The Competitive Impact of Targeted Television Advertisements Using DVR Technology

Bikram Ghosh, Michael R. Galbreth, and Guangzhi Shang
The Competitive Impact of Targeted Television Advertisements Using DVR Technology

Bikram P. Ghosh
Associate Professor
Department of Marketing
Moore School of Business
University of South Carolina, Columbia, SC 2908
Email: bghosh@moore.sc.edu

Michael R. Galbreth¹
Associate Professor
Department of Management Science
Moore School of Business
University of South Carolina, Columbia, SC 2908
Email: galbreth@moore.sc.edu

Guangzhi Shang
Ph.D. Candidate
Department of Management Science
Moore School of Business
University of South Carolina, Columbia, SC 2908
Email: guangzhi.shang@grad.moore.sc.edu

¹Corresponding author
ABSTRACT

Digital video recorders (DVRs) are an emerging technology that is fundamentally changing the competitive landscape in industries that advertise on television. Perhaps the most familiar impact of DVR technology is that it enables consumers to avoid advertisements by fast forwarding through them. However, this “zipping” of ads is only one aspect of the impact of DVR technology. DVRs also collect a wealth of information at the consumer level that can be used by firms to target their advertisements more effectively. We examine how this targeting capability moderates the impact of ad avoidance in a competitive setting. Insights are provided on how best to manage this emerging technology in terms of the key managerial decisions of product pricing and advertising efforts, as well as its impact on profits in a competitive (duopoly) marketplace. [Submitted: December 28, 2011. Revised: July 2, 2012; November 21, 2012. Accepted: February 1, 2013.]

Subject Areas: Ad Skipping Behavior, Digital Video Recorder, Product Competition, Targeted Advertisements

INTRODUCTION AND LITERATURE REVIEW

Digital video recorder (DVR) technology is fundamentally changing the nature of television advertising, and thus the competitive landscape in industries that advertise on television. The DVR has penetrated the home electronics market with surprising speed, becoming an integral part of the television viewing experience for many consumers. In fact, DVR penetration in the U.S. more than doubled, from 17% to 38% between 2007 and 2011 (Steinberg & Hampp, 2007; Nielsen, 2010). Given the important role of television advertising for many firms, a careful response to this trend is warranted. However, the impact of DVR technology on television advertising effectiveness is not clear.

A critical advantage of DVRs from the firm’s perspective is their ability to track viewing patterns at the individual consumer level, enabling firms to target their advertisements (ads)
to consumers who are more likely to be interested in their product. As noted by Wilbur (2008), DVRs enable “addressable advertising,” such as sending a specific ad only to viewers with certain income characteristics or who live in a certain area. This DVR-enabled targeting is the focal point of this article, as it represents a dramatic technology-driven departure from traditional competition between firms who use television advertising to reach potential consumers. Targeting enables firms to be more efficient in their advertising expenditures, concentrating ads on consumers who are more likely to be interested in their product. The impact of this improved targeting, however, must be considered in conjunction with the other key capability of DVR technology: the ability of consumers to avoid ads by fast forwarding, or “zipping” through them. We explicitly consider these two effects of DVR usage – targeting and ad avoidance – on pricing, advertising effort, and ultimately profits. As our analysis will show, when targeting is considered along with ad avoidance in a duopoly model, its impact on competition is not always intuitive, and it has important implications for managers striving to determine the appropriate response to the rapid penetration of DVR technology.

A preview of our main insights is provided here. First, we show that the ability to target ads through DVR technology leads to a monotonic and positive effect of DVR penetration on equilibrium television advertising effort, even when explicitly considering that some ads might be avoided by consumers. In other words, firms should respond to increased DVR penetration by increasing advertising effort. Second, equilibrium prices initially decrease with DVR penetration, but beyond a threshold, prices increase with penetration. That is, a firm should pair its increased ad effort with decreased prices to a point, but as DVR penetration becomes higher (and thus ad targeting improves), the firm should begin to increase prices. Finally, equilibrium profits are initially harmed by DVRs, but beyond a threshold we show that DVR usage is profit-enhancing for competing firms. From a marginal profit perspective, improved targeting is not able to offset ad avoidance when relatively few consumers use DVRs, but as DVRs begin to saturate the market, and ads become highly targeted, DVR usage begins to impact profits positively.
This article directly relates to two streams of research: selective targeting and consumer avoidance of ads. Although targeted advertising in the specific case of online ads (e.g. banner ads) has been receiving growing attention (Manchanda, Dube, Goh, and Chintagunta (2006) includes a recent example), the literature on selective/customized targeting that is not specific to the online context is relatively scant. Gal-Or and Gal-Or (2005) examined the case where a media technology provider acts as an ad distributor for multiple product advertisers, showing how it utilizes its ad exposure discriminating capacity to attenuate price competition in the product market. Iyer, Soberman, and Villas-Boas (2005) studied how retailer-advertisers discriminate consumers on price offers and ad exposure. In this article we assume a similar context to that of Iyer et al. (2005), but our focus is on the effect of DVR technology targeting on competition, and thus the potential for avoidance of targeted ads plays a central role in our framework.

Academic research on the causes and effects of ad avoidance has expanded dramatically in the past decade, as new ad avoidance technologies, such as DVRs, have become available. Wilbur (2008) summarized the motivations for avoiding TV ads as follows: substitute activities, boring ads, worn-out ads, and no interest in product. As noted by Speck and Elliott (1997), consumers can avoid ads through cognitive, behavioral, and mechanical means. The mechanical means enabled by a DVR tend to be the easiest way of avoiding ads among the three (Kelly, Kerr, & Drennan, 2010). Several recent papers focus on the impact of this type of mechanical ad avoidance. For example, Stühmeier and Wenzel (2010) examined how ad avoidance differentiates the profitability of regular TV and DVR-based viewing from a media provider’s perspective. Ghosh and Stock (2010) studied the effects of DVR penetration on product market competition and consequently firm profits. Teixeira, Wedel, and Pieters (2010) found that brand “pulsing” can be an effective way of keeping viewers’ attention when faced with ad avoidance. None of these papers, however, explicitly considered that DVRs, while facilitating ad avoidance, also enable firms to target their ads more effectively. In general, while the literature has established that selective targeting can affect retailer profit
positively and ad avoidance can do so negatively, no research has examined their joint effects in a unified analytical framework. A complete understanding of the impact of the DVR technology in a competitive setting requires such an examination, and it is this research gap that we address in this article.

**MODEL AND ANALYSIS**

Similar to Soberman (2004) and Grossman and Shapiro (1984), advertising in our model only plays an informative role, as opposed to a persuasive one. Informative ads tell consumers about the existence and attributes of a product, but they do not influence consumers’ valuations of it. In this context, we model product differentiation by locating consumers uniformly along the Hotelling line, with two competing firms, \( A \) and \( B \), located at its endpoints. The strength of differentiation between the products (i.e. the Hotelling ‘travel’ cost) is \( t \). Given a base valuation of the product \( v \), consumers located at \( x \in [0, 1] \) have utility \([v - tx - p_A]\) of buying from \( A \) and utility \((v - t(1 - x) - p_B)\) of buying from \( B \). We follow the larger literature on informative advertising by assuming that consumers ex ante are not aware of any product characteristics, and that the ads provided by the firms in our model are the only source of product information.

