ mode resulting in a bipolar shock that expands self-similarly.

The general conclusions of Blondin et al. [2] are that the shock perturbations, regardless of their origin (either from nonradial acoustics inside the shock, or from the fluctuations present in the upstream region) will eventually result in dramatic change to the shock structure. The resulting morphology is dominated by low order modes, with higher order modes contributing only on small scales in the turbulent post-shock region. The authors stress the point that this evolution is not due to convection, but due to a “vortical-acoustic” feedback. Such a feedback mechanism was originally identified by Foglizzo and collaborators [9, 5, 7] in the context of accretion onto compact objects, and provided a starting point for extensive studies of SAS instability for core-collapse supernova applications [2, 8, 22, 36, 6].

Following Foglizzo’s work, SASI systems are characterized by the following key properties. The first is the stationarity of the spherical shock. Of main importance

here is that the timescale of the vortical-acoustic cycle, τ , is on the order of the post-shock advective timescale, τ_{adv} . Therefore, the shock must remain stationary for an amount of time longer than τ , so that global structure has time to cohere. [27] reported stationary shock times of 200-800 ms in their study of ccSN explosion, depending on the mass of the progenitor. Additionally, advection times are between 30 and 50 ms. The above results provide a guidance as to what conditions a successful supernova SAS experiment must produce. Following the findings of [27], we conclude that the SAS ccSN experiment would need to produce stationary shock times of approx. $4\text{--}27 \tau_{\text{adv}}$.

The second requirement for the supernova SAS experiment is that the flow be sufficiently decelerated. In the supernova case, gravity combined with neutrino cooling result in a deceleration of the flow at lower radii. In the experimental setting, gravity is not present and it is possible the flow will continue to accelerate in a nozzle-like fashion. It is conceivable that one could try to replicate this behavior via some type of cooling, but it may not be necessary as the vortical-acoustic cycle can exist in accelerated, shocked Bondi accretion. We aim to clarify this point in the current study.

2.3 Experimental Setting

The design of the SAS experiment relevant to the ccSN application poses unique challenges. Consider that the conservation of the energy of a fluid element is given by the Bernoulli equation,

$$\frac{1}{2}u^2 + \frac{\gamma}{\gamma - 1} \frac{P}{\rho} - \frac{GM}{r} = \text{constant} \quad (2.4)$$

In the supernova setting, the gravity effectively acts as the energy sink term with the flow settling down deep in the potential well. The situation is dramatically different in the experiment where sufficiently strong gravity enabling settling of the flow is not available. In the absence of the accretor (central obstacle), the flow

Figure 2.1: Structure of the shock bounded region of the exploding core-collapse supernova. The region near the center-bottom represents the nascent protoneutron star (PNS). The region just outside the PNS is characterized by intense neutrino-matter interactions and deposition of the neutrino energy. The net heating in this region can be seen as a boiler plate which drives convection in the shocked material. The convection is organized on large scales and strongly deforms the shock front.

resembles more of a supersonic nozzle flow rather than accretion. In addition, the accreted shocked gas of the SN progenitor is subject to both cooling and heating (by neutrinos). In contrast, the flow in the experimental setting is, unless specifically designed, adiabatic.

The lack of both gravity and energy source terms in the experimental setting are a possibly overwhelming discrepancy for reproducing ccSN behavior. However, as we mentioned before, the original SAS model by Blondin et al. [2] does not require heating or cooling. Instead, the apparent source of the instability seems purely hydrodynamic in origin and is related to the accumulation of turbulent kinetic energy in the shocked gas. Then, the primary roles of heating and cooling are relegated to creating conditions favorable to convective instability. This, in turn, triggers the large scale shock oscillations, as we explain below.

The aforementioned accumulation of turbulent energy in the post-shock region had been originally discovered by Fogliizzo et al. [5]. Blondin et al. [2] use this

mechanism to explain SAS in the ccSN setting (SAS scenario). Their reasoning can be summarized as follows.

Consider a nonradial perturbation of the shock, resulting in a misalignment between the shock normal and the velocity of the infalling matter. As the material passes through the shock, vorticity is generated and advected downstream, increasing the turbulent kinetic energy. This increase in turbulence results in an acoustic “rumbling” which drives pressure waves towards the shock, thereby causing more local deformation. In principle, this becomes a cycle in which the increase of turbulent activity serves to redistribute energy over time.

If the above scenario holds true, one should be able to reproduce the behavior of SAS in a laboratory setting. As we mentioned, the shocked gas in ccSN decelerates as it nears the inner boundary. This deceleration of the flow aids the accumulation of the perturbations, in particular of the turbulent energy required in the SAS scenario. This accumulation of perturbations is essential for successful experiments of SAS.

Figure 2.2: High Energy Density Physics counterpart. (top) Initial state of the system. Energy is deposited at the inner and outer surfaces. The two supersonic, ablative flows are then driven together. Given suitable conditions it is possible for this to evolve into a system with one standing shock. (bottom) Final steady shock configuration. Material from the outer layer travels through the quasi-steady shock and exits through the vent.

CHAPTER 3

SAS EXPERIMENT DESIGN STUDY

There exist two well-known numerical solutions involving spherical shocks. The first problem is the strong, point-like explosion, also known as the Taylor-Sedov problem [37]. The second problem was introduced by Guderley [11] and is concerned with the evolution of the imploding spherical shock. The imploding case shares important characteristics with inertial confinement fusion applications and early phases of the stellar core-collapse [13, 12, 33, 19, 18]. The Taylor-Sedov point-like explosion was originally obtained to characterize nuclear explosions and is now frequently used in the analysis of supernova remnants.

Both of the aforementioned problems have been extensively used as benchmarks for shock hydrocodes. Due to its similarity, the SAS design problem is no less challenging. In particular, the numerical scheme not only must resolve the nearly stationary shock well but also produce an oscillation-free post-shock flow.

3.1 Mathematical Considerations

$$\begin{aligned}
 \frac{d}{d\xi} (\rho u A) &= 0 \\
 \frac{d}{d\xi} (u^2 \rho A) + A \frac{dP}{d\xi} &= \rho A f \\
 \frac{d}{d\xi} [u A (\rho E + P + \rho \Phi)] &= 0
 \end{aligned} \tag{3.1}$$

We investigate the stability properties and flow structure of the setting by numerically integrating the ODE system (Eqn. 3.1) obtained from the steady-state system of Eqn. 2.1 in a one dimensional coordinate system with arbitrary cross-sectional area, $A(\xi)$. The system is subjected to a body force, $f(\xi)$, and a potential energy function, $\Phi(\xi)$. This first order system requires one set of boundary conditions for ρ , u , and P . For the problem at hand, this was chosen to be either at the inner boundary or at the post-shock shock front, depending on the particular study of interest. By choosing to place the boundary conditions at the inner boundary, one may strictly enforce the transonic condition for a given post-shock region thickness, but it will not guarantee satisfying the critical Mach number condition. On the other hand, selecting the values at the shock front guarantees that the Mach number is above the critical value, but does not guarantee that flow stays subsonic for a given shock thickness. In order to evaluate the flow variables for the upstream pre-shock environment the post-shock values are first obtained at the shock. Then, application of Eqn. 2.3 determines the pre-shock values at the jump. From here, numerical integration can then be continued

Figure 3.1: Hydrodynamic structure of the standing accretion shock. (a) Supernova setting. (b) Proposed experimental system. Shown are the density, (negative) velocity, pressure, and entropy distributions as a function of radius. Note that in the supernova case the vertical scale is logarithmic and the inner region is not shown. In this case the solution has a settling character while the shocked material accelerates and leaves through the inner boundary in the experimental setting. The flow in both cases is isentropic. Also note that the post-shock region is relatively thin in the considered experimental setting. See the text for a detailed discussion.

to arbitrary pre-shock heights. See Fig. 3.1 for a comparison of the resulting solutions for the system of Blondin et al. and the experimental system.

