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Piezohydraulicactuator Development Foractive Microjet Flow Control

Fei Liu
FLORIDA STATE UNIVERSITY
COLLEGE OF ENGINEERING

PIEZOHYDRAULIC ACTUATOR DEVELOPMENT FOR ACTIVE MICROJET FLOW CONTROL

By

FEI LIU

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The members of the Committee approve the Thesis of Fei Liu defended on September 17, 2008.

William S. Oates  
Professor Directing Thesis

Jonathan Clark  
Outside Committee Member

Farrukh S. Alvi  
Committee Member

The Office of Graduate Studies has verified and approved the above named committee members.
This thesis is dedicated to my advisor Dr. Oates. This would not have been possible without his support.
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ABSTRACT

This thesis describes the development of a new piezohydraulic actuator for integration into a microjet flow control system. The piezohydraulic actuator utilizes a lead zirconate titanate (PZT) stack actuator and a hydraulic amplification design to achieve relatively large displacements required to control flow through a microjet with an orifice diameter of approximately 400 µm. Displacement amplification of 81 times the stack actuator displacement was achieved using a dual-diaphragm design to seal the fluid within a converging nozzle.

The piezohydraulic actuator is characterized and compared to a system dynamic model to identify underlying subsystem behavior that contributes to the system level piezohydraulic actuation. System dynamic modeling has been previously used to estimate the dynamic performance of similar piezohydraulic pump systems [1] [2] [3]. These models are useful for identifying subsystem dynamics and design optimization. Here, a similar modeling framework is implemented and extended to include nonlinear and hysteretic ferroelectric behavior using a homogenized energy framework [4] and hyperelastic behavior of the rubber diaphragm material [5]. It is shown that minor loop hysteresis of the piezoelectric stack actuator and nonlinear deformation of the diaphragm play an important role in the large piezohydraulic displacement amplification.
CHAPTER 1

INTRODUCTION

Flow control theory and actuator development have been the subject of intense research for more than a decade for applications on various aircraft structures including fixed wings, cavity bays, rotor blades, and impinging jets. A number of actuation techniques have been proposed which include passive systems, active open-loop, and closed-loop flow control designs. Passive systems such as micro vortex generators create undesirable drag and suffer from robustness over a broad range of operating conditions [6]. Active open-loop actuators are often limited in bandwidth and require large power. Closed-loop control designs can reduce the amount of power required [7], but complexities associated with achieving broadband capabilities have limited its implementation in many applications.

Current open-loop and closed-loop systems typically utilize actuators that provide steady mass flow, pulsed mass flow, or zero-net mass injection (synthetic jets). Maximum flow control is typically achieved by strategically placing the actuator(s) at the initial point of flow separation to interact with the shear boundary layer so that the minimum amount of control effort is expended. Steady mass injection increases the shear boundary and reduces undesirable vortices whereas pulsed mass injection includes a steady flow component and an oscillatory component that is believed to enhance flow control [8], although the physical mechanism is still unclear. Synthetic jets, typically actuated with piezoelectric diaphragms, produce no net mass flow and control flow through second order effects [9].

The major drawbacks of these systems are the lack of actuation capability and bandwidth limitation. Pulsed mass flow actuators such as oscillating fluidic actuators [10] can operate at high frequency (low kHz regime) but are limited to a narrow frequency range. Similarly, plasma or spark jet actuators [11] can operate at high frequency, but the pulse flow is limited to the combustion process and heat transfer rate. These narrow bandwidth controllers often lead to
splitting the resonant peaks (spillover) [8]. Piezoelectric composite actuators used in synthetic jets have provided reasonable flow control in the subsonic regime; however, the force output and narrow bandwidth limit raises questions about its robustness in a broad range of operating conditions. Recently, a new membrane actuator based on ferromagnetic shape memory alloy composite was applied to produce strong synthetic jet flow, but it is limited to the high frequency regime [12].

In contrast to pulsed flow, steady flow microjets have demonstrated significant improvement in reducing undesirable flow characteristics on a broad range of aircraft structures including cavities, inlet designs, and impinging jets. In these systems, an array of orifices (each with diameter of 400 $\mu$m) is strategically placed on an aircraft surface and steady blowing from the orifices reduces flow separation control and acoustic emissions. Implementation of steady flow microjets have demonstrated significant reductions in the overall sound pressure level by 11 dB in a supersonic cavity flow application [13]. However, different cavity geometries required different microjet mass flow rates to obtain similar reductions in the SPL. Similarly, Lou et al. [14] have demonstrated that steady-flow microjets can significantly reduce acoustic fields near an impinging jet for short take-off and vertical landing (STOVL) aircraft, but flow control using the microjets varied as the aircraft approached the ground. This has motivated the need for active microjet actuators that can be applied to aircraft structures operating under different external flow conditions.

To address this issue, a new piezohydraulic actuator is developed for integration into a microjet flow control system as illustrated in Figure 1.1 in this thesis. Piezoelectric materials are well known for their broadband (1Hz to $\sim$1kHz) electro-mechanical actuation characteristics which have been utilized in a number of compact actuator devices [15] [16] [17]. The large forces and small displacements exhibited by piezoelectric stack actuators are ideal for broadband nanopositioning applications [18]. In applications where larger displacement (1 mm) is desired, amplification techniques or frequency rectification is often employed [16] [19] [1] [2] [20] [3]. A hydraulic amplification technique is employed here to provide a compact throttling mechanism that can control a microjet with an orifice diameter of approximately 400 $\mu$m.
Figure 1.1: Schematic of two microjet flow applications: (a) Impinging jet nozzle using an array of microjets placed along the circumference of the nozzle to reduce noise and lift loss. (b) Microjet arrays implemented along a ramp to control separation flow.
CHAPTER 2

PIEZOELECTRIC MATERIALS AND APPLICATIONS

2.1 History of piezoelectricity

In the 18th century, crystals of certain minerals were known to generate electrical charge when heated. But actual “piezoelectricity” was first discovered by two brothers Pierre and Jacques Curie in 1880 [21]. After the discovery of the phenomenon of piezoelectricity, the European scientific community established the knowledge for identification of piezoelectric crystals. In the following 25 years, a more complete framework was made which defined the 20 out of the 32 natural crystal classes, in which piezoelectric effects occur [22].

The first noteworthy application for piezoelectric devices was an ultrasonic submarine sonar detector, which was developed in France during World War I [23]. Many kinds of piezoelectric devices were developed following World War I, such as accelerometers, microphones, bender element actuators, ultrasonic transducers, etc. [24]

During World War II, certain ceramic materials, which exhibited dielectric constants up to 100 times higher than common cut crystals, were discovered. They provided a new way for developing piezoelectric devices- namely, tailoring a material to a specific application [22].