Consider a market of size 1 where some fraction \( \alpha \) of consumers use a DVR, with the remaining \((1 - \alpha)\) non-DVR users watching traditional, “live” television. Each firm’s decision regarding advertising effort is denoted \( \phi_i \). Ads reach non-DVR users uniformly, with all non-DVR users having a probability \( \phi_A \) of receiving ads from firm \( A \) and \( \phi_B \) of receiving ads from firm \( B \). We assume that DVR users have a probability \( \beta \in (0, 1) \) of viewing (that is, not avoiding) the ads sent to them. We acknowledge that non-DVR users might also avoid ads (perhaps for example by leaving the room). For model parsimony and without loss of generality, we normalize the ad avoidance rate for non-DVR users to zero (Ghosh & Stock, 2010). In our main analysis we do not consider the possibility that a consumer’s \( \beta \) might be
linked to her preference for the product – that is, she might be more likely to stop zipping an ad, rewind, and watch the ad if she notices that the product appeals to her (Bernoff, 2004). However, in Lemma 2 in Appendix B we examine this selective avoidance context using an established approach from prior literature, and we confirm that all of the following results continue to hold under selective avoidance. Thus, we use the more parsimonious constant $\beta$ formulation in our main model.

As discussed above, two attributes distinguish DVR users from non-users – the ability to be targeted by specific ads and the ability to avoid ads by zipping through them. The targeting of ads to consumers who are more interested in a product is facilitated by the richness of data captured by a DVR. Given this targeting ability, we assume that, for the portion of the market that is using a DVR, consumers who prefer a firm's product have a higher probability of receiving its advertising. Since our model will also include the additional complexity of ad avoidance, we model targeting ability in a parsimonious way, defining a consumer located at $x$ to have probabilities $\phi_A(1 - x)$ and $\phi_B x$ of being reached by firms $A$ and $B$, respectively. This captures the idea that, if a consumer prefers Firm $i$'s product, more of Firm $i$'s ads will be targeted to her. The stronger this consumer preference, the more of the firm's ads the consumer will see. Note that this formulation assumes each firm chooses a single advertising effort $\phi_i$ for both targeted and non-targeted ads. This assumption is relaxed later in this paper in an extension, wherein the insights provided below are shown to exist when each firm chooses two separate levels of advertising effort, one for targeted ads and one for non-targeted ads.

Next, we incorporate the ad avoidance rate by assuming that the DVR user actually watches the ads sent to her by firms $A$ and $B$ with probabilities $\beta \phi_A (1 - x)$ and $\beta \phi_B x$, respectively. Finally, we include a cost of advertising. As noted by Araman and Popescu (2010), in upfront sales of television advertising (which make up the majority of television ad sales in North America and Europe), broadcasters typically sell impressions (or "eye-balls"), and firms pay a price based on the number of viewers that will see the ad. We
assume that the cost of reaching additional consumers is quadratically increasing with the form \( \theta \) (number of consumers)^2. A convex cost of ad reach is well-established in the literature (Grossman & Shapiro, 1984; Soberman, 2004, among others). Furthermore, we note that both linear and concave cost functions will result in a simple corner solution for ad expenditure, which is not particularly insightful (firms should choose the maximum ad expenditure if the marginal cost is not too high, otherwise they should choose zero ad expenditure). Thus, the only interesting case is that of convex cost, which is also the most well-established functional form in the related literature.

To obtain the total amount of ads sent by each firm, we accumulate both targeted (DVR) and non-targeted ads across the Hotelling line. In total, DVR users receive \( [\alpha \int_0^1 \phi_A(1 - x)dx = \frac{\alpha \phi_A}{2}] \) from firm A and \( [\alpha \int_0^1 \phi_B x dx = \frac{\alpha \phi_B}{2}] \) from firm B. Non-DVR users receive \( (1 - \alpha)\phi_A \) and \( (1 - \alpha)\phi_B \) from firms A and B respectively. Thus, we can define a total advertising cost function that explicitly considers DVR-based ad targeting:

\[
C(\alpha, \theta, \phi_i) = \theta \left( \frac{\alpha \phi_i}{2} + (1 - \alpha)\phi_i \right)^2 = \theta \phi_i^2 \left( 1 - \frac{\alpha}{2} \right)^2
\]

Note that the cost of sending ads given by Equation (1) is decreasing in DVR penetration \( \alpha \). In other words, as in previous work on targeted advertising (Gal-Or & Gal-Or, 2005), we model the fact that a benefit of targeted advertising is reduced ad costs.

We begin by deriving the demand faced by firm A (since the two firms are identical except for their positioning on the Hotelling line, a separate analysis for firm B is not needed). To derive firm A’s demand, we distinguish between consumers who are partially informed vs. fully informed about the competing products. Partially informed consumers represent those who view firm A’s ads but not firm B’s, whereas fully informed consumers view both firms’ ads. We assume that a consumer will only purchase an item if she has been made aware of it (i.e. if she has actually viewed an ad). Thus, the two firms only compete directly in the fully informed segment of the market. In line with Grossman and Shapiro (1984) and
Soberman (2004), we assume the partially informed fraction of the market is fully served (i.e., \( v - t - p_i > 0 \)). The assumption implies that the degree of differentiation between the product, \( t \), is relatively low (exact bounds on \( t \) are given in Lemma 1). Note that there are three possible types of partially informed consumers: (i) DVR users targeted by both firms but who avoid \( B \)'s ad, (ii) DVR users targeted solely by firm \( A \) who do not avoid \( A \)'s ad, and (iii) non-DVR users reached solely by firm \( A \). Define, \( Q_A^P \) as the demand for firm \( A \) from partially informed consumers, explicitly considering both DVR-based ad targeting and the possibility of ad avoidance:

\[
Q_A^P = \alpha \int_0^1 \left( (\beta(1 - \beta)\phi_A \phi_B x(1 - x)) + (\beta \phi_A (1 - x)(1 - \phi_B x)) \right) dx + (1 - \alpha)\phi_A (1 - \phi_B) \tag{2}
\]

Furthermore, there are two types of fully informed consumers: (i) DVR users targeted by both firms who do not avoid either ad, and (ii) non-DVR users reached by both firms. A fully informed consumer compares the utilities derived from buying from \( A \) and \( B \) and chooses rationally. Let \( x_i = (p_B - p_A + t)/2t \) denote the consumer on the Hotelling line who is indifferent between buying from \( A \) and \( B \). Consumers located between 0 and \( x_i \) buy from \( A \), with the rest buying from \( B \). Define \( Q_A^F \) as the demand firm \( A \) faces from fully informed consumers:

\[
Q_A^F = \alpha \int_0^{x_i} \left( \beta^2 \phi_A \phi_B x(1 - x) \right) dx + (1 - \alpha)\phi_A \phi_B x_i \tag{3}
\]

The profit function for firm \( A \) is therefore,

\[
\Pi_A = (Q_A^P + Q_A^F)(p_A - c) - C(\alpha, \theta, \phi_A) \tag{4}
\]

Given these demand and profit functions, equilibrium price, advertising effort, and profit are provided in the following lemma. The proof of the lemma, along with conditions of
existence and uniqueness, are provided in Appendix C.

**Lemma 1:** For $\theta \in \left[\theta^c, \theta^b\right]$, and $t \in [t^c, \overline{t}]$ and $\beta \geq \frac{12t}{3v-3c-t}$ there exists a unique pure strategy symmetric equilibrium and it is given by

$$p^* = c + 2\frac{\theta t(2 - \alpha)}{\sqrt{\theta t(4 - \alpha(4 - \beta^2))}}$$

$$\phi^* = \frac{6t(2 - \alpha(2 - \beta))}{t(6 - \alpha(6 - \beta^2)) + 3(2 - \alpha)\sqrt{\theta t(4 - \alpha(4 - \beta^2))}}$$

$$\Pi^* = \frac{9\theta t^2(2 - \alpha)^2(2 - \alpha(2 - \beta))^2}{(t(6 - \alpha(6 - \beta^2)) + 3(2 - \alpha)\sqrt{\theta t(4 - \alpha(4 - \beta^2))})^2}$$

where $\theta^c = \frac{1}{9}t(6 - \beta)^2; \overline{\theta}^c = \frac{(v - t - \epsilon)^2}{4t}; \overline{t}^c = \frac{2(v - \epsilon - c)}{9}; \overline{t}^c = \frac{3(v - \epsilon)}{13}$

This equilibrium result provides a rich context in which to examine the impact of DVR-enabled ad targeting on the strategic decisions, and ultimately profits, of competing firms. We begin by summarizing the equilibrium managerial response, in terms of pricing and advertising effort, to the improved targeting enabled by increased market penetration of DVR technology:

**Proposition 1:** In the unique symmetric equilibrium,

(a) prices decrease with DVR penetration $\alpha$ when the level of penetration is below a threshold, $\alpha^*$, and increase otherwise, where $\alpha^* = \frac{2\beta^2}{4-\beta^2}$.

