Mitigation of Vortex Induced Response in Long Span Bridges

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MITIGATION OF VORTEX INDUCED RESPONSE IN LONG SPAN BRIDGES

By

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I dedicate this manuscript to my parents and my guru, for everything, they bestowed on me.
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Span length of bridges is ever increasing. With the increase in span, bridges are becoming more slender, making them more prone to wind-induced forces and vibrations. Some bridges show significant wind-induced vibrations after the construction, which were not anticipated during the design. In order to improve the performance of these bridges, two strategies of retrofit are commonly used. First, tuned mass dampers (TMDs) may be added to dissipate the energy. Second, cross-section of the bridge may be retrofitted so that it is aerodynamically more favorable. Both methods are effective in reducing the magnitude of the wind-induced vibration. Among many types of wind-induced vibrations, this thesis deals with vortex-induced vibrations. The various parameters involved in retrofit have conflicting objective. For example if the maximum displacement is required to be reduced the cost involved is more. In such a case of conflicting objectives, multi-objective optimization can be used to find various solutions from the solution space. These solutions are termed as Pareto-optimal solutions. There is no specific solution which can be called as the best solution due to conflicting objectives. Multi-objective optimization gives the various options to choose from, to arrive at a decision for a particular real world problem. In our case, the problem is to decrease the magnitude of vortex-induced vibrations. The conflicting objectives are the cost of the retrofit and the displacement in terms of the performance. In thesis, the multi-objective optimization will be used to obtain various strategies of retrofit under conflicting objectives. The approach is illustrated with an example bridge modeled after Rio-Niterói Bridge.
CHAPTER 1

INTRODUCTION

The maximum span lengths of the bridges have been extended in the past few decades. The Akashi-Kaikyo Bridge has central span 1991 m. Even longer bridges like Messina Straits (3300 m), Gibraltar Straits (5000 m) are planned [1]. Aerodynamic instability increases with increase in unsupported span length. Therefore, the success of such long span bridges depends on efficient wind resistant design. There are some bridges constructed few decades back, like Bronx-Whitestone Bridge [13], Rio-Niteroi Bridge [7] have shown significant wind-induced oscillations which were not anticipated during design phase.

Wind direction and the behavior of the bridge deck under the wind forces set the bridge into four major aerodynamic responses such as flutter, galloping, buffeting and vortex shedding. The type of behavior of a bridge deck can be exhibited exclusively or a bridge deck may exhibit more than one type of responses at the same time. Tacoma Narrow Bridge is an example that failed due to the flutter at only 43 mph wind. Trans-Tokyo Crossing Bridge in Japan has exhibited the combined vortex and galloping during wind tunnel test on two dimensional sectional models. Rio-Niterói Bridge in Rio de Janeiro has exhibited the vortex shedding at wind speed as low as 14 m/s [4]. Wind tunnel test performed on the sectional model of Trans-Tokyo Bay Crossing Bridge has demonstrated that both vortex-induced vibration and galloping would occur in bridge girder at the same time [6]. This coupling of different types of response leads to the complexity in analysis.

The solution to attenuate the aerodynamic response of a bridge deck lies in retrofitting the cross section and thereby improving its’ aerodynamic performance. A section with better aerodynamic performance reduces the load to the bridge, enabling reduced response even with the similar stiffness. Another approach in retrofit is to increase the stiffness in desired direction without necessarily changing the aerodynamic performance. The cross sectional retrofit has limitations in terms the structural modification and overall weight of a bridge structure. Sometimes structural modification may not have significant effect, in improving the aerodynamic performance of a
bridge. In such cases, along with the cross sectional retrofit, energy absorbing units called as dampers are used.

In retrofitting method some steel members are added to steel girder of the bridge deck in stiffen the cross section. Aerodynamic appendages are attached in order to decrease the aerodynamic response. These aerodynamic appendages are in the form of steel plates, flaps and fairings which modify the ambient flow of wind around the bridge deck. This modification of wind flow decreases force exerted on the bridge deck and thereby stability is achieved. It is found that, in case of some bridges complete stability cannot be achieved even after selecting aerodynamically favorable cross section and adding aerodynamic appendages. This is exhibited in Trans-Tokyo Bay Crossing Bridge in Japan. Various mitigation strategies were tested, to suppress the vortex-induced vibration exhibited during wind tunnel test, including various combinations of aerodynamic appendages. As a result, tuned mass dampers were installed in the box girder of bridge deck after completion of bridge.

The main focus of this research work is on study and analysis of vortex induced oscillation and mitigating the response with tuned mass dampers. The number of dampers and their locations are decided by studying the structural, numerical, mathematical modeling, and comparing the results with the wind tunnel testing on a simplified model of the bridge structure.
CHAPTER 2

TYPES OF AERODYNAMIC RESPONSES

Depending on the direction of wind, excitation and behavior of a bridge deck, the aerodynamic response of a bridge is generally classified into four different types as Flutter, Galloping Buffeting and Vortex induced vibration.

2.1 Flutter:

Flutter is the structural dynamic instability resulting from interaction of structure and flow of air in which violent vibration of structure occurs with rapidly increasing amplitude. Flutter occurs when the torsional and flexural stiffness are close in magnitude. In this type of aerodynamic instability, simultaneous torsional and vertical vibrational modes are active. Flutter can occur due to single or two coupled modes. In case of Akashi-Kaikyo Bridge in Japan, coupling observed between a pair of modes during the design [1]. In practical situation aeroelastically influenced flutter is dominated by two modes, if it is not induced by single mode negative damping. In multimode flutter problem, the lowest solution for natural circular frequency gives flutter critical condition. The mode corresponding to the solution of natural circular frequency is the predominant mode in flutter condition.

In order to analyze and study a flutter problem, wind tunnel testing, mathematical and structural model is generally used. Since bridge flutter is generally dominated by fundamental modes, the lowest vertical and torsional modes are used for structural modeling. Apart from streamlining the deck section, the bridge flutter can be decreased by increasing the structural stiffness by choosing torsionaly stiff structure. This approach leads to increased weight and cost.

The bridge flutter can be suppressed by using tuned mass dampers (TMD). The performance of TMD can be judged on the basis of minimum flutter velocity. The research conducted on Golden Gate Bridge has established the fact that the minimum flutter velocity increases with the mass to moment ratio [2]. In case Golden Gate Bridge flutter onset velocity was as low as 22.9 m/s
during the design stage. The flutter velocity of the Seohae Bridge in Korea is almost 56.7 m/s which is more than the design velocity. It is established that the additional margin in flutter velocity can be obtained using TMDs [2].

2.2 Galloping:

Galloping is the structural dynamic instability due to high amplitude oscillation resulting due to wind, harmonic loading, exclusively in vertical mode, if the torsional stiffness is much larger than the flexural stiffness. In this type of aerodynamic instability the oscillation are perpendicular to wind direction. For structures with low damping such as suspension bridges, galloping can occur at low, steady or unsteady wind speed. In case of suspension bridges, galloping is attributed to galloping of the suspended cable and bridge deck.

Changing the cross section and size can reduce the galloping effect. Passive or semi-active tuned mass dampers suspended from the deck can reduce this non-linear vibration. Attempts to introduce passive or semi-active damping into stay cable of suspension bridge were made by various researchers. It has been proved that, damping is very effective factor in increasing the critical wind speed at which galloping occurs.

2.3 Buffeting:

Buffeting is the structural dynamic instability resulting from the fluctuations in velocity of oncoming wind. Wind turbulence and self excited vibration which depends on the motion of the deck, sets bridge deck in buffeting response. For small span length bridge, buffeting response does not lead to catastrophic failure. However, with the increase in span length, buffeting response increases significantly which may lead to serious fatigue damage to structural components and connections, instability of vehicles and pedestrian [3]. Scanlan and Gade [14] extended their research results for flutter instability to buffeting response. They believed that self excited forces would affect the buffeting response of the bridge deck and hence suggested considering the aerodynamic forces due to wind turbulence and the aeroelastic forces due to motion of the deck together in buffeting analysis. Theories given by Scanlan and Davenport as
referred by the author of [3], are the combination of numerical, experimental and analytical approaches. In modern long span bridges the natural frequency and mode shapes are determined by finite element method. In the development of the buffeting theory, it is assumed that the load may be calculated from the instantaneous velocity pressure and the appropriate load coefficient can be obtained from the static tests.

2.4 Vortex induced vibration:

Vortex shedding is the structural dynamic instability resulting from the separation of air flow on the surface of a structure causing vortices to shed alternatively on either side of the structure. Refer figure 2.1, 2.2 and 2.3. The vortices formed give rise to fluctuating across wind forces, cross sectional torsional moments, accompanied by fluctuating vertical or rotational displacements. When Strouhal vortex frequency is close to natural frequency, the vortex shedding is controlled by the structure at the natural frequency called as lock-in frequency. This results in large resonant vibrations with amplitude being at maximum.

Vortex induced vibrations in a structure are attributed to low damping and slenderness of the structure. This fact is confirmed in case of Rio-Niteroi Bridge. The bridge exhibited the vortex-oscillations at low cross wind velocity. The amplitude of vibration depends on the wind direction. This fact is corroborated by the study done on Trans-Tokyo Crossing Bridge by the author of [6]. The data was collected for over seven month period before installation of TMD. Large vortex-induced vibrations occurred by the wind within 20° on either side of transverse axis of bridge.

Vortex induced oscillations can be reduced by choosing the appropriate cross sectional shape for bridge deck [4]. Vortex induced vibrations in bridge girder can be controlled either by aerodynamic or mechanical measures. In aerodynamic measures flaps and fairings are attached to modify the flow, thereby decreasing the vibrations in bridge deck. In mechanical measure tuned mass dampers are used to suppress the vortex induced vibrations. In order to assess the vortex induced oscillations of a bridge deck, numerical, mathematical simulations are used.
These simulation results are normally cross checked with wind tunnel test. In the analysis of vortex shedding, wind speed along the bridge is disregarded and only cross wind is considered.

Figure 2.1: Kármán vortex street off the coast of Rishiri Island in Japan [16]

Figure 2.2: Vortices behind the circular cylinder [17]

Figure 2.3: Schematic representation of vortices around bridge deck [4]
CHAPTER 3

VOXER-INDUCED VIBRATION IN LONG SPAN BRIDGES

3.1 Introduction:

When a wind passes over a bluff body, flow separation occurs. Alternate vortices are then formed on either surface of slender structures like bridge deck, resulting in the formation of low pressure zones. Due to asymmetric nature of these vortices, different lift forces develop on each side of structure. This results in a motion transverse to the flow of wind. If the frequency of vortex shedding matches the resonance frequency of the structure, the structure will begin to resonate and the structures’ movement becomes self sustaining. This self sustaining movement in-turn changes the flow of wind around the structure. This alternate motion induces fatigue stresses in the components of bridge deck which can prove fatal to the overall safety of the bridge if neglected. Vortex-induced vibrations in modes higher than the second mode are found to have little effect on the fatigue limit state because they occur rarely. However, when such vibration does occur, the response amplitude of the girder is relatively large and locally induced stresses exceed the yield stress of the girder [5]. Even if the response amplitude is not deterrent to the bridge structure, it may render the bridge unserviceable for its’ intended purpose. Being a one of the major sources of wind-induced vibrations, the objective of this thesis is, to study and analyze the vortex induced vibrations, which play vital role in dynamic analysis of suspension bridge.