The system in Eqn. 3.1 was integrated using an adaptive step size 4/5 Runge-Kutta scheme [38]. In verifying Blondin et al.’s [2] solution to the system with gravity, a 1/5 stiff solver [38] produced marginally better results at small radii. Both solvers used an absolute error tolerance of 1×10^{-9} and relative error tolerance of 1×10^{-8} .

3.1.1 Analytic Constraints

Post-Shock Aspect Ratio. There are two physical constraints that will allow us to investigate characteristics of the experimental system analytically. The first constraint is that the flow at the inner boundary is subsonic ($M < 1$). The second restriction is that solutions are physically admissible. Formally, it is possible to obtain solutions with negative pressures when the upstream Mach number falls below a certain critical value.

Using the Prandtl-Meyer relation [1] and Eqn. 2.2 the following relation between the pre-shock and post-shock pressures can be obtained as a function of post-shock Mach number (for a full derivation see [1]),

$$P_{\text{pre}} = \frac{2\gamma M_{\text{post}}^2 - (\gamma - 1)}{\gamma + 1} P_{\text{post}} \quad (3.2)$$

Therefore, the only admissible post-shock Mach numbers are those which produce positive pre-shock pressures. In particular, if we allow the pre-shock pressure to be zero (cold, infalling gas in the supernova setting), we may define the critical post-shock Mach number, M_{crit} , such that $M_{\text{post}} \geq M_{\text{crit}}$, as

$$M_{\text{crit}} = \sqrt{\frac{\gamma - 1}{2\gamma}} \quad (3.3)$$

Note that this value depends only on the compressibility of the gas (γ) and not a particular state realization (pressure, velocity, etc.).

Figure 3.2: Mach number in the post-shock flow at the shock front as a function of γ . Note that for $\gamma = 4/3$ and $5/3$ the critical Mach number is relatively high.

In Fig. 3.2 the values of the critical Mach number as a function of the constant ratio of specific heats is shown. As the equation of state softens (γ decreases) the range of admissible values for M_{crit} increases, making all solutions admissible (with $\text{Ma} < 1.0$) in the limit as $\gamma \rightarrow 1$. As the equation of state becomes increasingly stiff the upper limit of the admissible range increases. For $\gamma \rightarrow \infty$, the viable Mach numbers must obey $M_{\text{admissible}} \in [1/\sqrt{2}, 1)$.

Assuming the flow is isentropic, it is possible to derive a relation uniting the ratio of area at the shock height to the critical area where the flow at the outlet becomes Mach 1. This relationship depends only on γ and the Mach number at the desired radius.

$$\frac{A_{\text{shock}}}{A_{\text{inner}}} = \left(\frac{\gamma + 1}{2} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{\left(1 + \frac{\gamma-1}{2} M_{\text{post}}^2 \right)^{\frac{\gamma+1}{2(\gamma-1)}}}{M_{\text{post}}} \quad (3.4)$$

Nondimensionalizing the thickness gives us the aspect ratio of the post-shock region as a function of the outflow section and shock radii.

$$\alpha = \frac{r_{\text{shock}}}{r_{\text{inner}}} \quad (3.5)$$

Combining Eqns. 3.3, 3.4, & 3.5 it is possible to determine the maximum attainable

Figure 3.3: Critical solution profiles for varying γ . For each of these solution the interior Mach number reaches 1.0 at the left, implying that these are the “thickest” admissible solutions for the specified γ . Note that for larger γ individual profiles tend to differ from one another, whereas smaller γ ’s produce pressure-density and Mach-velocity profiles that are extremely similar.

aspect ratio as a function of γ .

$$\alpha_{\text{crit}}^2(\gamma) = \left(\frac{\gamma + 1}{2} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{\left(1 + \frac{\gamma-1}{2} M_{\text{crit}}^2(\gamma) \right)^{\frac{\gamma+1}{2(\gamma-1)}}}{M_{\text{crit}}(\gamma)} \quad (3.6)$$

Therefore, the aspect ratio of the domain must obey $1 < \alpha < \alpha_{\text{crit}}(\gamma)$. A plot of the critical aspect ratio can be seen in Fig. 3.4. A diagram showing how different aspect ratios appear with a fixed inner radius is shown in Fig. 3.5.

Figure 3.4: Relation between the critical aspect ratio (α_{crit}) and the ratio of specific heats (γ). As $\gamma \rightarrow \infty$, $\alpha_{crit} \rightarrow 0$. Therefore, for stiff equations of state (i.e. it is difficult to compress the gas) the post-shock region is thin. On the other hand, as $\gamma \rightarrow 1$, the allowed compression of the gas increases, which is characteristic of isothermal processes.

Figure 3.5: Two-dimensional depiction of the thickness of the post-shock region for three aspect ratios. The numbers in each section are their respective α values.

Euler Number and Pre-Shock Mach Number. Instead of using basic physics constraints we can also turn to known HEDLA values to aid investigation. In order to accomplish this two new elements from experiments need to be incorporated. The first is the Euler number described in Sec. 1.5. This quantity provides a measure of the degree of similarity between flows in the HEDP laboratory and flows in astrophysical settings. The supernova explosion models of Kifonidis [21] and Scheck [35] indicate that Euler numbers immediately downstream from the shock are approximately 0.6-0.8. To err on the side of caution, we will consider $0.5 \leq Eu \leq 0.9$ to be a reasonable range of Euler numbers.

The second new element is the Mach number of the flow immediately upstream of the shock (M_{pre}) and is representative of the “strength” of the discontinuity. In

ccSN scenarios M_{pre} can be on the order of 10-100. [19] To put a range of feasible pre-shock Mach numbers into perspective, previous experiments have generated up to approximately Mach 10 reasonably. This value can be raised to 20 in certain situations. Therefore, if we consider that we, at a minimum, want relatively strong shocks we can use $M_{\text{pre}}^{\text{min}} = 3$ as a lower bound for our parameter studies. On the other side, being cautious it is reasonable to take $M_{\text{pre}}^{\text{max}} = 10$ as the upper bound.

Using the Prandtl-Meyer relation given in the previous section it is possible to construct a relation between the pre-and-post-shock Mach numbers,

$$M_{\text{post}}^2 = \frac{1 + \frac{\gamma-1}{2}M_{\text{pre}}^2}{\gamma M_{\text{pre}}^2 - \frac{\gamma-1}{2}} \quad (3.7)$$

The post-shock Mach number can be rewritten as

$$M_{\text{post}} = \text{Eu} / \sqrt{\gamma} \quad (3.8)$$

When this is substituted into Eqn. 3.7, the Euler number becomes a function of only M_{pre} and γ ,

$$\text{Eu} = \sqrt{\gamma \left(\frac{1 + \frac{\gamma-1}{2}M_{\text{pre}}^2}{\gamma M_{\text{pre}}^2 - \frac{\gamma-1}{2}} \right)} \quad (3.9)$$

In order to look at the extremes of Eqn. 3.9 we can take limits with respect to the pre-shock Mach number. Specifically, we are interested in the behavior as we approach a transonic regime (Mach number of 1) and an infinite Mach number. Note that because the expression for Eu is a rational function, the limit to infinity should be a horizontal asymptote. Additionally, the orders of the numerator and denominator are equivalent, implying the function is monotonic, and therefore the asymptote must be either an upper or lower bound,

$$\lim_{M_{\text{pre}} \rightarrow \infty} \text{Eu}(\gamma, M_{\text{pre}}) = \sqrt{\frac{\gamma-1}{2}} \quad (3.10)$$

$$\lim_{M_{\text{pre}} \rightarrow 1} \text{Eu}(\gamma, M_{\text{pre}}) = \sqrt{\gamma} \quad (3.11)$$

For a fixed γ , $(\gamma - 1) < 2\gamma$, implying that the post-shock Euler number must decrease as the pre-shock Mach number increases, which implies that the asymptote is a lower bound. As we consider γ values between 1.0 and 2.0, it appears Euler number values will span from 0.0 to $\sqrt{2}$, $\forall M_{\text{pre}} > 1$. This analysis shows that it's possible to encompass the required range of Euler numbers, but the “sweet spot” satisfying both conditions is best analyzed graphically. Figure 3.6 shows, for several values of γ , the relation given by Eqn. 3.9. The ranges for both parameters have been shaded and the area where they overlap indicates viable experimental territory.