(b) advertising effort always increases with DVR penetration $\alpha$.

The equilibrium price result in Proposition 1 is driven by the overall nature of the competition between the firms. A well-established result in the literature on informative ads is that competition increases with the proportion of fully informed consumers, since these consumers are aware of both products and hence firms must compete aggressively to capture them (Soberman, 2004). Partially informed consumers have the opposite effect on competition – overall competition decreases with the share of partially informed consumers, since
these consumers are aware of only one firm’s product, and thus that firm effectively exercises monopoly pricing power over them. As DVR penetration increases, more consumers avoid ads, and thus the fully informed segment shrinks while the partially informed segment grows, reducing competition. We show that this competition reduction, which should enable price increases, is moderated (and sometimes completely offset) by the competitive impact of the targeting capabilities of DVR technology.

Targeting implies that firms can concentrate their ads on DVR owners who prefer their product (i.e. are closer to them on the Hotelling line). It follows directly that the advertising efforts of the two firms only overlap, and consumers might be fully informed, around the middle of the Hotelling line. Price elasticity of demand is the highest for precisely these consumers, since they have no clear preference between the products. Now, as $\alpha$ increases, so does the firm’s targeting ability, and thus fewer consumers are fully informed. In this way, the targeting impact of DVR technology is analogous to the ad avoidance impact – reduce the size of the fully informed segment, and thus relax competition. However, although better targeting does shrink the size of the fully informed segment, it also results in the segment becoming increasingly composed of only the most price-sensitive consumers (those near the center of the Hotelling line). Note that this compression of demand from the fully informed segment to the most price-sensitive consumers is driven by the specific nature of targeted advertising, which is a novel aspect of our model of DVR usage. We find that the increasing price sensitivity of these fully informed consumers exerts downward pressure on prices, which moderates the DVR’s competition-reducing effect. Furthermore, the price-sensitivity effect actually outweighs the shrinking of the fully informed segment, and thus equilibrium prices in a competitive setting decrease with DVR penetration, when DVR usage is below the threshold level $\alpha^*$.

Proposition 1 also provides insights into the effect of DVR penetration on the equilibrium advertising efforts of competing firms. From the firm’s perspective, since DVRs enable firms to target ads to consumers that are more likely to be interested in their product (a good
thing), while also facilitating ad avoidance (a bad thing), an intriguing question arises: should television advertising efforts increase or decrease as DVR usage becomes more widespread? Our results suggest that ad efforts should increase with DVR penetration. This monotonic result is driven by two benefits of targeted advertising, which we find to always outweigh the negative impact of ad avoidance. First, targeted ads enable firms to focus their advertising efforts on DVR owners who are already inclined to choose their product. Secondly, targeted advertising is a more efficient approach, since more targeting ability enables a firm to reach the same number of consumers with fewer ads. This leads directly to a lower advertising cost. In total, DVR technology enables firms to lower their advertising costs while increasing the likelihood that an ad will be viewed by consumers that prefer their product. These benefits are so strong that firms will monotonically increase advertising effort as DVR penetration increases. Simply put, the more DVR users in the marketplace, the more effort firms will expend to reach them through television advertising.

We conclude our analysis by examining how DVR penetration impacts equilibrium profits. We find that, although the targeting benefits lead firms to consistently increase advertising effort as DVR penetration increases, the additional effort does not always lead to higher profits, as shown in the following proposition:

**Proposition 2:** In the unique symmetric equilibrium,

(a) profits are initially decreasing with DVR penetration $\alpha$.

(b) profits are increasing for sufficiently high levels of DVR penetration $\alpha$.

The intuition for this result derives from the fact that the profit benefits of targeting (lower ad costs, more selective placement of ads) are minimal when DVR usage is very low, since very little targeting is possible in that case. However, we find that there is a point above which DVR penetration is sufficiently high such that these targeting benefits begin to outweigh the negative consequences of DVR-based ad avoidance, and profits increase in $\alpha$. The profit result in Proposition 2 is also consistent with our finding that price competition first intensifies, then lessens, as DVR penetration increases. We find that, as expected, the
pattern of profits is similar to that of price competition – initially falling, then rising with DVR penetration.

(PLACE FIGURE 1 HERE)

Propositions 1 and 2 are summarized in Figure 1, which provides a complete graphical depiction of our findings regarding the impact of DVR penetration on prices, advertising effort, and ultimately profits in a competitive setting (parameters used in the figure: \( v = 1, c = 0.1, t = 0.16, \beta = 0.96, \theta = 0.5 \)). Additional graphs for other levels of \( \theta \) can be found in Appendix C.

EXTENSION: DIFFERENTIATED AD EFFORT

In this section, we extend our analysis to consider the case where firms separately determine the efforts directed to targeted and non-targeted advertising. That is, each firm no longer decides on a single \( \phi_i \). Instead, we use \( \phi_{Ti} \) and \( \phi_{Ni} \) to denote the level of ad effort of firm \( i \) for targeted and non-targeted ads, respectively. Under this setup, each firm’s cost function reflects the differential levels of ad reach for the two segments (i.e., there is an additional decision variable). For firm \( A \) we have

\[
C(\alpha, \theta, \phi_{TA}, \phi_{NA}) = \theta(\alpha \frac{\phi_{TA}}{2} + (1 - \alpha)\phi_{NA})^2
\]

Next, the demand from both partially informed \((Q^P_A)\) and fully informed \((Q^F_A)\) consumers should also reflect differential levels of ad reach. In Equations (2) and (3), we replace \( \phi_i \) with \( \phi_{Ti} \) for demand from DVR users and with \( \phi_{Ni} \) for demand from non-DVR users. Other aspects of the model remain the same. Thus, firm \( A \)’s profit function can be straightforwardly derived from Equation (4).

For \( \theta \in [\bar{\theta}, \bar{\bar{\theta}}], \ t \in [\bar{t}, \bar{\bar{t}}], \) and \( \beta \in [\frac{12}{13}, 1] \), the pure strategy symmetric equilibrium is given by
\[ p^* = c + \frac{12(1-\alpha)\alpha(1-\beta)(3\beta - 2)\theta + (6\alpha + 4\beta - 4\alpha\beta)\sqrt{\theta[t(4+5\alpha)\beta^2 - 36(1-\alpha)\alpha(1-\beta)^2\theta]}}{(4+5\alpha)\beta^2} \]

\[ \phi_D^* = \frac{6\beta[t\beta + 4(1-\alpha)(1-\beta)\theta] - 6\beta[t(4+5\alpha)\beta^2 - 36(1-\alpha)\alpha(1-\beta)^2\theta]}{t\beta^2 - \beta[9\alpha + 4(1-\alpha)\beta^2]t} \]

\[ \phi_N^* = \frac{2[-t\beta^2 + 9\alpha(1-\beta)\theta + \beta\sqrt{\theta[t(4+5\alpha)\beta^2 - 36(1-\alpha)\alpha(1-\beta)^2\theta]}]}{9\alpha\theta - \beta^2[t - 4(1-\alpha)\theta]} \]

where \( \theta = \frac{t\beta^2}{3\alpha(1-\beta)^2} \), \( \bar{\theta} = \frac{(v-c-t)^2\beta^2}{4t} \), \( \bar{t} = \frac{(v-c)(1-\beta)}{2-\beta} \), and \( t = \frac{(v-c)(1-\beta)}{4-3\beta} \).

