2016

Using Partially Observed Markov Decision Processes (POMDPs) to Implement a Response to Intervention (RTI) Framework for Early Reading

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USING PARTIALLY OBSERVED MARKOV DECISION PROCESSES (POMDPs) TO IMPLEMENT A RESPONSE TO INTERVENTION (RTI) FRAMEWORK FOR EARLY READING

By

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A Dissertation submitted to the
Department of Educational Psychology and Learning Systems
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

2016

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This dissertation is dedicated

to my parents,

Binnaz Doganay and Muammer Tokac.
ACKNOWLEDGMENTS

Thank you to all my family, friends, colleagues, and mentors, who have guided, inspired, motivated, and supported me through this long and tiring journey. This dissertation would not have been completed without assistance and encouragement of many people. First of all, I would like to thank Dr. Russell Almond, my advisor and chair of my dissertation committee for his support, guidance, and encouragement throughout my doctoral studies. I also would like to thank Dr. Betsy Becker, one of my dissertation committee members and the department chair for her support and encouragement. I would like to express my sincere gratitude to the rest of my dissertation committee members, Dr. Insu Paek and Dr. Young-Suk Kim for their thoughtful and valuable comments. They all encourage me to pursue and complete my dissertation. I cannot thank enough all other people who have not left me alone in this journey and they were always available whenever I need their help and support; Dr. Peggy Stillwell, Selen Razon, Itay Basevitch, Bernd Weiβ, Matt Glaser, Todd Fausel, Frank Baker, Christopher Thompson and Josh DeSha. Finally, I want to thank my parents who taught me never give up pursuing my goals.
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LIST OF ABBREVIATIONS

FCRR: Florida Center for Reading Research

JAGS: Just Another Gibbs Sampler

MH: Metropolis-Hasting algorithm

MDP: Markov Decision process

MCMC: Markov Chain Monte Carlo

NWF: Nonsense Word Fluency

POMDP: Partially Observed Markov Decision Process

PSF: Phoneme Segmentation Fluency

RD: Reading Difficulties

RTI: Response to Intervention

SD: Standard deviation

SE: Standard error
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<tr>
<td>$S_t$</td>
<td>A state at time $t$</td>
</tr>
<tr>
<td>$\overline{S_t}$</td>
<td>The predicted state vector at time $t$</td>
</tr>
<tr>
<td>$a_t$</td>
<td>An action at time $t$</td>
</tr>
<tr>
<td>$o_t$</td>
<td>An observation at time $t$</td>
</tr>
<tr>
<td>$p(s' \mid s,a)$</td>
<td>The probability of transitioning from state, $s$ to state, $s'$ after taking action $a$</td>
</tr>
<tr>
<td>$f(o \mid s,a)$</td>
<td>The probability of an observation $o$ given that action $a$ is taken in state $s$</td>
</tr>
<tr>
<td>$c(a)$</td>
<td>The applied instruction cost of taking action or activity $a$ in state $s$</td>
</tr>
<tr>
<td>$u(S_T)$</td>
<td>The utility of applied instruction or intervention $T$</td>
</tr>
<tr>
<td>$R_{Ph,t}$</td>
<td>phonological decoding</td>
</tr>
<tr>
<td>$R_{PhA,t}$</td>
<td>Phonological awareness</td>
</tr>
<tr>
<td>$Y_{NWF,t}$</td>
<td>The test score for Nonsense Word Fluency (NWF) skill of phonological decoding</td>
</tr>
<tr>
<td>$Y_{PSF,t}$</td>
<td>The test score for Phoneme Segmantation Fluency (PSF) skill of phonological awareness</td>
</tr>
<tr>
<td>$N$</td>
<td>Total sample size</td>
</tr>
<tr>
<td>$M$</td>
<td>Overall time points</td>
</tr>
<tr>
<td>$K$</td>
<td>The Kalman gain</td>
</tr>
<tr>
<td>$F_t$</td>
<td>The state transition matrix</td>
</tr>
<tr>
<td>$B_t$</td>
<td>The action matrix</td>
</tr>
<tr>
<td>$w_t$</td>
<td>The vector including the process noise terms for each parameter</td>
</tr>
<tr>
<td>$O_t$</td>
<td>The real measurement (observation) vector</td>
</tr>
<tr>
<td>$\overline{O_t}$</td>
<td>The predicted measurement (observation) vector</td>
</tr>
<tr>
<td>$H_t$</td>
<td>The transformation matrix</td>
</tr>
<tr>
<td>$v_t$</td>
<td>The vector including the measurement noise terms for each observation</td>
</tr>
<tr>
<td>$d_a$</td>
<td>The duration of meeting time at action $a$</td>
</tr>
<tr>
<td>$f_a$</td>
<td>The frequency with which the group meets at action $a$</td>
</tr>
<tr>
<td>$g_a$</td>
<td>Size of the group</td>
</tr>
<tr>
<td>$r_i$</td>
<td>The reliability of instrument $i$ at any time point</td>
</tr>
<tr>
<td>$R_{nm}$</td>
<td>The reading ability of individual $n$ on measurement occasion $m$</td>
</tr>
<tr>
<td>$\Delta T_m$</td>
<td>The elapsed time period between measurement occasions $m$ and $m-1$</td>
</tr>
<tr>
<td>$\gamma_{0m}$</td>
<td>An average growth rate on measurement occasion $m$</td>
</tr>
<tr>
<td>$\gamma_{a(n,m)}$</td>
<td>A tier-specific growth rate</td>
</tr>
<tr>
<td>Symbol</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{a(n,m)}$</td>
<td>The length of time between measurements occasions</td>
</tr>
<tr>
<td>$Y_{numi}$</td>
<td>Observation for individual $n$ at measurement occasion $m$ on instrument $i$</td>
</tr>
<tr>
<td>$R_i$</td>
<td>The reliability of instrument $i$ at any time point</td>
</tr>
<tr>
<td>$Var_{n(.)}$</td>
<td>The variance of individuals</td>
</tr>
<tr>
<td>$\hat{R}$</td>
<td>Latent reading variable for POMDP-RTI</td>
</tr>
<tr>
<td>$\bar{R}$</td>
<td>Latent reading variable for Current – time only RTI</td>
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ABSTRACT

This dissertation explored the efficacy of using a POMDP to select and apply appropriate instruction. POMDPs are a tool for planning: selecting a sequence of actions that will lead to an optimal outcome. RTI is an approach to instruction, where teachers craft individual plans for students based on the results of progress monitoring test. The goal is to determine whether the plans crafted by a POMDP model in a RTI setting offer advantages over the current practice that uses simple cut score methods. Two simulated data sets were used to compare the two approaches; the model had a single latent reading construct and two observed reading measures: Phoneme Segmentation Fluency (PSF) for phonological awareness and Nonsense Word Fluency (NWF) for phonological decoding. Simulation studies evaluated the POMDPs forecasts of the students’ end-of-year reading performance, and the studies compared how the students were placed into instructional groups using the two approaches. The POMDP-RTI model forecasted the students PSF and NWF scores for later time periods based on their scores in the previous time period as well as a forecast standard deviation. In the study, 91% of the estimated PSF scores and 94% of the estimated NWF scores fell within two standard deviations of observed PSF and NWF scores. The assignment to tiers was very different (after the initial time block) with just over half the students assigned differently under the two models at the last time point. The growth was better under the POMDP-RTI approach with a difference in mean reading ability of 0.49 on a standardized scale. The gain occurs because the POMDP model can take into account past observations and instructional history in its forecasts.
CHAPTER 1
INTRODUCTION

1.1 Purpose of the Study

According to Matthews (2015), statistics gathered by local school districts reflect that roughly 30% of their first-grade students read below grade level standards. Moreover, Landerl and Wimmer (2009) reported that 70% of struggling readers in first grade continued to struggle in eight grade when no intervention was provided. Mastropieri, Scruggs, and Graetz (2003) argued that reading is the main problem for most students with learning disabilities.

Response to Intervention (RTI) is an educational framework which is designed to identify students’ difficulties in reading and math, and prevent them as early as possible by using more intensive instruction for students who need it. Using RTI, teachers are able to identify struggling students with reading difficulties based on how they respond to effective instruction or intervention (Compton et al., 2006; Fuchs et al., 2003; Lyon et al., 2003).

The RTI approach divides instruction into Tiers, and each tier includes different interventions or instruction. The RTI process starts with screening tests which monitor the general knowledge and skills of all students in the class. The progress monitoring tests are administered on multiple occasions during a school year. The progress monitoring test results provide teachers with a rough estimate of each student’s proficiency that guides the assignment of students into appropriate tiers of instruction. The RTI approach uses the most recent monitoring test results to assign students to an appropriate tier.

In order to implement an RTI, an educator needs a guideline, a set of rules for assigning students to tiers based on the monitoring tests, creating a plan for each student. Most currently implemented RTIs use only the most recent test scores to make placement decisions.
Considering the entire history (both students’ previous screen-tests results and changes in instruction) should provide better estimates of current proficiency, and hence better plans for students. Almond (2007) suggested that this could be done using a partially observed Markov decision process (POMDP) — partially observed, because the true student proficiency is latent; a decision process, because the instructors decide what instruction or intervention to use between measurement occasions.

A POMDP is a probabilistic and sequential model designed to be used for planning. A POMDP can be in one of a number of distinct states at any point in time, and its state changes over time in response to events (Boutilier, Dean, & Hanks, 1999). “An action is a particular kind of event instigated by an agent in order to change the system’s [POMDP’s] state.” (Boutilier, Dean, & Hanks, 1999, p.5). For this study, the agent was an instructor, states referred to students’ reading-proficiency levels, and actions were the assignment of students to tiers. If the applied instruction in each tier is successful, then positive change occurs in the students’ levels of proficiency. If the instruction is not successful or a student is assigned to the wrong tier, the proficiency level might remain unchanged or slightly increase or decrease. Learning might still occur but just not as much as learning with a proper instruction. The rate of growth might be slow or only slightly positive. The students’ responses to instruction often provide important clues about both their proficiency and specific learning problems, if they have any (Marcotte & Hintze, 2009). A student’s current proficiency level and the choice of instruction determine a new probability distribution for the student’s ability at the end of the intervention period in each tier.

One noteworthy difference between a traditional RTI approach and a POMDP-RTI model is that the traditional RTI approach uses only the latest test results to identify students’
proficiencies and assign them to an appropriate tier (Nese et al., 2010). Therefore, we call it a Current-time-only RTI model. On the other hand, a POMDP-RTI model is the combination of a periodically applied monitoring test, the learning progression, and the RTI into a POMDP model. Additionally, it considers students’ entire histories when determining appropriate interventions at different time points in order to identify their current abilities and forecast their future abilities. Therefore, a POMDP-RTI model may be slightly better than a current-time-only RTI model.

This dissertation explored the efficacy of using a POMDP to determine whether the plans crafted by a POMDP model in an RTI setting offer advantages over the current practice that uses simple cut score methods to forecast student reading proficiency at the end of each school year, during the first year of reading instruction. The POMDP model evaluated students’ current performance and forecast end-of-year reading performance on specific target skills (Nonsense Word Fluency (NWF) for phonological decoding and Phoneme Segmentation Fluency (PSF) for phonological awareness) based on data collected via periodically applied assessments from reading classes. Research questions included:

- Are the POMDP-RTI model parameters properly identified, or are additional constraints necessary to obtain model parameter estimates?
- How accurate are the POMDP forecasts at each time point?
- How appropriate are students’ tier assignments based on their latent reading scores in the POMDP-RTI model, compared to their tier assignments based on their observed scores in the current-time-only RTI model?
- Do the number of times a monitoring test is applied and the interval length between repeated monitoring tests affect the POMDP forecasts at each time
point and students’ tier assignments based on their latent reading scores in the POMDP-RTI model?

1.2 Overview of Chapters

Chapter 2 reviews the previous and current literature on reading, learning progressions, reading learning progressions, and the RTI approach. In addition, the Markov Decision Process, POMDPs, and Markov Chain Monte Carlo (MCMC) estimation methods are considered in this chapter. Chapter 3 describes the methodology of the study including the models to be used and their parameterization, the design of the simulated data sets, and estimation of parameters using MCMC simulation. Chapter 4 presents the study results and finally Chapter 5 presents the discussion.
CHAPTER 2

REVIEW OF LITERATURE

In order to answer the research questions in Chapter 1, Chapter 2 addresses the previous and current literature about the RTI approach and POMDPs. I also present some background on reading, learning progressions, reading learning progressions and Markov decision processes as well as Markov Chain Monte Carlo (MCMC) estimation methods.

2.1 Reading

Torgesen (2004) asserts that reading consists of five components in a multidimensional construct: phonological awareness, phonological decoding, fluency, vocabulary, and reading comprehension. These five fundamental reading components are necessary for producing effective instructional programs and materials (Road & Associates, 2004).

*Phonological awareness* is the ability to determine and manage individual phonemes in words. It is one of the elemental components required to develop proficient decoding skills. For example, a student with keen phonemic awareness recognizes three sounds in the word *book* and can replace the /b/ with /t/ to make the word *took* (Shore & Sabatini, 2009).

*Phonological decoding* is an essential skill when a reader encounters an unknown word. Phonological decoding relates to the written word and is the relationship between a specific letter and its sound. By using phonological decoding skills, a reader can focus on the specific sound of each letter or combination of letters when having difficulty decoding a whole word. For instance, a child who is unable to recognize the word *chat*, might focus on each letter and phoneme, /ch/ /a/ /t/, to assign appropriate sounds for each letter or combination of letters. By combining these sounds, the child can read the whole word *chat* (National Institute of Child Health and Human Development [NICHD], 2000).
Fluency is the reader’s ability to read a text smoothly and accurately. Fluent reading consists of numerous cognitive activities, each requiring a level of automaticity that allows the reader to coordinate the execution of responding (Fuchs, Fuchs, Hosp, & Jenkins, 2001). A fluent reader is expected to show expression, intonation, and pacing those sounds naturally—much as he or she would speak aloud (NICHD, 2000).