Figure 3.6: Plot of the post-shock Euler number as a function of pre-shock Mach number for multiple γ values. Overlap of shaded regions indicates feasible experimental conditions.

3.1.2 Laboratory Influence on Semi-Analytic Boundary Conditions

While it is possible to analytically extract some information about the experimental system from the Euler equations, they do not allow simple closed-form solutions. The alternative is to numerically integrate the system of Eqn. 3.1. Solutions obtained in this manner allow us to investigate the effects that conditions at the nozzle boundary (the outflow) have on pertinent quantities at the shock front.

In Sec. 3.1 we investigated the parameter α that describes the thickness of the post-shock region and its relation to the Mach number immediately downstream from the shock. The aim of this portion of the study is to determine if conditions prevalent in HEDP experiments are compatible with the envisioned SASI experiment. The primary concern at this point is that densities, pressures, and temperatures are compatible with this constraint on aspect ratio. Therefore, the deliverables of this section are a set of ranges of primitive variables (ρ, P, u) ranges for use as boundary conditions for the ODE system. The requirements on these ranges are that they are both a subset of HEDP ranges and fulfill experimental requirements.

Semi-analytic Setup. With our deliverable quantities determined we can begin to probe the steady one-dimensional Euler equations numerically. In order to integrate the system we need to specify the values of density, velocity, and pressure at some location in the domain. We consider the post-shock region to be the most important part of the problem. Therefore, our two most reasonable choices are to specify the values at the outlet or at the downstream side of the shock. The simplest choice is to define the values at the outlet (inner radius). The inner boundary has the implicit constraint that the flow be subsonic, i.e. the Mach number is less than one. For HEDP experiments, the temperature (T) is $\sim 10^6$ K. A diagram of relevant HEDP parameters is given in Fig. 3.7. In order to keep the study general we do not want to make assumptions regarding the composition of the fabricated target at this point. This can be done by constructing a composition-independent temperature measure. We start with the ideal gas law (Eqn. 3.12), which provides the relation between the

Figure 3.7: Astrophysical and HEDLA parameter diagram.

density, pressure, and temperature.

$$P = \frac{R}{\mu} \rho T \quad (3.12)$$

Using CGS units, μ is the molar mass in units of g/mol , R is the ideal gas constant ($8.3145 \times 10^7 \text{ erg} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}$), we estimate temperature using Eqn. 3.12 as a function of density and pressure.

$$\hat{T} \equiv \frac{P}{\rho} \approx \frac{10^{14}}{\mu} \quad (3.13)$$

For typical composition the molar mass is between 1 and 250 g/mol , while simple molecules range between 10 and 1000 g/mol . Thus, if the experimental system uses only simple compounds then $10^{14} < \hat{T} < 10^{17}$. In our study we will investigate a somewhat broader range, 10^{12} to 10^{19} .

Without constraining ourselves to tight ranges on the primitive variables at this point it is possible to preliminary test the the state values at the inlet to determine whether there exists a combination that satisfies $10^{12} < \hat{T} < 10^{19}$. Using the values

from Tab. 3.1 the values for $\hat{T}$ were computed using the Latin Hypercube Sampling (LHS) algorithm to sample the parameters (Sec. A.2). The probability distributions shown in Fig. 3.8 are for a run of 500,000 samples with $\gamma = 5/3$. The profiles obtained with $\gamma = 4/3$ are identical.

Table 3.1: Initial sampling region to determine temperature constraint feasibility.

Quantity	Minimum	Maximum
ρ	10^0	10^4
$-u$	10^{-10}	10^{10}
P	10^{12}	10^{18}

Our sample produced about 250,000 samples with Mach numbers less than 1 at the nozzle boundary. Out of the resulting group of 250,000, approximately 34,000 (13.6%) lie in the desired $\hat{T}$ range. Figure 3.8 provides the probability distribution of the temperature as well as the histograms of the primitive variables. Figure 3.9 is a histogram of the Mach numbers produced from the sample. Note the favoring of low Mach number values, while Mach number values greater than 0.5 are roughly equiprobable.

While these results provide a first insight into the distribution of viable parameters at the inner boundary, it does not help clarify how these values are correlated. In order to graphically investigate this Fig. 3.10 shows the parallel coordinate plots for both the primitive variables and the derived quantities. From the primitive variable plot, it can be seen that density and pressure are strongly, positively correlated. Density then appears to be anticorrelated with velocity but positively correlated with pressure. For the derived quantities $\hat{T}$ and velocity have a very strong positive correlation (nearly 1:1). Temperature appears to be uncorrelated with the Mach number at the nozzle boundary (M_{outlet}), while velocity is strongly, positively correlated.

Figure 3.8: Probability distribution estimate of $\hat{T}$ and histograms of primitive variables for which $10^{12} < \hat{T} < 10^{19}$. Only results with Mach less than 1.0 at the outflow are shown here.

Figure 3.9: Histogram of M_{outlet} . Only Mach numbers greater than 0.1 at the outflow are shown here. This is to aid in the readability of the plot, as the bulk of the runs have Mach numbers less than 0.1.

Figure 3.10: Parallel coordinate plots of the primitive and derived variables at the inlet. These plots are intended to qualitatively demonstrate the correlation between the different quantities. (top) Coloring by the density, the plot shows that density is positively correlated to pressure, while density and pressure are negatively correlated to velocity. (bottom) Coloring by $\hat{T}$ there is a strong positive correlation (roughly 1:1) between $\hat{T}$ and the velocity, while neither of these variables are correlated to M_{outlet} .

Table 3.2: Correlation coefficients and p-values for the primitive quantities at the inner boundary. (a) Pearson correlation coefficients. (b) P-values for the correlation. Note that all variables are significantly correlated to each other.

(a)

Primitive Correlation Coefficients			
	ρ	P	u
ρ	1.0000	0.3937	-0.3605
P	0.3937	1.0000	0.6406
u	-0.3605	0.6406	1.0000

(b)

Primitive Correlation P-Values			
	ρ	P	u
ρ	1	0	0
P	0	1	0
u	0	0	1

Table 3.3: Correlation coefficients and p-values for the derived quantities at the inner boundary. (a) Pearson correlation coefficients. (b) P-values for the correlation. Note that M_{outlet} is not significantly correlated to $\hat{T}$.

(a)

Derived Correlation Coefficients			
	$\hat{T}$	u	M_{outlet}
$\hat{T}$	1.0000	0.9245	-0.0079
u	0.9245	1.0000	0.3739
M_{outlet}	-0.0079	0.3739	1.0000

(b)

Derived Correlation P-Values			
	$\hat{T}$	u	M_{outlet}
$\hat{T}$	1	0	0.3069
u	0	1	0
M_{outlet}	0.3069	0	1

Table 3.4: Ranges on the primitive variables at the inner radius for the full semi-analytic runs.