In contrast to bulk material, many device concepts utilizing ferroelectric materials will likely use alternately layered structures where dimensions are minute enough to observe lattice and domain structure size effects [25], such as ferroelectric random access memories. Thin films are used as a platform to explore size effects in ferroelectrics in which significant changes in the material physics can occur relative to bulk ferroelectrics [25]. Furthermore, the application of ferroelectric films on microsystems has been explored [26]. Sensors and actuators are added to MEMS by depositing ferroelectric films on silicon or other substrates. The material can provide large output force, has a broad bandwidth and low power consumption. Meantime, high frequency electric components, like microwave electronics, are miniaturized and integrated onto one substrate.
by utilizing ferroelectric films to reduce cost and size of devices [27].

2.2 Piezoelectric and ferroelectric constitutive behavior

Ferroelectric materials exhibit spontaneous polarization at temperatures below the Curie point, and the polarization can be reoriented by applied electric fields or mechanical stresses [21]. They produce voltage under stretching or compression and yield strain and stress by applied electric fields. The previous phenomenon is called direct effect and the latter is the converse effect. The converse effect of ferroelectric materials is detailed in this chapter since this property is the one that is employed in our specific application.

Ferroelectric materials exhibit hysteresis and constitutive nonlinearities as illustrated in the schematic Figure 2.1 [4]. At point A, the direction of all dipoles is upward for sufficiently large positive electrical field. As the positive electrical field decreases to point B, a non-zero polarization and strain remains in the material at zero electric field. This state is defined as the remanent polarization and remanent strain respectively. Changes in the polarization (P) and uniaxial strain ($\varepsilon$) which result from small electric fields are approximately linear and reversible. As the electrical field is applied in the negative direction and approaches point C, large amounts of polarization begin to occur. The negative strains are observed at point C due to intermediate 90$^\circ$ dipole switching. By decreasing the electric field past the negative coercive field ($-E_c$), more dipoles switch downwards as the internal material state evolves to point D. At this point, the polarization points in the opposite direction whereas the uniaxial strain is equal to the strain at point A. The same behavior occurs from point D back to point A, only the direction of the applied field and corresponding dipoles are opposite from the previous process.

The most common ferroelectric compound employed for smart material applications is PZT (lead zirconate titanate, xPbTiO$_3$—(1-x)PbZrO$_3$) alloys. By adjusting the molar fraction of PbZrO$_3$ and PbTiO$_3$, strong electromechanical coupling and large dielectric constants can be achieved [22]. Optimal compositions are typically achieved along the morphotropic phase boundary where both tetragonal and rhombohedral phases coexist. The characteristics described previously for ferroelectric materials are applicable to PZT alloys.
2.3 Linear and nonlinear constitutive models

2.3.1 Linear constitutive model

For low input electric fields, the electric field-electric displacement and electric field-strain relations are approximately linear as illustrated in Figure 2.1. In the linear approximation, the elastic, piezoelectric and dielectric coefficients are treated as constant values. The linear constitutive law governing piezoelectric materials is

\[ \varepsilon_{ij} = s_{ijkl}^{E} T_{kl} + d_{kij} E_{k} \quad (2.1) \]

\[ D_{i} = d_{ijkl} T_{kl} + \kappa_{ik}^{J} E_{k} \quad (2.2) \]
where $\varepsilon_{ij}$ is infinitesimal strain, $s^E_{ijkl}$ is compliance at constant field, $T_{kl}$ is stress, $d_{ki j}$ is the piezoelectric tensor, $E_k$ is the electric field, $D_i$ is the electric displacement and $\kappa^T_{ik}$ is the dielectric permittivity at constant stress. The strain and electric displacement is defined from a reference state which is typically the remanent strain and polarization.

### 2.3.2 Homogenized energy model

Ferroelectric materials are often nonlinear when driven at moderate to large field levels. Minor loop hysteresis can also occur under uni-polar fields which requires more advanced constitutive relations. To accommodate this behavior within a constitutive modeling framework, additional thermodynamic relations must be introduced. One example is homogenized energy modeling. The homogenized energy modeling framework is based on stochastic distributions of a reduced set of material parameters describing inhomogeneities typically present in polycrystalline ferroelectric materials [4] [28]. A local variation in the externally applied field is often approximated by a normal distribution, which is denoted by an interaction field. This is used to describe variations in the internal field due to inhomogeneities. The sum of the interaction field and applied field define the effective field. A local variation in the average coercive field is introduced and distributed about a log-normal distribution. A general distribution may also be implemented and fit to data for improved model estimates. Ferroelectric switching is determined by comparing the distributions of effective fields and coercive fields such that when a local effective field is larger than a local coercive field, local polarization switching occurs, as illustrated in Figure 2.2. Macroscopic polarization is computed by homogenizing the local polarization over the probability distributions of the interaction fields and coercive fields. A detailed description of this modeling framework can be found in [4].

Helmholtz energy used in the homogenized energy model is defined as a function of the polarization

$$
\Psi_E(P) = \begin{cases} 
\frac{\kappa_e}{2} (P \pm P_R)^2, |P| > P_I \\
\frac{\kappa_e}{2} (P_I - P_R)(P^2 - P_R), |P| < P_I 
\end{cases}
$$

(2.3)

where $P_I$ denote the inflection point, $P_R$ represents the remanent polarization at which the minimum of $\Psi$ occurs. The Gibbs energy is related with Helmholtz energy

$$
G_E = \Psi_E - EP
$$

(2.4)
The local polarization \( \tilde{P} \) is determined directly by energy minimization \( \frac{\partial G_E}{\partial \tilde{P}} = 0 \). Legendre transforms analysis is considered for the energy minimization, which is detailed in Appendix A. This yields the general form

\[
\tilde{P}(E) = \frac{1}{\eta} E + P_R \delta
\]

(2.5)

where \( \delta = -1 \) for negatively oriented dipoles and \( \delta = 1 \) for those with positive orientation.

