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ANALYSIS OF AFTEREFFECT PHENOMENA AND 
NOISE SPECTRAL PROPERTIES OF MAGNETIC HYSTERETIC 
SYSTEMS USING PHENOMENOLOGICAL MODELS OF HYSTERESIS

By

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To God be the glory, great things He has done. To my loving parents, Dr. Mrs. Omobola Oluyinka Adedoyin (Iya Rainbow) and Prof. John Akintayo Adedoyin, who willingly sacrificed their dreams to provide the best possible education I could have dreamed of. To Uncle Dayo and Aunty Mfon Adedoyin, I sincerely thank you for your countless support and advice over the years. To my siblings, I thank you for your continued prayers and undying support. To Deolu, thank you for opening my eyes to the wonderful world of mathematics and physics, I will never be able to repay you for all your support and mentorship. Finally, to my entire family and friends, I thank you for all the love and support you have shown me throughout my collegiate career.
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# LIST OF SYMBOLS

- \(a\) Fitting parameter in the Langervin model of hysteresis
- \(\alpha\) Ratio of applied magnetic field energy to thermal energy. Also denotes the mean-field parameter. Used to denote demagnetizing energy parameter in the Jiles-Atherton model of hysteresis.
- \(a_{\phi}\) Two dimensional function of \(\phi\) used as a fitting parameter in the Jiles-Atherton model
- \(\alpha_{\phi}\) Two dimensional function of \(\phi\) used as a fitting parameter in the Jiles-Atherton model
- \(A_i\) Fitting parameters in the Hodgdon model (\(i=1,\ldots,4\))
- \(b\) Inverse correlation time for the Ornstein Uhlenbeck stochastic input signal
- \(B_{bp}\) Fitting parameter used in the Hodgdon model
- \(B_{ci}\) Induction parameter at the closure point used in the Hodgdon model
- \(\beta\) Relaxation time parameter
- \(c\) Denotes the reversible and irreversible components of magnetization in the Jiles-Atherton model of hysteresis
- \(c_{\phi}\) Two dimensional function of \(\phi\) used as a fitting parameter in the Jiles-Atherton model
- \(c_r\) Ratio of the domain or grain geometry in the Energetic model of hysteresis
- CPM Classical Preisach model of hysteresis
- \(\hat{\Gamma}\) Scalar hysteresis operator
- \(\Gamma_{\alpha(r)}\) Denotes the three dimensional hysteresis operator
- \(\hat{\Gamma}_{\phi}\) Denotes the scalar hysteresis operator along direction \(\phi\)
- \(\hat{\gamma}_{\alpha}\) Elementary hysteresis operator for the reversible component of magnetization in the Preisach model with critical field \(H_\alpha\)
- \(\hat{\gamma}_{\alpha\beta}\) Elementary hysteresis operator for the irreversible component of magnetization in the Preisach model with critical fields \(H_\alpha\) and \(H_\beta\)
- \(d\dot{x}\) Denotes the Itô stochastic differential equation
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\delta$</td>
<td>Denotes parameter that introduces hysteresis into the Hodgdon model and the Jiles-Atherton model of hysteresis</td>
</tr>
<tr>
<td>$\delta_\phi$</td>
<td>Introduces hysteresis in the case of the two dimensional Jiles-Atherton model</td>
</tr>
<tr>
<td>$\Delta v$</td>
<td>Volume of independent magnetic moments</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>Time increment – numerical implementation</td>
</tr>
<tr>
<td>$\exp$</td>
<td>Exponential function</td>
</tr>
<tr>
<td>$E[\bullet]$</td>
<td>Denotes the expected value of the argument.</td>
</tr>
<tr>
<td>$\text{EM}$</td>
<td>Energetic model of hysteresis</td>
</tr>
<tr>
<td>$f$</td>
<td>Output variable of a hysteretic system</td>
</tr>
<tr>
<td>$f_i$</td>
<td>Denotes the irreversible magnetization process in the classical Preisach model</td>
</tr>
<tr>
<td>$f_0$</td>
<td>Value of the output at the last reversal point in the Energetic model</td>
</tr>
<tr>
<td>$f_r$</td>
<td>Denotes the reversible magnetization process in the classical Preisach model</td>
</tr>
<tr>
<td>$\text{FC}$</td>
<td>Field-Cooling</td>
</tr>
<tr>
<td>$\text{FORC}$</td>
<td>First order reversal curve</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Dissipation constant related to the variance of stochastic input signal</td>
</tr>
<tr>
<td>$g$</td>
<td>Adaptive constant used to fit ideal magnetization curves to measured data in the framework of the Energetic model</td>
</tr>
<tr>
<td>$g_0$</td>
<td>Adaptive constant used to fit ideal magnetization curves to measured data in the framework of the Energetic model under no stress</td>
</tr>
<tr>
<td>$G$</td>
<td>Total free energy of a volume of magnetic moments</td>
</tr>
<tr>
<td>$\text{GPM}$</td>
<td>Generalized Preisach model of hysteresis</td>
</tr>
<tr>
<td>$h$</td>
<td>Parameter related to nonlinear material behavior in the Energetic model</td>
</tr>
<tr>
<td>$h_0$</td>
<td>Parameter related to nonlinear material behavior in the Energetic model under no stress</td>
</tr>
<tr>
<td>$H$</td>
<td>Vectorial applied magnetic field</td>
</tr>
<tr>
<td>$H_a$</td>
<td>Applied magnetic field</td>
</tr>
<tr>
<td>$H_\alpha$</td>
<td>Upper critical fields</td>
</tr>
<tr>
<td>$H_\beta$</td>
<td>Lower critical fields</td>
</tr>
<tr>
<td>$H_c$</td>
<td>Coercive field</td>
</tr>
<tr>
<td>$H_g$</td>
<td>Arbitrary value of magnetic field used in the Energetic model</td>
</tr>
</tbody>
</table>
\(H_0\) Center of the irreversible Presiach distribution

\(H_r\) Arbitrary value of magnetic field used in the Energetic model

\(H_\sigma\) Irreversible Presisach distribution standard deviation for a uniform distribution

\(H_{\alpha c}\) Irreversible Presisach distribution standard deviation along the coercive field axis

\(H_{\alpha i}\) Irreversible Presisach distribution standard deviation along the interactive field axis

\(H_{\alpha r}\) Fitting parameter for the reversible Preisach distribution

\(H_x\) Component of magnetic field along the x direction

\(H_y\) Component of magnetic field along the y direction

HM Hodgdon model of hysteresis

i.i.d. Independently and identically distributed

JAM Jiles-Atherton model of hysteresis

\(k\) Domain wall pinning parameter in the Energetic model and Jiles-Atherton model

\(k_B\) Boltzmann constant \((1.3806503 \times 10^{-23} \text{JK}^{-1})\)

\(k_0\) Domain wall pinning parameter of the Energetic model under no stress

\(\kappa\) Fitting parameter used in the Energetic model to calculate initial magnetization curve

\(K_1\) Anisotropy constant for temperature dependent magnetic processes

\(k_\varphi\) Two dimensional function of \(\varphi\) used as a fitting parameter in the Jiles-Atherton model

\(\lambda_s\) Magnetostriction constant at saturation

\(L\) Denotes the Langevin function

\(L_\varphi\) Langevin function dependent on angle \(\varphi\)

LM Langevin model of hysteresis

\(m\) Normalized magnetization

\(m_g\) Normalized magnetization from arbitrary magnetic field value \(H_g\)

\(m_0\) Normalized value of the output at the last reversal point in the Energetic model

\(m_r\) Normalized magnetization from arbitrary magnetic field value \(H_r\)

\(M\) Average magnetization of a volume of magnetic moments

\(M_g\) Magnetization corresponding to arbitrary magnetic field \(H_g\)

\(M_i\) Component of magnetization along the x,y, or z axis
Remanent magnetization. Also denotes magnetization corresponding to arbitrary magnetic field $H_r$

$M_s$ Magnetization of saturation value

$M_{s0}$ Magnetization of saturation at $T = 0$

$M_T$ Truncated Fourier transform of the output process $m(t)$ of a hysteretic system

$M_x$ Component of magnetization along the x direction

$M_y$ Component of magnetization along the y direction

$M_z$ Component of magnetization along the z direction

$\mu_0$ Permittivity of free space

$\mu_i$ Rate-independent parameter used in Hodgdon model

$n_i$ Specific direction of magnetization of each statistical domain in the Energetic model

$N$ Number of independent magnetic moments

$N_e$ Total demagnetizing factor in the Energetic model

$N(0,1)$ Random variable normally distributed with zero average and unit variance

PM Preisach model of hysteresis

$P$ Irreversible component of the Preisach distribution function

PSD Power spectral density

$P$ Time average power of a truncated stochastic signal $x_T$

$q$ Parameter related to pinning in the Energetic model

$q_0$ Parameter related to pinning in the Energetic model under no stress

$r$ Unit vector specifying direction of three dimensional hysteresis operator

$\rho$ Denotes the probability distribution function of the stochastic input signal

$R$ Reversible component of the Preisach distribution function

$S$ Scalar viscosity coefficient. Also denotes power spectral density of the output of a hysteretic system

$S_i$ Component of viscosity coefficient along the x, y, or z axis

$S_M$ Power spectral density of the output process $m(t)$ of a hysteretic system

$S_0$ Universal curve after data collapse phenomena

$S^+$ Denotes area of switched up hysterons in the Preisach plane
\( S^- \)  Denotes area of switched down hysterons in the Preisach plane

\( S_{3D} \)  Effective three dimensional viscosity coefficient

\( \sigma_c \)  Irreversible Presisach distribution standard deviation

\( \sigma^2 \)  Variance of stochastic input signal

\( \sigma \)  Standard deviation of the stochastic input signal. Also denotes Diffusion coefficient for the Ornstein Uhlenbeck stochastic input signal.

\( t \)  Time in arbitrary units (a.u.)

\( T \)  Absolute temperature. Also denotes finite time period. Used to denote the limiting triangle in the Preisach model.

\( T_c \)  Curie temperature

\( T_0 \)  Room temperature in Kelvins

\( v_i \)  Statistical domains (small volumes) in the framework of the Energetic model

\( v_p \)  Poisson ratio

\( v \)  Volume of a magnetic particle

\( \omega \)  Angular frequency (in arbitrary units in this dissertation)

\( w_d \)  Energy density of the demagnetizing field in the Energetic model

\( w_H \)  Total energy density of the applied magnetic field in the Energetic model

\( w_i \)  Energy density of the irreversible statistical domains in the Energetic model

\( w_M \)  Energy density of the ferromagnetic material in the Energetic model

\( w_r \)  Energy density of the reversible statistical domains in the Energetic model

\( W \)  Wiener process

\( \hat{\phi} \)  Unit vector along the direction specified by polar angle \( \phi \)

\( \tilde{x}(t) \)  Stochastic input used to model presence of thermal agitation

\( x_{app} \)  Denotes the external applied magnetic field

\( x \)  Input variable of a hysteretic system

\( x_0 \)  Constant applied magnetic field. Also denotes the mean value of the stochastic signal.

\( x_d \)  Drift coefficient for the Ornstein Uhlenbeck stochastic input signal

\( x_T \)  Truncated signal of \( x \) between two finite time periods

\( X \)  Fourier transform of a stochastic signal \( x \)
$X_T$  Fourier transform of a truncated signal $x$

$\dot{\chi}$  Generalized (differential) susceptibility

$\chi_c$  Differential susceptibility at coercivity

$\chi_i$  Initial susceptibility

$Z$  Multiple independent moment partition function

ZFC  Zero field cooling

$Z_m$  Single moment partition function

$\langle \bullet \rangle$  Denotes the expected value of the argument
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ABSTRACT

A robust and computationally efficient Monte-Carlo based technique is developed to analyze the magnetic aftereffect and noise passage phenomena in magnetic hysteretic systems by using phenomenological models of hysteresis. The technique is universal and can be applied to model the aftereffect and noise passage phenomena in the framework of both scalar and vector models of hysteresis. Using this technique, we analyze a variety of magnetic viscosity phenomena. Numerical results related to the decay of the magnetization as a function of time as well as to the viscosity coefficient are presented. It is shown that a log\(t\) (logarithmic time) - type dependence of the average value of the magnetization can be predicted qualitatively in the framework of phenomenological models of hysteresis, such as the Preisach, Energetic, Jiles-Atherton, and Hodgdon models.

The basic assumption of the techniques developed in this dissertation is that the total applied field is equal to the external applied field plus a random perturbation field. The total magnetic field is used as input in the scalar or vector models of hysteresis (vector models of hysteresis are defined in this dissertation as a superposition of scalar models of hysteresis distributed along all possible spatial directions). A statistical approach is developed to compute the average value and direction of the magnetization vector as a function of time. Whereas in the case of isotropic materials the magnetization vector usually moves on a straight line oriented towards the direction of the applied field, in the case of anisotropic materials the magnetization vector can switch from one easy axis to another and cross the direction of the applied field. It is shown that, depending on the initial hysteretic state, the trajectory of the magnetization vector can deviate substantially from the straight line, which is a pure vectorial relaxation effect.
The vectorial properties of magnetic viscosity and data collapse phenomena are also investigated. The definition of the viscosity coefficient, which has been traditionally used to model aftereffect phenomena in scalar magnetic systems, is generalized in order to describe three dimensional systems, where both the direction and the magnitude of the magnetization vector can change in time. Using this generalization of the vector viscosity coefficient, we have analyzed data collapse phenomena in vectorial magnetization processes. It was found that the traditional bell-shaped curves of the scalar viscosity coefficient as a function of the applied field can have one or more maxima in the case of vectorial systems. The data collapse phenomena seem to apply to simple magnetization processes (such as first-order rotational reversal curves); however, it cannot be generalized to more complex magnetization processes because of the relatively complicated magnetization dynamics.

In the final part of this dissertation we present a statistical technique based on Monte-Carlo simulations, which we developed to compute the spectral densities of the output variable in phenomenological models of hysteresis. The input signal is described by an Ornstein-Uhlenbeck process and the magnetization is computed by using various phenomenological models of hysteresis: the Energetic, Jiles-Atherton, and Preisach models. General qualitative features of these spectral densities are examined and their dependence on various parameters is discussed. For values of the diffusion coefficient near and smaller than the coercive field, the output spectra deviate significantly from the Lorentzian shape, characteristic to the input process. The intrinsic differences between the transcendental, differential, and integral modeling of hysteresis yield significantly different spectra at low frequency region, which reflect the long-time correlation behavior.
CHAPTER 1

HYSTERETIC SYSTEMS

1.1 Introduction

The hysteresis phenomena is observed in a system where a quasi-static cyclic variation of the input causes a corresponding lagging cyclic variation of the output. Systems that exhibit this phenomena are referred to as hysteretic systems. In this dissertation we wish to analyze several aspects of this phenomena that are present in magnetic hysteretic systems. Although the numerical simulations presented in the dissertation focus on magnetic hysteretic systems, the algorithms and numerical techniques developed here can also be applied to study of hysteresis phenomena in many other areas such as earthquake science, neuroscience, and economics [1, 2].

The behavior of most magnetic systems is governed by the hysteresis phenomena. In fact, all applications of ferromagnetic and ferritic systems rely heavily on the hysteresis phenomena in one way or the other [3] making the hysteresis phenomena a very important subject area to study. The study of the hysteresis phenomena has attracted many researchers for a number of years however only recently have there been advances made to understand the physical mechanisms involved in this phenomena [4-7]. The existing studies reveal that the hysteresis phenomena is due to the multiplicity of metastable energy states in a magnetic hysteretic system. In other words, the state of a magnetic system can change from a higher energy metastable state to a lower one as the hysteretic system attempts to reach the thermal equilibrium state.
In this dissertation, we consider magnetic hysteretic systems where the input is the magnetic field and the output is the total magnetization induced by the applied magnetic field input. We note here that the application of a magnetic field on a magnetic material will cause a net magnetization in the system. However, over time this magnetization changes gradually and irreversibly due to the movement of the hysteretic system from one metastable energy state to another brought about by thermal perturbation. This change between metastable energy states is referred to as the magnetic aftereffect phenomena, thermal relaxation phenomena, or magnetic viscosity phenomena. The thermal relaxation phenomenon is a highly unpredictable phenomenon. The lack of mathematical tools to describe this phenomenon makes the study of hysteretic systems a difficult one. In fact, the behavior of a hysteretic system can often be described as a highly complex non-linear stochastic process. Our attempt to investigate the physical mechanism of such a system involves modeling and simulating magnetic hysteretic systems by using various phenomenological models of hysteresis which are discussed in the next section.

