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Salt Finger Fluxes in a Laminar Shear Flow

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THE FLORIDA STATE UNIVERSITY

COLLEGE OF ARTS AND SCIENCES

SALT FINGER FLUXES IN A LAMINAR SHEAR FLOW

By

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To Isabela and Silvia.

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ABSTRACT

This dissertation presents an experimental study of the buoyancy fluxes produced by salt fingers in the presence of a laminar shear flow. An annular tank was used in the experiments. Sugar and salt were used as the diffusing components. Salt fingers, initially aligned in the vertical, were observed to tilt when the shear flow was imposed. The fingers were also observed to be 3-dimensional structures at all times during the experiments. A consistent decrease in the salt fluxes (F_T) was measured as the Reynolds number (R_e) was increased by increasing the shear velocity magnitudes. Through a regression analysis, the salt flux was found to depend upon R_e as $AR_e^{-0.025}$, $BR_e^{-0.1}$ and $CR_e^{-0.34}$, when the density ratio R_ρ was equal to 1.2; 1.54 and 2.1 respectively. The constants A, B, and C are determined experimentally. A polynomial expression that encompasses the dependence of F_T with R_e and R_ρ is also suggested. A linear stability theory for fingers tilted by a shear flow together with a flux reduction mechanism, proposed by Krishnamurti (personal communication), are suggested as possible explanations for the experimental results.

CHAPTER 1

INTRODUCTION

In the 1960 article, "The Salt Fountain and Thermohaline Convection", Melvin Stern showed how opposing stratifications of two component species could drive convection if their diffusivities differed. Stommel *et al.* (1956) had earlier noted that there was significant potential energy available in the decrease of salinity with depth found in much of the subtropical ocean. While they suggested that a flow (the salt fountain) would be driven in a thermally-conducting pipe, it was Stern who realized that the two orders of magnitude difference in heat and salt diffusivities allowed the ocean to form its own pipes. These later came to be known as "salt fingers" and its role as a mixing mechanism in the ocean started to receive attention. In the same 1960 paper Stern also identified the potential for an oscillatory instability, when cold and fresh waters overly warm and salty waters. This type of double diffusive process, "diffusive-convection", was a few years later demonstrated by Turner and Stommel (1964) through laboratory experiments and by Walin (1964) by means of an instability analysis. In this work we will focus on the importance of the salt finger case of double diffusive processes. As we will discuss later, the subtropical ocean is unstable to this process and water mass properties should be affected by its presence.

1.1 The salt finger mechanism

Salt fingers are formed when a layer of fluid is stably stratified by heat and unstably stratified by salt, and the vertical density distribution is gravitationally stable. The density of the seawater is determined by its temperature (T) and salinity (S) as given by the linear approximation of the equation of state:

$$\rho = \rho_0(1 - \alpha T + \beta S), \tag{1.1}$$

where $\alpha = -(\frac{\partial \rho}{\partial T}) / \rho_0$ and $\beta = (\frac{\partial \rho}{\partial S}) / \rho_0$ are the thermal expansion and the salinity contraction coefficients and ρ_0 is a constant density of reference. The occurrence of the salt finger instability is due to the fact that T and S diffuse at different molecular rates. Figure 1.1 is a schematic representation of the finger instability. A two-layer fluid interface of hot and salty solution overlying relatively cold and fresh one is shown. Suppose that, at a time $t = t_0$ a fluid parcel in the cold and fresh layer is displaced upwards. In the upper layer, at $t = t_1$, this parcel equilibrates its temperature towards its new environment but mostly retains its low salinity, thereby becoming less dense than its neighbors and continuing to rise. Similarly, the two neighbor parcels loose heat but retain their relatively high salinity and sink. As a result the instability takes the form of alternating columns of up-going cold and fresh and down-going hot and salty fluid as showed when $t = t_2$ (Stern, 1960; Turner, 1967; Baines and Gill, 1968; Stern and Turner, 1969; Linden, 1973; Schmitt, 1979). The finger instability is driven by the potential energy stored in the vertical salt stratification. This energy is released by the instability, which can grow only by the lateral diffusion of heat between consecutive finger columns. Heat, advected in the down-going fingers, diffuses laterally and thus is partially returned in the up-going fingers. Salt, on the other hand, diffuses more slowly and lateral transfers can be neglected. Therefore, the vertical buoyancy transport due to salt is larger than the one due to heat and hence, the salt fingers lead to a net up-gradient buoyancy flux.

Figure 1.1: Sketch of the salt finger instability.

The original instability analysis was obtained by Stern (1960) and later was examined in more detail by Baines and Gill (1969). A brief description of the instability condition is given as follows. Let's consider a layer of fluid with depth d , with linear temperature and salinity gradients (in the undisturbed state), and lying between two horizontal boundaries. In these boundaries temperature and salinity values are prescribed and stress free conditions are assumed. The relevant Boussinesq perturbation equations in non-dimensional form are

$$\frac{1}{\sigma} \left(\frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} \right) = \nabla^2 u', \quad (1.2)$$

$$\frac{1}{\sigma} \left(\frac{\partial v'}{\partial t} + \frac{\partial p'}{\partial y} \right) = \nabla^2 v', \quad (1.3)$$

$$\frac{1}{\sigma} \left(\frac{\partial w'}{\partial t} + \frac{\partial p'}{\partial z} \right) = R_T T' - R_S S' + \nabla^2 w', \quad (1.4)$$

$$\left(\frac{\partial}{\partial t} - \nabla^2 \right) T' = -w', \quad (1.5)$$

$$\left(\frac{\partial}{\partial t} - \tau \nabla^2 \right) S' = -w', \quad (1.6)$$

where u' , v' and w' are the perturbation velocity components, p' is the perturbation pressure and T' and S' are the vertical temperature and salinity perturbation variables. The important non-dimensional parameters are:

$$R_T = \frac{g\alpha \frac{\partial \bar{T}}{\partial z} d^4}{\nu \kappa_T} \quad (\text{temperature Rayleigh number}); \quad (1.7)$$

$$R_S = \frac{g\beta \frac{\partial \bar{S}}{\partial z} d^4}{\nu \kappa_T} \quad (\text{salinity Rayleigh number}); \quad (1.8)$$

$$R_\rho = \frac{R_T}{R_S} \quad (\text{density ratio}); \quad (1.9)$$

$$\sigma = \frac{\nu}{\kappa_T} \quad (\text{Prandtl number}); \quad (1.10)$$

$$\tau = \frac{\kappa_S}{\kappa_T} \quad (\text{diffusivity ratio}), \quad (1.11)$$

where g is the gravitational acceleration; $\frac{\partial \bar{T}}{\partial z}$ and $\frac{\partial \bar{S}}{\partial z}$ are the undisturbed temperature and salinity vertical gradients; ν is the coefficient of kinematic viscosity of water; κ_T and κ_S are the molecular diffusivity of heat and salt respectively. The general procedure in the instability

analysis is to examine solutions in which the perturbation variables are given by functions of the vertical coordinate $(u'(z), v'(z), w'(z), p'(z), S'(z), T'(z))$, times $\exp[i(kx + ly) + \lambda t]$. In these solutions k and l are the horizontal finger wave-numbers and λ is the growth rate, which may be a complex number. By substituting the solutions above into the governing equations a polynomial equation in λ is found. The marginally stable condition is obtained by taken the real part of λ as equal to zero. The condition for finger instability is given by:

$$1 \leq R_\rho \leq \frac{1}{\tau}. \quad (1.12)$$

Although the original stability analysis was obtained for heat and salt components, the same reasoning applies to a system with two solutes: an unstably stratified slower diffusing solute and a stably stratified solute with a larger diffusion coefficient. In laboratory experiments salt fingers are usually studied with two solutes, salt and sugar instead of heat and salt. This is due to the fact that heat losses to the environment are difficult to control. Salt diffusivity is only three times that of sugar whereas the ratio of heat to salt diffusivities ($1/\tau$) is about 80. Nonetheless, many of the qualitative features of the heat-salt system are exhibited by the sugar-salt system (Taylor and Veronis, 1996).

1.2 Salt fingers in the ocean

In much of the subtropical ocean, evaporation exceeds precipitation at the same time that heating exceeds cooling. This produces a warm and salty water mass at the surface that overlies cooler, fresher water from higher latitudes. Such configuration makes all the central upper waters of the subtropical gyres favorable to the occurrence of salt finger instability (Schmitt, 2003). In regions where destabilizing salinity vertical gradients almost density compensate stabilizing temperature ones,

$$R_\rho = \frac{\alpha \frac{\partial T}{\partial z}}{\beta \frac{\partial S}{\partial z}} \simeq 1, \quad (1.13)$$

the salt fingers exist within high gradient steps $O(1-5)m$ deep separating mixed layers $O(10-100)m$ deep in the adjacent fluid, forming what is called thermohaline staircase, when a series of such layers and interfaces are found with depth. In a staircase, fingers within the high gradient interfaces produce an unstable buoyancy flux, which drives large-scale overturning

in the adjacent mixed layers. The staircase pattern was a key that lead scientists to identify this ocean phenomena as due to salt fingers, which were known to produce a similar pattern from laboratory experiments.

Persistent staircases have been well documented in the Tyrrhenian Sea within the Mediterranean, below the warm salty Mediterranean outflow water in the eastern North Atlantic, and in the western tropical North Atlantic (Figure 1.2). In the Tyrrhenian layers, the temperature and salinity of the layers are observed to remain constant for many years. The Mediterranean outflow steps appear to have a lateral coherence of 50 to 100 *km*, and have been identified several times. In the western tropical North Atlantic, the layers appear to be a permanent feature of the thermocline, with observations available over the past 3 decades (Schmitt, 1995).

Within the density steps in the thermohaline staircase, the presence of salt fingers has been proven by shadowgraph images. In a shadowgraph system, collimated light passes through a transparent, inhomogeneous medium, is refracted, and then falls on a screen producing a shadow of the inhomogeneities in the medium. A commonly observed shadowgraph is the image of wind ripples on a sand bottom in shallow water. The light from the Sun, being refracted by the ripples, is concentrated beneath the crests and diffused beneath the troughs. The uneven illumination on the bottom follows the ripples on the surface (Williams, 1974a). Shadowgraphs of salt fingers are able to reveal the density contrast between the up-going (cold and fresh) and down-going (hot and salty) finger columns. Figure 1.3, from Williams (1974a) shows images of a sharp finger interface in the Mediterranean outflow.

Salt fingers were also observed in regions where the classical thermohaline staircase pattern was absent. St.Laurent and Schmitt (1999) observed several finger patches through shadowgraphs profiles in the east North Atlantic at the Canary Basin region as we shall see later. Furthermore, substantial evidence is available that salt fingers are one of the mixing mechanisms operating on finescale intrusions at oceanic fronts (Marmorino, G.O., 1991; Bianchi, Piola and Colino, 2001).

Figure 1.2: From Schmitt (1981); Examples of thermohaline staircases.

Figure 1.3: From Williams (1974a); Images of a sharp finger interface at 1263 *m* in the Mediterranean outflow. The interface step is about 1 *m* deep. The images are approximately 5 *cm* diameter.