3.2 Case Studies:

Several bridges have exhibited aerodynamic instability resulting due to vortex shedding that was not anticipated during the design and construction phase. Few decades back, it was difficult to predict and estimate the exact behavior of bridge deck under influence of fluctuating wind-induced force.
3.2.1 Great Belt Bridge:

During the final phase of deck erection and surfacing of suspended spans of the Great Belt East Bridge, vertical oscillations with low frequency were observed. On site monitoring and wind speed measurement were cross checked with the wind tunnel test results. The observed oscillations were attributed to periodic von Kármán type vortex shedding in bridge girder. Though the large scale amplitude were found to be harmless from structural point of view, they were unacceptable considering possible physiological impact on users of the bridge [5]. The bridge is a three span box girder suspension bridge with 2 end spans 535 m long and central span 1624 m long. The bridge carries the 4 lane motorway across the international shipping route of the Great Belt, Denmark. Study has shown that the bridge deck was set to oscillations at moderate wind speed of 5 m/s – 10 m/s when wind was flowing perpendicular to the span of bridge. This behavior was confirmed in wind tunnel testing of bridge during design of bridge.

To monitor the structure under the influence of wind force, the system designed with accelerometers to trigger when the main span encountered vertical oscillations exceeding an rms level of 0.035m/s² sustained for period more than 5 minutes. Total of 287 events consisting of one or more 50 minutes of vibration recording were obtained. Total of 89 events were established to be due to wind. 10 vibrations events were recorded for wind direction perpendicular to the bridge span. The wind speed was recorded between 4 m/s – 12 m/s with large amplitude harmonic oscillations associated with the vertical mode. The time trace presented in [5] indicates the vortex shedding excitation almost sinusoidal with single well defined eigen modes.

To mitigate the vortex induced oscillations in bridge deck, guide vanes were attached to the bridge deck to make the cross section of bridge deck aerodynamically more favorable. The guide vanes composed of bent plate attached to the main span girder along the bottom plate. For design purpose, an empirical one-degree of freedom system oscillator model was applied to wind-tunnel for prediction of full scale bridge response. The results have shown that the maximum deflection of section model decreases with the damping ratio [5].
3.2.2 Trans-Tokyo Bridge:

Trans-Tokyo Bay Highway Crossing [6], completed in 1997, is 11 km in total length. Total route includes ten span continuous steel box girders with total length of 1630 m. The two longest spans of the bridge measure 240 m. In this bridge, significant wind-induced vortex oscillations were observed with peak wind velocity of 16-17 m/s in the direction perpendicular to the length of bridge. The maximum amplitude to the bridge deck was set, noted to be exceeding 50 cm.

In the design phase of bridge, various wind tunnel tests were carried out on two-dimensional as well as three dimensional sectional models. Complete mitigation of vibration was not achieved even after testing various aerodynamic strategies. As a result, consideration was given to the addition of tuned mass dampers (TMDs) to the bridge deck after completion. In the design process, wind tunnel tests were performed on the sectional and full bridge models in uniform and turbulent flows. To bring the vortex induced oscillations due to first and second mode vibration, to acceptable limit 16 TMDs were installed. Vibrations due to higher modes were controlled by attaching vertical plates [6].

In two dimensional wind tunnel tests on sectional model without aerodynamic measures, the maximum amplitude of vertical flexural-induced vibrations was 34 cm and the vortex vibration occurred at wind speed of 35 – 40 m/s. The critical wind speed for vortex to occur was 67.7 m/s [6]. When appendages like fairings, double flaps, skirt and their combinations were used the amplitude was about 18 to 40 cm for model studied in [6] which was more than designed value of 10 cm. In three dimensional tests varying depth of girder was taken in account by authors of [6]. The aerodynamic response due to vortex shedding was not improved and vortex-induced oscillations were noted at velocity much below the designed wind velocity 67.7 m/s.

To suppress the vortex oscillations, tuned mass dampers (TMDs) for this bridge were designed to accommodate inside the box girder. Two degree of freedom system was modeled to design the TMDs. TMDs for the first and second modes were designed independently. The parameters like target damping, variation of natural frequency, mass ratio, variation of mass of bridge were assumed based on the field test and the requirements to design the TMDs for the first and second modes separately [6]. After the addition of TMDs the maximum acceleration reduced to 25.4 L
from 191 L and displacement amplitude reduced to 5.4 cm from 40.7 cm [6]. Phase lag of 90° shown in time history analysis indicates the motion of TMDs effectively acting as damping force. To control the aerodynamic vibration due to higher modes the various wind tunnel tests were performed on complete bridge model. The amplitude in the third mode was reduced to about 80% and that in the fourth mode reduced to 69% [6].

3.2.3 Rio-Niterói Bridge:

Rio-Niterói Bridge [7] in Rio de Janeiro, brought into service in March 1974. Length of the bridge is 13.3 km, spans across the Guanabara bay in Rio de Janeiro. Bridge is prestressed concrete structure except its three central spans made of twin steel box girders. Average girder height to span ratio is 1/45. Whenever cross wind velocity reaches 14 m/s, this lightly damped structure exhibits vortex-induced oscillations in first vertical bending mode with large amplitude oscillations in frequency lock-in state. Heavy traffic traversing the link spans’ expansion joint also set the bridge in the same vertical mode as wind-induced vortex shedding. Adding aerodynamic appendages would have increased the overall weight of this heavily loaded bridge. Addition of control devices was considered as the appropriate approach for controlling the aerodynamic vibrations. Mathematical 3D FEM model was calibrated in terms of experimentally measured frequencies and associated oscillation modes for analyzing the performance with active and passive controls. From the obtained controlled response, as compared to the uncontrolled response of the original structure it has been established that active control has a better performance than the passive control in reducing and controlling the amplitude of vortex-induced oscillations [7].

3.3 Analysis of Vortex-Induced Vibration:

The Vortex-Induced aerodynamic instability of a suspension bridge can be analyzed by considering a continuous line-like structure. For analysis, wind flowing in the direction perpendicular to the length of bridge is considered. The effects arising from the wind flowing along the length of bridge is disregarded. The wind flowing across the bridge deck gives rise to the alternating vortices on either surface of the bridge deck. These vortices in turn imparts the
across wind forces ($q_v$) and cross sectional torsion moment ($q_\theta$) in the section under consideration. These forces are accompanied by fluctuating vertical displacement ($r_z$) and the rotational displacement ($r_\theta$). The vortex shedding frequency ($f_s$) is the frequency of the vortices generated due to across wind. The vortex shedding frequency ($f_s$) is the function of cross-sectional characteristic of the bridge deck and the constant, called as Strouhal number ($S_t$). The equation is for shedding frequency is

$$f_s = S_t \frac{V}{D}$$

(3.1)

where

- $V =$ Across wind velocity
- $D =$ Depth of bridge girder

When frequency of vortex-induced vibration becomes equal to the eigen frequency of the structure, a state called as resonance occurs. The vibrations at the resonance are detrimental to the safety and stability of the structure. As from the dynamics, we know that there is an eigen frequency associated with a mode of vibration and there can be more than one mode of vibration associated with the structural system. The frequency of vibration increases with the increase in velocity of impounding wind. For next eigen frequency there is an associated resonant frequency. At resonance, shedding frequency stay close to eigen frequency for certain increase in velocity. This is called as lock-in. At lock-in two load effects are more predominant. Fluctuating loads are co-related to span wise direction and the significant motion induced part is added. It implies that the load is not load induced at lock-in does not depend on the velocity or frequency of vibration. But when the fluctuations becomes large these effects starts decreasing. The total load induced in the structural element and response of the structure can be ascribed the combination of net motion independent part and the motion induced part. The motion induced part of the load is due to the structural displacement and the velocity.

Author of [10] referred the theory, developed by Vickery and Basu, to describe the net motion independent cross sectional load spectra. Mathematically it is given as follows.
\[
\begin{bmatrix}
S_{qz}(\omega) \\
S_{q\theta}(\omega)
\end{bmatrix} = \frac{(1/2)\rho V^2}{\sqrt{\pi} \omega_s} \left[ \frac{(b \partial \sigma_{qz})^2}{b_x} e^{-\left(1 - \frac{1}{b_x^2}\right)^2} \right] \left[ \frac{(b^2 \sigma_{q\theta})^2}{b \theta} e^{-\left(1 - \frac{1}{b \theta^2}\right)^2} \right]
\]

(3.2)

\(S_{qz}\) is the spectra for across wind forces and \(S_{q\theta}\) is the spectra for cross sectional torsion moment.

The corresponding co-spectra is given as

\[
\hat{C} \sigma_{q_m}(\Delta x) = \cos \left(\frac{2 \Delta x}{3 \lambda_m D}\right) e^{-\left(\frac{\Delta x}{3 \lambda_m D}\right)^2}
\]

(3.3)

Where

- \(m = z\) for vertical displacement
- \(m = \theta\) for rotational displacement
- \(\sigma_{q_m}\) = non-dimensional root mean square lift or torsional coefficient
- \(b_m\) = non-dimensional load spectrum band width parameter
- \(\lambda_m\) = non-dimensional coherence length scale
- \(\Delta x\) = span-wise separation.

Lift and torsional moment coefficient \((\sigma_{q_m})\) increases with the bluffness of the structure. \(bz\) attains the value between 0.1 and 0.3 and \(\lambda_m\) has values between 2 to 5.

The motion induced part of the load is attributed to the lock-in condition at eigen frequencies. The total damping available can be thought of algebraic summation of the damping available with the structural system and the negative damping. Thus the aerodynamic motion dependant part is introduced in analysis. The total modal damping ratio associated with the mode \(i\) is given as

\[
\zeta_{tot_i} = \zeta_i - \zeta_{ae_i}
\]

In the following discussion the analysis shown in [10] is summarized.
3.3.1 Aerodynamic derivatives:

Aerodynamic derivatives are basically the coefficients that are required for full frequency domain description of motion induced dynamic forces associated with structural velocity and displacement. They are represented as three by three matrices generally in normalized form. All the elements of these matrices are non-dimensional coefficients specific to drag, lift and torsional moments. The aerodynamic derivatives $H_1^*$ and $A_2^*$ are responsible for aerodynamic damping related to vertical direction and in torsion.

$$C_{ae} = \frac{\rho B^2}{2} \omega_i(V) \tilde{C}_{ae}$$

$$K_{ae} = \frac{\rho B^2}{2} \left[ \omega_i(V) \right]^2 \tilde{K}_{ae}$$

Considering only vortex shedding response, the aerodynamic derivatives for the damping coefficients are given as

$$\tilde{C}_{ae} \approx \frac{\rho B^2}{2} \omega_i(V) \begin{bmatrix} 0 & 0 & 0 \\ 0 & H_1^* & 0 \\ 0 & 0 & B^2 A_2^* \end{bmatrix} \quad \text{and} \quad K_{ae} \approx 0$$

(3.6)

Where $H_1^* = K_{az} \left[ 1 - \left( \frac{\sigma_2}{a_z D} \right)^2 \right]$ and $A_2^* = K_{a\theta} \left[ 1 - \left( \frac{\sigma_\theta}{a_\theta} \right)^2 \right]$.

$K_{az}, K_{a\theta}$ are velocity dependant damping coefficients.

Assuming $\omega_i(V = 0)$, the aerodynamic damping $\zeta_{ae_i}$ is given as

$$\zeta_{ae_i} = \frac{\tilde{C}_{ae_i}}{2 \omega_i \tilde{M}_i}$$

(3.7)

The modal motion induced (aerodynamic) damping

$$\tilde{C}_{ae_i} = \int \varphi_i^T C_{ae} \varphi_i \, dx$$

(3.8)

The modal motion induced (aerodynamic) mass

$$\tilde{M}_i = \tilde{m}_i \int \varphi_i^T \varphi_i \, dx$$

(3.9)

$$\begin{align*}
\tilde{m}_i &= \frac{\tilde{M}_i}{\int \varphi_i^T \varphi_i \, dx} \\
\tilde{m}_i &= \frac{\tilde{M}_i}{\int (\phi_y^2 + \phi_z^2 + \phi_\theta^2) \, dx}
\end{align*}$$

13
where \( \tilde{m}_i \) is evenly distributed and modally equivalent mass associated with mode \( i \). Substituting these values in equation for aerodynamic damping we get

\[
\zeta_{ae_i} = \frac{\rho B^2 L_{exp} \int (H_1 \phi_z^2 + B^2 A_2 \phi_\theta^2) dx}{4\tilde{m}_i \int (\phi_z^2 + \phi_\theta^2 + \phi_\phi^2) dx}
\]  

(3.10)

\( K_{a_m} \) (\( m = z \) or \( \theta \)), are the coefficients that account for the accelerating part of the motion induced load when the velocity of flow (\( V \)) becomes equal to the resonance velocity (\( V_{rt} \)) at eigen frequency of the vibration. They are the characteristic of cross section of the body under consideration and also the function of \( V \) and the resonance frequency of the eigen mode. \( a_z D, a_\theta \) are associated with the self limiting nature of vortex shedding and represents the upper limit for the vertical or rotational displacement that can occur for a given particular set of bridge and aerodynamic parameters.

Though the vortex shedding effects are the functions of various structural parameters, they also depend on the fluid characteristics such as Reynolds number and the kinematic viscosity of the incoming flow. It is found that, the most of the bridge structure exhibit the vortex induced oscillations under the smooth flow conditions.

The aerodynamic response are generally studied under three parts depending on the complexity of the problem. First part is single mode single component response calculation. In this part, it is assumed that the eigen frequencies are well spaced out. The coupling between the modes is ignored and each mode shape contains only one component i.e. any mode shape is studied as purely vertical, horizontal or torsional. The total effects are obtained as the sum of contribution from each mode. Second part is single mode three component response calculation. In this approach, it is assumed that the eigen frequencies are well spaced out on frequency vs. response graph and each mode shape contain three displacement components. Third part is multimode response calculation. The single mode single response can be looked upon as the specific case of multimode response.
3.3.2 Multimode response calculation:

For given arbitrary horizontal, vertical or torsion mode shape $\phi_i(x)$ with eigen frequency $\omega_i$ and damping ratio $\zeta_i$, the time domain displacement response contribution is given by

$$r_i(x, t) = \phi_i(x) \eta_i(t).$$

The equation for multimode response is given by $r(x, t) = \Phi(x) \eta(t)$. As we are considering multimode response, the generalized terms are given as under.

Response $r(x, t) = [r_y, r_z, r_\theta]^T$

Matrix containing all mode shapes $\Phi(x) = [\phi_1 \ldots \phi_i \ldots \phi_{N_{mod}}]$

Vector containing generalized coordinates $\eta(x) = [\eta_1 \ldots \eta_i \ldots \eta_{N_{mod}}]^T$

Here $\phi_i(x) = [\phi_y \phi_z \phi_\theta]^T_i$ are the vectors containing mode shapes in three directions. $N_{mod}$ are the number of modes considered for analysis. Now we know that instantaneous cross sectional load is the algebraic summation of load resulting from the flow induced constant part and motion induced fluctuating part. Constant part is given by $q(x, t)$ and the motion induced part is given by $q_{ae}(x, t, \dot{r}, \dot{\theta}, \ddot{r})$.

Here $q(x, t) = [q_y \ q_z \ q_\theta]^T$ and $q_{ae}(x, t, \dot{r}, \dot{\theta}, \ddot{r}) = [q_y \ q_z \ q_\theta]^T_{ae}$

Thus $q_{tot} = q(x, t) + q_{ae}(x, t, \dot{r}, \dot{\theta}, \ddot{r})$

The time domain modal equilibrium equation is given by

$$\ddot{\mathbf{M}}_0 \ddot{\eta}(t) + \mathbf{C}_0 \dot{\eta}(t) + \mathbf{K}_0 \eta(t) = \mathbf{Q}(t) + \mathbf{Q}(t, \eta, \dot{\eta}, \ddot{\eta}) \quad (3.11)$$

Where, $\dddot{\mathbf{M}}_0, \dddot{\mathbf{C}}_0, \dddot{\mathbf{K}}_0$ are $N_{mod}$ by $N_{mod}$ diagonal matrices for mass damping and stiffness of the system.

$$\dddot{\mathbf{M}}_0 = \text{diag} [ \dddot{M}_i ] \quad \text{where} \quad \dddot{M}_i = \int \phi_i^T \mathbf{M}_0 \phi_i \, dx$$

$$\dddot{\mathbf{C}}_0 = \text{diag} [ \dddot{C}_i ] \quad \text{where} \quad \dddot{C}_i = 2 \dddot{M}_i \omega_i \zeta_i$$

$$\dddot{\mathbf{K}}_0 = \text{diag} [ \dddot{K}_i ] \quad \text{where} \quad \dddot{K}_i = \omega_i^2 \dddot{M}_i$$
Flow induced $N_{mod}$ by 1 load vector is given by

$$\bar{Q}(t) = [\bar{Q}_1 ... \bar{Q}_l ... \bar{Q}_{N_{mod}}]^T \quad \text{Where} \quad \bar{Q}_l = \int (\varphi_l^T \cdot q) \, dx$$

Taking Fourier transform on both side of the time domain equilibrium equation

$$\left(-\bar{M}_0 \omega^2 + \bar{C}_0 i \omega + \bar{K}_0\right) \cdot \alpha_\eta(\omega) = \alpha_{\bar{q}}(\omega) + \alpha_{\bar{q}_{ae}}(\omega, \eta, i, \bar{\eta}) \quad (3.12)$$

where

$$\alpha_\eta = \begin{bmatrix} a_{\eta_1} & ... & a_{\eta_i} & ... & a_{\eta_N} \end{bmatrix}^T$$
$$\alpha_{\bar{q}} = \begin{bmatrix} a_{\bar{q}_1} & ... & a_{\bar{q}_l} & ... & a_{\bar{q}_N} \end{bmatrix}^T$$

As modal frequency domain motion induced load proportional to and in phase with structural displacement, velocity, and acceleration $\alpha_{\bar{q}_{ae}}$ is given as

$$\alpha_{\bar{q}_{ae}} = (-\bar{M}_{ae} \omega^2 + \bar{C}_{ae} i \omega + \bar{K}_{ae}) \cdot \alpha_\eta \quad (3.13)$$

Where $\bar{M}_{ae}$, $\bar{C}_{ae}$, $\bar{K}_{ae}$ are $N_{mod}$ by $N_{mod}$ matrices as follows

$$\bar{M}_{ae} = \begin{bmatrix} \ddots & \ddots \cdots \cdots & \ddots \\ \ddots & \ddots \cdots \cdots & \ddots \\ \ddots & \ddots \cdots \cdots & \ddots \\ \ddots & \ddots \cdots \cdots & \ddots \end{bmatrix} \quad \bar{C}_{ae} = \begin{bmatrix} \ddots & \ddots \cdots \cdots & \ddots \\ \ddots & \ddots \cdots \cdots & \ddots \\ \ddots & \ddots \cdots \cdots & \ddots \\ \ddots & \ddots \cdots \cdots & \ddots \end{bmatrix} \quad \bar{K}_{ae} = \begin{bmatrix} \ddots & \ddots \cdots \cdots & \ddots \\ \ddots & \ddots \cdots \cdots & \ddots \\ \ddots & \ddots \cdots \cdots & \ddots \\ \ddots & \ddots \cdots \cdots & \ddots \end{bmatrix}$$

The element on row $i$ and column $j$ is given by

$$\begin{bmatrix} \bar{M}_{ae_{ij}} \\ \bar{C}_{ae_{ij}} \\ \bar{K}_{ae_{ij}} \end{bmatrix} = L_{exp} \int \begin{bmatrix} \varphi_l^T \bar{M}_{ae} \varphi_j \\ \varphi_l^T \bar{C}_{ae} \varphi_j \\ \varphi_l^T \bar{K}_{ae} \varphi_j \end{bmatrix} \, dx \quad (3.14)$$

Where $\bar{M}_{ae}$, $\bar{C}_{ae}$, $\bar{K}_{ae}$ are 3 by 3 motion dependent cross sectional load coefficient matrices

$$\bar{M}_{ae} = \begin{bmatrix} \ddots & \ddots \cdots \cdots \ddots \\ \ddots & \ddots \cdots \cdots \ddots \\ \ddots & \ddots \cdots \cdots \ddots \\ \ddots & \ddots \cdots \cdots \ddots \end{bmatrix} \quad \bar{C}_{ae} = \begin{bmatrix} \ddots & \ddots \cdots \cdots \ddots \\ \ddots & \ddots \cdots \cdots \ddots \\ \ddots & \ddots \cdots \cdots \ddots \\ \ddots & \ddots \cdots \cdots \ddots \end{bmatrix} \quad \bar{K}_{ae} = \begin{bmatrix} \ddots & \ddots \cdots \cdots \ddots \\ \ddots & \ddots \cdots \cdots \ddots \\ \ddots & \ddots \cdots \cdots \ddots \\ \ddots & \ddots \cdots \cdots \ddots \end{bmatrix} \quad \text{where} \quad n,m = y,z,\theta$$

Gathering terms in above equations we get

$$[-(\bar{M}_0 - \bar{M}_{ae}) \omega^2 + (\bar{C}_0 - \bar{C}_{ae}) i \omega + (\bar{K}_0 - \bar{K}_{ae})] \cdot \alpha_\eta(\omega) = \alpha_{\bar{q}}(\omega) \quad (3.15)$$
\( \mathbf{K}_0 = \text{diag} \left[ \omega_i^2 \mathbf{M}_i \right] \) and \( \mathbf{C}_0 = \text{diag} \left[ 2\mathbf{M}_i \omega_i \zeta_i \right] \)

Pre multiplying by \( \mathbf{K}_0^{-1} \) using reduced modal load vector

\[
\mathbf{a}_q(\omega) = \mathbf{K}_0^{-1} \mathbf{a}_0(\omega) = \left[ \ldots \frac{\int \varphi_i^T(x) a_q(x, \omega) \, dx}{\omega_i^2} \ldots \right]^T
\]

(3.16)

Where \( \mathbf{a}_q(x, \omega) = \begin{bmatrix} a_{q_y} & a_{q_z} & a_{q_\theta} \end{bmatrix}^T \) and \( \mathbf{a}_\eta(\omega) = \tilde{\mathbf{H}}_\eta(\omega) \cdot \mathbf{a}_\xi(\omega) \)

\[
\tilde{\mathbf{H}}_\eta(\omega) = \left\{ \mathbf{I} - \mathbf{K}_0^{-1} \mathbf{K}_{ae} - \left( \text{diag} \left[ \frac{1}{\omega_i^2} \right] - \mathbf{K}_0^{-1} \mathbf{M}_{ae} \right) \omega^2 + \left( \text{diag} \left[ \frac{2\zeta_i}{\omega_i} \right] - \mathbf{K}_0^{-1} \mathbf{C}_{ae} \right) i\omega \right\}^{-1}
\]

(3.17)

is called as frequency response matrix. ‘\( \mathbf{I} \)’ is identity matrix. Making the following substitutions

\( \mathbf{\mu}_{ae} = \text{diag}[\omega_i^2](\mathbf{K}_0^{-1} \mathbf{M}_{ae}) \)

\( \mathbf{K}_{ae} = \mathbf{K}_0^{-1} \mathbf{K}_{ae} \)

\( \zeta_{ae} = \frac{1}{2} \text{diag}[\omega_i](\mathbf{K}_0^{-1} \mathbf{C}_{ae}) \)

\( \mathbf{\zeta} = \text{diag}[\zeta_i] \)