1.2 Models of hysteresis

The existing models of hysteresis can be classified in two categories, according to how much physical information they use or provide to predict the magnetization curves.

a) The first category of models of hysteresis includes the so called physical models of hysteresis which start from first principles and are based on a rigorous physical description of the phenomena involved. These models are usually based on various approximations of the Hamiltonian of the magnetic system under consideration or on other micromagnetic approximations (such as the Landau-Lifshitz or Landau-Lifshitz-Gilbert equations) and can be solved numerically only for very simple magnetic systems such as one or a few particle systems. Unfortunately, these models become computationally very demanding when applied to more realistic magnetic systems, particularly when we would like to analyze the magnetic system over a longer period of time.
b) The second category of hysteresis models include *phenomenological models* that are based on relatively simple analytical (mathematical) equations and do not contain much fundamental physics. These models are also called mathematical models of hysteresis and are usually much simpler to use when modeling and simulating real magnetic systems. Phenomenological models of hysteresis can be expressed as differential, integral, or even algebraic equations for the magnetization as a function of the magnetic field. The advantage of phenomenological models of hysteresis is that they are usually much easier to implement numerically and the total computational time is orders of magnitude smaller than in the case of physical models. For this reasons, phenomenological models of hysteresis are widely used today to model electronic or mechanic devices containing magnetic or other hysteretic materials. Examples of such devices are transformers, motor and actuators, piezoelectric devices, etc.

### 1.3 Motivation and methodology

The magnetic aftereffect phenomena in magnetic systems have drawn much attention in the last decade mainly because of their negative implications in the magnetic recording industry. At temporally constant external conditions the total magnetization of a magnetic system exhibits a relatively slow but irreversible loss of its magnetic history, which is usually detrimental for the time reliability of recording information.

Although it has been firmly established that the origin of aftereffect phenomena lies in the multiplicity of metastable states of hysteretic materials, the mathematical analysis of these phenomena is still far from being complete. The main reason for the relatively slow progress in the mathematical understanding of aftereffect phenomena is mostly related to the large number and strong nonlinearity of integro-differential equations that usually govern the relaxation phenomena and limit the theoretical findings and computational simulations to very simplified magnetic systems.
The relatively few attempts in the literature to model the vectorial properties of thermal relaxation are based on the solution of the Landau-Lifshitz equation, which is randomly perturbed by writing it in the form of a “Langevin” equation [8]. This approach can be applied to ensembles of a small number of monodomain ferromagnetic particles; however it cannot be applied to more complex magnetic systems such as bulk materials or systems of multidomain particles because of the difficulty in solving the Landau-Lifshitz equation. In this dissertation we generalize the concept of random perturbation fields and use it in phenomenological vector models of hysteresis [9-12]. In this way we combine the strong mathematical foundation of random perturbation fields used in micromagnetic modeling with the power and computational efficiency of phenomenological models of hysteresis.

The existing approaches for the analysis of the magnetic aftereffect phenomena model relaxation using two different approaches. The first approach [13,14] is based on thermal activation models and is usually applied to “non-interacting particle” magnetic systems or bulk materials. The second approach is based on the assumption that the effect of random thermal agitations is equivalent to the effect of a stochastic magnetic field superimposed on an external applied deterministic magnetic field. The second approach is a more general approach since it does not require any assumption about the nature of the magnetic system. This approach was first introduced by Neel [15,16] and then developed and applied to the Preisach model by Mayergoyz et al. [17] who established analytical equations for the decay of the average value of the output of a hysteretic system described by rectangular hysteresis loops. Although very powerful, the technique developed in [17] can only be applied to hysteresis models that are based on the superposition of elementary hysteresis loops, such as the classical and generalized Preisach models of hysteresis [12]. It is practically impossible to find analytical equations for the decay of the average value of the output signal in systems described by other phenomenological models of hysteresis like the Energetic [9], Hodgdon [11], and Jiles-Atherton [10] models of hysteresis.

The common feature of the existing approaches is that they mainly focus on the analysis of scalar thermal relaxation, in which the magnetization is measured only in one direction. The direction of magnetization usually coincides with the direction of the applied magnetic field, which makes
it possible to model thermal relaxation phenomena by using scalar models of hysteresis. However, this approach is appropriate for isotropic magnetic systems, in which the direction of applied magnetic field and magnetization are fixed. This approach is inappropriate for anisotropic magnetic systems or systems in which the magnetization is changing in time. Therefore, a full vectorial characterization of the thermal relaxation phenomena is essential when considering materials in which both the direction and the magnitude of the magnetization vector can change. In this dissertation we use an algorithm based on Monte-Carlo methods [18] to analyze the magnetic aftereffect phenomena by using vector phenomenological models of hysteresis. This Monte-Carlo based approach has the advantage that it is universal in the sense that it can be applied to any vector model of hysteresis. The scalar analysis and modeling of thermal relaxation phenomena can be regarded as a special case of the more general vector analysis.

1.4 Outline

This dissertation is structured as follows. The main phenomenological models of hysteresis used in the dissertation are presented in Chapter 2. The first part of this chapter presents the main scalar models of hysteresis; the second part presents their vectorial generalization. The last two sections of Chapter 2 present our approach to model rate-dependent magnetization process. Particular attention is given to the relaxation-time and effective-field approximations that we have implemented to describe rate-dependent phenomena. The last section of Chapter 2 presents temperature and mechanical stress dependent simulations performed by using the Energetic model of hysteresis. This model has the advantage that it can describe the effects of temperature and mechanical stress on the magnetization curves by using simple analytical equations. Our technique for the modeling and simulation of the magnetic aftereffect phenomena is developed in Chapter 3 for both scalar and vector phenomenological models of hysteresis. At the end of Chapter 3, we also present the scalar and vector results for the data collapse phenomena exhibited in magnetic hysteretic models. The filtering properties of phenomenological hysteretic systems are presented in Chapter 4. Conclusions are drawn in Chapter 5.
CHAPTER 2

PHENOMENOLOGICAL MODELS OF HYSTERESIS

In this Chapter we present the most commonly used phenomenological models of hysteresis. In Section 2.1 we present the hysteretic operator and the generalized susceptibility of the scalar models of hysteresis. In the same section we present the scalar phenomenological models of hysteresis with special emphasis on the Preisach model of hysteresis. Vector models of hysteresis are introduced in Section 2.2 as a superposition of scalar models of hysteresis. In Section 2.3 we present dynamic models of hysteresis followed by temperature and stress dependent hysteresis in Section 2.4.

2.1 Scalar models of hysteresis

Let us denote by \( x(t) \) the input variable of a hysteretic system and by \( f \) the output variable (such as polarization or magnetization). The input variable can be the magnetic field in the case of magnetic systems, the electric field in the case of piezoelectric systems, and the applied force in the case of mechanical systems. The hysteretic operator \( \hat{\Gamma} \) is defined as (see Figure 1a):

\[
f = \hat{\Gamma}x.
\]  

(1)

The generalized (differential) susceptibility, which is the rate the output varies with respect to the input, is defined only on those regions where function \( f \) is differentiable as (see Figure 1b):

\[
\hat{\chi} = \frac{df}{dx}.
\]  

(2)
Various phenomenological models of hysteresis use different equations to define either operator $\hat{\Gamma}$ or the generalized susceptibility as a function of input variable $x(t)$. In the following subsections we present a few such models that are often used to describe magnetic hysteresis. However, note that some of these models (e.g. the Preisach model) are often used to describe hysteresis in many other areas such as mechanics, economics, or biology.

![Diagram](image.png)

Figure 1. The hysteric operator $\hat{\Gamma}$ (a) and the (differential) susceptibility operator $\hat{\chi}$ (b). In the case of vector hysteresis models the input and output variables are two or three dimensional vectors.

### 2.1.1 The Langevin model

The Langevin model (LM) is one of the simplest models that describes the magnetization as a function of the applied magnetic field and is usually applied to super-paramagnetic materials in which there are no hysteresis losses. The model can also be applied to ferromagnetic materials at high temperatures, where the coercive field is negligible. In the framework of the Langevin model the output variable is expressed analytically in terms of the input variable by using:

$$f = M_s \left[ \coth \left( \frac{x}{a} \right) - \frac{a}{x} \right],$$

where $M_s$ is the saturation value of the output and $a$ is a fitting parameter related to the initial susceptibility of the material. Although the Langevin model does not describe scalar hysteresis (or branching) it is usually regarded as a zero-order hysteresis model on which other, more advanced phenomenological model of hysteresis are based. In fact, as it will be shown later in the dissertation, the dynamic and vector Langevin models can describe branching, for which reason it is often introduced in literature as a model of hysteresis. The Langevin model is often
the starting point for other models of hysteresis such as the Jiles-Atherton model or some versions of the Hodgdon model.

### 2.1.1.1 Derivation of Langevin equation for a system of magnetic dipoles

To derive of the Langevin equation let us assume that we have a magnetic system consisting of an assembly of identical permanent magnetic moments [3]. We also assume that the magnetic moments do not interact with each other and, therefore, we can neglect the magnetostatic dipolar coupling. The thermal behavior of an entire magnetic system is then considered to be a summation of the thermal behaviors of each magnetic moment. The thermal behavior of a single magnetic moment is computed from a single moment partition function, which is defined as:

$$Z_m(H_a, T) = \int_0^{2\pi} \int_0^\pi e^{\frac{\mu_0 m H_a \cos(\theta)}{k_b T}} \sin \theta d\theta d\varphi,$$

where $H_a$ is the applied magnetic field, $T$ is the absolute temperature, $\theta$ is the angle between the magnetization vector $m$ and the applied magnetic field, and the ratio of the energy due to the applied magnetic field to the thermal energy of each magnetic moment is given by $\frac{\mu_0 m H_a \cos(\theta)}{k_b T}$. By performing the integration in (4), the partition function becomes:

$$Z_m(H_a, T) = \frac{4\pi \sinh(\alpha)}{\alpha},$$

where $\alpha = \left(\frac{\mu_0 m H_a}{k_b T}\right)$.

The total partition function of a magnetic system with $N$ independent magnetic moments is the product of each single moment partition functions:

$$Z = [Z_m(H_a, T)]^N.$$

The Langevin equation computes the magnetization of a small volume of magnetic moments by minimizing the total free energy of the small volume. The total free energy of the magnetic moments can be computed by using:
The average magnetization of the small volume of magnetic moments is:

\[ M(H_a,T) = -\frac{1}{\mu_0\Delta V} \left[ \partial G(\mu H_a, kT) / \partial H_a \right] \], \hspace{1cm} (8)

where \( \Delta V \) is the volume occupied by the magnetic moments. Introducing (5)-(7) into equation (8), we obtain the following expression for the average magnetization of the system:

\[ M(H_a,T) = M_s \left[ \coth(\alpha) - \frac{1}{\alpha} \right] = M_s \left[ \coth\left( \frac{\mu_0 m H_a}{kT} \right) - \frac{k_T H_a}{\mu_0 m} \right] \], \hspace{1cm} (9)

where \( M_s = \frac{N m}{\Delta V} \). The last expression is identical to (3) in which \( a = \frac{\mu_0 m}{kT} \) and \( x = H_a \).

### 2.1.2 The Energetic model

#### 2.1.2.1 Model definition

The Energetic model (EM) (also known as the Hauser model) is a phenomenological model that can describe the magnetization of ferromagnetic materials as a function of the external applied magnetic field, temperature, and mechanical stress [9, 19-22]. The magnetization state is computed by minimizing the total energy density of the entire ferromagnetic material [19-21]:

\[ w_T = w_H + w_M \], \hspace{1cm} (10)

where \( w_H \) is the total energy density of the applied magnetic field. The latter term \( w_M = w_d + w_r + w_i \) is the energy density of the ferromagnetic material, where \( w_d, w_r \) and \( w_i \) are the energy densities of the demagnetizing fields, reversible, and irreversible statistical domains, respectively. In the framework of the Energetic model the magnetic material is divided into small volumes \( v_i \) (statistical domains) and each small volume has a specific direction of magnetization \( n_i \). The total magnetization (output variable) can be expressed as [23-26]:

\[ f = M_s \left| \sum_{i=1}^{N} v_i n_i \right| \], \hspace{1cm} (11)
where \( M_s \) is the magnetization of saturation. Therefore, the direction of the total magnetization of the material is computed by summing over all small statistical domains to compute the entire magnetization of the material. In the case of isotropic Energetic model the computations are usually simplified by considering only two directions of magnetization, parallel and antiparallel to the direction of the magnetic field. By using classical statistical methods it can be shown that the output variable is given analytically as a function of the input variable (which is the applied field) by simple algebraic equations. The initial magnetization curve is [9]:

\[
x = N_e M_s m + \text{sgn}(m) h \left[ \left(1 + m\right)^{1+m} \left(1 - m\right)^{1-m} \right]^{\frac{q}{2}} - 1 \right]
\]

\[
+ \text{sgn}(m-m_0) \left[ \frac{k}{\mu_0 M_s} + c_r H_r \right] \times \left[ 1 - \kappa e^{-\frac{q}{M_s} |m-m_0|} \right],
\]

(12)

where

\[
m = \frac{f}{M_s}.
\]

(13)

\( M_s \) is the saturation value of the output, \( N_e, h, g, c_r, k, \) and \( q \) are fitting parameters, and \( m_0 = \frac{f_0}{M_s} \) is the normalized value of the output at the last reversal point. Parameter \( \kappa \) depends on the past history of the material and, at each reversal point, \( \kappa \) is modified as:

\[
\kappa = 1 - \kappa_0 e^{-\frac{q}{M_s} |m-m_0|}.
\]

(14)

The simulation always starts with the initial curve \( (m_0 = 0, \kappa = 1) \). At each reversal point \( \kappa \) is calculated by (14) and \( m_0 \) is set to the actual value of \( m \) at this point. The hysteretic state at any moment in time is completely described by the absolute values of the input variable, output variable, parameter \( \kappa \), and direction (increasing or decreasing) of the input variable. Figure 2 shows sample magnetization curves obtained in HysterSoft by using the Energetic model for a sample barium ferrite material.
2.1.2.2 Noniterative parameter identification technique

In this subsection we present a noniterative technique for the computation of parameters of the Energetic model. This presentation follows closely the analysis presented in [27].

The technique described below computes the model parameters by using the following data: the coercive field $H_c$, the initial susceptibility in the demagnetized state $\chi_i$, the differential susceptibility at coercivity $\chi_c$, the remanent magnetization $M_r$, the magnetization $M_s$ corresponding to a magnetic field of arbitrary value $H_s$ on the ascending curve of the major loop, and the magnetic field $H_r$ on the ascending curve corresponding to a magnetization equal to $M_r$. These variables can be expressed as functions of the six model parameters by using the following equations that can be easily derived from the model’s basic equations presented in [9]:

![Energetic Model](image)

Figure 2. Major hysteresis loop (a) and sample magnetization curves (b) computed by using the Energetic model.
\[
H_e = \frac{k}{\mu_0 M_0} \left( 1 - 2e^{-\frac{q}{2}} \right),
\]  
(15)

\[
\frac{1}{\chi_i} = N_e + \frac{qk}{\mu_0 M_s^2},
\]  
(16)

\[
\frac{1}{\chi_c} = N_c + \frac{qk}{\mu_0 M_s^2} e^{-\frac{q}{2}},
\]  
(17)

\[
N_e M_s m_r + hG(m_r) - F(-m_r) = 0,
\]  
(18)

\[
N_e M_s m_s + hG(m_s) + F(m_s) = H_g,
\]  
(19)

\[
N_e M_s m_r + hG(m_r) + F(m_r) = H_e,
\]  
(20)

where

\[
m_s = M_s / M_s, 
\]  
(21)

\[
m_r = M_r / M_s, 
\]  
(22)

\[
F(m) = \left[ \frac{k}{\mu_0 M_s} + c, hG(m) \right] \left\{ 1 - 2\exp\left[ -\frac{q}{2} (1+m) \right] \right\},
\]  
(23)

and

\[
G(m) = \left[ (1+m)^{1-m} (1-m)^{1-m} \right]^\frac{q}{2} - 1 > 0.
\]  
(24)

Equations (15)-(20) represent a system of six nonlinear equations that need to be solved for the six model parameters. All model parameters should be positive in order for the model to describe magnetic hysteresis with positive differential susceptibilities. The conditions under which Equations (15)-(20) have no solution, one solution, or multiple solutions are described in the following. For now, let us note that Equations (15)-(17) involve only parameters \( k \), \( q \), \( N_e \) so these three equations can be solved first. Hence, the following subsection focuses on the computation of \( k \), \( q \), \( N_e \) alone; after that, we also present the technique for the computation of the remaining parameters.
A. Computation of $k$, $q$, and $N_c$

Equations (16) and (17) can be rearranged to obtain the following equation that needs to be solved numerically for $q$:

$$
\left( \frac{1}{\chi_i} - \frac{1}{\chi_c} \right) M_s = q \frac{1 - e^{-\frac{q}{2}}}{1 - 2e^{-\frac{q}{2}}}.
$$

(25)

Function $H(q) = q \frac{1 - e^{-\frac{q}{2}}}{1 - 2e^{-\frac{q}{2}}}$ is represented in Figure 3. The region for which $q < 2 \ln 2$ is not important to us because here $H(q)$ is negative, which implies that $\chi_i > \chi_c$. Hence, we focus only on the region where $q > 2 \ln 2$ and for which function $H$ has a minimum at $q_{\text{min}} \approx 2.668$.