The homogenized polarization is defined by

\[
\bar{P} = \int_{-\infty}^{\infty} \int_{0}^{\infty} \nu(E_I, E_c) \tilde{P}(E + E_I; E_c, \xi) dE_I dE_c
\]

(2.6)

where \( \tilde{P}(E + E_I; E_c, \xi) \) is the local polarization, \( E \) is the applied field, \( E_I \) is the interaction field, \( E_c \) is the coercive field, and \( \xi \) is an initial set of variants defining the local polarization. Rate
dependent hysteresis can also be included in the local polarization; see [4] and [29] for details. The probability distribution is denoted by $\nu(E_I, E_c)$ and defined by

$$\nu(E_I, E_c) = C e^{-E_I^2/2b^2} e^{-ln(E_c/\bar{E}_c)^2}$$

(2.7)

where $b$ is the variance of the normal distribution and $c$ defines the variance of the log-normal distribution with maximum value at $\bar{E}_c$. The constant $C$ ensures integration to unity.

### 2.4 Piezoelectric stack actuators

Piezoelectric stack actuators are used in a number of actuator devices where small displacements and high forces are needed. In order to satisfy the demands of stringent speed and accuracy specifications, piezoelectric stack actuators play a critical role in the design of stages for nanopositioning [18]. Another example is a piezohydraulic pump which utilizes a piezoelectric actuator to achieve large force and large displacement by a step-and-repeat operation process [1] [2].

Piezoelectric stack actuators consist of a number of thin layers (~1 mm) of piezoelectric material separated by electrodes, see Figure 2.3. Each layer is polarized along the direction of its thickness. The displacement of the stack actuator is controlled by the external field and is a function of the number of active layers. The inactive layers and surrounding insulating material can affect the constitutive response of the piezoelectric actuator. In this thesis, all modeling results are based on effective piezoelectric and elastic coefficients for stack actuators.

Figure 2.3: Illustration of a piezoelectric stack actuator cross-section. The polarization is in the direction of the applied field, $E_x = -\frac{\partial \phi}{\partial x}$, where $\phi$ is the electric potential.
Since the piezoelectric stack actuator can provide large forces over a broad frequency regime, it is used in this thesis to construct a piezohydraulic amplification device. This device would potentially inherit broader bandwidth relative to many traditional actuators and also have the actuation capability to achieve relatively large displacements required for throttling a microjet flow control device.
A new piezohydraulic actuator was designed and fabricated for next generation microjet flow control. A hydraulic amplification structure was utilized to overcome the issue of small displacements of the piezoelectric stack actuator and to provide a compact throttling mechanism that can control a microjet with an orifice diameter of approximately 400 µm. The piezohydraulic actuator includes a piezoelectric stack actuator, piston, dual rubber diaphragm assembly, hydraulic fluid, and aluminum structural housing. A cross section view of the actuator design is illustrated in Figure 3.1. The critical components of this device are described as follows. All dimensions are in inches unless otherwise noted.

3.1 Piezoelectric stack actuator

The piezoelectric stack actuator (Kinetic Ceramics), as described in Section 2.4, is used to convert electrical energy to mechanical work. The large forces afforded by the stack actuator are ideal for hydraulic amplification designs. The stack actuator implemented in this device is 22 mm long and has a circular cross section with a diameter of 19 mm. Small alumina disks were bonded to both sides of the stack for electric insulation. The schematic of the piezoelectric stack actuator is illustrated in Figure 3.2.

3.2 Piezohydraulic actuator structural design

In order to assemble the piezoelectric stack actuator, piston and sealed hydraulic fluid together to achieve the fundamental function of the piezohydraulic actuator, an aluminum structural housing was designed and illustrated in Figure 3.3. The cylinder head and threaded end cap are shown in Figure 3.4 and 3.5 respectively. The internal wall of its bottom end was threaded for the end cap
Figure 3.1: A cross section of the piezohydraulic actuator

Figure 3.2: The schematic of the piezoelectric stack actuator (Kinetic Ceramics, Inc. 2004)
assembly to position the stack actuator within the aluminum structure. The piston was placed in the top section, which is assembled with the cylinder head.

## 3.3 Cylinder head design

A cylinder head was constructed to seal the hydraulic fluid. A converging nozzle was used for hydraulic amplification of this device. Four holes were drilled and tapped on the top plane which were used to connect microjet interface with the cylinder head using bolts. An external rubber diaphragm was sandwiched in between the cylinder head and the microjet interface. A bleed valve is located on the top of the cylinder head for evacuating entrained air. An additional hydraulic fitting was used for charging the system with fluid. The cylinder head design is shown in Figure 3.4.
3.4 Threaded end cap design

A threaded end cap was used for supporting the piezoelectric stack actuator inside the structural housing. Two holes were drilled for electrical connections to the piezoelectric stack actuator. The threaded end cap is shown in Figure 3.5.

3.5 Diaphragm material

A dual-diaphragm design (as shown in Figure 3.1) was employed to seal a low viscosity hydraulic fluid (Dow Corning 200, viscosity—0.65 cSt) inside the cylinder head. Rubber was chosen as the material for the internal diaphragm since it can easily deform to conform to the shape of the internal geometry. This is believed to reduce losses from air around the perimeter of the piston. The same rubber material was used for the external diaphragm so that relatively large displacement would be achieved when driving the piezoelectric actuator.
In order to detect the displacement of the rubber material, a preliminary microjet interface was utilized to clamp the rubber material, as seen in Figure 3.6. A tiny pin will be placed through the central hole in contact with the rubber for probe measurement, which will be detailed in Chapter 4.

3.7 Piezohydraulic Actuator Assembly

The prototype of the piezohydraulic actuator assembly is given in Figure 3.7, which has an overall length of 66 mm and the largest diameter of 67 mm with a total weight of 360g. The dual-diaphragm configuration seals the low viscosity silicone oil inside the cylinder head and provides a simple mechanical design for controlling air through a microjet.
Figure 3.6: Microjet interface design
Figure 3.7: The prototype of the piezohydraulic actuator. A bleed valve is located on the top left for purging air. The rubber diaphragm is located underneath the top plastic microjet interface.
CHAPTER 4

DEVICE CHARACTERIZATION

In this chapter, the experimental setup and characterization of the piezohydraulic device is given.

4.1 Experimental Setup

4.1.1 Hydraulic system

The hydraulic system used to characterize and control the actuator is illustrated in Figure 4.1. This system was set up to charge the piezohydraulic actuator with fluid, control the biased pressure, and purge air from the device. A hydraulic accumulator was used to control bias pressure in the piezohydraulic actuator and a fluid reservoir was connected to a vacuum pump to purge air from the system. The accumulator can be replaced with another one with reduced size, as shown in Figure 4.2. Shut-off valves were used to pressurize the system and evacuate entrained air using a vacuum pump. A pressure gauge was used to monitor bias pressure in the piezohydraulic cylinder head. During operation, the shut-off valve between the fluid reservoir and the piezohydraulic actuator was closed and the shut-off valve to the accumulator was left open. During operation, hydraulic fluid was prevented from flowing back into the accumulator by including a check valve in the hydraulic circuit.