Quantity	Minimum	Maximum
ρ	10^0	10^4
$-u$	10^5	10^9
P	10^{12}	10^{18}

3.1.3 Semi-Analytic Simulations

Summary of Analytic Results. Before continuing on, let us summarize the results obtained so far. Section 3.1 provided analytic insight into the behavior of a standing spherical shock based on the physical assumption that the pressure upstream of the shock is positive. In particular, it was shown that the post-shock Mach number (M_{post}) must be greater than some critical Mach number (M_{crit}). Furthermore, a simple expression for M_{crit} was provided in Eqn. 3.3, which is dependent only upon the ratio of specific heats of the gas (γ). Therefore, in order to provide physically meaningful solution profiles, the post-shock Mach number must obey $M_{\text{crit}}(\gamma) < M_{\text{post}} < 1$.

From this result, it was possible to determine a critical aspect ratio for the post-shock region (α_{crit}). This is a supremum on the upper bound that allows subsonic flow at the lower boundary. As this aspect ratio is determined solely by the critical Mach number it must also be a function only of γ . Figures 3.5 and 3.4 depict this relation. Taking into consideration that we are concerned with realistic systems, the post-shock region must have a substantive thickness. This leads to the result that $1 < \alpha < \alpha_{\text{crit}}(\gamma)$.

The final result provided by the analytic study is that the post-shock Euler number is a function of only the pre-shock Mach number and γ . Combining this relation with estimated bounds on the Euler number and Mach number for HEDP experiments, it was possible to see which combinations of M_{pre} and γ produce experimentally feasible scenarios possible in the lab. Figure 3.6 illustrates this result.

Conditions at the Inner Boundary for Laboratory Setting. The second set of flow parameter bounds comes from the analysis mentioned in Sec. 3.1.2. These results help connect the experimental conditions and boundary conditions required by the Euler equations. One of the first outcomes of that section is the material-independent temperature that allows us to perform these studies without delving into the details of target fabrication (Eqn. 3.13). Combining this with the density and pressures estimated to be possible, this study provides us with probability estimates for each primitive variable. While it is nice to know the distributions of density and pressure, the only range in the experiment not known a priori was for the velocity.

Therefore, the most helpful result following from the analysis was a narrowing of the range of feasible velocities at the inner boundary. The end result is having feasible ranges for all three primitive variables (ρ, P, u) at the inner boundary. These ranges are summarized in Tab. 3.4. With this information, it is possible to integrate the system of Eqn. 3.1 and obtain full solution profiles.

Full Semi-Analytic Simulations. At this point we have at our disposal three independent results: a set of feasible primitive variables at the inner domain boundary, an upper bound on the thickness of the domain, and a relation between the pre-shock Mach number and the experimentally significant Euler number. What is still missing is the link between these results. The aim of this section is to produce a final set of primitive variable parameters for the full, time-dependent simulations.

In order to proceed forward, simulations of the entire one-dimensional, steady-state system must be performed. By using the current parameters it is possible to integrate Eqn. 3.1 starting from the inner boundary as described in Sec. 3.1. The first analytic result regulates how far we can integrate away from the inner radius. This distance provides the radial position of the shock. Once the solution is obtained at the shock front, it can be reconciled with the second analytic constraint that the pre-shock Mach number is in the proper range.

Generation of the full runs utilized LHS, again assuming uniform distributions of parameters. This may appear asinine as distributions for the parameters were

generated in the previous section. The rationale behind assuming uniform profiles once more is two-fold. First, the distributions determined previously are not, in general, independent from one another (see Tab. 3.2). In order to use LHS with distributions in a simple manner, the probability distributions of the parameters must be uncorrelated. This allows the practitioner to permute the bins to obtain samples as outlined in Sec. A. The problem of LHS with Dependence (LHSD) is an active area of research that goes beyond the scope of this work. Second, uniform distributions provide a consistency check with the previous results.

The above procedure separates the original boundary condition parameter space into viable and non-viable groups. The set of viable parameters are those which satisfy all previously mentioned criteria. All results were obtained for $\gamma = 5/3$.

Figure 3.11: Relation between viable primitive variables at the outflow boundary and the aspect ratio of the post-shock region for $\gamma = 5/3$. Note the correlation with respect to M_{inner} . The lower bound is due to the restriction on the pre-shock Mach number. The upper bound is a result of the post-shock Mach number having a lower bound.

Figure 3.12: Relation between viable primitive variables at the outflow and the pre-shock Mach number for $\gamma = 5/3$. Notice that the pre-shock Mach number ends up controlling the lower bound on the inner Mach number.

Figure 3.13: Relation between viable primitive variables and the temperature at the outflow for $\gamma = 5/3$. There is a tight distribution of $\hat{T}$ with respect to the inner velocity. This can be explained by the inner Mach number having a relatively tight bound ($0 < M_{\text{inner}} = |u|/\sqrt{\gamma\hat{T}} < 1$). Note that in practice the lower bound on M_{inner} is not actually 0, but is near 0.45.

3.2 Numerical Simulations

3.2.1 Simulation Setup

In our design study the flow equations are solved using the FLASH code [10]. The hydro kernel of the code is the PPM solver [4]. The simulations were performed in spherical geometry in one and two spatial dimensions. We imposed a supersonic inflow at the outer boundary, and allowed for free outflow at the inner boundary. We imposed reflecting boundary conditions in the lateral direction.

3.2.2 One-Dimensional Models

Radial Stability to Upstream Perturbations. We investigated the radial stability properties of the SAS experimental designs by means of numerical simulations for admissible sets of initial and boundary conditions identified in the previous section. In the simulations the upstream density at the supersonic inlet were perturbed around its nominal steady-state values. Velocity and pressure were not perturbed.

Perturbations to the density at the inlet had the form

$$\rho_1(t) = \rho_0 [1 + \kappa \sin(\omega t)] \quad (3.14)$$

where ρ_1 is the perturbed value, ρ_0 is the steady-state value at the supersonic inlet, κ is the perturbation amplitude, and ω is the perturbation frequency.

In order to have a rough estimate on the bounds for the perturbation frequency we start with the case of SASI in ccSN, where formation of post-shock flow structures occur within $10\tau_{\text{adv}}$. If the development of instabilities happen within 10 advective times, only 10 or so perturbation waves will be able to fit in the post-shock region. Therefore, we have a reasonable upper bound on the number of perturbations in the post-shock. In order to provide a safety margin, we will use a maximum of 20 perturbations in the post-shock flow. On the lower end of the frequencies, we allow for at least 1 perturbation wave in the post-shock region. We can express

Table 3.5: Parameter space for the time-dependent, one-dimensional simulations.

Quantity	Minimum	Maximum
γ	$5/3$	$5/3$
ρ	10^0	10^4
$-u$	10^5	10^9
P	10^{12}	10^{18}
α	1.0	1.2
β	0.05	1.0
κ	0.0	0.25

the perturbation frequency, ω , as a function of the average steady-state, post-shock velocity and a fraction of the steady-state, post-shock width,

$$\omega = 2\pi \frac{|\bar{u}_{\text{post}}|}{\beta L_{\text{post}}} \quad (3.15)$$

In Eqn. 3.15, β controls how many perturbations, $N_{\text{waves}}^{\text{post}}$, can fit into the post-shock region. The above two values are related, $\beta N_{\text{waves}}^{\text{post}} = 1$. The ranges for all parameters in the one-dimensional runs are given in Tab. 3.5.