The frequency response matrix is given as follows

\[
\tilde{\mathbf{H}}_\eta(\omega) = \left\{ \mathbf{I} - \mathbf{K}_{ae} - \left( \omega \cdot \text{diag} \left[ \frac{1}{\omega_i} \right] \right)^2 \cdot (\mathbf{I} - \mathbf{\mu}_{ae}) + 2i\omega \cdot \text{diag} \left[ \frac{1}{\omega_i} \right] \cdot (\mathbf{\zeta} - \zeta_{ae}) \right\}^{-1}
\]

(3.18)

\[
\mathbf{\mu}_{ae} = \begin{bmatrix} \cdots & \mu_{ae_{ij}} & \cdots \end{bmatrix}, \quad \mathbf{K}_{ae} = \begin{bmatrix} \cdots & \mathbf{k}_{ae_{ij}} & \cdots \end{bmatrix}, \quad \zeta_{ae} = \begin{bmatrix} \cdots & \zeta_{ae_{ij}} & \cdots \end{bmatrix}
\]
\[
\mu_{ae_{ij}} = \frac{\mathcal{M}_{ae_{ij}}}{\mathcal{M}_i} = \frac{L_{exp} \int \Phi_i^T \mathbf{M}_{ae} \Phi_j \, dx}{\mathcal{M}_i} \tag{3.19}
\]

\[
\kappa_{ae_{ij}} = \frac{\mathcal{K}_{ae_{ij}}}{\omega_i^2 \mathcal{M}_i} = \frac{L_{exp} \int \Phi_i^T \mathbf{K}_{ae} \Phi_j \, dx}{\omega_i^2 \mathcal{M}_i} \tag{3.20}
\]

\[
\zeta_{ae_{ij}} = \frac{\mathcal{C}_{ae_{ij}}}{2\omega_i \mathcal{M}_i} = \frac{L_{exp} \int \Phi_i^T \mathbf{C}_{ae} \Phi_j \, dx}{2\omega_i \mathcal{M}_i} \tag{3.21}
\]

The response spectral density is given by

\[
S_\eta(\omega) = \lim_{T \to \infty} \frac{1}{\pi T} (\mathbf{a}_\eta^* \cdot \mathbf{a}_\eta^T)
\]

\[
S_\eta(\omega) = \lim_{T \to \infty} \frac{1}{\pi T} \left[ (\mathbf{\tilde{H}}_\eta \mathbf{a}_Q)^* (\mathbf{\tilde{H}}_\eta \mathbf{a}_Q)^T \right]
\]

\[
S_\eta(\omega) = \mathbf{\tilde{H}}_\eta^* \left[ \lim_{T \to \infty} \frac{1}{\pi T} (\mathbf{a}_Q^* \cdot \mathbf{a}_Q^T) \right] \cdot \mathbf{\tilde{H}}_\eta^T
\]

\[
S_\eta(\omega) = \mathbf{\tilde{H}}_\eta^* \cdot S_\tilde{Q} \cdot \mathbf{\tilde{H}}_\eta^T \tag{3.22}
\]

Where \( S_\tilde{Q} \) is \( N_{mod} \) by \( N_{mod} \) normalized modal matrix

\[
S_\tilde{Q}(\omega) = \lim_{T \to \infty} \frac{1}{\pi T} (\mathbf{a}_\tilde{Q}^* \cdot \mathbf{a}_\tilde{Q}^T)
\]

\[
= \lim_{T \to \infty} \frac{1}{\pi T} \left( \begin{bmatrix}
\mathbf{a}_{\tilde{Q}_1}^* \\
\mathbf{a}_{\tilde{Q}_1}^* \\
\vdots \\
\mathbf{a}_{\tilde{Q}_1}^* \\
\mathbf{a}_{\tilde{Q}_{N_{mod}}}^*
\end{bmatrix} \cdot \begin{bmatrix}
\mathbf{a}_{\tilde{Q}_1} \\
\mathbf{a}_{\tilde{Q}_1} \\
\vdots \\
\mathbf{a}_{\tilde{Q}_1} \\
\mathbf{a}_{\tilde{Q}_{N_{mod}}}
\end{bmatrix} \right)
\]

\[
=> S_\tilde{Q}(\omega) = \begin{bmatrix}
S_{\tilde{Q}_{\tilde{Q}_1}}(\omega) \\
S_{\tilde{Q}_{\tilde{Q}_1}}(\omega) \\
\vdots \\
S_{\tilde{Q}_{\tilde{Q}_1}}(\omega)
\end{bmatrix} \tag{3.23}
\]
Whose elements on row i and column j are given by

\[ S_{\hat{q}_i, \hat{q}_j}(\omega) = \lim_{T \to \infty} \frac{1}{\pi T} \left( a^*_q(\omega) \cdot a^*_q(\omega) \right) \]

\[ = \lim_{T \to \infty} \frac{1}{\pi T} \left( L_{\exp} \int \frac{\phi^T(x) a_q(x, \omega) \, dx}{\omega_i^2 M_i} \cdot L_{\exp} \int\left[ \phi^T(x) a_q(x, \omega) \right]^T \frac{dx}{\omega_j^2 M_j} \right) \]

\[ = \lim_{T \to \infty} \frac{1}{\pi T} \left\{ L_{\exp} \int \frac{\int [\phi^T(x_1) a_q(x_1, \omega)] [\phi^T(x_2) a_q(x_2, \omega)]^T \, dx_1 \, dx_2}{\omega_i^2 M_i} \frac{dx_1 \, dx_2}{\omega_j^2 M_j} \right\} \]

\[ = \frac{L_{\exp} \int \phi^T(x_1) \left[ \lim_{T \to \infty} \frac{1}{\pi T} a_q(x_1, \omega) a_q^T(x_2, \omega) \right] \phi_j(x_2) \, dx_1 \, dx_2}{\omega_i^2 M_i} \frac{dx_1 \, dx_2}{\omega_j^2 M_j} \]  

(3.24)

The elements of \( S_\hat{q}(\omega) \) are given by

\[ S_{\hat{q}_i, \hat{q}_j}(\omega) = \frac{L_{\exp} \int \phi^T_i(x_1) S_{\hat{q}q}(\Delta x, \omega) \phi_j(x_2) \, dx_1 \, dx_2}{\omega_i^2 M_i} \frac{dx_1 \, dx_2}{\omega_j^2 M_j} \]  

(3.25)

Where \( \Delta x = |x_1 - x_2| \) and \( S_{\hat{q}q}(\Delta x, \omega) \) is the spectral density matrix of cross sectional load

\[ S_{\hat{q}q}(\Delta x, \omega) = \lim_{T \to \infty} \frac{1}{\pi T} \left[ a^*_q(x_1, \omega) a^*_q(x_2, \omega) \right] \]

(3.26)

\[ S_{\hat{q}q}(\Delta x, \omega) = \lim_{T \to \infty} \frac{1}{\pi T} \begin{bmatrix} a^*_y a_y & a^*_y a_z & a^*_y a_q \\ a^*_q a_y & a^*_q a_z & a^*_q a_y \\ a^*_Q a_y & a^*_Q a_z & a^*_Q a_y \end{bmatrix} \]

(3.27)

A three by \( N_{\text{mod}} \) matrix associated with a chosen span-wise position \( x_r \)
\[ \Phi_r(x_r) = [\Phi_1(x_r) \ldots \Phi_i(x_r) \ldots \Phi_N(x_r)] \]

\[
= \begin{bmatrix}
\phi_y(x_r) & \cdots & \phi_y(x_r) \\
\phi_z(x_r) & \cdots & \phi_z(x_r) \\
\phi_\theta(x_r)_1 & \cdots & \phi_\theta(x_r)_N
\end{bmatrix}
\]

(3.29)

Three by three cross spectral density matrix of the unknown modal displacements \( r_y, r_z \) and \( r_\theta \) at \( x = x_r \)

\[
S_{rr}(x_r, \omega) = \begin{bmatrix}
S_{ryr_y} & S_{ryr_z} & S_{ryr_\theta} \\
S_{rxr_y} & S_{ryr_z} & S_{rxr_\theta} \\
S_{r_\theta r_y} & S_{r_\theta r_z} & S_{r_\theta r_\theta}
\end{bmatrix}
\]

(3.30)

\[
S_{rr}(x_r, \omega) = \Phi_r(x_r) \cdot S_\eta(\omega) \cdot \Phi_r^T(x_r)
\]

(3.31)

where \( S_\eta(\omega) \) is given as

\[
S_{rr}(x_r, \omega) = \Phi_r(x_r) \left[ \hat{H}_\eta^* \cdot S_\eta(\omega) \cdot \hat{H}_\eta^T(\omega) \right] \cdot \Phi_r^T(x_r)
\]

(3.32)

Above equation is applicable to any linear load on a line-like structure. If the cross coupling between the \( q_z \) and \( q_\theta \) is disregarded then \( S_{qq}(\Delta x, \omega) \) is given as

\[
S_{qq}(\Delta x, \omega) \approx \begin{bmatrix}
0 & 0 & 0 \\
0 & s_{q_zq_z} & 0 \\
0 & 0 & s_{q_\theta q_\theta}
\end{bmatrix}
\]

(3.33)

Where, the cross spectra \( S_{q_zq_z} \) and \( S_{q_\theta q_\theta} \) are given by

\[
S_{q_zq_z} = S_{q_z}(\omega) \cdot \hat{C}_{q_z}(\Delta x)
\]

(3.34)

\[
S_{q_\theta q_\theta} = S_{q_\theta}(\omega) \cdot \hat{C}_{q_\theta}(\Delta x)
\]

(3.35)

The single point spectra are defined as \( S_{q_z}, S_{q_\theta} \) and the reduced co-spectra \( \hat{C}_{q_z}, \hat{C}_{q_\theta} \) are given by Vickery and Basu. (Equation 3.2 and 3.3). The elements in cross spectral density matrix \( S_{\tilde{q}} \) are reduced to
\[ S_{\tilde{p},\tilde{q}}(\omega) = \frac{L_{\text{exp}} \int \int \Phi_i^T(x_1) \cdot S_{qq}(\Delta x, \omega) \cdot \Phi_j(x_2) \, dx_1 \, dx_2}{(\omega_i^2 \tilde{M}_i)(\omega_j^2 \tilde{M}_j)} \]

\[ = L_{\text{exp}} \int \left\{ \phi_{iz}(x_1)\phi_{jz}(x_2)S_{qzqz} + \phi_{i\theta}(x_1)\phi_{j\theta}(x_2)S_{q\theta q\theta} \right\} \, dx_1 \, dx_2 \]

\[ = S_{qz} \cdot L_{\text{exp}} \int \phi_{iz}(x_1)\phi_{jz}(x_2) \hat{C}_{qz} \, dx_1 \, dx_2 + S_{q\theta} \cdot L_{\text{exp}} \int \phi_{i\theta}(x_1)\phi_{j\theta}(x_2) \hat{C}_{\theta q\theta} \, dx_1 \, dx_2 \]

\[ = \frac{S_{qz} \cdot L_{\text{exp}} \int \phi_{iz}(x)\phi_{jz}(x) \, dx + S_{qz} \cdot L_{\text{exp}} \int \phi_{i\theta}(x)\phi_{j\theta}(x) \, dx}{(\omega_i^2 \tilde{M}_i)(\omega_j^2 \tilde{M}_j)} \] (3.36)

It is assumed that the integral length-scale of the vortices \( \lambda D \) is small as compared to the flow exposed length \( L_{\text{exp}} \) of the structure. As \( q_z, q_{\theta} \) are caused by the same vortices, their coherence properties are likely to be identical.

\[ S_{\tilde{p},\tilde{q}}(\omega) = \frac{2\lambda D \left[ S_{qz} \cdot L_{\text{exp}} \int \phi_{iz}(x)\phi_{jz}(x) \, dx + S_{qz} \cdot L_{\text{exp}} \int \phi_{i\theta}(x)\phi_{j\theta}(x) \, dx \right]}{(\omega_i^2 \tilde{M}_i)(\omega_j^2 \tilde{M}_j)} \] (3.37)

Due to orthogonality of mode shapes, \( S_{\tilde{q}_i} \) is diagonal matrix.

\[ S_{\tilde{q}_i} = \text{diag}[S_{\tilde{q}_i}] \text{ where} \]

\[ S_{\tilde{q}_i}(\omega) = \frac{2\lambda D \left[ S_{qz}(\omega) \cdot L_{\text{exp}} \int \phi_{iz}^2 \, dx + S_{q\theta}(\omega) \cdot L_{\text{exp}} \int \phi_{i\theta}^2 \, dx \right]}{(\omega_i^2 \tilde{M}_i)^2} \] (3.38)

The calculation of the spectral response matrix is given in equations 3.30, 3.31, 3.32. \( \tilde{H}_{\eta} \) and \( S_{\tilde{q}} \) are diagonal if the above simplification holds.

\[ S_{rr}(r, \omega) = \Phi_r(x_r) \cdot \text{diag}[S_{\eta_i}(\omega)] \cdot \Phi^T_r(x_r) \]
\[ N_{\text{mod}} \]
\[ = \sum_{i=1}^{N_{\text{mod}}} \varphi_i(x_r) \varphi_i^T(x_r) S_{\eta_i}(\omega) \]

\[ = \sum_{i=1}^{N_{\text{mod}}} \begin{bmatrix} 
\phi_i^2(x_r) & \phi_i(x_r)\phi_z(x_r) & \phi_i(x_r)\phi_\theta(x_r) \\
\phi_i(x_r)\phi_z(x_r) & \phi_z^2(x_r) & \phi_z(x_r)\phi_\theta(x_r) \\
\phi_i(x_r)\phi_\theta(x_r) & \phi_z(x_r)\phi_\theta(x_r) & \phi_\theta^2(x_r) 
\end{bmatrix} . S_{\eta_i}(\omega) \] (3.39)

where

\[ S_{\eta_i}(\omega) = |\tilde{H}_{\eta_i}(\omega)|^2 . S_{\tilde{Q_i}}(\omega) \] (3.40)

\[ \tilde{H}_{\eta_i}(\omega) = \left[ 1 - \left( \frac{\omega}{\omega_i} \right)^2 + 2i \left( \zeta_i - \zeta_{ae_i} \right) \cdot \frac{\omega}{\omega_i} \right]^{-1} \] (3.41)

\[ \zeta_{ae_i} = \frac{\rho B^2 H_1^* L_{\text{exp}} \int \phi_i^2(x) dx + B^2 A_2^* \int \phi_i^2 \phi_\theta^2 dx}{L \left( \phi_i^2 + \phi_z^2 + \phi_\theta^2 \right) dx} \] (3.42)

The corresponding covariance response matrix \( \text{Cov}_{rr}(x_r) \) for dynamic response at span-wise position \( x_r \) is given by

\[ \text{Cov}_{rr}(x_r) = \int_0^\infty S_{rr}(x_r, \omega) d\omega \]

\[ = \begin{bmatrix} 
\sigma_{r_x r_x}^2 & \text{Cov}_{r_y r_x} & \text{Cov}_{r_y r_\theta} \\
\text{Cov}_{r_y r_x} & \sigma_{r_z r_z}^2 & \text{Cov}_{r_z r_\theta} \\
\text{Cov}_{r_y r_\theta} & \text{Cov}_{r_z r_\theta} & \sigma_{r_\theta r_\theta}^2 
\end{bmatrix} \]

\[ = \sum_{i=1}^{N_{\text{mod}}} \begin{bmatrix} 
\phi_i^2(x_r) & \phi_i(x_r)\phi_z(x_r) & \phi_i(x_r)\phi_\theta(x_r) \\
\phi_i(x_r)\phi_z(x_r) & \phi_z^2(x_r) & \phi_z(x_r)\phi_\theta(x_r) \\
\phi_i(x_r)\phi_\theta(x_r) & \phi_z(x_r)\phi_\theta(x_r) & \phi_\theta^2(x_r) 
\end{bmatrix} . \sigma_{\eta_i}^2 \] (3.43)

Where \( \sigma_{\eta_i}^2 = \int_0^\infty S_{\eta_i} d\omega \) is the variance contribution due to mode \( i \).
In structural engineering total dynamic response can be looked upon as the summation of the response from low frequency background part and narrow banded resonant part. It is customary to discard the low frequency, steady background part and to consider only resonant part.

\[
\sigma^2_{\eta_i} = \int_0^\infty S_{\eta_i} d\omega
\]

\[
= \int_0^\infty |\tilde{H}_{\eta_i}(\omega)|^2 S_{\tilde{Q}_i}(\omega) d\omega
\]

\[
\approx \int_0^\infty |\tilde{H}_{\eta_i}(\omega)|^2 d\omega S_{\tilde{Q}_i}(\omega)
\]

\[
= \frac{\pi \omega_i S_{\tilde{Q}_n}(\omega_i)}{4(\xi_i - \xi_{ae_i})}
\] (3.44)

where

\[
S_{\tilde{Q}_i}(\omega_i) = \frac{2\lambda D}{(\omega_i^2 M_i)^2} \frac{(\rho V^2 B/2)^2}{\sqrt{\pi} \omega_s} \left\{ \frac{\sigma^2_{qz}}{b_z} L_{exp} \int \phi_{i_z}^2 dx e^{-\left(\frac{1-\omega_i}{\omega_s}\right)^2} \right\}
\] (3.45)

From the equation 3.2 given by Vickery and Basu substituting for \( S_{qz} \) and \( S_{q\theta} \) we get the following expression

\[
S_{\tilde{Q}_i}(\omega_i) = \frac{2\lambda D}{(\omega_i^2 M_i)^2} \frac{(\rho V^2 B/2)^2}{\sqrt{\pi} \omega_s} \left\{ \frac{\sigma^2_{qz}}{b_z} L_{exp} \int \phi_{i_z}^2 dx e^{-\left(\frac{1-\omega_i}{\omega_s}\right)^2} \right\}
\] (3.46)
The circular frequency \( \omega_s = 2\pi f_s \). As the aerodynamic derivatives \( H^*_i \) and \( A^*_i \) are the functions of \( \sigma_{r_x r_x} \) and \( \sigma_{r_y r_y} \), the iteration will depend on difference between \( \zeta_i \) and \( \zeta_{ae_i} \).

### 3.3.3 Single mode single component response calculations:

The single mode single component response can be looked upon as the special case of multimode response analysis. The response in single mode single component is considered relevant for displacement in vertical direction or in torsion. The displacements in the direction, either \( z \) or \( \theta \) are relevant in single component single mode analysis. Thus \( \Phi_i(x) \approx \begin{bmatrix} 0 & \phi_z & 0 \end{bmatrix}^T \\ or \quad \Phi_i(x) \approx \begin{bmatrix} 0 & 0 & \phi_\theta \end{bmatrix}^T \) are considered for the analysis.

The off diagonal element in equation 3.39 is discarded rendering all the covariance terms obsolete. The cross spectral density matrix \( S_{rr} \), contains only diagonal terms representing response variance of the excitation.

The response spectrum associated with an arbitrary mode is given as

\[
S_{\eta_n}(\omega) = \phi_n^2(x) \left| \bar{H}_{\eta_n}(\omega) \right|^2 S_{\theta_n}(\omega)
\]  

(3.47)

The displacement variance associated with an arbitrary mode is given as

\[
\sigma_{\eta_n}^2 = \int_0^{\infty} S_{\eta_n}(\omega) d\omega
\]

(3.48)

\( n = z \) or \( \theta \) depending on the type of displacement under consideration.

\[
\bar{H}_{\eta_n}(\omega) = \left[ 1 - \left( \frac{\omega}{\omega_n} \right)^2 + 2i \left( \zeta_n - \zeta_{ae_n} \right) \frac{\omega}{\omega_n} \right]^{-1}
\]

(3.49)

\[
S_{\theta_n}(\omega) = 2\lambda D \frac{S_{\eta_n}(\omega) L_{exp} \int \phi_n^2(x) dx}{\left( \omega_n^2 \bar{H}_{\eta_n} \right)^2}
\]

(3.50)
Aerodynamic damping properties for single mode single component can be expressed as follows from the multimode analysis:

\[
\zeta_{ae_z} = \frac{\tilde{C}_{ae_{zz}}}{2\omega_z \bar{M}_z} = \frac{\rho B^2 H_1^*}{4\bar{m}_z} \frac{L_{exp} \int \phi_z^2 \, dx}{\int_0^L \phi_z^2 \, dx}
\]

\[
= \frac{\rho B^2 K_a_z}{4\bar{m}_z} \left[ 1 - \left( \frac{c_z}{a_z D} \right)^2 \right] \frac{L_{exp} \int \phi_z^2 \, dx}{\int_0^L \phi_z^2 \, dx}
\]

(3.51)

and

\[
\zeta_{ae_{\theta}} = \frac{\tilde{C}_{ae_{\theta\theta}}}{2\omega_\theta \bar{M}_\theta} = \frac{\rho B^4 A_2^*}{4\bar{m}_\theta} \frac{L_{exp} \int \phi_\theta^2 \, dx}{\int_0^L \phi_\theta^2 \, dx}
\]

\[
= \frac{\rho B^4}{4\bar{m}_\theta} K_a_\theta \left[ 1 - \left( \frac{c_\theta}{a_\theta} \right)^2 \right] \frac{L_{exp} \int \phi_{\theta}^2 \, dx}{\int_0^L \phi_{\theta}^2 \, dx}
\]

(3.52)

As vortex shedding induced dynamic response of the structure is narrow banded and resonant, steady state background part is usually neglected.

\[
\sigma_n^2 = \int_0^\infty S_{r_n}(\omega) \, d\omega \approx \phi_n^2(x_r) \int_0^\infty |\tilde{H}_{r_n}(\omega)|^2 \, d\omega S_{\tilde{q}_n}(\omega)
\]

\[
\Rightarrow \sigma_n^2 = \frac{\phi_n^2(x_r) (\pi \omega_n S_{\tilde{q}_n}(\omega_n))}{4(z_n - \zeta_{ae_n})}
\]

(3.53)

Where \( n = z, \theta \)

Above equation gives displacement variance associated with the direction under consideration for single mode single component response calculation.
CHAPTER 4

MITIGATION OF VORTEX-INDUCED VIBRATIONS USING A HYBRID APPROACH

4.1 Cross-sectional retrofit:

It is well known that, the aerodynamic response exhibited by a bridge deck can be controlled by using aerodynamically favorable cross section. When air flows around the aerodynamically favorable section, the flow remains in streamlined laminar state. The flow separation and thereby the differential air pressure creation is prevented. This situation refers to ideal fluid flow condition and the flow with low velocities. Providing such cross section is not always possible due to structural and construction difficulties in practice. In last few decades of bridge engineering, some bridges have shown the aerodynamically induced oscillations which were not anticipated up to the extent it was actually recorded. The increased length of suspended span aggravated these vibrational problems. To mitigate the aerodynamic response of the bridge deck, cross sectional retrofitting proves the acceptable solution. The cross sectional retrofit can be employed by addition of stiffening plate girders to the supporting deck of the bridge. This improves the overall stiffness of the bridge deck and thereby imparts the aerodynamic stability to the structure. The drawback of this approach is the increased weight of the structure, which is the main concern in bridge engineering. The bridges which cannot be made with streamlined cross section, aerodynamic mitigation measures are employed in order to alter the flow of wind around the bridge deck. These aerodynamic mitigation measures are basically aerodynamic appendages like fairings, double flaps.

4.2 Tuned-mass damper:

While designing the Trans-Tokyo Bridge [6] in Japan, wind tunnel tests were conducted on the model of bridge section with various types of appendages and their combinations. Despite these trials, it was found that the aerodynamically induced vortex shedding vibration cannot be suppressed to the acceptable limit. So, it was decided to install tuned mass dampers
(TMDs) along with aerodynamic appendages. TMDs were employed to control the vibrations due to first and the second mode.

For analysis, the simple spring mass system as shown below can be considered.

\[ m \rightarrow k(y_2 - y_1) \quad g(t) \]
\[ c(y_2 - y_1) \]

\[ ky_1 \quad cy_1 \]
\[ M\ddot{y}_1 \quad f(t) \]

\( y_1 \) = displacement of structure
\( y_2 \) = displacement of dampers
\( g(t) \) = force on dampers
\( f(t) \) = force acting on structure

Mass ‘M’, stiffness ‘k’ and the coefficient of damping ‘c’ of the system are in opposite direction to the corresponding parameters of tuned mass dampers (TMD).

\[ M\ddot{y}_1 + Cy_1 + Ky_1 = C(\dot{y}_2 - \dot{y}_1) + k(y_2 - y_1) + f(t) \]
\[ M\ddot{y}_2 + C(\dot{y}_2 - \dot{y}_1) + k(y_2 - y_1) = g(t) \]

Let \((y_2 - y_1) = z \) then \((y_2 = z + y_1)\)

\[ M\ddot{y}_1 + Cy_1 + Ky_1 = Cz + k_z + f(t) \quad \text{(4.1)} \]
\[ M\ddot{z} + Cz + Kz = -m\ddot{y}_1 + g(t) \quad \text{(4.2)} \]

Adding 4.1 and 4.2 and considering \(g(t) = 0\) for non-earthquake condition we get

\[ (M + m) \ddot{y}_1 + Cy_1 + Ky_1 = f(t) + g(t) - m\ddot{z} \quad \text{(4.3)} \]

Multiply equation 4.3 by \(\ddot{y}_1\) we get

\[ (M + m)\dddot{y}_1 + Cy_1^2 + Ky_1\dddot{y}_1 = f\dddot{y}_1 - m(\dddot{y}_1) \quad \text{(4.4)} \]
\[ C\dddot{y}_1^2 = f\dddot{y}_1 - m(\dddot{y}_2\dot{y}_1 - \dddot{y}_1\dot{y}_1) \quad \text{(4.5)} \]

For damping maximum energy dissipation \(y_2\) and \(y_1\) has phase angle of 90°
\(\dddot{y}_1\dot{y}_1 = 0\) and \(\dddot{y}_2, y_1\) are in phase.
\[ C \ddot{y}_1 + m(\ddot{y}_2 \dot{y}_1) = f \dot{y}_1 \]
\[ C + m \frac{\ddot{y}_2 \dot{y}_1}{\ddot{y}_1} = f \dot{y}_1 \]

\( C + m \frac{\ddot{y}_2 \dot{y}_1}{\ddot{y}_1} \) is effective damping. The simplified design for TMD can be arrived at by considering the single degree of freedom system. In this approach the modal mass is obtained by using a mode shape vector normalized with respect to the location of TMD. The overall effective damping of the system is calculated as
\( \xi = \xi_s + \xi_{eff} \)

Where
\( \xi_s \) = sum of structural damping and aerodynamic damping
\( \xi_{eff} = \frac{1}{2} \sqrt{\frac{\mu}{\frac{\mu}{2} + \frac{1}{2}}} \]

\( \mu = \frac{m_t}{m_i} \)

\( m_t \) = mass of TMD
\( m_i \) = modal mass

### 4.3 The proposed approach:

As observed in Trans-Tokyo Bridge [6] in Japan, TMDs were employed to suppress the vibrations due to first and second mode. The vibrations resulting due to the higher modes were controlled by the combination of aerodynamic appendages. This hybrid approach is found to be useful in existing bridges to suppress the aerodynamically induced vibrations. This approach has been proposed in this research work. The case study is done to establish the efficiency, feasibility and the challenges in this approach. Depending upon the ambient condition, static and dynamic parameters of bridge and economy, the amount of cross sectional retrofit and the number of TMDs are decided. The objectives to be achieved in this problem are of conflicting nature. If the deflection of the bridge deck to be restricted to the certain acceptable level, the cost increases. On the other hand, if the cost is required to bring down to a certain level, the compromise has to be made in terms of maximum allowable deflection. If the torsional stiffness is required to
increase, the addition of members would increase the weight of the bridge. Thus, the one objective stiffness, dominates the other objective weight. The problem becomes more complex when the TMDs are used in addition to cross sectional retrofit. Another degree of complexity is added when a bridge deck exhibits more than one response of comparable magnitudes as observed in case of Trans-Tokyo bridge while wind tunnel testing on sectional models. Such conflicting objective can be achieved by the repeated application of single objective optimization technique. However the application of single objective based approach becomes more tedious and computationally inefficient in the case of multiple objectives of conflicting nature. In such situations multi-objective genetic algorithms comes handy and useful to achieve the optimization. The multiple solutions are obtained by this technique are called as Pareto-optimal solutions. There is no solution which can be termed as the best solution to our problem. Instead, it gives the options to choose from the solution set, to suit the situation and the decision making.

4.4 Multi-Objective Genetic Algorithm:

Many real-world optimization problems have multiple objectives. The difficulty arises due to optimality conditions for multiple objectives. Multiple solutions are due to the fact that no one solution can be an optimal solution for multiple conflicting objective situations. As mentioned earlier, in case of bridge deck, the deflection and the cost are the two objectives which are of conflicting nature. The cost increases if the deflection is required to be keep within certain acceptable limit. In such cases no single solution can be absolute optimum. The decision has to be made on the basis of specific requirement governed by nature of the problem to be solved. In order to come up with the solution space for such problems, evolutionary algorithms such as Multi-Objective Genetic Algorithms are developed. Before the advent of multi-objective optimization algorithm, multi-objective optimization problems have been suitably converted to single objective optimization problems [8]. The drawbacks of this approach are the tediousness and the computational inefficiency as it requires multiple runs for multiple objectives. There are many multi-objective optimization algorithms have been developed by various researchers to find the solutions closer to Pareto-optimal front and having wider spread. The author of [8], has suggested the Non-Dominated sorting Genetic Algorithm (NSGA), to find a biased distribution
of solution to the Pareto-optimal region. The NSGA allows more solutions to be found according to the users’ preference.

The concept can be explained with the help of Figure 4.1. The figure represents the solution to the real world problem of deflection of a bridge deck against the cost involved. The solution points along A, B, C are called as Pareto-optimal solutions resulting from the multi-objective non-dominated sorting genetic algorithm. They are also called as Pareto-optimal front. All the individual points along the solution front are the possible solutions. The point “A” represent the solution where the deflection is minimized (optimized) but the cost is high. The point “C” depicts the situation where cost is minimal and the deflection is maximized. Thus these two objectives cost and deflection conflicts with each other. The point “D” represents the solution which looks better than solution point “A” in terms of cost, but cannot be said for sure. There are solutions represented by the points like “B” which are more acceptable than solution point “D” in terms of both the objectives. Solution points like “D” are called as dominated solutions and the solution points along the A, B, C are referred as non-dominated solutions. A solution is said to dominate the other if that solution is no worse than the other solution in all the objectives involved the problem. The other condition to be satisfied is the solution should be strictly better than the other solution at least in one of the objectives under consideration.

Figure 4.1: Schematic representation of Pareto-optimal solution
Srinivas and Deb [15], implemented the non-dominated sorting genetic algorithm. In their approach, the population is ranked according to non-domination level and then the sharing function is used to assign the fitness to each individual. According the stated procedure, the solutions are marked dominated and non-dominated. All the non-dominated solutions are considered as first non-dominated front and procedure is followed to find the successive level of non-dominated solutions. After assigning the ranks to the solution sets, fitness is assigned according to non-domination level. The individual with higher level gets the lower fitness. Fitness to the individual levels is assigned according to the sharing function method [8] and crowding of solutions in the solution front.

Once the Pareto-optimal set of the solution is available from the multi-objective optimization, multi-criterion decision making is conducted. It is expected from the end user to decide preference of a solution appropriate to the requirement. In order to decide upon the solution, two approaches are employed. They are broadly categorized as post-optimal techniques and optimization-level techniques. In post-optimal techniques the methods used are compromise programming, marginal rate of return, weighted average [8]. The method of compromise programming picks the solution from the Pareto-optimal set which is at a minimum location from a reference point. Author of [8] has given the mathematical expression to ascertain the location of reference point according the requirement of decision maker. In marginal rate of return method, the amount of improvement in one objective function is obtained by sacrificing the performance in other object. The solution having maximum rate of return is chosen by this method. In weighted average method, user specified weighted average is used to decide a solution closer to optimum solution. In optimization level techniques two methods are mentioned by authors of [8]. First is utility functions and the second is biased sharing approach. In utility function method, the two solutions having the same utility function value have the same user preference. This way multiple objectives reduce to single objective. The construction of utility function is subjective to the user. Authors of [8] have proposed the Biased sharing approach which uses the Euclidian distance with biased or unequal weight to the objective functions. The method ensures that the highest priority objective function always gets a weight of one whereas the other objective functions get the weight between zero and one. The procedure with example has been explained [8].
CHAPTER 5

ILLUSRATIVE EXAMPLE

As explained in chapter 4, the goal of this research work is to mitigate the vortex induced response in long span bridges using the hybrid approach, where the mitigation is achieved through cross-sectional retrofit and the addition of dampers. The approach is explained with the help of case study done on Rio-Niterói bridge. The Rio-Niterói bridge is located across the Guanabara bay in Rio de Janeiro. Most of the bridge is constructed with prestressed concrete structures except the three central spans. The central spans are of twin steel box girders. These three central girders have 200m, 300m, and 200m lengths. The girders’ height to span ratio is 1/45. Due to such a small ratio, these spans are slender and used to set in vertical bending motion even at low wind speed of 50km/h. For sustained velocities of 60km/h, the bluff box section often experienced the vortex-induced vibrations in the first vertical bending mode. Though the torsional displacement was restrained for the wind less than 200km/h, by bracing the two steel box birders, the vortex induced vibration at lower modes were significant. All these efforts resulted in, to explore the option of employing the control devices such as dampers.

5.1 The approach:

The bridge model for illustration purpose is devised based the Rio-Niterói bridge. The overall approach is broadly divided into four sub-tasks.

- Development of vortex-induced vibration analysis using Matlab.
- Data collection and processing,
- Optimizing the solution using Non-dominated Sorting Genetic Algorithm (NSGA-II).

These sub-tasks are briefly discussed in the following.
5.1.1 Development of vortex-induced vibration analysis using Matlab:

Matlab coding is used to simulate the maximum deflection based on the data and analysis. The analysis and the procedure to find the aerodynamic response is discussed in chapter 3. Specifically, single-mode single-component analysis described in 3.3.3 is used. Based on the procedure, the flow chart for coding is prepared and Matlab coding for maximum response is done. The flow chart is shown in figure 5.1.

5.1.2 Data collection and processing:

Too analyze vortex-induced vibration using the developed code, various physical properties such as depth, width of bridge deck, span length, weight per unit length, mode shapes and natural frequencies are required [10]. They are obtained from a structure modeled after the Rio-Niterói Bridge.

The aerodynamic derivatives are processed based on data and mathematical equations as discussed in chapter 3 and used in numerical evaluation. In case of parametric study or prediction of behavior of structure, wind tunnel test are done and aerodynamic coefficients are evaluated and used in modeling. In our case data is collected from wind tunnel test on a bridge model performed by Dr. Soon-Duck Kwon, Chonbuk National University, Korea and then used in illustrative example.

5.1.3 Optimization and Non-Dominated Sorting Genetic Algorithm (NSGA-II):

The illustrative example being multi-objective problem, the solution is optimized using NSGA-II algorithm. The Matlab code for maximum response is integrated with the NSGA-II to find the optimized solution. Optimized solution is illustrated in the form of graph for displacement vs cost. The optimized solution gives the option to choose from the set of solutions. The allowable deflection is used to select a particular solution and the corresponding cost can be estimated. After first set of optimized solution, the simulation is run again by modeling the dampers. The
The response of the bridge deck to any destabilizing force is complex phenomenon. Also the overall response of the bridge deck in practical can be the result of the combination of different modes of vibration associated with different resonance frequencies. The degree of complexity increases when the overall response of the structure is the result of coupling between the two or more modes or frequencies. If a certain type response is controlled with the help of certain retrofit measure, the other type of response may not be suppressed to the desired level or may not be suppressed at all. That particular response may need different retrofit approach or it may require changing the values of design parameters and redesigning the structure. This renders the approach to be of iterative nature. Thus when more than one mode and response components are involved, it is called as multimode multi component analysis.

Here, we are analyzing vortex induced oscillations and response of bridge deck in vertical direction only. We are considering one mode of vibration that is vertical oscillation. The bridge deck system modeled here is the case of single mode single component system. The numerical example is discussed in next section.
Figure 5.1: Flow chart

1. **START**
2. Bridge Parameters - L, B, D, f₀, dr, φ, ζ, Nm
3. Aerodynamic Data - Vmax, Vmin, Vme, ρ, H₁⁺, Sₜ, q₂, b₂ λ, Ns
4. Read and process modes
   - Modal_Mass (∫ ϕ² m dx or ∫ ϕ'' m ϕ dx)
   - modal_int_coef (∫ ϕ₂ dx)
   - eq_mass (m̃ₖ) = (∫ ϕ² m dx) / (∫ ϕ₂ dx)
5. Read and process aerodynamic coefficient H₁⁺
6. Want to plot mode shapes?
   - Yes
     - Plot Mode Shapes
   - No
Figure 5.1: Flow chart continued

Aerodynamic Damping Ratio
Multimode response calculation
\[ \zeta_{ae_i} = \frac{\rho B^2}{4\tilde{m}_i} \frac{H^i_{1, \text{exp}} \int \varphi^2_i \, dx + B^2 A^2_i \int \varphi^2_{i_\theta} \, dx}{\int (\varphi^2_{i_\theta} + \varphi^2_{i_\theta} + \varphi^2_{i_\theta}) \, dx} \]

Single mode response calculation \[ \zeta_{ae_i} = \frac{\rho B^2}{4\tilde{m}_i} \frac{H^i_{1, \text{exp}} \int \varphi^2_i \, dx}{\int \varphi^2_i \, dx} \]

Want to Plot aerodynamic damping?

Yes

Plot aerodynamic damping ratio

No

Frequency response function (H)
Multimode response calculation
\[ \tilde{H}_\eta(\omega) = \{I - (\omega \cdot \text{diag} \left[ \frac{1}{\omega_i} \right])^2 + 2i \omega \cdot \text{diag} \left[ \frac{1}{\omega_i} \right] (\zeta - \zeta_{ae_i}) \}^{-1} \]

Single mode response calculation
\[ \tilde{H}_{\eta_n}(\omega) = \{1 - \left( \frac{\omega}{\omega_n} \right)^2 + 2i (\zeta_n - \zeta_{ae_n}) \left( \frac{\omega}{\omega_n} \right) \}^{-1} \]
Response calculation

\[
\begin{bmatrix}
S_{q_z}(\omega) \\
S_{q_\theta}(\omega)
\end{bmatrix} = \frac{(1\rho V^2)^2}{\sqrt{\pi} \omega_s} \begin{bmatrix}
\left(\frac{b \sigma_{q_z}}{b_z}\right)^2 e^{-\left(\frac{1 - \omega}{\omega_s}\right)^2} \\
\left(\frac{b \sigma_{q_\theta}}{b_\theta}\right)^2 e^{-\left(\frac{1 - \omega}{\omega_s}\right)^2}
\end{bmatrix}
\]

\[
\sigma_{r_n}^2 = \frac{\phi_n^2(x_r) \pi \omega_n S_{\tilde{q}_n}(\omega_n)}{4(\zeta_n - \zeta_{ae_n})}
\]

where \( n = z \) for vertical displacement
\( n = \theta \) for rotational displacement

Here \( n = z \) as we are considering vertical response only

\[
S_{\tilde{q}_n}(\omega) = 2\lambda D \frac{S_{q_{n,\text{Lip}}}}{(\omega_n^2 \beta_n)^2} \int \phi_n^2(x) \, dx
\]
5.2 Numerical examples:

The same bridge model is used for all examples. Two different aerodynamic coefficients are used to test the proposed method for different cases.

5.2.1 Model definition:

**Bridge:**
Length = 850 m  
Width of bridge deck = 25.9 m  
Depth of bridge deck = 7.42 m

**Wind:**
Density = 0.001225 t/m³

**Aerodynamic data:**
Natural frequencies $f_1 = 0.32$ Hz; $f_2 = 0.55$ Hz; $f_3 = 0.76$ Hz;  
Damping ratio $\zeta = [0.005 \ 0.005 \ 0.005]$;  
Modal mass $[\ 2864.8 \ 2552.9 \ 2189.3]$;

**Non-retrofitted section:**
Strouhal number $S_t = 0.145$  
Non-dimensional root mean square lift coefficient $q_z = 0.4$  
Non-dimensional load spectrum band width parameter $b_z = 0.15$  
Non-dimensional coherence length scale $\lambda = 2$

**Retrofitted section:**
Strouhal number $S_t = 0.1$  
Non-dimensional root mean square lift coefficient $q_z = 0.1$  
Non-dimensional load spectrum band width parameter $b_z = 0.1$  
Non-dimensional coherence length scale $\lambda = 2$
Modes shapes:

First three vertical modes are considered as shown in figure 5.2

5.2.2 Aerodynamic coefficients:

The proposed method is first tested using synthetic aerodynamic coefficients shown in figure 5.3. Horizontal axis indicates normalized velocity, vertical axis indicates aerodynamic coefficient for lift.
The base line problem:

The following figure 5.4 and 5.5, illustrates the base line problem. The figures show the typical response of a bridge deck with respect to wind velocity. For illustration purpose the section selected, are the sections where the response is maximum under no retrofit condition. From the figure 5.2, these sections are 160m, 425m and 680m. The maximum response of these sections is plotted in figure 5.4. The figure 5.5 shows the typical response of a bridge deck when all the sections are retrofitted. The response at the same sections is plotted with respect to wind velocity.

The maximum response in original (no retrofit) condition at 160m is found to be 0.34m at the wind speed of 40m/s. When all the sections are retrofitted, the maximum response at the same section, at the same speed is found to be 0.045m. The responses at the other sections are found to be reduced to great extent in case of retrofitted section.

Figure 5.4: Response of original section (Example 1)
Figure 5.5: Response of retrofitted section (Example 1)

**Optimization with NSGA-II:**

For optimization purpose the cost function is defined as the length of retrofit multiplied by cost per unit length of retrofit and number of dampers required multiplied by the cost per dampers. The optimization is done with the NSGA-II code. The parameters used by NSGA-II are number of generations and number of individuals. The number of individuals indicates the end of sections which is length of retrofit in the pre-described given range and number of generations indicates repetition of optimization process to get fully optimized solution. The number of sections to be retrofitted is the input along with the range of sections. The cost function is devised as the amount of cross sectional retrofit and number of dampers multiplied by the respective costs. The code selects the amount of section based on the algorithm developed and optimizes the solution.

The analysis was done for population equal to 50 and generations equal to 2000. The cost of retrofit is set as $3000 per meter of length and the cost per damper is set as $100000. The optimization run gives the 50 different solutions with cost and corresponding deflections.
Cross-sectional retrofit:

The code was first run for the cross sectional retrofit only. The result of the simulation run is as shown in figure 5.6. The graph shows the optimized solution for cost in million dollars vs displacement in meters. The solution point ‘A’ gives the displacement of 0.16m and the corresponding cost of retrofit is 0.19 million dollars. For the solution point ‘B’ the displacement is restricted to 0.075m but the cost involved is around 0.6 million dollars. The solution point ‘C’, has the displacement of 0.04m and the cost involved is 1.25 million dollars. From the graph it is evident that the amount of retrofit involved is less in case of solution point ‘A’ whereas it is more in case of solution point ‘C’. The displacement in case of solution point ‘A’ is more whereas it is restricted to lower value in case of solution point ‘C’. Thus it can be inferred that, as the solution is optimized by NSGA-II, the solution points are shifted towards the axis.

Cross sectional retrofit and tuned mass dampers (TMD):

The code was run for the cross sectional retrofit and tuned mass dampers (TMD). To simulate the dampers the additional damping parameters are modeled in the code. The result of simulation is presented in figure 5.7. The graph shows the optimized solution for the cost vs displacement. The point ‘A’ indicates the optimized solution when displacement is 0.13m and the corresponding cost is 0.3 million. For the solution point ‘B’ the displacement is decreased to 0.09m and cost is increased to 0.6 million dollars. The point ‘C’ indicates the solution with minimum displacement of 0.04m and the cost involved is 2.4 million dollars.
In figure 5.6, for the case of cross sectional retrofit, the graph is parallel to cost function at the displacement equal to 0.015m. It indicates that the solution cannot be optimized further. For the
given set of bridge data, and the disturbing force, the vertical displacement of 0.045m will always present even though the amount cost of retrofit (cost) is increased. The client has to choose the solution point depending on the requirement from the solution point ‘C’ to solution point ‘A’.

In figure 5.7, for the case of cross sectional retrofit and dampers, the graph is parallel to the cost axis at the displacement of 0.04m. For all the solution points above the solution point ‘C’, the displacement cannot be decreased even with the more cost input. The client has to choose the solution from solution pint ‘C’ to solution ‘A’. There is sudden decrease in displacement from the solution point ‘A’ to solution point ‘B’ without significant increase in cost. This behavior indicates that for the given system, the dampers are active for the solution points between ‘A’ and ‘C’.

All the solution points between ‘C’ and ‘A’ are the correct solutions. This solution set is called as Pareto-optimal solution set. By comparing both the graph it can be inferred that for the given situation, the addition of dampers reduces the response of bridge to great extent with the reduced amount of retrofit. It is evident from the graphs that the Pareto-optimal front is shifted towards both the cost and displacement axis in case of combined approach of cross-sectional retrofit and dampers, compared to the case of cross-sectional retrofit only. The evidence of optimization can further be corroborated by the presence of more solution point in the region near to both the objective function axis.

5.2.3 Analysis using measured and synthesized aerodynamic coefficients:

Aerodynamic coefficients for non retrofitted (original) and retrofitted section is as shown in figure 5.8. Original section coefficients are obtained from the wind tunnel test performed by Dr. Soon-Duck Kwon, Chonbuk National University, Korea. The retrofitted section coefficients are synthesized based on the reasoning that retrofits removes the change in coefficients due to the vortex-induced vibration. Horizontal axis indicates normalized velocity, vertical axis indicates aerodynamic coefficient for lift. Here the velocity is normalized by circular frequency.
5.2.3 Base line problem:

Figure 5.9 and 5.10 illustrates the base line problem for second example. For illustration purpose the section selected, are the sections where the response is maximum under no retrofit condition. From the figure 5.2, these sections are 160m, 425m and 680m. Figure 5.9 shows the graph for rms displacement vs wind speed for no retrofit condition. Figure 5.10 shows the graph for rms displacement vs wind speed when all the sections are retrofitted.

Under no retrofit condition, the maximum response at 160m is found to 0.34m at the wind speed of 40m/s. The response at 425m is 0.26m when the wind speed reaches to 17m/s. The response at 680m is 0.31m when the wind speed reaches to 29m/s. When all the sections are retrofitted, the response is found to be reduced compared to no retrofit condition as shown in figure 5.10.
Figure 5.9: Response of original section (Example 2)

Figure 5.10: Response of retrofitted section (Example 2)
Optimization with NSGA-II code example 2:

Figure 5.11 shows the optimized solution for cross sectional retrofit. For illustration purpose, three solution points are discussed here. Solution point ‘A’ has the displacement of 0.24m and the corresponding cost involved is 0.5 million dollars. For the solution point ‘B’, the displacement is 0.19m and the cost is 0.75 million dollars. For the solution point ‘C’ the displacement is 0.16m and the cost required is 1.6 million dollars. From the figure 5.11 it follows that the further reduction in displacement is not possible even with the increase in the cost.

Figure 5.12 shows the optimized solution when cross sectional retrofit and damper both are employed together. The solution point ‘A’ shows the displacement of 0.075m with the cost 0.9 million dollars. The solution point ‘B’ shows the optimized solution where the displacement is 0.06m and the cost required is 1.25 million dollars. For the solution point ‘C’ 0.04m and the cost involved is 1.8 million dollars.

The graph in case of cross-sectional retrofit and TMD acting together is found to be shifted towards both the axis compared to the case of cross sectional retrofit only. It indicates the evidence of higher level of optimization in case of hybrid approach of cross sectional retrofit and TMD, compared to the case of cross sectional retrofit only. The graph becomes parallel to the cost axis at the displacement of 0.04m. It indicates that, for the given set of system, further optimization is not possible.
Figure 5.11: Optimized solution- cross sectional retrofit (Example 2)

Figure 5.12: Optimized solution- cross sectional retrofit and dampers (Example 2)
5.3 Acceptance criterion for allowable displacement:

The criterion for the acceptance for the allowable response of a bridge varies with the purpose for which the bridge is intended. For the bridges in United States of America, the guidelines for the deflection criterion have been given in ASSHTO 4th Edition 2007. As per the clause C2.5.2.6.2, the deflection limit for vehicular and general purpose bridge is span/800. For vehicular and pedestrian load the deflection limit is restricted to span/1000. According to the norms set by various states, this deflection criterion varies to suit the requirement.

The acceptance criterion for the bridge deflection can be looked upon with the acceptable acceleration to the users of the bridge. The criterion for acceptable acceleration depends on human perception. The acceleration criterion in North America and England is given by author of [11]. In North America [12] the comfort level criterion adopted is the maximum peak acceleration should be less than 50mg for the wind speed less than 13m/s and less than 100mg for wind speed between 13m/s and the design wind speed of the bridge. In England the maximum peak acceleration should be less than 40 mg for wind speed lower than 20m/s. The author of [11], has proposed formula for the acceptable acceleration.

\[ A_{peak} \leq 0.1g e^{0.9163(U-U_d)/(U_d-20)} \]  \hspace{1cm} (5.1)

Where
\[ U = \text{wind speed} \]
\[ U_d = \text{designed wind speed} \]

The North American criterion [12] for acceleration criteria is summarized as follows

\[ A_{peak} \leq 0.04g \text{ for } U \leq 20 \text{ m/s} \]  \hspace{1cm} (5.2)

\[ A_{peak} \leq 0.1g \text{ for } 20 \text{ m/s} < U \leq U_d \]  \hspace{1cm} (5.3)

Where \( A_{peak} \) is the maximum peak acceleration in m/s\(^2\) of the bridge deck and g is the acceleration due to gravity.
Illustration:

The equation of motion for steady state response is given as

\[ U = C \sin \omega t + D \cos \omega t \]  

(5.4)

1\textsuperscript{st} differentiation gives

\[ \dot{U} = C \omega \cos \omega t - D \omega \sin \omega t \]  

(5.5)

2\textsuperscript{nd} differentiation gives

\[ \ddot{U} = -C \omega^2 \sin \omega t - D \omega^2 \cos \omega t \]  

(5.6)

\[ \ddot{U} = -\omega^2 (C \sin \omega t + D \cos \omega t) \]  

(5.7)

Substitute equation 5.4 in 5.7

\[ \ddot{U} = -\omega^2 U \]  

(5.8)

Thus the acceleration can be expressed as product of square of natural circular frequency and displacement of bridge deck. Rearranging equation 5.8 we get

\[ U = \frac{-\ddot{U}}{\omega^2} \]  

(5.9)

Taking absolute value

\[ U = \frac{\ddot{U}}{\omega^2} \]  

(5.10)

In order to illustrate the approach for acceptable displacement criteria and the cost involved we will take the sample data from illustrative example 1.

We will consider the first vibration mode of the bridge deck. The frequency corresponding to first mode is 0.32Hz

\[ f_1 = 0.32 \text{Hz} \]
\[ \omega = 2\pi f_1 \]
\[ \omega = 2.01 \text{ rad/s} \]
According to North American standard maximum acceleration for the wind speed more than 20m/s and less than design speed say 50 m/s is given by equation 5.3

\[ \ddot{U} = 0.1g \]
\[ \ddot{U} = 0.1 \times 9.81 \]
\[ \ddot{U} = 0.981 \text{ m/s}^2 \]

From equation 5.10

\[ U = \frac{0.981}{(2.01)^2} \]
\[ U = 0.2428 \text{ m.} \]

For given bridge, according to the North American criteria the acceptable displacement is 0.2428m. The figure 5.13 shows the optimized solution for cross-sectional retrofit and dampers, where the RMS displacement is multiplied by the peak factor of 1.5 to obtain the maximum acceptable displacement. According to North American criteria, the acceptable displacement is 0.2428m and the corresponding cost is 0.25 million dollars.

As there are many uncertainties like variable wind speed, assumption of strouhal number depending on the cross sectional shapes, assumption of values for lift coefficient, band width parameter, coherence length, multiple solutions rather than single solution should be evaluated. Depending upon the requirement any solution point can be selected on the left hand side of point ‘A’ shown. The bell-curve in figure 5.13 schematically illustrates why the decision is necessary due to uncertainties. Also, if the budget permits, further reduction of the displacement may be desirable. The proposed multi-objective approach enables the decision-maker to easily compare various candidates.
Figure 5.13: Acceptance criteria for optimized solution- cross sectional retrofit and dampers

Decision required due to the uncertainties.

Acceptable displacement according to North American standard.
CHAPTER 6

CONCLUSION AND FUTURE WORK

Conclusion:

The approach to mitigate the vortex induced response in a bridge deck is presented in this research work. The relevance of the research topic is touched upon at the start of research work. The fact is corroborated with the reference to the practical example in the past. The overall response of a bridge deck to the wind force is a complex phenomenon. The complexity in the analysis arises due to the coupling of more than two modes of vibration. The analysis presented here is single mode single component analysis. It is discussed briefly with the help of realistic assumption and mathematical equations based on dynamics. Based on analysis, the flow chart and Matlab code is developed. The Matlab code gives the maximum rms response of the bridge at the desired section. The displacement of bridge deck is function of velocity of wind. In practical situation it may not be possible to mitigate the response by the cross-sectional retrofit alone or a certain type of response may not be suppressed by one certain type of retrofit alone. In such cases, combined approach of cross-sectional retrofit and addition of tuned mass dampers (TMD) can be used. The possibility and the effectiveness of combined approach is studied and validated with the help of two examples in the present research work. First the base line problem is discussed with example and graph for the case of cross sectional retrofit only. Then the Matlab code was used to analyze the vortex induced response. The retrofit of bridge being situation of multiple objectives of conflicting nature, suitable optimization technique is required for complete analysis. The Matlab code for vortex induced response is integrated with the Non-dominated Sorting genetic Algorithm (NSGA-II) in order to optimize the solution. The computational methodology is developed by setting the cost function against the maximum rms displacement. The results are discussed with the help of graphs. Though all the Pareto-optimal solutions resulted by optimization code are correct, not a single solution can be termed as the best solution for a problem. The selection of a solution can be done based on the specific requirement. The process is discussed with the help of an example of North American criteria for acceptable acceleration and displacement.
Future work:

The research work presented here, is for single mode single component analysis of vortex induced vibration. This work can further be extended to multi-mode, multi-component analysis of vortex induced vibrations. The Matlab code can be further upgraded, to incorporate the multi-mode, multi-component analysis. For complete analysis of the response, the present work can be extended to the buffeting response of the bridge.
APPENDIX

NOMENCLATURE

ae Indicates terms related to aerodynamic
B Width of bridge
b\textsubscript{z} Non-dimensional load spectrum bandwidth coefficient (value 0.1 – 0.3)
D Depth of bridge
f\textsubscript{0} Natural frequency of vibration
H\textsubscript{\eta}(\omega), H\textsubscript{\eta}(\omega) Frequency response function or matrix
H\textsubscript{1} Aerodynamic derivatives related to pure vertical displacement
I Identity matrix
I\textsuperscript{\text{f}} Imaginary number (\sqrt{-1}), index variable
L, L\textsubscript{exp} Length, wind exposed length of bridge
m, M Mass or mass matrix
\bar{m}\textsubscript{i} Equivalent modal mass
modal\_int\_coef Modal integral coefficient
n Index number
Nm No. of modes in vertical direction
Ns No. of sections with different aerodynamic coefficient
Q, Q Wind load or wind load vector at system level
q, q Wind load or wind load vector at cross sectional level
q\textsubscript{z} Non-dimensional rms lift coefficient
r, r Cross sectional displacement or rotation, displacement vector
S, S Auto or cross spectral density or cross spectral density matrix
S\textsubscript{t} Strouhal number
V\textsubscript{\text{max, min, me}} Cross wind velocity maximum, minimum, mean
x,y,z Coordinate axis
\zeta or \zeta Damping ratio or damping ratio matrix
\zeta\textsubscript{ae} Aerodynamic damping
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>Generalized coordinates or vector containing $N_{\text{mod}}$ $\eta$ components.</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Non-dimensional coherence length scale (value 2 – 5)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Air density</td>
</tr>
<tr>
<td>$\phi_y, \phi_z, \phi_\theta$</td>
<td>Mode shape component in $y$, $z$, $\theta$ direction</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Circular frequency of the (rad/s)</td>
</tr>
<tr>
<td>$\omega_i$</td>
<td>Still air eigen frequency associated with mode shape $i$</td>
</tr>
<tr>
<td>$\omega_s$</td>
<td>Circular frequency associated with vortex shedding</td>
</tr>
<tr>
<td>$x_r$</td>
<td>Chosen Span wise position for response calculation</td>
</tr>
<tr>
<td>$\sigma, \sigma^2$</td>
<td>Standard deviation, variance</td>
</tr>
<tr>
<td>$\phi^T$</td>
<td>Transposed of a vector or matrix</td>
</tr>
<tr>
<td>$^\wedge$</td>
<td>Indicates normalized quantity</td>
</tr>
<tr>
<td>$\sim$</td>
<td>Indicate modal quantity</td>
</tr>
</tbody>
</table>
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BIOGRAPHICAL SKETCH

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Atul Sudhakar Patil graduated with bachelors’ degree in civil engineering from University of Mumbai, India, in December 1999. He then worked in a capacity of site engineer with construction firms. While working as site engineer, he got interested in designing aspect of civil engineering structures and then worked with different designing, consulting firms as design engineer. He joined as a masters’ student at Florida State University in Fall 2008. He conducted the research under the guidance of Dr. Sungmoon Jung. His research interests lies in earthquake engineering, structural engineering, designing of reinforced concrete and steel structures.