Figure 3. (Color online) Function $H(q)$ that appear in the right-hand-side of Equation (25). The initial susceptibility $\chi_i$ predicted by the model is larger than the differential susceptibility at coercivity $\chi_c$ in region one and smaller than $\chi_c$ in region 2.
Equation (25) implies \( \left( \frac{1}{\chi_i} - \frac{1}{\chi_c} \right) \frac{M_s}{H_c} \geq H(q_{\text{min}}) \approx 4.153 \), which shows that the space of physical parameters that the energetic model can describe is bounded:

\[
\frac{1}{\chi_i} > \frac{1}{\chi_{i,\text{max}}} = \frac{1}{\chi_c} + \frac{4.153M_s}{H_c}.
\]  

(26)

Initial susceptibilities that do not satisfy this inequality cannot be modeled by the Energetic model of hysteresis. Once Equation (25) is solved for \( q \), parameters \( k \) and \( N_c \) can be found from:

\[
k = \frac{\mu_0M_sH_c}{1 - 2e^{-q/2}},
\]  

(27)

\[
N_c = \frac{1}{\chi_i} - \frac{qk}{\mu_0M_s^2}.
\]  

(28)

Figure 4. (Color online) Separation of physical parameters in the \( \chi_i - \chi_c \) space in parameters that cannot be modeled (region 1) and parameters that can be modeled by the EM (regions 2 and 3). For relatively large values of \( \chi_i \) the model predicts a negative demagnetizing factors \( N_c \) as implied from Equation (28).
Note that, as long as inequality (26) is satisfied, Equation (25) will always have two solutions and each of them can be computed by using the standard bisection technique.

The space of possible physical parameters for a material with $H_c = 7.76$ A/m (such as Armco FeSi steel sheets) is represented in Figure 4 by regions 2 and 3. The initial susceptibility and the susceptibility at coercivity of the FeSi steel sheets are $1.35 \times 10^4$ and $9.7 \times 10^3$, respectively. The model parameters that we obtain by using (25), (27), and (28) are $q = 15.3$, $k = 15.9$, and $N_e = 10^{-6}$.

In the following we describe the technique for the computation of parameters $h$, $g$, and $c_r$. We consider two cases that appear often in the literature: (1) the case when $c_r = 0$ which is also called the \textit{classical} Energetic model, and (2) the case when $c_r > 0$, which gives the \textit{generalized} Energetic model.

**B. Computation of $h$ and $g$ in the classical Energetic model ($c_r = 0$)**

In the case of the \textit{classical} Energetic model $c_r = 0$ and Equations (18)-(20) simplify substantially. By substituting $h$ from (18) into (19) we obtain the following equation that should be solved numerically for $g$:

$$H_g = N_e M_g + \frac{G(m_g)}{G(m_r)} \left[ \frac{k}{\mu_0 M_s} Q(m_r) - N_e M_s m_r \right] + \frac{k}{\mu_0 M_s} Q(m_g),$$ \hspace{1cm} (29)

where

$$Q(m) = 1 - 2 \exp \left[ -\frac{q}{s} (1 + m) \right].$$ \hspace{1cm} (30)

Once parameter $g$ is computed, parameter $h$ can be found by solving Equation (18). We obtain:

$$h = \frac{k}{\mu_0 M_s} \frac{Q(-m_r)}{G(m_r)} - \frac{N_e M_s m_r}{G(m_r)}.$$ \hspace{1cm} (31)

Equations (25), (27)-(31) are the equations that need to be solved sequentially in order to
compute all the model parameters. In the case of Armco FeSi steel sheets we obtain \( h = 0.29 \) A/m and \( g = 7.12 \).

It is important to analyze now the conditions under which \( h > 0 \) and \( g > 0 \). The condition \( h > 0 \) implies that the remanent magnetization of the material should be smaller than some limiting value \( M_{r,\text{max}} \), which can be found by solving

\[
\frac{k}{\mu_0 M_s} Q\left(-M_{r,\text{max}} / M_s\right) > N_e M_{r,\text{max}}.
\]

Due to the monotonicity of functions \( G \) and \( Q \), if \( M_r < M_{r,\text{max}} \) Equation (31) will always have unique solution. The condition \( g > 0 \), on the other hand, gives a limited set of values for \( H_g \) for any fixed \( M_g \). After some algebraic manipulations one can distinguish two cases:

1) If \( M_r < M_g \), then \( H_g \) should satisfy

\[
N_e M_g + \frac{k}{\mu_0 M_s} Q\left(m_g\right) > H_g \geq N_e M_g + h G\left(m_g\right)\Big|_{g=0} + F\left(m_g\right),
\]

which provides a lower limit for the possible values of the applied fields for which magnetization can be \( M_g \).

2) If \( M_r > M_g \), then \( H_g \) is bounded from both above and below, and

\[
N_e M_g + \frac{k}{\mu_0 M_s} Q\left(m_g\right) > H_g \geq N_e M_g + h G\left(m_g\right)\Big|_{g=0} + F\left(m_g\right).
\]

These conditions are important for solving the identification problem because magnetic materials that do not satisfy them cannot be modeled by the energetic model. For instance, in the case of Armco FeSi steel sheets, the space of available physical parameters is given by region 2 in Figure 5.
Figure 5. (Color online) Possible values of the magnetic field $H_g$ corresponding to a magnetization of $M_g = 1.49 \times 10^9$ A/m. Only parameters $(H_g, M_r)$ in region 2 can be described by the classical Energetic model.

C. Computation of $h$ and $g$ in the generalized Energetic model ($c_r > 0$)

In the case of the generalized energetic model $c_r > 0$ and parameters $g$, $h$, and $c_r$ can be computed from the following equations:

$$H_g = N_c M_g + \frac{G(m_g)}{G(m_r)} + \frac{k}{\mu_0 M_s} Q(m_g) + I(m_r, m_g), \quad (32)$$

$$h = \frac{1}{G(m_r)} \left[ \frac{H_r Q(-m_r)}{Q(m_r) + Q(-m_r)} - N_r M_r \right], \quad (33)$$

$$c_r = \frac{1}{fG(m_r)} \left[ \frac{H_r}{Q(m_r) + Q(-m_r)} - \frac{k}{\mu_0 M_s} \right], \quad (34)$$

which can be obtained from Equations (18)-(20). In the last equations we introduced the following notation:
\[ I(m_r, m_g) = H_r \frac{Q(-m_r) + Q(m_g)}{Q(m_r) + Q(-m_r)} - \frac{k}{\mu_0 M_s} Q(m_g) - N_r M_r m_r. \]  

(35)

Note that Equation (32) is a transcendental equation in \( g \) and should be solved numerically. By using the same line of reasoning as in the previous subsection we can derive the conditions for which \( h \) and \( g \) are positive. Condition \( h > 0 \) implies that

\[ \frac{(H_r - N_s M_r)Q(-m_r) - N_s M_r Q(m_r)}{Q(m_r) + Q(-m_r)} > 0, \]  

(36)

which shows that the remanent magnetization should lie outside the interval \([M_{r,\min}, M_{r,\max}]\), where \( M_{r,\min} \) and \( M_{r,\max} \) are the roots of the numerator and denominator in (36), respectively. It can be shown that both the numerator and the denominator in (36) have unique a solution in the interval \([0, M_s]\), which confirms the existence of the limits of interval \([M_{r,\min}, M_{r,\max}]\).

The possible values of \((H_g, M_g)\) that can be described by the generalized energetic model can be found by imposing \( g > 0 \) in Equation (32). It can be shown again that the possible values of \( H_g \) that can correspond to a measured value of the magnetization equal to \( M_g \) depend on the whether \( M_g \) is larger or smaller than \( M_r \). Equations somewhat more complex than the ones derived in the previous subsection hold in this case as well: if \( M_r < M_g \), \( H_g \) should be larger than some minimum value that can be computed by setting \( g = 0 \) and solving (32) for \( H_g \); if \( M_r > M_g \), \( H_g \) should lay between two critical values that can be computed by solving (32) for \( H_g \) in the limits \( g = 0 \) and \( g = \infty \). Figure 6 shows the space of available physical parameters for the Armco FeSi steel sheets in the \( M_r - H_g \) plane. Applying equations (32)-(34) we obtain \( h = 0.217 \text{ A/m}, \ g = 8.24, \) and \( c_e = 1.32 \).
The Hodgdon model is a mathematical model of hysteresis, which describes the magnetization curves by using relatively simple mathematical equations for the differential susceptibility [11, 28-32]. Although the model is not based on any physical assumptions it has been shown that it can describe remarkably well a large variety of magnetization processes in hysteretic inductors and magnetic cores [33]. The model can also describe simple dynamic magnetization processes by using more complex fitting formulas. In this dissertation we use a simplified version of the Hodgdon model, in which the time dependent phenomena are not taken into account. The main equations are presented below.

The output variable satisfies the following differential equation [33]:

$$\frac{df}{dx} = \frac{1}{\alpha \delta \left[ F(f) - x \right] + G(f)}^{-1},$$

(37)
where $\delta$ is equal to 1 if $x$ is increasing in time and -1 if it is decreasing and $F$ and $G$ are some piecewise smooth material functions. The model assumes the following expressions for $F$ and $G$:

$$F(f) = \begin{cases} 
A_1 \tan(A_2 f) & \text{if } |f| \leq B_{bp} \\
A_1 \tan(A_2 B_{bp}) + (f - B_{bp})/\mu_s & \text{if } f > B_{bp} \\
-A_1 \tan(A_2 B_{bp}) + (f + B_{bp})/\mu_s & \text{if } f < B_{bp}
\end{cases} \tag{38}$$

$$G(f) = \begin{cases} 
F'(f) \cdot \left[1 - A_4 \exp\left(-\frac{A_2 |f|}{B_{cl} - |f|}\right)\right] & \text{if } |f| < B_{cl} \\
F'(f), & \text{if } |f| \geq B_{cl}
\end{cases} \tag{39}$$

where $\alpha$, $\mu_s$, $A_1$, $A_2$, $A_3$, $A_4$, $B_{bp}$, $B_{cl}$ are some material parameters (rate-independent parameters); $B_{cl}$ is the induction at the closure point (where the anhysteretic curve reaches the major loop). The hysteretic state at any moment in the Hodgdon model is completely described by the absolute values of the input variable, output variable, and direction (increasing or decreasing) of the input variable. Figure 7 shows sample magnetization curves computed by using the Hodgdon model in HysterSoft.
2.1.4 The Jiles-Atherton model

The Jiles-Atherton model (JAM) of hysteresis (also called the Jiles model) is a phenomenological model that describes the change in the state of the magnetization of ferromagnetic materials by using simple differential equations for the magnetization as a function of time [10, 34-41]. Like in the case of the Energetic model, the Jiles-Atherton model takes into consideration the applied field, demagnetization, and domain pinning energies [42-50].

The output variable satisfies the following differential equation:

$$\frac{df}{dx} = (1-c) \frac{L(x + \alpha f) - f}{k (1-c) \text{sgn}(x) - \alpha [L(x + \alpha f) - f]} + c \frac{dL(x)}{dx},$$  \hspace{1cm} (40)$$

where $L$ is the “anhysteretic” curve, which is assumed to be a Langevin function:

$$L(x) = M_s \mathcal{L} \left( \frac{x}{a} \right) = M_s \left[ \coth \left( \frac{x}{a} \right) - \frac{a}{x} \right], \hspace{1cm} (41)$$

Figure 7. Major hysteresis loop (a) and sample magnetization curves (b) computed by using the Hodgdon model.
and \( \delta \) is equal to 0 if \( \delta \left[L(x+\alpha f)-f\right]\leq 0 \) and 1 otherwise. In the above equations, \( M_s \) is the saturation value of the output and \( a, c, \alpha, \) and \( k \) are some parameters that can be identified by fitting simulations to the experimental values of the hysteresis curves. Parameter \( \alpha \) is introduced to account for the demagnetization energy, parameter \( k \) is introduced to account for domain pinning, and parameter \( c \) is introduced to separate explicitly the reversible and irreversible components of the magnetization.

The hysteretic state of the Jiles-Atherton model is completely described by the absolute values of the input variable, output variable, and direction (increasing or decreasing) of the input variable. Figure 8 shows sample magnetization curves computed by using the Jiles-Atherton model in HysterSoft.

![Figure 8](image.png)

Figure 8. Major hysteresis loop (a) and sample magnetization curves (b) computed by using the Jiles-Atherton model.
2.1.5 The Preisach model

The Preisach model (PM) is probably the most efficient and commonly used phenomenological model to describe magnetic hysteresis. Numerous versions of the PM used to describe various effects of hysteresis in magnetic systems exist in the literature. However, in this Section we consider only two such versions. The simplest version of the PM is the classical Preisach model (CPM), which is also used as the foundation for developing other variations of the PM such as the moving PM or vector PM. The CPM was developed to describe the magnetization process in particulate media where the change in magnetization is solely due to the switching of the magnetic dipole moments of each particle from one state to another. Later it was found that the CPM had an important limitation in that the CPM was inadequate to describe reversible magnetization processes (such as reversible rotations of the magnetic dipole moments). This limitation is relaxed in the generalized Preisach model (GPM). The GPM of hysteresis [51-62] is among the most used phenomenological model of hysteresis in the literature because of its ability to describe accurately reversible and irreversible magnetization processes [63-67].

In the framework of the Preisach-type models of hysteresis, the total magnetization is modeled by a superposition of elementary hysteresis operator (hysteron). In this way, the magnetization of each hysteron is summed to compute the magnetization of the entire magnetic material. Based on the magnitude and the history of the magnetic field applied to the system each hysteron can either be saturated positively or negatively. If the history of the magnetic field is known and the distribution (coordinate of each hysteron in the magnetic system) of the hysterons within the magnetic systems can be found, then the magnetization can be computed easily for each hysteron. Once the magnetization of each hysteron is computed then a summation of the magnetization of all the hysterons provides the magnetization of the magnetic system.

2.1.5.1 Classical Preisach model (CPM)

The output function (magnetization) for the CPM can be computed by the following double integral [12]:

$$ f = \int_{H_\alpha=H_\beta} P(H_\alpha, H_\beta) \hat{x}_{\alpha\beta} x \ dH_\alpha dH_\beta, $$

(42)
where \( P(H_\alpha, H_\beta) \) is the Preisach distribution function, \( \hat{\gamma}_{\alpha\beta} \) is the elementary hysteresis operator (hysteron) and \( H_\alpha \) and \( H_\beta \) are the upper and lower critical fields respectively, where \( H_\alpha \geq H_\beta \) (Figure 9). Each hysteron can be characterized by an ideal rectangular hysteresis loop shown in Figure 9 and each hysteron has a unique \( H_\alpha \) and \( H_\beta \) value that specifies the location of the hysteron within the magnetic system.