It can be shown that this equilibrium price is initially decreasing in \( \alpha \) and is increasing in \( \alpha \) for sufficiently high \( \alpha \). That is, \( \frac{\partial p^*}{\partial \alpha} |_{\alpha=0} < 0 \) and \( \frac{\partial p^*}{\partial \alpha} |_{\alpha=1} > 0 \). For both DVR and non-DVR segments, equilibrium ad effort is increasing initially in \( \alpha \) and is increasing in \( \alpha \) for sufficiently high \( \alpha \). That is, \( \frac{\partial \phi_D^*}{\partial \alpha} |_{\alpha=0}, \frac{\partial \phi_D^*}{\partial \alpha} |_{\alpha=1}, \frac{\partial \phi_N^*}{\partial \alpha} |_{\alpha=0}, \) and \( \frac{\partial \phi_N^*}{\partial \alpha} |_{\alpha=1} \) are all positive. In other words, the insights into managerial decisions provided by our main model (Proposition 1) do not substantively change under this model extension, lending robustness to our main results.

**SUMMARY AND CONCLUSION**

The impact of DVR technology on consumer behavior (skipping ads) is well-known. In this paper we expand the understanding of the competitive impact of DVR usage by acknowledging that ad avoidance is just part of the DVR story. A complete understanding must also consider the targeting capability of the technology, which increases the efficiency of advertising expenditures and enables firms to reach consumers that are more likely to be interested in their product. The multifaceted impact of DVRs on competition makes it difficult to determine the appropriate managerial response to the increasing penetration of DVRs. This study presents a model that explicitly considers both ad targeting and ad avoidance in a competitive framework, enabling us to provide insights into how managers in a competitive environment should respond to the changing landscape of consumer behavior.
marketplace should respond to this emerging technology.

The first suggestion from our results is that, as DVR penetration increases, the best competitive response for managers is always to increase their television advertising efforts (an increasing proportion of which will be targeted). This is a new, straightforward insight regarding the impact of DVR-based targeting on the advertising strategy of firms. The best competitive price response for firms, however, is more nuanced. As DVR penetration increases and firms are better able to target their television advertising, they should initially decrease prices. However, when considering ad targeting in this competitive context, we find a simple threshold above which the equilibrium competitive response is to increase prices as DVR penetration continues to expand. Finally, we find that, although low levels of DVR penetration will initially hurt firm profits, as DVR usage, and thus targeted advertising, becomes very widespread, it will lead to higher profits in equilibrium.

This paper presents the first model of the impact of DVR technology on competition that explicitly captures the emerging ability of firms to target advertising to DVR users. From a research perspective, this model can serve as a foundation for future work in the area and can be enhanced as our understanding of the capabilities and competitive implications of DVR-based targeting continues to evolve. The insights provided here are also relevant for practitioners, as more firms become interested in leveraging the targeting capabilities enabled by the DVR technology.

REFERENCES


Bernoff, J. (2004). The mind of the DVR user: Media and advertising. Forrester Research (September 8).


## APPENDIX A: NOTATION

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>DVR penetration</td>
</tr>
<tr>
<td>$\beta_i(x)$</td>
<td>Probability of watching firm $i$’s ad for a consumer located at $x$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Average probability of watching firm $i$’s ad</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Selectivity in ad avoidance</td>
</tr>
<tr>
<td>$x$</td>
<td>Consumer’s Hotelling line location</td>
</tr>
<tr>
<td>$v$</td>
<td>Base product valuation</td>
</tr>
<tr>
<td>$t$</td>
<td>Strength of product differentiation</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Coefficient of advertising cost</td>
</tr>
<tr>
<td>$c$</td>
<td>Unit production cost</td>
</tr>
<tr>
<td>Decision Variables</td>
<td></td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>Ad effort of firm $i$</td>
</tr>
<tr>
<td>$p_i$</td>
<td>Price of firm $i$</td>
</tr>
</tbody>
</table>
APPENDIX B: SELECTIVE AVOIDANCE

In this appendix we model the context in which the viewership parameter $\beta$ is linked to a consumer’s preferences – that is, the stronger her preference for a product, the less likely she is to skip ads for it. In other words, some product characteristics, which might be observed as the ad is being skipped, might make the consumer more likely to rewind and watch the ad if they match the consumer’s preferences more closely. This is based on the idea, described in detail by Bernoff (2004), that consumers sometimes rewind an ad that they had been zipping if they notice that the product might appeal to them. Formally, we relax our assumption in the main model that all consumers along the Hotelling line have the same probability $\beta$ of viewing firm $i$’s ad. Denote the probability that a consumer views an ad from firm $i$ as $\beta_i$.

To reflect preference-related selectivity in skipping ads, we use the approach of Ghosh and Stock (2010), defining $\beta_A = \beta + \varepsilon(1 - 2x)$ and $\beta_B = \beta - \varepsilon(1 - 2x)$. Note that $\beta_i$ is bounded by 0 and 1, which implies $\beta \in (\varepsilon, 1 - \varepsilon)$ and $\varepsilon \in (0, \frac{1}{2})$. Under this formulation, consumers who prefer A’s product (i.e. have a lower value of $x$) are more likely to view A’s ads and skip B’s ads. $\varepsilon$ captures the level of selectivity, where a larger $\varepsilon$ means a larger role played by a consumer’s horizontal preference $x$ in determining the chance of avoiding an ad. Note that, when $\varepsilon = 0$, this formulation reverts to our main model (i.e. $\beta_A = \beta_B = \beta$).

Due to symmetry, an analysis of firm A’s problem will suffice. Substituting $\beta$ with $\beta_A$ and $\beta_B$ in Equations (1) and (2), we obtain demands from the partially and fully informed segments, $Q_P^A$ and $Q_F^A$. $\Pi_A$ is then calculated by substituting $Q_P^A$ and $Q_F^A$ into Equation (3), and the following pure strategy symmetric equilibrium emerges. Conditions for existence and uniqueness are provided in the proof of Lemma 2.

**Lemma 2:** Assume that consumers selectively skip ads. Then for $\theta \in [\theta_1, \theta_2]$, $t \in [t_1, t_2]$,
\( \beta \in [\sqrt{11} \varepsilon, 1 - \varepsilon] \), and \( \varepsilon \leq \frac{1}{1+\sqrt{11}} \), the pure strategy symmetric equilibrium is given by

\[
p^* = c + \frac{2(2 - \alpha)\theta t}{\sqrt{t\theta[4 - \alpha(4 - \beta^2)]}}
\]

\[
\phi^* = \frac{10t[6 - \alpha(6 - 3\beta - \varepsilon)]}{t[30 - \alpha(30 - 5\beta^2 + \varepsilon^2)] + 15(2 - \alpha)\sqrt{t\theta[4 - \alpha(4 - \beta^2)]}}
\]

\[
\Pi^* = \frac{25t^2\theta(2 - \alpha)^2[6 - \alpha(6 - 3\beta - \varepsilon)]^2}{\{t[30 - \alpha(30 - 5\beta^2 + \varepsilon^2)] + 15(2 - \alpha)\sqrt{t\theta[4 - \alpha(4 - \beta^2)]}\}^2}
\]

where \( \theta_s = \frac{t(5(6 - \beta)\beta + \varepsilon)(10 + \varepsilon)^2}{(15\beta)^4} \), \( \overline{\theta}_s = \frac{(v - e - t)\beta^2}{4t} \), \( t_s = \frac{15(v - e)\beta^2[5(3 - \beta)\beta + \varepsilon(5 + \varepsilon)]}{[30(\beta + \varepsilon)(e + 10)]^2 - 5\beta^2[10(\beta^2 + 15\beta + (5 - \varepsilon))]} \), and \( \overline{t}_s = \frac{15(v - e)\beta^2}{5\beta(12 + \beta) + 2e(10 + \varepsilon)} \).