As children start learning to read, they realize that the words (vocabulary) in a text correspond to the spoken words they encounter in daily life. Children can comprehend the written words more easily when they make these words a part of their oral language. As children increase their vocabulary level in daily speech, their reading levels improve as well (NRP, 2000).

Reading comprehension is defined as the interaction between reader and text. Comprehension is more than simply decoding words on a page. While decoding is a process that uses knowledge of letters and sounds in order to identify the words, comprehension includes the intentional thinking process that happens when a student reads (NRP, 2000).

For children at young ages, the first two components, phonological decoding and phonological awareness play an initial role to generate the remaining three reading components: fluency, vocabulary, and reading comprehension (Al Otaiba et al., 2011; Rock, 2007). A lack of either phonological decoding or phonological awareness affects the other components and causes reading difficulties. Because of the criticality of the early development of reading skills, instructors should identify children with reading difficulties and intervene early to prevent reading difficulties (Catts & Hogan, 2003).

The evidence shows that if the five critical components of reading (i.e., phonological awareness, phonological decoding, fluency, vocabulary, and reading comprehension) are
presented at the beginning of reading instruction, fewer than 2% to 6% of students have chronic difficulty learning to read (Torgesen, 2004). Meta-analyses of reading instruction indicate that phonemic awareness and phonological decoding instruction during the first two school years are critical for later reading performance (Rock, 2007). Therefore, the selection and delivery of appropriate instruction are crucial when addressing reading difficulties. That is, a student may be misidentified as having a learning disability when the reading difficulty actually results from exposure to inappropriate reading instruction. Students who experience reading difficulties under the regular literacy curriculum may benefit from specific instructional interventions beyond the regular curriculum (Fowler & Scarborough, 1999; Taylor & Olson, 1995).

2.1.1 Learning Progressions

According to Stevens, Shin, Delgado, Krajcik, and Pellegrino (2007), learning progressions are descriptions of how students gain more expertise within a discipline over a period of time: "They represent not only how knowledge and understanding develop, but also they predict how knowledge builds over time" (p.2).

A well-designed learning progression provides a number of benefits to instructors for instructional planning (Heritage, 2008). These paths toward mastery of content knowledge enable instructors to focus on important learning goals in the domain, focusing their attention on what the student needs to learn (Heritage, 2008). Furthermore, learning progressions enable instructors to make effective instructional decisions and modify instructional activities to meet the needs of students at different achievement levels (Heritage, 2008). An appropriate learning progression consists of proper sequential units. Those are manageable for the instructor. Popham (2007) describes a learning progression as a “carefully sequenced set of building
blocks that students must master a route to a more distant curricular aim. The building blocks consist of subskills and bodies of enabling knowledge” (p. 83).

Figure 1: Learning progression with building blocks

The figure above presents the building blocks of a learning progression. The nodes marked with circles (○) represent latent variables, those with triangles (▼) represent observable variables; and those with diamonds (◇) represent instructor decision variables.

At the end of each block, the instructor assesses the student’s knowledge and skills by applying an assessment to measure what and how much the student has learned. The length of each block might be a day, a week, or a month, depending on the content of the block, the instructional design, and the teacher’s decisions. Based on the assessment results, the instructor can identify and monitor struggling students. The instructor can use the results to make instructional decisions and to choose interventions that are needed for these students. The activities depicted in Figure 1 represent the changes in instruction or interventions that are selected by instructors to help struggling students close the gap between current performance...
and desired performance. While learning progressions are applied in other fields in education (e.g., science, mathematics, and language arts) reading is the focus of this study.

2.1.2 Reading Learning Progression

The reading learning progression identifies the continuum of reading strategies, behaviors, and skills needed to meet with grade expectations (Renaissance Learning, 2011). The progression is research-based, promoted by the student data which is obtained from periodically applied assessment during a year, and developed by adjusting instruction or creating new instruction (Emma Wilson Elementary School, 2011).

Reading learning progressions help instructors to monitor how learning generally will develop, and consequently to create or use ongoing assessment “probes” (e.g., Keeley, Eberle, & Farrin, 2005; OGAP, 2008; Rose, Minton, & Arline, 2007). Such probes can reveal where particular students might be at any point in time along the learning continuum (Hess, 2008). Moreover, the purpose of having a reading learning progression is to identify students with reading difficulties early so the teacher can monitor their progress throughout the academic year (Kim & Petscher, 2010). Implementation of reading learning progressions in schools has helped to raise the literacy achievement levels of students who are struggling to learn how to read (Greaney & Tunmer, 2010). Therefore, it is important to include a discussion of reading progressions in this chapter in order to underscore the importance of early identification and intervention.

2.1.3 Response to Intervention

Response to Intervention is an approach designed to identify students with reading difficulties and intervene as early as possible (Al Otaiba et al., 2011). By using RTI, instructors can identify students with potential reading difficulties by considering their lack of progress or
slow rate of growth, and then provide instruction and/or intervention (Kim & Petscher, 2010). The RTI approach supports different instructional interventions for individual students based on their observed needs (Gresten et al., 2008). Moreover, in order to monitor students who do not respond to intervention or instruction, researchers believe both performance levels and growth rates of students in an RTI approach should be considered (Fuchs et al., 2003; Fuchs & Fuchs, 1998; McMaster, Fuchs, Fuchs, & Compton, 2005; Speece & Case, 2001).

RTI divides the students into multiple tiers. Tier 1 consists of students who can improve their knowledge and skills with general instruction and minimal intervention. Tier 2 includes students who fall below the benchmark standard when exposed to the general classroom curriculum and instructional strategies or struggle to keep up with their peers (Greenwood et al., 2011). For students who receive intensive intervention in Tier 2, yet continue to demonstrate little or no progress, more intensive and individualized interventions are provided in Tier 3 for those students (Greenwood et al., 2011).

The potential benefits of using an RTI approach are greatest for students who have been deprived of key social-emotional and/or early literacy experiences that could have prevented the need for special education services for reading difficulties (Greenwood et al., 2011). Moreover, for children with difficulties, the RTI approach is claimed to improve a student’s abilities and eliminate the student’s difficulties by providing intensive interventions individually and seamlessly (Fletcher & Vaughn 2009).

According to the National Center on Response to Intervention (2012), three different types of assessment methods are commonly used in the RTI approach: summative, diagnostic, and formative. While formative assessment is the most commonly used method in an RTI framework, circumstances could call for the use of another method or a combination.
Typically, a universal monitoring test is administered to the whole class to make Tier1/Tier2 placement decisions and formative assessment is used at the end of each block to monitor student progress. In this study, the monitoring test was used to make Tier1/Tier2 placements. Because of its common use in RTI and its relevance to the purpose of this study, formative assessment was used to monitor student progress. In the RTI approach, formative assessments are applied on multiple occasions during a school year in order to monitor students’ progress before and after each period.

According to the model used by the IRIS Center (Fuchs & Fuchs, 2014), RTI components, universal monitoring, and tiers 1, 2 and 3 are presented in Figure 2 below.

Figure 2: Class-wide assessment and Tiers 1, 2 and 3 (IRIS Center, 2014)

Indications are that applying the RTI approach in reading classes have promised for decreasing the number of students with reading difficulties and improving overall reading outcomes in the elementary grades (e.g., Gersten et al., 2008). In many research studies, the positive effect of RTI on early identification of the students who have reading difficulties and targeted reading interventions for them is exhibited (Cavanaugh, Kim, Wanzek, and Vaughn,
2004; Mathes et al., 2005; Torgesen et al., 1999; Vellutino, Scanlon, Small, and Fanuele, 2006; Vellutino, Scanlon, Zhang, & Schatschneider, 2008).

According to Greenwood (2010) state directors and coordinators of early childhood programs report that a lack of tier-2 intervention strategies and the lack of evidence-based tier-1 programs are among the most significant challenges to the RTI approach. Basically, these challenges result from limitations in available criteria to identify students who have reading difficulties via an RTI approach (Hale, 2008). In addition, measures used in an RTI approach often lack adequate reliability and validity. Even when reliable and valid measures are used, it is unclear whether responses are evaluated based on local or national norms (Hale, 2008).

2.2 Markov Decision Processes

Ross (1983) has a clear explanation for sequential decision-making problems such as the Markov Decision process (MDP): “A problem typical of those with which we are concerned involves a process that is observed at the beginning of a discrete time period to be in a particular state. After observation of the state, an action must be chosen; and based only on the state at that time and the action chosen, an expected reward is earned and the probability distribution for the next state is determined.” (p.1) As a formulated sequential decision-making problem (e.g. Sutton (1997); Feinberg and Shward (2002)), MDP consists of the following five elements (Lee, 2009):

States: $S_i$ indicates a state at time $t$

Actions: $a_i$ indicates an action at time $t$.

$\Pr(S_{t+1} | S_t, a_t)$: Transition probability function with performing action $a_t$ from state $S_t$ to $S_{t+1}$ at time $t$. For discrete states and actions the transition probability function notation can be simplified as $P_{ij}(a) = P\{S_{t+1}=j| S_t=i, a_t=a\}$
$c(a)$ represents the cost of taking action or activity $a$ in state $s$;

$u(S_t)$ is the utility function to monitor a possible magnitude of risk of the decision makers to have the state at time $t$.

Periodically applied assessments can allow for monitoring of the students’ growth. Also, they can inform the instructor about the effects of prior instruction or intervention selection, thus aiding in future planning. Assume that an assessment is given periodically at multiple time points, $t_1, t_2, t_3,...$. Based on the assessment results, the instructor can make instructional changes or choose activities that help to improve the students’ learning. A MDP model that consists of periodically applied assessments at multiple time points, and forecasts the results of educational actions, permits the educator to produce optimal educational plans. The usefulness of the MDP has been proven in a variety of sequential planning applications where it is critical to account for uncertainty in the process (Puterman, 1994).

The proficiency variable $S_t$ can be defined as discrete or continuous. In a discrete model, the ability is usually represented as an integer that can take on any one of $M$ possible states, 1, 2, 3, ..., $M$. After observing the state of a process, the instructor needs to decide upon an action which is indicated with $a$, they will be carried out until the next time point. If the student is in state $i$ at time $t$, and action $a$ is chosen by the instructor, then the next action of the next state is decided according to transition probability $P_{ij}(a)$ (Ross, 2000). Let $S_t$ denote the state of the process at time $t$ and $a_t$ the action chosen at time $t$ by the instructor, then

$$P_{ij}(a) = P\{S_{t+1}=j \mid S_t=i, a_t=a\}$$

Hence, a function of transition probabilities is identified based on the present state and the subsequent action (Ross, 2000). If the state is taken as a continuous variable, then the transition probability matrix becomes a function: $f(x \mid s, a) = Pr (S_{t+1}=s \mid S_t = s, a_t=a)$.
Almond (2007b) used Figure 3 to present the Markov Decision Processes for student learning. Each time slice or box represents a short period of time, $t = 1, 2, 3, \ldots n$, and the student’s proficiencies are assumed to be constant at each time point of the measurement. That is, each time slice might represent a periodic event, such as the first half of the spring semester of the academic year. The variable $S$ represents the students’ proficiency which is a latent variable at each time slice. The variable $O$ represents the observed outcomes from some assessment tasks assigned during that time slice. In Almond (2007b) paper, actions in Ross’s paper (2000) are called “activities”, which are instructional tasks chosen by the instructor and applied between time slices.

In this study, the student proficiency variable $S$ is not directly observable, but it is partially observable in light of applied assessments. Therefore, the partially observable Markov Decision Process (POMDP) is particularly appropriate for this application.

2.2.1 Partially Observed Markov Decision Process

Instructors are unable to directly observe student abilities. That is, their observations are limited to the students’ performance on assessments. Because the students’ abilities are latent variables, the Partially Observable Markov Decision Process (POMDP; Boutilier, Dean,
& Hanks, 1999) is appropriate. The POMDP includes a probabilistic, sequential, decision-theoretic approach and involves individualized teaching activities (Rafferty et al., 2011). Specific components of POMDP include an assessment to measure the effects of instruction on students’ learning. The student’s current state and applied instruction jointly determine a probability distribution over the student’s possible next states (Boutilier, Dean, & Hanks, 1999).

The empirical study conducted by Rafferty, Brunskill, Griffiths and Shafto (2011) shows that using POMDP improves students’ learning. Rafferty et al. (2011) found that framing teacher action planning as a POMDP could accelerate students’ “alphabet arithmetic” learning relative to baseline performance. According to Rafferty et al. (2011), POMDP has some advantages over other decision-theoretic approaches: “POMDPs can use sophisticated models of learning, rather than assuming that learners' understanding can be directly observed or approximated by a large number of features. In contrast to approaches that only maximize the immediate benefit of the next action, POMDPs reason about both the immediate learning gain and the long-term benefit to the learner after a particular activity.” (p. 1 and p. 2)

A POMDP consists of the following components:

- $S$ is a set of states, $s$;
- $A$ is a set of actions or activities, $a$;
- $O$ is a set of observations, $o$;
- $p(s' \mid s, a)$ is a transition model that gives the probability of transitioning from state, $s$ to state, $s'$ after taking action $a$;
- $f(o \mid s, a)$ is an observation model that gives the probability of an observation $o$ given that action $a$ is taken in state $s$;
$c(a)$ represents the cost of taking action or activity $a$ in state $s$;

$u(S_T)$ is the utility function to monitor whether an applied instruction or intervention works well to help students meet targeted proficiency standards.