![Elementary hysteresis rectangular loop representing the magnetization of each hysteron.](image)

Figure 9. Elementary hysteresis rectangular loop representing the magnetization of each hysteron. The ascending branch is traced for all hysterons with upper switching fields lower than the input however the descending branch is traced if the input is lower than the lower switching field.

Each hysteron can be in one of two states, that is, they can either be switched “up” or “down” depending on the input applied to the system. All hysterons that have \( H_\alpha \) smaller than the value of the input will be in the “up” state, i.e. \( \hat{\gamma}_{\alpha\beta} x = 1 \). All hysterons that have \( H_\beta \) larger than the value of the input will be in the “down” state., i.e. \( \hat{\gamma}_{\alpha\beta} x = -1 \). If the value of the input is between
and \( H_\beta \) then that hysteron is either in the “up” or “down” state, depending on the past history of the input. If a hysteron is in the “down” state and value of the input is increased, then the magnetization follows the ascending branch “abcde” (see Figure 9). Similarly, if a hysteron is in the “up” state and value of the input is decreased, the magnetization follows the descending branch “edfba”.

### 2.1.5.2 Generalized Preisach model (GPM)

In the framework of the generalized Preisach model the output variable is given by [68-78]:

\[
f = f_i + f_r = \int_{H_{a}>H_{\beta}} P(H_{a}, H_{\beta}) \hat{\gamma}_{a\beta} x dH_{a} dH_{\beta} + \int_{-\infty}^{\infty} R(H_{a}) \hat{\gamma}_{a} x dH_{a}. \tag{43}
\]

The first term in (43) is similar to the double integral from the CPM and describes irreversible magnetization processes \((f_i)\). The second term describes reversible magnetization processes \((f_r)\). In equation (43) \( \hat{\gamma}_{a\beta} \) are elementary operators over the ensemble of elementary hysteresis hysterons represented by rectangular hysteresis loops (see Figure 9) with \( H_{a} \) and \( H_{\beta} \) as up and down switching values and with saturation values defined by \( \hat{\gamma}_{a\beta} x = 1 \) if \( x > H_{a} \), \( \hat{\gamma}_{a\beta} x = \pm 1 \) if \( H_{a} > x > H_{\beta} \) and \( \hat{\gamma}_{a\beta} x = -1 \) if \( x < H_{\beta} \). \( \hat{\gamma}_{a} \) are the set of step operators defined as \( \hat{\gamma}_{a} x = 1 \) if \( x < H_{a} \) and \( \hat{\gamma}_{a} x = -1 \) if \( x > H_{a} \). \( P(H_{a}, H_{\beta}) \) and \( R(H_{a}) \) are the irreversible and reversible Preisach distribution functions respectively.

The irreversible component of the Preisach distribution \( P(H_{a}, H_{\beta}) \) is usually assumed to be the product of a normal function in the interaction field and a normal or log-normal function in the coercive field [79-93]:

- **uniform distribution:**

\[
P(H_{a}, H_{\beta}) = \begin{cases} 
\frac{2M_{s} S}{H_{\sigma}^2}, & \text{if } |H| < H_{\sigma} \\
0, & \text{otherwise} 
\end{cases}, \tag{44}
\]
• normal-normal distribution:

\[
P(H_a, H_\beta) = \frac{M_s S}{2\pi H_\sigma H_\sigma c} \cdot \exp \left[ -\left( \frac{H_a + H_\beta - 2H_0}{2H_\sigma^2} \right)^2 \right] \cdot \exp \left[ -\left( \frac{H_a - H_\beta}{2H_\sigma c} \right)^2 \right],
\]

(45)

• normal-lognormal distribution:

\[
P(H_a, H_\beta) = \frac{M_s S}{\sqrt{2\pi} \sigma_a \sigma_\alpha \left( H_a - H_\beta \right)} \times \\
\exp \left[ -\left( \frac{H_a + H_\beta}{2H_\sigma^2} \right)^2 - \left( \frac{\ln \left( \frac{H_a - H_\beta}{2H_0} \right)}{2\sigma_c^2} \right)^2 \right],
\]

(46)

where \( H_\sigma, H_\sigma c, H_\sigma, \) and \( \sigma_c \) are related to the distribution standard deviations and \( H_0 \) is the center of the distribution.

The reversible component of the Preisach distribution \( R(H_a) \) is usually considered to be one of the following functions:

• uniform distribution:

\[
R(H_a) = \begin{cases} 
\frac{M_s (1 - S)}{2H_\sigma}, & \text{if } |H| < H_\sigma, \\
0, & \text{otherwise}
\end{cases}
\]

(47)

• normal distribution:

\[
R(H_a) = \frac{M_s (1 - S)}{\sqrt{2\pi} H_\sigma} \cdot \exp \left[ -\frac{H_a^2}{4H_\sigma^2} \right]
\]

(48)

• exponential distribution:

\[
R(H_a) = \frac{M_s (1 - S)}{2H_\sigma} \exp \left[ -\frac{|H_a|}{H_\sigma} \right]
\]

(49)
• Cauchy distribution:

\[
R(H_\alpha) = \frac{M_s (1 - S)}{\pi H_{\sigma r}} \frac{1}{1 + \left( \frac{H_\alpha}{H_{\sigma r}} \right)^2},
\]

(50)

where \( H_{\sigma r} \) is a fitting parameter. Restricting attention to bivariate functions simplifies the identification problem and the numerical simulations significantly [67, 91, 94-97]. Commercial software packages usually integrate these equations as much as possible analytically in order to decrease the computational burden. HysterSoft [98] implements fast algorithms for the integration of these functions for all possible distribution presented in (44)-(50) as well as for the case in which the distribution is given by using discrete points. Figure 10 shows the major and minor hysteresis loops computed in HysterSoft by using the Preisach model. Figure 11 shows a set of first-order-reversal-curves (FORCs) computed by the model. It should be noted that, if carefully calibrated, the Preisach model can describe exactly any experimental set of FORCs. This had made the Preisach model very popular in the magnetic hysteresis community.

Figure 10. Major hysteresis loop (a) and sample magnetization curves (b) computed by using the Preisach model.
2.1.5.3 Computation of the magnetization process

In this section we present the computation of the magnetization process by using the GPM of hysteresis. For clarity a geometric representation of (43) will be used. In equation (43) the integrals are computed over a region where \( H_\alpha \geq H_\beta \).

Figure 12 shows the Preisach distribution function \( P(H_\alpha, H_\beta) \) as a function of the upper switching field \( H_\alpha \) and the lower switching field \( H_\beta \). The plane \( H_\alpha - H_\beta \) is called the Preisach plane. The entire region where \( H_\alpha \geq H_\beta \) is highlighted and will be referred to as the limiting triangle. In the case of the CPM the hysterons are distributed only inside the limiting triangle while in the case of GPM they are distributed both inside the limiting triangle and on the hypotenuse of the limiting triangle. The hysterons within the limiting triangle all have a unique \( (H_\alpha, H_\beta) \) coordinate where \( H_\alpha \geq H_\beta \). The magnetization of the hysterons inside the limiting
triangle contributes to the irreversible component of magnetization due to the fact that all hysterons in this region have non-zero hysteresis width loops. The reversible component of the magnetization is computed by using hysterons distributed along the hypotenuse of the limiting triangle where $H_\alpha = H_\beta$. We assume that there are no hysterons distributed outside the $H_\alpha \geq H_\beta$ region because we only consider counterclockwise hysteresis loops in our analysis.

![Preisach Plane](image)

**Figure 12.** Distribution of magnetic hysteresis operators within Preisach plane (lower right angle triangle).

Now let us consider the case when a strongly negative magnetic field (the magnitude of the input is lower than the minimum value of the upper critical field $H_\alpha$ of all hysterons) is applied to a magnetic system causing all the hysterons to be switched down therefore placing the entire system in a negative saturation state. If the input is then monotonically increased to some value equal to $H$ then all those hysterons whose switching fields are lower than the applied field $H$ will be switched up and the other hysterons will remain switched down. This divides the Preisach plane into two regions as shown in Figure 13. The red colored region (denoted by a ‘+’
sign) denotes switched “up” hysterons while the blue colored region (denoted by a ‘-‘ sign) denotes the switched “down” hysterons. If the applied magnetic field is varied continuously, that is, monotonically increased or decreased then the Preisach plane will be similar to the figure shown in Figure 14.

![Preisach Plane Diagram](image)

Figure 13. Hysterons switched "up" or "down" after magnetic field is applied to a system initially at a negative saturation state.
The line of separation between the positive $S^+$ and the negative $S^-$ portions of the Preisach plane is called stairline where the vertices represent the past extremum values of the input. All hysterons below the stairline are considered to be switched up and the rest are switched down.

At any instant in time the reversible component of the magnetization can be computed as:

$$f_r = \int P(H_a, H_b) \hat{\gamma}_{\alpha\beta} x dH_a dH_b + \int P(H_a, H_b) \hat{\gamma}_{\alpha\beta} x dH_a dH_b.$$  \hspace{1cm} (51)

For all hysterons in the $S^+$ region $\hat{\gamma}_{\alpha\beta} x = +1$; similarly all the hysterons in the $S^-$ region where $\hat{\gamma}_{\alpha\beta} x = -1$ therefore (51) can be reduced to:

$$f_r = \int P(H_a, H_b) dH_a dH_b - \int P(H_a, H_b) dH_a dH_b.$$  \hspace{1cm} (52)
Equation (52) shows that the instantaneous value of the magnetization depends on a particular subdivision of the Preisach plane into positive and negative regions. The subdivision solely depends on the shape of the stairline.

### 2.1.5.4 Computation of the Preisach distribution function

In order to compute the magnetization state of a system the Preisach distribution function \( P(H_\alpha, H_\beta) \) needs to be determined. The most often used identification technique for the Preisach distribution function uses the first-order reversal-curves (FORCs), which are defined in Figure 15. A FORC is obtained by:

a) Initially placing the material in a state of positive (or negative) saturation,

b) Monotonically increasing (after negative saturation) or decreasing (after positive saturation) the magnetic field to a particular value, and

c) Monotonically increasing (or decreasing) the magnetic field until the material is saturated.

d) The process is repeated depending on the number of FORC to be generated. The more FORCs generated the more accurate the Preisach distribution function.

Consider a magnetic material that is placed in a negative saturation state. If the applied magnetic field is monotonically increased to some value \( x = H_\alpha \) then the resulting magnetization can be computed and denoted as \( f_{H_\alpha} \). The resulting Preisach plane is shown in Figure 16. Following the monotonic increase, we then monotonically decrease the input to some value \( x = H_\beta \). The corresponding Preisach plane after the monotonic decrease is shown in Figure 17. At this point a first order reversal curve will be formed. Let us denote by \( f_{H_\alpha H_\beta} \) the output along the first order reversal curve.
Figure 15. First order reversal curves obtained from a system initially at positive saturation.

Figure 16. Preisach plane showing monotonic increase of input after negative saturation.
Figure 17. Preisach plane showing monotonic decrease of input after a monotonic increase as shown in Figure 16.

Figure 18. Differential triangle representing the history of the input.
The difference between the two resulting magnetization processes described by Figure 16 and Figure 17 can be expressed as
\[ f_{H_{\alpha},H_{\beta}} - f_{H_{\alpha},H_{\beta}'} = \int_{T(H_{\alpha},H_{\beta})} P(H_{\alpha},H_{\beta}) \, dH_{\alpha} \, dH_{\beta} \]
where \( T(H_{\alpha},H_{\beta}) \) is the triangular shaded area shown in Figure 18. We next define the function
\[ F(H_{\alpha}',H_{\beta}') = \frac{1}{2} \left( f_{H_{\alpha}',H_{\beta}'} - f_{H_{\alpha},H_{\beta}'} \right), \]
which can be expressed in terms of the Preisach distribution function \( F(H_{\alpha}',H_{\beta}') = \int_{T(H_{\alpha}',H_{\beta}')} P(H_{\alpha},H_{\beta}) \, dH_{\alpha} \, dH_{\beta} \). Therefore, the Preisach distribution function can be expressed as
\[ P(H_{\alpha}',H_{\beta}') = \frac{1}{2} \frac{\partial^2 f_{H_{\alpha},H_{\beta}}}{\partial H_{\alpha} \partial H_{\beta}}. \] (53)

Note that the first derivative of \( f_{H_{\alpha},H_{\beta}} \) with respect to \( H_{\alpha}' \) is equal to \( \frac{\partial f_{H_{\alpha},H_{\beta}}}{\partial H_{\alpha}'} = \tan \theta(H_{\alpha}',H_{\beta}') \), where \( \theta(H_{\alpha}',H_{\beta}') \) is the angle between the input axis and the tangent to the FORC at the point where \( x = H_{\alpha}' \). Finally, the Preisach distribution function can be expressed as:
\[ P(H_{\alpha}',H_{\beta}') = \frac{1}{2} \frac{\partial \tan \theta(H_{\alpha}',H_{\beta}')}{\partial H_{\beta}}. \] (54)

Equations (53) and (54) are often used to compute the Preisach distribution experimentally. HysterSoft implements fast techniques to compute the second order derivative of \( f_{H_{\alpha},H_{\beta}} \) from any experimental set of FORCs.

### 2.2 Vector models of hysteresis

In this section we consider the case of vector models of hysteresis. In the case of vector models of hysteresis the input and output of the hysteretic system represented in Figure 1(a) are two or three-dimensional vectors. Next we define the vector models of hysteresis by using the approach of superposition that has been developed by Mayergoyz [60, 62, 99-108]. In the past this approach has been applied only to the Preisach model of hysteresis. Recently, we have generalized it to other models of hysteresis [18, 109].

35
2.2.1 Two-dimensional models of hysteresis

Two-dimensional vector models of hysteresis are constructed as a two-dimensional superposition of scalar models of hysteresis [12]:

\[ f(t) = \int_{-\pi/2}^{\pi/2} \hat{\phi} \hat{\Gamma}_\phi [x(t) \cdot \hat{\phi}] d\phi, \] (55)

where \( \hat{\phi} \) is the unit vector along the direction specified by polar angle \( \varphi \), \( x \) is vector input variable, and \( \hat{\Gamma}_\phi \) is the scalar hysteresis operator along direction \( \varphi \). \( \hat{\Gamma}_\phi \) is the scalar hysteresis operator that can be any of the models presented in previous sections.

In order to understand how equation (55) can be implemented in numerical simulations, we present next the main equations of the vector Jiles-Atherton model of hysteresis. The same analysis can be extended to any other model. In the case of the vector Jiles-Atherton model of hysteresis each model parameter in equation (40) depends on an additional angular parameter \( \varphi \):

\[ \frac{df}{dx} = (1 - c_\varphi) \delta_\varphi \frac{L_\varphi (x + \alpha_\varphi f) - f}{k_\varphi (1 - c_\varphi) \text{sgn}(\dot{x}) - \alpha_\varphi [L_\varphi (x + \alpha_\varphi f) - f]} + c_\varphi \frac{dL_\varphi(x)}{dx}, \] (56)

where the Langevin function \( L_\varphi \) also depends on angle \( \varphi \):

\[ L_\varphi(x) = M_s \mathcal{L} \left( \frac{x}{a_\varphi} \right) = M_s \left[ \coth \left( \frac{x}{a_\varphi} \right) - \frac{a_\varphi}{x} \right], \] (57)

and \( \delta_\varphi \) is equal to 0 if \( \dot{x} [L_\varphi (x + \alpha_\varphi f) - f] \leq 0 \) and one otherwise. In the above equations, \( M_s \) is the saturation value of magnetization and \( a_\varphi \), \( c_\varphi \), \( \alpha_\varphi \), and \( k_\varphi \) are some functions of \( \varphi \) that can be identified by fitting simulations to the experimental values of the magnetization. It should be noted that in the case of the scalar Jiles-Atherton model of hysteresis \( a \), \( c \), \( \alpha \), and \( k \) are the classical model’s parameters, however, in the framework of the vector model these parameters become functions of angle \( \varphi \) ranging from \( -\pi/2 \) to \( \pi/2 \).
2.2.2 Three-dimensional models of hysteresis

The three-dimensional vector model is defined as [12]:

\[
 f(t) = \iiint_{\mathbb{R}^3} r \Gamma_{\alpha(r)} \left[ r \cdot x(t) \right] dS,
\]

where the integration is taken over the unit sphere. The parameter vector \( \alpha \) is considered to depend on the direction in which the integration is performed. It should be mentioned that a similar vector model was introduced in [12] for the Classical Preisach model. However, due to the high computational cost required by the Preisach model this technique is very difficult to implement in real-time simulations. Given the past and current values of the input \( x(t) = [H_x(t), H_y(t), H_z(t)] \) the \( x, y, \) and \( z \) components of the output in three-dimensional models can be computed by using the following equations:
\[
\begin{align*}
  f_x(t) &= \iiint_{\Gamma_{a(\theta, \phi)}} h(t) \sin^2 \theta \cos \phi \, d\theta \, d\phi, \\
  f_y(t) &= \iiint_{\Gamma_{a(\theta, \phi)}} h(t) \sin \theta \sin \phi \, d\theta \, d\phi, \\
  f_z(t) &= \frac{1}{2} \iiint_{\Gamma_{a(\theta, \phi)}} h(t) \sin 2\theta \, d\theta \, d\phi,
\end{align*}
\]
where \( h(t) = x_x(t) \sin \theta \cos \phi + x_y(t) \sin \theta \sin \phi + x_z(t) \cos \theta \).