1.3 A brief overview of finger flux laws determined through laboratory experiments

Of principal interest to oceanographers is to quantify the heat and salt fluxes produced by salt fingers since these modify water-masses. This was originally attempted through laboratory experiments in which heat- and salt-fluxes were measured between two convecting layers separated by a thin finger interface (Turner, 1967; Linden, 1973; Schmitt, 1979a; MacDougall and Taylor, 1984; Taylor and Bucens, 1989). Turner (1967) assumed that the salt flux (F_s) is proportional to $\Delta S^{4/3}$ and hence, independent of any vertical length scale. This would imply that the salt Nusselt number N_S , defined as the ratio of F_S to the diffusive flux, must be proportional to the salt Rayleigh number R_S to the 1/3 power, where

$$N_S = F_S / \kappa_S (\Delta S / d),$$

$$R_S = g \beta \Delta S d^3 / \kappa_T \nu.$$

In the expressions above ΔS is the salinity vertical difference between the two layers, and d is the layer depth.

Applications of the $\Delta S^{4/3}$ law to finger favorable density steps in the ocean have been found to overestimate fluxes by a factor of $\simeq 30$ (Gregg and Sanford, 1987; Lueck, 1987; Kunze, 1987; Hebert, 1989; Bianchi, Piola and Collino, 2001).

In the light of the $\Delta S^{4/3}$'s law for heat and salt, Stern and Turner (1969) assumed that the sugar fluxes through a salt-sugar interface were given by $\beta F_S = C(\beta \Delta S)^{4/3}$, where S now stands for the sugar concentration; $C = C(R_\rho, \tau, \nu / \kappa_T)$ is a constant to be determined experimentally; $\tau = \kappa_S / \kappa_T$ is the ratio of molecular diffusivities; ν is the kinematic viscosity; ΔS and ΔT are the sugar and salt concentration differences between layers in grams of solute per gram of solution. By the same assumption, the mass flux ratio $r = \alpha F_T / \beta F_S$ will depend upon R_ρ , τ and ν / κ_T only. Stern and Turner (1969) found $C \simeq 10^{-2} g cm^{-2} s^{-1}$ and a flux ratio $r = 0.92 \pm 0.02$ over the range $1.1 < R_\rho < 2.2$. Lambert and Demenkow (1972) found the same value of r and C ranging from 0.5×10^{-3} to $0.75 \times 10^{-3} g cm^{-2} s^{-1}$. In both studies the authors concluded that C , which gives the sugar fluxes, didn't vary significantly with R_ρ . Griffiths and Ruddick (1980) found a sugar-salt flux ratio decreasing from $r \simeq 0.94 \pm 0.01$

at $R_\rho = 1.02$ to $r = 0.88 \pm 0.01$ at $R_\rho = 2$. The sugar flux coefficient C was also found to decrease by two orders of magnitude with increasing R_ρ , varying approximately as R_ρ^{-6} in the range $1.2 < R_\rho < 2$.

The growth of sugar-salt fingers was investigated by Taylor and Veronis (1996) in the density ratio range $2.5 < R_\rho < 3$. At this values, long and slender fingers were generated at the interface. Measurements showed that the salt finger interface grew according to the molecular diffusive spread of the faster diffusing component (salt). When the initial R_ρ was set closer to unity, fingers were seen to penetrate into the reservoirs very rapidly causing a convective overturning of the top and bottom reservoirs. In the range $1.5 < R_\rho < 3$, the sugar fluxes were seen to vary by two orders of magnitude.

Krishnamurti (2003) explored the dependence of the finger fluxes with depth by performing experiments using three tanks, which have same width and length but different heights. The experiments were conducted using sugar and salt as diffusing components and the initial vertical stratification was produced by linear profiles of sugar and salt. The end values of the initial sugar and salt concentration profiles were maintained during the experiments. The overall sugar and salt Rayleigh numbers ($R_S = g\beta\Delta S d^3 \kappa_s \nu$ and $R_T = g\beta\Delta T d^3 \kappa_t \nu$) were in the range 10^{12} to 10^{15} and R_ρ in the range 1.1 to 1.3. Flow visualization, via shadowgraphs, showed many distinctive flow regimes at different parameter ranges R_S and R_T . This regimes could be roughly categorized as straight fingers, lumpy fingers, wavy fingers, and staircases of alternating finger and convective layers. The overall flux measurements of sugar and salt fluxes (F_S and F_T respectively) in the staircase regime, were found to be depth dependent, differently from previous studies. The main result regarding the overall Nusselt number ($N_S = F_S / \kappa_S (\Delta S / d)$) is that it varies inversely with the number n of finger interfaces plus convecting layers across the depth (h) through which ΔS is imposed. Internal flux measurements for salt across finger interfaces were expressed as

$$F_{S,i} \sim \frac{\Delta S}{h_i} R_{S,i}^a R_{\rho,i}^b \quad (1.14)$$

for the i th layer. A least-squares fit to a linear regression equation gave $a = 0.18$ to 0.19 , and $b = 1.8$ to 2.1 .

1.4 Salt fingers and shear

As part of the Caribbean Sheets and Layers Transect project (C-SALT), in November 1985, 17 profiles were made during 5 days in the center of the thermohaline staircase field east of Barbados. A Multi-Scale Profiler (MSP) and an acoustic current meter, which allowed velocity measurements to a vertical resolution of 0.5 m were used in the analysis. Conductivity and temperature were observed with Sea-Bird sensors, a fast thermistor, and a Neil Brown conductivity cell. Nine density interfaces occurred between 300 and 600 m depth with $R_\rho \simeq 1.6$ (Figure 1.4). Typical height and velocity differences across the interfaces were 2 m and $15 - 25\text{ mm s}^{-1}$ respectively, which results in a shear of $O(10^{-2}\text{ s}^{-1})$ (Gregg and Sanford, 1987).

Figure 1.4: From Greg and Sanford (1987); Summary of the profile with the most clearly defined layers. Data are plotted with a resolution of 1 KPa (0.1 m) between 3 MPa (300 m) and 6 MPa (600 m). East (U) and North (V) velocity components were measured with an acoustic current meter corrected for instrument motion, and checked against electromagnet velocity data. The Brunt Vaisala frequency (N) and shear were computed over successive 5 KPa (0.5 m) intervals. Both show sharp spikes, sometimes only 0.5 m thick, across the interfaces.

The shear was found to be near inertial and the Richardson number, $R_i = N^2 / [(U_z)^2 + (V_z)^2]^{1/2}$, was ≥ 1 for over 90% of the interfaces. $R_i < 0.25$ is a necessary, but not sufficient condition for shear instability. The rate of viscous dissipation of turbulent kinetic energy (ϵ) averaged across the entire staircase was low, corresponding approximately to the sensor noise level. In the convecting layers, the turbulent kinetic energy equation reduces to the simple balance $\epsilon \simeq J_b$, where ϵ is the rate of viscous dissipation of turbulent kinetic energy and J_b is the buoyancy flux across the finger interfaces. In the same staircase region, shadowgraph images from C-SALT (Kunze et al., 1987) revealed small-scale laminae tilted $\pm 10^\circ$ from the horizontal in the finger-favorable interfaces (Figure 1.5). This was surprising as earlier shadowgraphs revealed more vertically oriented structure (Schmitt, 1994), as we have previously shown, similar to the ones usually observed in the laboratory. The tilted laminae structures were identified as fingers that have been tilted by an internal wave shear.

In light of the above results, kunze (1990) presents a time dependent mathematical model, where fingers were allowed to be tilted by shear. In this model the fingers consist of alternating finite-length columns of up- and down-going fluid, embedded in uniform background vertical gradients of temperature, salinity and shear. This shear was set to be near inertial, as found in the C-SALT staircase, but numerical runs with a steady shear were also performed. The parameters such as, density ratio, R_i , and background temperature and salinity gradients were set to values typical of the interfaces in the thermohaline staircase east of Barbados. The model results were computed as time-evolutions of the normalized finger interface height h/h_o and vertical velocity w/w_o , where h_o and w_o are initial values. In the case of steady shear, 2 and 3-dimensional finger structures were consider. When fingers were set as 2-dimensional (finger sheets; Linden, 1974) their growth was the same as in the case of unsheared fingers. When fingers were 3-dimensional, the calculations showed that the finger interface almost did not grow. The fingers were interpreted to tilt over very rapidly (less than two local buoyancy periods) until they line up horizontally, which caused their vertical velocity to vanish. The author concluded that, unless they form sheets aligned with the shear, as suggested by Linden (1974), the fingers did not produce significant fluxes in the presence of a steady shear.

Figure 1.5: From Kunze et al. (1987), four images of optical microstructure from a free fall shadowgraph profiler deployed in the water column east of Barbados. Each image is 10 cm in diameter and represents a view through 60 cm of water. The four examples were thought to correspond to stratified turbulence (upper left), a shear billow (upper right), a diffusively unstable interface (lower left) and what may be shear-tilted salt fingers (lower right).

In the North Atlantic Tracer Release Experiment (NATRE; St.Laurent and Schmitt, 1999) within the central waters of the North Atlantic subtropical gyre at the Canary basin, salt fingers tilted $10^\circ - 20^\circ$ from the horizontal were the most frequently observed structures, through shadowgraphs, Figure 1.6, when turbulence was absent. In this study a High Resolution Profiler, HRP was used. This profiler produced measurements of temperature, conductivity and velocity at both finescale and microscale resolutions. The strength of salt-finger mixing has been diagnose through the use of a non-dimensional parameter (the dissipation ratio), which is related to the ratio of diffusivities for heat and buoyancy. By examining the dissipation ratio in a parameter space of density ratio (R_ρ) and Richardson number (R_i), the signal of salt fingers was discerned even under conditions where turbulent mixing also occurred. The fingers were found to be the dominant mixing mechanism when $R_\rho \leq 2$ and $R_i \geq 1$. The classical thermohaline staircase pattern caused by the up-gradient density flux was not observed and the fingers were seen to occur in patches. The authors concluded that this was likely because mixing by turbulence, which in their study was consistent with $R_i < 1$, was sufficiently strong to disrupt the formation of permanent step and layer system. The suppressing of finger activity by the presence of turbulence is known from laboratory studies (Linden, 1971).

Figure 1.6: From St.Laurent and Schmitt (1999); Shadowgraph images of optical microstructure that were obtained during the NATRE HRP survey. The tilted laminae shown here were observed throughout the thermocline. The circular window has a diameter of 10cm , and the optical features have a characteristic wavelength of 0.5 to 1.0cm . Laminae tilted 10° to 20° from the horizontal (a) were the most frequently observed orientation, although filaments with vertical alignment (b) were also observed. The images were obtained near 300m depth.

The first laboratory attempt to study the interaction between salt fingers and a laminar shear flow was done by Linden (1974). It is also to date the only systematic laboratory study done in this subject. His work was focused on determining whether or not salt fingers can exist in the presence of a steady shear flow. The problem was also examined theoretically through of linear and finite amplitude stability analysis.

A two-layer initial configuration was considered and the fluid was assumed to flow between free, conducting boundaries at $z = \pm \frac{d}{2}$ (z represents the vertical axis increasing upwards and d is the depth of each layer), which were chosen for convenience. The upper boundary is maintained at concentration values ΔS and ΔT , both positives, relative to the lower boundary. The linear equation of state was considered.