The results of our investigations on the radial stability are shown in Fig. 3.14. The response of the shock to the upstream perturbations can be characterized by comparing the frequency of shock oscillations to the frequency of the upstream perturbations. We found that in our time-dependent simulations of the experimental system with perturbations the shock eventually develops a slow drift with time, and eventually leaves the domain. Due to this behavior, we had to construct a suitable metric for characterizing shock stability. For the present application, we used the instantaneous shock radius. The shock was considered steady if its position remained within $\pm 5\%$ of its initial radius. Before the frequency of the shock front could be determined, the drift over time needed to be removed. This was accomplished using a Butterworth high-pass filter [24] with a cut-off frequency of 30, which defined a heuristic upper bound for the shock drift. Due to sampling the shock position at unequal time intervals, the Lomb-Scargle method of least-squares spectral analysis [32] was used to

Figure 3.14: Radial oscillation frequency of the shock as a function of perturbation frequency. Note that the bulk of the results (approximately 100 simulations) lie near (1,1), but some do not. These outlying results are numerical artifacts of the method used to determine the frequency on the discrete mesh.

construct the power spectrum. This also enables hypothesis testing by providing p-values.

The data points that are not consistent with 1:1 coupling of the shock oscillations due to the upstream perturbations have p-values values multiple orders of magnitude greater than the other results. This suggests that they are most likely numerical artifacts. We have confirmed this through the visual inspection of the corresponding hydrodynamic models by failing to find evidence for perturbation growth in the post-shock region. From these results, we conclude that the outlying models are not indicative of the true behavior of these model experimental systems.

Advective Timescales. Of particular interest from the experimental design point of view is the amount of time the shock remains steady. The theory by Foglizzo

Figure 3.15: Scatter plot of the number of stable advective crossing times as a function of initial aspect ratio. Simulations were run until the shocks moved $\pm 5\%$ from their initial position. The spread of α can be interpreted as the aspect ratio not being a unique function of the problem parameters (initial conditions).

predicts that at least 10 advective crossing times (τ_{adv}) are needed for new, SASI-like flow features to develop. This measure nondimensionalizes the real evolutionary time by breaking it into increments based on how long it takes a fluid parcel to travel from the shock front to the inner boundary. Therefore, if the shock stays stable for enough advective times, the full system should be able to develop pertinent features.

For given post-shock flow conditions, the advective timescale directly depends on the systems aspect ratio, i.e. how large the post-shock region is. The simulations' advective crossing times are shown as a function of the unperturbed aspect ratio of the shock in Fig. 3.15. Note that as the aspect ratio increases, the number of sustainable advective times decreases. The distribution shows significant spread, which is due to α not being a unique function of the problem parameters (i.e. initial flow conditions).

3.2.3 Two-Dimensional Models

Until this point the focus has been on strict quantitative measures of feasibility. It has been shown that, in one-dimension, solutions in spherical geometries are stationary for timescales greater than those predicted to be required by theory and observation. We have also refined the initial parameter space to couple constraints imposed by both the laboratory and the true physical setting. The final portion of this work is dedicated to determining if the predicted behavior of post-shock turbulence buildup is observed. In order to investigate this topic, multidimensional models need to be constructed.

In our two dimensional studies, we used a smaller sample of admissible designs than in one dimension. The summary of the initial conditions for these experiments are shown in Tab. 3.6.

Table 3.6: Parameters of select two-dimensional models.

Quantity	Model A	Model B	Model C	Model D	Model E	Model F
α	1.05	1.05	1.125	1.125	1.2	1.2
β	-	0.5	0.25	0.75	0.1	0.1
κ	0	0.05	0.1	0.1	0.05	0.25

Upstream Perturbations. The systems were perturbed by imposing variations in the upstream density. Similar to the one dimensional case, the two dimensional perturbations are of the form

$$\rho_1(t, \theta) = \rho_0 [1 + \kappa \sin(\omega t) \sin(\eta \theta)] \quad (3.16)$$

where the previously defined parameters from Eqn. 3.14 are the same, and the new parameters η and θ are the angular frequency in the lateral direction and the angle describing the lateral direction, respectively. Such a function produces an egg crate-like, or bubble-like, distribution about the nominal value ρ_1 . The goal of introducing these lateral oscillations is an attempt to help trigger vorticity generation at the

shock front. The lateral frequency was chosen to maintain a roughly 1:1 aspect ratio in dimension for each of the peaks.

When the egg crate reaches the shock, the differing densities will cause the local shock speed to deviate from zero, as shown in the Rankine-Hugoniot relations (Eqn. 2.2). In particular, high density bubbles will cause the shock to recede, with lower density bubbles causing it to expand. As the radially moving material crosses the now oblique shock it will experience nonuniform accelerations. The shearing force caused by this interaction should cause the infalling material to spin, resulting in a vorticity increase. While this is theoretically feasible, the question remains whether or not this momentum transfer will be sufficient to produce interesting behavior.

The outcome of these simulations point to non-unique behavior of the system

Figure 3.16: Snapshot of Model F profiles. (top left) Density (top right) Pressure (bottom left) Radial Velocity (bottom right) Entropy $\propto P/\rho^\gamma$

for the parameters given in Tab. 3.6. Therefore, in lieu of presenting flow fields for each parameter sample, a snapshot of Model F is presented in Fig. 3.16. With regard to timescales, the two-dimensional simulations appeared comparable to their one-dimensional counterparts.

Flux Decomposition. More insight into the nature of the multidimensional characteristics of our designs can be obtained by analyzing individual components of the energy flux vector [16]. This procedure is routinely applied in the analysis of convective flows in stellar astrophysics [30, 29].

Averaging the multidimensional energy equation (Eqn. 2.1. over the angular coordinate in two-dimensional spherical geometry, one arrives at

$$\frac{\partial}{\partial t} \left[\overline{\rho c_v T} + \frac{1}{2} \overline{\rho \vec{v} \cdot \vec{v}} - \overline{\rho g r} \right] + \frac{\partial}{\partial r} (F_C + F_K) = 0 \quad (3.17)$$

where the overbar denotes the lateral mean of the quantity and $\vec{v} = (u, w)$, with u and w as the radial and lateral components of velocity, respectively. In Eqn. 3.17, the first term represents the time derivative of the total energy density, including the internal energy density, the kinetic energy density, and the gravitational energy density. The second term denotes the spatial changes of energy fluxes in the domain, with F_C the convective (or enthalpy) flux and F_K the kinetic flux. These quantities are defined as

$$F_C = -c_p \overline{\rho u T'} \quad (3.18)$$

$$F_K = -\frac{1}{2} \overline{u (\rho \vec{v} \cdot \vec{v})} \quad (3.19)$$

In the above equations, primed quantities denote the fluctuation about the lateral mean (e.g. $q' = q - \bar{q}$). The total energy flux in the domain is given by $F_C + F_K$ and is not, in general, constant.

In addition to the above decomposition, one may write the equation for the conservation of kinetic energy (E) as

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial r} (F_K + F_P) - (E_B + E_P) = 0 \quad (3.20)$$

Figure 3.17: Flux distribution for Model F.

The second term is the divergence of the kinetic and pressure fluxes. The quantity F_P describes the flux of acoustic (pressure) energy and is defined as

$$F_P = \overline{uP'} \quad (3.21)$$

The third term is an expression of the rates of work performed on a fluid element, with E_B being the buoyant work, or work done by the mean pressure, and E_P is the work done by fluctuating pressure. These two quantities are defined as

$$E_B = g\overline{\rho'u} \quad (3.22)$$

$$E_P = \overline{P'(\nabla \cdot \vec{v})} \quad (3.23)$$

The total energy flux (F_T) may be averaged over time. Assuming the system reaches a statistical steady state, sufficiently long time averages should yield

$$\langle F_T \rangle = \langle F_K \rangle + \langle F_C \rangle \quad (3.24)$$

at any radial coordinate in the domain. While this implies that $\langle F_T \rangle$ is constant throughout the domain, $\langle F_K \rangle$ and $\langle F_C \rangle$ may vary as a function of position.