4.1.2 Data acquisition system

The piezohydraulic actuator was characterized at different bias pressures and voltage amplitudes. A dSpace data acquisition system (DS1005 DSP board), controlled by Matlab and Simulink, was used to collect actuator displacement data. The piezoelectric stack actuator was driven by a 1000V/7A switching power supply (PEIZOMechanik) which amplified the voltage applied to the
Figure 4.1: The schematic of the hydraulic system used to charge the piezohydraulic actuator with fluid and remove entrained air.

piezoelectric stack actuator. A Lion Precision capacitor probe with a sensitivity of 10 V/mm was used to detect displacement of a light-weight pin (1 g) that was placed in contact with the external diaphragm, see Figure 4.3. A schematic of the cross-section showing the deformed rubber, needle and capacitor probe is given in Figure 4.4. The data acquisition system and drive electronics are shown in Figure 4.5. The amount of displacement amplification is estimated by measuring free displacement of the piezoelectric stack actuator and comparing the results with piezohydraulic device actuation. The same experimental set-up was used to measure free displacement of the stack actuator.

4.2 Experimental Result

The piezoelectric stack actuator and hydraulic actuator displacements were characterized at different voltage amplitudes and frequencies. In addition, different bias pressures were applied to the piezohydraulic system. Sinusoidal wave forms were applied to the stack actuator with voltages
Figure 4.2: The hydraulic system used to charge the piezohydraulic actuator with fluid and remove entrained air.

Table 4.1: Peak stack actuator displacements and diaphragm displacements versus voltage inputs at 1 Hz

<table>
<thead>
<tr>
<th>Voltage</th>
<th>400V</th>
<th>600V</th>
<th>800V</th>
<th>1000V</th>
</tr>
</thead>
<tbody>
<tr>
<td>PZT displacement (µm)</td>
<td>9</td>
<td>14</td>
<td>18</td>
<td>23</td>
</tr>
<tr>
<td>Diaphragm displacement (µm)</td>
<td>730</td>
<td>1047</td>
<td>1209</td>
<td>1339</td>
</tr>
<tr>
<td>Amplification gain</td>
<td>81</td>
<td>75</td>
<td>67</td>
<td>58</td>
</tr>
</tbody>
</table>

ranging up to 1000V (2 MV/m) at 1 Hz. A set of experimental data of the piezoelectric stack actuator and piezohydraulic displacements at different operating conditions is given in Table 4.1. The hydraulic amplification gain ranged between 58 and 81 and increased as the voltage decreased.

The nonlinear and hysteretic responses of the piezoelectric stack actuator and piezohydraulic actuator displacements are illustrated in Figure 4.6 and 4.7, respectively. Optimum performance at each voltage amplitude was obtained by applying relatively small changes in the bias pressure as listed in Figure 4.7. The mechanisms contributing to nonlinear and hysteretic actuation are proposed to be predominantly related to nonlinear deformation of the rubber diaphragm and minor
Figure 4.3: A Lion Precision capacitor probe used to detect displacements of a light-weight needle.

Figure 4.4: A schematic of the cross-section showing the deformed rubber, needle and capacitor probe.
Figure 4.5: The schematic of the drive electronics and data acquisition system used to characterize the piezohydraulic actuator.

Table 4.2: Comparison of the piezohydraulic actuator with other PZT actuators in terms of DAF

<table>
<thead>
<tr>
<th>Displacement Amplification Techniques</th>
<th>DAF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multi-flexure-hinge based amplifier [30]</td>
<td>20</td>
</tr>
<tr>
<td>Inchworm-type actuator [31]</td>
<td>5.7</td>
</tr>
<tr>
<td>Dual-diaphragm sealed converging nozzle design in this thesis</td>
<td>81</td>
</tr>
</tbody>
</table>

hysteresis of the piezoelectric stack actuator. This behavior is quantified and compared with data using a nonlinear system dynamic model described in Chapter 5.

The piezohydraulic actuator was compared with some other PZT actuators in terms of displacement amplification factor (DAF), which is defined as a ratio of output and input displacements, as seen in Table 4.2.
Figure 4.6: Response of the piezoelectric stack actuator for a sinusoidal input at 1 Hz.
Figure 4.7: Response of the piezohydraulic actuator for a sinusoidal input at 1 Hz.
CHAPTER 5

MODELING

In this chapter, the displacement of the piezohydraulic actuator was modeled to quantify the underlying mechanisms contributing to the device performance. This is implemented using a lumped parameter approach [32] [33]. Nonlinear and hysteretic behavior associated with the PZT stack actuator and rubber diaphragm are integrated into the model and coupled with fluid dynamics behavior in the cylinder head. Each subsystem is described and then formulated as a coupled set of equations to predict piezohydraulic actuation.

The state space model is developed with reference to Figure 5.1. The stack actuator is modeled as a damped oscillator which includes linear piezoelectric coupling and nonlinear and hysteretic ferroelectric behavior. During the application of an applied field, fluid is forced through the cylinder head which is modeled as fluid capacitive elements with flow resistance. The fluid forces the rubber diaphragm to deform which is described by a nonlinear damped oscillator. The nonlinear effective stiffness of the diaphragm is quantified using a hyperelastic constitutive relation. The governing equations describing the piezoelectric stack actuator, fluid coupling, and rubber diaphragm are presented as subsystems and then combined into a matrix of equations for numerical implementation.

5.1 Piezoelectric Stack Actuator Subsystem

The dynamic behavior of the piezoelectric stack actuator/piston subsystem was modeled as a damped oscillator actuated by an applied voltage. Linear piezoelectricity is initially considered. Nonlinear ferroelectric behavior is later incorporated into the model to accommodate minor loop hysteresis shown in Figure 4.6. The stack actuator drives a piston mass that is coupled to the hydraulic fluid as illustrated in the Figure 5.1 schematic. Kelvin-Voigt damping is included in the
Figure 5.1: The electro-fluid-mechanical system dynamic model for the piezohydraulic actuator.

subsystem model to account for internal friction. The resulting equation of motion is

$$m_p\ddot{x}_1 + c_p\dot{x}_1 + k_p x_1 = cV - P_1A_{pi}$$

(5.1)

where $m_p$ is the effective mass of the piston and stack actuator, $c_p$ is the damping coefficient, and $k_p$ is the stiffness of the stack actuator. The forces include piezoelectric coupling given by the coefficient $c$ and voltage input $V$ and the force from the fluid pressure $P_1$ acting over the piston area $A_{pi}$.