The differences between utility and cost functions are the total rewards for getting the student to an appropriate action or activity. In order to address how utility and cost functions work in the POMDP model, the following example might be helpful.

For instance, in Figure 4, for possible students’ reading abilities, $S_H$ represents students with high reading proficiency at time $t$ and $S_L$ represents students with the low proficiency at time $t$. For assigning each student to each tier, their probability functions: $Pr(S_H \mid a_1)$ equals to $P1$ and $Pr(S_L \mid a_2)$ equals to $P1$.

![Figure 4: The total reward at the end of the state at time t](image)

The instructors should assign students to Tier 1 if the product of the probability function with the total reward for $S_H$ in Tier 1 is bigger than the product of probability function with the total reward of $S_L$ students in Tier 2,

$$P1 * u > P2 * (u - c).$$  \hspace{1cm} (1)
According to Rafferty et al. (2011), many teachers can benefit from this framework and easily formalize their teaching tasks by using it. The learner knowledge is modeled as a state $s$. Then the transition model indicates how a teaching action stochastically affects the learner knowledge, and the observational model provides the probability that the learner will give a particular response to a teaching action based on his or her current understanding (Rafferty et al., 2011). Rafferty et al. (2011) propose using the expected amount of time for the learner to complete the activity successfully as the cost for each action. If a learner knows or is familiar with the concept, the action cost will decrease and drop to zero. Therefore, a sound policy or instructional aim is to select actions or activities to minimize the expected time needed for the learner to understand the concept successfully (Rafferty et al., 2011).

2.2.2 Kalman Filter

The Kalman filter (Kalman & Bucy, 1961) is an algorithm that is also a known linear quadratic estimation. It uses a series of measurements obtained over time, including random variations (noise), and estimates unknown variables that tend to be more precise than those based on a single measurement alone (Maybeck, 1999). The Kalman filter provides exact inference in a linear dynamical system such as a hidden Markov model, but it requires that the state space of the latent variables is continuous and that all the latent and observed variables have a Gaussian distribution (Faragher, 2012). According to Almond (2011), “Kalman filters apply in the case in which the competency growth and the evidence models are linear functions.” (p. 15) Kalman filters assume that competency growth and evidence-model parameters are known and that they belong to a single individual. The main point of using this filter is to estimate the individual’s true score (Almond, 2011). Based on Faragher (2012), the
Kalman filter assumes that the state at a time $t$ grows from the prior state at time $t-1$, and the state prediction equation is

$$
\overline{S}_t = F_t S_{t-1} + B_t a_t + w_t,
$$

(2)

where

- $\overline{S}_t$ is the predicted state vector including the student’s reading proficiencies at time $t$,
- $a_t$ is the action vector including the different instructional activities for students at different reading proficiency levels,
- $F_t$ is the state transition matrix which applies the effect of each state parameter (i.e. students reading proficiencies) at time $t-1$ to the state at time $t-1$,
- $B_t$ is the action matrix which applies the effect of each different instructional parameter in the vector $a_t$ to the state vector $S_t$, and
- $w_t$ is the vector including the process noise terms for each parameter in the state vector. It is assumed that the process noise is drawn from a zero-mean multivariate normal distribution with covariance matrix $Q_t$.

Based on the equation below, the measurement (observation) of the student’s reading proficiencies can be predicted at the end of each state as

$$
\overline{O}_t = H_t S_t + v_t,
$$

(3)

where

- $\overline{O}_t$ is the predicted measurement (observation) vector,
- $H_t$ is the transformation matrix that conveys the state vector parameters into the measurement domain, and
\( \nu_t \) is the vector including the measurement noise terms for each observation in the measurement vector. The measurement noise is assumed to be zero-mean Gaussian, as was the process noise, with covariance \( R_t \).

In order to estimate student reading proficiency in the next state space at time \( t \), we use

\[
\text{Est}[S_t] = \overline{S}_t + K(O_t - \overline{O}_t) \tag{4}
\]

where

\( S_t \) is the estimated state vector including the students’ reading proficiencies at time \( t \),
\( O_t \) is the real measurement (observation) vector
\( \overline{O}_t \) is the predicted measurement (observation) vector as defined in (3), and
\( K \) is the Kalman gain, which is a correction term that makes the estimated state more reliable if the predicted measurement and state vectors are not predicted correctly.

After obtaining the measurement vector, and determining the action vector at time \( t \) with known state at \( t-1 \), the probability distribution for the state at time \( t \) is determined as

\[
P(S_t | S_{t-1}, a_t, O_t) \tag{5}
\]

2.2.3 Markov Chain Monte Carlo (MCMC)

Because I have prior distributions of all parameters plus a set of observations (the sequence of observations and actions), I used Bayesian analysis to estimate each parameter in this study. By using Bayesian analysis, I was able to use the entire posterior distribution for each of the model parameters. According to Lynch (2007), there are three main benefits of using the entire posterior distribution.

- By summarizing the entire posterior distribution for a parameter, normality of the distribution can be directly assessed.
• Having the entire posterior distribution for a parameter allows additional tests and summaries which are not available under the classical likelihood-based approach.

• Distributions of quantities which may not be directly estimated from the original model can be easily obtained by transforming the distributions for the parameters in the model.

Bayesian analysis uses a variety of approximation and sampling methods in order to run the integration necessary to summarize the posterior distributions (Lynch, 2007).

Sampling methods can be considered as an alternative to approximation methods. The logic behind the sampling method is to simulate a sample of size \( n \) from the distribution of interest and apply discrete formulas to the sample in order to approximate the integrals of interest (Lynch, 2007).

The estimated mean is

\[
\int xf(x)dx \approx \frac{1}{n} \sum x,
\]

and the variance is

\[
\int (x - \mu)^2 f(x)dx \approx \frac{1}{n} \sum (x - \mu)^2.
\]

Bayesian inference involves establishing a model, obtaining a posterior distribution for parameters of interest, generating samples from the posterior distribution, and applying discrete formulas to the samples from these posterior distributions to summarize the knowledge about the parameters, (Lynch, 2007; Geyer, 2010).

In this study, Markov Chain Monte Carlo (MCMC) algorithms are transformed Bayesian inference, by generating samples from posterior distributions of model parameters (Roberts & Rosenthal, 2004). The name MCMC consists of two parts. The first part is Markov Chain which refers to a stochastic process in which the outcome of the previous experiment
can affect only the outcome of next experiment. The second part, Monte Carlo refers to the random simulation.

A **Markov Chain** is a sequence of random variables $\theta^0, \theta^1, \ldots$, for which, for any $t$, the distribution of $\theta^t$ given all previous $\theta^i$’s depends only on the most recent value, $\theta^{t-1}$, and more than one sequence can be drawn at the same time (Gelman, Carlin, Stern & Rubin, 2004). For each $t$, $t = 1, 2, 3\ldots$, of each sequence, $\theta^t$ is produced by starting at some point $\theta^0$ and then drawing $\theta^t$ from a transition distribution, $T_t(\theta^t | \ theta^{t-1})$ that only depends on $\theta^{t-1}$ as stated before (Gelman, Carlin, Stern & Rubin, 2004). The Markov Chain converges to a unique stationary distribution that is the posterior distribution ($p(\theta | y)$) by constructing the transition probability distribution. According to Geyer (2010), in order to have the desired stationary distributions, the Markov Chain central limit theorem should hold for one initial distribution and transition probability. It can hold for all initial distributions and that same transition probability (Geyer, 2010; Meyn & Tweedie, 1993, Proposition 17.1.6) and asymptotic variance will be the same for all initial distributions afterward. A Markov Chain is used when it is not possible to sample $\theta$ directly from $p(\theta | y)$. Therefore, an iteration process is applied to get closer to $p(\theta | y)$ by drawing each step of the process from a distribution that becomes closer to $p(\theta | y)$ (Gelman, Carlin, Stern & Rubin, 2004).

The **Gibbs sampler** is one of the most preferred MCMC algorithms; it iteratively updates all components of $\theta_T$ by drawing samples from their full conditional probability distributions (Fahrmeir & Tutz, 2001). It is also called alternating conditional sampling (Gelman, Carlin, Stern & Rubin, 2004).

The **Metropolis-Hasting algorithm** (MH) is a general name for the Markov chain simulation methods that are used for drawing samples from Bayesian posterior distributions.
The Gibbs sampler can be considered as a special case of Metropolis-Hasting that is a more appropriate sampling method if the full conditionals are standard distributions such as multivariate normal, inverse gamma, etc. (Fahrmeir & Tutz, 2001).
CHAPTER 3

METHODS

This dissertation explored the efficacy of using a POMDP to determine whether the plans crafted by a POMDP model in an RTI setting offer advantages over the current practice that uses simple cut score methods to forecast student reading proficiency at the end of each school year, during the first year of reading instruction. This dissertation consists of two studies in order to answer the research questions. Study 1 is a study to measure an overall parameter recovery by checking the percentages of failures in chi-square statistics. Study 2 consists of two related studies, Study 2(a) and Study 2(b). Study 2(a) examines the model data fit via cross-validation; the latest estimated PSF and NWF scores were compared with the latest obtained simulated data scores. In addition, Study 2(a) compares the assignment of POMDP-RTI model with the observed cut-score condition, which is business-as-usual in the Current-time only-RTI model. Study 2(b) compares two different implementations of the POMDP-RTI model.

Study 1 answered the following research question:

– Are the model parameters properly identified, or are additional constraints necessary to obtain model parameter estimates?

Study 2 (a) answered the following research questions:

– How accurate are the POMDP forecasts at each time point?

– How appropriate are students’ tier assignments, based on their latent reading scores in the POMDP-RTI model, compared to their tier assignments based on their observed scores in the Current-time only-RTI model?
Study 2 (b) answered the following research question:

– Do the number of times a monitoring test is applied and the interval length between repeated monitoring tests affect the POMDP forecasts at each time point and students’ tier assignments based on their latent reading scores in the POMDP-RTI model?

Two simulated datasets were used in both Study 1 and Study 2 in order to answer each research question. The initial values of the parameters were based on a longitudinal Florida Center for Reading Research (FCRR) study of reading proficiency (Al Otaiba, Folsom, Schatschneider, Wanzek, Greulich, Meadows, & Li, 2011) and data sets were simulated based on the Almond (2007) model using parameters derived from the Al Otaiba et al (2011) study in order to produce realistic data for answering the research questions posed above.

This chapter describes the steps used to generate and analyze the simulated data: Section 3.1 describes the model for the study. Section 3.2 describes data generation and parameter estimation for the model.

3.1 The Models of the Study

Figures 5 presents two different implementations of the periodically applied monitoring tests, and the POMDP-RTI model. This model used in this study to simulate the data and is also the model which is fit to the simulated data. Implementation 1 in Figure 5 consists of three learning blocks and the progress monitoring tests, and Implementation 2 in Figure 5 consists of four learning blocks and monitoring tests. The only difference between the two implementations lies in the number of measurement occasions, and an adjustment to the elapsed time of each block so that each implementation takes up 1 school year.
Implementation 1 in Figure 5 was used in all three studies, Study 1, Study 2(a) and Study 2(b). Implementation 2 in Figure 5 was used only in Study 2(b) to examine the effect of increasing the number of decision points on student performance.

In these implementations, the monitoring test was applied at the end of each block. During Block 1, the entire class received general instruction as a Tier 1 group. After a monitoring test administered at the end of the block at $t_1$, struggling students and those who were believed to have reading difficulties were identified and assigned to Tier 2 so that they could receive help and support through intensive instruction or intervention. In the business-as-
usual condition, the students were assigned to each tier based on a cut score on the observed monitoring test (the median of the applied monitoring test results) for the Current-time only-RTI model. The total reward (Section 2.2.1) was used to assign the students to each tier for the POMDP-RTI model in Study 2. During Block 2 and later blocks, each student received either Tier 1 or Tier 2 instruction, according to the decision rule used in the study condition. Tier 1 is for the students who are not experiencing difficulty and/or slow growth in reading compared to their classmates. Tier 2 is for the students who are struggling to read and having some reading difficulties. At the end of Block 2, a second monitoring test is applied at \( t_2 \).

This study incorporates the POMDP model into an RTI approach. The POMDP consists of the components below:

- \( M \) is the set of blocks, \( m \); implementation 1 has three blocks and implementation 2 had four blocks;

- \( R \) is a latent variable which represents the students’ reading proficiencies; \( R_{nm} \) is the unidimensional latent reading proficiency for student \( n \) at measurement occasion \( m \).

- \( A \) is the set of actions, activities to which a student can be assigned in each block. For simplicity, the only two actions considered are assignment to Tier 1 or Tier 2. The notation \( a_{(n,m)} \) is the action assigned to student \( n \) at measurement occasion \( m \).

- \( Y \) is the set of observations; students’ reading test scores after applying two monitoring tests: \( Y_{nmi} \) is the observation for individual \( n \) at measurement occasion \( m \) on observed variable \( i \).
\( p(R' \mid R, a) \) is the transition model that gives the probability of transitioning from R at the end of a block to \( R' \) at the end of the next block given the student was assigned to Tier \( a \) (equation 6 below).

\( f(Y \mid R) \) is an observation model that gives the probability of an observation \( Y \) given that the student has reading ability \( R \) (equation 7).

\( c( a) \) represents the model cost of assigning a student to tier \( a \);

\( u(R_M) \) is the utility function to assign a value to the eventual reading ability a student obtains. The absolute value of this function is not as important as the relative value (e.g., how much better is \( R_M=0.5 \) than \( R_M=0.1 \), and how does this increase in value compare to the cost function \( c(a) \)).