As described by Linden, the non-dimensional, Boussinesq, linearized equations of momentum, heat and salt conservation in a steady shear flow are given by:

$$\left[\frac{1}{\sigma} \left(\frac{\partial}{\partial t} + \sigma R_e U(z) \frac{\partial}{\partial x} \right) - \nabla^2 \right] \nabla^2 w' + \frac{R_e}{\sigma} \frac{d^2 U(z)}{dz^2} \frac{\partial w'}{\partial x} - R_T \nabla_H^2 T' + R_S \nabla_H^2 S' = 0, \quad (1.15)$$

$$\left[\frac{\partial}{\partial t} + \sigma R_e U(z) \frac{\partial}{\partial x} - \nabla^2 \right] T' + w' = 0, \quad (1.16)$$

$$\left[\frac{\partial}{\partial t} + \sigma R_e U(z) \frac{\partial}{\partial x} - \tau \nabla^2 \right] S' + w' = 0, \quad (1.17)$$

where w' is the vertical velocity (Linden, 1974).

The boundary conditions at $z = \pm \frac{1}{2}$ are stress free:

$$w' = w'_{zz} = 0, \quad (1.18)$$

and

$$T' = S' = 0. \quad (1.19)$$

The non-dimensional parameters describing the flow are the same as in the unsheared case presented earlier with the addition of the Reynolds number (R_e),

$$R_T = \frac{g \alpha \Delta T d^3}{\nu k_T}; \quad (1.20)$$

$$R_S = \frac{g \beta \Delta S d^3}{\nu k_S}; \quad (1.21)$$

$$\sigma = \frac{\nu}{k_t}; \quad (1.22)$$

$$R_e = \frac{U(d/2)d}{\nu}; \quad (1.23)$$

$$\tau = \frac{k_S}{k_T}; \quad (1.24)$$

where the dimensionless mean flow $U(z)$ has been scaled by its velocity at $z = \pm \frac{d}{2}$.

As already described for the unsheared case, the general procedure is to examine solutions of the form

$$w'(x, y, z, t, y) = w(z) \exp i(kx + ly - \lambda t); \quad (1.25)$$

$$T'(x, y, z, t, y) = T(z) \exp i(kx + ly - \lambda t); \quad (1.26)$$

$$S'(x, y, z, t, y) = S(z) \exp i(kx + ly - \lambda t); \quad (1.27)$$

where the marginally stable case is given by taking the real part λ as equal to zero.

For stationary disturbances, i.e., $\lambda = 0$, the solutions above admit two special cases; transverse modes ($l = 0$) with no variation in the cross stream direction, and longitudinal modes ($k = 0$) with no variation downstream. The linear stability analysis for small shears revealed that transverse modes, which are perpendicular to the direction of the mean shear flow, are inhibited by the shear, and that the preferred mode of instability is a 2-dimensional mode parallel to the mean flow (longitudinal modes, named finger sheets). In the instability calculations, k is scaled as $\frac{\partial}{\partial x}$. Thus, the substitution of $k = 0$ in the governing equations above reduces them to those for a 2-dimensional disturbance in the absence of shear. The finite amplitude properties of the transverse modes were investigated by expanding the dependent variables in powers of the perturbation amplitude ε . It was shown that the sheets were stable to $O(\varepsilon^3)$, and that the first finite amplitude correction only affects the vertical structure and not the horizontal plan form.

The laboratory experiments were carried out in a rectangular channel, which allowed two layers of fluid to be set up in a counterflow arrangement. Fingers were allowed to form at an interface between these two layers with the shear being produced across the interface by the bulk motion of the layers. The apparatus, Figure 1.7, was 1.3 m in length, 10 × 10 cm in cross section, and was divided into an upper and lower layers by a horizontal splitter plate.

FIGURE 1 A diagrammatic representation of the flow channel.

Figure 1.7: From Linden (1974); Apparatus sketch.

Each section of the channel has an inlet at mid depth, which was supplied with fluid at given steady flow rates controlled by constant-head tanks and flow meters. A central portion of the splitter plate could be removed horizontally through one side wall of the tank producing a measurement section of 30 *cm* in length. This was done to allow the two fluids to come in contact.

Both, heat-salt and salt-sugar components were used in the experiments but we will concentrate in the sugar-salt case for further comparison purposes. For this case the Rayleigh numbers (R_T , R_S) were of order 10^{11} , approximately. The range of values used for the density ratio and Reynolds number parameters were

$$R_\rho = \frac{\alpha \Delta T}{\beta \Delta S} = 1.07; 1.2; 1.68; 1.76, \quad (1.28)$$

$$R_e = \frac{U(d/2)d}{\nu} = 0; 75; 163; 400, \quad (1.29)$$

where $\tau = k_S/k_T \simeq 1/3$ and $Pr = \nu/k_T \simeq 1000$.

The vertical difference in horizontal velocity measured at the center of the layers (ΔU) ranged from 0.04 – 0.7 *cm/s*. In all experiments, the interface was stable to shear instabilities via Richardson number ($R_i = \frac{g \Delta \rho d}{\rho_0 (\Delta U)^2}$) criteria, $R_i > 6.2$. The salt and sugar fluxes across the interface were estimated by taking bulk samples of the upper and lower layers at the outlets and measuring the conductivity and density of each layer.

The author describes that, for all flow rates used, strong convective motion was observed in both layers on either side of a distinct interface region of some 0.5 – 1.5 *cm* in thickness. The interface appeared to become thinner with increasing flow rates. Observations of the plan form, through shadowgraphs, revealed that, when there was no shear, the plan form was essentially square cells with arbitrary orientation. As the shear is increased the first effect was an immediate alignment of these cells into rows parallel to the mean flow (Figure 1.8). This parallel rows would correspond to the longitudinal modes, which are dominant in the presence of the shear as predicted by the linear stability theory of Linden. The main quantitative experimental result was that the sugar and salt fluxes were seen to increase with increasing shear. Figure 1.9, from Linden's original article, shows the sugar and salt flux values normalised with respect to their values at zero shear. We can see that the increase in the salt and sugar fluxes with ΔU was not monotonic for any of the R_ρ values used. A

slight increasing trend appears to happen in the sugar fluxes when $R_\rho = 1.2$; 1.68; 1.76, which correspond to the closed circles, triangles and squares respectively. On the other hand, a decreasing trend seems to occur, for example, when $R_\rho = 1.07$ (closed diamonds) and, for the salt fluxes, when $R_\rho = 1.2$ (open circles). The scatter observed in the data makes its interpretation quit difficult.

Figure 1.8: From Linden (1974); Shadowgraph image of the longitudinal modes (finger sheets parallel to the mean flow) taken from the top of the apparatus.

FIGURE 4 The fluxes of sugar and salt, normalised with respect to the value at zero shear, plotted against the velocity difference ΔU . There are four values of $\alpha\Delta T$ and $\beta\Delta S$ represented here: $\alpha\Delta T = 18.0\%$, $\beta\Delta S = 15.0\%$ —●: $\alpha\Delta T = 37\%$, $\beta\Delta S = 21.0\%$ —■: $\alpha\Delta T = 74.0\%$, $\beta\Delta S = 44.0\%$ —▲ $\alpha\Delta T = 15.0\%$, $\beta\Delta S = 14.0\%$ —◆. Closed symbols refer to the sugar flux and open ones to the salt flux. The broken line represents the flux reduction due to the shadowzone when $w = 0.01$ cm/sec and $h = 1.5$ cm.

Figure 1.9: From Linden (1974); Sugar and salt flux measurements as they varied with ΔU .

Wells (2001) performed laboratory experiments, in which a vertically sinusoidal shear flow was initially introduced in a rectangular tank while filling it with a finger favorable profile of sugar and salt through the two bucket method. This arrangement produced a sheared flow with vertical wavelength of $\sim 120\text{ mm}$ and maximum horizontal velocities of around $\pm 2.5\text{ mm s}^{-1}$. Shadowgraph images showed sheared salt fingers tilted to $\pm 45^\circ$ and bands of different light intensity in the vertical indicating that there were variations in horizontally averaged density gradient (layering). This layering of the density gradient was observed only for $R_\rho < 1.5$, where the buoyancy flux is larger. The formation of 2-dimensional finger sheets aligned with the shear was not observed in any of the experiments.

It seems that the condition under which fingers remain as 3-dimensional structures or turn into 2-dimensional ones, in the presence of a shear flow, may be an important point in determining their ability to flux. The fingers may also occur as both structures but at different parameter values. The tilted fingers observed in the C-SALT survey may be associated with the small scale shear due to internal waves propagating at near inertial frequencies as suggested by Kunze (1987). The same explanation may be applicable to the NATRE observations. If that is the case, a laboratory attempt to quantify the effects of a shear flow upon the finger fluxes may help to bring some clarifications to the above discussion. In this scenario, the work of Linden (1974) could represent a first qualitative step when a steady shear flow is considered. However, we believe that the scatter observed in the flux measurements made his results inconclusive. Hence, in this study, we aim to determine, from laboratory experiments, a flux law relation that describes the finger flux dependence on R_e and R_ρ in the presence of a laminar shear flow.

To answer this question, we employ an annular apparatus. We believe that the annular geometry represents an improvement over the rectangular geometry previously employed by Linden. The reasons for such choice will be discussed later in this work.

By monitoring R_T , R_S and R_ρ variation with time and, by carefully controlling the imposed values of the R_e parameter we seek to quantify the salt and sugar finger fluxes in the presence of a shear flow and hence, determine flux laws. The materials and methods used in this study are described in the next chapter.

CHAPTER 2

MATERIALS AND METHODS

In this chapter we propose to use an apparatus with annular geometry to determine the salt finger fluxes in a shear flow. The apparatus consists of a bottom reservoir separated from a working section, above it, by a porous membrane. Another porous membrane at the top of the working section separate it from the top reservoir above it. The top reservoir rotates at a controlled rate above the working section while the bottom reservoir is held motionless.

We first discuss the apparatus design by means of an scale analysis in 2.1. Then, in 2.2, we provide detailed information about the dimensions of the annular tank and some of the materials used. A description of the experimental procedure is given in 2.3 along with additional materials and methods used to perform internal flux measurements. In the last section, 2.4, we describe the Particle Image Velocimetry method (PIV) used to determine the vertical velocity profile, which forms in the working section.