The constants $c$ and $k_p$ were derived from the linear constitutive law of the piezoelectric material as previously given by Equation 2.1.

In the uniaxial case considered here, Equation 2.1 reduces to

$$\varepsilon_{33} = s_{3333}^E T_{33} + d_{333} E_3$$

(5.2)

where $\varepsilon_{33}$ is the axial strain component, $T_{33}$ is the axial stress component, $E_3$ is electric field component, $s_{3333}^E$ is the compliance component at fixed electric field, and $d_{333}$ is the piezoelectric
Table 5.1: The stack actuator/piston subsystem parameters.

<table>
<thead>
<tr>
<th>PIEZOELECTRIC STACK ACTUATOR</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>PZT short circuit modulus</td>
<td>80 GPa</td>
</tr>
<tr>
<td>Piezoelectric constant</td>
<td>$525 \times 10^{-12} \text{m/V}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PISTON</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>58 g</td>
</tr>
<tr>
<td>Stiffness</td>
<td>$1.026 \times 10^9 \text{N/m}$</td>
</tr>
<tr>
<td>Damping</td>
<td>$1.026 \times 10^5 \text{N.s/m}$</td>
</tr>
</tbody>
</table>

coefficient. Equation 5.2 is written in terms of force, displacement, and voltage as

$$F = \frac{A_{PZT}E d_{333}}{L} V - \frac{A_{PZT}E L}{x_1} = cV - k_p x_1$$  \hspace{1cm} (5.3)$$

where the constant $c$ is equivalent to $n A_{PZT} d_{333}/E_{3333} L$, with the number of layers of the stack actuator ($n$), its cross-sectional area ($A_{PZT}$), and the total length of the stack actuator ($L$). The spring constant, $k_p$, from Equation 5.1, is equivalent to $A_{PZT}/E_{3333} L$. The equivalent modulus of the stack actuator was estimated using published short circuit modulus of PZT. The effective piezoelectric constant for the stack actuator was obtained by measuring stress free displacement at each electric field as previously given in Table 4.1. The parameter values used in the linear stack actuator model are given in Table 5.1.

5.2 Nonlinear Ferroelectric Behavior

A ferroelectric homogenized energy model was introduced into the system dynamic model and compared to the linear piezoelectric model to quantify its effect on piezohydraulic actuation. Ferroelectric nonlinearities and hysteresis are primarily due to domain wall motion during quasi-static electro-mechanical loading and can lead to minor loop hysteresis at moderate to large field inputs. The introduction of this behavior in the system dynamic model is shown to play an important role in accurately predicting the piezohydraulic actuation previously shown in Figure 4.7.

Equation 5.1 is modified to include ferroelectric switching behavior as a function of polarization

$$m_p \ddot{x}_1 + c_p \dot{x}_1 + k_p x_1 = \ddot{a}_1 \ddot{P}(E) - P_1 A_{pi}$$  \hspace{1cm} (5.4)$$

27
Table 5.2: The homogenized energy parameters.

<table>
<thead>
<tr>
<th>E_c</th>
<th>1MV/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>0.8MV/m</td>
</tr>
<tr>
<td>c</td>
<td>$5 \times 10^3$</td>
</tr>
<tr>
<td>$\tilde{a}_1$</td>
<td>$6.2 \times 10^4$ Nm$^2$/C</td>
</tr>
</tbody>
</table>

where $\tilde{a}_1$ is the effective piezoelectric coefficient, $\tilde{P}$ is the homogenized polarization in the direction of applied field. The homogenized polarization was previously defined in Equation 2.6 and given here for convenience.

$$\tilde{P} = \int_{-\infty}^{\infty} \int_{0}^{\infty} v(E_I, E_c) \tilde{P}(E + E_I, E_c, \xi) \, dE_I \, dE_c$$  \hspace{1cm} (5.5)

A normal/log-normal probability distribution is implemented and denoted by $v(E_I, E_c)$. The form of this distribution is

$$v(E_I, E_c) = Ce^{-E_I^2/2b^2} e^{-[ln(E_c/E_c)]^2}$$  \hspace{1cm} (5.6)

where $b$ is the variance of the normal distribution and $c$ defines the variance of the log-normal distribution with maximum value at $\tilde{E}_c$. The constant $C$ ensures integration to unity. The values for the homogenized energy model are given in Table 5.2. The homogenized energy model is compared to the linear piezoelectric model and stack actuator displacement measurements at 1 Hz as shown in Figure 5.2.

### 5.3 Fluid Subsystem

The lumped parameter fluid subsystem was developed for coupling the fluid with the actuator/piston and rubber diaphragm subsystems. The hydraulic fluid volume was divided into two subdomains to accommodate flow resistance in the cylinder head as the fluid flows through the converging nozzle as shown in Figure 5.3. An effective fluid bulk modulus is defined for the cylinder head and for the second fluid element in contact with the external diaphragm. The governing equations for each fluid element are given as follows. The fluid dynamic relations are based on mass flow rate continuity and the definition of bulk modulus

$$\sum \dot{m}_j = \frac{d(\rho V)}{dt}$$  \hspace{1cm} (5.7)
Figure 5.2: Comparison of the linear piezoelectric model, nonlinear ferroelectric homogenized energy model and stack actuator displacement at 1 Hz.

\[ \beta_f = \rho_0 \frac{dP_i}{d\rho} \]  

(5.8)

where \( \rho_0 \) is the nominal fluid density, \( \dot{m} \) is mass flow, \( V \) is the fluid volume and \( P_i \) is pressure in the cylinder head \((i=1)\) and the fluid element in contact with the rubber diaphragm \((i=2)\). The bulk modulus of the fluid is denoted by \( \beta_f \).

These equations can be combined to obtain a first order dynamic relation governing changes in the pressure in response to a mass flow

\[ \frac{\rho_0 V_0}{\beta_{eff}} \dot{P_i} = C_{eff} \dot{P_i} = \sum \dot{m}_j = \frac{\Delta P}{R_f} \]  

(5.9)

where the fluid capacitance is denoted by \( C_{eff} \) which is proportional to the nominal density and volume of the chamber \( V_0 \) and indirectly proportional to an effective fluid bulk modulus \( \beta_{eff} \). The fluid resistance is denoted by \( R_f \) and is estimated based on losses due to the converging nozzle geometry in the cylinder head.
Figure 5.3: The hydraulic cylinder head and diaphragm configuration. The external diaphragm is in an equilibrium deformed state due to an internal biased pressure.