### 3.2 Data Generation

The initial values of the parameters were based on a longitudinal FCRR study of reading proficiency (Al Otaiba et al., 2011) and data sets were simulated based on the Almond (2007) model using parameters derived from the Al Otaiba et al (2001) study in order to produce realistic data for answering the research questions posed above. The reading scale scores, the correlation coefficients between reading tests, and target growth in each time period seen in the FCRR study were initial values for the simulation. The original data were collected at three-time points (fall, winter, and spring) during a longitudinal FCRR study of reading proficiency (Al Otaiba et al.). Multiple measures of reading proficiency were collected at each time point, including Phoneme Segmentation Fluency, Nonsense Word Fluency, Picture Vocabulary, Letter-Word Identification (Woodcock & Johnson, 2007), and Letter Naming
Fluency (DIBELS, based on the research of Marston & Magnusson, 1988). The data for each measurement are currently available from 224 kindergarten students.

The simulated data set consisted of simulated students’ observed proficiency scores on NWF and PSF components of reading proficiency assessments administered at three or four-time points during a school year of kindergarten. The simulated data has the same scale as the measures from the Al Otaiba et al. (2011) study. The NWF and PSF were scaled based on the Woodcock-Johnson (Woodcock, 1999) W score distribution. The correlation coefficients between two variables were computed and was Pearson $r = .65$ (p < 0.01). In this study, the correlation coefficient value was also .65.

3.2.1 $W$ scale ($W$ score)

The $W$ scale is an equal-interval scale; that is, a given interval represents the same amount of difference in the skill or ability measured regardless of the interval’s location along the scale (Jaffe, 2009). Therefore, the $W$ scale is useful for reporting each student’s growth in a skill, ability, or knowledge area (Jaffe, 2009). Both proficiency and observable variables were continuous and they were scaled based on $W$ scales. The $W$ scales were developed by Richard Woodcock and Marshall Dahl (1971) for the WJ-III assessment. The $W$ scale is centered on a value of 500 for each test and ranges from 430 to 550, although the range can contract or expand based upon the skills being measured (McGrew & Woodcock, 1989). For this study, the initial $W$ scale scores were distributed in the range 470 to 530. In the $W$ scale, the median value of a population’s $W$ score is the reference score to compare the skill level of each student with that of the peer group (Woodcock, 1999). The $W$ scores can be converted to other scales such as standard scores, percentile ranks, and relative proficiency indices (McGrew & Woodcock, 2001). Woodcock (1978, 1999) asserts that if a student’s $W$ score on the test is
higher than the reference $W$ score, the probability of success for that student is greater than for his or her peers. Conversely, if the $W$ score is lower than the reference $W$ score, the probability of success is decreased. For the purpose of this study, the median value of the $W$ score was considered the reference criterion for grouping students into Tier 1 and Tier 2.

While the observed variables were on the $W$ scale, the latent variable had a standardized scale. The latent reading variable $R$ was obtained as a standardized scale score in order to prevent a possible convergence problem. Moreover, I thought that having standardized scores for the latent reading variable would be easier to interpret compare to $W$ scale score.

The simulated data set consisted of students’ $W$ scores on NWF and PSF tests administered at $t_1$, $t_2$, $t_3$ and $t_4$ (i.e., fall, winter, spring, and summer). According to the FCRR data set (Al Otaiba, 2007), the correlation coefficient between these two skills is about .65 based on the $W$ score. The implementations in Figure 5 were used to simulate the data sets.

Each simulated data set included 300 students. I considered the time point, assessment administration time and target proficiency score as variables in each model. The condition of three measurement occasions with two different tiers, Tier 1 and Tier 2 presented in Figure 5, were compared to one with four measurement occasions with two different tiers in Figure 5.

### 3.2.2 Proficiency Growth Model

The model from which the data was simulated had a single latent proficiency variable and two observable measures: PSF and NWF. In the current study, this single latent proficiency variable was a reading variable, and a unidimensional model of reading was used and represented by a single continuous variable: $R_{nm}$, the reading ability of individual $n$ on measurement occasion $m$. In this case, $N$ was 300 students and $M$ represented the three time points, $t_1$, $t_2$, $t_3$, for Implementation 1 and four time points, $t_1$, $t_2$, $t_3$, $t_4$, for Implementation 2.
3.2.3 Model for Growth

This study assumed that a teacher provided general instruction to all the students until the first time point, $t_1$, and that the initial ability distribution was normal, $R_0 \sim N(0,1)$. After analyzing the results of assessments administered at $t_1$, the teacher delivered additional and more intensive instruction to students who were assigned to Tier 2, but delivered only general instruction to students in Tier 1.

The tier for student $n$ determines action is used, which is represented by $a(n,m)$. Following this logic, for measurement occasion $m > 1$,

$$R_{nm} = R_{n(m-1)} + \gamma_{a(n,m)} \Delta T_m + \eta_{nm},$$

where $\eta_{nm} \sim N(0, \sigma_{a(n,m)}\sqrt{\Delta T_m})$,

and where $\Delta T_m$ represents the elapsed time period between measurement occasions $m$ and $m-1$ for Tier 1 and Tier 2. In this study, each school year was equal to 1, and $\Delta T_m$ was fixed and equal to $1/M$ (for implementation 1, $M = 3$; for implementation 2, $M = 4$). The parameter $\gamma_{a(n,m)}$ is a tier-specific growth rate and it was fixed and had two different initial values for each tier. I set $\gamma_{am} = 0.9$ for Tier 1, and $\gamma_{am} = 1.2$ for Tier 2. I used $k_a$ in order to represent the tier-specific growth rate at each time point. If $k_1 = \gamma_{1m} / \Delta T_m$ for Tier 1, and $k_2 = \gamma_{2m} / \Delta T_m$ for Tier 2 then I assumed that $k_2 > k_1$. The residual standard deviation, $\sigma_{a(n,m)}\sqrt{\Delta T_m}$, depends on both a tier-specific rate, $\sigma_{a(n,m)}$, and the length of time, $\Delta T_m$, between measurements. $\sigma_{a(n,m)}$ was fixed to 1 for both tier-specific rates.

3.2.4 Evidence Models

The evidence model involved two independent regressions, one for each observed variable $i$. These two observable variables were chosen because they are critical reading components for later reading performance in the first two years of elementary school (Rock,
Let $Y_{nmi}$ be the observation for individual $n$ at measurement occasion $m$ on observed variable $i$ of the proficiency variables, then:

$$R_{nm} \sim N(0,1)$$

$$Y_{nmi} = a_i + b_iR_{nm} + \varepsilon_{nmi},$$

$$\varepsilon_{nmi} \sim N(0, \omega_i).$$

The reliability of the instruments can be used to determine $b$ and $\omega$. The reliability of an observed variable $i$ at any time point was represented as $r_i$. In classical test theory, the reliability is the squared correlation coefficient between the true score and the observed score of the student. This definition translates into an equation as

$$r_i = 1 - (\text{Var}_n(\varepsilon_{nmi}) / \text{Var}_n(Y_{nmi}))$$

where $\text{Var}_n(.)$ indicates that the variance comes from individuals (where measurement occasion and instrument are considered as constant). Then

$$b_i = \frac{\sigma_{Y_i}}{\sigma_{R_i}} \sqrt{r^2} \quad \text{and}$$

$$\omega_i = \sigma_{Y_i} \sqrt{1 - r^2}$$

In order to make $r_i = .45$ at each time point, $t_m$, for the measurement of each skill on observed variable $i$, $b_i = .98$ and $\omega_i = .65$ was used at $t_m$.

### 3.2.5 Decision Rules

In order to answer the research question in Study 2, two different decision rules for assigning students to each tier were compared: a) the decision was based on cut scores on the current observed scores and b) the Bayes decision was calculated by the POMDP using the expected utilities and costs.

Many RTI implementations used the reference $W$ score (general class median score, Section 3.2.1) as a cut score for assigning each student to either the Tier 1 or Tier 2 group. For
instance, if a student’s $W$ score on the NWF test is lower than the cut score for NWF, the student was assigned to Tier 2 for NWF.

The POMDP forecasts expected learning under each possible outcome and assigns students to tiers in a way that balances the expected learning gains with the cost of instruction. The utility function is the expected gain at the last time point and the cost function is the sum of costs of applied instruction at each state. The benefit is always higher for Tier 2, as is the cost. However, the cost exceeds the utility of the benefit for some regions of the distribution because the utility is nonlinear, while for other regions it does not. For instance, the net utility might be low for a student who already has a high proficiency level but is inappropriately assigned to Tier 2, as the additional benefit would be less than the added cost.

The contact hours with the instructor drive the cost of each block (Section 2.2.1). Cost is high for more intensive instruction in Tier 2, and it is really low for Tier 1. The cost function consists of three components: the frequency with which the group meets, $f_a$, the duration of the meeting time, $d_a$, and size of the group, $g_a$ (Almond & Tokac, 2014). In this study, the frequency with each group meets was assumed to be 100 times, the meeting duration time was assumed to be 15 minutes, and the size of group was assumed to be 150 students for Tier 2 in each block. Therefore, the cost value was fixed for Tier 2 and $c(a) = 0.1$. These values were not considered for Tier 1 because Tier 1’s cost was set to zero. Then

$$c(a) = f_a d_a / g_a,$$

represents the model cost of taking action or activity $a$ in state $s$.

The utility function is

$$u(R_M) = \logit^{-1}(\alpha(R_M - \beta)).$$
In this equation $\alpha$ and $\beta$ are fixed; $\beta$ is a proficiency target, which is on the scale of the internal latent variable $R_M$. Specifically $\beta = 0.5$ for Tier 1 and $\beta= 0.1$ for Tier 2. Also, $\alpha$ is a slope parameter, and $\alpha= 0.8$ for both Tier 1 and Tier 2. High values of $\alpha$ favor bringing students near proficiency standards above the proficiency target $\beta$, while low values of $\alpha$ give more weight to enriching students at the high end of the scale and providing remediation at the low end of the scale (Almond & Tokac, 2014).

In this case, the total reward is $u(R_M) - c(a)$. The difference between the utility function and the cost function is the total reward for getting the student to proficiency level Tier 1 using instruction $a_1$. The reward is the basis for the assignment of each student to Tier 1 or Tier 2. The POMDP model forecasts the expected reward, and balances that with cost during each period.

### 3.2.6 Simulation Design

The music tutoring simulation described in Almond (2007) was used as a starting point for the data simulation process in this dissertation, and the $R$ scripts from that paper were adapted to fit the current study. Following the FCRR study, data were generated for three and four measurement periods. Two conditions were also considered in the data simulation process. These conditions were

- number of measurement occasions, $M$, which is three for Implementation 1 (Figure 5a) in Study 1 and Study 2 and four for Implementation 2 (Figure 5b) in Study 2(b),

- administration time of measurement occasions in Study 2.
For each condition, I a) drew a set of parameters, b) simulated two sets of data according to these parameters, c) estimated the model parameters from the data, and d) evaluated the model using cross validation.

a) Drew a set of parameters: The parameters, $\gamma_{am}$ in equation 6 (Section 3.2.3) and $a_i$, $b_i$ in equation 7 (Section 3.2.4) were drawn as the proficiency model parameters.

b) Simulated two sets of data according to these parameters: Two different data sets were simulated, the data set based the POMDP-RTI model and the data set based on the Current-time only RTI model. In order to simulate the data sets, the latent reading variable $R_{nm}$ in equation 6 and the observed reading variable $Y_{nm}$ in equation 7 were used. The cut score was used as the decision rule for Current-time only RTI while POMDP estimation used for the POMDP-RTI model.

c) Estimated the model parameters from the data: The simulated data was used to estimate model parameters. MCMC was used to estimate the drawn parameters. JAGS was the software to run the MCMC analysis. The estimated parameters for equation 6 and 7 were compared in order to answer the Study1 question.

d) Evaluated the model using cross validation: The estimated scores were obtained by removing the last observed PSF and NWF scores at time 3 from the simulated data set. The Kalman filter was run on the observed scores of simulated data set at time 1 and time 2. Based on the obtained Kalman filter estimation scores, an MCMC run and estimated scores for time 3, as well as the model parameters and $\hat{R}_{nm}$ (the average proficiency of each student at each time point) were based on the sampling.

The initial value of the simulated data student distribution at time 0 was based on the FCRR data set. Based on the FCRR data set (Al Otaiba, 2007), the correlation coefficient
between NWF and PSF was 0.65. The simulated data set consisted of students’ W scores on the reading test, which consists of NWF and PSF subtests administered at \( t_1, t_2 \) and \( t_3 \) in *Implementation 1* and at \( t_1, t_2, t_3 \) and \( t_4 \) in *Implementation 2*. At each time point, the correlation coefficient between NWF and PSF was around 0.65 and the same growth and measurement error residuals in equation 6 were used for both NWF and PSF.

The proficiency growth model and evidence model parameters were obtained by the MCMC simulation analysis and *JAGS* was used to run the MCMC simulation analysis. *JAGS* is the abbreviation for “Just Another Gibbs Sampler,” and it is a program for analyzing Bayesian hierarchical and graphical models via MCMC simulation (Plummer, 2003). In MCMC simulation analysis, the four independent sequences of the Markov chain, parallel chains for the model, and 500000 of iterations for adaptation were run by using random starting positions to see whether the model converged. The rationale for having four independent sequences of Markov chains is that multiple independent sequences are necessary to identify disparities among the chains (Gelman, Carlin, Stern & Rubin, 2004).

After obtaining the estimated parameters, I evaluated the accuracy of model estimation by applying cross validation. In cross-validation methods, predictions of each part of the partitioned data are compared to predictions of the rest of the data replications and the whole data set (Gelman et al., 2004).
CHAPTER 4

RESULTS

Chapter 4 describes the results of the analysis. This dissertation consists of two studies in order to answer the research questions. Study 1 is a study to measure an overall parameter recovery by checking the percentages of failures in chi-square statistics. Study 2 consists of two related studies, Study 2(a) and Study 2(b). Study 2(a) is a study to examine the model data fit via cross-validation; the latest estimated PSF and NWF scores were compared with the latest obtained simulated data scores. In addition, Study 2(a) is a study to compare the assignment of POMDP-RTI model with the observed cut-score condition, which is business-as-usual in the Current-time only-RTI model. Study 2(b) is a study to compare two different implementations of POMDP-RTI model.