2.1 Scale analysis

The suitable working section dimensions and Reynolds number values used in the experiments are investigated by performing an scale analysis of the Navier Stokes momentum equations 2.8 to 2.11 in cylindrical coordinates as illustrate in figure 2.1. We take the density as a constant and assume that the motion is azimuthally independent and steady. The non-dimensional variables are defined as follows:

Figure 2.1: Sketch of the cylindrical geometry.

$$r^* = \frac{r}{R} = (1 + \xi^*), \quad (2.1)$$

$$\xi^* = \frac{x}{R}, \quad (2.2)$$

$$z^* = \frac{z}{L}, \quad (2.3)$$

$$u^* = \frac{u}{V_o}, \quad (2.4)$$

$$v^* = \frac{v}{V_o}, \quad (2.5)$$

$$w^* = \frac{w}{V_o}, \quad (2.6)$$

$$P^* = L \frac{P}{\rho_o V_o \nu}. \quad (2.7)$$

The non-dimensional equations governing the velocity components and pressure field are given by:

$$\epsilon u \frac{\partial u}{\partial \xi} + w \frac{\partial u}{\partial z} - \frac{\epsilon v^2}{(1 + \xi)} = \epsilon \gamma \frac{\partial P}{\partial \xi} + \gamma \epsilon^2 \left(\frac{\partial^2 u}{\partial \xi^2} + \frac{1}{(1 + \xi)} \frac{\partial u}{\partial \xi} - \frac{u}{(1 + \xi)^2} \right) + \gamma \frac{\partial^2 u}{\partial z^2}, \quad (2.8)$$

$$\epsilon u \frac{\partial v}{\partial \xi} + w \frac{\partial v}{\partial z} + \epsilon \frac{uv}{(1 + \xi)} = \gamma \epsilon^2 \left(\frac{\partial^2 v}{\partial \xi^2} + \frac{1}{(1 + \xi)} \frac{\partial v}{\partial \xi} - \frac{v}{(1 + \xi)^2} \right) + \gamma \frac{\partial^2 v}{\partial z^2}, \quad (2.9)$$

$$\epsilon u \frac{\partial w}{\partial \xi} + w \frac{\partial w}{\partial z} = \gamma \frac{\partial P}{\partial z} + \gamma \epsilon^2 \left(\frac{\partial^2 w}{\partial \xi^2} + \frac{1}{(1 + \xi)} \frac{\partial w}{\partial \xi} \right) + \gamma \frac{\partial^2 w}{\partial z^2}, \quad (2.10)$$

$$\left(\frac{u}{(1+\xi)} + \frac{\partial u}{\partial \xi} \right) \epsilon + \frac{\partial w}{\partial z} = 0, \quad (2.11)$$

where the star symbols are neglected for convenience. The variables u , v , and w are the total zonal, azimuthal and vertical velocity components respectively. P is the total pressure, z is the non-dimensional vertical coordinate, R is the radius of the outer cylinder and ΔR is the gap width between the inner and the outer cylinder. Hence, $\xi = \frac{x}{R}$ is the non-dimensional gap coordinate, which ranges from 0, at the inner wall, to $\frac{\Delta R}{R}$ at the outer wall.

The important non-dimensional parameters are:

$$R_e = \frac{V_0 L}{\nu} = 1/\gamma; \quad \epsilon = \frac{L}{R}, \quad (2.12)$$

where V_0 is the magnitude of the azimuthal top reservoir velocity at $x = \Delta R$, L is the depth of the working section, and ν is the molecular kinetic viscosity of water. Thus γ is the inverse of the Reynolds number and ϵ is a small parameter, which appears due to the cylindrical geometry of the experimental apparatus.

We search for asymptotic limiting cases in order to approximate the equations 2.8 to 2.11 to the equations governing the classical Couette Flow in a straight channel. We focus on low Reynolds number flow, where the balance is given by pressure and viscous forces. In these sense, the following limiting cases are considered;

- $\gamma \rightarrow \infty$ faster than $\epsilon \rightarrow 0$

The equations 2.1.8 to 2.1.11 reduce to:

$$\epsilon \gamma \frac{\partial P}{\partial \xi} + \gamma \epsilon^2 \left(\frac{\partial^2 u}{\partial \xi^2} + \frac{1}{(1+\xi)} \frac{\partial u}{\partial \xi} + \frac{u}{(1+\xi)^2} \right) + \gamma \frac{\partial^2 u}{\partial z^2} = 0, \quad (2.13)$$

$$\gamma \epsilon^2 \left(\frac{\partial^2 v}{\partial \xi^2} + \frac{1}{(1+\xi)} \frac{\partial v}{\partial \xi} + \frac{v}{(1+\xi)^2} \right) + \gamma \frac{\partial^2 v}{\partial z^2} = 0, \quad (2.14)$$

$$\gamma \frac{\partial P}{\partial z} + \gamma \epsilon^2 \left(\frac{\partial^2 w}{\partial \xi^2} + \frac{1}{(1+\xi)} \frac{\partial w}{\partial \xi} \right) + \gamma \frac{\partial^2 w}{\partial z^2} = 0. \quad (2.15)$$

$$\frac{\partial w}{\partial z} = 0; \quad (2.16)$$

Equation 2.14 allows us to obtain a non-dimensional solution for $v(\xi, z)$. Considering the following boundary conditions:

$$v(\xi, z) = 0, \text{ at } z = 0, \quad (2.17)$$

$$v(\xi, z) = 1, \text{ at } z = 1, \quad (2.18)$$

$$\frac{\partial v(\xi, z)}{\partial \xi} = 0, \text{ at } \xi = 0, \frac{\Delta R}{R}, \quad (2.19)$$

the solution gives

$$v(\xi, z) = z(1 + \xi). \quad (2.20)$$

The linear dependence in z corresponds to the solution of the Couette Flow problem in an straight channel. The remaining term, inside the parenthesis, accounts for the radial dependence.

In the second case, we examine:

- $\gamma = O(1)$ and $\epsilon \rightarrow 0$

The equations 2.8 to 2.11 reduce to:

$$w \frac{\partial u}{\partial z} = \gamma \frac{\partial^2 u}{\partial z^2} \quad (2.21)$$

$$w \frac{\partial v}{\partial z} = \gamma \frac{\partial^2 v}{\partial z^2}; \quad (2.22)$$

$$w \frac{\partial w}{\partial z} = \gamma \frac{\partial P}{\partial z} + \gamma \frac{\partial^2 w}{\partial z^2}; \quad (2.23)$$

$$\frac{\partial w}{\partial z} = 0; \quad (2.24)$$

Thus, for the azimuthal and radial components, the leading order corresponds to a balance between viscous and advection terms. From 2.24 we get that w is independent of z , i.e, $w=W_0(\text{constant})$. To satisfy the boundary conditions 2.17 and 2.18, the only possible value for W_0 is zero and then we, again, recover the equations governing the Couette Flow problem in a straight channel.

Therefore, based on the analyses above, we design L as smaller as possible than R , i.e, $\epsilon < 1$ in the experiments. Also, we make the range of R_e values as close as possible of $O(1)$.

2.2 Apparatus

A schematic 2-dimensional diagram of the experimental apparatus is shown in Figure 2.2. The dimensions of the annular tank are 6.625" in high, 18.000" of external diameter (OD) and 17.500" internal diameter (ID) at the outer wall, and 7.875" OD and 7.375" ID at the inner wall. The tank has a Plexiglas lid closing the bottom end and an open top. Inside this annular tank, a bottom reservoir and a working section are separated as follows. Two Plexiglas rings of 1.75" high and 0.375" thick, tangent to the outer and inner walls, are placed at the bottom of the tank. An O-ring is fitted in a groove along the whole extension of the inner and outer Plexiglas rings in order to tightly hold the edges of a porous membrane, which separates the fluid on the bottom reservoir from the fluid on the working section. This membrane is a sheet of Versapor 800 filter. It has 0.8 micron pore size, and thickness 0.010 *cm* when stretched. The membrane allows salt and sugar to diffuse through in accordance with the concentration gradients across it (Krishnamurti, 2003).

The top reservoir is another annulus made out from a three inches thick block of Plexiglas. the reservoir is 2" deep and it has a radial gap width of 3.75". The OD and ID dimensions are properly chosen to fit the upper part of the annular tank. The top reservoir is closed at the top end and open at the bottom, where an O-ring is fitted in a groove along the inner and outer edges to hold a second porous membrane.

The design described above allows a 2" deep working section in the central part of the annular tank. The gap width of the working section is 4.8" while the top and bottom reservoir gaps are 3.75" wide. This difference, $\simeq 1"$, may cause undesirable flow due to lateral density differences between the fluids nearby the outer and inner regions of the working section. To eliminate this inconvenient, two Plexiglas rings of 2" depth and 0.375" thickness each are placed tangentially to the outer and inner walls. The outer ring can be adjusted radially in order to reduce the working section width to the same length of the bottom and top reservoirs.

The adaptation of stirring devices to the top and bottom reservoirs is done as follows. In the top reservoir, three rotating paddles equally spaced along the azimuthal direction are placed inside it through an $\simeq 0.25"$ diameter hole. Each paddle is driven by a Synchron AC Timing motor (see www.herbach.com, Model H1-14 for details), which operates at 30 RPM. Previous tests using dye tracer showed that, at this frequency, no

2D Schematic diagram of the apparatus

Figure 2.2: 2-dimensional Schematic diagram of the apparatus

noticeable fluid is forced through the membrane due to the motion of the stirring paddles. The Synchron motors are fed by a stationary power supply (Electro-craft Corporation, model E-550-M). The connection between the rotating and the stationary frameworks is made possible by employing a slip ring, Model 305 from Mercotac Incorporated (see <http://www.mercotac.com/html/threeconductor.html> for details). This device can be used in any electromechanical system that requires unrestrained, continuous rotation while transferring power from a stationary to a rotating structure. In the bottom reservoir, we use three Teflon-coated magnetic stirring bars. These bars are 2" long by 0.25" wide and they are mounted on an acrylic cradle. These stirring bars are driven from outside the tank by the motor driven rotation of three other magnetic bars, which are placed right beneath them.

The methods of internal field measurements used in this work required many types of materials. They are described along with the work procedure in the next section.

2.3 Procedure

In preparing the solutions, we used distilled water, cane sugar and kosher salt. Approximately 525 liters of distilled water, 25 *kg* of sugar and 21 *kg* of salt were consumed in the 35 experiments performed. Kosher salt was used as it is free of calcium silicate often found in table salt; the former makes clear solutions while the latter does not (Krishnamurti, 2003).

The apparatus is filled as follows. One bucket containing sugar solution and a second one containing salt solution are placed on a shelf positioned 125 *cm* above the level of the annular tank. Initially, the bottom reservoir is completely filled with the desired salt concentration solution. Then, the membrane is stretched over the fluid and an O-ring is set in its groove along the inner and outer Plexiglas rings to hold the edges of the membrane. This makes a flat and rigid bottom for the working section. The working section is filled with a two-layer fluid. First the sugar solution (less dense) is poured in and then, the salt solution (more dense) is gravitationally inserted beneath it at a filling rate of 2.0 – 2.5 *cm/hour*. In the last step, the top reservoir, with the membrane already in its groove, is placed on top of the annular tank and filled with the desired concentration of sugar solution. The volumes of the fluid solutions at the top and bottom reservoirs are the same.

In the procedure for collecting data, fluid samples of 50 *ml* volume are withdrawn from the top reservoir through a syringe connected to an outlet. Conductivity, density and temperature readings are taken, at a 15 minutes time interval, and then the samples are returned to the reservoir with the smallest loss of volume possible. At the bottom reservoir, measurements are taken just to ensure mass conservation. Hence, samples are collected only at the initial and final instants of the experiments. An immersion-probe-type conductivity meter with three significant figures of precision is employed (see model 1056 at <http://www.amberscience.com/instruments> for details). The conductivity readings of the top and bottom layer solutions are converted to salt concentration using the polynomial equations of state of Ruddick and Shirtcliffe (1979) and the results are shown in Figures 2.3 and 2.4. The upper panel shows the conductivity against salt concentration for the three different sugar concentration values of the top reservoir solution used in the experiments, $S = 45 \text{ ppt}$, $S = 64 \text{ ppt}$ and $S = 83 \text{ ppt}$. The lower panel shows the curve when no sugar molecules are present ($S = 0 \text{ ppt}$), which approximately represents the bottom reservoir conditions.