The fluid resistance is approximated by an inviscid fluid by employing Bernoulli’s equation. Based on an inviscid flow problem, the flow resistance in the cylinder head is determined from the slope of the pressure versus flow rate due to losses in a converging nozzle. This is described by

\[
\Delta P = \frac{\rho Q^2}{2} \left( (1 + K_L) \frac{1}{A_r^2} - \frac{1}{A_{pi}^2} \right)
\]

where the pressure drop is associated with the change in area from the piston face \(A_{pi}\) to the orifice area \(A_r\) and the volume flow rate is denoted by \(Q\). A loss coefficient \(K_L\) of 0.5 was used to model a sharp corner [34].

The flow resistance based on Equation 5.10 is therefore

\[
R_f = \frac{d(\Delta P)}{dQ} = Q \left( (1 + K_L) \frac{1}{A_r^2} - \frac{1}{A_{pi}^2} \right)
\]

The linear dependence on flow rate was found to be negligible in the system dynamic model and was therefore set to a nominal value based on an average flow rate within the operating regime of the device.

The fluid capacitance for the lumped fluid elements in Figure 5.3 is determined based on a nominal density and an effective bulk modulus. The effective bulk modulus of the fluid in the
cylinder head was based on prior modeling of a piezoelectric pump which included experimental measurements of the effective bulk modulus of the fluid system to be 70 MPa using the same silicon oil that was used in these experiments [15]. The published bulk modulus of the fluid is 1150 MPa. The reduced value assumes a level of entrained air remains in the system after vacuuming or compliance in the internal rubber diaphragm seal. The capacitance is also dependent on the volume of the cavity. A nominal volume is used in the cylinder head since the displacement of the stack actuator is small. The fluid volume in contact with the rubber diaphragm is updated on each load step due to finite deformation of the top diaphragm.

The two equations describing the fluid behavior in each lumped fluid element was obtained from the continuity equation

\[ C_1 \dot{P}_1 = \dot{m}_1 - \dot{m}_2 = \dot{m}_1 - \frac{P_1 - P_2}{R}, \quad (5.12) \]

\[ C_2(x_2) \dot{P}_2 = \dot{m}_2 - \rho A_r \dot{x}_2 = \frac{P_1 - P_2}{R} - \rho A_r \dot{x}_2, \quad (5.13) \]

\[ C_2(x_2) = \rho (A_r x_2 + V_0) / \beta, \quad (5.14) \]

where \( x_2 \) corresponds to radial deformation of the rubber diaphragm as described in the following section. This gives a change in fluid capacitance as the volume changes. \( C_2 \) is expressed in terms of \( x_2 \) in Equation 5.14, where \( V_0 \) is the nominal volume of the fluid element in contact with the external diaphragm. The volume of the fluid changes with a shape of a circular cylinder by approximation. The mass flow rate \( \dot{m}_1 \) is determined by the velocity of the piston/stack actuator assembly.

### 5.4 Rubber Diaphragm Subsystem

The piezohydraulic actuator exhibited significant departures from linearity which is partially due to a minor loop hysteresis of the stack actuator. However, this does not fully explain the sharp anhysteretic behavior exhibited by the rubber diaphragm displacement. Therefore, finite deformation of the external rubber diaphragm was investigated. A hyperelastic constitutive law was integrated into the system dynamic model to incorporate this behavior into the model.

The rubber diaphragm subsystem was modeled as a second-order mass-spring-damper with a driving force provided by the pressure of the hydraulic fluid. The governing equation is

\[ m_r \ddot{x}_2 + c_r \dot{x}_2 + k_r(x_2)x_2 = (P_2 - P_0)A_r, \quad (5.15) \]
which includes an effective mass \((m_r)\) with nonlinear stiffness \((k_r(x_2))\) and a linear Kelvin-Voigt damping coefficient \((c_r)\). The displacement was produced by the pressure \((P_2)\) of the hydraulic fluid applied to the rubber area \((A_r)\). A second force was generated by the external ambient pressure \((P_0)\).

Hyperelastic constitutive behavior is incorporated into the model using Ogden’s model \([8]\) to simulate the rubber diaphragm pressure versus stretch constitutive behavior. The deformation is approximated by assuming a spherical shape and uniform radial deformation. Using this approximation as described in \([8]\) pp. 239-242, the constitutive relation between inflation pressure and circumferential stretch is

\[
P_2 = 2\frac{H}{R} \sum_{p=1}^{3} \mu_p (\lambda^{\alpha_p-3} - \lambda^{-2\alpha_p-3})
\]

where \(\mu_p\) are shear moduli and \(\alpha_p\) are dimensionless constants. The initial thickness of the rubber diaphragm is \(H\), the initial radius (zero-pressure) of the rubber diaphragm is \(R\), and \(\lambda\) is the circumferential stretch which is defined by \(\lambda=1+x_2/R\). Thus, \(x_2\) is the radial displacement from the initial radius \(R\). The values of the constants were obtained by fitting to the piezohydraulic actuator results; see Table 5.3. In order to compare to the theoretical value, the shear modulus of the rubber material in this model was calculated by

\[
2\mu = \sum_{p=1}^{3} \alpha_p \mu_p
\]

According to the magnitudes in Table 5.3, the shear modulus is 0.533 MPa, which is comparable to the theoretical value of 0.6 MPa when Young’s modulus and Poisson’s ratio are 1.7 MPa and 0.5, respectively.

The set of parameters in Table 5.3 give rise to the inflation pressure versus stress behavior illustrated in Figure 5.4. The constitutive response has been plotted for a range of stretch values corresponding to the deformation predicted to occur in the piezohydraulic actuator.

The tangential stiffness is implemented in the system dynamic model based on Equation 5.16. This stiffness is

\[
k_r(\lambda) = \frac{dP_2}{d\lambda} \bigg|_{\lambda_0} \frac{A_r}{R}
\]

about an operating point \(\lambda_0\). The stiffness can be written as a function of the displacement \(x_2\) using the relation \(\lambda=1+x_2/R\). The nonlinear stiffness of the rubber diaphragm corresponds to the inflation pressure versus stretch plot in Figure 5.4. The rubber diaphragm starts to become unstable as the
Table 5.3: The rubber diaphragm subsystem parameters

<table>
<thead>
<tr>
<th>RUBBER</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>2.1 mm</td>
</tr>
<tr>
<td>Thickness</td>
<td>0.6 mm</td>
</tr>
<tr>
<td>Density</td>
<td>1346 kg/m³</td>
</tr>
<tr>
<td>Damping</td>
<td>0.603 N.s/m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DIMENSIONLESS CONSTANTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>α₁</td>
</tr>
<tr>
<td>α₂</td>
</tr>
<tr>
<td>α₃</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SHEAR MODULI</th>
</tr>
</thead>
<tbody>
<tr>
<td>µ₁</td>
</tr>
<tr>
<td>µ₂</td>
</tr>
<tr>
<td>µ₃</td>
</tr>
</tbody>
</table>

stretch becomes 1.13 where the stiffness approaches zero. The system bandwidth corresponding to the initial stiffness in the model (\(k_r=12062\) N/m) of the rubber diaphragm is illustrated in Figure 5.5. From Figure 5.5, the bandwidth is approximately 1kHz. Dynamic response of the nonlinear system is given after comparing the model to quasi-static measurements.

5.5 Overall Subsystem

The fully coupled system dynamic model of the piezohydraulic actuator system consist of the equations of motion for the piezoelectric stack actuator/piston, fluid in the cylinder head, and rubber diaphragm subsystems. By utilizing Equations 5.1, 5.12, 5.13 and 5.15, the state space model was developed as a sixth order system using the linear piezoelectric constitutive relation. The ferroelectric model is developed by replacing Equation 5.1 with the homogenized energy model given by Equation 5.4. The second-order differential equations for the stack actuator and rubber are written as two first-order differential equations for convenience in simultaneously solving the system of equations. The system of differential equations is written in state space form as

\[
\dot{x}(t) = A(x)x(t) + [B(u)](t)
\]  

(5.19)
Figure 5.4: Inflation pressure $P_2$ versus stretch $\lambda$ based on parameters in Table 5.3 and the hyperelastic constitutive law given by Equation 5.16.

\[ y(t) = Cx(t) \quad (5.20) \]

and subjected to a set of initial conditions $x(0) = x_0$. The state vector $(x)$ includes the displacements and velocities of the stack actuator and rubber diaphragm and pressures within the cylinder head. The matrix $A(x)$ is a function of the rubber diaphragm displacement due to the finite deformation of the external rubber diaphragm. The input operator $(B(u))$ is written as a nonlinear function of polarization when the ferroelectric homogenized energy model is implemented. The form of these expressions is given in the Appendix B.

Due to the nonlinearity of the differential equations, the system of equations is solved numer-
Figure 5.5: The range of the system bandwidth according to the range of the nonlinear rubber diaphragm stiffness.

Numerically using a temporal discretization at time steps denoted by \( i=1,\ldots,N \) and time step \( \Delta t \). The solution of the state equations at the end of each time step is used as the initial condition for the next iteration. Using the central difference method, the discrete form of Equation 5.19 is

\[
x_{i+1} = \left( I - \frac{\Delta t}{2} A_{i+1} \right)^{-1} \left[ \left( I + \frac{\Delta t}{2} A_i \right) x_i + \Delta t B(u_i) \right]
\]

where the subscripts denote each time step defined over the time interval \([t_0, t_f]\) with uniform mesh having a size \( \Delta t \) at points \( t_0, t_1, \ldots, t_N = t_f \). Note that this is an approximate central difference algorithm since the state matrix \( A_{i+1} \) would normally enter into the equation inside the inverse operator; however, it is not known \textit{a priori}.

The system level performance is simulated by comparing linear piezoelectric model to nonlin-
Figure 5.6: System dynamic model comparisons of the piezohydraulic actuator at 1 Hz. Minor loop ferroelectric hysteresis of the stack actuator is shown to affect piezohydraulic actuation.

ear ferroelectric model. All other parameters were the same. As shown in Figure 5.6, significant differences in piezohydraulic actuation is predicted when minor loop ferroelectric hysteresis is incorporated into the model. This is due to the hydraulic amplification of the device which amplifies the minor loop ferroelectric stack actuator displacement. Smaller minor loop hysteresis is predicted when linear piezoelectricity is modeled which is most likely due to snap buckling of the diaphragm as discussed in the following section. Good estimates are predicted by the ferroelectric model at 2 MV/m; however, the model did not predict lower voltages as well. The differences in model predictions and experimental results are discussed in the following section.

The actuator response presented in Figure 5.6 includes an internal bias pressure of 406 kPa and an external ambient pressure of 0 kPa. When the actuator is integrated within a microjet system, piezohydraulic actuation must be able to work against external pressure to control microjet flow. This is typically 207 kPa (30 psia). Simulations using the nonlinear ferroelectric model predict sufficient robustness to control flow in the presence of ambient pressure. This is illustrated in
Figure 5.7: Simulations using the nonlinear ferroelectric system dynamic model with different external pressures. This simulates the effect of static pressure applied to the microjet.

Figure 5.7 by comparing zero ambient pressure to an external ambient pressure of 690 kPa (100 psia). Slight variations in the hysteresis are shown, although the same maximum displacement amplification is achieved.

### 5.6 Dynamic Model Predictions

The nonlinear ferroelectric model (homogenized energy model) was utilized to predict the dynamic behavior of the piezoelectric stack actuator and rubber diaphragm. A comparison of the piezoelectric stack actuator and nonlinear ferroelectric model at 10Hz, 200Hz, and 300Hz is illustrated in Figure 5.8. Furthermore, the nonlinear ferroelectric model of the diaphragm response at high frequencies is illustrated in Figure 5.9. From Figure 5.8, the minor loop hysteresis becomes larger and the displacements of the stack actuator become smaller as the driving frequency increases. But the overall performance of the stack actuator exhibits broad bandwidth. As illustrated from Figure 5.9, the piezohydraulic actuator shows robustness under high frequencies.
Table 5.4: The constitutive model parameters used in the homogenized energy model. See [4] for definitions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_c$</td>
<td>1 MV/m</td>
</tr>
<tr>
<td>$b$</td>
<td>0.8 MV/m</td>
</tr>
<tr>
<td>$c$</td>
<td>0.5 MV/m</td>
</tr>
<tr>
<td>$\tilde{a}_1$</td>
<td>$6.0 \times 10^4$ N$m^2$/C</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$1.0 \times 10^7$</td>
</tr>
<tr>
<td>$P_r$</td>
<td>0.4 C/m$^2$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.10 ms$^{-2}$/kg·K</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.02 sec</td>
</tr>
</tbody>
</table>

Figure 5.8: Comparison of the piezoelectric stack actuator and nonlinear ferroelectric model for different frequencies.
Figure 5.9: Nonlinear ferroelectric model of the diaphragm response at high frequencies.
CHAPTER 6
DISCUSSION

The piezohydraulic actuator has provided significant displacement amplification necessary to throttle a microjet with a 400 µm diameter. Key contributions to piezohydraulic actuation have been identified by formulating a system dynamic model that includes nonlinear and hysteretic electro-fluid-mechanical coupling. The underlying mechanisms were determined to be strongly related to the nonlinear and hysteretic response of the piezoelectric stack actuator and finite deformation of the external rubber diaphragm.

The piezohydraulic actuation of this device is believed to be partially related to minor loop hysteresis of the piezoelectric stack actuator. Whereas the hydraulic amplification increases actuation, it also amplifies the ferroelectric hysteresis shown in Figure 5.2. However, broadband nonlinear control designs can be implemented to mitigate this behavior [35] [36]. Furthermore, single crystal ferroelectric relaxor stack actuators which exhibit smaller hysteresis [37] under unipolar fields may also improve open loop performance.

The modeling results also illustrate that the rubber constitutive behavior is critical to achieve significant displacement amplification. The constitutive behavior of the rubber illustrated in Figure 5.4 shows a critical stretch at approximate 1.13 where the material becomes unstable to further increases in pressure. This creates a dynamic snap phenomenon which leads to large deformation to another equilibrium stretch of approximately 1.20. This “snap buckling” behavior gives rise to hysteresis and large deformation independent of the minor loop piezoelectric stack actuator hysteresis. The model predictions and experimental results compare well for a field input of 2 MV/m using this constitutive model; however, lower input fields were not able to accurately predict the experimental results. This is most likely due to the hyperelastic constitutive equation or the approximation of the deformation as an ideal spherical shape change. In the device, the rubber diaphragm was constrained to deform up a cylindrical cavity. More work is required to resolve
these issues.

It should also be noted that the reliability of the rubber material will be important for pulsed actuation where pulsed frequencies on the order of 100-1000 Hz are desired for bench top microflow experiments and scaled wind tunnel experiments. Whereas preliminary calculations using the system dynamic model predict a bandwidth of approximately 1 kHz, the deformation of the rubber diaphragm will introduced significant reliability challenges for the rubber material. However, the microjet orifice is 400 µm; therefore, smaller deformation will be sufficient for flow control applications which may reduce mechanical fatigue.

The device presented here is expected to provide an important laboratory testing device for integration into bench top and wind tunnel microjet experiments to elucidate how pulsed actuation interacts with different aircraft control surfaces. Many of these problems are still poorly understood in the subsonic and supersonic flow regimes. New knowledge on broadband pulsed flow actuation is anticipated to provide important microflow information for designing next generation adaptive flow control actuators for enhanced separation control and noise reduction.

6.1 Recommendations

On the next step for this research, the bench-top flow and wind tunnel tests need to be conducted using the piezohydraulic actuator. The microjet interface needs to be modified for fluid visualization and outlet flow field measurements by an acoustic probe. The distance from the point at which the diaphragm controls the fluid to the outlet is critical, which determines the resistance of the channel and can dampen the pulsed flow actuation. Meanwhile, the initial deformation of the rubber under optimum biased pressure should not block microjet flow. Furthermore, the microjet interface should be able to clamp the rubber diaphragm on top of the cylinder head tightly to avoid slipping out during operation.
CHAPTER 7

CONCLUSION

A piezohydraulic actuator has been designed, characterized, and modeled for future integration into a microjet flow control system. Significant displacement amplification has been achieved sufficient to throttle a microjet actuator. The device presented here is expected to provide an important testing device for integration into bench top and wind tunnel microjet experiments to elucidate how micropulsed actuation interacts with different aircraft control surfaces. Many of these problems are still poorly understood in the subsonic and supersonic flow regimes. New knowledge on broadband pulsed flow actuation is anticipated to provide important microflow information for designing next generation adaptive flow control actuators for enhanced separation control and noise reduction relevant to a number of aircraft control surfaces such as jet inlets, cavity bays, rotor blades, and impinging jets.
Consider the Gibbs energy relation

\[ G_E = \psi_E - EP \]  \hspace{1cm} (A.1)

which is a function of the independent variable \( E \) and dependent variable \( P \). To quantify the dependence of \( E \) on \( P \), and hence formulate \( G \) solely in terms of \( E \), Legendre transform analysis is considered. Legendre transforms map functions in a vector space to functions in the dual space. Within the realm of model development for smart systems, Legendre transforms are employed when defining thermodynamic potentials and establishing the correspondence between Lagrangian and Hamiltonian frameworks. Legendre transform maps convex functions to convex functions and is self-dual or an involution.
APPENDIX B

THE EXPRESSIONS IN THE STATE SPACE FORM

The expressions of $x(t)$, $A(X)$ and $B(u)$ in Equation 5.19 for the state space model are given as follows.

$$x(t) = \begin{pmatrix} x_1 \\ \dot{x}_1 \\ P_1 \\ P_2 \\ x_2 \\ \dot{x}_2 \end{pmatrix}$$  \hspace{1cm} (B.1)

$$A(x) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -k_p/m_p & -c_p/m_p & -\lambda_{pi}/m_p & 0 & 0 & 0 \\ 0 & \rho_\psi A_{pi} / c_1 & -1/C_f K_f & 1/C_f K_f & 0 & 0 \\ 0 & 0 & 1/C_2(x_2) K_f & -1/C_2(x_2) K_f & 0 & -\rho_0 A_f/C_2(x_2) \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \lambda_r/m_r & -k_r/m_r & -c_r/m_r \end{pmatrix}$$ \hspace{1cm} (B.2)

$$B(u) = \begin{pmatrix} 0 \\ c_1/m_p \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} V + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\lambda_r/m_r \end{pmatrix} P_0$$ \hspace{1cm} (B.3)

In the case of the ferroelectric model, the voltage input is replaced by a field input which is a nonlinear function of the polarization

$$B(u) = \begin{pmatrix} 0 \\ \tilde{a}_1/m_p \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tilde{P}(E) + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \lambda_r/m_r \end{pmatrix} P_0$$ \hspace{1cm} (B.4)
REFERENCES


BIOGRAPHICAL SKETCH

Fei Liu

Was born on September 27, 1983 in Hubei, China. He graduated from E-nan High School in June of 2001. Following high school he enrolled at Huazhong University of Science and Technology (HUST), China in September of 2001. In June of 2005, he completed his Bachelors degree in Naval Architecture and Ocean Engineering and Bachelors degree in Optoelectronic Engineering. He was accepted into the graduate Mechanical Engineering program in Florida State University in Spring 2007.