To address the research questions in Studies 1 and 2, the simulated data were analyzed to evaluate the efficacy of implementations 1 and 2 according to the analytical approach proposed in Chapter 3.

4.1. Study 1

- Are the model parameters properly identified, or are additional constraints necessary to obtain model parameter estimates?

An MCMC simulation analysis was performed to address this question. In particular, an assessment of model convergence was conducted by running simulations for the four independent sequences of the Markov chain and the parallel chains for the model with 300 of iterations for adaptation and 500000 iterations for updating by using random starting positions (Neal, 2010). The convergence of the sequence of draws and the entire set of posterior
distributions for the estimated model are provided by the equation 7 where the intercept and slope are represented by a\textsubscript{1} and b\textsubscript{1} for PSF, respectively, and a\textsubscript{2} and b\textsubscript{2} for NWF, respectively.

The trace and density plots for the intercepts and slopes for both PSF (a[1], b[1]) and NWF (a[2], b[2]) are shown in Figure 6. The density plots displayed a nearly symmetric unimodal distribution, showing the parameters have a normal distribution.

Table 1 shows the parameter distributions of the four different simulated data sets. The overall mean (SD) of a\textsubscript{1} and a\textsubscript{2} was 497.69 (0.23) and 499.79 (0.479) respectively. Meanwhile, the overall mean (SD) of b\textsubscript{1} and b\textsubscript{2} was 2.94 (0.2) and 7.98 (0.33), respectively. The initial expected values were 500 for intercepts and 0.98 for slope parameters.

Table 1: Mean and standard deviation (SD) of the model intercepts and slopes for four simulated data sets

<table>
<thead>
<tr>
<th></th>
<th>(\mu_1 (\sigma_1))</th>
<th>(\mu_2 (\sigma_2))</th>
<th>(\mu_3 (\sigma_3))</th>
<th>(\mu_4 (\sigma_4))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_1)</td>
<td>497.71 (0.160)</td>
<td>497.74 (0.152)</td>
<td>497.90 (0.207)</td>
<td>497.35 (0.167)</td>
</tr>
<tr>
<td>(a_2)</td>
<td>499.84 (0.200)</td>
<td>499.93 (0.113)</td>
<td>500.35 (0.409)</td>
<td>498.88 (0.179)</td>
</tr>
<tr>
<td>(b_1)</td>
<td>3.06 (0.177)</td>
<td>3.12 (0.181)</td>
<td>3.07 (0.181)</td>
<td>2.81 (0.158)</td>
</tr>
<tr>
<td>(b_2)</td>
<td>8.27 (0.15)</td>
<td>8.45 (0.188)</td>
<td>8.31 (0.197)</td>
<td>7.62 (0.139)</td>
</tr>
</tbody>
</table>

At the end of the analysis, the MCMC output provided a complete parameter set that consisted of four simulated separate distributions for each parameter, and each distribution consisted of 1666 cases. Therefore, I had four different obtained distributions for each \(a_i\), \(a_2\), \(b_1\) and \(b_2\). In the next step, I used equation 12 to utilize \(\chi^2\) statistics to evaluate the overall parameter recovery. In equation 12, \(x_i\) is each obtained distribution with mean \(\mu_i\) and \(\sigma_i^2\) from each simulated data set for each parameter.

\[
\chi^2 = \frac{(x_1 - \mu_1)^2}{\sigma_1^2} + \frac{(x_2 - \mu_2)^2}{\sigma_2^2} + \frac{(x_3 - \mu_3)^2}{\sigma_3^2} + \frac{(x_4 - \mu_4)^2}{\sigma_4^2} = \sum_{i=1}^{4} \frac{(x_i - \mu_i)^2}{\sigma_i^2} \tag{12}
\]
Figure 6: Trace and density plots of parameter estimates for the model
Within the framework of this analysis, the percentage of failures in the $\chi^2$ statistics was monitored using the cumulative distribution function and the Q-Q plot to assess whether the parameters had been properly identified. Moreover, a reference $\chi^2$ was created for comparison with the obtained $\chi^2$ for interpreting parameter identification. In Figures 7 through 10, the red and green lines represent the obtained and reference $\chi^2$ probability density distributions, respectively.

Figure 7: Chi-Square probability density function for $a_1$

Figure 8: Chi-Square probability density function for $a_2$
The probability density function (PDF) describes the relative likelihood of a random variable of having a given value (Lynch, 2007). A nearly perfect $\chi^2$ distribution was observed for $a_1$ (Figure 7) and $b_1$ (Figure 9). Meanwhile, the $\chi^2$ distribution showed a fairly good but multimodal correspondence for $a_2$ (Figure 8) and $b_2$ (Figure 10) for df = 4. I am not sure what caused the multimodal distributions for $a_2$ and $b_2$; however, it might be due to identifiability of the components under symmetric priors in the MCMC output even though having multimodal distributions did not affect the results.
In this analysis, the $\alpha$-level was set at .05, and 95% of the simulated $\chi^2$ values was assumed to fall between $\chi^2_{.025} = 11.143$ and $\chi^2_{.975} = 0.484$. The percentage of values were falling outside of this range was subsequently calculated to monitor the percentage of failures.

According to the assumption, the sample is drawn from a normally distributed population is critical (Raftery and Lewis, 1995). If I want the parameters properly identified then sample sizes of $a_1$, $a_2$, $b_1$, $b_2$ had to be drawn from a normally distributed population, this quantity has a chi-square distribution with 4(n-1) degrees of freedom in this study. Since the distribution is chi-square distribution with $df = 4$, I expected to have higher percentage (roughly between .925 and .975 (Raftery and Lewis, 1995)) of the chi-square values between the 95% interval of critical chi-square values, 0.484 and 11.143. Thus, the 95% confidence interval for $\sigma^2$ was calculated to measure the distribution of the population. The following equation was used for the calculation:

$$
\frac{(n-1)Sd^2}{\chi^2_{.025}} < \sigma^2 < \frac{(n-1)Sd^2}{\chi^2_{.975}},
$$

where “n-1” is df and “SD” is the standard deviation of the sample ($SD_{a1} = 2.74$, $SD_{a2} = 2.92$, $SD_{b1} = 2.87$ and $SD_{b2} = 2.61$). Based on this calculation, there was 95% confidence that the standard deviation of the population of $a_1$ was between 1.64 and 7.88; $a_2$ was between 1.74 and 8.39; $b_1$ was between 1.9 and 5.84; and $b_2$ was between 1.73 and 5.31.

According to the density function in Figure 7 and the Q-Q plot in Figure 12, the $\chi^2$ distribution fell largely within the range of 0 to 11, with some outliers occurring above and below critical $\chi^2$ values for $df = 4$. To obtain more specific data on the percentage of failures, I generated an accumulative distribution function for $a_1$ (Figure 11).
The cumulative distribution function (CDF) represents the probability that the values for a variable will be less than or equal to a critical value. The x-axis represents the range for a given probability function; the y-axis represents the probability, which must fall between zero and one. The green line in Figure 11 represents the reference \( \chi^2 \) cumulative distribution while the black line represents the obtained \( \chi^2 \) cumulative distribution. According to the cumulative distribution function in Figure 11, 96% of the value of \( \chi^2 \) statistics fell within a band going from 0.484 and 11.143, with only 4% occurring as the higher values of \( \chi^2 \) values which were not falling between 0.484 and 11.143.

The quantile-quantile or Q-Q plot is a graphic method of assessing the validity of a distributional assumption for a data set. The y-axis represents the reference chi-square distribution while the x-axis represents the simulated chi-square distribution. If the data follow the assumed distribution, the points on the Q-Q plot should approximate a straight line. In Figure 12, an almost straight line was indeed observed for the Q-Q plot.
Based on the density function in Figure 8 and the Q-Q plot in Figure 14, the $\chi^2$ distribution for the parameter $a_2$ fell largely within the range of 0 to 11, with some outliers occurring above the critical $\chi^2$ values for df = 4. To acquire more specific data on the percentage of failures, I generated a cumulative distribution function for $a_2$ (Figure 13).

According to the cumulative distribution function in Figure 13, 94% of the value of $\chi^2$ statistics fell between 0.484 and 11.143, 6% occurring as the higher values which were not falling between 0.484 and 11.143.
If the data conform to the assumed distribution, the points on a Q-Q plot are predicted to approximate a straight line. As shown in Figure 14, a nearly straight line on the left but a distinct curve on the right of the plot was observed for the Q-Q plot. This distinct curve might be one of the results of having the identifiability problem of the components under symmetric priors in the MCMC output as it is observed in Figure 8.

Based on the density function in Figure 9 and the Q-Q plot in Figure 16, the $\chi^2$ distribution for the parameter $b_1$ fell mostly within the range of 0 to 11, with some outliers occurring above and below the critical $\chi^2$ values. Meanwhile, according to the density function in Figure 10 and the Q-Q plot in Figure 18, the $\chi^2$ distribution for the parameter $b_2$ occurred mostly within the range of 0 to 11. To obtain more specific data on the percentage of failures, I generated cumulative distribution functions for both parameters (Figure 15 for $b_1$ and Figure 17 for $b_2$).

Based on the cumulative distribution function in Figure 15, almost 95% of the values of $\chi^2$ statistics fell within 0.484 and 11.143, with 5% of the distribution representing the higher values which were not between 0.484 and 11.143.
In this study, I anticipated the points on a Q-Q plot would approximate a straight line. Indeed, as shown in Figure 16, a nearly straight line with a single outlier was observed for the Q-Q plot. Moreover, according to the cumulative distribution function in Figure 17, about 96% of the value of $\chi^2$ statistics fell between 0.484 and 11.143, with only 4% of the distribution representing the higher values which were not falling between 0.484 and 11.143.
As shown in Figure 18, the Q-Q plot of the Chi-Square distribution for $b_2$ showed a slight deviation from a straight line and a distinct curve on the right of the plot. This distinct curve might be one of the results of the identifiability problem of the components under symmetric priors in the MCMC output as it is observed in Figure 10.
4.2. Study 2 (a)

- How accurate are the POMDP forecasts at each time point?

To check the model-data fit via cross-validation, the estimated scores were obtained by removing the last observed PSF and NWF scores at time 3 from the simulated data set. The Kalman filter applied to the observed scores in the simulated data set at time 1 and time 2 in order to reduce the noise of observed scores and obtain the each student’s true skill scores for PSF and NWF. After the application of the Kalman filter, an MCMC was run and PSF and NWF scores were estimated for time 3, as well as the model parameters and $\hat{R}_{nm}$ (the average proficiency of each student at each time point) based on the sampling. The obtained parameters were 498.6 (0.23) and 500 (0.24) for $a_1$ and $a_2$ and 3.75 (0.18) and 4.08 (0.17) for $b_1$ and $b_2$ for the original simulated data, respectively. Meanwhile, the obtained parameters were 498.82 (0.25) and 500.1 (0.24) for $a_1$ and $a_2$ and 4.12 (0.19) and 4.13 (0.17) for $b_1$ and $b_2$ after estimating the scores at time 3, respectively. Table 2 presents the comparison of the values of the original simulated data set scores at time 3 and the estimated time 3 scores. I called the original simulated data set scores at time 3 observed scores and called the estimated time 3 scores estimated scores.

Table 2: Comparison of Observed and Estimated scores summary

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Min</th>
<th>1 Quartile</th>
<th>Median</th>
<th>Mean</th>
<th>3 Quartile</th>
<th>Max</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSF</td>
<td>Observed</td>
<td>483</td>
<td>496</td>
<td>498</td>
<td>498.4</td>
<td>501</td>
<td>515</td>
</tr>
<tr>
<td></td>
<td>Estimated</td>
<td>492.2</td>
<td>497.7</td>
<td>500</td>
<td>500.3</td>
<td>502.5</td>
<td>517</td>
</tr>
<tr>
<td>NWF</td>
<td>Observed</td>
<td>492</td>
<td>498</td>
<td>500</td>
<td>500.5</td>
<td>503</td>
<td>517</td>
</tr>
<tr>
<td></td>
<td>Estimated</td>
<td>493.6</td>
<td>499.1</td>
<td>501.4</td>
<td>501.7</td>
<td>503.9</td>
<td>518.4</td>
</tr>
</tbody>
</table>

Table 3 provides a comparison of the observed and estimated scores for 30 randomly selected individuals. Values in bold font reflect differences between estimated and observed scores that are one or two standard deviations.
Table 3: Comparison of the Observed and Estimated scores of 30 randomly selected cases