An specific gravity balance, which allows density measurements up to a precision of $\pm 0.0001 g/L$ is used. By knowing the density and the salt concentration, the sugar concentration can also be obtained from the polynomial equations of Ruddick and Shirtcliffe (1979). The reservoirs are mechanically stirred at all times during the experiments. Previous tests done by inserting dye in the top reservoir through a point source reveal that less than 5 minutes are need for total spread of the dye in the reservoir.

The fluxes are determined as follows. Calling T^{tr} the salt concentration on the top reservoir for example, its time rate of change is given by:

$$\frac{dT^{tr}}{dt} = \frac{F_T}{h}, \quad (2.25)$$

where F_T is the salt flux into the top reservoir, via salt fingers and diffusion, and h is the depth of the top reservoir. A similar relation applies to the time rate of change of the sugar concentration.

The experimental parameter values explored in this work are density ratio, $R_\rho = 1.2; 1.54; 2.1$. The correspondent values of the sugar Rayleigh number ($R_S = \frac{g\beta\Delta S h_{ws}^3}{k_T\nu}$) are $R_S \simeq 12 \times 10^{10}; 9.4 \times 10^{10}; 7 \times 10^{10}$. The salt Rayleigh number defined as $R_T = \frac{g\alpha\Delta T h_{ws}^3}{k_T\nu}$ is not varied between experiments. In all the experiments the initial R_T value is $R_T \simeq 4.8 \times 10^{10}$. In the expressions above g is the gravitational acceleration, α and β are the fractional change in density due to changes in the salt and sugar concentration respectively; ΔT and ΔS are the initial salt and sugar concentration difference; $k_T = 10^{-5} cm^2/s$ is the molecular salt diffusion coefficient; $h_{ws} \simeq 5.1 cm$ is the working section depth and $\nu = 10^{-2} cm^2/s$ is the kinetic viscosity coefficient of water.

The ω values (top lid rotation speed) are approximately set within the range 0 to $0.02 rad/s$. This latter value was found by Krishnamurti and Zhu (1996) as the limit beyond which radial motions (secondary circulation) become important. In this work heat and momentum transport were studied in sheared Rayleigh-Benard convection. An experimental apparatus with same dimensions as the one proposed here was used. A rotation was imposed to the bottom lid while the top one formed part of a torsion balance for the measurement of the torque exerted by the water and hence, the momentum flux through the water. The behaviour of the dynamic viscosity coefficient with the rotation speed, found in this study, is shown on Figure 2.5.

Figure 2.3: Conductivity versus salt concentration curves for a sugar-salt solution at $S = 45\text{ ppt}$, $S = 60\text{ ppt}$ and $S = 77\text{ ppt}$.

Figure 2.4: Conductivity versus salt concentration curve for $S = 0\text{ ppt}$.

Figure 2.5: From Krishnamurti and Zhu (1996); Viscosity coefficient versus rotation speed (ω).

The Reynolds number in our experiments is computed by $Re = \frac{U h_{ws}}{\nu}$, where $U = \omega \times r_m$ is defined as the tangential velocity at mid radius distance ($r_m = 17$ cm).

2.4 Internal velocity measurements: The PIV technique

In order to determine the velocity field produced in the working section due to the imposed rotation speed of the top reservoir we employ the Particle Image Velocimetry (PIV) method. PIV is a flow velocity measurement technique on a plane sheet of laser light used to illuminate a flow seeded with tracer particles. These particles are silver-coated hollow glass spheres whose density should ideally match the one of the fluid being studied. The positions of the particles are recorded at two different instants of time (approximately 1/3 of a second apart) on a high-resolution digital camera, which is oriented 90 degrees to the

Figure 2.6: 2D Sketch (top view) of the PIV technique with the experimental apparatus immersed in the square tank.

plane of the light sheet. From a top view, this plane is seen as a straight line crossing the annulus and the camera shots are taken along the radial direction as shown on Figure 2.6. The recorded images are analysed using the PIV software (Christensen, Soloff and Adrian, 2001), which determines the instantaneous horizontal and vertical velocity components as functions of position. This is done basically by determining the average displacement of the particles over each small interrogation region in the images (two frame cross correlation method).

The cylindrical geometry of the working section causes undesirable distortion in the light path while photographing the flow. The light, initially propagating through the air media, reaches the cylindrical wall and penetrates the fluid in the working section. The refractive index of the sugar-salt solution ($n_{su} \simeq 1.35$ at 10% concentration) differs from air ($n_{air} \simeq 1$) in about 35%. This difference is proportional to the light path deviation. Thus, in order to minimize that, we place the apparatus inside a 20" length ($\simeq 4"$ deep) acrylic square tank filled with water. The difference in the refractive index of water ($n_{water} \simeq 1.33$) and the sugar-salt solution is only about 2%.

As an illustration, we end this chapter by showing a few photographs of the experimental apparatus. Figure 2.7 shows a top view of the annular tank used in the experiments. At

the top we see the electric motors, which drive the stirring paddles in the top reservoir. The wire connections of the motors to the slip ring device, placed at the center, are also shown. Figure 2.8 shows a front view of the annular tank. The regions occupied by the top reservoir, working section, and bottom reservoir (mostly covered by a ruler) are shown. A syringe connected to the bottom reservoir, which was used to take fluid samples, and the square tank where the annular tank is placed can also be seen. A diagonal view of the tank, in Figure 2.9, shows the plane of laser light used to illuminate the tracer particles in the working section.

Figure 2.7: Top view of the annular tank.

Figure 2.8: Front view of the annular tank showing the separation between the top reservoir, the working section and the bottom reservoir.

Figure 2.9: Side view of the annular tank. The green plane of laser light used to illuminate the tracer particles in the flow is shown.

CHAPTER 3

RESULTS

In this chapter we describe our experimental results. In section 3.1 we show the velocity maps computed from the PIV analysis. In section 3.2 we enumerate our experimental observations and present the results for the finger fluxes. We also present the least square method used to determine the polynomials that govern the salt flux dependence on the Reynolds number and on density ratio.

3.1 The velocity field

A computation of the shape of the velocity profile formed across the working section is performed for different values of the imposed top lid speed (ω). Figure 3.1 shows a snapshot map of the velocity vector in the working section with a constant density fluid, when $\omega = 0.003 \text{ rad/s}$, which corresponds to $Re \simeq 25$. In the plane of the laser light used to illuminate the tracer particles, an organized azimuthal flow was observed at all depths in the working section. The maps correspond to a portion of the light plane of $\simeq 3 \text{ cm}$ depth by 1.2 cm width. This plane of light intercepts the working section, approximately, at mid-radius. A coherent flow to the right with vector magnitudes decreasing from top to bottom can be seen. In the lower panel, Figure 3.2, the velocity vectors were averaged horizontally. The velocity (U_{mean}) is seen to decrease from approximately $2.6 \times 10^{-3} \text{ cm/s}$ at the top of the profile to about $0.7 \times 10^{-3} \text{ cm/s}$ at the bottom. Some wiggles and spikes can be observed along the profile. However their deviation from a linear profile is, at most, one order of magnitude smaller than the averaged velocity values in the profile. In each plot, the amplitude of the most pronounced spike is taken as the precision to within the velocity profile is computed. In Figures 3.3 and 3.4, ω was doubled to 0.006 rad/s , which corresponds

to $R_e \simeq 50$. Again, the velocity map shows an organized flow to the right. The horizontally averaged profile shows velocity magnitudes decreasing approximately from $4 \times 10^{-2} \text{ cm/s}$ at the top, to $1.3 \times 10^{-2} \text{ cm/s}$ at the bottom, in agreement with the increase by a factor of 2 in ω .

Figure 3.1: Vector map of the instantaneous velocity in the working section. The fluid in the working section was set with constant density. The map corresponds to a region of 3 cm depth by 1.2 cm wide; $\omega = 0.003 \text{ rad/s}$ ($R_e \simeq 25$).

Figure 3.2: Horizontally averaged profile of the instantaneous velocity in the working section (U_{mean}) when $\omega = 0.003 \text{ rad/s}$ ($R_e \simeq 25$) and constant density.

Figure 3.3: Vector map of the instantaneous velocity in the working section. The fluid in the working section was set with constant density. The map corresponds to a region of 3 cm depth by 1.2 cm wide; $\omega = 0.006 \text{ rad/s}$ ($R_e \simeq 50$).

Figure 3.4: Horizontally averaged profile of the instantaneous velocity in the working section (U_{mean}) when $\omega = 0.006$ rad/s ($R_e \simeq 50$) and constant density.

Figure 3.5 and 3.6 show a snapshot map of the velocity vector and the correspondent velocity profile in the working section, which now contains fingers extending approximately from top to bottom. The density ratio was set as $R_\rho = 1.2$ and $\omega = 0.003 \text{ rad/s}$, i.e, $R_e \simeq 25$. Both plots show similar features to the ones already observed in the correspondent non-finger case (constant density). Again, an organized flow to right with velocity magnitudes decreasing from top to bottom can be seen. The horizontally averaged profile shows velocity magnitudes decreasing approximately from $2.7 \times 10^{-2} \text{ cm/s}$ at the top, to $0.9 \times 10^{-2} \text{ cm/s}$ at the bottom similarly to the non-finger case. The small wiggles and spikes are, again, observed to occur along the profile. When the imposed ω is doubled the results, once again, show an increase by a factor of $\simeq 2$ in the velocity magnitudes as observed in Figure 3.7 and 3.8.

Figure 3.5: Vector map of the instantaneous velocity in the working section. The fluid in the working section is finger favorable and $R_\rho = 1.2$. The map corresponds to a region of 3 cm depth by 1.2 cm wide; $\omega = 0.003 \text{ rad/s}$ ($R_e \simeq 25$).

Figure 3.6: Horizontally averaged profile of the instantaneous velocity in the working section (U_{mean}) when $\omega = 0.003$ rad/s ($R_e \simeq 25$) in the presence of fingers, $R_p = 1.2$.

Figure 3.7: Vector map of the instantaneous velocity in the working section. The fluid in the working section is finger favorable and $R_\rho = 1.2$. The map corresponds to a region of 3 cm depth by 1.2 cm wide; $\omega = 0.006 \text{ rad/s}$ ($R_e \simeq 50$).

Figure 3.8: Horizontally averaged profile of the instantaneous velocity in the working section (U_{mean}) when $\omega = 0.006$ rad/s ($R_e \simeq 50$) in the presence of fingers, $R_\rho = 1.2$.

A computation of the radial component of the velocity is also done. Figure 3.9 and 3.10 show the radial velocity map when $R_e = 50$; 220. No significant radial circulation can be observed. Within the range $0 < R_e < 220$, the radial velocities are almost two orders of magnitude smaller than the velocity magnitudes computed for the azimuthal component.

Figure 3.9: Vector map of the radial velocity in the working section when $R_e = 50$.

Figure 3.10: Vector map of the radial velocity in the working section when $R_e = 220$.

3.2 Finger fluxes

In many repetitions of the experiments some qualitative observations were made. We summarized them as follows:

- By the time we finish filling up the the reservoirs, the fingers already occupied most of the depth of the working section. They are between 3.5 to 5 *cm* tall and approximately 1 *mm* wide.
- When the top lid rotation starts the fingers tilt in the direction of the imposed velocity. This tilting is not uniform along the depth of the fingers, being more noticeable at their upper $\simeq 1$ *cm*, where they appeared not only inclined but also curved.
- The tilt of the fingers appears to increases with increasing magnitude of the top lid rotation velocity. This increase also leads to a more intense curving of the upper finger region.
- The fingers appear to be 3-dimensional structures in the experiments regardless of the presence or not of the imposed top lid velocity.