Note that in the case of three-dimensional vector models of hysteresis each parameter of the scalar phenomenological model under consideration depends on two angles \( \varphi \) and \( \theta \). For instance, the Langevin function in the case of Jiles-Atherton model becomes:

\[
L_{\varphi, \theta}(x) = M_x \left[ \coth \left( \frac{x}{a_{\varphi, \theta}} \right) - \frac{a_{\varphi, \theta}}{x} \right].
\]

### 2.3 Dynamic models of hysteresis

Dynamic models of hysteresis describe the hysteresis phenomena by also taking into consideration the rate at which the input variable is changing in time [12, 110-113]. Two models that are commonly used in the literature for the simulation of dynamic hysteresis are the mean-field and the effective time approximation, which are presented in the next two subsections.

#### 2.3.1 Mean field approximation

In the mean field approximation the input variable \( x \) is modified to also depend on the output variable \( f \) and the rate of variation of the output variable \( \dot{f} \). Denoting the effective value of the input by \( x_{\text{eff}}(t) \):

\[
\begin{align*}
  f(t) &= \dot{x}_{\text{eff}}(t), \\
  x_{\text{eff}} &= x + F(f, \dot{f}),
\end{align*}
\]
where $F$ is a function of the output variable $f$ and of its derivative with respect to time, $\dot{f}$. Equations (63) and (64) represent a system of nonlinear, coupled equations that should be solved for $f$. In the framework of the Preisach model, this system is a system of integro-differential equations, while in the framework of the Jiles, Energetic and Hodgdon models it is a system of differential equations.

2.3.2 Relaxation time approximation

In the relaxation time approximation, output variable $f(t)$ can be described by the following first-order differential equation:

$$\frac{df}{dt} = -\frac{f(t) - f_x(t)}{\beta}, \quad (65)$$

where $\beta$ is a relaxation time parameter. The generalized susceptibility can be computed by using:

$$\chi(t) = \frac{h(t) - h_x(t)}{\beta \dot{x}(t)}, \quad (66)$$

where $\dot{x}(t)$ is the derivative of the input variable with respect to time. Equations (65), (66), and (1) are solved to compute the output variable $f$ as a function of $x(t)$. These equations are integrated numerically in HysterSoft by using finite differences. Figure 20 presents sample simulations for the magnetic hysteresis (magnetization as a function of the magnetic field) using the two approximations for a barium ferrite.
2.4 Temperature and stress dependent hysteresis

Many scalar models of hysteresis in the literature implement the effects of temperature and mechanical stress on magnetization curves. One of the most used approaches is based on modifying the scalar Energetic model parameters as a function of the temperature and mechanical stress. This approach is summarized in this section and follows the work presented in [25, 111, 114-117].

2.4.1 Temperature dependent magnetic processes

To perform temperature dependent magnetization processes, one assumes that the magnetization of saturation depends on temperature $T$ as:

$$ M_s(T) = M_{s0} \left(1 + \frac{T}{2T_C}\right) \sqrt{1 - \frac{T}{T_C}}, $$

where $M_{s0}$ is the magnetization of saturation at $T = 0$ and $T_C$ is the Curie temperature. The anisotropy constant and the magnetostriction constant at saturation $\lambda_s$ are assumed to depend on the temperature as:
\[ K_1(T) = K_1(T_0) \left[ 1 + \alpha_K (T - T_0) \right], \]  
(68)

and

\[ \lambda_s(T) = \lambda_s(T_0) \left[ 1 + \alpha_{\lambda_s} (T - T_0) \right], \]  
(69)

respectively, where \( \alpha_K \) and \( \alpha_{\lambda_s} \) are constants that should be identified at \( T_0 = 300 \text{K} \). The effects of the temperature on the barium ferrite are shown in the simulations presented in Figure 21(a). The total area of the magnetic hysteresis loop is decreased due to the lowering of the magnetization at saturation and the decrease in the anisotropy constant of the barium ferrite, which results in the decrease of the coercive field.

Figure 21. Temperature (a) and mechanic stress (b) effects on a barium ferrite.

### 2.4.2 Stress dependent magnetic processes

Let us denote by \( k_0 \), \( q_0 \), \( g_0 \), and \( h_0 \) the parameters of the Energetic model under no stress, and introduce the following parameters:

\[ c_s = \frac{k_0}{\lambda_s^2 E_Y}, \]  
(70)
\[ c_q = q_0 \frac{K_1}{k_0}, \quad (71) \]
\[ c_q = g_0 \frac{\mu_0 M_s^2}{K_1}, \quad (72) \]
\[ c_h = \frac{h_0}{M_s} \left( c_r + 1 \right) \left( e^{g_0 \ln 2} - 1 \right) + \frac{c_r \lambda_s^2 E_y}{\mu_0 M_s^2} + N, \quad (73) \]

where \( E_y \) is the Young modulus, \( K_1 \) is the magnetic anisotropy constant, and \( \lambda_s \) is the magnetostriction constant at saturation. Then, the model parameters under stress level \( \sigma \) (measured in Pa) are evaluated as:

\[ k = c_k \lambda_s^2 E_y, \quad (74) \]
\[ q = c_q \frac{2K_1 + 3 \left( 1 + v_p \lambda_s \right) \sigma}{2c_k \lambda_s^2 E_y}, \quad (75) \]
\[ c_g = \frac{2K_1 + 3 \left( 1 + v_p \right) \lambda_s \sigma}{2\mu_0 M_s^2}, \quad (76) \]
\[ h = \frac{\mu_0 M_s^2 \left( c_h - N \right) - c_k \lambda_s^2 E_y}{\mu_0 M_s \left( c_r + 1 \right) \left( e^{g_0 \ln 2} - 1 \right)}, \quad (77) \]

where \( v_p \) is the Poisson ratio (for permalloy \( v_p = 0.32 \)). The effects of mechanical stress on the major hysteresis loop of a barium ferrite are shown in Figure 21 (b) for \( 10^6 \) Pa, \( 3 \times 10^6 \) Pa, \( 10^7 \) Pa, and \( 4 \times 10^7 \) Pa.

In many applications it is required to perform simulations in which both the temperature and the mechanical stress are taken into consideration. A simulation of such an example is presented in Figure 22 for a barium ferrite that forms the core of an RLC circuit in which we have induced dumped oscillations. The magnetic field is represented as function of time on the inset in these figures. No stress and no dynamic effects are considered in the simulations presented in left figure. No stress is assumed in middle figure, and \( \sigma = 4 \times 10^7 \) Pa is applied on the magnetic material in the simulations presented on the figure on the right. The total computation time for the simulations presented in Figure 22 is less than one second on a 2 GHz one processor computer.
Figure 22. Barium ferrite subject to both mechanical stress and high rates of variation of the applied magnetic field. The simulations were performed for an RLC circuit. The magnetic field is represented in the inset as a function of time.
CHAPTER 3

THERMAL RELAXATION PHENOMENA

In this chapter we present our technique for the modeling and simulation of the thermal relaxation phenomena. Section 3.1 presents the Monte-Carlo algorithm that we have developed for the analysis of thermal relaxation phenomena. Then, we present numerical simulation results for thermal relaxation in scalar (Section 3.2) and vector models of hysteresis (Section 3.3). Particular attention is given to presenting a geometrical interpretation of the loss of magnetic memory in the framework of the scalar Preisach model of hysteresis. Chapter 3 is then concluded by a presentation of the results for the data collapse phenomenon in both scalar and vector models of hysteresis on Section 3.4.

3.1 Monte-Carlo algorithm for the analysis of thermal relaxation phenomena

In order to model the magnetic aftereffect phenomena in hysteretic systems we assume that the presence of thermal agitations is equivalent to the effect of a stochastic input \( \dot{x}(t) \) superimposed on the applied magnetic field \( x_{app} \). Consequently, the magnetic system is driven by the following stochastic process:

\[
x(t) = x_{app} + \dot{x}(t), \quad \langle \dot{x}(t) \rangle = 0 ,
\]  

(78)
where \( \langle \tilde{x}(t) \rangle \) is the expected value of \( \tilde{x}(t) \). In order to simplify the problem we will assume what is usually done in the literature that process \( x(t) \) is:

1. stationary,
2. the joint distribution of any finite set \( x(t_1), x(t_2), x(t_3) \ldots \) is zero, and
3. the statistical properties of \( x(t) \) are independent of the orientation of the x, y, and z axes.

It should be noted that in the framework of the Monte-Carlo method presented in this section each of the previous assumptions can be relaxed. As shown in the classical work of Brown [8] for systems of small noninteracting magnetic particles, as well as by Neél [15] in a more phenomenological framework, the correlation function (variance) of \( x(t) \) is proportional to the absolute temperature, i.e.:

\[
\langle x^2(t) \rangle = \frac{2k_B T \eta}{V}, \quad (79)
\]

where \( k_B T \) represents the thermal energy, \( V \) is the volume of a particle, and \( \eta \) is the dissipation constant. We will also assume that this equation remains valid for any magnetic material and use it in analyzing temperature dependent magnetization processes.

The average values of the output magnetization in the case of vector models can be computed by taking the average of the left and right-hand sides of equations (55) and (58):

(2-D models) \[
\langle f(t) \rangle = \frac{1}{\pi/2} \int_{-\pi/2}^{\pi/2} \hat{\phi} \langle \Gamma_{\varphi}[x(t) \cdot \hat{\phi}] \rangle d\varphi, \quad (80)
\]

(3-D models) \[
\langle f(t) \rangle = \iiint_{|r|=1} r \langle \Gamma_{\alpha(r)}[r \cdot x(t)] \rangle dS. \quad (81)
\]

From (80) and (81) we note that the whole problem of computing the magnetization can be simplified to the evaluation of \( \langle \Gamma_{\varphi}[x(t) \cdot \hat{\phi}] \rangle \) in the case of two-dimensional models and \( \langle \Gamma_{\alpha(r)}[r \cdot x(t)] \rangle \) in the case of three-dimensional models. Computing averages in equations (80) and (81) is in general difficult to perform analytically for any scalar hysteresis operator \( \Gamma_{\varphi} \) but it can easily be done numerically by using statistical Monte-Carlo methods. To this end we
assume that noise $x(t)$ is a discrete-time, independent and identically distributed (i.i.d.) random process. The probability distribution functions of the components of $x(t)$ are assumed to be normal and the variance of the distribution proportional to the temperature as defined in equation (79). In this way both the time and temperature are expressed in arbitrary units and for a quantitative analysis the model should be carefully calibrated.

The numerical algorithm for the calculation of “creep” of the magnetization as a function of time can be summarized as follows (see also Figure 23):

a) First, a large number of input processes $x(t)$ are generated.

b) The output variable (magnetization) is computed numerically for each such process by using a given vector model.

c) The average value of the magnetization $\langle f(t) \rangle$ is computed by averaging the solutions computed at the previous step.

Figure 23. Algorithm for the analysis of thermal relaxation (aftereffect) phenomena.
3.2 Thermal relaxation in scalar models of hysteresis

The technique described in the previous section has been numerically implemented and used to compute the dependence of the output of magnetic systems as a function of time. The output variable in magnetic systems can be identified as the magnetization of a magnetic sample, while the input variable is the applied magnetic field which, in our simulations, is considered to be a discrete-time i.i.d. random process. The probability distribution function of the input variable is assumed to be normal with standard deviation $\sigma$ and mean $x_0$:

$$
\rho(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-x_{app})^2}{2\sigma^2}\right].
$$

(82)

We have considered two initial states that are obtained as follows:

*State 1*: We saturate the sample by applying a strong magnetic field and then apply a magnetic field equal to some value $x_0$ (see Figure 24). The magnetic field is kept constant at this value and the magnetization is measured as a function of time.

*State 2*: We use a decreasing alternative magnetic field that initially has a relatively high magnitude and is decreased slowly to zero. After that, we apply a constant magnetic field $x_0$ and measure magnetization as a function of time (see Figure 25).
Figure 24. “State 1” is obtained by applying a positive saturating magnetic field and then a magnetic field equal to \( x_0 \).

Figure 25. “State 2” is obtained by a.c. demagnetizing the magnetic material and then applying a magnetic field equal to \( x_0 \).

Figure 26 presents the change of the magnetization as function of time computed in the framework of the Preisach model of hysteresis after the first Monte-Carlo simulation and after 500 Monte-Carlo simulations by assuming that the initial state is “State 1” [Figure 26 (a)] and
“State 2” [Figure 26 (b)]. The results of the analytical computations (see Appendix A) are represented by dash lines, while the results of the Monte-Carlo simulations are represented by continuous lines. As expected, the results obtained by using the Monte-Carlo method converge to the analytical results when the number of total simulations is increased. It is interesting to observe that the time dependence of the magnetization depends on the past history of the magnetic sample. If the initial state is “State 1” the average value of the magnetization is monotonically increasing, while if the initial state is “State 2” it is decreasing. However, the expected value of the magnetization after long periods of time (when $t$ goes to infinity) is the same, and does not depend on the past magnetic history of the sample. This effect was also observed by Mayergoyz [12].

Figure 26. Thermal relaxation of magnetization simulated using the Preisach model of hysteresis. The initial hysteretic state in the simulations is State 1(a) and State 2 (b).

Similar simulation experiments have been conducted by using the Jiles-Atherton model where equation (40) was integrated numerically by using standard quadratures and the average value of
the magnetization was computed assuming that the initial state was State 2. The model parameters used in the simulations were chosen as \( a = 41, k = 39, c = 0.85, M_s = 10^5 \), and \( \alpha = 0 \). The results of the simulations are represented in Figure 27(b) after the first Monte-Carlo simulation and after 100 Monte-Carlo simulations. It is remarkable that, although the Jiles-Atherton model is a fairly simple model that displays only local memory, the model still predicts a \( \log t \) - dependence of the magnetization. The asymptotic value of the magnetization can be computed by using a relatively small number of Monte-Carlo simulations as compared to the Preisach model.

Similar results to the ones presented in Figure 27(b) have also been obtained in the case of the Hodgdon model (37)-(39). These results are represented in Figure 27(a) for the following model parameters \( \alpha = 1, A_1 = 0.374, A_2 = 7.5 \times 10^{-7}, A_3 = -0.769, A_4 = 4.7 \times 10^8 \), and \( B_\alpha = 2 \times 10^9 \). It is apparent from the simulations that the computational burden is somewhat heavier that in the case of the Jiles-Atherton model.

Figure 27. Thermal relaxation of magnetization simulated using the Hodgdon (a) and Jiles-Atherton (b) models. The initial hysteretic state in the simulations is State 2.
3.2.1 Loss of magnetic memory in the Preisach model of hysteresis

One of the most interesting phenomena observed during thermal relaxation in hysteretic systems is the loss of magnetic memory in time. As shown in Figures 28-45 the magnetic history in the case of the Preisach model is given by the line of separation between positive and negative hysterons. At time $t = 0$ the effect of thermal relaxation phenomena can be neglected and the line of separation is well-defined. The hysterons above this line are all in the +1 state, while the hysterons below are all in the -1 state. After $t = 0$ the line of separation can change its position and usually becomes undeterministic.