<table>
<thead>
<tr>
<th>ID</th>
<th>Observed (PSF)</th>
<th>Estimated (PSF)</th>
<th>Error</th>
<th>Error²</th>
<th>Observed (NWF)</th>
<th>Estimated (NWF)</th>
<th>Error</th>
<th>Error²</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>500</td>
<td>502.91</td>
<td>2.91</td>
<td>8.47</td>
<td>500</td>
<td>504.33</td>
<td>4.33</td>
<td>18.75</td>
</tr>
<tr>
<td>24</td>
<td>504</td>
<td>501.82</td>
<td>-2.18</td>
<td>4.75</td>
<td>501</td>
<td>503.26</td>
<td>2.26</td>
<td>5.11</td>
</tr>
<tr>
<td>44</td>
<td>491</td>
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<td>9.98</td>
<td>99.60</td>
<td>500</td>
<td>502.38</td>
<td>2.38</td>
<td>5.66</td>
</tr>
<tr>
<td>48</td>
<td>501</td>
<td>502.56</td>
<td>1.56</td>
<td>2.43</td>
<td>503</td>
<td>503.97</td>
<td>0.97</td>
<td>0.94</td>
</tr>
<tr>
<td>55</td>
<td>499</td>
<td>498.79</td>
<td>-0.21</td>
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<td>496</td>
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<td>4.21</td>
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<td>500.09</td>
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<td>504.46</td>
<td>4.46</td>
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<td>497.55</td>
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<td>110</td>
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<td>29.48</td>
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<td>4.58</td>
<td>495</td>
<td>494.29</td>
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<td>0.50</td>
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<td>0.64</td>
<td>0.41</td>
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<td>2.04</td>
<td>4.16</td>
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<td>5.19</td>
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<td>0.01</td>
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</tr>
<tr>
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<td>4.73</td>
<td>22.37</td>
<td>496</td>
<td>496.13</td>
<td>0.13</td>
<td>0.02</td>
</tr>
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<td>3.71</td>
<td>13.76</td>
<td>498</td>
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<td>3.12</td>
<td>9.73</td>
</tr>
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<td>0.47</td>
<td>0.22</td>
<td>502</td>
<td>504.87</td>
<td>2.87</td>
<td>8.24</td>
</tr>
<tr>
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<td>505</td>
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<td>0.52</td>
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<td>506.95</td>
<td>4.95</td>
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<tr>
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<td>218</td>
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<td>3.88</td>
</tr>
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<td>3.68</td>
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<td>9.80</td>
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<td>502.8</td>
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<td>23.04</td>
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<tr>
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<td>1.21</td>
<td>1.46</td>
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<td>0.63</td>
<td>0.40</td>
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<tr>
<td>284</td>
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<td>0.85</td>
<td>0.72</td>
<td>503</td>
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<td>287</td>
<td>501</td>
<td>498.43</td>
<td>-2.57</td>
<td>6.60</td>
<td>503</td>
<td>499.81</td>
<td>-3.19</td>
<td>10.18</td>
</tr>
</tbody>
</table>

Bias(30)      1.41      2.23
MSE(30)        8.84      9.25

The Observed PSF and NWF scores and estimated PSF and NWF scores were expected to show the scores had comparable values. According to differences between observed and estimated scores, I computed the mean square errors. The overall obtained mean square error was 10.1 for PSF and 4.88 for NWF (typical W scale SD = 9). Aside from a few exceptions,
91% of estimated PSF scores and 94% of estimated NWF scores fell within two standard deviations. As seen in Figures 19 and 20, the results of the observed and estimated scores revealed the distributions of the PSF and NWF scores were a little wider between first and third quartiles for estimated scores than for the observed scores.

Figure 19: The distribution of Estimated and Observed scores based on PSF

Figure 20: The distribution of Estimated and Observed scores based on NWF

In Figures 21 and 22, the blue line represents the identity line with slope is 1 and intercept is 0. It was anticipated the scatter of the scores would be located near the blue line.
Meanwhile, the red line represents their actual nonparametric-regression smooth which is gamLine and it fits a generalized additive model and allows including a link and error function. Red-dotted lines represent the first and third quartiles of the red line. Although exceptions were present, most of the upper end student PSF and NWF scores were on or in proximity to the blue line. As seen in Figures 21 and 22, POMDP-RTI model did nearly accurate estimation for upper end student scores.

Figure 21: Comparison of Estimated and Observed PSF scores

Figure 22: Comparison of Estimated and Observed NWF scores
When I compared the box plots of the observed and estimated scores, I saw that the distributions of observed PSF and NWF scores had a little wider spread based on their first and third quartiles than the distributions of estimated scores as seen from the bar graphs in Figure 21 and Figure 22. In Figures 21 and 22, POMDP-RTI model did nearly accurate estimation for upper-end student scores. However, for the lower-end students in Tier 2, I can say that the model overestimated the growth rate for the students in Tier 2. The reason might be using the two different fixed initial tier-specific growth rates for Tier 1 and Tier 2 in equation 6; however only Tier 1 growth rate was estimated for both tiers at time 2 and time 3 in order to avoid having a possible convergence problem. I estimated the student scores based on Tier 1 growth rate value because I claimed that POMDP-RTI model accurately assigns the students with reading difficulties to proper tier; therefore, the upper end student scores were my main focus in this study. The student scores estimated based on Tier 1 growth rate parameter.

- How accurately are students assigned to each tier based on their latent reading score in the POMDP-RTI model as compared to their observed score in the Current-time only-RTI model?

The students in the simulated dataset were assigned to Tier 1 or Tier 2 based on either the (1) cut score of Current-time only-RTI model or (2) POMDP estimate of POMDP-RTI model (Table 4). There were an equal number of students between tiers for both selection criteria at time 2. However, at time 3, a greater number of students were present in Tier 1 when the categorization was based on the POMDP scores than on the cut scores.

An MCMC run and model parameters were obtained for the cut scores and POMDP estimates. In addition, the latent student true reading variables \( \tilde{R}_{nm} \) and \( \tilde{R}_{nm} \) were calculated.
based on estimated variables $\tilde{R}_{nm}$ and $\hat{R}_{nm}$ scores and the assignment of students into tiers according to the cut scores and POMDP estimates, respectively. The obtained parameters were 498.6 (0.23) and 500 (0.24) for $a_1$ and $a_2$ and 3.75 (0.18) and 4.08 (0.17) for $b_1$ and $b_2$, respectively, for the POMDP estimates. Meanwhile, the obtained parameters were 500.1 (0.31) and 500 (0.37) for $a_1$ and $a_2$ and 4.92 (0.16) and 6.25 (0.18) for $b_1$ and $b_2$, respectively, for the cut scores.

Table 4: Comparison of the number of PSF and NWF scores between tiers categorized by cut scores or POMDP estimates

<table>
<thead>
<tr>
<th>Method</th>
<th>Tier</th>
<th>PSF Time 2</th>
<th>NWF Time 2</th>
<th>PSF Time 3</th>
<th>NWF Time 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>POMDP</td>
<td>Tier 1</td>
<td>150</td>
<td>149</td>
<td>181</td>
<td>181</td>
</tr>
<tr>
<td></td>
<td>Tier 2</td>
<td>150</td>
<td>151</td>
<td>119</td>
<td>119</td>
</tr>
<tr>
<td>Cut Score</td>
<td>Tier 1</td>
<td>150</td>
<td>149</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>Tier 2</td>
<td>150</td>
<td>151</td>
<td>150</td>
<td>150</td>
</tr>
</tbody>
</table>

A comparison was made between the latent reading scores of randomly selected students. The standard error (SE) of the scores were found to be around 0.0005 and 0.0009 when tier assignments were based on the cut scores and POMDP estimates, respectively. Table 5 compares the achieved abilities for these selected students. The numbers in parentheses represent what PSF and NWF tiers each reading score belong.

Table 5: Comparison of random $R_{nm}$ scores from two models

<table>
<thead>
<tr>
<th>ID</th>
<th>$R_{n2}$ (PSF-NWF)</th>
<th>$R_{\tilde{n}2}$ (PSF-NWF)</th>
<th>$R_{n3}$ (PSF-NWF)</th>
<th>$\tilde{R}_{n3}$ (PSF-NWF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.668 (2-2)</td>
<td>-1.5 (2-2)</td>
<td>0.062 (1-1)</td>
<td>-1.231 (2-2)</td>
</tr>
<tr>
<td>54</td>
<td>0.114 (1-1)</td>
<td>-0.776 (1-1)</td>
<td>-0.305 (1-1)</td>
<td>-1.102 (2-2)</td>
</tr>
<tr>
<td>56</td>
<td>0.266 (2-2)</td>
<td>-0.647 (2-2)</td>
<td>-0.952 (1-1)</td>
<td>-0.141 (2-2)</td>
</tr>
<tr>
<td>74</td>
<td>0.12 (1-1)</td>
<td>-0.739 (1-1)</td>
<td>-0.222 (1-1)</td>
<td>-0.363 (2-2)</td>
</tr>
<tr>
<td>87</td>
<td>1.359 (2-2)</td>
<td>-1.414 (2-2)</td>
<td>1.041 (1-1)</td>
<td>-1.184 (2-2)</td>
</tr>
<tr>
<td>105</td>
<td>-0.257 (1-1)</td>
<td>0.659 (1-1)</td>
<td>-0.872 (2-2)</td>
<td>0.089 (1-1)</td>
</tr>
<tr>
<td>132</td>
<td>0.024 (1-1)</td>
<td>-0.564 (1-1)</td>
<td>-0.404 (1-1)</td>
<td>-1.114 (2-2)</td>
</tr>
<tr>
<td>163</td>
<td>-0.444 (1-2)</td>
<td>-0.036 (1-2)</td>
<td>-0.081 (2-2)</td>
<td>-0.484 (1-1)</td>
</tr>
<tr>
<td>166</td>
<td>0.435 (1-1)</td>
<td>-0.321 (1-1)</td>
<td>0.705 (1-1)</td>
<td>-0.481 (2-2)</td>
</tr>
<tr>
<td>187</td>
<td>-0.202 (2-2)</td>
<td>-0.061 (2-2)</td>
<td>0.196 (2-2)</td>
<td>-0.835 (1-1)</td>
</tr>
<tr>
<td>217</td>
<td>0.978 (2-2)</td>
<td>-0.407 (2-2)</td>
<td>-0.088 (1-1)</td>
<td>-0.545 (2-2)</td>
</tr>
<tr>
<td>226</td>
<td>-0.463 (2-1)</td>
<td>-0.185 (2-1)</td>
<td>-0.056 (2-2)</td>
<td>-1.38 (1-1)</td>
</tr>
<tr>
<td>252</td>
<td>-0.323 (2-2)</td>
<td>-0.269 (2-2)</td>
<td>0.713 (2-2)</td>
<td>-0.981 (1-1)</td>
</tr>
</tbody>
</table>
As seen from Table 5, the students showed better reading improvements and more consistent reading scores when categorized by POMDP estimates than by cut scores. For instance, one of the students (case 7), after instruction was applied for both PSF and NWF in Tier 2, the student score ($R_{72}$) was 0.688 at time 2. The student was transferred to Tier 1 and had a score of 0.062 at time 3. By contrast, the same student ($R_{72}$) showed a score of -1.5 at time 2 after being transferred to Tier 2 for both PSF and NWF. The student remained in Tier 2 with a score of -1.231 at time 3.

POMDP-RTI and Current-Time only RTI models consisted of Tier 1 and Tier 2 for each PSF and NWF. Therefore, in POMDP-RTI and Current-Time only RTI models, four different tier assignments occurred. The students were assigned to Tier 1 of both PSF and NWF, Tier 2 of both PSF and NWF at time 2 and time 3. Also, some students were assigned to Tier 1 of PSF and Tier 2 of NWF or Tier 2 of PSF and Tier 1 of NWF at time 2 and time 3 as seen one of the students (case 226) in Table 5. In order to make the students’ assignments more understandable, the situation of Tier 1 of PSF and Tier 2 of NWF or Tier 2 of PSF and Tier 1 were called Mix Tiers.

Figures 29 and 30 present the true $\hat{R}$ score distributions based on tiers at time 2 and time 3. Figures 31 and 32 present the true $\tilde{R}$ score distributions based on tiers at time 2 and time 3. As seen in Figures 23 and 24, the majority of $\hat{R}$ score distributions were distributed between -2 to 2, and they substantially overlapped. As observed in Figures 25 and 26, most $\tilde{R}$ score distributions were distributed between -3 to 3 and the overlapping region was smaller.
compare to \( \hat{R} \) score distributions in Figure 23 and 24. From the overlapping region of \( \hat{R} \) score distributions at time 2 and time 3, it is safe to say that number of students in Tier 1 and Mix Tiers increased by using the POMDP-RTI model. On the other hand, from the \( \hat{R} \) score distributions at time 2 and time 3, the number of students in Mix Tiers and Tier 2 increased by using Current-time only RTI model. As mentioned before, Figure 23 and Figure 24 were plotting based on the true \( \hat{R} \) score; however estimated \( \hat{R} \) score was used to assign each student to each tier. Therefore, the measurement error between true and estimated \( \hat{R} \) score might be the main reason of having the substantially overlapped tier distributions in Figure 23 and Figure 24 while I was expecting to have nearly two separate tier distributions. In addition, the decision rule in POMDP-RTI model works based on utility function and estimated \( \hat{R} \) score, another reason might be using the two different fixed initial tier-specific growth rates for Tier 1 and Tier 2 in equation 6; however estimating the tier growth rate parameters based on only one tier-specific growth rate value for both tiers at time 2 and time 3 as mentioned before. Also I used the same fixed slope parameter value for each tier in utility function in equation 11. Appendix B includes the expected utility plots for each tier at time 2 and time 3.

Figure 23: \( \hat{R} \) score distributions at time 2 for Tier 1, Mix Tiers and Tier 2
Figure 24: $\tilde{R}$ score distributions at time 3 for Tiers 1, Mix Tiers and Tier 2

Figure 25: $\tilde{R}$ score distributions at time 2 for Tiers 1, Mix Tiers and Tier 2

Figure 26: $\tilde{R}$ score distributions at time 3 for Tiers 1, Mix Tiers and Tier 2
Table 6 presents the average reading scores belong to each model and a number of students in each tier. The assignment to tiers was very different (after the initial time block) with just over half the students assigned differently under the two models at the last time point. The growth was better under the POMDP-RTI approach with a difference in the mean reading ability of .49 on a standardized scale. In Table 6, “N” represents a number of students in each tier of PSF and NWF, “R̅mean” represents the reading proficiency score mean of each tier group, and “Number of Non-Matching Students” represents a number of students assigned differently to Tier 1, Tier 2 and Mix Tiers in each model.