Figures 3.11, 3.12 and 3.13 show the changes in the salt concentration with time measured in the top reservoir for $R_\rho = 1.2; 1.54; 2.1$ respectively. In each plot, the different curves contain 11 to 14 data measurements and each curve corresponds to different Reynolds number values. The time scale, $\tau = h_{ws}^2/\nu$, of the flow adjustment in the working section was estimated as $\simeq 40$ *min*. Hence, the first three salt concentration measurements, which correspond to the initial 45 minutes of experiments, were discarded.

In all plots a monotonic increase in the salt concentration with time can be observed. This is the direct result of the vertical salt transport by the fingers from the bottom to the top reservoir. We also note that, in each of the three plots, a general and systematic decrease in the slope of the curves is seen to occur as R_e increases.

Figure 3.11: Concentration of salt versus time in the top reservoir; $R_p=1.2$. The curves are offset by a factor of 0.2 *ppt*.

Figure 3.12: Concentration of salt versus time in the top reservoir; $R\rho=1.5$. The curves are offset by a factor of 0.1 *ppt*.

Figure 3.13: Concentration of salt versus time in the top reservoir; $R_p=2.1$. The curves are offset by a factor of 0.05 ppt.

A polynomial fit using the least square method (see, Emery and Thomsom (2001), for details) was performed to determine the slope associated with each salt concentration curve. The fluxes were obtained by multiplying each slope value by the depth of the fluid in the top reservoir ($h = 3.8 \text{ cm}$). Figures 3.14, 3.15 and 3.16 show the resulting salt fluxes plotted against the Reynolds number (R_e). In all these plots, a general decrease in the salt fluxes can be observed as R_e increases.

When $R_\rho = 1.2$, the first point, which corresponds to the non-shear experiment shows the highest flux value. The first four points at $0 < R_e < 53$ show a consistent decrease in F_T as R_e increases. When $53 < R_e < 76$, F_T remains approximately constant. In the next three consecutive points, $76 < R_e < 155$, the salt flux fluctuates and then F_T decreases again when $155 < R_e < 216$. The cut off value where, radial motions are expected to become noticeable, should occur when $R_e \simeq 270$.

When $R_\rho = 1.5$ we, again, observe a consistent decrease in the salt flux with the first four R_e values. The next following point, $R_e = 76$ shows a small increase in the F_T value. Then a salt flux decrease is again observed when $76 < R_e < 155$. When $155 < R_e < 216$ the salt flux slightly increases. Then F_T is seen to decrease again when $R_e > 216$ following an approximately linear pattern as seen in the last three points.

When $R_\rho = 2$, an monotonic decrease in the salt flux as R_e increases can be observed for all data points when $R_e < 216$. Beyond this value, F_T appears to slightly increase with a linear pattern.

The variation of F_T with R_ρ at fixed R_e values is shown in Figure 3.17. The topmost curve shows the F_T behaviour when no velocity difference is imposed ($R_e = 0$). When $R_e > 9.5$ the F_T curves become closer, mainly between $R_\rho = 1.2$ and $R_\rho = 1.54$. When $R_e > 95$ the salt flux curves mostly overlap at all three R_ρ values. Even though we explored only three R_ρ values in the present study, it is clear that the salt fluxes decrease with increasing R_ρ . F_T is seen to vary by approximately one order of magnitude in the range $1.2 < R_\rho < 2.1$.

Figure 3.14: Salt flux in the top reservoir at $R\rho = 1.2$.

Figure 3.15: Salt flux in the top reservoir at $R\rho = 1.54$.

Figure 3.16: Salt flux in the top reservoir at $R_\rho = 2.1$.

Figure 3.17: Salt flux varying with R_p at different Re values.

From dimensional analysis, the salt flux F_T across the working section may be expressed as

$$F_T = k_T \frac{\Delta T}{h_{ws}} f(R_e, R_\rho, R_T, \tau, Pr) \quad (3.1)$$

where $k_T \frac{\Delta T}{h_{ws}}$ is the diffusive flux, which was constant $\simeq 10^{-4} \text{ ppt cm/s}$ in all experiments; h_{ws} is the depth of the working section; f is a function of the dimensionless numbers R_T , the salt Rayleigh number, R_ρ , the density ratio and R_e , the Reynolds number; τ is the diffusivity ratio k_S/k_T ; Pr is the Prandtl number ν/k_T . In the sugar-salt system, $\tau \simeq 0.3$ and $Pr = 1000$. The initial value of the salt Rayleigh number, $R_T = 4.78 \times 10^{10}$, was the same in all experiments. Variations of R_T with time during a single experiment occur due to the small run down of the initial ΔT concentration. This variations were less than 10% in all experimental runs and hence, R_T is taken as a constant in the following analysis. First we express the function f , for each R_ρ , as a function of R_e only, writing this as a power law:

$$f \sim (R_e)^a \quad (3.2)$$

Thus

$$F_T \sim k_T \frac{\Delta S}{h_{ws}} (R_e)^a \quad (3.3)$$

To determine the exponent a we take the logarithm of equation 3.3 and re-arrange to display linear equations:

$$Z_n = aX_n \quad (3.4)$$

where

$$\begin{aligned} Z_n &= \ln(F_T) - \ln(k_T \frac{\Delta T}{h_{ws}}), \\ X_n &= \ln(R_e). \end{aligned}$$

A least-squares fit of the data to the linear regression gives a as shown in Table 3.1. This table also shows the constant A , which was determined from the experiments. In this calculations we only use F_T values for which $R_e \leq 216$. As can be seen from Table 3.1, not only the Reynolds number exponent (a) but also the experimental constant (A) depend upon R_ρ . The values of a are obviously negative since the salt fluxes were found to decrease with increasing R_e . This values are seen to become more negative as R_ρ increases. The constant A is seen to increase as R_ρ increases.

Table 3.1: F_T dependence with R_e for fixed R_ρ values.

R_ρ	1.2	1.54	2.1
A	11 ± 0.06	8.1 ± 0.15	6.2 ± 0.2
a	-0.025 ± 0.004	-0.1 ± 0.01	-0.34 ± 0.01

The salt fluxes, as a function of R_ρ , are then expressed as:

$$F_T(R_\rho = 1.2) = 11 \left(k_T \frac{\Delta T}{h_{ws}} \right) R_e^{-0.025} \quad (3.5)$$

$$F_T(R_\rho = 1.54) = 8.1 \left(k_T \frac{\Delta T}{h_{ws}} \right) R_e^{-0.1} \quad (3.6)$$

$$F_T(R_\rho = 2.1) = 6.2 \left(k_T \frac{\Delta T}{h_{ws}} \right) R_e^{-0.34} \quad (3.7)$$

The results from Table 3.1 are plotted in Figures 3.18, 3.19 and 3.20 superposed to the F_T versus R_e curves. At R_e beyond $\simeq 216$, a different power fit is obtained for all the R_ρ values studied. The dashed red lines correspond to a least square fit of the F_T values when $R_e > 216$. For this points the exponent a was found as equal to 0.024, -0.22 and -0.05 for $R_\rho = 1.2; 1.54; 2.1$ respectively.

In order to express the mathematical dependence of a and A with R_ρ we perform a polynomial fit using the values showed on Table 3.1. The fitted polynomials are showed in Figures 3.21 and 3.22. Thus, the mathematical equation that encompass the complete F_T dependence with R_e and R_ρ should be represented as:

$$F_T = k_T \frac{\Delta T}{h_{ws}} A(R_\rho) (R_e)^{a(R_\rho)} \quad (3.8)$$

where $A(R_\rho)$ and $a(R_\rho)$ are respectively given by:

$$A(R_\rho) = 5.84R_\rho^2 - 24.6R_\rho + 32.2 \quad (3.9)$$

$$a(R_\rho) = -0.234R_\rho^2 + 0.424R_\rho - 0.198 \quad (3.10)$$

Figure 3.18: Power law fit of the salt flux in the top reservoir at $R_\rho = 1.2$.

Figure 3.19: Power law fit of the salt flux in the top reservoir at $R_p = 1.54$.

Figure 3.20: Power law fit of the salt flux in the top reservoir at $R_p = 2.1$.

Figure 3.21: Polynomial fitting of the R_e exponent (a) to R_ρ . In the equation above $x = R_\rho$ and $y = a$.

Figure 3.22: Polynomial fitting of the experimental constant (A) to R_p . In the equation above $x = R_p$ and $y = A$.

Thus, for any value of R_ρ and R_e within the range $1.2 < R_\rho < 2.1$ and $0 < R_e < 216$ the finger salt flux (F_T) can be determined through 3.8, where a and A are given by 3.9 and 3.10. It should be noticed that A and a are also a function of τ , P_r and R_T . The formers are constants. The latter, as already mentioned, varied by less than 10% during experiments and hence, was taken as a constant in the analysis.

The F_T dependence with R_e and R_ρ is shown in Figure 3.23 . We see that the salt flux map is a 3-dimensional surface, which tilts down as we move to higher R_ρ and R_e values. This is also shown when the F_T values are plotted as contour lines over the R_e versus R_ρ plane, Figure 3.24. The contours are almost horizontal at high R_e values indicating low sensitivity of F_T when R_e is changed. However, for values of $R_e < 80$ the F_T curves change quit significantly and start bending upwards.

Figure 3.23: 3-dimensional representation of the polynomial fit of the salt flux to R_e and R_ρ . The salt flux units are $\text{ppt} \times \frac{\text{cm}}{\text{s}}$.

Figure 3.24: Contour map of the polynomial fit of the salt flux to R_e and R_ρ . The salt flux units are $ppt \times \frac{cm}{s}$.

The salt Nusselt number (N_T), defined as the ratio of the total salt flux measured and the molecular diffusive flux ($N_T = F_T / k_T \frac{\Delta T}{h_{ws}}$), is showed in Figures 3.25, 3.26, 3.27. The N_T values decrease with increasing R_e in a pattern very similar to the one already described for F_T . When $R_\rho = 1.2$ and $R_e < 200$ we see that the N_T values range from a maximum $N_T \simeq 11.3$, corresponding to the non-shear experiment, to a minimum of $N_T \simeq 9.6$ at $R_e \simeq 216$. When $R_\rho = 1.54$ the values range from $N_T \simeq 7.1$ to $N_T \simeq 4.8$, and when $R_\rho = 2.1$ N_T ranges from $\simeq 3.6$ to $\simeq 1$. This latter value indicates that at $R_e \simeq 216$ the salt flux approach the molecular diffusive flux ($N_T = 1$). This low value should be found at higher R_ρ in experiments without an imposed shear.

Figure 3.25: Salt Nusselt number at $R_\rho = 1.2$

Figure 3.26: Salt Nusselt number at $R_\rho = 1.54$

Figure 3.27: Salt Nusselt number at $R_\rho = 2.1$

In all repetitions of our experiments the density measurements taken at the bottom and top reservoirs solutions were observed to vary only in the fourth decimal place regardless of the presence of an imposed velocity. Therefore, no significant variations in density could be measured during the 4 hours duration of the experiments. Figure 3.28 shows the time rate of change of density in the top reservoir for the case of $R_e = 0$ and $R_\rho = 2.1$, for example. The increase in the measured density due to the increase in salt concentration is compensated by the decrease in the inferred density due to the decrease in the sugar concentration keeping the measured density of the solution approximately constant. Some experiments were run for up to 18 hours with no measurable change in density. Further considerations about this results will be made in the next chapter.