As shown in the proof of the above lemma, all propositions in this paper continue to hold when selective avoidance is explicitly modeled, lending robustness to our findings.

### APPENDIX C: PROOFS

**Proof of Lemma 1**

We derive the equilibrium profile by a simple profit maximization. In the following, we ensure the uniqueness of the equilibrium and the stability of the equilibrium. The structure of the proof is similar to that of Ghosh and Stock (2010).

We first prove that all prices greater than \( v - t \) are never observed in equilibrium in the sense that, given the conditions in Lemma 1, all such prices are strictly dominated. When firm \( i = A, B \) charges \( p_i > v - t \), the appropriate profit function is such that the market for partially informed consumers is not completely covered. Hence, the demand and profit function for firm \( i \) ought to accommodate for that.

In a symmetric equilibrium when both firms charge \( p_i = p = v - t \) we have, dropping the subscript \( i \) from prices:
\[
\frac{\partial \Pi_i}{\partial p} = \frac{-12(2(v-2t-c)(1-\alpha) - t\alpha\beta)\phi_i + (12(v-2t-c)(1-\alpha) + (3c + t - 3v)\alpha\beta^2)\phi_i\phi_{-i}}{24t}
\]  

(C1)

where the subscript \(-i\) refers to the rival of firm \(i\).

We first attempt to show that the above expression is strictly negative for all \(p > (v-t)\). Notice that the sign of the above expression depends on the numerator. However, getting a necessary condition to evaluate whether the numerator is positive or negative is fairly cumbersome. Thus, we look for a sufficient condition for \(\frac{\partial \Pi_i}{\partial p} < 0\) for all prices greater than \((v-t)\). One can easily verify that for all \(\phi_i, \phi_{-i}, \alpha \in (0,1)\) as long as

\[
t < \bar{t} = \frac{3}{13}(v-c) \text{ and } \beta \in \left[\frac{12t}{3v - 3c + t}, 1\right]
\]

(C2)

The above restriction is imposed not only to get a sufficient condition for dominance but also to ensure existence of interior equilibrium in this case. One can also verify that \(\frac{\partial^2 \Pi_i}{\partial p^2} < 0\) for all prices greater than \((v-t)\), implying that \(\frac{\partial \Pi_i}{\partial p}\) has a supremum at \(p = v-t\). This fact along with (C2) means that the profit for either firm is falling for all prices greater than \((v-t)\) because the first derivative is negative for such prices and it gets even more negative as price increases in that range. In other words, all prices greater than \((v-t)\) are never observed in equilibrium, since all prices above \((v-t)\) are strictly dominated for either firm. However, it can still be the case that \(p = v-t\) is a viable deviation for the firms. Below, we show that such a deviation to \(p = v-t\) for either firm is not profitable. Before that, we derive the equilibrium profile as stated in Lemma 1 above for values of \(t\) stated in expression C2 above.

We restrict \(\theta\) such that, in equilibrium, advertising effort given in Lemma 1 is less than 1, which yields the following condition:

\[
\theta \geq \theta^*_L = \frac{t(6 - \alpha(6 - (6 - \beta)\beta))^2}{9(2-\alpha)^2(4-\alpha(4-\beta^2))}
\]

(C3)
Given that condition (C3) is fairly cumbersome, we simplify the above threshold using the following steps. First, we note that for $\beta > 0.14 \left(\frac{\partial^2 \theta}{\partial \alpha^2} \geq 0\right)$ and the above threshold does not possess an interior maximum and hence one needs to look at the extreme values of $\alpha$ to get the maximum on the threshold. $\theta^c_L|_{\alpha=0} = \frac{t}{4} \leq \theta^c_L|_{\alpha=1} = \frac{(6-\beta)^2}{9}t$. Second, we notice that $\beta \geq \frac{12t}{3v-3c+t}$ automatically implies $\beta > 0.14$ because a sufficient condition for existence is $t > \frac{2(v-c)}{9}$ (see the condition for non-deviation below). Therefore, we get the following more stringent, however simpler lower cutoff value of $\theta$:

$$\theta^c = \frac{(6-\beta)^2}{9}t$$  \hfill (C4)

Given the equilibrium profile, next we demonstrate that deviation to $p = v - t$ is not profitable for either firm; this is because as stated above the candidate pure strategy equilibrium must be stable to defections by either firm to $v - t$.

The profits earned by charging $v - t$ are

$$\Pi(\phi^*, v - t) = \left(\alpha \int_0^1 [\beta(1-\beta)\phi^* (1-x) + \beta \phi^*(1-x)(1-\phi^*)(1-\phi^*)]dx + (1-\alpha)\phi^*(1-\phi^*)\right)$$

$$+ (v - t - c) - \theta \left(1 - \frac{\alpha}{2}\right)^2 \phi^*$$

(C5)

Therefore, $\Pi(\phi^*, p^*) > \Pi(\phi^*, v - t)$ is necessary for a pure strategy in prices, where $\Pi(\phi^*, p^*)$ is the equilibrium profit. Substituting equilibrium $\phi^*$ in $\Pi(\phi^*, v - t)$ and computing $\Xi_D = \Pi(\phi^*, p^*) - \Pi(\phi^*, v - t)$ yields the following:

$$\Xi_D = \frac{3t(2 - \alpha(2-\beta))^2(6\theta t(2 - \alpha)^2 - (c + t - v)(t(6 - \alpha(6 - \beta^2)) - 3(2 - \alpha)\sqrt{\theta t(4 - \alpha(4-\beta^2)))})}{\left(t(6 - \alpha(6 - \beta^2)) + 3(2 - \alpha)\sqrt{\theta t(4 - \alpha(4-\beta^2)))}\right)^2}$$

(C6)

Note that sign of ($\Xi_D$) depends on the numerator of $\Xi_D$. Given that $\theta \geq \theta^c_L$ and $\beta \in$
Having established the stability of the candidate equilibrium, next we establish that the equilibrium is unique.

In order to ensure uniqueness of the symmetric pure strategy equilibrium, we obviously need to look at the corner symmetric equilibrium wherein \( p_i = p_{-i} = v - t \). All other prices are not viable because the profit function is globally concave in prices interior to the interval \([0, v - t]\). Notice above, we established the fact that unilateral deviation to \( p = v - t \) is not profitable to firms. However, that does not rule out a stable equilibrium at \( p = v - t \). In other words, it can still be the case that both firm have a stable corner equilibrium at \( p = v - t \).

For such a corner symmetric equilibrium we need the following to hold:

\[
\frac{\partial \Pi_i}{\partial p_i} \bigg|_{p_i \leq (v-t)} > 0, \quad \frac{\partial \Pi_i}{\partial p_i} \bigg|_{p_i > (v-t)} < 0
\]  

(C8)

To rule out such a corner equilibrium, we need to show that the above condition cannot hold, which in turn guarantees the equilibrium we stated in Lemma 1 is unique in the interval \([0, v - t]\).

We know, from above that \( \frac{\partial \Pi_i}{\partial p_i} \bigg|_{p_i > (v-t)} < 0 \). This leaves us only to consider the case for \( p_i \leq v - t \),

\[
\frac{\partial \Pi_i}{\partial p_i} \bigg|_{p_i, p_{-i} \leq (v-t)} = \frac{\phi_i(3(c-v)(4 - \alpha(4 - \beta^2))\phi_{-i} + t(24 + \alpha(-24 + \beta(12 + \beta\phi_{-i}))))}{24t}
\]  

(C9)
Even though $t$ is small, for $\phi_{-i}$ arbitrarily small, it can be that $\frac{\partial \Pi_i}{\partial p_i} > 0$ for prices close to but less than $(v-t)$. In other words, for low values of $\phi_{-i}$ it might be possible that $p_i = v-t$ is a symmetric equilibrium for both firms.

Focusing on the case when $p_i = v-t$, the objective function for each firm can be derived by substituting that condition in the appropriate profit function. Thus, the objective profit function for firm $i = 1, 2$ as a function of advertising reach is as follows:

$$\Pi_i = \frac{\phi_i(-6\theta(2-\alpha)^2\phi_i + 2(c + t - v)(6(-2 + \phi_{-i}) + \alpha(12 - 6\beta - 6\phi_{-i} + \beta^2\phi_{-i})))}{24} \quad (C10)$$

Maximizing $\Pi_i$ given in Equation (C10) with respect to $\phi_i, i = A, B$ and then setting $\phi_i = \phi_{dev}^*$ in equilibrium we get

$$\phi_{dev}^* = \frac{6(v - t - c)(2 - \alpha(2 - \beta))}{6\theta(2 - \alpha)^2 + (v - t - c)(6 - \alpha(6 - \beta^2))} \quad (C11)$$

Substituting (C11) into (C9) we get $\frac{\partial \Pi_i}{\partial p_i|_{p_i \leq (v-t)}}$. One can verify that $\frac{\partial \Pi_i}{\partial p_i|_{p_i \leq (v-t)}} < 0$ for

$$\theta < \theta_H^* = \frac{(v - t - c)^2(4 - \alpha(4 - \beta^2))}{4t(2 - \alpha)^2} \quad (C12)$$

In other words, for $\theta < \theta_H^*$, $\frac{\partial \Pi_i}{\partial p_i|_{p_i \leq (v-t)}} < 0$ which implies that we obtain a contradiction to (C8). Therefore, for such values of $\theta$ there does not exist a symmetric corner equilibrium with $p^* = v-t$, and the equilibrium is unique as long as (C12) holds.

As a final step to complete Lemma 1, we simplify the restriction by finding the minimum of $\theta_H^*$ in $\alpha$. However, $\frac{\partial^2 \theta_H^*}{\partial \alpha^2} < 0$ which means that $\theta_H^*$ does not possess an interior minimum.
We look at the extremes and find that $\theta_H^c$ attains a minimum at $\alpha = 1$. Hence, 

$$\theta_H^c|_{\alpha=1} = \bar{\theta}^c,$$  

where,

$$\bar{\theta}^c = \frac{(v - t - c)^2}{4t}\beta^2 \quad \text{(C13)}$$

Finally, for the interval $[\theta^c, \bar{\theta}^c]$ to have a positive measure we need $\beta \geq \frac{12t}{3v - 3c + t}$.

This completes the proof of Lemma 1. Next, we perform comparative statics around the equilibrium, which establishes the results stated in Propositions 1 and 2. □

(PLACE FIGURE C1 HERE)

**Proof of Lemma 2**

The proof of Lemma 2 follows the same structure of that for Lemma 1. The equilibrium profile is attained through profit maximization. In the following, we ensure that such a profile is unique and deviation-proof. Specifically, we check three issues: (i) domination of price above $v - t$, (ii) non deviation to the corner, and (iii) uniqueness of the equilibrium profile.

**Domination of price above $v - t$.** Given full market coverage, we want no symmetric equilibria to exist for $p_i > v - t$. To ensure this, we refute all the possible symmetric equilibria for $p_i \in (v - t, +\infty)$. The appropriate demand function for $p_i > v - t$ is obtained through replacing $1$ with $\frac{v - p_i}{t}$ in the integral of Equation (2). As a result, profit function, Equation (4), should also accommodate this change, that is, plugging in the new $Q_A^p$. In order for all the prices above $v - t$ not to be observed in symmetric equilibrium, one sufficient condition is $\frac{\partial \Pi}{\partial p_i}|_{p_i = p_j = p} < 0$ for $p > v - t$. We show two things in the following. First, $\frac{\partial \Pi}{\partial p_i} < 0$
at $p = v - t$. Second, $\frac{\partial^2 \Pi}{\partial p_i^2} < 0$ for $p > v - t$.

In a symmetric equilibrium when both firms charge $p = v - t$, we have

$$
\frac{\partial \Pi}{\partial p_i} |_{p_i=p_j=v-t} = \frac{60[-2(v-c-2t)(1-\alpha) + \alpha \beta \phi_i + 20t \alpha \epsilon \phi_i + [60(v-c-2t)(1-\alpha) - 5(3v-3c-t)\alpha \beta^2 + 2t \alpha \epsilon^2] \phi_i \phi_j]}{120t}
$$

(C14)

The sign of the above expression depends on the numerator, which is strictly negative for

$$
t < t_{\text{max}} = \frac{15(v-c)\{8 - 4\phi - \alpha[8 - (4 + \beta^2)\phi]\}}{120(2 - \phi) - 20\alpha(12 - 3\beta - \epsilon) + \alpha[5\beta^2 + 2(60 + \epsilon^2)]\phi}
$$

(C15)

Observe the above threshold is a function of $\alpha$, and $\phi$. We simplify $t_{\text{max}}$ in two steps. First, due to the fact that $t_{\text{max}}$ is decreasing in $\phi$, it is smallest when $\phi = 1$.

$$
t_{\text{max}}|_{\phi=1} = \frac{15(v-c)[4 - \alpha(4 - \beta^2)]}{120(1 - \alpha) + \alpha[5\beta(12 + \beta) + 2\epsilon(10 + \epsilon)]}
$$

(C16)

Second, observe $t_{\text{max}}|_{\phi=1}$ is continuous and strictly decreasing in $\alpha$. Thus, we attain the infimum for $t_{\text{max}}$, $\bar{t}_s$, by substituting $\alpha = 1$, where

$$
\bar{t}_s = \inf(t_{\text{max}}) = \frac{15(v-c)\beta^2}{5\beta(12 + \beta) + 2\epsilon(10 + \epsilon)}
$$

(C17)

Next, we show $\frac{\partial^2 \Pi}{\partial p_i^2} < 0$ for $p > v - t$. The sufficient condition for this is $\frac{\partial^2 \Pi}{\partial p_i^2} |_{p=v-t} < 0$.
and \( \frac{\partial^2 \Pi_i}{\partial p_i^2} \) strictly decreases with \( p \) for \( p > v - t \).

\[
\frac{\partial^2 \Pi_i}{\partial p_i^2} \bigg|_{p \to v - c} = -\frac{8t[2 - \alpha(2 + \beta - \varepsilon)]\phi - t[12 - \alpha(12 + 9\beta^2 - 8\varepsilon^2)]\phi^2 + 8(v - c)\alpha(\beta - \varepsilon)[1 - (\beta + \varepsilon)\phi]\phi}{8t^2}
\]

(C18)

As long as \( t < \overline{t}_s \), the above expression is negative. To see \( \frac{\partial^2 \Pi_i}{\partial p_i^2} \) is a decreasing function in \( p \) for \( p > v - t \). Replace \( p \) with \( v - t + \zeta \) in \( \frac{\partial^2 \Pi_i}{\partial p_i^2} \), where \( \zeta > 0 \). Differentiate \( \frac{\partial^2 \Pi_i}{\partial p_i^2} \) with respect to \( \zeta \). We can show that the derivative is negative for \( \varepsilon < \frac{\beta}{\sqrt{\Pi}} \) and \( t < 2\frac{(v-c)}{\sqrt{t}} \). Furthermore, \( t < 2\frac{(v-c)}{\sqrt{t}} \) is automatically implied by \( t < \overline{t}_s \).

To summarize, it requires \( t < \overline{t}_s \) and \( \varepsilon < \frac{\beta}{\sqrt{\Pi}} \) to guarantee all the prices above \( v - t \) are never observed in symmetric equilibria.

Since no equilibrium exists for \( p > v - t \), we search for an interior equilibrium in the interval \((0, v - t]\). Maximization of the profit function assuming full market coverage yields the equilibrium profile in Lemma 2. It can be confirmed that the Hessian matrix is negative semi-definite at the equilibrium profile. Furthermore, we restrict the cost of advertising, \( \theta \), such that \( 0 < \phi^* < 1 \). Intuitively, a sufficiently low cost of advertising will lead to the maximum ad reach level, \( \phi = 1 \), being optimal. Thus, we look for a threshold on \( \theta \) that ensures the optimal ad reach is interior. The lower bound for \( \theta \) is \( \theta_{Ls} \).

\[
\theta > \theta_{Ls} = \frac{t[30(1 - \alpha) + 5\alpha(6 - \beta)\beta + \alpha(10 + \varepsilon)\varepsilon]^2}{225(2 - \alpha)^2(4 - 4\alpha + \alpha\beta^2)}
\]

(C19)

We simplify \( \theta_{Ls} \), which is an increasing function of \( \alpha \), to obtain the lower bound on \( \theta \).

\[
\theta_s = \frac{t[5(6 - \beta)\beta + \varepsilon(10 + \varepsilon)]^2}{225\beta^2}
\]

(C20)
Non deviation. Deviating to anywhere in the interval \((0, v-t)\) is not profitable for either firm. Thus, we need only to check whether setting \(p = v-t\) by either firm generates more profit. The profits earned by charging \(p = v-t\) are

\[
\Pi_i(\phi^*, v-t) = \{ \alpha \int_0^1 [\beta_A(1-\beta_b)\phi^* x(1-x) + \beta_A\phi^*(1-x)(1-\phi^*)]dx + (1-\alpha)\phi^*(1-\phi^*)\}(v-t-c) - \theta \left(1 - \frac{\alpha}{2}\right)^2 \phi^{*2}\]
\]

(C21)

It requires \(\Pi_i^* > \Pi_i(\phi^*, v-t)\) to ensure the stability of pure strategy equilibrium in price, where \(\Pi_i^*\) is the optimal profit reported in Lemma 2. Define \(\Xi_D = \Pi_i^* - \Pi_i(\phi^*, v-t)\) and plug in \(\phi^*\).

\[
\Xi_D = \frac{5t(6-\alpha(6-3\beta + \varepsilon))^2((v-c-t)[t(30-30\alpha + 5\alpha\beta^2 - \alpha\varepsilon^2) + 15(2 - \alpha)\sqrt{t\theta(4 - 4\alpha + \alpha\beta^2)}] + 30t\theta(2 - \alpha)^2]}{3[t(30 - \alpha(30 - 5\beta^2 + \varepsilon^2)] + 15(2 - \alpha)\sqrt{t\theta(4 - 4\alpha + \alpha\beta^2)})^2}
\]

(C22)

Note that sign of \(\Xi_D\) depends on

\[
(v-c-t)[t(30 - 30\alpha + 5\alpha\beta^2 - \alpha\varepsilon^2) + 15(2 - \alpha)\sqrt{t\theta(4 - 4\alpha + \alpha\beta^2)}] + 30t\theta(2 - \alpha)^2 \quad (C23)
\]

Given that \(\theta > \theta_s\), sign of the above polynomial is strictly positive for

\[
t > t_s = \frac{15(v-c)\beta^2[5(3-\beta)\beta + \varepsilon(5 + \varepsilon)]}{[30\beta + \varepsilon(\varepsilon + 10)]^2 - 5\beta^2[10\beta^2 + 15\beta + (5 - \varepsilon)\varepsilon]} \quad (C24)
\]
We also verify that $T_s - t_s$ strictly has a positive measure for $\varepsilon < \frac{\beta}{\sqrt{11}}$.

**Uniqueness.** Having established the stability of equilibrium profile, we move on to its uniqueness. As profit maximization has already searched the interior of $(0, v - t)$, we need only to check whether $p_i = p_j = v - t$ could be an equilibrium. For $v - t$ to be a symmetric equilibrium price, the following need to hold:

$$\frac{\partial \Pi_i}{\partial p_i|_{p_i = v-t}} > 0, \frac{\partial \Pi_i}{\partial p_i|_{p_i = v-t}^{-}} < 0$$

(C25)

Our strategy is to come up with a contradiction to Equation (C25), so that the corner solution $p_i = p_j = v - t$ is not stable. We know from the proof of domination of price above $v - t$ that $\frac{\partial \Pi_i}{\partial p_i|_{p_i = v-t}^{-}} < 0$ is true. In the following, we derive a region of parameters which can contradict $\frac{\partial \Pi_i}{\partial p_i|_{p_i = v-t}^{-}} > 0$.

In the corner equilibrium where both firms charge $v - t$, profit as a function of $\phi_i$ and $\phi_j$ is obtained through substituting $p_A$ and $p_B$ with $v - c$ in Equation (4).

$$\Pi_i|_{p_i=p_j=v-t} = \frac{\phi_i\{(v - c - t)[60(2 - \phi_j) + 2\alpha(30\beta - 60 + 10\varepsilon - 5\beta^2\phi_j + 30\phi_j + \varepsilon^2\phi_j)] - 30(2 - \alpha)^2\theta\phi_i\}}{120}$$

(C26)

Optimizing $\Pi_i|_{p_i=p_j=v-t}$ on $\phi_i$ and invoking symmetry, we get

$$\phi_i^{dev} = \frac{10(v - c - t)[6 - \alpha(6 - 3\beta + \varepsilon)]}{(v - c - t)[30 - \alpha(30 - 5\beta^2 + \varepsilon^2)] + 30(2 - \alpha)^2\theta}$$

(C27)
Substituting (C27) into \( \frac{\partial \Pi}{\partial p_i}|_{p_i \rightarrow (v-t)^-} \), we obtain

\[
\frac{\partial \Pi_i}{\partial p_i}|_{p_i \rightarrow (v-t)^-} = \frac{25(v - c - t)[6 - a(6 - 3\beta - \varepsilon)]^2((v - c - t)^2[4 - \alpha(4 - \beta^2)] - 4t(2 - \alpha)^2\theta}}{2t{(v - c - t)[30 - \alpha(30 - 5\beta^2 + \varepsilon^2)] + 30(2 - \alpha)^2\theta}^2}
\]

(C28)

As long as \( \theta < \theta_{H_s} = \frac{(v-c-t)^2[4-\alpha(4-\beta^2)]}{4t(2-\alpha)^2} \), (C28) is negative, which gives a contradiction to \( \frac{\partial \Pi}{\partial p_i}|_{p_i \rightarrow (v-t)^-} > 0 \). Observe that \( \theta_{H_s} \) does not posses an interior minimum in \( \alpha (\frac{\partial^2 \theta_{H_s}}{\partial \alpha^2} < 0) \).