In order to study the loss of magnetic memory in the Preisach model we have used the Monte-Carlo method to simulate a large number of magnetic relaxations and have taken the average of the state of each hysteron at the end of the simulations. In this way we can compute the expected value of each hysteron in the Preisach plane for any initial magnetic state. The results of these simulations are presented in Figures 28-45 for different initial states. Figures 28-45 represent the state of the Preisach plane and the Preisach distribution after thermal relaxation has occurred. The simulations were performed at the temperatures indicated at the top of each figure. The inset of these figures shows the state of the Preisach plane at $t = 0$. The total magnetization of the material is obtained by performing the integration in (43). The blue colored areas (above the line of separation) represent regions where the hysterons are magnetized in the negative direction (switched “down”), while the red colored areas represent regions where the hysterons are magnetized in the positive direction (switched “up”).

The initial magnetic state (before relaxation) for the simulations presented in Figures 28-33 was obtained by a.c. demagnetizing the magnetic material, i.e. by applying an initially strong a.c. magnetic field whose magnitude is slowly decreasing to zero. The initial state for the simulations presented in Figures 34-39 was obtained by applying strong magnetic fields to saturate the magnetic material and, then, decreasing the field to zero (remanent state). Finally, the initial state for the simulation presented in Figures 40-45 was obtained by applying a strong magnetic field to saturate the material, then applying a negative magnetic field approximately equal to the
coercive field, and eventually applying a positive magnetic field. The initial state in this last type of simulations is very close to the d.c. demagnetized state.

It is interesting to observe that the effect of thermal relaxation in the Preisach plane is to “erase” the separation line between the positive and negative hysterons. It is also interesting to observe that thermal relaxation can also change the expected position of the coordinates of the separation line, like in the case when the initial state of the magnetic material was the remanent state (see Figures 34-39).

Finally, let us note that the effect of the time is not always to decrease the total magnetization of the material (like in the simulations presented in Figures 28-39 but it also can increase the value of the total magnetization (see Figures 40-45).

Figure 28. Effect of thermal relaxation on the Preisach plane when the initial state is the a.c. thermally demagnetized state at a temperature of 25 a.u.
Figure 29. Effect of thermal relaxation on the Preisach distribution when the initial state is the a.c. thermally demagnetized state at a temperature of 25 a.u.

Figure 30. Effect of thermal relaxation on the Preisach plane when the initial state is the a.c. thermally demagnetized state at a temperature of 50 a.u.
Figure 31. Effect of thermal relaxation on the Preisach distribution when the initial state is the a.c. thermally demagnetized state at a temperature of 50 a.u.

Figure 32. Effect of thermal relaxation on the Preisach plane when the initial state is the a.c. thermally demagnetized state at a temperature of 100 a.u.
Figure 33. Effect of thermal relaxation on the Preisach distribution when the initial state is the a.c. thermally demagnetized state at a temperature of 100 a.u.

Figure 34. Effect of thermal relaxation on the Preisach plane when the initial state is the remanent state at a temperature of 25 a.u.
Figure 35. Effect of thermal relaxation on the Preisach distribution when the initial state is the remanent state at a temperature of 25 a.u.

Figure 36. Effect of thermal relaxation on the Preisach plane when the initial state is the remanent state at a temperature of 50 a.u.
Figure 37. Effect of thermal relaxation on the Preisach distribution when the initial state is the remanent state at a temperature of 50 a.u.

Figure 38. Effect of thermal relaxation on the Preisach plane when the initial state is the remanent state at a temperature of 100 a.u.
Figure 39. Effect of thermal relaxation on the Preisach distribution when the initial state is the remanent state at a temperature of 100 a.u.

Figure 40. Effect of thermal relaxation on the Preisach plane when the initial state is the d.c. demagnetized state at a temperature of 25 a.u.
Figure 41. Effect of thermal relaxation on the Preisach distribution when the initial state is the d.c. demagnetized state at a temperature of 25 a.u.

Figure 42. Effect of thermal relaxation on the Preisach plane when the initial state is the d.c. demagnetized state at a temperature of 50 a.u.
Figure 43. Effect of thermal relaxation on the Preisach distribution when the initial state is the d.c. demagnetized state at a temperature of 50 a.u.

Figure 44. Effect of thermal relaxation on the Preisach plane when the initial state is the d.c. demagnetized state at a temperature of 100 a.u.
3.2.2 Viscoelastic coefficients in scalar models of hysteresis

The viscosity coefficient is used to characterize the rate at which the magnetization changes its value during thermal relaxation. In order to define the viscosity coefficient let us note that the magnetization usually varies as $\log t$ (see Figure 46). Starting from an initial magnetic state (that can be demagnetized or not) we apply a constant magnetic field $H$ and measure the value of the magnetization as a function of time $M(t)$. The (scalar) viscosity coefficient $S$ is defined as:

$$M(t) \sim M(0) + S(H,T)\log t,$$

where $S(H,T)$ is the viscosity coefficient (also known as the aftereffect decay coefficient), $t$ is the time, and $T$ is the absolute temperature. This logarithmic dependence has been observed often in both theoretical computations and experimental data. The viscosity coefficient can be expressed as:

$$S = \frac{d\{f(t)\}}{d\{\log t\}}.$$
The viscosity coefficient computed using the Preisach model is plotted in Figure 47 when the initial state of the magnetic sample is State 1 obtained as described in the previous section. Abscissa $x_0$ denotes the value of the magnetic field at which the gradual change in the magnetization is observed. The model parameters were the same as in the previous section. The results of the viscosity coefficients obtained by using the Preisach, Jiles-Atherton, and Hodgdon models are plotted in Figure 48 when the initial state of the magnetic sample is State 2. It is important to observe that the Jiles-Atherton and Hodgdon models predict results that are in good qualitative agreement with those obtained by using the generalized Preisach model. The symmetry of the viscosity coefficient plotted in Figure 48 can be interpreted based on the symmetry of the experimental setup.

Figure 46. Thermal relaxation of magnetization on a log scale. The scalar viscosity coefficient is defined as the slope of these curves.
Figure 47. Viscosity coefficients as a function of the magnetic field computed by using the Preisach model. The initial state was obtained by coming from positive saturation to a given value of the applied field (State 1).
3.2.3 Zero-field cooling simulations

It is very interesting to observe that the relatively simple Monte-Carlo approach presented in this dissertation can be extended to simulate field-cooling (FC) and zero-field-cooling (ZFC) phenomena in hysteretic systems. In this section we present such simulation results related to the analysis of the output of a hysteretic system as a function of the temperature. It has been shown
that the variance of the noise that is superimposed on the input variable is linearly dependent on temperature (see equation (79)). Therefore, variance $\sigma^2$ should be proportional to the temperature of the magnetic sample. By using this assumption we have conducted simulation experiments in which we start from an initial magnetic state and increase the “temperature” from 0 to relatively high values.

In ZFC experiments the hysteretic sample is cooled down to very low temperatures, after which a constant magnetic field $x_0$ is applied and the temperature is increased gradually while the magnetization is measured as a function of the temperature. The results of these simulations are represented in Figure 49 for the Preisach model of hysteresis.

![Figure 49. Zero-field curve simulated by using the Preisach model.](image)
3.3 Thermal relaxation in vector models of hysteresis

Thermal relaxation is a vectorial phenomenon because the random fluctuation fields (thermal agitation) are vector quantities. There is no reason for these fields to act in only one particular direction that is the direction of the external applied field. For this reason it is important for us to analyze the vector effects of thermal relaxation. As it will be shown in this section, new interesting phenomena appear in thermal relaxation if more than one direction is considered.

3.3.1 Simulation results for the magnetization dynamics

The technique described in the previous section has been numerically implemented in HysterSoft and used to compute the dependence of magnetization as a function of time. In the following we present simulation results obtained with the Preisach, Energetic, and the Jiles-Atherton models. In all the cases the components of the vector magnetization in the $x$ and $y$ directions were computed by using (55). The total magnetization was computed by averaging each magnetization process for 500 times. The results of the simulations are represented in Figure 54 for the Preisach, Energetic, and Jiles-Atherton models. The figure presents the trajectory of the magnetization in the $M_x-M_y$ plane as a function of time for three different magnetic histories of the applied field. Three initial states are considered in these figures:

(a) In simulations A the magnetic sample is first demagnetized by a constantly rotating and decreasing from saturation magnetic field (see Figure 50), after which a constant field $H = (H_x, H_y)$ is applied and the magnetization is measured as a function of time (see Figure 51).

(b) In simulations B, the magnetic sample is first demagnetized by a constantly rotating and decreasing from saturation magnetic field (see Figure 51), after which the magnetic field was increased along the $x$-axis to some fixed value $H_x$ and then changed to $(H_x, H_y)$ (see Figure 52).

(c) In simulations C, the magnetic material was saturated by applying a strong magnetic field in the $x$-direction and then decreased to $(H_x,0)$ and eventually changed to $(H_x,H_y)$ (see Figure 53).
Figure 50. A.C. demagnetization obtained by applying a constantly rotating and decreasing field from saturation.

Figure 51. Procedure used to obtain the initial magnetization state in simulations A. After an a.c. demagnetization we apply a magnetic field \( H = (H_x, H_y) \).
Figure 52. Procedure used to obtain the initial magnetization state in simulations B. After an a.c. demagnetization the magnetic field was increased along the \(x\) axis to some fixed value \(H_x\) after which another magnetic field \(H_y\) was applied along the \(y\) direction.

Figure 53. Procedure used to obtain the initial magnetization state in simulations C. First, a strong magnetic field was applied in the \(x\) direction to saturate the material, after which it was decreased to \((H_x,0)\) and eventually changed to \((H_x,H_y)\).
Figure 54. Magnetization creeps in isotropic materials.

The magnetic histories of the three simulations are different. However the final value of the applied field is always the same \((H_x, H_y)\) for all the simulations (A, B, and C). The values of \(H_x\) and \(H_y\) are chosen to be comparable to the saturation field of the material. The simulations presented in Figure 54 show that the trajectories of the magnetization vector are usually straight lines in the \(M_x - M_y\) plane. Moreover, after long periods of time the magnetization vector goes asymptotically to the value of the magnetization corresponding to an applied field of \((H_x, H_y)\) on the anhysteretic curve.

In Figure 55 the simulation results of the magnetization components are represented as a function of time for an anisotropic magnetic material. The easy axis of the material is the x-axis, while the hard axis is the y-axis. Note that, although the magnetization was initially oriented in the positive
direction, due to thermal agitation, the magnetization has eventually switched its orientation in the opposite direction. Moreover, since the material is anisotropic, the magnetization vector can cross the direction of the applied magnetic field vector. This is a pure anisotropic phenomenon and which is not met in the case of isotropic materials. In the simulations presented in Figure 54 the magnetization vector was moving to the direction of the applied field, but never crossed it.

Figure 55. Simulation for the magnetic relaxation for an isotropic (1→2) and anisotropic (3→4) magnetic materials. The trajectory of the magnetization might be curved particularly for anisotropic materials.

Another important property of the thermal relaxation phenomena in anisotropic materials is that the trajectory of the magnetization “creep” can deviate substantially from a straight line. This property can be observed on the inset presented in Figure 55 and can be attributed to the fact that in anisotropic materials the x and y components of the magnetization might deviate from the
classical log$t$ curve. Once the magnetization vector crosses the hard axis of the magnetic material, it will start moving much faster towards the direction of the easy axis.

### 3.3.2 Definition of viscosity coefficient in vector magnetization

The definition given for the viscosity coefficient given at the beginning of this section can be applied only to magnetic systems in which the direction of the magnetization vector is constant. This usually happens in only in isotropic systems and when the direction of the magnetic field has varied along one fixed direction. If this condition is not satisfied the magnetization vector might change the direction in time and (83) cannot be used. This discussion prompts us to look for a more general definition of the viscosity coefficient that can be reduced to the scalar one in the case of scalar magnetization.

The generalization of scalar viscosity coefficient to vector magnetic systems is challenging particularly because the trajectory of the magnetization in the case of vectorial relaxation phenomena can deviate substantially from a straight line (see Figure 55). One way to generalize the viscosity coefficient to vectorial magnetization processes is to consider that (83) can be generalized for each component of the magnetization, i.e.:

$$M_i(t) \sim M_i(0) + S_i(H,T) \log t,$$

where $i = x, y, z$, and $S_i(H,T)$ are the viscosity coefficients on the three axes. However, if we consider that (85) is true in all three spatial directions one can show that the trajectory of the magnetization should always be a straight line, which is not true for many anisotropic materials (see Figure 55). Indeed, if the previous equation holds in any direction, then the infinitesimal change of the magnetization during time $dt$ is

$$dM_i = S_i(H,T) \frac{dt}{t}, \quad t > 0.$$  

Therefore, the slope of the line in the 3-D space given parametrically by $[M_x(t), M_y(t), M_z(t)]$ is constant in time (for instance, $\frac{dM_x}{dM_y} = \frac{S_x(H,T)}{S_y(H,T)}$ is time independent). The only way to make
the trajectory of the magnetization not to follow a straight line would be to consider that the three viscosity coefficients are time-dependent.

A better approach to generalize the scalar viscosity coefficient to vectorial systems is to look at the total length of the trajectory described by the magnetization vector, which we refer to as the magnetization creep:

$$\Delta M(t) = \int_{t_1}^{t_2} dM = \int_{t_1}^{t_2} \sqrt{dM_x^2 + dM_y^2 + dM_z^2}$$

(87)

$$= \int_{t_1}^{t_2} \sqrt{\left(\frac{dM_x}{dt}\right)^2 + \left(\frac{dM_y}{dt}\right)^2 + \left(\frac{dM_z}{dt}\right)^2} dt$$

As has been recently shown in [107] the magnetization creep depends logarithmically on time and:

$$\Delta M(t) = S_{3D}(H,T) \log \frac{t}{t_1},$$

(88)

where $S_{3D}(H,T)$ is an “effective” viscosity coefficient. The magnetization creep computed using (87) is represented in Figure 56. The inset of this figure shows the magnetization creep as a function of time on a log scale.

Finally, let us remark that definition (88) seems to be a natural generalization of the scalar definition (83). If the components of the magnetization satisfy (85), the effective viscosity coefficient can be written as

$$S_{3D} = \sqrt{S_x^2 + S_y^2 + S_z^2}.$$
73

3.4 Data collapse phenomena

It has been shown that the viscosity coefficient in scalar models of hysteresis can be scaled as:

$$S(H,T) = S_0 \left( \frac{H}{f(T)} \right) g(T),$$

(89)

where $f$, $g$, and $S_0$ are some given functions. The universality of this factorization has been referred to as data collapse in magnetic systems. Practically, if we represent the viscosity
coefficient as a function of $H$ for different temperatures, all curves collapse into one universal curve $S_0$ after appropriate scaling of the two axes. Such an example is presented in Figure 57, in which we show the viscosity coefficient as a function of the applied field for different temperatures. These curves have been computed for an isotropic material in which the direction of the field is fixed. The data collapse phenomenon has been verified by both theoretical computations and experimental measurements [12] in isotropic materials by using scalar models and assuming that the direction of the magnetization is parallel to the direction of the applied field.

Figure 57. Viscosity coefficient as a function of the magnetic field for an anisotropic magnetic system measured at different temperatures. Only one peak appears because the direction of the magnetic field was fixed and the magnetization was not subject to rotational changes. The inset shows the data collapse property.
In order to study the data collapse phenomena in vector magnetic systems we have simulated viscosity magnetization processes by using a Preisach-type vector hysteresis model implemented recently by us [18, 118]. The presence of thermal agitations is taken into account by using a stochastic input \( \tilde{x}(t) \) superimposed on the applied magnetic field \( x_{\text{app}} \). Consequently, the magnetic system is driven by a random field \( x_{\text{app}} + \tilde{x}(t) \), where the expected value of \( \tilde{x}(t) \) is equal to zero and the correlation function of \( \tilde{x}(t) \) is proportional to the temperature [8]. The thermal relaxation was computed by using Monte-Carlo simulations and taking averages in order to find the expected value of \( f \).