Figure 3.17 shows the flux distribution for Model F. Note that the pre-shock flow is dominated by kinetic energy. As the material passes through the shock, its energy is converted into pressure energy. Shortly thereafter it is distributed primarily to convective flux. The dominance of convective energy flux continues to the inner boundary. In the supernova SASI setting, this convective flux would then be converted into another form of energy flux. Because the convective flux is not sandwiched between the kinetic layer and another layer, this is representative of behavior that is not expected in the SASI scenario.

CHAPTER 4

DISCUSSION

4.1 Analytic and Semi-Analytic

Post-Shock Aspect Ratio. In the first part of the analytic results presented above, it was determined that the relative thickness of the post-shock region is bounded from above if only $\text{Ma} < 1$ flows are considered. It was shown that this bound is a function of only gas composition and is independent from any absolute geometric coordinate. In order to explain why this is, let us look at Fig. 3.3. These solutions bear a striking resemblance to the flow profile in the converging section of a converging-diverging nozzle. In particular, nozzle is it most similar to is the Venturi tube, and in the extreme case discussed below, the de Laval nozzle. A nozzle system such as this consists of two sections. The first is a converging section in which the cross-sectional area decreases in the direction of the flow. The second is the diverging section, where the cross-sectional area grows. Joining these two pieces, at the minimum cross-sectional area, is the throat. Prior to the throat the profiles of density and pressure steadily decrease, while the speed of the flow increases.

The explanation of this may be interpreted as follows. From the steady-state continuity equation, the mass flux must be constant ($\rho u A = \text{const}$). As the flow approaches the throat the cross-sectional area decreases, which implies that ρu must increase. The flow near the throat has gained momentum, but the nozzle system is assumed to be adiabatic, i.e. no energy is entering or exiting the system through the boundaries. Therefore, with a fixed energy budget, energy must be drawn from other

Figure 4.1: Diagram of a de Laval nozzle that resembles the experimental system under consideration.

sources to allow for the increase in momentum. The only other energy term in this system is the thermal energy. Thus, the increase in momentum leads to a decrease in temperature, which is equivalent to a decrease in pressure.

When the flow reaches the throat there are two possible scenarios. The first is the Venturi tube case, where the flow remains subsonic in the diverging section. The second is the de Laval nozzle, in which conditions are right to make the flow supersonic in the diverging section. If this case is to occur, the flow must become transonic at the throat (“choked” flow). Equation 3.4 makes the assumption of choked flow, and allows us to determine the aspect ratio of the post-shock domain, i.e. the relative width between the start of our nozzle and the throat. What this does not address is why we have not allowed the flow to become supersonic again, and instead have chosen a design with a vent to an ambient area.

Let us try to think about how a spherical geometry can be viewed as a converging-diverging nozzle. As a fluid parcel travels toward the origin along a ray, the surface area of the spherical shell it resides in decreases. This is the convergent portion of the nozzle. As the parcel moves toward lower radii, its Mach number increases, until it

eventually reaches 1. Once this happens, the parcel should transition into supersonic. But, as it moves through the throat it never sees the surface area increase. Being unable to expand, the flow is stuck in a transonic state for the remainder of its journey. If one takes into account a constant mass flow rate as well as this new fixed Mach number, it can be readily seen that $P\rho \propto 1/A^2$. This implies that in order to sustain flow to arbitrarily small radii, the combined density and pressure must go to infinity. This is, in practice, infeasible. Therefore, the most reasonable conclusion is to allow for a divergent portion at some point.

The only practical way to allow a divergent portion in an experimental setting is to allow venting into a relatively open area after some small radius. If this divergent behavior is allowed, the throat must occur at this location. Therefore, we need only consider situations up to this critical point, given by the critical aspect ratio.

Euler Number and Pre-Shock Mach Number. The second result from the analytic considerations gave a first insight into the interplay between HEDLA constraints and astrophysical constraints. Figure 3.6 illustrates the combination of constraints and what range it is possible to probe with experimental studies.

Looking at the limits of Euler number with respect to pre-shock Mach number (Eqns. 3.10), it is clear that it will not be possible to span the entire range of Euler numbers we'd like to. Particularly, for a $\gamma = 4/3$ gas it is near impossible to probe the space of interest. For $\gamma = 5/3$, the range of Euler numbers possible is only roughly 0.575-0.625. This is a far cry from 0.5-0.9, and only barely includes the lower end of the current full simulation results.

Regardless of the limited range, these results do not rule out the possibility of performing any experiments. They do, however, indicate that higher γ materials are required in order to even probe the lower end of the range of Euler numbers.

Joining HEDLA Conditions to Experimental Requirements. The first results generated in the semi-analytic section consists of a basic range of primitive variables at the inner boundary. By factoring out any dependence on specific material,

it was possible to produce generic results independent of fabrication. The molar-mass corrected temperature ($\hat{T}$) is normally distributed, with its mean near the lower range of desired temperatures to fit with expected HEDLA conditions. While we only anticipate utilizing a certain subset of the total temperature range possible, it shows that it may be possible to generate low temperature environments. While not directly applicable to our current problem, this result may prove useful to other projects.

What was not addressed in the results but should be noted here is that the range of velocity values approaches $0.1c$. This is a result of placing no a priori constraints on the velocity, and therefore these results fall out. However, this is physically infeasible. In practice, it is possible to expect velocities of 10^6 cm/s, give or take an order of magnitude. Examining the distributions in Fig. 3.8 it can be seen that the range of velocities include these practical values.

Full Semi-Analytic Simulations. There are two interesting results that emerge when all constraints are put together. The first is the relation between viable aspect ratios and the Mach number at the inner boundary (Fig. 3.11). The upper bound on this relation may be found following method similar to the analytic section. Instead of going through this exercise, it may be more worthwhile to discuss what this implies. As the inner Mach number approaches the critical Mach value, the only way this situation can occur is if the post-shock region has zero width ($\alpha = 0$). As the flow becomes choked ($M_{\text{inner}} = 1$) with $M_{\text{post}} = M_{\text{crit}}$, the critical aspect ratio is reached.

The lower bound is explained by the restriction placed on M_{pre} . By having a fixed lower bound on the pre-shock Mach number, the post-shock Mach number becomes bounded from above. Equation 3.7 defines this relation. The lower bound on M_{post} is given in Eqn. 3.3. These two bounds account for the banding behavior of $\alpha(M_{\text{inner}})$. This behavior may pose problems to controlling the system in the laboratory, as control must be fairly precise.

The second result is the relation between viable temperatures and the primitive variables at the inner boundary, depicted in Fig. 3.13. Of these results, the velocity benefits from a very tight distribution. Due to this, one of the primary design decisions

should be to decide on a temperature and determine its corresponding velocity. The goal then is to determine the proper combination of parameters that correspond to the temperature and puts the aspect ratio where the designer wants it.

4.2 One-Dimensional Models

Stationarity. The calculations performed by Foglizzo [6] based on theory and empirical observations indicate that at least $10 \tau_{\text{adv}}$ is required for important post-shock flow features to develop. Therefore, the experimental shocks should be able to remain quasi-stationary for this amount of time. This must also occur in the presence of perturbations. This is due to non-uniform infalling density in the SASI case, and perturbations caused by either intent or fabrication in the experimental case.

From Fig. 3.15, it can be seen that experimental cases are able to sustain longer than required stationarity. This result appears to only have a weak, negative correlation with initial steady aspect ratio. It does not appear to correlate to any other system parameter. A possible reason for this result is that as aspect ratio increases the flow experiences increasing curvature. In other words, at low α values the system is essentially Cartesian, while increasing α values makes the flow appear spherical. The Cartesian case is unconditionally stable, while the results here indicate that for spherical flows the shock can be pushed onto unstable trajectories. It is not clear if it is even possible to obtain a stable configuration for indefinitely long times in spherical geometry.

Coupling of Radial Shock Oscillations to Density Perturbations. In the event that the post-shock flow develops the “rumbling” of SASI, the flow structure in the post-shock should grow. It is reasonable that these new structures could perturb the shock at frequencies different from the driving frequency. Therefore, the radial movement of the shock should decouple from the upstream perturbations. However, our simulation results (Fig. 3.14) indicate that the shock oscillates with the frequency equal to the driving frequency. This result points to the conclusion that the motion of

Figure 4.2: Comparison of one dimensional perturbations. In the experimental system, perturbations are shocked and then transported from the domain. In the ccSN scenario the post-shock flow is rich in behavior and contains multiple secondary shocks that will interact with the standing shock.

the shock evolves with no noticeable feedback from the perturbed post-shock region.

Comparison to the ccSN Setting. The results coming out of the one-dimensional models have indicated that there may be something fundamentally different between the experimental setting and SASI. Our results show that the shocks are quasi-stationary for long enough periods of time. They also show that no post-shock features are developing. The first result is consistent with SASI, whereas the second result is on. In order to take a closer look, let us compare our solutions to ccSN solutions.

Figure 4.2 shows the perturbed variable profiles for both ccSN and experimental settings. As a pre-shock fluid parcel crosses the SASI surface it is compressed and decelerates. As this fluid parcel abruptly slows, it is forced forward by the material behind. This causes it to be pushed directly into the parcel before it. Energy extraction due to gravity allows the packets to pile up at lower radii, increasing the pressure and density and making it progressively more difficult for a newly shocked packet of gas to fall through the atmosphere. As the post-shock area near the shock is pelted with infalling bubbles, the inability for the gas to freely fall causes the jostling to produce shock waves. These shocks interact in the post-shock region, causing the shock to rumble. This rumbling, however, is stable and does not eventually cause

expansion of the shock. These results are in agreement with Blondin et al. [2]

The rationale for this failure to trigger explosion is that in one dimension the flow is still allowed to exit the domain relatively unimpeded. Despite the shockwaves behind the SAS, there is no mechanism to stop the material from falling. In multiple dimensions, however, the growth of turbulent energy in the post-shock begins hampering the radial velocity of the accreting material by creating a turbulent “wall” of fluid. Over time, this wall begins to grow, stopping increasing amounts of material and simultaneously increasing the intensity of interactions with the SAS.

For the experimental setting, note that post-shock perturbations spurred by pre-shock oscillations behave like the ccSN setting for only small periods of time after being shocked. The lack of gravity in this scenario does not induce strong outward gradients of pressure and density, and in fact its nonexistence allows the opposite. Therefore, when a fluid parcel crosses the shock and interacts with the packet in front of it, the earlier shocked material tends to have less thermal pressure. Thus, the newly shocked gas will, essentially, push the “older” gas down stream instead of being forced to slow down. This is, potentially, the most counterintuitive part of nozzle flow. In addition, the aspect ratios in experimental flows are quite small and therefore any potentially critical compression effects caused by geometry at small radii are not noticeable.

Thus, for experimental setups to be effective, there should be some way to either a) recreate the net effect of gravity or b) cause a turbulent kinetic energy buildup without gravity.

4.3 Two-Dimensional Models

Morphology of Models. As seen in Fig. 3.16 the density profile has a noticeable egg crate distribution in the pre-shock, with pressure and radial velocity having constant (or nearly constant) distributions. As denser material passes through the shock one can see the depression of the shock surface. Alternatively, low density material ends up sucking the shock away from the interior, causing it to bow outward. Therefore, as the radial position of the shock varies roughly like a sine wave as a function of angle. This agrees with our predicted behavior from the previous section.

The differing density regions appear to become Richtmeyer-Meshkov unstable, the impulsive acceleration version of the Rayleigh-Taylor instability. The higher density peaks end up turning inside out and “shredding,” while the lower density valleys are compressed together and become bubbles. This process should induce vortical perturbations to the post-shock region.

By looking at both the radial and lateral velocity distributions, the adjacent hill and valley fluid packets begin to bleed together. This bleeding quickly becomes a sloshing motion in the lateral direction, with two adjacent packets colliding, bouncing apart, and then interacting with the other adjacent packet in a similar way. No large-scale turbulent behavior is able to form before the perturbations are advected from the domain, if they would even cohere at all.

If we investigate the entropy distribution, we can note that it behaves in a similar manner to the nominal density and pressure profiles. That is, it reaches its peak at the shock and gradually decreases as the radius becomes smaller. This entropy profile indicates that the post-shock region is stable to convective motions. Therefore, it is unlikely that these vortical disturbances will form the “wall of turbulence” indicated as a contributing factor in ccSN.

Interpretation of Flux Results. In the SASI scenario, it is theorized that a layer of turbulent, convective flow exists in the post-shock region. This unstable convective layer ultimately impedes settling of the infalling gas, creating a runaway

energy build up that revives the shock. While this occurs, there is still settling of material onto the PNS surface. Therefore, at smaller radii one should witness a return to a kinetically dominated (in terms of flux) state.

From the flux analysis, we note that no unstable convective layer emerges. While there is an increase in the convective flux behind the shock, the third layer of kinetically dominated flow does not manifest. In order to explain this, we consider the entropy profiles of each setting. In the supernova setting, a large scale negative entropy gradient exists in the post-shock region. This is due to redistribution of the fluid flow energy components. This behavior could be understood as follows.

Consider adding air to a hot air balloon while standing on the surface of the earth. The local bubble of higher entropy gas (the balloon) has a preferred state at higher altitudes, which manifests itself as buoyant forces that cause it to rise. In the experimental system, there is a positive entropy gradient in the post-shock region. This infers that the preferred state of shocked fluid parcels is to continue falling towards the inner boundary.

CHAPTER 5

CONCLUSIONS

First, it has been determined that the upper bound on the thickness of the post-shock region is only a function of the gas composition (γ). In particular, for large γ values the post-shock aspect ratio goes to zero. Conversely, for small values the aspect ratio diverges to infinity. For reasonable values, the aspect ratio is forced to be relatively small (below 1.2), which gives thin, almost Cartesian post-shock regions.

The second is the direct relation between the Euler number of the supernova setting and the pre-shock Mach number in the laboratory. This relationship shows that there are only certain γ values that are supported. In particular, $\gamma = 4/3$ allows only a small segment of the desired quantities to be explored. If $\gamma = 5/3$, it is possible to explore Euler numbers for the entire available range of pre-shock Mach numbers. This does limit the Euler numbers though, as the relationship between the Euler number and Mach number is one-to-one for a fixed γ . Specifically, for $\gamma = 5/3$, Euler numbers between 0.55 and 0.6 can be probed, which at the low-end of possible ccSN values.

It was shown that it is possible to generate a viable range of parameters that encompassed both HEDLA and ccSN constraints, which led to an investigation in one-dimension. From here, the shock was perturbed by upstream density fluctuations. The result was that the radial frequency of the shock oscillation corresponded 1:1 with the frequency of the upstream perturbations. This indicated that, in one-dimension, no post-shock features were being formed that allowed the shock motion to decouple from the upstream. This inquiry also led to the conclusion that the shock is able

to remain quasi-stationary for “long enough”. This means that the nominal shock position remained quantitatively close to its initial value for times comparable to at least 10 post-shock advective times.

With these results in hand, two-dimensional simulations were performed to determine if the predicted ccSN post-shock behavior manifested in the experimental system. The result is that the perturbations passing through the shock exhibit Richtmeyer-Meshkov shredding. The resultant flow field is then advected downstream and exits the lower boundary before any structure can cohere.

By looking at the flux decomposition, we see that there is no sandwiched convectively unstable layer in the post-shock. This implies that the flow does not return to a kinetic flux dominated state at lower radii. While there is a jump in the convective flux, the failure to return to kinetic flow indicates that there is no “wall” building like is theorized in the SASI formulation.

CHAPTER 6

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APPENDIX A

NUMERICAL PARAMETER SPACE SAMPLING

In order to sample our parameter space we employ the Latin Hypercube Sampling (LHS) technique. Originally described by McKay et al. [28] and subsequently worked on by Iman et al. [17].

In order to justify the decision to use LHS, two alternative methods need be described. First is the method of random sampling. The most naive way to implement this method is to first obtain bounds on the parameter space. One then samples points in the space by randomly choosing their coordinates via a uniform distribution (per component). In order to enhance this method, one may assign probability distributions to each component individually or use multivariate distributions. By doing so one may sample the space in a more targeted and importance-based manner. For uniform distributions this method does not guarantee that extreme values of the parameter space are sampled, potentially missing informative information. As a point of definition, we define an extreme value to be a parameter value such that the probability of selecting it at random from a nonuniform distribution is "small". An example would be the two tails of a Gaussian distribution, where the values do not have a high chance of being chosen but may produce interesting information by virtue of being less likely.

The second method is stratified sampling. The simplest implementation of this starts by dividing the parameter space into equal bins. From here, each bin is visited and the centroid of the bin is chosen to be the sample. Like random sampling this may

be improved upon by dividing the bins into equiprobable sizes. This method guarantees the coverage of extreme parameter values, but at a significant cost. Consider the simple case of a parameter hypercube of dimension M , with M bins per dimension. This results in a total of MN samples, which for large numbers of parameters at even meager values of N may quickly become unreasonable to use. While decreasing M (with a fixed M) is a possibility, one quickly runs into the problem of undersampling the domain.

In order to demonstrate the difficulty in increasing the number of parameters sampled, consider a uniform stratified sampling division with $N_1 = 10$ and $M_1 = 3$, and each parameter spans $[0, L]$. Now, the maximum distance between the centers of any two adjacent bins (diagonals included) is given by

$$\begin{aligned}
D(i, j) &= \sqrt{\sum_{k=1}^M (x_k^i - x_k^j)^2} \\
\Rightarrow \max(D(i, j) | j \in \text{Neighborsof } i) &= \sqrt{\sum_{k=1}^M \Delta x^2} \\
&= \sqrt{\sum_{k=1}^M \left(\frac{L}{N}\right)^2} \\
&= L \frac{\sqrt{M}}{N} \tag{A.1}
\end{aligned}$$

The results of Eqn. A.1 imply that in order to maintain the same maximum distance between samples while increasing M_1 to M_2 by a factor of k , the number of bins per dimension must increase to $N_2 = \sqrt{k}N_1$. Therefore, the total number of bins increases from $N_1^{M_1}$ to $(\sqrt{k}N_1)^{M_2}$. For our initial conditions given above, increasing to $M_2 = 6$ by a factor $k = 2$ changes the number of total samples from $O(1 \times 10^3)$ to $O(1 \times 10^7)$, an increase of four orders of magnitude. This many runs is impractical from both a computational angle and a data analysis angle.

Latin hypercube sampling provides an alternative to these two methods by both ensuring the extremes in the parameter space are covered and that number of re-

quired runs is minimized. Like stratified sampling the parameter space is divided into N equiprobable bins per dimension, with the hypercube H being of M dimensions (parameters) and indexed in each dimension from 1 to N . One then creates an index set for each dimension such that $I_i = 1, 2, \dots, N$. If these bins were visualized using these index lists, one would have a line of N bins along a diagonal H , spanning only that line in the parameter space. In order to span the entire space we permute the elements in each I_i independently from any other set. This essentially shuffles the order our bins appear as we travel along our index sets, forcing them to span the space.

Once these permuted index sets are generated, one may traverse them to locate bin B_j , such that $B_j := H(I_1(j), I_2(j), \dots, I_M(j))$. If we perform this operation for all $j \in [1, N]$ the set of all parameter sample bins B is populated.

Now recall that all possible sample points have an equal probability of being sampled. Therefore, by sampling any point randomly (from a uniform distribution) inside a bin B_j , we will obtain the parameter sample P_j . Performing this procedure on all bins produces the parameter sample points as input to our analysis.

In summary, random sampling allows, in general, smaller sampling sizes but does not guarantee that unlikely parameter compositions will be probed. On the other hand, stratified sampling guarantees sampling at cost of performing more runs. The multiple runs produced by stratified sampling are typically quite similar to their neighbors (if the space being sampled is smooth), resulting in redundant data that does not shed light on the problem. Latin Hypercube Sampling provides a hybrid method between the aforementioned methods, guaranteeing the sampling of outliers while simultaneously reducing the number of simulation runs required.

Figure A.1: Comparison of parameter space sampling for random sampling, stratified sampling, and Latin hypercube sampling. (a) Points in the parameter space are chosen randomly (either uniformly or weighted by some probability distribution). (b) Stratified sampling consists of dividing the parameter space into discrete bins. Once it has been subdivided, the parameters corresponding to each bin are chosen as the sample parameters. (c) Latin Hypercube Sampling divides the parameter space into bins (like stratified sampling), but each bin is weighted such that every interval is equiprobable. Only a subset of available bins are chosen for the parameter sample.

Figure A.2: Diagram of the LHS algorithm. (a) Initial subdivision of parameter space. (b) The first bin is chosen. (c) A randomly selected point inside the bin is chosen as the parameter sample. (d) Another bin is chosen. (e) Parameter sample is randomly chosen. (f) The process repeats until the Latin hypercube is complete.

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BIOGRAPHICAL SKETCH

I was born in Alameda, CA on December 18, 1987. It was a fairly cold day, with a mean temperature of 47° F. Looking back, it seemed warmer. My first handful of years were fairly uneventful. I was shuttled around quite a bit, moving both inside California and to Arizona. Eventually my position in the frame of reference of a fixed coordinate system stabilized with me ending up in rural Colorado.

After a reasonable number of years, I ended up relocating to Tallahassee, FL the summer before sixth grade. I then attended Betton Hills Preparatory School, a K-8 core knowledge school. I owe this school a debt of gratitude in making me the student that I became later in life. This was not, however, reflected in the next two years of my life. I ended up leaving traditional high school after one month, and instead worked under the table at Betton Hills as an office assistant/handyman/computer technician. I was biding my time until I could implement my larger plan. My grand plan fell into place two weeks after my 16th birthday. I obtained my GED and began attending the local community college as a full student. I had no idea what I wanted in life, but with two years ahead of my peers, I felt like I had some time to decide.

I was upheaved again that summer by moving to Oklahoma. I then convinced the University of Central Oklahoma to let me in, and so began my career as an education major. That about wraps up my story...

...except, I left UCO with a degree in Engineering Physics - Mechanical Systems and a passion for mathematics and computer simulations. This drove me to where I am now, Florida State University, unexpectedly back in Tallahassee, FL. In the process, I brought along an Oklahoman, my wife, April. Now we're wrapped up.