Looking at the two extremes, we found that \( \theta_{H_s} \) is smaller at \( \alpha = 1 \). The upper bound on \( \theta \) is given as

\[
\overline{\theta}_s = \theta_{H_s}|_{\alpha \rightarrow 1} = \frac{(v - c - t)\beta^2}{4t}
\]

(C29)

We further verify that \( \overline{\theta}_s \) is strictly larger than \( \underline{\theta}_s \) in (C29). Thus, provided \( \theta < \overline{\theta}_s \), equilibrium profile in Lemma 2 is unique.

In summary, Equations (C17) and (C24) give the upper and lower bound on \( t \). Equations (C27) and (C29) give the bounds on \( \theta \). Furthermore, \( \beta \geq \sqrt{\Pi}\varepsilon \), together with \( \beta \in [\varepsilon, 1 - \varepsilon] \), implies \( \beta \in [\sqrt{\Pi}\varepsilon, 1 - \varepsilon] \) and \( \varepsilon \leq \frac{1}{1+\sqrt{\Pi}} \). These are the parameter conditions we give in Lemma 2.

Finally, we show that insights in Propositions 1 and 2 are preserved with the current more general setup. We start with confirming Proposition 1. Observe that the current equilibrium price is exactly the same as that in the constant \( \beta \) case. Thus, we only need to
look at equilibrium ad reach. Differentiating $\phi^*$ with respect to $\alpha$, we get

$$\frac{\partial \phi^*}{\partial \alpha} = \frac{15t^2(5\theta[12(4 - \beta)\beta + 16\varepsilon + \alpha^2(4 - \beta^2)(6 - 3\beta - \varepsilon) - 2\alpha(4 - \beta^2)(3 + 3\beta + \varepsilon)] + 4[5(3 - \beta)\beta + \varepsilon(5 + \varepsilon)]\sqrt{t\theta[4 - \alpha(4 - \beta^2)]^2}}{t^2(30 - \alpha(30 - 5\beta^2 + \varepsilon^2)) + 15(2 - \alpha)\sqrt{t\theta[4 - \alpha(4 - \beta^2)]^2} \sqrt{t\theta[4 - \alpha(4 - \beta^2)]}}$$

(C30)

The sign of the above depends on

$$sgn[12(4 - \beta)\beta + 16\varepsilon + \alpha^2(4 - \beta^2)(6 - 3\beta - \varepsilon) - 2\alpha(4 - \beta^2)(3 + 3\beta + \varepsilon)]$$

(C31)

which is strictly positive for $\beta \in [\sqrt{\Pi}\varepsilon, 1 - \varepsilon]$, and $\varepsilon \leq \frac{1}{\sqrt{\Pi}}$. Thus $\phi^*$ increases with $\alpha$.

Next, we examine the two extremes of equilibrium profit (Proposition 2). Differentiate $\Pi^*$ with respect to $\alpha$ and look at the $\alpha \to 0^+$ and $\alpha \to 1^-$.

$$\frac{\partial \Pi^*}{\partial \alpha} \bigg|_{\alpha \to 0^+} = -\frac{2t^2\theta\{2t[15 + 5(3 - \beta)\beta - \varepsilon(5 + \varepsilon)] + 5[3(2 - \beta)^2 + 4\varepsilon]\sqrt{t\theta}}{15(t + 2\sqrt{t\theta})^3} < 0$$

(C32)

For $\beta \in [\sqrt{\Pi}\varepsilon, 1 - \varepsilon]$ the above is strictly negative.

$$\frac{\partial \Pi^*}{\partial \alpha} \bigg|_{\alpha \to 1^-} = \frac{25t^3(3\beta + \varepsilon)\theta\{15\theta[3(2 - \beta)^2\beta + (4 + \beta^2)\varepsilon] + 2(-15\beta^3 - 5\beta^2(6 + \varepsilon) + 3\beta(30 + \varepsilon^2) + \varepsilon(30 + 6\varepsilon + \varepsilon^2)]\beta\sqrt{t\theta}}{\beta\sqrt{t\theta}(5\beta^2 - 1\varepsilon t^2 + 15\beta\sqrt{t\theta})^3} > 0$$

(C33)

For $\beta \in [\sqrt{\Pi}\varepsilon, 1 - \varepsilon]$ the above is strictly positive. Therefore, by continuity of $\Pi^*$, there must exist intervals above 0 for which $\Pi^*$ decreases with $\alpha$ and below 1 for which $\Pi^*$ increases with $\alpha$. 

29
Proof of Proposition 1

(i) First we differentiate the equilibrium price with respect to $\alpha$. It can be seen that the equilibrium price is increasing for $\alpha \geq \alpha^* = \frac{2\beta^2}{4-\beta^2}$ for $\beta \in \left[\frac{12}{3v-3c+t}, 1\right]$ and $t < \frac{3}{13}(v-c)$.

(ii) Secondly, we differentiate the equilibrium ad reach with respect to $\alpha$ which yields the following expression:

\[
\frac{\partial \phi^*}{\partial \alpha} = \frac{(3t^2(3\theta(-4+\beta)\beta + \alpha^2(-2+\beta)^2(2+\beta) + 2\alpha(-2+\beta)(1+\beta)(2+\beta)) - 4(-3+\beta)\beta\sqrt{t(4-\alpha(4-\beta^2)))}}{(\sqrt{4-\alpha(4-\beta^2))) (t(6-\alpha(6-\beta^2)) + 3(2-\alpha)\sqrt{t(4-\alpha(4-\beta^2)))}}^2
\]

Whether the above is negative or positive depends on the numerator of the above equation which is strictly positive for $\beta \geq \beta_{\alpha} = \left(\frac{\sqrt{5} - 9}{2}\right)$. Second, $\beta \geq \frac{12}{3v-3c+t}$ directly implies $\beta > \beta_{\alpha}$ because a sufficient condition for existence is $t > \frac{2(v-c)}{9}$, which is violated if $\beta < \beta_{\alpha}$. In other words, for such values of $\beta$, ad effort is monotonically increasing in $\beta$. This completes the proof of Proposition 1. ■

Proof of Proposition 2

It is cumbersome to obtain closed form conditions on changes in profit function with respect to $\alpha$. Thus, to get the result in Proposition 2, we look for non-empty intervals around the extreme values to see how equilibrium profit changes for low and high values of $\alpha$.

(i) We differentiate the equilibrium profit expression stated in Lemma 1 above with respect to $\alpha$ and evaluate around $\alpha = 0, 1$.

\[
\frac{\partial \Pi^*}{\partial \alpha} \bigg|_{\alpha=0} = -\frac{2t^2\sqrt{t}(3\theta(2-\beta)^2 + 2\sqrt{t}(3 - (3-\beta)\beta))}{3(t + 2\sqrt{t})^3} < 0
\]

By continuity of $\Pi^*$ in $\alpha$, it follows that, given the above expression there exists a non-
empty interval around $\alpha$ close to 0 such that $\frac{\partial \Pi^*}{\partial \alpha} < 0$ in that interval.

$$\frac{\partial \Pi^*}{\partial \alpha} |_{\alpha \to 1} = \frac{9t^2 \beta \sqrt{\theta t}(3\theta(2 - \beta)^2 + 2\beta \sqrt{\theta t}(6 - (2 + \beta)\beta))}{(t\beta^2 + 3\sqrt{\theta t}\beta^2)^3} > 0$$  \hspace{1cm} (C36)

Again by continuity of $\Pi^*$ in $\alpha$, there exists a non-empty interval around $\alpha$ close to 1 such that $\frac{\partial \Pi^*}{\partial \alpha} > 0$ in that interval. The above facts establish the result stated in Proposition 2. $\blacksquare$
Figure 1: Impact of DVR penetration in a competitive context.
Figure C1: Impact of DVR penetration in a competitive context.