Next, we present the results of simulations for the data collapse phenomena. We have performed the following two types of simulation experiments:

a) Simulation 1: first we saturate the material in the positive direction, then we apply a field \( H_0 \) along the x-axis, followed by a series of holding fields \( H_i \) in the y-direction, \((H_0, H_i)\); after applying each field \( H_i \) we wait for thermal relaxation (see Figure 58).

b) Simulation 2: we come again from positive saturation to a field \( H_i \) along the x-axis, after which we apply a field of the same magnitude \( H_i \) along the y-direction, and finally removing the field along the x-direction, so that the final values of the field is \((0, H_i)\); then, we wait for thermal relaxation (see Figure 59).
Figure 58. Procedure used to obtain the initial magnetization state in simulation 1. The material was saturated in the positive direction, then a field $H_0$ was applied along the x-axis, followed by a series of holding fields $H_i$ in the y-direction, $(H_0, H_i)$; after applying each field $H_i$ we wait for thermal relaxation.

Figure 59. Procedure used to obtain the initial magnetization state in simulation 2. The material was saturated in the positive direction, then a field $H_i$ was applied along the x-axis, after which we apply a field of the same magnitude $H_i$ along the y-direction, and finally removing the field along the x-direction, so that the final values of the field is $(0, H_i)$; then, we wait for thermal relaxation.

These simulation experiments were performed for values of $H_i$ ranging from positive to negative saturation. Each time we were measuring the viscosity coefficient by using (88), where
\( H = H_i \). Note that the first type of experiment corresponds to a first-order rotational magnetization process, while the second type corresponds to a second-order process.

The viscosity coefficients obtained by using the first type of simulations (a) at different temperatures are represented in Figure 60. The inset of this figure shows the scaled curves of the viscosity coefficients. It is remarkable that even for this relatively simple type of experiment we obtain two peaks for the viscosity curves. This is a purely vectorial effect that can be attributed to rotational changes of the magnetization during aftereffect. It is also interesting to observe that the two peaks correspond to values of the magnetic field that are close to plus and minus the values of the coercive field of the material, which in our simulations was approximately 60 A/m. All the viscosity curves seem to scale into one universal curve just like in the case of scalar relaxation phenomena.

In Figure 61 we have represented the viscosity coefficients obtained using the second type (b) of magnetization processes at the same temperatures. The viscosity coefficient curves show two or even three maxima, depending on the values of the temperatures. At low temperatures, the viscosity coefficient curves show only two maxima because the thermal agitation is not strong enough to clear the magnetic history completely. At high temperatures the viscosity coefficient curves show three maxima because the random agitation field can clear all reversal points in the magnetic history. It is apparent from these simulations that the more complex the magnetization processes are the more maxima can appear on the viscosity curves. In addition, the data collapse phenomena are more difficult to observe for higher rotational reversal-curves.
Figure 60. Viscosity coefficient as a function of the magnetic field measured at different temperatures. The inset shows the data collapse property.
Figure 61. Viscosity coefficient as a function of the magnetic field measured at different temperatures. The multiplicity of peaks is a pure vectorial effect and its origin is attributed to rotational changes of the magnetization during aftereffect.
CHAPTER 4

SPECTRAL ANALYSIS OF HYSTERETIC SYSTEMS

In Chapter 3 we have introduced the Monte-Carlo technique for the analysis of aftereffect phenomena and loss of magnetic memory in magnetic materials. In this Chapter we extend this Monte-Carlo technique to analyze the effects of noisy inputs in hysteretic systems. In Section 4.1 we present the methodology for computing the power spectral density of the output of a noisy hysteretic systems. We then present the existing analytical results in the existing literature in Section 4.2. In Section 4.3 we present simulation results obtained by using our Monte-Carlo algorithm and compare these results to analytical results. Finally, in Sections 4.4. and 4.5, we analyze the effects of temperature and mechanical stress on the power spectral curves of hysteretic systems.

4.1 Introduction

Hysteretic systems driven by stochastic inputs have attracted a large amount of research interest over the last few years particularly because of the fundamental challenges posed by noise and fluctuations in ultra-small high density magnetic devices. The existing literature shows that relatively simple hysteresis models can describe a large range of noise induced phenomena from the occurrence on 1/f noise and thermal relaxations [119-121] to coherence and stochastic resonances [122, 123].

One of the most important properties that describes the noise passage through a hysteretic system is the power spectral density $S(\omega)$ of the output variable (usually magnetization). Therefore, in
the next few subsections we develop an efficient numerical algorithm for the computation of the \( S(\omega) \) in hysteretic systems based on statistical Monte-Carlo techniques.

The computation of the spectral density of stochastic signals is of considerable mathematical complexity especially when considering nonlinear systems. Work in this area has been done by van Vleck [124] and Uhlenbeck [125, 126] who derived analytical results for the computation of the spectral density for nonlinear systems. Recently, closed form analytical results for the spectral density of the output process have been derived for complex hysteretic systems that can be described through Preisach model as a weighted superposition of symmetric rectangular operators [127, 128]. However the extension of these analyses to other models of hysteresis has not been performed due to the non-Markovian nature of the output process of the hysteretic system. In this Chapter, the spectral density analysis is extended to phenomenological models of hysteresis, such as the Energetic model, Jiles-Atherton model, and the generalized Preisach model. General qualitative features of these spectral densities are examined and their dependence on various parameters is discussed. The intrinsic differences between the phenomenological models of hysteresis are well exposed when the systems are driven by noisy inputs and their stochastic behaviors are compared against each other.

4.1.1 Computation of the power spectral density using statistical methods

In this subsection we present the technique for the computation of the power spectral density (PSD) of a stochastic signal in general, by using statistical averaging methods. Consider that \( x(t) \) describes a stochastic signal, which can be either the input (e.g. magnetic field) or the output signal (e.g. magnetization) of the magnetic hysteretic system. The Fourier transform of \( x(t) \) is:

\[
X(\omega) = \int_{-\infty}^{\infty} x(t) \exp^{-j\omega t} dt.
\]  

(90)

The time domain signal \( x(t) \) can always be recovered by taking the inverse Fourier transform on \( X(\omega) \):
\[
x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega.
\] 

(91)

In real-world applications we expect the signal \( x(t) \) to exist only between two finite periods of time. If we denote by \( x_T(t) \) the truncated signal of \( x(t) \) between two finite time periods then \( x_T(t) \) can be expressed as:

\[
x_T(t) = \begin{cases} 
  x(t), & -T < t < T \\
  0, & \text{elsewhere}
\end{cases},
\]

(92)

where the finite time period \( T \) must be large enough to contain as much of the signal information as possible. The Fourier transform of the stochastic signal can then be computed by using the truncated signal \( x_T(t) \):

\[
X_T(\omega) = \int_{-\infty}^{\infty} x_T(t)e^{-j\omega t} dt = \int_{-T}^{T} x(t)e^{-j\omega t} dt,
\]

(93)

where \( X_T(\omega) \) represents the Fourier transform of \( x_T(t) \) over the time period limits shown in equation (93).

In order to compute the PSD it is instrumental now to look at the time-average power \( P(t) \) (defined as the energy divided by time) of \( x_T(t) \). We can write that:

\[
P(t) = \frac{1}{2T} \int_{-T}^{T} x_T^2(t)dt = \frac{1}{2T} \int_{-\infty}^{\infty} x_T^2 dt = \frac{1}{4\pi T} \int_{-\infty}^{\infty} |X_T(\omega)|^2 d\omega,
\]

(94)

where we have used Parseval’s theorem to relate the power in the time domain to the power in the frequency domain. Although equation (94) gives the time-average power of \( x_T(t) \), this equation does not quite provide the information we wish to know for two reasons. Firstly, the power equation requires that period \( T \) be large enough so that all the signal power can be included in the analysis. Secondly, the power computed using (94) does not represent the average power of the stochastic process but rather the time-average power of a single stochastic signal. To relax the first limitation we let the period \( T \) approach infinity by taking the limit
\( T \to \infty \); to avoid the second limitation, we compute the expected value of \( P(t) \) for the stochastic process:

\[
E[P(t)] = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} E[x^2(t)] dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} E\left[\left|X_T(\omega)\right|^2\right] d\omega.
\]

(95)

The last expression allows us to identify the PSD of the stochastic signal \( x_T(t) \):

\[
S(\omega) = \lim_{T \to \infty} \frac{E\left[\left|X_T(\omega)\right|^2\right]}{2T},
\]

(96)

where we have used that:

\[
E[P(t)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) d\omega.
\]

(97)

In this dissertation we use equation (96) to compute the PSD of any random signal. The Fourier transform of \( x_T(t) \) is computed by using the Fast Fourier Transform (FFT).

### 4.1.2 Numerical generation of Ornstein-Uhlenbeck processes

In the analysis presented in this chapter we assume that the stochastic input signal \( x(t) \) is described by the Itô stochastic differential equation:

\[
dx(t) = -b[x(t) - x_0] dt + \sigma \cdot dW(t),
\]

(98)

where \( x_0 \) and \( \sigma \) are the drift and diffusion coefficients, respectively, \( \frac{1}{b} \) has the meaning of the correlation time, and \( W(t) \) is the Wiener process. Known as Ornstein-Uhlenbeck (OU) noise [129], this random process represents a more realistic noise model than white noise because of the decay of its spectrum at high frequency ensures a finite power of the stochastic signal. In magnetic systems, the input consists of the internal or external noise superimposed on the deterministic applied magnetic field. The initial condition \( x_0 \) in (98) is assumed to be equal to the value of the external applied magnetic field.
In order to generate numerically OU processes it is convenient to use finite differences and
- discretize the left-hand-side (98) by using \( dx(t) = x(t_n) - x(t_{n-1}) \),
- express the time increment as \( dt = \Delta t = t_n - t_{n-1} \).

Employing that
\[
W(s + t) = W(s) + N(0,1)t^{1/2}. \tag{99}
\]
where \( N(0,1) \) is a random variable normally distributed with zero average and unit variance, one obtains the following approximate updating formula:
\[
x(t + \Delta t) \approx x(t) - b[x(t) - x_0] \Delta t + \sigma \cdot N(0,1)(\Delta t)^{1/2}. \tag{100}
\]
Although Eq. (100) has often been used in the literature to generate OU processes, it is reliable only when \( \Delta t \) is relatively small. An exact updating formula has been derived in [98] by integrating (1) and by using the properties of normal variables:
\[
x(t + \Delta t) = x(t)e^{-\Delta t} + \left[ \frac{\sigma^2}{2b} (1 - e^{-2\Delta t}) \right]^{1/2} N(0,1), \tag{101}
\]
in which it is assumed that \( x_0 = 0 \). As expected, this updating formula is reduced to (100) when \( \Delta t \ll 1/b \). It is noteworthy that Eq. (101) splits explicitly the random process into two terms: the first one is the mean and the second one is the standard deviation of \( x(t) \). Since the time step \( \Delta t \) is usually constant, the factors in (101) can be computed in advance and stored to increase the computational efficiency. This latter approach has been used in our study to generate OU processes numerically.

### 4.1.3 Spectral properties of Ornstein-Uhlenbeck processes

Next we present the spectral characteristics of the OU processes and present a few sample numerical generated processes. Figure 62 presents a normally distributed white, i.i.d. noise used to further generate the Wiener and OU processes. Sample Wiener processes generated by using equation (99) are presented in Figure 63. Figure 64 presents sample OU processes generated by using the algorithm presented in the previous subsection.
Note that the variance of the Wiener process increases in time, which makes this noise inappropriate to describe thermal fluctuations in real systems.

Figure 62. Normally distributed random signal (white noise) where $x(t) \in [-1,1]$.

Figure 63. Plot of the Wiener process.
It is important to note that the magnitude of a signal generated using the OU process does not diverge in time (Figure 65) and the spectrum of an OU process decays at high frequencies (Figure 66). This makes OU processes ideal to describe real systems in which the total energy of the noise is finite.

A few properties the OU processes are summarized below [130]:

- The average value of the signal:
  \[ E(x(t)) = x_0 e^{-bt} \]  
  (102)
  Note that as \( t \) goes to infinity, \( E(x(t)) = 0 \).

- The variance of the signal:
  \[ \text{Var}(x(t)) = \frac{\sigma^2}{2b} \left(1 - e^{-2bt}\right) \]  
  (103)
  Note that as \( t \) goes to infinity, \( \text{Var}(x(t)) = \frac{\sigma^2}{2b} \).

- The covariance of the signal
  \[ \text{Cov}(x(t+h), x(t)) = \frac{\sigma^2}{2b} e^{-bh} \left(1 - e^{-2bh}\right) \]  
  (104)
  Note that as \( t \) goes to infinity, \( \text{Cov}(x(t+h), x(t)) = \frac{\sigma^2}{2b} e^{-bh} \).

- The spectral density of the signal
  \[ S(\omega) = \frac{\sigma^2}{b^2} \frac{1}{1 + \left(\frac{\omega}{b}\right)^2} \]  
  (105)
Figure 64. Plot of the Ornstein-Uhlenbeck process described by equation (98) where $b = 1$ and $\sigma = 0.1$.

Figure 65. Plot of the Ornstein-Uhlenbeck process (red) with a Wiener process (blue).
Figure 66. Spectral density of an Ornstein-Uhlenbeck process where $x_0 = 1$ and $\omega_0 = 2000 \text{ rad} / \text{s}$.

4.1.4 Monte-Carlo technique for computing power spectral density

To compute the spectral density of the output process, we first generate a sufficiently large number of realizations of the stochastic input signal underlying equation (100). We then compute the corresponding output signals of each input signal generated. The FFT technique is then used to evaluate the spectral density of each output signal. Finally, we average the output spectra to obtain their expected values. The spectral density of the magnetization $m(t)$ of the hysteretic system can be expressed as

$$S_m(\omega) = \lim_{T \to \infty} \frac{E\left[|M_T(\omega)|^2\right]}{2T}$$

where $M_T(\omega) = \int_{-T}^{T} m(t)e^{-j\omega t} dt$ is the truncated Fourier transform of the output process $m(t)$ and $E[\cdot]$ denotes the expected value of the enclosed quantity. This algorithm has been implemented numerically in HysterSoft [98] and is used to compute the noise spectral density of the output signal for various hysteretic systems. In the simulations presented in this chapter, we have computed the spectrum of the output signal by averaging over 500 statistical (Monte-Carlo)
simulations. The numerical complexity required to compute the spectrum of the output signal is $N \times \log(N) \times n$, where $n$ is the number of signals generated. The total time to evaluate $S_m(\omega)$ on a one-processor computer operating at 3 GHz is of the order of seconds. The flowchart for the Monte-Carlo technique used is presented below in Figure 67:

![Flowchart](image)

Figure 67. Monte-Carlo algorithm used to compute the spectral density of the output process.
4.2 Analytical results for the power spectral density

In this section we present analytical results for the spectral density in two cases. In the first case, we consider that the magnetic hysteretic system is driven by a step function input generated using an OU process. In the second case, the input is a symmetric rectangular loop.

4.2.1 Analytical results for the power spectral density of a step function

In this subsection we consider the case where the hysteretic system is a step function (also known as the hard limiter system) characterized by the simple step function with $m(t) = 1$ if $h(t) \geq 0$ and $m(t) = -1$ if $h(t) < 0$ as shown below in (Figure 68). By following the work of van Vleck [124], the spectral density for the output $(m(t))$ process can be computed analytically by using:

$$S_m(\omega) = \frac{4}{\pi} \int_0^\infty \arcsin(e^{-h}) \cos(\omega t) dt.$$  \hspace{1cm} (106)

Figure 68. Plot of the step function for which analytical results of the power spectral density of the output exist in the literature.
4.2.2 Analytical results for the power spectral density of a rectangular loop

We next consider the hysteretic system is the elementary hysteretic operator shown in Figure 69.

![Figure 69. Symmetric rectangular loops with switching fields α and β for which analytical expressions of the cross-spectral density are available.](image)

It is proved in [128] that the spectral properties for a rectangular loop driven by an Ornstein-Uhlenbeck input can be computed by using the theory of stochastic processes on graphs. From this theory, closed-form analytical expressions for the correlation spectra of the output signal of two rectangular loops are obtained in terms of parabolic cylinder functions.