Figure 3.28: Time rate of change of the measured density due to changes in the salt concentration (dashed circles); density (dashed stars); and the inferred density due to changes in the sugar concentration (dashed squares), when $R_\rho = 2.1$ and $R_e = 0$, in the top reservoir solution.

CHAPTER 4

DISCUSSION AND CONCLUSIONS

In this chapter we discuss the results for the finger fluxes in the presence of a shear flow described in the previous chapter. In section 4.1 we present the results of an instability model for tilted fingers and suggest a physical mechanism that may explain the finger flux reduction with the imposed shear as observed in our experiments. In section 4.2 we discuss how the results obtained in this study may be applicable towards ocean studies. In section 4.3 we enumerate our conclusions and then in 4.4 we make suggestions for future work.

4.1 Finger fluxes

In Figure 3.11, corresponding to $R_\rho = 1.2$, the salt concentration versus time is seen to curve downwards. This shows that the time rate of change of salt concentration is slightly larger at earlier stages of the experiment than at later ones. This is expected since there is a small run down of the initial sugar and salt concentrations with time during the experiments. The curving is more evident when $R_\rho = 1.2$, which corresponds to the largest concentration changes with time. At this density ratio, the salt concentration in the top reservoir changes, on an average, from 0 to $\simeq 4.3 \text{ ppt}$ during the four hours of the experiments. This corresponds to a reduction of $\simeq 8.4\%$ of the initial vertical salt concentration difference (ΔT). In Figure 3.12, when $R_\rho = 1.54$ the curving is again observed but with less intensity. In this case the salt concentration changes from 0 to $\simeq 2.3 \text{ ppt}$ during the four hours, which corresponds to a ΔT reduction of $\simeq 4.5\%$. At $R_\rho = 2.1$, Figure 3.13, the salt concentration appears to curve slightly upwards, i.e, the small decrease in the time rate of salt transport, observed in the previous cases at the later stages of the experiments does not occur. The averaged changes are from 0 to 0.8 ppt , corresponding to a ΔT and ΔS reduction of $\simeq 1.6\%$.

The changes in the salt concentration with time when $R_\rho = 2.1$ are slower than in the previous two cases because the finger fluxes are smaller. A longer experimental run would more likely show a similar decrease at later times as observed at $R_\rho = 1.2$ and $R_\rho = 1.54$.

As shown in the Chapter 3, the salt fluxes were computed by the slope of the salt concentration versus time at different R_e values. To evaluate the effect of the observed curving in the determination of the fluxes, a computation of the slopes was done for shorter time intervals in the salt concentration plots (corresponding to data points from 120 to 180 *min*, for example). No differences in the relative magnitude of the fluxes were found. Hence, we conclude that the curving observed in Figures 3.11, 3.12 and 3.13 was small enough to justify the use of a linear regression to compute the salt fluxes.

Figures 3.14, 3.15 and 3.16 show the decrease of the salt fluxes as the imposed velocity difference increases for the three R_ρ cases studied. Deviations from a monotonic decrease are seen in the cases where $R_\rho = 1.2$ and $R_\rho = 1.54$. In the first case, the salt flux fluctuates in the interval $53 < R_e < 122$. In the latter, a small increase in F_T occurs at $R_e \simeq 75$. This may indicate some transition state in the F_T versus R_e dependence. However, no other evidence of such transition was observed at this R_e values or in its vicinity. Hence, we then opted for smoothing the F_T dependence with R_e . Figures 3.18, 3.19 and 3.20 show that this smoothing can be accomplished within the error bars.

The observed decrease of the salt fluxes with increasing R_e may be closely related to the experimental observations that the fingers are seen to tilt in the direction of the imposed flow. We now present a theoretical model for finger fluxes proposed by Krishnamurti (personal communication). The model consists of two steps: (i) a linear stability analysis, (ii) the thermohaline staircase model of flux reduction (Krishnamurti, 2003).

Krishnamurti (personal communication) used the salt finger model of Howard and Veronis (1987) for the case of no salt diffusion ($\kappa_S = 0$), and computed the density in up and down going fingers, where the equilibrated temperatures are given by hyperbolic and trigonometric functions. The temperature T is given by

$$T = Q \left[\frac{(\cosh x \cos(\pi b - x) + \cos x \cosh(\pi b - x))}{\cosh \pi b + \cos \pi b} \right], \quad (4.1)$$

where πb is the finger width and $Q = \beta \Delta S / \alpha L \bar{T}_z$ in up-fingers, and the sign is reversed in

down-fingers. L is the buoyancy boundary layer thickness

$$L = \left(\frac{4\nu\kappa_T}{g\alpha\bar{T}_z} \right)^{1/4}, \quad (4.2)$$

where κ_T is the thermal diffusivity; ν is the kinematic viscosity; g is the acceleration of gravity; α and β are the thermal expansion and salinity contraction coefficients respectively; ΔS is the salinity of the top reservoir and $\bar{T}_z$ is the mean vertical temperature gradient.

The salinity in the down-fingers is taken as ΔS and, in the up-fingers as $S = 0$. If the negatively buoyant down-fingers were placed horizontally over positively buoyant up-fingers also placed horizontally, calculations show that convective instability would result if the Rayleigh number R , determined by the density difference $\Delta\rho$ between the up and down fingers and the finger width $\delta/2$, exceeded some critical value R_C :

$$R = g \frac{\Delta\rho\delta^3}{\rho_0\kappa_T\nu} \geq R_C. \quad (4.3)$$

If the fingers were not horizontal but at an angle θ to the vertical then g may be replaced by $g\sin\theta$ in 4.3. Thus, a critical angle θ_C for convective instability would be given. Calculations indicate $\theta_C \sim 30^\circ$ to 40° depending on density ratio R_ρ and finger width $\delta/2$.

Linear instability theory does not imply flux increase or reduction. However, we will now argue that stirring by convection, as depicted in the right side diagram of figure 4.1, would result in flux reduction (Krishnamurti, 2003). The down-fingers carry salinity ΔS from the top reservoir, and the up-fingers carry zero salinity from the bottom reservoir as shown in figure 4.1 (left side diagram), resulting in a certain salt flux F_0 . If the up and down fingers are perfectly mixed as in figure 4.1 (right side diagram), then the up-going fingers reaching the top reservoir would have salinity $\Delta S/2$, that is, one half of the ΔS sent down in the down-fingers is returned to the top reservoir, resulting in flux $F_0/2$. In general if there are n convecting zones the flux is reduced to F_0/n (Krishnamurti, 2003).

Near onset of instability $R \geq R_C$, the resulting mixing may be less efficient than when $R \gg R_C$. As a measurement of this effect, we note that as the angle θ is increased the potentially unstable region, where denser fluid overlies less dense fluid is also increased. The potentially unstable region is indicated in Figure 4.2 by the hatch shading. In figure 4.2(a), 20% of the area is potentially unstable; in figure 4.2(b), 50%; in figure 4.2(c), 94% is potentially unstable.

Figure 4.1: From Krishnamurti (2003); Schematic representation of the finger flux reduction due to convection.

Figure 4.2: Schematic representation of the convecting area between two consecutive finger columns according to Krishnamurti (personal communication).

This model suggests that it is this increasing partial instability that accounts for the monotonic decrease in flux with increasing shear. However, the asymptotic reduction at large shear would be $F_0/2$ if one convection zone fills most of the layer (except where fingers emerge from each boundary.) This may be the case at small enough R_ρ , where the destabilizing component is relatively large. However, the asymptotic reduction in the flux may be $F_0/3$ if the convection is unable to penetrate the two layers (as may be the case at large R_ρ) and the fluid state consists of fingers out of the top boundary, a convection layer below this, fingers at an interface, convection below the interface and fingers at the bottom boundary. The two mixing zones, if thoroughly mixed, would lead to a flux of $F_0/3$. We note from the experiments that the observed fluxes decrease monotonically with the imposed shear R_e and that the ratio of the observed "asymptotic" flux (at $R_e = 500$) to the unsheared flux F_0 varies with R_ρ as follows:

Table 4.1: The salt flux dependence with R_ρ at $R_e = 500$ normalised by the flux at no shear.

R_ρ	$F_n = \frac{F_{R_e 500}}{F_{R_e 0}}$
1.2	0.8
1.5	0.6
2.0	0.3

The alignment of the fingers into rows (2-dimensional sheets) parallel to the flow described by Linden was not observed in our experiments. In our study, the fingers appear to remain 3-dimensional regardless of the presence or not of the imposed velocity in agreement with Wells (2001). Figure 4.3 is a shadowgraph obtained by Krishnamurti (personal communication) of sheared salt fingers, viewing the fluid perpendicularly to the mean flow showing clear 3-dimensional structures. In this experiment hot salty water, at temperature $T_0 + \Delta T$, and salinity $S_0 + \Delta S$, was initially placed in the region above the lower solid black line in the figure. Below this line, water at T_0 , $S_0 = 0$ flows from left to right at a controlled speed ($R_e \simeq 100$). The upper shadowgraph was at larger R_ρ and the sheared fingers appear stable. The lower shadowgraph at smaller R_ρ shows convective instability.

Figure 4.3: From Krishnamurti (personal communication); Shadowgraphs of sheared heat and salt fingers viewed from the side. Flow is from left to right. The top shadowgraph shows fingers tilted by the shear flow. The bottom one, which correspond to a different R_ρ , shows convection.

Differently from Linden's experiments, the working section was confined between two closed reservoirs in our experiments. Thus, it was not possible to study the plan form of the fingers from top view shadowgraphs. We have no means to argue against Linden's observations but we believe that two important facts should be mentioned. The results of the stability analysis performed by Linden show that the longitudinal modes are preferred to grow at marginally unstable conditions. However, both his and ours experiments were conducted at Rayleigh numbers high above critical corresponding to a possibly different stability regime, where finger sheets may not necessarily be present. Secondly, the plan view of the 2-dimensional sheets obtained through shadowgraphs are vertically integrated images. From thermal convection studies, convecting cells are known to be aligned into rows parallel to the mean flow when a shear is imposed. Thus, the 2-dimensional structures observed by Linden may be a signature of these aligned cells in the convecting fluid above and below the finger interface.

The R_e , R_T and R_ρ values in this work were set in the same range as in Linden's experiments. Why are the flux results different ? Unlike in our annular geometry, we believe the imposition of an horizontal velocity in Linden's experiments causes downstream variations in the ΔS and ΔT vertical differences from that imposed at the inlets. The top "reservoir" of sugar fluid for example, initially of vertical concentration difference ΔS_V^i , where i refers to inlet, enters the test section at speed U (Figure 4.4). It is in contact with a stationary lower "reservoir" of fluid, which has initially zero sugar concentration. A finite travel distance L of fluid in the top "reservoir" leads to horizontal concentration gradients in this "reservoir" on the scale L because of the sugar losses to the lower layer via salt fingers.

The horizontal variation in the sugar concentration ΔS_H as it varies with horizontal coordinate x is

$$\Delta S_H(x) = F_S \frac{x}{U} \frac{1}{\Delta z}, \quad (4.4)$$

where F_S is the flux of sugar, and the constant Δz is the depth of the top reservoir. The vertical sugar concentration difference $\Delta S_V(x)$ as it varies with position x is

$$\Delta S_V(x) = \Delta S_V^i - \Delta S_H(x), \quad (4.5)$$

which gives

$$\Delta S_V(x) = \Delta S_V^i - F_S \frac{x}{U} \frac{1}{\Delta z}. \quad (4.6)$$

Figure 4.4: Schematic representation of the concentration changes on a parcel of fluid moving downstream over a finite length contact section.

Ideally we would want $\Delta S_H = 0$ or at least $\Delta S_H \ll \Delta S_V^i$. We note from the equation above that

- The larger is x , the smaller will be ΔS_V ,
- The smaller is U , the smaller is ΔS_V ,
- The larger is F_S , the smaller is ΔS_V .

We also note that F_S itself is a function of U and may lead to a complicated dependence of ΔS_V upon U but, this is not considered here. Equation 4.6 shows that the fingers close to the inlet ($x \simeq 0$) feel a vertical sugar concentration $\Delta S_V(x) \simeq \Delta S_V^i$ at all times. However, the fingers close to the outlet ($x \simeq L$) feel $\Delta S_V(x) \simeq \Delta S_V^i - \Delta S_H(x)$. Taking $x = L$ (30 cm) and writing $F_S = w \Delta S_V(x)$, where w is the vertical velocity within the fingers, a substitution of F_S into 4.6 gives

$$\Delta S_V(x) = \frac{\Delta S_V^i}{1 + \frac{w}{U} \frac{L}{\Delta z}}. \quad (4.7)$$

To estimate the magnitudes of the changes in $\Delta S_V(x)$, let's take the closed circles plotted on Figure 1.9 for example. They refer to an initial vertical sugar concentration difference of $\Delta S_V^i = 15 \text{ ppt}$ and $R_\rho = 1.2$. The first data point on the left corresponds to $U = 0.04 \text{ cm/s}$. Linden (1974) has estimated the vertical velocity within the fingers as $w = 0.01 \text{ cm/s}$. Taking w and $\Delta z = 5 \text{ cm}$ into 4.7 gives that the initial sugar concentration ($\Delta S_V^i = 15 \text{ ppt}$) would change to $\Delta S_V \simeq 6 \text{ ppt}$, which corresponds to a variation of $\simeq 60\%$. The same computation done for the rightmost data point, which corresponds to the highest velocity $U \simeq 0.46 \text{ cm/s}$, gives a concentration variation of only $\simeq 12\%$. This would correspond to a final value of $\Delta S_V \simeq 13.2 \text{ ppt}$ in the top reservoir. Therefore, the effective value of $\Delta S_V(x)$, which corresponds to $\Delta S_V(x)$ averaged over the distance L , should be smaller at lower values of the velocity U than at higher ones. This may explain the increase in fluxes with increasing U observed by Linden.

In all the experiments performed in this study we found that as time progressed, the salinity in the top reservoir increased monotonically. Yet, measurements showed that the density remained unchanged to the fourth decimal place with time. This same constancy in density was first observed by Krishnamurti (2003). In her experiments, while the salt concentration was seen to increase with time in the top reservoir due to the finger transport, the density remained unchanged for approximately the first 5 to 10 days. A possible explanation for such an effect may be given in terms of a chemical bond theory of density of water (Dougherty, 2001). According to this study, the addition of salt to water perturbs the hydrogen bond strength of the water molecules causing changes in the density of the water. It is still not clear how this process could compensate the decrease in the density of the top reservoir solution, which is required to happen by the finger mechanism, in order to hold the density of this solution approximately constant with time. This chemical mechanism can not be expected to hold to all salt concentrations. These concentrations however, were not realized within the four hours of duration of our experiments or even when the experiments were run up to 18 hours. Due to this phenomenon, inferences of sugar fluxes and flux ratio from our experiments could not be obtained.

4.2 Ocean aspects and their relation with the present study

The first important question we should address is how the results obtained in this study

for sugar-salt fingers in the presence of a shear flow can be related to heat-salt fingers, which corresponds to the ocean scenario. The determination of a laboratory flux law that encompasses variations of the τ parameter is a major point to be attacked. Numerical computations of heat and salt finger fluxes by Stern et. al, 2001 shown that the heat fluxes of 2 and 3-dimensional salt fingers increase with decreasing τ for $R_\rho = 2$ and $Pr = 7$. If $\tau \rightarrow 1$, the relatively rapidness with which the faster component diffuses laterally between two neighboring finger columns would be diminished. Thus, an increase in the vertical transport of the faster diffuser should be observed. On the other hand, the slower diffusive component would diffuse more laterally, causing a decrease in its vertical transport. Thus, the magnitudes of heat and salt finger fluxes would certainly differ from the flux magnitudes in the sugar and salt system. Thus, even when a shear flow is imposed, we may qualitative expect heat and salt fingers to be affected in the same way as sugar and salt ones.

In the oceans, internal waves are observed to be a conspicuous feature. They are also seen to propagate mostly at near inertial frequencies. If most of the small scale shear is due to such waves (as usually found in the ocean), tilted salt fingers should be observed almost everywhere. However, as previously mentioned, they were only observed in the shadowgraph images from the C-SALT and NATRE surveys. This images are sensitive to the focal length of the system. As discussed by Schmitt (1994), the shadowgraph system used at the C-SALT survey was designed to best resolve relatively short focal-length structure. The earlier system of Williams (1974), with which tilted fingers were not observed, had focal lengths 2 to 10 times greater. It may be possible that the difference in the two designs could account for the observed finger tilt. Kunze (1990) suggests that below the Mediterranean salt tongue and in the Tyrranean Sea, shear may be weak enough that fingers can grow without being significantly tilted. According to him this would require shears less than $0.1N$.

As described in chapter 1, conditions favorable to salt fingering are very common in the main thermocline of the subtropical gyres. In 90% of the subtropical Atlantic Ocean the main thermocline has a density ratio less than 2.3. At $24^\circ N$ in the Atlantic, Schmitt (1990) found that 95% of the upper kilometer is fingering favorable, with over 65% having a density ratio between 1.5 and 2.5. The Pacific is less conducive to salt fingering, but the Indian Ocean also harbors vast regions of low density ratio (Schmitt, 1994). In the Atlantic ocean salt fingers have, so far, been studied in regions that altogether corresponds to only $\simeq 2\%$ of the total area of the Atlantic (not counting its adjacent seas). In the Pacific ocean, a

solely observational effort was performed by Gargett and Schmitt (1982). We are, up to date, unaware of observational studies of salt fingers in the Indian Ocean.

It has been established that the strength of the thermohaline circulation depends on the magnitude of the vertical (diapycnal) mixing coefficient (Bryan, 1987; Zhang, Schmitt and Huang, 1999). Uncertainties about the magnitude of finger fluxes leads to associated uncertainties in the magnitudes of the temperature and salinity diapycnal mixing coefficients that may be key to the accuracy of numerical models. We do hope that the constant technological advances in the development of higher frequency temperature and conductivity sensors, acoustic velocity profilers, and under water shadowgraph and PIV set ups will improve, qualitatively and quantitatively, the acquisition of ocean data in small scales and, consequently, reveal much more about double diffusive processes.

4.3 Conclusions

In the present work we have performed laboratory experiments on salt fingers in the presence of a steady shear flow. Sugar (the slower diffuser) and salt (the faster diffuser) were used as density components. A quantitative study of the salt fluxes was performed and a polynomial dependence of the salt flux with Reynolds number and density ratio was determined. An investigation of the shape of the vertical velocity profile across the fingers was also performed. Then, we presented the mathematical model proposed by Krishnamurti (personal communication) for finger flux reduction by an imposed shear to qualitatively explain our experimental results. The following conclusions were made:

- The salt flux (F_T) was found to decrease with increasing Reynolds numbers (R_e) and increasing density ration (R_ρ) for sugar and salt fingers. A flux law describing the F_T as a function of R_e is given by:

$$F_T(R_\rho = 1.2) = 11 \left(k_T \frac{\Delta T}{h_{ws}} \right) R_e^{-0.025} \quad (4.8)$$

$$F_T(R_\rho = 1.54) = 8.1 \left(k_T \frac{\Delta T}{h_{ws}} \right) R_e^{-0.1} \quad (4.9)$$

$$F_T(R_\rho = 2.1) = 6.2 \left(k_T \frac{\Delta T}{h_{ws}} \right) R_e^{-0.34} \quad (4.10)$$

- The F_T dependence with R_e and R_ρ together is suggested as:

$$F_T = k_T \frac{\Delta T}{h_{ws}} (5.84R_\rho^2 - 24.6R_\rho + 32.2) R_e^{(-0.234R_\rho^2 + 0.424R_\rho - 0.198)}$$

- The finger fluxes of salt in the presence of a shear flow were found to approach molecular flux values when $R_\rho = 2.1$ and $R_e \simeq 216$.
- An instability model shows that a convective instability should occur between neighbor fingers when they are tilted beyond a critical angle $\theta_C = 30 - 40^\circ$, depending on density ratio and finger width. This model suggests that a partial increase of the this instability accounts for the monotonic decrease in flux with increasing shear observed in our experiments.

4.4 Future work

- A flux that can describe the dependence of the finger fluxes with the diffusive ratio $\tau = \kappa_T/\kappa_S$ is needed. This can be accomplished through laboratory experiments in which finger fluxes would be computed for several different pairs of density components. This study would make flux results obtained for the sugar-salt system in laboratory to be quantitatively applicable to the ocean scenario.
- The same laboratory study performed for salt fingers here can be extended to the oscillatory diffusive case, where cold and fresh waters (salt solution) overlies relatively warmer and saltier ones (sugar solution). This study would bring more clarifications on the dynamical role of shear acting upon double diffusive mixing mechanisms.
- The decrease of the salt fluxes with increasing R_e , quantified through the linear regression analysis is related to the experimental observations that the fingers are seen to tilt in the direction of the flow when a vertical velocity difference is imposed. In the experiments, this tilt was seen to increase with increasing the magnitude of ΔU , which leads to further decrease of the fluxes. A quantitative study of finger tilt angles when different velocity magnitudes are imposed might help on future mathematical models of tilted finger fluxes.

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BIOGRAPHICAL SKETCH

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Alexandre M. Fernandes was born in Rio de Janeiro (RJ, Brazil) on October 29, 1974. In 1993 he was accepted in the Federal University of Rio de Janeiro, where he graduated with a Teaching Certificate degree in Physics 5 years later. In January 1999 he moved to Sao Paulo (SP, Brazil) and started his Masters program at the University of Sao Paulo. In 2001 he obtained the degree of Masters in Physical Oceanography working with Dr. Ilson C. A. da Silveira. In January 2002 he entered in the PhD program at the Department of Oceanography at Florida State University. Since then, he has been working on double diffusive processes with Dr. Ruby Krishnamurti. His research interests are focused in small scale mixing processes in the ocean due to double diffusion, turbulence, and intrusions.

Alexandre is the father of a lovely child, Isabela Fernandes, who has been a constant source of motivation in his life.