Table 6: Comparison of POMDP-RTI and Current-Time only-RTI models based on average reading scores R̅

<table>
<thead>
<tr>
<th>Tiers (PSF-NWF)</th>
<th>POMDP-RTI</th>
<th>Current - Time Only RTI</th>
<th>Number of Non-Matching Students</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time 2</td>
<td>Time 3</td>
<td>Time 2</td>
</tr>
<tr>
<td>1-1</td>
<td>14</td>
<td>0.62</td>
<td>136</td>
</tr>
<tr>
<td>1-2</td>
<td>9</td>
<td>0.72</td>
<td>45</td>
</tr>
<tr>
<td>2-1</td>
<td>8</td>
<td>-0.86</td>
<td>45</td>
</tr>
<tr>
<td>2-2</td>
<td>14</td>
<td>0.35</td>
<td>74</td>
</tr>
</tbody>
</table>

Both the POMDP-RTI and the Current-time only-RTI model assign the students to each tier and they aim to help more struggling students to make them move from Tier 2 to Tier 1. In order to determine which model made a perfect job of assigning more students from Tier 2 to Tier 1 at the end of a year, their agreements with their ideal placements were calculated.

Ideal placements were based on the true reading values (standardized scale scores) but as the true reading value was different in the two simulations (because students received
different instruction), there were two sets of ideal placement, one for each model, ideal placement for POMDP-RTI and ideal placement for Current-time only RTI.

In the ideal placement, the proficiency scale was truncated as the ideal cut scores for being in Tier 1 of PSF and NWF, Tier 2 of PSF and NWF or Mix Tiers. The ideal cut scores were assigned based on true reading values, the values higher than the value of 0.1 were considered as Tier 1, lower than the value of -0.4 were considered as Tier 2 and the values between them considered Mix Tiers. After ideal placement was done for each model, the agreement between classifications of each model with their ideal placement calculated.

Table 7 presents the percentage of agreement between the POMDP-RTI assignment with its ideal placement and also between the Current-time only RTI assignment with its ideal placement. The POMDP-RTI model assigned the exactly the same students to Tier 1 and Tier 2 with its ideal placement while only 81% and 85% of the ideal placement students in Tier 1 and Tier 2 respectively were exactly matched with the Current-time only-RTI model assignment.

Table 7: Agreement between Ideal, POMDP-RTI and Current-time only RTI

<table>
<thead>
<tr>
<th></th>
<th>POMDP-RTI</th>
<th>Ideal</th>
<th>Agreement</th>
<th>Current-Time only RTI</th>
<th>Ideal</th>
<th>Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tier 1</td>
<td>136</td>
<td>118</td>
<td>100%</td>
<td>107</td>
<td>89</td>
<td>81%</td>
</tr>
<tr>
<td>Mix</td>
<td>90</td>
<td>138</td>
<td>65%</td>
<td>86</td>
<td>143</td>
<td>40%</td>
</tr>
<tr>
<td>Tier 2</td>
<td>74</td>
<td>44</td>
<td>100%</td>
<td>107</td>
<td>68</td>
<td>85%</td>
</tr>
</tbody>
</table>

From the agreement between a model and its ideal placement, the ideal Tier 1 placement rate (lambda) was computed by using equation 13 to assess how much the POMDP-RTI and the Current-time only-RTI models successfully assigned students to their Tier 1. In equation 13, $k$ is the marginal probability of each agreement and $i$ is the each tier.

$$
\lambda = \frac{\sum_i k_{i} - k_{\text{Tier}1}}{1 - k_{\text{Tier}1}}
$$

(13)
The lambda gets a value between 0 and 1. If it is 1, it means that the model did a perfect job of assigning students to the ideal tier which is Tier 1 in this study. Therefore, the POMDP-RTI model did a better job of assigning students to the ideal tier according to the comparison of the Tier 1 ideal placement rate of each model, (lambda = 0.74 for POMDP-RTI, lambda = 0.51 for Current-time only RTI).

4.3. Study 2 (b)

- Do the number of times a monitoring test is applied and the interval length between repeated monitoring tests affect the POMDP forecasts at each time point and students’ tier assignment based on their latent reading scores in the POMDP-RTI model?

The analysis for Study 2 (a) was applied for implementation 2 as well. At the end of running simulations for the four independent sequences of the Markov chain and the parallel chains for the model, with 300 iterations for adaptation and 500000 iterations for updating by using random starting positions, I obtained a convergence of the sequence of draws for implementation 2 as I did for implementation 1 in Study 1.

The obtained parameters were 498.4 (0.21) and 500.1 (0.23) for $a_1$ and $a_2$ and 3.52 (0.17) and 4.06 (0.17) for $b_1$ and $b_2$, respectively. For evaluating the implementation 2 forecasting with respect to implementation 1, the obtained PSF and NWF scores from implementation 2 were compared with estimated PSF and NWF scores from implementation 2. Table 8 provides a comparison of the implementation 2 observed and estimated scores of the same 30 randomly selected individuals represented in Table 3. The value in bold font shows that the difference between estimated and observed score was one or two standard deviations.
As performed for implementation 1 in Study 2 (a), the observed PSF and NWF scores were compared with estimated PSF and NWF scores. As previously, it was expected each type of score would be comparable between the data sets.

Table 8: Comparison between Observed and Estimated scores for implementation 2

<table>
<thead>
<tr>
<th>ID</th>
<th>Observed (PSF)</th>
<th>Estimated (PSF)</th>
<th>Error</th>
<th>Error2</th>
<th>Observed (NWF)</th>
<th>Estimated (NWF)</th>
<th>Error</th>
<th>Error2</th>
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</thead>
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<td>500.3</td>
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<td>501.9</td>
<td>3.80</td>
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<td>6.76</td>
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<td>500.3</td>
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<td>500.7</td>
<td>5.10</td>
<td>26.01</td>
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<td>5.70</td>
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<td>1.69</td>
<td>502.0</td>
<td>500.9</td>
<td>-1.10</td>
<td>1.21</td>
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<td>503.8</td>
<td>500.9</td>
<td>2.90</td>
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<td>2.00</td>
<td>4.00</td>
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<td>3.90</td>
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<td>0.36</td>
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<tr>
<td>132</td>
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<td>498.6</td>
<td>504.1</td>
<td>5.50</td>
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</table>

Bias(30) 1.60
MSE(30) 11.34

According to differences between observed and estimated scores, the computed mean square error was 9.49 for PSF and 4.7 for NWF. Aside from a few exceptions, 90% of PSF scores and
83% of NWF scores from estimated scores fell within two standard deviations. As seen in Figures 27 and 28, the results of the observed and estimated scores revealed the distributions of the PSF and NWF scores were a little wider between first and third quartiles for observed scores than for the estimated scores.

Figure 27: The distribution of Estimated and Observed scores based on PSF at time 3

Figure 28: The distribution of Estimated and Observed scores based on NWF at time 3

Figure 29 and 30 shows that most of the upper-end student scores were located near the blue identity line for both the PSF and NWF scores. When I compared the all 300 scores, I saw that
observed PSF and NWF scores had a little wider distribution from first quartile to third quartile than the estimated scores as seen from the bar graphs in Figure 30 and 31 and Table 9.

![PSF Scores](image1)

**Figure 29: Comparison of Estimated and Observed PSF scores**

![NWF Scores](image2)

**Figure 30: Comparison of Estimated and Observed NWF scores**

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Min</th>
<th>1 Quartile</th>
<th>Median</th>
<th>Mean</th>
<th>3 Quartile</th>
<th>Max</th>
<th>SD</th>
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<tr>
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<td></td>
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<td></td>
<td></td>
<td></td>
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<td>498.9</td>
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<td>Estimated</td>
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<td>501.9</td>
<td>515.3</td>
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<tr>
<td>NWF</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>499.1</td>
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<tr>
<td>Estimated</td>
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<td>501.7</td>
<td>503.7</td>
<td>518.2</td>
<td>3.31</td>
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</tbody>
</table>

*Table 9: Comparison of Observed and Estimated scores summary for implementation 2*
To evaluate whether *implementation 2* makes a significant difference in the students’ $\hat{R}$ score at the end of the year, a paired t-test was conducted to compare the end-of-the-year $\hat{R}$ scores of *implementation 1* and the end-of-the-year $\hat{R}$ scores of *implementation 2*. According to the paired t-test result, there was a significant different between the *implementation 1* reading proficiency scores (M= 0.103, SD = 0.81) and the *implementation 2* reading proficiency scores (M= -0.235, SD = 0.77); $t(299) = 10.44$, $p < 0.001$, $d= 0.4$.

Figure 31 and Figure 32 present a comparison of the end-of-the-year $\hat{R}$ scores from *implementations 1* and 2.

![Figure 31: Comparison of the end-of-the-year $\hat{R}$ scores](image)

Based on these results, *implementation 1* had the higher average $\hat{R}$ scores compare to the end-of-the-year $\hat{R}$ scores of *implementation 2* even though *implementation 1* had a higher SD and a wider score distribution compare to SD of *implementation 2*. The green line in Figure 32 represents the best-fitting line and I observed that the scatter of all the scores was located near except the low-end and high-end scores.
Figure 32: The scatter plot of the end-of-the-year $\hat{R}$ scores

Table 10 presents the comparison of implementation 1 and 2 based on average $\hat{R}$ scores at the end of the year. Based on the tables, the average $\hat{R}$ scores decreased in all tier groups. The number of students decreased in Tier 1 and Tier 2 while it increased in the Mix Tiers groups in implementation 2 compare to implementation 1.

Table 10: Comparison of Implementation 1 and 2 based on average reading scores $\hat{R}$ at the end of the year

<table>
<thead>
<tr>
<th>Tiers (PSF-NWF)</th>
<th>Implementation 1</th>
<th>Implementation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time 3</td>
<td>Time 4</td>
</tr>
<tr>
<td>N</td>
<td>$R_{\text{mean}}$</td>
<td>N</td>
</tr>
<tr>
<td>1-1</td>
<td>136</td>
<td>0.32</td>
</tr>
<tr>
<td>1-2</td>
<td>45</td>
<td>-0.46</td>
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<td>2-1</td>
<td>45</td>
<td>0.53</td>
</tr>
<tr>
<td>2-2</td>
<td>74</td>
<td>-0.21</td>
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</table>
CHAPTER 5

DISCUSSION

In chapter 5, I provide social significance of the study and present a summary, interpretation of the analysis of simulated data results and conclusion. Moreover, the limitations and future directions were also discussed.

5.1. Summary and Conclusion of the Results

This dissertation compares the POMDP-RTI model with the Current-time only-RTI, to determine whether the plans crafted by a POMDP model in an RTI setting offer advantages over current practice which uses the simple cut score method. This study does this through simulation studies based on the numbers obtained from fitting the POMDP model to a group of kindergarten students in an earlier RTI study (Al Otaiba et al., 2011).

An MCMC simulation analysis was performed by running simulations for four independent Markov chains starting from random positions with 300 iterations for adaptation and 500000 iterations for updating by using random starting positions.

Study 1:

As seen in Figure 6, the posterior distributions of the model parameters converged. In addition, the Chi-square cumulative distributions and Q-Q plots of Chi-square distributions of each parameter in Figure 11 to 18 showed that the model parameters were properly identified. According to the percentages of the simulated $\chi^2$ values, 96% and 94% of the values of $\chi^2$ statistics fell within 0.484 and 11.143 for the intercepts of PSF and NWF respectively, and 95% and 96% of the value of $\chi^2$ statistics fell within 0.484 and 11.143 for the slopes of PSF and NWF respectively. Thus, the posterior distributions of the model parameters converged and the model parameters were properly identified since there were 6% failures at maximum.
Study 2:

a.1) In Table 3, the POMDP-RTI model forecasting looks pretty accurate for 30 randomly selected cases according to the mean square errors. The obtained overall mean square error was 10.1 for PSF and 4.88 for NWF. Therefore, as presented in Figures 19 and 20, aside from a few exceptions, 91% of estimated PSF scores and 94% of estimated NWF scores fell within two standard deviations of observed PSF and NWF scores which prove that the POMDP-RTI model forecasted at each time point pretty accurately.

a.2) Table 4 and Table 6 indicates that students who were assigned to each tier based on their latent reading score in the POMDP-RTI model showed better performance compared to the students who were assigned to each tier based on their observed score in the Current-time only-RTI model. For each tier at time 3 in Table 4, the POMDP-RTI model had more students in Tier 1 than Tier 2 compared to the Current-time only-RTI model. Furthermore, the graphs in Figure 23 and Figure 24 presents the latent reading score distributions at time 2 and 3 for the POMDP-RTI model whereas Figure 25 and Figure 26 present the latent reading score distributions at time 2 and 3 for the Current-time only-RTI model. According to those figures, it is safe to say that latent reading scores in the POMDP-RTI model were closer to each other than the latent reading scores in the Current-time only-RTI model. Also the number of students increased in Tier 1 and Mix Tiers of POMDP-RTI when the number of students increased in Tier 2 and Mixed Tiers of the Current-time only-RTI model. According to the average latent reading score, growth was better under the POMDP-RTI approach, with a difference in the mean reading ability of 0.49 on a standardized scale. In addition, computed lambda values showed that the POMDP-RTI model did a better job of assigning students to the ideal tier according to the comparison of the Tier 1 placement rate of each model, lambda = 0.74 for
POMDP-RTI, lambda = 0.51 for Current-time only RTI. Appendix A includes the graphs of \( R_{nm}^{\sim} \) and \( R_{nm}^{\tilde{\sim}} \) of each student at each state of Implementation 1 for more detailed information about students’ growth.

b) Implementation 2 converged as well as Implementation 1. Having one more learning block in the POMDP-RTI model or applying the assignment earlier because of having four blocks in the model for Implementation 2 made a significant difference in the average latent reading scores of the students at the end of the year, compared to Implementation 1. The average latent reading score difference was 0.34 between the two Implementations with \( t(299) = 10.44, p < 0.001 \). The students had lower end of year proficiency levels in Implementation 2 compared to Implementation 1. As seen in Table 10 and Figure 32, the mean differences are mostly seen on low-end and high-end students’ latent reading scores. The POMDP-RTI model forecasting in Implementation 1 seemed more accurate than Implementation 2 according to a comparison of Figure 19 and 20 with Figure 28 and 29. For Implementation 2, the computed mean square error was 9.49 for PSF and 4.7 for NWF. Aside from a few exceptions, 90% of PSF scores and 83% of NWF scores from estimated scores fell within two standard deviations. For Implementation 1, the computed mean square error was 10.1 for PSF and 4.88 for NWF. Aside from a few exceptions, 91% of estimated PSF scores and 94% of estimated NWF scores fell within two standard deviations. Therefore, the POMDP-RTI model forecast better in Implementation 1 than Implementation 2.

5.2. Social Significance of the Study

The Mastropieri, Scruggs, and Graetz (2003) study indicates that reading is the main problem for the majority of the students with learning disabilities. The researchers reveal that reading difficulties are observed as a core symptom for 80% of the students with the learning
disabilities (Berdine, 2003). Therefore, learning disability is a result of a student with low-reading achievement (Shore & Sabatini, 2009). Hence, reading difficulties might be used as a symptom to figure out whether students have a learning disability. In this case, teachers might be able to diagnose the students with learning disabilities by considering their reading difficulties.

To identify the students with reading difficulties, response to intervention (RTI) is one of the frameworks to consider and modify the instruction. By using RTI, teachers are able to determine and identify the struggling students with reading difficulties or disabilities based on their limited response to an effective instruction or intervention (Compton et al., 2006; D. Fuchs et al., 2003; Lyon et al., 2003). Periodically applied assessment in the RTI framework helps instructors to determine the students’ current ability levels, and make decisions that may affect the outcomes for the group of students. In fact, the student’s response to instruction often provides important clues about their proficiency and specific learning problems (Marcotte & Hintze, 2009). Many researchers advocate using periodically applied assessments (Black & Wiliam, 1998; Wiggins, 1998), which a teacher can then use to modify weekly or monthly instruction in RTI framework.

A proper integration of RTI, periodic assessment, and student reading ability across time requires a model in order to monitor and predict how student reading proficiency changes over time. Almond (2007) suggested that this could be done using a POMDP model. Periodically measuring the students reading proficiency, the involving different instruction or intervention according to students’ proficiency level made the POMDP-RTI model for this study.
The POMDP-RTI model consists of multiple measurement occasions and instructor decision processes. By using the model, instructors are able to monitor struggling students at each measurement occasion based on their response to the applied instruction and they can decide whether Tier 1 instruction or Tier 2 instruction is necessary to improve the struggling students’ performance to catch up their peers.

5.3. Limitations and Future Directions

Limitations

This dissertation demonstrated that the POMDP-RTI model properly works to select and apply an appropriate instructional approach to increase the students reading proficiency level at different time points. Also it forecasts students reading proficiency at the end of a school year. However, only simulated datasets were used in this dissertation. In a real life situation, creating two different instructions as Tier 1 and Tier 2 and using them through a school year might provide different results than the simulated data sets. In the simulated data sets, I assumed that all students had the same initial reading proficiency level. However, real students might have different initial reading proficiency levels at the beginning of a school year in a real life situation.

The limitation of obtained results from using POMDP depends critically on the fixed model parameters such as cost and effectiveness of intervention and utility of various outcomes. For instance, I used the two different fixed initial tier-specific growth rates for Tier 1 and Tier 2 in equation 6; however the student scores were estimated based on only one estimated tier-specific growth rate value for both tiers at time 2 and time 3 in order to avoid having a possible convergence problem. This might be the reason of having the overestimated student scores in Tier 2 as seen in Figure 21 and Figure 22 in Study 2(a) and Figure 29 and
Figure 30 in Study 2(b). In addition, I used the same fixed slope parameter value for each tier in the utility function in equation 11. This also might be the reason for having the substantially overlapping tier distributions in Figure 23 and Figure 24, while I was expecting to have nearly two separate tier distributions. Additionally, in Study 2(b), I used the same fixed parameter values, which were used in the utility and cost functions of the POMDP-RTI model in Study (a). However, Implementation 2 consisted of four blocks which decreased the time length of each block in the model and number of students in Tier 2 decreased in time. This might directly influence the cost of each block, the frequency with which the group meets, and the duration of the meeting time in equation 10. This missed point might be one of the reasons for the significant difference in the average proficiency levels of the students at the end of the year in Implementation 2.

Future Directions

Based on the mentioned limitations above, the obvious next step is to determine the appropriate fixed parameters for utility and cost functions and to use two estimated tier-specific growth rates for each tier in equation 6 after fixing the convergence problem and run the model again to show how much these parameters affect the overestimated Tier 2 growth rate and the substantially overlapped tier distributions in POMDP-RTI model.

The benefit of using POMDP-RTI model as opposed to a simple cut score depends critically on model parameters such as cost and effectiveness of intervention and utility of various outcomes. Students’ responses to these fixed instructions might change based on their backgrounds and initial reading proficiency levels in real life situation. In addition, a school policy might not allow or limit teachers to have these kinds of fixed instructions, which might be the biggest limitation of this study. Thus, another future step is to use the POMDP-RTI
model to answer the research questions by collecting and using a real data set. The POMDP-RTI model might be used at a selected couple of pilot schools, and a similar analysis can be done by using the collected data from these schools.

In this study, I only examined the students’ reading proficiency growth during a school year. One of the future steps will be the possible application of Hierarchical Linear Models considering the students’ reading proficiency growth under different teachers and school instructions. Differences among schools and teachers might influence students’ current reading performance and their school performance.
APPENDIX A

STUDENTS GROWTH IN IMPLEMENTATION

The graphs in Figures 33-36 and 37-40 present the $R_n^m$ and $R_{nm}$, respectively, for each student in each time. The red lines and dots represent the student in Tier 1; the green lines and dots, the student in Tier 2.

Figure 33: PSF and NWF for Tier 2 at time 2 and for Tier 1 at time 3

The $\hat{R}$ scores of students who were exposed to PSF-Tier 2 and NWF-Tier 2 at time 2 and PSF-Tier 1 and NWF-Tier 1 at time 3 are shown in Figure 33. A total of 42 and 83 students were assigned to the PSF-Tiers and NWF-Tiers, respectively. For both PSF and NWF, with the exception of a single student for PSF, all of the $\hat{R}$ scores increased at time 2 after the instructor used Tier 2 instruction. However, aside from a few exceptions for both PSF and NWS, the scores showed a decrease at time 3. For PSF, the scores ranged from -1.5 to 0.5 at time 1 and from -1 to 3 at time 2, therefore showing an overall increase. Meanwhile, the scores ranged from -1 to 2 at time 3. For NWF, the scores ranged from -2.5 to 0 at time 1, from 0 to 3 at time 2, and from -1 to 2 at time 3.
The $\hat{R}$ scores of students who were exposed to PSF-Tier 2 and NWF-Tier 2 at time 2 and 3 are presented in Figure 34. A total of 108 and 68 students were assigned to the PSF-Tiers and NWF-Tiers, respectively. For both PSF and NWF, the majority of students showed increased $\hat{R}$ scores at time 2 when the instructor used Tier 2 instruction and had consistent or increased scores at time 3. For PSF, the scores ranged from -3 to 0.5 at time 1, from -1.5 to 2 at time 2, and from -2 to 2 at time 3. For NWF, the scores ranged from -3 to 0 at time 1, from -1.5 to 0.5 at time 2, and from -2 to 1.5 at time 3. Most students showed overall increased scores from time 1 to time 3.
The $\hat{R}$ scores of students who were exposed to PSF-Tier 1 and NWF-Tier 1 at time 2 and 3 are shown in Figure 35. A total of 139 and 98 students were assigned to PSF-Tier 1 and NWF-Tier 1, respectively. For both PSF and NWF, some students showed increased scores at time 2, but many had either consistent or decreased scores. For time 3, the majority of scores was either consistent or decreased for PSF and was decreased for NWF. For PSF, the scores ranged from -0.5 to 2.5 at time 1, from -0.5 to 4.5 at time 2, and from -2 to 4 at time 3. For NWF, the scores ranged from 0 to 3 at time 1, from 0 to 4.5 at time 2, and from 1.5 to 4.5 at time 3.
The $\tilde{R}$ scores of students who were exposed to PSF-Tier 1 and NWF-Tier 1 at time 2 and PSF-Tier 2 and NWF-Tier 2 at time 3 are presented in Figure 36. A total of 11 and 51 students were assigned to the PSF-Tiers and NWF-Tiers, respectively. For PSF, with the exception of a single student, all students showed decreased $\tilde{R}$ scores at time 2. With the exception of two students, the students had scores that were either consistent or increased at time 3 as compared to time 2. The scores ranged from -0.25 to 1 at time 1 and from -1.25 to 1 at time 2, thus showing a decrease. The scores ranged from -1 to 0.75 at time 3. For NWF, almost every student showed decreased $\tilde{R}$ scores at time 2. The scores were mostly either consistent or increased at time 3 compared to time 2. The scores ranged from 0 to 1 at time 1 and from -1.5 to 0 at time 2, thus showing an overall decrease. The scores were between -2 to 1 at time 3. Most student scores were observed to increase in the presence Tier 2 instruction.
Figure 37: PSF and NWF for Tier 2 at time 2 and Tier 1 at time 3

The $\bar{R}$ scores of students exposed to PSF-Tier 2 and NWF-Tier 2 at time 2 and PSF-Tier 1 and NWF-Tier 1 at time 3 are shown in Figure 37. A total of 45 and 38 students were assigned to the PSF-Tiers and NWF-Tiers, respectively. Most students showed increased $\bar{R}$ scores at time 2 when the instructor started using Tier 2 instruction. However, the scores mostly decreased at time 3. For PSF, the scores ranged from -1.5 to 0 at time 1 and from -1.5 to 1.5 at time 2, therefore showing increased values. The scores ranged from -2 to 1 at time 3. For NWF, the scores ranged from -1.5 to 0.5 at time 1, from 0.5 to 1.5 at time 2, and from -2 to 0.5 at time 3.
The \( \tilde{R} \) scores of students who were exposed to PSF-Tier 2 and NWF-Tier 2 at time 2 and 3 are presented in Figure 38. A total of 105 and 113 students were assigned to the PSF-Tiers and NWF-Tiers, respectively. For both PSF and NWF, although some students showed increased \( \tilde{R} \) scores at time 2, many had either decreased or consistent scores. The majority of scores were decreased at time 3. For PSF, the scores ranged from -4 to 0 at time 1, from -3.5 to 1 at time 2, and from -3.5 to 1 at time 3. For NWF, the scores ranged from -4 to 0 at time 1 and from -3.5 to 0 at time 2, therefore showing an overall increase. The scores were between -3.5 to 1 at time 3.
The $\tilde{R}$ scores of students who were exposed to PSF-Tier 1 and NWF-Tier 1 at time 2 and 3 are presented in Figure 39. A total of 105 and 112 students were assigned to PSF-Tier 1 and NWF-Tier 1, respectively. For PSF, although a small percentage of students showed increased scores, most students had either a decreased or consistent score at time 2 or 3. The scores ranged from -0.5 to 4 at time 1, from -1.5 to 4 at time 2, and from -2 to 4 at time 3. For NWF, most students showed consistent or decreased scores, but some had increased at time 2. The majority of scores were decreased at time 3. The scores ranged from 0 to 4 at time 1, from -0.5 to 4 at time 2, and from -2 to 4 at time 3.

![Figure 40: PSF and NWF Tier 1 at time 2 and Tier 2 at time 3](image)

The $\tilde{R}$ scores of students who were exposed to PSF-Tier 1 and NWF-Tier 1 at time 2 and to PSF-Tier 2 and NWF-Tier 2 at time 3 are presented in Figure 40. A total of 45 and 37 students were assigned to the PSF-Tiers and NWF-Tiers, respectively. For both PSF and NWF, nearly every student showed a decreased $\tilde{R}$ score at time 2 in the presence of continued Tier 1 instruction. By contrast, the scores were largely consistent or increased at time 3 when compared those at time 2. For PSF, the scores ranged from -0.5 to 1.5 at time 1, from -1.5 to 1.5 at time 2, and from -1.5 to 1 at time 3. For NWF, the scores ranged from -0.5 to 1 at time 1, from -1.5 to -0.25 at time 2, and from -1.5 to 0.5 at time 3.
APPENDIX B

EXPECTED UTILITY

Figure 41: Expected Utility for Tier 1 at time 2

Figure 42: Expected Utility for Mix Tiers at time 2
Figure 43: Expected Utility for Tier 2 at time 2

Figure 44: Expected Utility for Tier 1 at time 3
Figure 45: Expected Utility for Mix Tiers at time 3

Figure 46: Expected Utility for Tier 2 at time 3
REFERENCES


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BIOGRAPHICAL SKETCH

Umit Tokac is from Nevsehir, Turkey. He received his bachelor’s and master’s degrees in Mathematics Education from Gazi University, Turkey and a second master’s degree in Measurement and Statistics from Florida State University. His primary research interests are educational statistics, Bayesian data analysis applications in education, and adapting artificial intelligence methods to education in order to measure and monitor learners’ current proficiencies, and forecast their future proficiencies. In his spare time, he likes to read non-fiction books, listen to indie rock music and play soccer.