Given two hysteretic systems describe by symmetrical rectangular loops with switching fields $\alpha$ and $\beta$, the cross-spectral density spectrum of the output is given by:

$$
S_{\alpha\beta}(\omega) = \frac{4\sigma \sqrt{b}}{\omega^2 \sqrt{\pi}} \int_{-\beta}^{\beta} e^{b(y-x)^2/\sigma^2} dy - \frac{2\sigma^2}{\omega} \text{Im} \left[ \frac{\partial G^0}{\partial x}(1,-\beta^+,\omega) \right] 
$$

$$
\quad - \frac{\partial G^0}{\partial x}(1,-\beta^-,\omega) + \frac{\partial G^0}{\partial x}(\beta^+ ,\omega) \right].
$$

(107)

for $\alpha < \beta$ and:
\[
S_{\alpha\beta}(\omega) = \frac{4\sigma \sqrt{b}}{\omega^3 \sqrt{\pi}} \int_{-\alpha}^{\alpha} e^{\frac{(y-x_0)^2}{\sigma^2}} dy - \frac{2\sigma^2}{\omega} \text{Im} \left\{ \sum_{\nu} \left[ \frac{\partial G^0}{\partial x}(1, i\alpha, -\beta^\nu, \omega) \right] \left[ \frac{\partial G^0}{\partial x}(1, i\alpha, \beta^\nu, \omega) \right] \right\}. \tag{108}
\]

For \( \alpha > \beta \), where \( G^0(y, \omega) \) is the solution of a differential equation associated with the OU diffusion process and which can be represented as parabolic cylindrical functions. Details about the derivation of (107) and (108) and the computation of function \( G^0(y, \omega) \) can be found in [128]. Somewhat simpler expressions can be obtained for the power spectral density \( S_{aa}(\omega) \).

### 4.3 Numerical results for the power spectral density

The reliability of our Monte-Carlo approach was tested against the analytical results presented in the previous section.

#### 4.3.1 Numerical results for the power spectral density of a step function

The analytical solution for the power spectral density was computed using (106). A comparison between the spectral densities computed using our Monte-Carlo technique and the analytical solution is plotted in Figure 70. The symbols and the continuous line represent the analytical and numerical solution respectively. The figure shows the very good agreement between the analytical and numerical results for all values of \( b \) and all frequencies.
4.3.2 Numerical results for the power spectral density of a rectangular loop

A second set of tests have been performed for the case where the input is an elementary hysteretic operator described by symmetric rectangular loops where the “up” switching field and “down” switching fields are equal in magnitude. We compare the results obtained from the analytical solution of the power density spectra using (107) and (108) to our Monte-Carlo simulations by plotting the solutions for selected values of the critical field $\alpha$ of the rectangular loop. The analytical and numerical solutions are plotted with the symbols and continuous lines respectively in Figure 71. The inset in Figure 71 shows the power spectral density at low
frequencies as a function of the critical field. This figure shows an excellent agreement between the numerical and analytical solutions, which proves the accuracy and reliability of our Monte-Carlo technique.

Figure 71. Power spectral density for a symmetric rectangular loop computed by using Monte-Carlo simulations (continuous lines). The symbols represent analytical results obtained in [128].
4.3.3 Numerical results for the power spectral density of phenomenological models of hysteresis

In this section we present the power spectral density results obtained in phenomenological models of hysteresis by using the Monte-Carlo technique. The simulation results presented were obtained by using the main phenomenological models of hysteresis discussed in Chapter 2 of this dissertation. In the simulations presented in this section we have used the Energetic model, Jiles-Atherton model, and the Generalized Preisach model. Since these phenomenological models usually describe magnetic hysteresis we have considered that the input signal is the external magnetic field $h(t)$ and the output signal is the magnetization $m(t)$. We have carefully identified the parameters of each model in order to describe a permalloy ferrite material. By carefully selecting these parameters, we ascertain that the hysteresis loops produced by each model are the same. For each model, we have used a coercive field value $H_c = 1.28 \, \text{A/m}$, saturation of magnetization $M_s = 7.72 \times 10^5 \, \text{A/m}$, and remanence value of $M_r = 3.95 \times 10^5 \, \text{A/m}$. The rest of the physical parameters and details about the permalloy ferrite material used in our simulations can be found in [131]. Also, the parameters of the Jiles-Atherton model are given in [131] where $k = 2.44 \, \text{A/m}$, $a = 4.36 \, \text{A/m}$, $\alpha = 1.7 \times 10^{-6}$ and $c = 0.49$. The parameters used for the EM were identified by using the technique presented in [27]. The parameters used are as follows: $h = 0.4 \, \text{A/m}$, $k = 1.2 \, \text{T}$, $g = 8.24$, $c_r = 0.02$, and $N_e = 3.5 \times 10^{-7}$. In the case of the GPM we have first simulated a set of 400 first-order-reversal-curves using the JAM (with the parameters given above) and then computed the Preisach distribution by using the identification technique presented in [1]. The Preisach distribution was then fitted to a two-dimensional normal distribution in order to speed up the computations by using (45) where $S = 0.88$, $H_{\alpha r} = 0.49 \, \text{A/m}$, $H_{\alpha c} = 2.23 \, \text{A/m}$, and $H_0 = 1.52 \, \text{A/m}$. The reversible component of the Preisach distribution was also approximated by a normal distribution using (48) where $H_{\alpha r} = 2.12 \, \text{A/m}$. For all the phenomenological models used in the simulations, the initial hysteretic state was assumed to be the a.c. demagnetized state.
Figure 72. Spectral density of the magnetization output by using the EM for different $\sigma$. 

$S_M(\omega)$
Figure 73. Spectral density of the magnetization output by using the JAM for different $\sigma$. 
Figures 72-74 show the power spectral density of the magnetization computed by using the Energetic model, Jiles-Atherton model and the generalized Preisach model, respectively for different values of the diffusion coefficient $\sigma$. All three figures show that at high frequencies most of the spectra decays as $1/f^2$. It is also very interesting to note that all the phenomenological models used predict a flat spectrum at low frequencies and large magnitudes of the input signal for large values of $\sigma$. Unlike the generalized Preisach model and Energetic model, the Jiles-Atherton model also predicts an increase in the power spectra at low frequencies even for relatively low values of $\sigma$. 

Figure 74. Spectral density of the magnetization output by using the GPM for different $\sigma$. 

4.4 The effect of temperature on the power spectral density of hysteretic systems.

In this section we analyze the effect of the temperature on the power spectral density of magnetic hysteretic systems by using the Energetic model of hysteresis. We have used the EM because it is has been often tested and employed to model temperature and mechanical stress dependent magnetization processes as discussed earlier in Chapter 2 of this dissertation. We have used the Monte-Carlo algorithm to compute the magnetization of the hysteretic system by varying the value of the temperature. The model parameters that we have used in these simulations are the ones used in Section 3 of this Chapter.

Figure 75. Power spectral density of the magnetization computed by using the EM for different values of absolute temperature (in arbitrary units). The Curie temperature of the material is $T_c = 226$ arbitrary units.
Figure 75 shows the spectral density of the magnetization (output signal) computed by using the Energetic model of hysteresis for values of absolute temperature ranging from $T = 5$ arbitrary units to $T = 226$ arbitrary units. All the curves accurately predict the 1/f decay at higher frequencies. It is interesting to observe that the initial value of the spectral density decreased for increasing values of temperature. As the temperature approaches the Curie temperature ($T_c = 226$ arbitrary units) the low-frequency value of the power spectral density is decreasing.

4.5 The effect of mechanical stress on the power spectral density of hysteretic systems.

Figure 76. Power spectral density of the magnetization computed by using the EM for different values of the mechanical stress.
In this section we present the results of the analysis of the effect of the mechanical stress on the power spectral density of magnetic hysteretic systems. As mentioned in the previous section we use the Energetic model of hysteresis because it has been extensively tested and calibrated in the literature for such simulations.

Figure 76 shows the simulation results for the power spectral density of the magnetization computed by using the Energetic model of hysteresis for a wide range of the mechanical stress. The simulations were performed for the room temperature. It is apparent from the figure that the spectrum at low frequencies is flat and decays as $1/f$ decay at higher frequencies. The spectral density at low frequencies decreases as the mechanical stress applied to the system is decreasing. The simulation also shows a slight decrease of the cutoff frequency with the decrease in the mechanical stress.
CHAPTER 5

CONCLUSION

In this dissertation we developed an approach to analyze the magnetic aftereffect phenomena and filtering properties of magnetic hysteretic systems. The numerical algorithms used for the simulations presented in this work have been implemented in HysterSoft [98].

We develop a simple yet efficient technique for characterizing vectorial models of hysteresis. The technique developed is universal and can be applied to any model of hysteresis. This way scalar models of hysteresis are a special case derived that can be derived from the vector models. In this work we have reviewed the main phenomenological models of hysteresis and studied the scalar and vectorial properties of thermal relaxation phenomena in magnetic materials. It was shown that these phenomena can be easily modeled by using standard phenomenological vector models of hysteresis, in which the expected value of the magnetization is computed through Monte-Carlo simulations. The gradual change of the magnetization as a function of time was computed by using scalar and vectorial hysteresis models based on the classical Preisach, Jiles-Atherton, and Energetic models. It was found that all models give results which are in qualitative good agreement with each other. Moreover, the well known $\log t$ - type dependence of the magnetization “creep” in isotropic materials can be qualitatively predicted by all phenomenological models presented. Depending on the initial hysteretic state the trajectory of the magnetization vector can deviate substantially from the straight line, which is a pure vectorial relaxation effect.
This work also provides a vectorial characterization of magnetic viscosity, which is very important for the analysis of aftereffect phenomena in anisotropic systems. We have shown that one cannot explain the curved magnetization trajectories observed in anisotropic systems using three scalar viscosity coefficients along the x, y, and z direction, respectively. Moreover, the data collapse phenomena that was observed in isotropic systems when the direction of the magnetic field is fixed cannot be observed in general when both the direction and the magnitude of the magnetization change in time but only in the case of simple rotational (such as first-order reversal) magnetization processes. The presented simulation results show that the dynamics of the magnetization during vectorial aftereffects is in general a complex process that can lead to multiple maxima on the viscosity curves.

A similar Monte-Carlo based technique was developed to study the noise spectral density of magnetic hysteretic systems by using the main phenomenological models of hysteresis. Monte-Carlo simulations provide a relatively fast and reliable way to analyze the power spectral densities of hysteretic systems. According to our analysis, the output spectra deviate significantly from the Lorentzian shape of the input process for values of the diffusion coefficient near and smaller than the coercive field. The intrinsic differences between the transcendental, differential, and integral modeling of hysteresis yield significantly different spectra at low frequency region, which reflect the diverse long-time correlation behavior. It is also apparent from this study that the spectral analysis is a powerful characterization tool that can be used to design filters based on hysteretic systems.
APPENDIX A: ANALYTICAL EQUATIONS FOR THE AVERAGE OF THE OUTPUT IN THE PREISACH MODEL

Suppose the input of the hysteretic system is a discrete-time i.i.d. random process. A full derivation of the equations for the average of the output variable is presented in [12] for a single rectangular loop and for the Preisach model. These equations are also summarized in this appendix.

A.1 The average value of the output for a single rectangular loop

The average value of the output can be computed by [12]:

\[
\langle f_n \rangle = \mathcal{G}(\alpha, \beta) - \zeta(\alpha, \beta) \right] r_{\alpha\beta} + \zeta(\alpha, \beta),
\]

(109)

where \(\alpha\) and \(\beta\) are the “up” and “down” switching values the elementary hysteretic loops, and:

\[
\zeta(\alpha, \beta) = \frac{P_\alpha - P_\beta}{P_\alpha + P_\beta},
\]

(110)

\[
r_{\alpha\beta} = 1 - P_\alpha - P_\beta,
\]

(111)

while \(P_\alpha\) and \(P_\beta\) are defined as follows:

\[
P_\alpha = \int_\alpha^\infty \rho(x)dx,
\]

(112)

\[
P_\beta = \int_{-\infty}^\beta \rho(x)dx.
\]

(113)

In eq. (109) \(\mathcal{G}(\alpha, \beta)\) is equal to 1 or -1, depending on the initial state (“up” or “down”) of the
A.2 The average value of the output for the GPM

In the case of the classical Preisach model of hysteresis the average value of the output variable can be computed by using the following equation:

\[
\langle f_n \rangle = \langle f_\infty \rangle + \int_{\alpha,\beta} P(\alpha, \beta) \left[ \vartheta(\alpha, \beta) - \zeta(\alpha, \beta) \right] \tau_{\alpha \beta}^n \, d\alpha \, d\beta, \tag{114}
\]

where \( \vartheta(\alpha, \beta) \) is equal to 1 or -1, depending on the state (“up” or “down”) of the elementary hysteresis loop with switching values equal to \( \alpha \) and \( \beta \) and

\[
\langle f_\infty \rangle = \int_{\alpha,\beta} P(\alpha, \beta) \zeta(\alpha, \beta) \, d\alpha \, d\beta + \int_{-\infty}^\infty R(\alpha)(2\mathbb{P}_\alpha - 1) \, d\alpha \tag{115}
\]

is the asymptotic value of the average value of the output variables. It should be noted that \( f_\infty \) does not depend on the history of the input variations but only on the Preisach functions and the probability density function of the input random variable.
APPENDIX B: MATLAB CODE USED TO GENERATE
ORNSTEIN-UHLENBECK STOCHASTIC INPUT PROCESSES

The following code was used to generate Ornstein-Uhlenbeck stochastic input signals (see equation (98) in Section 4.1.2). The finite difference equation that was used to generate this stochastic signal is:

\[ x(t_n) = x(t_{n-1}) - b\left[ x(t_{n-1}) - x_0 \right] \cdot \Delta t + \sigma \cdot \sqrt{\Delta t} \cdot dW, \]

where \( W \) is the Wiener process, \( \sigma \) is the diffusion constant, \( \frac{1}{b} \) is the time constant, \( x_0 \) is the drift coefficient, and \( \Delta t \) is the time step.

```
function[]=
Generate_Ornstein_Uhlenbeck_Process(OU_x0,OU_b,OU_sigma,npnts,ncurves)
```

%====================================================================
% FUNCTION NAME:    Generate_Ornstein_Uhlenbeck_Process
% % INPUT ARGUMENTS:
% %: OU_x0 := Initial condition (drift coefficient)
% %: OU_b := 1/OU_b has the meaning of correlation time
% %: OU_sigma := Ornstein Uhlenbeck diffusion coefficient
% %: npnts := Number of points to generate
% %: ncurves := number of OU curves to generate
% %====================================================================
% OUTPUT ARGUMENTS:
%                 N/A
%
% DESCRIPTION:
%  1.) Generate ncurves stochastic signals as OU processes
%  2.) Plot all the generated curves on the same figure

if(~exist('OU_x0'))
    OU_x0 = 0;
end

if(~exist('OU_sigma'))
    OU_sigma = 0.1;
end

if(~exist('OU_b'))
    OU_b = 1;
end

if(~exist('npnts'))
    npnts = 1000;
end

if(~exist('ncurves'))
    ncurves = 10;
end

delta_t = 1 / npnts;

x = zeros(1,npnts);
x(1) = OU_x0;
tmax = 1;
t = 0:tmax/(npnts-1):tmax;

%====================================================================
% Numerical Implementation of the Ornstein-Uhlenbeck process
% Plot all the curves on the same figure by using Matlab command "hold
% on"
%====================================================================

for j = 1:1:ncurves
    for i = 2:1:npnts
        x(i) = x(i-1) - OU_b*(x(i-1)-OU_x0)*delta_t + ...
              OU_sigma*sqrt(delta_t)*randn;
    end
    plot(t,x,'r','LineWidth',1)
    hold on
end
REFERENCES


BIOGRAPHICAL SKETCH

Ayodeji Adedoyin was born in Ibadan, Oyo State Nigeria (West Africa). He received his BS degree in Electrical Engineering from Florida Agricultural and Mechanical University in 2005. Then, he obtained his MS degree in Electrical Engineering in 2007. He is currently a PhD Candidate in the Electrical and Computer Engineering Department and also a Graduate Research Assistant in the Modeling and Simulating Group at Florida State University. His research interests are in chip design for submicron VLSI, nano-scale electromagnetic computations using FMM, thermal relaxation effect in high capacity magnetic devices and the integration of magnetic and semiconductors in producing MRAM devices.

Ayodeji has the following publications listed below:


