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Kindergarten Students Solving Mathematical Word Problems

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KINDERGARTEN STUDENTS SOLVING MATHEMATICAL WORD PROBLEMS

By

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DEDICATION

I dedicate this dissertation to my loves. I love you more - more than I ever thought possible, more today than yesterday, and more than I will ever be able to show you. God gave me you; for that I am profoundly grateful. I pray that you will pursue your dreams as you live life to the full.

My love will follow you wherever you are.
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ABSTRACT

The purpose of this study was to explore problem solving with kindergarten students. This line of inquiry is highly significant given that Common Core State Standards emphasize deep, conceptual understanding in mathematics as well as problem solving in kindergarten. However, there is little research on problem solving with kindergarten students. This study is one of a few to explore problem solving with kindergarten students.

This study explored problem solving with kindergarten children using pencil paper. The study also presents a preliminary scale for assessing students’ representation and problem solving. It explored the relationships between students’ levels of conservation, representation and problem solving. The data confirmed previous findings that stated kindergarten students can solve a wide variety of problems, including multiplication and division problems and that most students represent number in writing at or below their level of conservation. New findings from this data also suggest that there are levels of representation and problem solving through which students progress. However, problem solving is not a lockstep process through which students progress, rather, it is a complex individual process that combines factors related to the student, the problem and the problem context. This new way of conceptualizing problem solving has several implications that change the way problem solving is frequently taught. Approaching problem solving as a complex interwoven process between these three factors changes the focus to each individual student’s construction of logico-mathematical knowledge rather than the transfer of social knowledge from teacher to student. The data suggests that including a conference, like the one used in this study, between all students and the teacher is a crucial part of building the knowledge (mental structures) for all types of students. The instructional method of solving a wide variety of authentic and relevant mathematical word problems using pencil and paper utilized in this study is a viable means of improving students’ problem solving proficiency.

This study also presents the idea that assessing problem solving is possible using a scale. The preliminary scales created by this study are presented with the invitation that further study will be undertaken to confirm them. The hope is that future studies will continue to document the highly complex and sophisticated nature of problem solving skills that kindergarten students demonstrate when allowed to solve problems using their own strategies. Scales and taxonomies can be used to document these natural processes in the current climate of accountability.
CHAPTER ONE
INTRODUCTION

By reflecting global standards and goals, the Common Core State Standards (CCSS) represent current efforts to improve and reform how mathematics is taught in the United States (NGACBP & CCSSO, 2010). Previous reform efforts were initiated by the publication of the National Council of Teachers of Mathematics (NCTM) goals in 1989. Since that time, mathematics educators have agreed that problem solving plays a central role in the mathematics curriculum (Cohen, 1990; Goos, 2004; Hembree & Marsh, 1993; Muth, 1997; NCTM, 1989, 1991, 1995, 2000; Steinberg, Empson, & Carpenter, 2004; Voutsina, 2012; Winograd, 1991; Wyndhamn & Saljo, 1997). Additionally, mathematics educators have established that word problems are the primary method to teach problem solving (Depaepe, De Corte & Verschaffel, 2010; Verschaffel, De Corte & Lasure, 1994; Wynhamn & Saljo, 1997). Researchers have also come to the consensus that the problem solving steps are as follows: understanding the problem; representing the problem (constructing a mathematical model); reflecting on the problem representation; and then modifying it prior to selecting a solution strategy. The final step in problem solving is selecting and working a solution strategy (De Paepe et al., 2010; Willis & Fuson, 1988). Yet, despite such common understanding of the nature of problem solving, other issues and aspects associated with this approach to teaching mathematics are unclear.

1.1. Issues Identified in the Research

Despite consensus regarding mathematics goals and methods, research shows that students are not learning to be effective problem solvers (Boaler, 1998; Greer, 1997; Schoenfeld, 1988 & 1991; Verschaffel, De Corte & Lasure, 1994; Wyndhamn & Saljo, 1997). Many examples of the phenomenon of “suspension of sense-making” (Schoenfeld, 1991) can be found, where students answer word problems with a disregard for the reality represented in the problem, thus accepting nonsense answers (Greer, 1993, 1997; Schoenfeld, 1991; Verschaffel et al., 1994). Carpenter, Ansell, Franke, Fennema and Weisbeck (1993) document the tendency for students to abandon a “fundamentally sound and powerful problem solving approach in favor of mechanical application of arithmetic and algebraic skills” (Carpenter et al., 1993, p. 429). This conclusion has prompted researchers to search for mechanisms that will overcome these tendencies that are detrimental to problem solving, and that will help educators achieve the goals of problem solving, reasoning, and communication (Wyndhamn & Saljo, 1997).
1.1.1. Kindergarten Research Lacking

In the search for new approaches to learning and teaching that would produce effective problem solvers, researchers have investigated many topics such as the artificial nature of the problems (Verschaffel, De Corte & Lasure, 1994); culture of mathematics (Boaler, 1998; Wyndhamn & Saljo, 1997); situated learning (Lave, 1988); teachers’ expertise in mathematics; teachers’ knowledge about taxonomies of learning (Carpenter et al., 1993; Carpenter, Fennema & Franke, 1996, Carpenter, Franke, Jacobs & Fennema, 1998); and students’ beliefs about mathematics (Franke & Carey, 1997). Although, multiple studies on problem solving have investigated these topics for older students and adults (Boaler, 1998; Lave, 1988; Lave, Murtaugh & de le Rocha, 1984; Masingila, 1993; NAEP, 1983; Nunes, Schliemann & Carraher, 1993), kindergarten students have been all but omitted from the research on problem solving (Hembree, 1992; Hembree & Marsh, 1993). The majority of research on school-aged students has been conducted in first grade and above (Boaler, 1998; Baroody & Hume, 1991; Francisco & Maher, 2005; Hembree 1992; Hembree & Marsh, 1993; Verschaffel et al., 1994; Willis & Fuson, 1988; Wyndhamn & Saljo, 1997), thus leaving out a large population of students who are now included in public education. However, a search for research on kindergarten students solving word problems in ERIC did produce a few published articles (e.g., Carpenter et al., 1993; Kamii, 2000; Kamii & DeVries, 1973; Siegler & Svetina, 2006; Sinclair et al., 1983; Voutsina, 2012; Wilkins & Baroody & Tiilikainen, 2001). In many of these articles, authors present the stages through which students commonly progress as they are taught to problem solve (e.g. taxonomies of learning) and how teachers attempt to guide students through these states (e.g. the teacher’s use of taxonomies of learning), yet little is known about what is happening in the mind of kindergarten students (e.g. their mental processes) as they learn to solve word problems. This apparent gap in the research is concerning because of the emphasis the CCSS (NGACBP & CCSSO, 2010) has placed on problem solving in all grades, including kindergarten. The CCSS requires that kindergarten students be problem solvers, but there is a lack of research that informs us how they become problem solvers. This study reports the results of analyses of how kindergarten students solve word problems in classroom settings and how their problem solving is related to their levels of constructive abstraction and representation.
1.1.2. Predominant Instructional Strategies

The instructional strategies for teaching problem solving reported in the literature usually fall into one of three categories. The first is where students are provided with given cue word-operation hints. A second instructional approach is where students are provided with solution strategies that were created by someone else. Then, the third type of instructional strategy is when the teacher allows the students to create their own solution strategies. These three instructional approaches are described in the following sections.

1.1.2.1. Cue word-operation hints. The first category is teaching cue word–operation hints for solving problems. In cue word–operation hints, the relationship between the vocabulary in the problem and the operation needed to solve the problem is explicitly taught (Rosales et al., 2012; Schoenfeld, 1982). Students are taught to automatically perform a set operation when the cue word is present in the problem. (e.g. add when “all together” is in the problem, subtract for “how many more?” and “how many left?”). The concern with cue words is that students often implement the shortcut, without making sense of the problem. In one humorous example, the majority of students subtracted in a problem that mentioned a character Mr. Left, when that was not the correct operation. The students had been so conditioned to subtract when they saw the word, “left,” that they did not pay attention to the fact that it was a name in this case. It appears that the cue-word hints encourage students to think about the problem in a manner that ignores the realistic considerations in the problem (Verschaffel et al., 1994).

1.1.2.2. Solution strategies, algorithms and manipulatives. The second category occurs when students are given solution strategies that were formulated by someone else. These solution strategies can be in the form of algorithms (also called solution strategies) (Burns, 1994, 2007; Carpenter et al., 1983), manipulatives, or diagrams (Kamii et al., 2001; Threadgill-Sowder, Sowder, Moyer & Moyer, 1985). Algorithms, or solution strategies, present a well-defined procedure that, when followed exactly, guarantee a perfect outcome (Algorithms in Everyday Math, 2013). The literature shows that the teaching of algorithms can range from very didactic and formal to a more constructivist approach that links invented algorithms to conventional algorithms and fosters the construction of knowledge (Blume, 1981; Carpenter et al., 1988; Threadgill-Sowder et al., 1985).
Algorithms for younger children are often called solution strategies. When teaching solution strategies, teachers often emphasize strategy type and steps, guiding students through a lockstep progression from least to most difficult strategy. The teacher presents the word problem and then explains the solution strategies effective in solving the problems (Carpenter & Moser, 1984). The relevant information and operation is often extracted from the word problem for the students who are then expected to solve the problem using the proscribed method. Other solution strategies or methods are allowed, but not often used by students.

Older or more advanced students are taught how to represent these problems with simple algorithms, commonly called open number sentences (e.g., 8-3= _ ) (Blume, 1981; Carpenter et al., 1988). Older children are provided diagrams or models that someone else has created to represent the problem (Threadgill-Sowder et al., 1985). Following the same idea, younger children are often given manipulatives that represent the problem solution (to the adult) and shown how to count the manipulatives to get to desired answer (Burns, 1994; Carpenter & Moser, 1984; Empson, 1999; Francisco & Maher, 2005; Goos, 2004; Kamii, 2000; Madell, 1985; Voutsina, 2012; Willis & Fuson, 1988). For example, younger students are often given blocks, counters, buttons, or other manipulatives and told those items represent the objects in the problem (Burns, 1994; Carpenter & Moser, 1984; Empson, 1999; Francisco & Maher, 2005; Goos, 2004; Kamii, 2000; Madell, 1985; Voutsina, 2012; Willis & Fuson, 1988). Then the students are shown how to combine the groups of objects and count to find the answer. The problem with the giving of solution strategies is that the representations created by someone else often do not make sense to the problem solver.

1.1.2.3. Create your own solution strategy. A third category of instructional strategy is lightly documented in the literature. Students are sometimes encouraged to create their own algorithms, solution strategies, diagrams or choose their own manipulatives (Burns, 1994, 2007; Carpenter et al., 1983, Everyday Math, 2013, Kamii, 2000). While some researchers emphasize the use of manipulatives (Everyday Math, 2013 and CGI, 1999), Kamii (2000) encourages students to draw their own representations with pencil and paper. Everyday Math (2013) and Cognitively Guided Instruction (Carpenter et al., 1983), encourage a combination of students figuring out ways to solve problems and the teaching of conventional algorithms in a manner that allows for students to construct their own mathematical knowledge.
The current study is relevant in the context of these current research findings. The following section expands on these findings and identifies four specific issues that are addressed in the current study.

1.2. Statement of the Problem

The current study addresses four specific issues related to these research findings. These issues are explained in this section.

- Kindergarten students are underrepresented in research on problem solving.
- Predominant instructional strategies often skip over crucial first steps that have been identified for successful problem solving.
- Instructional strategies (diagrams, algorithms or solution sentences) do not always accurately represent the problem to the students.
- Instructional strategies promote attitudes that are harmful to deep conceptual mathematical learning.

1.2.1. Kindergarten Students Underrepresented in the Research

The first issue is that the majority of research has been conducted with older students and adults. This means that the 4,122,454 five-year-old children in the United States (Kids Count Data Center, 2011) who make up the first grade of the public school system are not adequately represented in current research. The kindergarten year marks an important stage in a child’s development. There is an undisputed major shift in cognition that occurs in children between the ages of five and seven. (Brookshire & Lake, 2010; Piaget, 1952; Hembree & Marsh, 1993; Sameroff & McDonough, 1994; White, 1965, 1970, as cited in Copple & Bredekamp, 2009). One way this shift is marked is by children’s ability to conserve number (Kamii, 2000; Piaget, 1941/65). Prior to this shift, kindergarten aged children are more like preschoolers than school age children (Copple & Bredekamp, 2009). Effectively teaching children who are in the process of making this shift requires highly educated teachers who meet the ever-changing capabilities of the students while ensuring vertical alignment with preschool and kindergarten curricula (Brookshire & Lake, 2010; National Association for the Education of Young Children, 2009).
Because teaching methods should be informed by research, it is important that all grade levels are represented by research that fosters deep conceptual learning rather than research that encourages simply rote computation. In addition, CCSS emphasize problem solving in kindergarten, which means this area will receive more attention as CCSS is implemented in schools (NGACBP & CCSSO, 2010). It will be important to introduce problem solving in ways that foster deep understanding rather than rote computation, as has been discovered in older students as they are taught problem solving.

1.2.2. Predominant Instructional Strategies Skip Crucial Steps

Most instruction in problem solving involves didactic strategies that focus on giving students strategies for solving problems. It appears that these didactic strategies are shortcutting the students’ processes that are necessary for constructing useful and durable knowledge about mathematics (Carpenter et al., 1983; Kamii, 2000; Willis & Fuson, 1988). When students are given solution strategies, they are deprived of the practice of figuring out how to solve problems, consequently missing the activity that builds their mathematical understanding. It is the students’ participation in the problem solving steps - representation of the problem scenario, the search for the solution strategy, and subsequent application of solution strategies - that builds mental structures necessary to become problem solvers (Briars & Larkin, 1984; Carpenter & Moser, 1984; Fuson, 1988; Riley, Greeno & Heller, 1983; Willis & Fuson, 1988). When the teacher gives the solution strategy, they are doing the first two problem solving steps, not the student.

As such students are deprived of the opportunity to represent the problem and search for solution strategies themselves. Recent research findings suggest that these are crucial steps that greatly contribute to their problem solving abilities (Kamii, 2000; Kato, Kamii, Tomiyama, and Nagahiro, 2002; Willis & Fuson, 1988). Hembree and Marsh (1992) state that students who adhered to a lock-step method perform less well than their peers who applied flexible methods. They also found that, “instruction in using a [didactic] method [algorithms or heuristics] produced performance that seemed no better than scores achieved through teacher supervised practice” (Hembree & Marsh, 1993, p. 158). Skipping these steps has been shown to encourage students to solve problems in superficial ways (Rosales et al., 2012). Students are deprived of
the exercise of representing the problem to themselves and searching for an appropriate strategy when the solution strategy or algorithm is given to them (Burns, 1994, 2007; Kamii, 2000; Willis & Fuson, 1988). That is, the solution strategy method forces students to skip the very process that enables them to become proficient problem solvers (Briars & Larkin, 1984; Carpenter & Moser, 1984; Fuson, 1988; Riley, Greeno & Heller, 1983; Willis & Fuson, 1988).

1.2.3. Representations Do Not Always Represent

The third issue addressed by the present study is also related to the instructional strategy of giving students manipulatives, prepared diagrams or algorithms as representations of the problem situation. Although the literature documents that students in kindergarten and first grade were often able to calculate the correct answers using models of the problems and solution strategies that were given to them, research indicates that students were merely completing exercises by rote instead of solving mathematical problems using deep conceptual understanding (Blume, 1981; Carpenter et al., 1983; Carpenter et al., 1988; Kamii, 2000; Kato et al., 2002).

Kamii’s (2000) and Kato, Kamii, Tomiyama, and Nagahiro (2002) offer several points that explain this. The first explains the term representation. When viewed in the light of constructivist theory, representation is something that happens within a person’s mind. An object does not represent something in reality until the person understands it to be a model of the real object. For example, a complex scientific model of a chemical reaction does not represent anything to me because it is above my level of understanding. However, it is meaningful to the scientists using it in their research because it is at their level of understanding. Those scientists can use the model to solve problems. It is the same with school mathematics. Students are often given manipulatives, solution strategies or algorithms that represent the problem to another person. They might be given a set of three and five blocks and told they represent the cars in the word problems. However, unless the blocks represent cars to the students, whatever they do with the blocks is rote calculation. Even if they can combine the blocks and correctly count them, the child is not problem solving, only computing by rote. Many of the manipulatives, diagrams, and algorithms that are given to students do not represent their own ideas about reality, so they cannot use them to problem solve only to compute by rote – mechanically performing an exercise from memory.
According to Piagetian theory (Kamii, 2000 and Kato et al., 2002), children’s understanding about reality is mediated by their mental processes, called “constructive abstraction”. Until children are at a certain level of constructive abstraction, they use their perception rather than logic to understand reality. Until children are conservers, conventional symbols (such as manipulatives or a diagram) do not necessarily represent an object in reality. Children are not able to understand someone else’s representation until they reach the level of understanding required to understand that representation. They may be able to compute using the given representation, but not understand the solution. Research by Kato and colleagues (2002) demonstrated this. They found that students can only represent at or below their level of constructive abstraction, not above. “The meaning students can give to conventional symbols [manipulatives, solution sentences, algorithms] depends on their level of abstraction” (Kato et al., 2002, p. 30). Hembree (1992) and Hembree and Marsh (1993) found in their related meta-analyses of problem solving research on K-4 students that conservers performed substantially better than non-conservers on mathematical word problems, which further supports the idea that level of constructive abstraction is related to problem solving. So if a student can only give meaning to symbols that are at or below his level of abstraction, it would be important to know his level of abstraction.

Research suggests that strategies taught to students are often above the students’ level of mathematical understanding (Kato et al., 2002), or “constructive abstraction” (Kamii, 2000), which leads students to approach problem solving as the rote application of mathematical concepts (Carpenter et al., 1983). The study by Carpenter et al. (1983) illustrated this point. In this study, students were given open number sentences (someone else’s representation of the problem situation). Over several months students were instructed in the use of these arithmetic sentences to solve word problems. Students were shown how to formulate an open number sentence (e.g., 8+___ = 12) using word problems. After several months of instruction, the researchers found that the students were able to write correct arithmetic number sentences as they were taught, however, the students wrote the number sentences after they solved the problem using another method. In other words, even though they could correctly complete the number sentence, the students did not use the number sentence to solve the problem, indicating that the number sentence was not related to the problem-solving process to the students.
(Carpenter et al., 1983). Carpenter et al. (1983) further suggested that the students were not able to use the number sentences because their problem solutions had not been coordinated with the number sentences. When interpreted in light of Kamii’s (2000) work, the number sentence was someone else’s representation of the problem, and it did not represent the problem situation to the students. The number sentences were above the students’ level of understanding so they did not use the number sentence to solve the problem even though they could solve the number sentence by a rote procedure (Kato et al., 2002). To the researchers, this represented a less than “entirely beneficial” (p. 70) shift in the students’ thinking from real life problem solving to the less desirable view of problem solving as the selection of the correct mathematical operation (Carpenter et al., 1983).

In summary, the use of representations that are not constructed by the learner encourages students to shift from direct modeling to mathematical technician, and thus learn to compute without understanding. This is the suspension of sense making phenomenon that the current wave of mathematics reform is fighting against. The CCSS and researchers seem to agree that educators need to focus more on the mental relationships students make (i.e., their abstraction) (Kato et al., 2002; NGACBP & CCSSO, 2010). This is the focus of the current study, to demonstrate the link among the child’s level of abstraction, representation and problem solving ability.

1. 2.4. Instructional Strategies Promote Harmful Habits

The fourth issue is that the predominant instructional strategies focus more attention on the student’s ability to apply a strategy than to the student’s construction of knowledge (Baroody & Hume, 1991). Rosales et al. (2012) describe these approaches as superficial (apply a strategy) versus genuine (construction of knowledge). This promotes the idea that mathematics is about producing the right answers and that producing the right answer means that students understand the concept (Baroody & Hume, 1991). The fallout from approaching mathematics superficially instead of genuinely has been well-documented (Rosales et al., 2012). In current times, CCSS is seeking to challenge this idea and change the way mathematics is taught with the publication of the new standards. “The [new CCSS] standards stress not only procedural skill but also conceptual understanding, to make sure students are learning and absorbing the critical
information they need to succeed at higher levels - rather than the current practices by which many students learn enough to get by on the next test, but forget it shortly thereafter, only to review again the following year” (NGACBP & CCSSO, 2010, p. 1). The above-reported issues regarding students solving word problems combined with the new focus on conceptual understanding leads to the purpose of this study.

1.3. Purpose of the Study

The basis of this study is that mathematics is commonly treated as social knowledge that is transmitted to the learner rather than as logico-mathematical knowledge that is constructed by the learner (Kamii, 2000). Commonly used didactic teaching strategies that focus on teaching solution strategies are shortcutting the student’s processes that are necessary for constructing knowledge. Teaching that encourages students to construct their own knowledge, such as self-selecting solution strategies and representations or creating their own algorithms, leads to more durable learning and understanding (Burns, 1994; Carpenter et al., 1983; Kamii, 2000). In the current study, I investigated kindergarten students as they engaged in solving word problems using self-selected solution strategies. The students’ self-produced solution strategies were identified and categorized using a scale created by the author. The students’ levels of constructive abstraction and representation have also been determined using established scales (Piaget, 1941/1965; Sinclair et al., 1983). The students’ levels in these three categories were compared and the relationships between these areas of development have been documented. The primary purpose of the current study is to report on the relationship between the students’ levels of mathematical problem solving, levels of abstraction/conservation, and levels of mathematical representation.

A related purpose of this study was to provide information about an instructional method that is being utilized by educators as they begin to implement the CCSS. The results of the study could be used to help teachers determine the appropriate level and strategy for teaching problem solving to young children. The participants in this study were kindergarten students in a school that was implementing the CCSS for the first time and espouses the philosophy that mathematics is a means to develop logico-mathematical knowledge. The school leadership chose instructional methods designed to foster student’s construction of knowledge such as solving word problems using self-selected solution strategies. As the relationship between a
student’s level of understanding, his level of representation, and his level of problem solving is investigated, valuable information can be gained toward the aim of attaining the goals articulated by the CCSS.

1.4. Significance of the Study

The current study demonstrates the legitimacy of the current wave of mathematics reform, marked by the publication and adoption of the CCSS. Current mathematics reform goals are strongly supported by decades of research (Burns, 2010; Burton, 2010; Carpenter et al., 1983; Piaget, 1941/1965; Polya, 1945; Schoenfeld, 1982; Verschaffel et al., 1994). However, when the literature is reviewed, it becomes obvious that problem solving in the kindergarten years has been understudied (Hembree, 1992), which means that there is no research on which to base the implementation of CCSS in kindergarten. The following sections further explain the literature related this perceived gap in the research.

1.4.1. Kindergarten Students in the Literature

The majority of research on students’ problem solving has been conducted with older students (e.g., first grade and above) (Carpenter et al., 1983; Hembree, 1992). It is hardly surprising that previous research has focused on older school-aged children as kindergarten is a relatively new addition to the compulsory public education scene. In 2011, Education Commission of the States (ECS) reported that kindergarten is mandatory in only 14 states. However, enrollment in kindergarten, and all pre-school programs, is currently at 60% and is a steadily increasing trend (Kids Count Data, 2007; NCES, 2011). In fact, the trend to compel school attendance at a lower age is indicated by the ECS (2011) data. The current study has the potential to provide a more complete picture about school children and word problems by providing information about a previously understudied population of the public education system.

1.4.2. Thinking Logically

The current wave of educational reform has focused on the need for students to think mathematically rather than merely to produce right answers to mathematical problems (NGACBP & CCSSO, 2010; Schwartz, 2010). The authors of the CCSS state it this way: The standards stress not only procedural skill but also conceptual understanding, to make sure students are learning and absorbing the critical information they need to succeed at higher levels - rather than the current practices by which many
students learn enough to get by on the next test, but forget it shortly thereafter, only to review again the following year (NGACBP & CCSSO, 2010, p. 1).

Because the current study is aligned with CCSS, there is great potential for it to help educators provide instruction that meets the international goals of deep conceptual problem solving not simply rote computation (NGACBP & CCSSO, 2010). When students are taught a strategy before the mental structures that are required to logically understand the strategy are present, students learn to mechanically duplicate a procedure, but not to transfer this knowledge to similar problems (Verschaffel et al., 1994; Wyndhamn, 1993). This is a particularly important point when considering instruction for kindergarten students because they are in a transitional period, most of which are just beginning to use logic to understand their numerical world (Hembree & Marsh, 1993). Logic, the ability to let reasoning guide understanding rather than empirical evidence, often requires ignoring empirical evidence, as in the case of conservation of number. Logic states that a group of objects retains its number as long as none are added or taken away from the group, regardless of the empirical configuration of that group. Researchers (Kato et al., 2002; Sinclair et al., 1983) demonstrated the connection between ability to conserve number and their ability to represent number. They found that children are only able to represent number at or below their level of conservation. In other words only children who can conserve number are capable of conventionally representing number (Kamii, 2000). This means that although non-conserving children can get the right answer by solving a problem using representations, diagrams or solutions strategies that are given to them, but they are merely completing a series of steps, not developing deep conceptual understanding. The current study has the potential for helping educators provide instruction that produces durable and lasting knowledge rather than temporary knowledge useful for passing a test and little else.

1.4.3. Information about Instructional Methods

There is an abundance of research on how elementary school-aged children solve problems using the solution strategy method and manipulatives (Carpenter, 1983; Carpenter et al., 1993), however, only a few studies on how kindergarten and grade 1 students solve problems using their own invented solution strategies (Kamii, 2000; Kamii & Lewis, 1993; Kamii, Lewis & Livingston, 1993; Threadgill-Sowder et al., 1985). In Young Children Reinvent Arithmetic, Kamii (2000) describes the progress that first graders made in arithmetic when engaged in a
mathematics program solely composed of math games and problem solving (Constructivist Group). The students from these classes were compared with students in a traditional class taught algorithms (Textbook Group). When compared on basic computation of number facts, the classes performed fairly similarly. However, when compared on problem solving, the Constructivist group gave more sophisticated answers that revealed deeper mathematical thinking. In his meta-analysis, Hembree (1993) noted that conservers performed better than non-conservers in problem solving and that the more the children developed in the concrete stage (marked by conservation) the better they became in problem solving.

The experiences that I have had as a classroom teacher reflect the research, which suggests that kindergarten students create complex solution strategies when they are given the opportunity to do so. The current research study intends to document the complex ways kindergarten students solve word problems when allowed to create their own solution strategies. This information could be used to inform mathematics education as it undergoes the current wave of reform as envisioned by CCSS.

1.4.4. Suggestions for Further Research

The current study also fills another gap in the existing research by following up on suggestions for further research that have been proposed in the literature (see Willis & Fuson, 1988). The current study supports the point that educators’ focus should be more on the students’ mental relationships than on finding the solutions (Kato et al., 2002) and further suggests that children’s problem solving using conventional symbols and algorithms depends on their level of abstraction. This idea is supported by Hembree (1993) who found that in problem solving, “conservers performed substantially better than children still in pre-operations,” and “children possessing high levels of skill in conservation and class inclusion scored well above children low in these skills” (p. 155). However, this idea has not been further researched. This study provides more information to educators about the connections between students’ mental relationships, their ability to represent number and their problem solving strategies. Further, it may present a practical method for assessing the child’s levels in these areas.

Willis and Fuson (1988) suggested that further research be conducted to determine a teaching method that focuses on student’s need to represent the problem situation instead of focusing only on the solution strategy (Willis & Fuson, 1988). While the current study does not
directly study a teaching method, it indirectly provides information about a teaching method for
problem solving that highlights children’s natural processes.

1.4.5. Research Methodology

The current study also has the potential to add to the body of research that is documenting
the new paradigm of research for early childhood education, called praxeological research. The
praxeological approach is “an emergent worldview and new scientific paradigm” (Formosinho
and Formosinho, 2012a, p. 471) for research that aims to change pedagogical practices and
educational contexts, provides a means to transform implicit knowledge about educational
realities into explicit knowledge by acknowledging the contributions possible when the research
is in close proximity to the research setting (Formosinho & Formosinho, 2013b). The invitation
has been proffered by the authors for other researchers to enter into the conversation exploring
the new paradigm (Formosinho & Formosinho, 2013b). The praxeological approach is further
discussed in section 3.1.2.4.

1.5. Theoretical Framework

This study is grounded in Piagetian theory and framework. A fundamental premise of
this study is that mathematics is logico-mathematical in nature rather than social. This is drawn
directly from Piaget’s types of knowledge as stated in his theory and supported by research
(Kamii, 2000; Kato et al., 2002).

1.5.1. Knowledge

According to Piaget’s theory, knowledge is constructed in the mind of the individual by
constructive abstraction. Piaget identified three categories of knowledge:

1. physical knowledge, knowledge of objects that can be known empirically;
2. social knowledge, knowledge of man-made conventions that differ from culture to
culture and can only be known when explained to the learner; and
3. logico-mathematical knowledge, knowledge that is constructed by the learner.

Logico-mathematical knowledge consists of the relationships that an individual
constructs in his own mind as he classifies information. Logico-mathematical knowledge is the
process of organizing or classifying empirical and social knowledge into a set of relationships
(Chandler, 2003; Kamii, 2000; Piaget, 1942, 1965). Both physical knowledge and social
knowledge are put into relationship through logico-mathematical knowledge (Piaget, 1942, 1965;
Ewing & Kamii, 1996; Kato, Kamii, Ozaki & Nagahiro, 2002). Knowledge of number is one a type of logico-mathematical knowledge that is constructed by each individual from within through the process of constructive abstraction (also called reflective abstraction or reflection). For example, when given a red and blue bear, they can be thought of as different, similar, or two. If we focus on the colors, they are different, if we focus on the animal, they are similar; and if we think numerically, there are two (Kato et al., 2002).

In relation to this study, this means that for a child to understand a mathematical concept, the child needs to mentally act on the problem. It is not sufficient for information to simply be transmitted by the teacher. In this study, this will be accomplished by having the child solve the problem using pencil and paper, using his or her own representations rather than the teacher’s representation (manipulatives or an algorithm).

1.5.2. Representation

Representations that come from a child, like drawings, always represent a child’s thinking about a concept. This is in stark contrast to manipulatives, or other conventional representations, that only sometimes represent number to a child - if that child has constructed that number. To explain further, the term “representation” is often used to describe what an object does; a flag represents its country; blocks represent number; = represents an equal equation. Some mathematical symbols represent operations and some represent number (Kato et al., 2002). According to Piaget (1945/1962), representation is what people do, not something a symbol or object does. A child represents what he understands about reality (Kamii, 2000; Kato et al., 2002). A student’s representations are, according to Piaget, how a person understands his world. To a child who has constructed four, four blocks might represent the quantity four. However, a child who has not constructed four might represent four with a group of objects (Kato et al., 2002). A child who sees five objects can represent the number five to herself if she has constructed the idea of five. A child who has not constructed the idea of five cannot represent the idea to herself or for anyone else, regardless of her ability to write the numeral five (Sinclair et al., 1983). Kato, Kamii, Ozaki and Nagahiro’s (2002) study of students’ representation demonstrates that students can only represent at or above their level of constructive abstraction.
In simple terms, a child can only represent a number that she has constructed. Research has demonstrated a correlation between a child’s ability to conserve number and that child’s ability to represent a number with a symbol (Kato et al., 2005). Children’s level of representation can go no higher than their level of conservation, regardless of their ability to write numerals from memory. Kindergarten students are just beginning to conserve, to use logic to understand their world (Hembree & Marsh, 1993). However, the predominant instructional methods require students to use advanced levels of logic by using conventional symbols (manipulatives, numerals and algorithms). The current study seeks to determine whether a kindergarten child’s ability to solve mathematical problems is mediated by his level of understanding (constructive abstraction).

1.5.3. Constructive Abstraction

According to Piaget (1945/1962), constructive abstraction, also called reflective abstraction, is the process through which logic is constructed. This is distinctively different from the idea that logic or understanding is transmitted to a child. Conservation is the ability to use logic over empirical information. Conservation marks the transition from Piaget’s preoperational stage to the concrete operations stage. Conservers and non-conservers see the same physical event and have different interpretations of the event. This is the fundamental difference between children who are in the preoperational stage and children in the concrete operational stage (Hembree & Marsh, 1993; Kamii, 2000). This is applicable to the present study because participants in this study are in this transitional stage which directly relates to their problem solving capacity.

1.6 Research Questions

The current study was guided by the following questions:

1. How do kindergarten students represent their mathematical thinking when solving word problems?

2. What levels of numerical representation do kindergarten students demonstrate when solving word problems?

3. What strategies are used by kindergarten students to solve mathematical problems?
4. What is the relationship between kindergarten students’ levels of constructive abstraction, levels of representation and level of problem solving?

1.7. Definition of Terms

Constructive abstraction – the process through which logic is constructed (Kamii, 2000).

Logico-mathematical Knowledge – the process of organizing or classifying empirical and social knowledge into a set of relationships (Chandler, 2003; Kamii, 2000, Piaget, 1942/1965).

Logico-mathematicizing – A verb, used by Kamii (2000), to describe the process of putting knowledge into relationships. A child who is putting physical and social knowledge into relationships is logico-mathematicizing his world.

Physical Knowledge – Knowledge that is gained by perception. Gravity and bouncing are examples of physical knowledge. A ball will fall to the ground and bounce when it is dropped. This can be observed. In contrast, the name for the action is social knowledge.

Problem Solving – the use of mathematics to solve a problem. Problem solving occurs across all domains of mathematics, algebra, geometry, measurement, data analysis and probability for all ages of students (Kamii, 2000; NCTM, 2000). Problems can be created at all levels across all domains.

Representation – the way a person sees reality (verb). A person represents what he or she understands about reality, not reality. There is mental representation and physical representation (noun). A student who has not constructed number will see or represent (verb) a group of 5 balls as a bunch of objects rather than exactly 5 balls. That same student may create a written representation (noun) of a bunch of circles (Kamii, 2000).

Sign – words or numerals that do not have a resemblance to the object represented. Symbols have their source in social conventions: They cannot be invented, but have to be taught (Kamii, 2000).

Social Knowledge – Knowledge about properties and conventions that varies by culture and can only be transmitted by someone. An example is color words. There is no way to reason the name for a color. Blue can only be learned if someone tells the learner that the color we know as blue is called “blue.”
Symbol – an object or picture used to stand for (represent) an object. The source is a person’s thinking. They are invented by the person without any instruction in social convention (Kamii, 2000).

Word Problems – the written form of a problem within a realistic context that requires mathematics to solve (Depaepe et al., 2010).

These definitions are presented in the sense of kindergarten students’ engagement with problem solving.

1.8. Summary

Although problem solving in kindergarten corresponds closely to mathematics goals set forth by CCSS and professional educator associations, there are very few research studies on problem solving with kindergarten students. This study was designed to explore the implementation of problem solving with kindergarten students.

This chapter defined problem solving, issues with problem solving that have been discussed in the literature for students in older grades, the need for investigating problem solving with this age group, and the theoretical framework upon which this study was built. Chapter two presents a review of the related literature and Chapter 3 provides a detailed description of the methodology. Chapter 4 describes the procedures used in developing a scale for rating kindergarten students’ problem solving. Chapter 5 presents the findings while Chapter 6 discusses the implications of these findings.
CHAPTER TWO
REVIEW OF LITERATURE

Problem solving has been at the forefront of mathematics education for most of the 20th century (Hembree, 1992), and it continues to remain there today with the introduction of the Common Core State Standards (NGACBP & CCSSO, 2010). It is suggested that problem solving has been studied at “levels seemingly unmatched in all mathematics teaching and learning” (Hembree, 1992, p. 242). Hembree asserted that the separation of time and circumstance under which this research has been conducted and reported increase the likelihood that insights in the research are being lost (Hembree, 1992). As such, this literature review was compiled and organized to present the insights about kindergarten students solving word problems that have been reported over time and different circumstances in a unified manner, thus situating the current study within the context of existing research (Creswell, 2007; Glatthorn & Joyner, 2005). The literature was searched extensively to select the studies most relevant to the research questions, regardless of date of publication. Guiding the study and related literature review were the following four questions:

1. How do kindergarten students represent their mathematical thinking when solving word problems?

2. What levels of numerical representation do kindergarten students demonstrate when solving word problems?

3. What strategies are used by kindergarten students to solve mathematical problems?

4. What is the relationship between kindergarten students’ levels of constructive abstraction, levels of representation and level of problem solving?

Initially the ERIC database was searched for studies specifically on kindergarten students and word problems. There were only two studies on kindergarten students’ problem solving found in this initial search, and therefore the search was broadened to examine problem solving in general. The results of the two studies on kindergarten students will be discussed along with the information gleaned from two meta-analyses of problem solving that were helpful in uncovering information from the problem solving literature (Hembree, 1992; Marsh and Hembree, 1993). Because of the limited number of studies on kindergarten problem solving,
this section will also include the results from studies on older students, as well as some theoretical/practical articles.

The following sections of the chapter will make connections between the literature and the current study. They begin with the literature on problem solving itself, moving to the issues with problem solving as used in the schools, then exploring research aimed at ameliorating these problems, suggesting how the current study can add to this body of research.

2.1. What is Problem Solving?

Problem solving, the use of mathematics to solve a problem, occurs across all domains of mathematics, algebra, geometry, measurement, data analysis and probability for all ages of students (Kamii, 2000; NCTM, 2000). Problem solving requires the conscious search for an appropriate action that will allow one to achieve a clearly conceived, but not immediately attainable aim (Hembree & Marsh, 1993; Polya, 1962). Word problems, setting a mathematical problem into a story using written words, offer only one context for problem solving and are the primary instructional tool for problem-solving in modern education (Babbitt & Miller, 1996; Burton, 1980). However, under the heading of problem solving, there are many different types of mathematical tasks that are labeled problems, but do not teach problem solving.

2.1.1. What is a Problem?

According to John Gough (1997), a problem is “a task, which, when we read it, see it, or hear it, we don’t immediately know how to do it or what to do, but we are willing to try” (p. 17). A problem is not an exercise, something we have previously learned how to do. It is not simply a hard question or one that takes trial and error those are tricks and puzzles. It is not a brain teaser or something that can only be reasoned by trial and error, these are logic problems. These take no reasoning and solutions cannot be generalized. A problem is not a project, which is simply a longer version of one of the examples above that takes several days to solve. Fermi problems, that require the solver to make estimates toward an approximate solution, are not true problems either. Certain types of word problems are not problems at all if they are reduced to a set of terms that are translated into arithmetic operations. Authentic problems must have a solution that can be generalized because the purpose of encouraging children to be problem
solvers is for them to be independent problem solvers “once they have escaped from school” (Gough, 1997, p. 22).

2.1.2. Types of Word Problems

There are two types of word problems: routine (also called standard problems) and non-routine problems (also called non-standard or process problems). Routine problems require that the solver translate statements into operations. These problems are straightforward problems that are easily translated into operations. This is because the numbers used in the problem are larger than the solvers’ representation of that number or because there are no set procedures for finding the solution in non-routine problems (Hembree & Marsh, 1993; Kamii, 2000). Kamii (2000) described non-routine problems as problems for which there is more than one answer or problems that require careful logico-mathematization. Table 1 presents examples of routine and non-routine problems. The literature (Carpenter et. al, 1983; Kamii, 2000) suggests using a combination of routine and non-routine problems to teach problem solving. While the CCSS does not offer examples of word problems, it does give guidelines that would suggest that a combination of routine and non-routine problems would best accomplish the totality of the goals – including the Standards for Mathematical Practice (NGACBP & CCSSO, 2010) as well as the kindergarten grade level standards and strands (NGACBP & CCSSO, 2010). Routine problems most closely relate to the grade level standards where non-routine problems would be more appropriate for reaching the mathematical practice goals as they foster deeper conceptual thinking.

Table 1
Types of Word Problems

**Routine Problems**

There were some robins and 13 cardinals in the tree. There were 19 birds in the tree. How many robins were in the tree? (CGI Workshop Guide, 1999)

Table 1 continued

<table>
<thead>
<tr>
<th>Routine Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Willy has 12 crayons. Lucy has 7 crayons. How many more crayons does Willy have than</td>
</tr>
</tbody>
</table>
Lucy? (CGI Workshop Guide, 1999)

Lucy has 8 fish. She wants to buy 5 more fish. How many fish would Lucy have then? (CGI Workshop Guide, 1999)

Max had some money. He spent $9 on a video game. Now he has $7 left. How much money did Max have to start with? (CGI Workshop Guide, 1999)

Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have altogether? (CGI Workshop Guide, 1999)

Non-routine Problems

Grandpa said he grew up in a house where there were 12 feet and one tail. Who could have lived with Grandpa? (Kamii, 2000)

Carl looked at the pigs and the chickens in the farmer’s backyard. He counted 8 heads and 22 feet. How many pigs and how many chickens were in the barnyard? (Kamii, 2000)

Write what you know about 10. (Kamii, 2000)

There are 6 boys in front of Lee and 3 girls behind him. How many children are in the line? How many of our classmates are not in the line? (Kamii, 2000)

I have some pennies, nickels and dimes in my pocket. I put three of the coins in my hand. How much money do you think I have in my hand? (Curriculum and Evaluation standards, NCTM, 1989, p. 24)

2.2. Findings from the Research in First Grade and Above

Since problem solving was put in the forefront of the mathematics curriculum, researchers have been attempting to discover mechanisms to explain and understand problem solving (Wyndhamn & Saljo, 1997). The explicit aim of much of this research was to make it simpler for students to acquire problem solving skills (Wyndhamn & Saljo, 1997 – c.f. Lester). Most of this research has been conducted with students in first grade and above.
Much of the research on problem solving in the early childhood years has been undertaken by a group of researchers who created the Cognitively Guided Instruction (CGI) professional development program. This program represents years of research of young children’s problem solving which resulted in the identification of word problems into problem types, scales of their relative difficulty, and taxonomies of the typical solution strategies children use (Carpenter, Ansell, Franks, Fennema & Weisbeck, 1993; Carpenter, Franke, Jacobs & Fennema, 1998; Carpenter, Hiebert & Moser, 1983; Carpenter, Moser & Bebout, 1988; Empson, 1995; Fennema, Carpenter, Levi, Frank & Empson, 1999; Franke & Cary, 1997). Other unrelated research has confirmed the validity of the problem types, scales, and taxonomies (e.g., students use derived facts to solve unknown problems; there is a typical order in which students learn sums - doubles, one plus a number, two plus a number, etc.) (Kamii, 1985; Suydam & Weaver, 1975).

While these and other, similar taxonomies and problem solving steps have made significant contributions to the field that demonstrate that they are quite effective in helping the teacher focus on individual students thinking, often, when they are implemented in the classroom they have been reduced to linear models of problem solving or formulas to be followed in lock-step fashion (Fennema, et al, 1999; Francisco & Maher, 2005; Lederman, 2009; Schoenfeld, 1987; Wilson, 1993). Many researchers (Francisco & Maher, 2005; NGACBP & CCSSO, 2010; Wilson, 1993) bemoan the fact that reducing expertise in problem solving to a formula goes against the goal of teaching students to think about problem solving. Explicit training in problem solving as solving a formula through which one masters a series of rigid steps (Wilson, 1993) has not improved the problem solving skills of students (Wyndhamn & Saljo, 1997). In fact, reducing word problem solving expertise to a formula, such as found in many modern classrooms, has been found to be harmful to mathematical reasoning. Francisco and Maher (2005) stated it quite clearly: “The reliance on word problems that can be unambiguously solved through computational proficiency and obvious arithmetic operations hinder mathematical reasoning where as emphasis on mathematical modeling and interpretive skills enhances it” (p. 362). To this end, many researchers and educators have put in place instructional strategies that support mathematical reasoning (Burns, 1994, 2007; Empson, 1999, 2001; Kamii, Lewis & Livingston, 1993; Muth, 1997).
2.2.1. Suspension of Sense-making

The tendency of students to create and accept nonsense solutions to word problems is well documented in the literature (Greer, 1993, 1997; Schoenfeld, 1991; Verschaffel, De Corte & Lasure, 1994). This tendency excludes real-world knowledge when confronted with problems and is referred to as suspension of sense making (Greer, 1993, 1997; Schoenfeld, 1991; Verschaffel, et al., 1994; Wyndhamn & Saljo, 1997). Research suggests that the suspension of sense making is prompted by factors surrounding the culture of school mathematics, such as the artificial nature of word problems themselves and the didactic contract that exists in classrooms where mathematics is understood to be artificial (De Paepe, De Corte & Verschaffel, 2010; Greer, 1997, Sowder, 1988, Wyndhamn & Saljo, 1997). Wyndhamn & Saljo (1997) suggest that the school culture of mathematics encourages the tendency of students to view mathematical problem solving as merely the application of the correct mathematical operation to complete a task. “This focus on the rules-based relationship between symbols often results in less of [sic] attention being paid to what the problem is really about” (1997, p. 362). A popular example of this is the case where students were asked to find the number of busses required to transport 1128 soldiers if 36 soldiers could ride in each bus. In the study only 75% of the students used division to find the answer and only 33% rounded up to the next whole number because they failed to consider that there cannot be a part of a bus (NAEP, 1983). Students often correctly solve certain kinds of problems, but are not able to apply the mathematical skills in other related problems. It appears that the focus on the rules rather than on the sense-making prohibits students from viewing problem solving as using mathematical reasoning to find a solution to a real-life problem (Francisco & Maher, 2005; Verschaffel et al., 1994; Wyndhamn and Saljo 1997).

The shift away from thinking of problem solving as mathematical reasoning that is promoted by didactic teaching of problem-solving solutions presents a distinct disadvantage to students, especially for complex forms of word problems (Willis & Fuson, 1988). Researchers generally agree on the following steps to solving word problems: 1. understand the problem, 2. represent the problem, sometimes described as constructing a mathematical model, 3. reflect on the problem representation, and 4. modify it prior to selecting a solution strategy and solving the problem (De Paepe et al., 2010; Willis & Fuson, 1988).
Young children initially solve problems by directly modeling the problem situation realistically using pencil and paper or manipulatives (Carpenter, 1983; Kamii, 2001). Research shows that older children who exhibit trouble with problem solving, no longer model the problem situation and resort to applying a mathematical operation even in cases when genuine modeling would be the appropriate solution strategy (De Paepe et al., 2010). Researchers suggest that an emphasis on mathematical modeling and interpretive skills enhances realistic mathematical reasoning while an emphasis on the rules of mathematical computation does not (Francisco & Maher, 2005; Greer, 1997; Verschaffel, 1997). However, the most common methods for teaching problem solving emphasize computation over mathematical reasoning.

### 2.3. Teaching Problem Solving

The literature reveals that problem solving is most commonly taught through three types of instructional strategies:

1. Teach cue word – operation hints;
2. Teach algorithms (also called solution strategies) by providing students manipulatives, diagrams, or other representations of the problem (Willis & Fuson, 1988); and
3. Encourage students to invent their own solution strategies.

In the first two cases, a didactic, mechanical approach is often used. However, there are cases where construction of knowledge is fostered when teaching solution strategies. The literature does not present information about the frequency of use of a didactic, mechanical approach or even suggest that teachers use one approach exclusively. However, the literature does give examples from research that demonstrate how these instructional approaches have led to a mechanical approach to problem solving rather than to deep conceptual thinking about problem solving. These examples from the literature will be shared in the section below as well as information from the literature that supports the instructional strategy of having students invent their own solution strategies.

#### 2.3.1. Cue Words

In cue word – operation hints the relationship between key words and the operations they usually represent are taught to students. For example, students are taught that the word “left” indicates subtraction. If you have 10 balls and someone takes 3 away, how many are left? However, Schoenfeld’s (1982) research demonstrated the inefficacy of this strategy. In
Schoenfeld’s (1982) study, when a word problem included a person with the name of Mr. Left, the majority subtracted when cued by the word left, even though this was not the correct operation.

2.3.2. Algorithms or Solution Strategies

Generally speaking, algorithms are established mathematical procedures that guarantee a particular outcome, when followed step-by-step (Burns, 1994; Everyday Math, 2013). Algorithms can be the rules for borrowing and carrying or the steps to long division - divide, multiply, subtract, bring down (Burns, 1994). When used as an instructional strategy, students generally memorize an algorithm, and are expected to use it to solve problems. In the literature about younger children, researchers generally speak of teaching solution strategies or solution sentences instead of using the term algorithm. However, solution strategies meet the definition of algorithm because, when the steps are followed exactly, a correct answer is guaranteed. Solution strategies can be written (number sentences) or appear in other forms (Blume, 1981; Carpenter et al., 1988). Because of the simplicity of the operations involved when teaching math to younger children, solution strategies also include teaching children to count out manipulatives for each number in the problem, then pushing them together and counting all of them for addition; counting on by holding up the first number on your fingers, then counting out the next number; or finding the first number on a number line and jumping forward the second number. For ease of understanding, these algorithms for simple operations with younger students will be called solution strategies throughout (Carpenter et al., 1983). In the solution strategy method the children are given word problems from of a certain problem type and then shown how to solve them using different solution strategies. The issues with the teaching of algorithms or solution strategies have been documented by both formal research and practitioner articles. These articles will be discussed next.

2.4. Carpenter and Colleagues

Research about use of problem solving and use of solution strategies with young children was largely undertaken by a group of researchers whose work contributed to the publication of a professional development program entitled, Cognitively Guided Instruction (CGI). The association among these researchers is unclear; however, the influence of their work on the other researchers is evidenced by the citations in their studies. (Carpenter, Franke, Jacobs &

2.4.1. Early Studies

Early studies (Carpenter & Moser, 1983, 1984) were important in that they documented how young children could solve addition and subtraction word problems without formal instruction. These studies demonstrated that students solved problems by directly modeling the action in a problem with counting strategies, manipulatives, fingers, or some internal representation (Carpenter et al., 1983; Carpenter et al., 1981; Carpenter, 1988). This conclusion was supported by other studies (Briars & Larkin, 1984; Riley, Greeno & Heller, 1983). After concluding that young children are successful at solving addition and subtraction problems without formal instruction, research progressed to formal instructional methods.

In other studies Carpenter, Heibert and Moser (1983) investigated first- and second-grade students’ addition and subtraction solutions before and after explicit instruction in the writing of solution sentences. In that study, findings revealed that although the students were able to use the more efficient open number sentence (algorithm) that they learned over several months of didactic teaching, they did not understand how to use arithmetic sentences to solve word problems. They noted that the students often wrote the number sentences after they had solved the problem, indicating that the number sentence was unrelated to the solution of the problem. A similar finding regarding students’ tendency to write an equation after solving the problem was also documented by Willis & Fuson (1988) in an unrelated study with second graders. Carpenter et al. (1983) attributed the students writing the number sentence after solving the problem to the fact that their problem solutions were not coordinated with the number sentences. In other words, the students could solve the problem using the given strategy without understanding what they were doing with the algorithm. Carpenter et al. (1983) suggested that the shift to a single solution strategy for subtraction may not be entirely beneficial because it could mean that children are beginning to view word problems as the selection of the correct mathematical operation rather than real life problem solving.

After presenting evidence that students do not see the connection between number sentences (their name for solution sentences/algorithms) they are taught and their own informal modeling, Carpenter et al. (1988) undertook a study of first and second graders’ ability to write a
specific type of number sentence: open number sentences (e.g., 5+ ____ = 8) and standard number sentences (e.g., 5+8=__). The results from this study showed that first grade students wrote number sentences that directly modeled the action in the problem, even when they had been taught to represent the problem in another form. This conclusion confirmed previous research that students pass through stages, the first of which is direct modeling. Students pass through more advanced stages in second grade, when they are no longer limited to direct modeling (Carpenter et al., 1988).

2.4.2. Kindergarten Research Study

Another study by Carpenter et al. (1993) was particularly significant to the present study because it offered the only study documenting kindergarten students as they solved word problems. This study investigated the problem-solving processes of kindergarten students who had been provided the opportunity to explore problem situations for a year. The study explored the potential for instruction based on the professional development program Cognitively Guided Instruction to build upon and extend their natural problem solving process of direct modeling. It also sought to explain students’ modeling using an analytic framework (Carpenter, 1993). This study is important in that it combines all aspects of this line of research: student thinking, the taxonomies of learning and the effects of this knowledge on instruction (Carpenter, Franke, Jacobs & Fennema, 1998; Carpenter, Hiebert & Moser, 1983; Carpenter, Moser & Bebout, 1988; Empson, 1995; Fennema, Carpenter, Levi, Frank & Empson, 1999; Franke & Cary, 1997).

“The kindergarten students in this study demonstrated remarkable success in solving word problems” (Carpenter et al., 1993, p. 438). Almost half of the students used valid strategies for all of the problems administered and almost two-thirds correctly solved seven problems or more. Almost 90% of kindergarteners used a valid strategy for the subtraction and multiplication problems and over half were successful at even the most difficult problem. These results demonstrated the wide range of problems that kindergarten students are capable of solving. The most difficult problem faced by the students was successfully answered by more than 50% of the students. The problem was: “19 children are going to the circus. 5 children can ride in each car. How many cars will be needed to get all 19 children to the circus?” The remainder caused no problem for most of students on this non-routine problem. Although not evenly comparable, these results demonstrate much higher success than the results of a similar question answered by 13-year-old students in an NAEP (1983) study. The question from the
NAEP study was: Find the number of busses required to transport 1128 soldiers if 36 soldiers could ride in each bus. Of the 13-year olds who answered this question, only 75% used division to solve the problem and only 33% rounded quotient up to the next whole number because the answer had to be a whole bus.

2.4.2.1. Relationship of studies to the present study. The findings from these studies are important to the present study when interpreted through a Piagetian lens. Research demonstrates that students pass through stages in their development of arithmetic (Briars & Larkins, 1984; Carpenter & Moser, 1984; Riley et al., 1983). It also demonstrates that students were taught to apply an algorithm to a problem. However, they were only able to effectively apply the algorithm to the problems that matched their level of mathematical thinking. When students are given a strategy to use, they are only able to successfully apply the strategy at their level of development. The first graders in this study were at the direct modeling stage, so they were only able to successfully represent and solve those problems whose action could be directly modeled. This supports the position of the present study that mathematical problem solving will also be directly related to developmental levels.

2.4.3. Cognitively Guided Instruction

Carpenter, Fennema, Franke, Levi, & Empson, published Cognitively Guided Instruction (CGI) in 1993. This professional development program helps teachers become familiar with the stages through which children pass when learning to solve problems, the strategies students use to solve problems, and the relative difficulty of those problems (Carpenter, Fennema, Peterson, Chaing & Loef, 1989; Fennema, Franke, Carpenter, Carey, 1993; Peterson, Fennema, Carpenter, Loef, 1989; Villasenor & Kepner, 1993). According to Villasenor and Kepner, CGI “builds on the knowledge that students already have and helps them analyze their own thinking” (1993, p, 62). The researchers repeatedly report that students were successful at solving problems after being taught the solution strategies by teachers who were involved in the CGI professional development program. The authors state that CGI is not a teaching method, that teachers are not told how to teach, but armed with information that should guide their teaching. Despite the fact that the intent is that teachers should not teach problem solving steps in a lockstep fashion, the teaching of problem solving steps is often done (Fennema et al., 1999; Willis & Fuson, 1988). However, as stated above by Carpenter et al., (1983), questions arise, even by the researchers, about the understanding with which the students compute the answers.
2.4.3.1. Relevance of CGI to the present study. The question that Carpenter and his colleagues (1983, 1988) raise about whether students are actually demonstrating understanding by writing number sentences as a part of the problem solving process is an important question for the present study in that it helped frame the theoretical framework. This question reflects the dilemma faced by educators, how to implement instructional practices that will both influence students to deeply understand mathematical concepts and move them toward curriculum goals. It is the perspective of this study that this question can be answered through the theoretical approach taken. The aim of education according to Piaget (Kamii, 2000) is autonomy, to teach students to think so they can operate in an interdependent fashion. When viewed this way, curriculum goals and deep conceptual thinking cease to be the ends, but are necessary components of the goal of autonomy. In other words, curriculum goals are the articulation of the steps taken when deep conceptual understanding is developed. Deep conceptual understanding is necessary for a person to be autonomous. It is an aim of the present study to suggest another instructional method that will answer this question.

These studies are also relevant to the present study because they influenced the methodology and procedures utilized in the study. The interviews utilized in these studies (Carpenter et al., 1988; Carpenter, et., al., 1993) were used as models for the conferences in the present study. The premise that teachers who are armed with knowledge about how students solve problems are more effective teachers and will produce students who are effective problem solvers also inspired me to undertake research that has the potential to add to teachers’ armory of knowledge about students’ thinking while problem solving.

2.5. Marilyn Burns

Marilyn Burns is a prolific author of practitioner articles regarding the teaching of mathematics. In the seminal article, *Arithmetic, the Last Holdout* (1994), she documented the changes that she and other teachers made in teaching arithmetic as they explored the findings of Madell (1985) and Kamii and Lewis (1991) in their classrooms. The article gives striking narrative reports of the variety of problem-solving methods used by students who were not taught one standard algorithm. For example, Burns reports that when given the word problem $54 + 28$, she found eight different methods for solving this problem after not doing any worksheets during the year. She describes each of the different methods used. She continues to describe the work of students solving division problems in a middle school class and percentages.
in middle school. In another article, *Looking at How Students Reason*, Burns (2005) documented and described what happens when teachers “delve into the thinking behind students’ answers…when answers are wrong…and correct” (p. 26). She used illustrations from students’ answers to demonstrate how allowing students to create their own solution strategies helps them become more flexible in their mathematical thinking. She described a fourth-grade class using fractions and a second-grade class exploring addition. In the second-grade class, the students were able to describe multiple methods for solving simple addition problems.

Burns’s suggestions for teaching influenced the methodology and procedures used in the present study. Notably, the following suggestions were incorporated into the present study: the use of non-routine problems (Burns, 1994, 2005, 2007), use of picture books to teach mathematics (Burns, 2010), and suggestions for teaching (Burns, 1994, 2005, 2007, 2010) The following specific teaching suggestions were utilized in the current study: ask students to explain their answers – both correct and incorrect answers (Burns, 2005, 2007), share solution strategies with the group (Burns, 2005), and utilize writing to help students keep track of their thinking (Burns, 2007).

### 2.6. Rob Madell and Constance Kamii

The work of Madell and Kamii also influenced the present study. As Burns (1984) noted in her article, Madell’s work also noted that students who had not been taught the algorithm for double column addition proceeded from left to right, instead of from right to left. Madell’s findings also inspired Kamii to pursue research on the use of algorithms. She has produced many publications on the topic such as *How Algorithms Un-teach Double Column Addition* and *Young Children Reinvent Arithmetic* (2002). In these studies she documents, in detail, the rich problem-solving that young children are capable of when given the chance. She presents compelling evidence that students who are given the opportunity to create their own problem solving strategies out-think those who only use an algorithm. She also demonstrates that the students who do not learn the conventional algorithms do equally well on standardized skills tests. Research supports the finding that the teaching of algorithms un-teaches the understanding of place value (Kamii, 1994, 2000; Kamii et. al., 1993; Madell, 1985). Students who were taught algorithms made more nonsensical mistakes when adding double digit numbers than students who were not taught algorithms. The students who were not taught algorithms
gave more logical answers than the students who were taught the algorithms (Kamii, 1994, 2000).

2.7. Perspective of this Study

Based on the information from the literature presented above, the perspective of the current study is that these didactic teaching methods treat word problem solving as social knowledge - knowledge that is given or transferred to the learner. Instead, mathematics and problem solving are logico-mathematical in nature and must be constructed within the mind of the learner by constructive abstraction (Kamii, 2000; Kamii & Lewis, 1990; Kato et al., 2002). Giving solution strategies or algorithms to students or teaching them key word strategies treats problem solving as social knowledge rather than logico-mathematical knowledge. It is not necessary to give a child the information to move him to the next stage or level, the stages of development are not steps teachers can force students through, rather students will construct knowledge out of necessity if they are allowed to think (Kamii, 2000; Madell, 1985). This assumption is supported by the larger body of research on didactic learning environments (De Smedt et al, 2010). Shrager and Sigler (1998) and Sigler and Araya (2005) argued that creating learning environments that explicitly teach and train students in strategies is not necessary, but that by putting students in learning environments that implicitly promote discovery of new, more advanced strategies, they may discover new strategies. They further stated that children develop and modify strategies with increased practice in executing the strategy (Shrager & Sigler, 1998; Sigler & Araya, 2005). This is not to say that students should never be exposed to conventional symbols or algorithms, but such types of social-knowledge should be carefully given when the need for that it arises and when the teacher has determined it is appropriate for the students’ level of constructive abstraction (Kamii, 2000). Kamii (2000) has documented in her research and I have seen in my classroom experiences that children will construct the need for numerals, conventional symbols, and algorithms as our ancestors did, when they are given experiences that promote this understanding. For example, when solving problems, students often get tired of drawing symbols for every child in the class, a well timed, “Can you think of a shorter way to show that?” may just be the question a child needs to begin to use numerals instead of drawing 20 pairs of feet that would be required to accurately represent the problem situation.

This concept follows with Piagetian theory that states that as children repeatedly experience something, they experience it differently each time, moving through levels of
wrongness. It is the confrontation with something new that does not fit with the current schema that creates disequilibrium, thus the need to search for knowledge. The process of organizing empirical information in logico-mathematical structures through the process of reflective abstraction is one that takes time and experience -- it cannot be forced. It also follows with Kamii’s (2000) and Kato et al.’s (2002) assertion, in the Piagetian tradition, that objects and symbols do not necessarily represent number to children, but rather, that children represent what they know about number. Further, that sometimes objects can be used by children to represent what they know, but their drawings always represent what they know about number. This conclusion lead researchers to question the appropriateness of using conventional symbols, algorithms, solution sentences or other notation systems with young children.

Kato, Kamii & Sinclair’s (2002) research demonstrated that a student’s ability to understand conventional symbols depends on that student’s level of understanding about number and operations. In other words, a student may be able to write numerals and copy algorithms (Sinclair, Siegrist & Sinclair, 1983), even producing the right answer to sums, without understanding what he or she is doing mathematically. A student may be able to combine two bears with three bears and count five to answer the related word problem. However, this can be done without understanding. The usefulness of manipulatives in solving a problem is directly dependent on the relationships students have made with them, through constructive abstraction (Kamii, 2000). Kamii’s (1994, 2000) research demonstrates that students go well beyond producing the right answer to word problems when they are encouraged to represent their thinking on paper and pencil (Kamii, 2000). This is the reason students usually choose to draw and write their solutions to word problems over using manipulatives (Kamii, 2000; Olivier et al., 1991).

It was the hypothesis of this study that if there is a relationship between a student’s level of constructive abstraction and representation of number, then there could also be a relationship between these factors and a child’s problem solving proficiency. This study followed Kato et al.’s (2002) suggestion that educators need to “focus more on the mental relationships students make (i.e., their abstraction) because the meaning students can give to conventional symbols [manipulatives, solution sentences, algorithms] depends on their level of abstraction” (Kato et al., p. 30), by demonstrating how a child’s problem solving is related to his level of conservation.
2.8. Summary

This chapter presented the definition of problem solving, the related findings that have been presented in the research regarding problem solving and the teaching of problem solving. The key literature providing theoretical framework for the methods of this study and the theoretical perspective of this study was described. This literature is important to the study because it demonstrates the connection and relevance of decades of educational literature from Piaget to the present educational context.

The next chapter outlines the methodology that was adopted in conducting the study. In doing so, the importance of including children’s voice as well as the reasons for choosing a naturalistic observational study is discussed.
CHAPTER THREE

METHOD

Although problem solving for all levels has been articulated as a goal by mathematics educators and is currently being brought into the spotlight by the CCSS, literature and research concerning the practice with kindergarten students is scarce, thus suggesting the need for further study and investigation. The purpose of this study was designed to fill this research gap by investigating how kindergarten students solve word problems. In doing so, the current study examined possible relationships between three aspects of students’ mathematical thinking: constructive abstraction/conservation, mathematical representation, and mathematical problem solving. Each of these aspects was measured by a task that was analyzed with a corresponding coding scale. These tasks and scales are described in the Procedures section. Scales from previous research (Kato et al., 2002) were used to code the conservation of number and mathematical representation tasks. The codes for problem solving were determined as part of this study. The process of creating the coding scheme for problem solving is detailed in Chapter 4. The results of the students' performance on the tasks were used to answer the following research questions:

1. How do kindergarten students represent their mathematical thinking when solving word problems?
2. What levels of numerical representation do kindergarten students demonstrate when solving word problems?
3. What strategies are used by kindergarten students to solve mathematical problems?
4. What is the relationship between kindergarten students’ levels of constructive abstraction, levels of representation, and levels of problem solving?

This chapter is organized into six sections: research in the early childhood setting, participants, setting, procedure, instruments, coding, and data analysis.

3.1. Research Methodology in the Early Childhood Setting

The nature of the research questions for this study made the selection of one well-established research methodology somewhat challenging. The limited number of studies on kindergarten problem solving (Hembree, 1992) demonstrated no single preferred method for conducting research on this topic. The research studies that were the inspiration for the current study did not solely rely on a named research methodology, but instead created their own unique
methodology. Kamii and her colleagues described their theoretical framework, procedures, and the data so that their methods could be replicated, but did not codify them as a formal methodology. This created the need to search for a well-established methodological guide that reflected the unique characteristics of undertaking research in the early childhood years. Anthony Pellegrini’s (2004) methodological guide for collecting observational data provided the method that would best answer the essential elements of the research questions posed (Creswell, 2007; Flick, 2009).

3.1.1. Naturalistic Descriptive Observational Study

Anthony Pellegrini’s (2004) methods are widely used in early childhood research and were selected for this study to enhance the rigor and sophistication of the research design (Creswell, 2006). Pellegrini’s (2004) research methodology served as the guiding framework for successfully combining elements of research methods from other established early childhood researchers (Kamii, 2000; Kato, Kamii, Tomiyama & Nagahiro, 2002; Piaget, 1842/1965; Sinclair, Seigrist and Sinclair, 1983. This section describes how Pellegrini’s methodology frames the current study, as well as how the work of the other researchers fits into the methodology of the collection of data for each individual task.

According to Pellegrini (2004), a naturalistic observational study is, “conducted where individuals and groups live and function, such as in homes, schools, and the work place. Research in natural settings is most dramatically contrasted with research in contrived or artificial settings, such as laboratory experiments” (p. 54). A naturalistic school setting is usually chosen to increase the likelihood that the observed behavior approximates the realistic behavior of children who are being studied, thus overcoming the limitations that a laboratory setting might impose (Pellegrini, 2004). “Field studies (or studies of people in their everyday environments) are primarily concerned with describing behavior as it occurs in its natural habitat and recognize limited internal validity” (Pellegrini, 2004, p. 5). However, descriptive research “…is an important, and necessary first step, [sic] of any scientific enterprise” (Pellegrini, 2004, p. 5) that allows the formulation of categories that will provide educators the opportunity to design appropriate instruction (Pellegrini, 2004). As there is limited research on how kindergarteners solve word problems, the first step of describing what they do when solving word problems was necessary. Further, the limitations to internal validity of the naturalistic setting were deemed
acceptable for the purposes of this study (i.e., providing information about how students solve mathematics word problems within a school context).

3.1.1.2. Perspective. Another important aspect of Pellegrini’s (2004) naturalistic observational study methodology that informed the research design of this study is the perspective of the researcher. The current naturalistic study was undertaken from the qualitative/insider/emic perspective rather than the quantitative/outsider/etic perspective. In this perspective, the researcher interacts with the students as an insider as a means for the participants to habituate to the presence of the researcher so their behavior will “approach the way they would act if [the researcher] were not there” (Pellegrini, 2004, p. 97).

Pellegrini (2004) dictates that research conducted strictly by the quantitative/outsider/etic perspective would consist of strict observation and the analysis of personal diary entries. In the context of this study, using the quantitative/outsider/etic perspective would mean that students would merely be observed as they solved the problems or that their papers would be analyzed with no input from the students (Pellegrini, 2004). In light of the purpose of this study, strictly observing the students while they were writing their solutions would not have been informative unless they were spontaneously explaining what they were drawing while they were drawing (Pellegrini, 2004). An alternative could have included an outside observer sit with the student encouraging him or her to engage in a “talk aloud” procedure or asking clarifying questions while the student was solving the problem. However, the time and resources necessary to record 22 students solving word problems simultaneously prohibited these options. This data gathering technique would have interfered with the purpose of the research to document what students do naturally when solving problems in the school setting. In addition, the presence of an outsider in the classroom would likely inhibit the student’s responses and keep the data from approaching reality (Pellegrini, 2004).

3.1.1.3. Procedures. The data collection methods and procedures met Pellegrini’s criteria for naturalistic, descriptive, insider/qualitative/emic observational study. The data collection process was naturalistic in the sense that it took place within the typical, every day school setting. Special care was taken to situate the data collection scenarios within a setting that was familiar to the students. For example, the students often completed individual work at tables in the classroom, followed by one-on-one or small group work with an adult either in the
classroom, across the hall, in the hall, or in the work room. So, the researcher and teacher followed the same format for the word problem task. The students solved the word problems in the classroom, and then moved to the work room for the one-on-one conference rather than completing the word problem task in a laboratory setting. Further enhancing the naturalistic quality of the data collection, the problem solving activities in which the students were engaged were a part of the math instruction procedure for the entire school. As a part of the transition to CCSS, all of the students in the school participate in a whole group math instruction time that concludes with the completing of math journals followed by a conference with a teacher.

3.1.1.4. Habituation period. An important part of the insider perspective is a habituation period (Pellegrini, 2004), also referred to as the desensitization period (Patton, 2002). In this study, I spent time interacting as a teacher with the students prior to and during data collection as a means of mutual habituation (Pellegrini, 2004). I was involved in various activities with the students such as reading to the students, leading group times, working with small groups of students, and walking them to special area classes. As a result, the students were familiar with me and interacted with me as an insider.

3.1.1.5. Data collection procedures. Pellegrini’s methodological primer provided specific procedures for observational data collection methods, which are descriptive in nature. Observation and interviews are direct methods, while document collection and video-taping are indirect observational data collection methods. Both types are useful and valid for collecting observational data. When directly observing behavior, it is necessary to make choices about what to observe and record. It is important to record all relevant details but impossible to record every detail in the sample selected. Pellegrini (2004) provided sampling rules.

3.1.1.6. Sampling. Focal person sampling with continuous recording rules were most applicable to this study. For this sampling method, one person is chosen to observe for a specified period and as much information as possible is recorded for the entire observation period. According to the topic of interest, the researcher determines the specified recording period. Continuous recording differs from interval recording, which is conducted for short intervals with breaks in between recording sessions. One disadvantage of this type of sampling is that it is time-consuming; however, the use of video equipment can minimize this disadvantage (Pellegrini, 2004). Observing multiple times and at different times of day and selecting the order in which people are observed were suggested as means to ensure that the data
were representational of the natural situation. In this study, special care was taken to ensure that the order of observation did not confound the results. To preserve the naturalistic nature of the study, and for practical implementation considerations, a formal rotating observation order list was not used for the conferences and tasks that were completed in the work room.

### 3.1.2. Research Context

Chapter two situated the study within the current research context. In this section I explain how the literature influenced my choice of methods, procedures and instruments used in this study. This is an important section because the methodology employed has not been codified into a formal methodology.

#### 3.1.2.1. Problem solving with young children.

The work of Constance Kamii inspired this research study wherein kindergarten students solved word problems. Kamii’s criteria for creating and selecting word problems and procedures were used as the model for the procedures in this study. In *Young Children Reinvent Arithmetic*, Kamii (2000) presented information about first graders doing problem solving as a part of a larger year-long study of the entire mathematics curriculum. The present study necessitated a more formal conference procedure and a rating scale for the sophistication of the problem solving strategies. As such, the methodology for the word problem solving task was field tested prior to data collection. An exploratory study was completed late in the spring semester of 2012. The pilot study was designed for me to become familiar with the infrastructure of the personnel at the data collection sight, to field test the procedures, and to determine whether the approach would elicit the type of responses from the participants that would answer the research questions (Pellegrini, 2004). A full description of the pilot study is attached as Appendix B.

#### 3.1.2.2. Conservation and representation.

The research of Kato, Kamii, Ozaki and Nagahiro, (2002) presented the relationships found between students’ levels of conservation and representation. This study influenced the present study in several ways. The idea to investigate possible relationships between problem solving, conservation and representation came from this study. If there was a relationship between conservation of number and representation and they are both used in problem solving, then there might be relationships between these factors if they are investigated as students problem solve. The tasks, procedures, and instruments of Kato et al. (2002) were adapted for this study. Kato et al. (2002) used Piaget’s well-established conservation
of number task. They adapted the representation task and scale from the study by Sinclair et al. (1983). I used these studies as a guide as I created scales for this current study.

### 3.1.2.3. Conferences.

The procedure I used for conducting one-on-one conferences was modeled after research by both the Carpenter et al. (1983) study and the Kamii (2000) study. Kamii’s (2000) descriptions of her conferences with her students, complete with quotations from conversations between the researcher and students, clearly outline procedures used in the research in her classroom. Carpenter et al. (1983) also did comparable conference procedures in a separate room removed from the classroom. The problem was re-read several times if needed, then the student solved the problem. If the researcher could not understand a child’s solution method, individual interviews (conferences) were undertaken. In Carpenter’s study the conferences were administered two ways, with a different amount of time elapsing between the problem solving session and conference. In session one the conferences were conducted on the same day as the task; however, in the second wave of data collection, conferences were held several days after the students completed the task. Although Carpenter et al. (1983) called them “interviews” they were very similar to Kamii’s (2000) “conferences” in that in both cases the researchers asked the students to describe how they found an answer and did not tell the students how to solve problems, but offered encouragement and clarification. The use of conferences was supported by the research of Brannin (1983) and Seigler & Svetina (2006). Brannin (1983) suggested that questioning children directly about their thinking gives a more complete picture of their thinking than post hoc analysis of their written work. Seigler & Svetina (2006) suggested that children sometimes use the correct procedure before they have conceptual understanding. They also suggest that receiving feedback might increase oscillation from a more sophisticated strategy to a less sophisticated strategy (Seigler & Svetina, 2006). For these reasons, the students and I engaged in conferences that were conducted in a separate location for the purpose of giving the student an opportunity to explain his solution strategies without interfering with the naturalistic classroom setting. The template for the Conference Protocol is attached as Appendix B.

### 3.1.2.4. Praxeological research.

I was especially drawn to the methodology utilized by Kamii in her work. The seamless way she integrated regular school-day activities, conferences, and formal Piagetian tasks into a natural cycle that could be used by educators appealed to me. Her credibility as a researcher has been established by the number of articles and books that have
been published by reputable journals and publishers. Up to this time no formal name had been applied to Kamii’s research methodology. However, a review of the literature revealed research utilizing similar methods using the insider perspective within European journals, in particular, refer to as praxeological research.

The limited number of research studies on problem solving in kindergarten may be due to the challenges posed by the lack of an insider perspective. Studies undertaken in laboratory settings often do not give the information about children in their natural educational settings. Children are not always able to articulate the concept being studied, so that information has to be inferred by the researcher. Outside researchers are unfamiliar with the children, so those inferences may be less accurate than inferences made by a researcher familiar with them. Quantitative information does not always answer the questions that need to be asked. The greater early childhood research community has been exploring solutions to these challenges.

The *European Early Childhood Education Research Journal* recently produced a special topics issue (2012) that posed praxeological research as a widely accepted solution to these challenges (Formosinho & Formosinho, 2012a). Praxeological research is presented as an emerging social sciences paradigm that marks a shift from the either/or paradigm wars of quantitative versus qualitative research. The editors present the disclaimer that praxeological research is an emerging world view and invite researchers into the discussion (Formosinho & Formosinho, 2012a) rather than presenting it as a final product. Praxeological research is a framework for research that is “concerned with study of change and praxis development” (Formosinho & Formosinho, 2012a, p. 471). It is participatory research undertaken by researchers embedded in a natural educational context, which includes the voices of the participants. It is not limited to one research methodology, such as case study, but is flexible according to the study and research topic, using multiple techniques of data collection. Formosinho & Formosinho (2012) maintain that rigor in praxeological research will come from detailed descriptions of actions and contexts that are shared with peers and close proximity marked by reflexive, ethical attachment rather than by the distance between the researchers and the object of research. “Praxiology transforms implicit knowledge into explicit knowledge” (Formosinho & Formosinho, 2012b, p. 597).

Formosinho & Formosinho, (2012a) suggested that research methodology should match the theory and philosophy of the research study. The above-mentioned aspects of praxeological
research fit the theoretical underpinnings of this study. The researcher participates in a reflexive, ethical way with the students. The close proximity and interaction with the students gives the researcher insight into the students’ thinking. It also gives ear to the students’ voices. Students are given the opportunity to describe their thinking and solutions. They are an active part of the change process; changes in their thinking can be a part of the research study, the researcher changes her questions based on their conversation, and the research has the capacity to change classroom practices. This study transforms implicit knowledge about problem solving into explicit knowledge about the practical situation. The use of multiple and unconventional research methods, such as using Piagetian tasks, individual problem solving, and conferences in the same study, are two other characteristics of this study that fit the emerging praxeological research paradigm.

3.2. Participants

The participants for this study were 19 kindergarten students from one kindergarten class in an accredited public school in the state of Georgia, hereinafter referred to as “Children’s Learning Center” or “CLC”. At the time of the study CLC housed 16 general education classes and one special needs kindergarten class. In addition, the school also housed a three-year-old special needs reverse inclusion program and a Pre-K Program. There was a total enrollment of 660 students. The school served 38 students in a special needs inclusion program, where the students received services within a regular education class.

3.2.1. Participant Selection

The number of participants selected for this study was limited by the number of students in one classroom and the practical considerations of collecting the data (i.e., the number of conferences that could be completed by a researcher within a regular school day without disturbing the natural flow of the classroom activities). An effort was made to include all students from the class in the study to maintain the naturalistic setting for data collection (Pellegrini, Symons & Hoch, 2004). Also, because this study was only intended to describe the population studied, not to be generalized to other populations, a small number (19) was deemed acceptable (Pellegrini, et al., 2004).

The kindergarten class selected by the principal for this study had an official enrollment of 23 students at the time of the study. However, four students were excluded from the study, leaving 19 student participants. Two regular education students were not given consent from
their parents to participate in the study, so their data was not included in this study. Two special needs students inclusion were excluded from the study due to considerations other than their special needs. The current research study did not exclude any student within a general education kindergarten class based on special needs qualification alone. One student was served in the room for only two hours of the day. The remainder of the day he is in another classroom. He was out of the room so frequently that the teacher does not count him in her routines. For example, she uses 22 as the number of students in her class for activities, such as word problems. He was not in the classroom during data collection times, so he was excluded from the activities of this study as it is a naturalistic study designed to capture the daily routines of the class. Another of the special needs inclusion students only spoke a few words each day. He was categorized as “non-verbal” by the teacher. As a result, I was not able to receive answers to questions about his written answers to the word problems during the conferences. His data was also excluded from the study. Five special needs inclusion students participated in the study. To ensure that no student felt excluded from the whole class activities that were a part of the study, data was collected from all students when they were present for data collection times.

3.2.2. Demographic Information

Nineteen students participated in this study. There were 10 males and 9 females in this group. Five students were special needs inclusion. The students were between 64 months (five years and four months) old and 75 months (six years and three months) old at the time of the study.

3.3. Setting

The setting for this study was one self-contained kindergarten classroom at CLC a public school in Georgia. When I asked the principal if I could conduct the research for this study at the school, she did not hesitate to agree. I chose the school as a potential site for this research study because of my long-standing relationship with the school. Over the past five years my relationship with the school has been comprised of being a parent of a student at CLC, I have served on volunteer committees, been a long-term substitute, provided professional development classes for teachers, spoken at PTA meetings and collaborated on the mathematics journals project. The administration fosters a climate of professional reflection and improvement which makes it an excellent setting for research. Teachers are eager and willing to participate in
research projects and collaborate well. CLC implements Developmentally Appropriate Practices and CCSS, which make it a good fit for the philosophy of this study.

The classroom was set up as a typical developmentally appropriate kindergarten classroom. There were table, chairs, centers, easels, blocks, books, a rug and beanbags nicely arranged throughout the room. There were enough places for each child in the class to sit in a chair at a table at one time. In addition there was a reading center, with books easily accessible on shelves and a child-sized sofa and beanbags. There was a writing center, with desk space for two students, paper, pencils, markers, scissors and other supplies within easy reach of students. The painting center was set up with two wall-mounted easels, paper, smocks, paint and brushes. Other centers included blocks, math center, science center, sensory table and a kitchen center. There were colorful rugs for whole group times; a smart board; displays of students’ work; computers; and many poems and charts hung within view on the walls.

As in every kindergarten class in the school, there was one lead teacher and one assistant teacher who worked with the students all day. However, throughout the day there were other teachers who worked with the class on a regular basis. There were often volunteers, interns and other visitors. There were rarely fewer than three adults in the room including speech teachers, inclusion teachers, and physical therapists. In the daily schedule there were times for whole group instruction and small group/individual instruction scheduled around recess, snack and lunch. Whole group instruction generally took place with the children sitting on the carpet and the teacher leading. The teacher had many personal resources that she used to supplement the school-provided resources. Whole group instruction times were organized and structured while allowing for appropriate student participation such as sing along songs, interactive white board activities, and student-led activities. Individual and small group work times (small groups and centers) were also organized and well managed. During center/work time, teachers worked at the tables with students, either individually or in small groups, on structured activities while the other students not in groups worked in centers of their choice. The classroom environment was often noisy as the students were engaged in different activities and projects, but the noise level signaled productive conversation as students and teachers worked together to complete projects and assignments.
As a part of implementing the Common Core State Standards, and reading some articles that I provided in conjunction with research for the current study, the kindergarten classes in the school implemented “Math Problem Solving Notebooks” this school year. After attending several CCSS workshops the school’s mathematics team wrote and/or compiled a collection of “problems of the day” designed to help the students meet the CCSS. Some examples of problems of the day follow:

1. There are 8 children in the bathroom and 3 more join them. Solve it!

2. Teacher: give students a handful of no more than 10 objects that are of different kinds. (e.g., bears, cubes, pennies). “What can you tell me about your objects?”

3. At centers, 7 boys were playing in puzzles and 6 girls were painting faces. Solve it.

4. Look at these two numbers 12 and 14. What do you see and understand?

Teachers and administrators regularly met to explore the students’ progress in their mathematical thinking and to determine whether the problem solving notebooks were, in fact, helping students become problem solvers. The current study was a natural fit for the school because of the mutual interest in kindergarten students’ mathematical thinking. The collaborative attitude of the administration and teacher meant that the teacher was willing to work with me in creating and carrying out the study.

Each morning the teachers followed the same routine for completing mathematics workbooks. During morning group time, the teacher completed the morning routine (greeting, songs, calendar, weather, book, etc.) and then moved to an activity that related to the problem of the day. Next, the teacher called the students to tables to work in groups where the problem of the day was read to them and then the students were directed to answer the problem. As the students completed the problem, the teacher observed and assisted the students individually. The teacher made notes about each student’s work. The notes were usually quotations of what the student said, or documentation of the student’s participation. After this small group time, students were dismissed to an activity, usually work time (centers).
3.4. Procedure

The following section will outline in detail the procedure followed within this research study. It will trace the steps of the study from the planning stages through to the data analysis.

3.4.1. Planning and Habituation Period

I visited the site five times over two weeks planning, visiting, observing and interacting with the students prior to data collection. This was done so that the students could become habituated to my presence as an observer/participant researcher in the classroom (Patton, 2002; Pellegrini, 2004). Data collection took two weeks. A chart listing the time on the site for visits and data collection can be found in Appendix D.

The research study began with a planning time with the classroom teacher and her teaching assistants. The group made decisions about the schedule and routines of data collection. We selected a topic for study, chose books, and collected possible word problems. Among other aspects, the group planned to fit the data collection into the regularly scheduled work times to keep the daily schedule as naturalistic as possible throughout the duration of the research study. We also planned other aspects of the study such as the habituation period (Patton, 2002; Pellegrini, 2004) and the conference protocol. We decided that the habituation period would be relatively short because students were accustomed to having multiple teachers and visitors working with them in the classroom. In this case, I visited the classroom two times, spending approximately an hour participating in class activities prior to data collection. On the third visit, the teacher formally introduced me and I read a book to the class on the mathematics topic, geometry. I spent time observing the small groups of students who were working with teachers and interacting with the students as they completed their work. On the third day, I stayed for half of the school day.

3.4.2. Data Collection

On the fourth visit I came to the classroom before the morning bell and sat at group time with the students. I participated with the students as the teacher led the morning group time. This time consisted of watching and participating with the school-wide good morning show broadcast, a greeting time, song time, calendar, weather, pledge, and number activities. The teacher introduced me to the students again. I read, *I Spy Shapes in Art* by Lucy Micklethwait. Then the students completed their math journal activity for the day. I moved around the room
talking with the students about their work, much as a teacher would. I stayed in the classroom for half of the school day, participating with the students.

My involvement followed a new pattern for the remainder of the study. I came in at the start of the day and participated in group time, with my participation as a teacher increasing each day. By the end of the research study, I was leading the entire group time. During the group time, I read a book on the topic of the week, shapes and geometry, and then introduced the task for the day. The students moved to their work space (tables) and worked individually on the day’s problem solving task. The entire class sat at tables while individually solving the word problem. The teachers sat with the students at s and encouraged them to complete their problem. As the students completed their task, I called them to the teacher workroom to complete a word problem conference or to complete a task. On word problem days, I returned to the classroom and conduct a summarizing session for the word problems.

The order of the completion of the tasks was important to ensure reliability of the data collection. The word problem tasks were distributed throughout the time of the study. The conservation task was done last because it would be best for me not to know which students were conservers until after the data were collected.

3.5. Instruments

There were four types of tasks that were given to the students. Three of the tasks, a numeral writing task, a representation task and a conservation task, were given one time to each child. The word problem task, the fourth type of task, was repeated five times. The following section will describe the instruments used to collect and analyze the data and the procedures used.

3.5.1. Numeral Writing Task

A student was called to the table and given a piece of paper and a pencil. The student was told, “Write the numeral for the number I call out.” The researcher called out the following numbers in the following order, 4, 3, 6, 8, 10, 22, 44, 30, 12, 16. I wrote notes on the paper to clarify answers if they were not clear. I also marked the answers on a recording sheet.

3.5.2. Representation of Groups of Objects Task

The procedure and coding scale that was reported in Kato et al. (2002) was used for this study. The participants were shown sets of common objects (e.g. a set of four plates) and asked
to record that information on paper (Kato et al., 2002; Sinclair, Siegrist & Sinclair, 1983). The following numbers of objects were presented to all the children in the same order: Four plastic dishes (4-6 inches), three spoons, six pencils and eight wooden blocks (2x2 or 3x3). I lined up the objects in front of each child while asking the child to “take a good look at these” (Kato et al., 2002, p. 35). Then I asked the child to, “Draw or write what is here on this sheet of paper so that your teacher will be able to tell exactly what I showed you.” I was careful not to use phrases such as how many, number, or anything that might have suggested quantities. If a child asked whether to draw or write, he was told, “You decide whether to draw or write or do both.”

When the child finished, he was asked, “Will your teacher be able to know what I showed you?” and “Is there anything you want to add?” If he answered yes, then the child was encouraged to modify his answer. On the first two occasions if the child answered that they wanted to add and wrote a number sentence, the question was modified to, “Is there anything else you want to draw or write on the paper?” If the answer was “no” I moved on to next set of objects on the list while giving the child a new piece of paper. This process was continued until all sets of objects were presented. If I was not able to identify the child’s drawing, she asked probing questions such as, “Tell me about your drawing,” or “Why did you draw that shape?”

3.5.3. Conservation of Number Task

The following section explains the procedures for the conservation of number task created by Jean Piaget (1941/1965). The task is patterned after the one used by Kato et al. (2002).

Equality - I made a row of 8 blue counters. Eight counters were used because numbers up to 7 are perceptual numbers, meaning they can be distinguished at a glance. However, numbers larger than seven are not distinguishable at a glance. The researcher asked the child to, “Put out the same amount of red counters.” The child’s response was recorded. If necessary, the red and blue counters were put into one-to-one correspondence and the child was asked, “Do these two rows have the same amount.”

Conservation - I said, “Watch carefully what I am going to do.” Then I manipulated the counters in front of the child, spreading the top row out. I then asked the child these questions while running her finger along the corresponding row: “Are there as many blue ones as red ones,
or are there more here (pointing to top row) or here (pointing to bottom row)?” “How do you know?”

Counter Suggestion - If the child gave a correct conservation answer with a logical explanation, I said, “But another child said there were more in this row (indicating the longer row) because this row is longer. What do you think? Are you right or is the other child right.”

If the child gave an answer of non-conservation, I reminded the child of the initial equality by saying, “But remember how you put a red counter in front of each blue one before? Another child said there were just as many red ones as blue ones now because all I did was move them. Who do you think is right, you or the other child?” (Sinclair, et.al, 1983).

I recorded the answers in writing during the task. After completing the task, the child was told he or she was finished, I turned the camera off and the child returned to the classroom.

3.5.4. Word Problem Task

This task was adapted from Kamii’s (2000) description of the problem solving tasks that were given to the first grade students in her study. Kamii’s (2000) word problem list was significantly more detailed and authentic because her study was a year-long study and involved all aspects of the mathematics curriculum. However, the scope of this study was limited only to word problems, rather than the entire mathematics curriculum, so the word problems were pre-selected according to the topic of study instead of growing out of everyday occurrences as occurred in Kamii’s (2000) study. The procedure for administering the word problems will be described in the following paragraph.

All of the students who were in the classroom participated in a group time where they were introduced to a problem solving scenario that served as a springboard to a word problem. For this study, the problem solving springboards were books or stories. I read a book to introduce the problem situation. For example, during the reading of Ten Apples Up on Top, the Dr. Seuss classic, I stopped on the pages where three main characters had 10 apples on top of their heads. I pointed out the illustrations and they talked about each one having 10 apples on top. After completing the book, I read the word problem to the students. I confirmed that the students comprehended the problem by asking them questions about it. After determining that the students could retell the problem sentence, I instructed them to solve the problem individually with paper and pencil at their tables. The students were instructed to, “Think about
how you would solve this problem. Show your thinking on the paper. What is the answer? How do you know? Make it so that if you showed the principal (friend, teacher, etc.) your paper, she would know what you were thinking.” The students then sat at their tables in their usual place and solved the problem in writing. The teachers spent time at each table monitoring the students. The teachers could re-read the problem for the students, but could not offer any assistance or suggestions for solving the problem. When a student said he or she was finished, the teacher wrote the time on the paper and collected the paper. The student then moved on to work time, where he or she chose a work station.

I collected a group of papers and began the individual conferences with students in the workroom. When the student and I sat at the table, I turned on the video camera and began the conference. I was careful to ensure that the same students were not called first (Pellegrini, 2000).

3.5.4.1. Word problem task conference. During the conferences, I asked semi-scripted questions designed to help the researcher understand the solution strategies drawn by the children. In an attempt to be rigorous in gathering data (Creswell, 2007), I followed a suggested conference protocol was followed as consistently as possible. This protocol is attached as Appendix B. However, other researchers (Fennema, Carpenter et al., 1999) point out that conferencing with students about their mathematical thinking with a predetermined set of questions is often hard to do. They further state that it is common practice for teachers to adapt questions during a conference based on the child’s solution of the problem. “Tell me about your picture.” “What was the problem?” “What is your answer?” “What is this?” “How did you get that answer?” “Will the principal be able to know what you did?” and “Is there anything you want to add?” are examples of questions that were asked. If the child answered, “yes,” then the child was encouraged to modify his answer (Kato, et al. 2002).

While this conference was intended to highlight the student’s thinking, in several cases, it served to foster a child’s logico-mathematical thinking (Kamii, 2000) and challenged his ideas. If a student wanted to change his answers he was encouraged to do so. I offered assistance and suggestions while following the suggestions for interacting with students, listed in Appendix B. Conferences were video recorded. Each conference lasted between two and eight minutes.
3.5.4.2. Word problems. The following word problems were selected for the study.
The word problems were created by the researcher and the teacher to match the current theme of study. The word problems were created to include routine and non-routine problems, problems with simple operations, two problems with addends under 10, problems with large numbers, problems using two, and a problem using ten.

1. You have 5 grapes for snack. The teacher gives you 8 more. How many grapes will you have then? How will you know? Show your thinking.

2. There are 22 children in our class. Everyone has two eyes. How many eyes do we have in our class?

3. There were a lot of shapes at the circus. There were 4 circles, 4 triangles, 4 squares and 4 rectangles. How many shapes were there all together? How do you know? Show your thinking.

4. There are 22 children in our class. Yesterday each child wore two gloves. How many gloves were there all together? How do you know? Show your thinking.

5. The lion, tiger and dog each have 10 apples on top. How many apples are up on top all together? How do you know? Show your thinking.

3.6. Coding
The following section outlines the procedures for coding the numeral writing task, the representation task and the conservation task. The coding for the word problem task was created as a part of this study and will be described in Chapter 4.

3.6.1. Numeral Writing Task Coding
I observed each student as he wrote the numerals and marked the codes on an answer sheet as well as making notes on the student’s answer sheet. Reversals of numerals were accepted as correct as these are typical for kindergarten students.
3.6.2. Representation Task Coding

The responses to the representation task were categorized according to the scale created by Kato et al. (2002) which is a modification of the scale created by Sinclair et al. (1983). This scale is described below. The scale attached as Appendix C shows the scale as reported in Kato et al. (2002).

Level 1: Global pre-numerical representation of quantity. Sublevel 1a indicates absence of one-to-one correspondence even when the number is smaller than five (e.g., draws five circles to represent five plates). Sublevel 1b means there is an absence of one-to-one correspondence when the number of objects is greater than five (e.g., child draws four circles to represent four dishes, but draws 12 shapes to represent eight blocks).

Level 2: Representation with one-to-one correspondence. Response type 2a represents that the student understands one-to-one correspondence with pictures (e.g., child draws six pencils to represent six pencils). Response type 2b indicates there is an understanding of one-to-one correspondence with numerals (e.g., child writes “1234” to represent four dishes).

Level 3: Representation with one numeral indicating the total quantity. Response type 3a indicates the writing of a numeral only (e.g., child writes “6” for six pencils). Response type 3b represents the writing of a numeral and name of object or a complete sentence that includes both (e.g., Child writes, “There are 6 dishes.”).

Notation type 2: Representation of the object kind. These notations are attempts to represent the actual object, its nature or shape or the name often used to describe it. Younger children most often produce a drawing. Older children make alphabetic writing attempts.

Notation type 3: One-to-one correspondence with non-numerals. Each object is represented by one abstract graphic symbol. The type of symbols used and their combinations vary. Some students use only one symbol, some students use different symbols for different objects.

3.6.3. Conservation Task Coding

The following section explains the conservation task coding levels as described by Kato et al. (2002).
Level 1: Children cannot make a set that has the same number. They put counters in the same space that the blue counters take up.

Level 2: Children can make a set that has the same number by using one-to-one correspondence, but cannot conserve equality. When asked the conservation question, they reply, “There are more red ones because the red line is longer.”

Intermediate level: Conservation develops over time. Intermediate children hesitate and keep changing their minds or give the correct answer without justification.

Level 3: Children are conservers. They give correct answers to the questions and do not change their answers, even with counter suggestions. They give reasonable explanations for why they think the two rows have the same quantity. “There’s just as many blue ones as red ones because you did not add any or take any away.” This is the identity argument. Or they say, “We could put all the red ones back the way they were before and you’ll see it’s the same number” (Kamii, 2000, p. 7-8). This is the reversibility argument.

3.6.4. Word Problem Task Coding

Because the creation of a coding scheme was an outcome of the current study, the coding scale will be described in Chapter 4, Results.

3.7. Data Analysis

3.7.1. Representation Task Analysis

The student response recording sheets were analyzed by the researcher according to the criteria noted above. If any portion of the recording sheet data were unclear, the researcher referred to the video recording to check for further information. Member checking was conducted by the classroom teacher. That is, the classroom teacher reviewed the responses of five students (one-fourth of the participants). The researcher and teacher agreed on the scoring for the students. The teacher reviewed all of the results from the researcher’s tasks and concurred with those scores as well.

3.7.2. Conservation of Number Task Analysis

The student response recording sheets were analyzed by the researcher according to the criteria noted above. If any portion of the recording sheet data were unclear, the researcher had the opportunity to review the video recording to check for further information. However, this
was not necessary. Member checking was conducted by the classroom teacher. That is, the classroom teacher reviewed the responses of all of the students and concurred that those were reasonable levels for the students. In addition, the researcher retested five students (one-fourth of the participants) in the presence of the classroom teacher, who confirmed the levels recorded. The teacher reviewed all of the results from the researcher’s tasks and concurred with those scores as well.

3.7.3. Word Problem Task Analysis

The word problem task analysis was created as a part of the present study. The description of this task analysis is detailed in Chapter 4.
CHAPTER FOUR
DEVELOPMENT OF CODING SCALES

The purpose of this study was to compare the relationships between students’ levels of conservation of number, representation of number of objects, and problem solving. This topic originated with my own experiences in teaching kindergarten students word problems. I learned about Kamii’s (2002) work as an undergraduate and have been using her instructional strategies for teaching for almost 20 years. Because of that experience, I had implicit knowledge about the ways young children solve mathematical problems and the patterns that the students and the class as a whole demonstrate in their problem solving. However, descriptions of these patterns were not explicitly documented in the literature. The literature does, however, present such information as it relates to students’ conservation and representation of number, as well as the related methodology and scales for analyzing data. As previously mentioned, Kamii’s (2002) book provided the methodology for teaching and investigating problem solving with young children. However, neither her research nor a broader search of the literature revealed scales for determining and documenting students’ levels of problem solving. As such, it became a purpose of this study to make explicit what kindergarten students do when they are solving mathematical word problems by providing descriptions of their problem solving experiences and to begin the process of developing a scale for determining levels of student problem solving. The purpose of such a study is to document what kindergarten students do when solving mathematical problems and create a means for recognizing and their recording gains in problem solving.

The perspective offered by this study is that problem solving is a complex process that involves an interaction between the student, the problem, and the problem context. Much of the research findings and information about classroom instruction tends to favor one aspect over the other, for example, focusing on providing a certain set of problems in a particular order, focusing on the teacher’s knowledge of the patterns children exhibit when solving routine problems, or correctly matching student level with the problem. However, just as mathematical problem solving is not simply the application of a mathematical algorithm, problem solving by kindergarten students is not a process in which they follow a series of steps. In other words, the teaching of problem solving cannot be reduced to an algorithm any more than problem solving can. The research presented in Chapter 2 sought to demonstrate that approaching problem solving as an algorithm or skill set has not been effective in enabling young children to become
effective problem solvers. This study therefore investigated factors surrounding the student, the problem and the problem context as a starting point for understanding problem solving with kindergarten students.

Essentially, problem solving entails an interaction between the student, the problem, and the problem solving context. Problem solving depends on how a student thinks about number (constructive abstraction), represents number, and represents the problem situation. The problem includes the word problem selected (type, difficulty, authenticity), the problem solving task (with manipulatives, on paper), and the social interaction within the setting (partners, individual, collaborative). The teacher’s knowledge of how children learn and solve mathematical problems as well as her interaction with students each play a part in the teaching of word problems. This study aims at providing information about how the interaction of the student, the problem and the problem solving context influence and potentially affect kindergarten students’ problem solving. Given the complexity and individual nature of problem solving and that it does not progress in a linear fashion, it is often difficult to recognize and document a student’s progress toward becoming a proficient problem solver. This study is an attempt to address these issues by providing information that will help a teacher identify and document kindergarten students’ progress when solving mathematical word problems. This purpose is grounded in a Piagetian framework that encourages teachers to provide opportunities for students to construct knowledge, and not push them to a higher level.

4.1. Data Collection

4.1.1. Pre- and Post-Conference Designation

The students’ written or drawn problem solving solutions were analyzed and assigned a pre-conference or post-conference code. This section describes the criteria for coding data as pre- or post-conference. When a student indicated he or she had finished solving the problem, he or she gave the paper to the teacher or to me. I copied the papers to preserve the students’ independent solutions. This set of papers became the pre-conference data. I then began the conferences with the students. I would turn on the video camera prior to begin the conference with a student. During the conference the student’s original paper and several pencils were made available. I began the conference by prompting the student to explain his or her drawing by saying, “Tell me about your paper.” The student would then be given an opportunity to explain the solution he or drew on the pre-conference papers. I wrote down any information
necessary to document the student’s explanation of his or her solution. This was done so as to include the student voices in the study, rather than simply relying on my interpretation of the drawing alone.

Many times, the student’s explanations changed how I would have coded a pre-conference answer. For example, the conference with AX resulted in information that led to me to change the coding of the pre-conference solution. AX’s solution is shown in Figure 1. In this example, AX wrote, “84,” as the answer for Problem 2 (There are 22 children in our class. Everyone has two eyes. How many eyes do we have in our class?). Without his explanation in the conference “84” would have been coded as an incorrect answer. However, when AX responded to my question, “How did you get your answer?” he stated “by counting to 48.” I thus realized that he had simply made an age-appropriate inversion. Even though this information was gathered during the conference, it was simply explanatory information and therefore considered pre-conference information.

The same conference with AX also revealed a simple counting error. This particular student counted the eyes as he was showing me how he arrived at his answer of 48. He accurately counted up to 39 with one-to-one correspondence. Then, he hesitated and looked at me saying “40” with a questioning tone as if he was not sure. He then skip counted by twos as he touched one face for each number that he said. “40…42…44…46…48.” This meant that his final answer would be incorrect, despite the fact that he was counting using one-to-one correspondence. I knew the class had been practicing counting by twos in math time, so I had him count again and counted aloud with him when he neared 40, “38…39…40…41…” then I stopped counting aloud. This time he arrived at 44 as his answer. He then wrote, “44” as his answer. Thus, I recorded this answer as a correct answer pre-conference because the error was a simple clerical error, not an error in thinking. There were no further changes to his paper as a result of the conference. This decision was made because of my familiarity with kindergarten students and with this individual student. He frequently had problems with the language of mathematics, but not in understanding number. Lapsing into a common classroom routine, like counting by twos, is a common occurrence in kindergarten, especially when counting higher numbers in the 40s.
In summary, if students verbally corrected a clerical mistake (such as a numeral inversion or reversal), or demonstrated that they miscounted prior to the conference, these were not considered changes that resulted from the conference. This was information that was necessary for me to correctly interpret the written solution provided prior to the conference. As such clarifying explanations that resulted in changes were coded as pre-conference answers.

JR’s answers for problem three offer an example of both, Figure 2 and Figure 3. When explaining his answer, I re-stated the question, “There were a lot of shapes at the circus. There were 4 circles, 4 triangles, 4 squares and 4 rectangles. How many shapes were there all together?” He said, “11.” I then pointed to where he had written 11 at the bottom of his paper saying, “I see 11 right here.” I then asked him to show me how he got 11. He touched the shapes he had drawn and started counting, “1…2…” Then started over, “1…2…3…” and started again haltingly, “1…2…3…4…5…6…” Then he looked at me. I said, “7,” not noticing that he had skipped a shape when he looked up at me. He counted 11 again. I suggested that he do it again, and that I would write what he said. This time he touched each shape and counted 1 to 12 without hesitating. He then wrote “12” at the bottom of the paper as his answer. It was my decision that his writing 11 was a miscounting not because he did not understand the number of objects he had drawn, but rather because he got distracted in his counting. Because the change
he made was a clerical change, not representing a change in his thinking, I recorded this as an explanation, not a post-conference change.

Figure 2
JR’s Solution Pre-Conference: 4 circles, 4 triangles, 4 squares and 4 rectangles

The changes JR made when he realized that he left out the set of rectangles was recorded as a post-conference change. When we re-read the problem and I asked him to show me each set of shapes, he said he did not have any rectangles and added those to his drawing. Thus, a change or addition to a paper was considered a post-conference change if a student changed his thinking about his solution strategy as a result of the conference.

Figure 3
JR’s Solution Post-Conference: 4 circles, 4 triangles, 4 squares & 4 triangles
4.2. Procedure for Data Analyses

There were no procedures for the analysis of students’ problem solving reported in the literature on this topic. Consequently, specific procedures were developed for the current study and these are described in the following section. Also included are the rationale and research base for the procedures that were adopted. As a reminder, the word problems were:

Problem 1: You have 5 grapes for snack. The teacher gives you 8 more. How many grapes will you have then? How will you know? Show your thinking. (Standard word problem - Join result unknown taken from CGI).

Problem 2: There are 22 children in our class. Everyone has two eyes. How many eyes do we have in our class? (multiplication/repeated addition, dealing with 2s).

Problem 3: There were a lot of shapes at the circus. There were 4 circles, 4 triangles, 4 squares and 4 rectangles. How many shapes were there all together? How do you know? Show your thinking. (standard word problem, adding four addends).

Problem 4: There are 22 children in our class. Yesterday each child wore two gloves. How many gloves were there all together? How do you know? Show your thinking. (multiplication/repeated addition, dealing with 2s).

Problem 5: The lion, tiger and dog each have 10 apples on top. How many apples are up on top all together? How do you know? Show your thinking. (multiplication/repeated addition, dealing with 10s)

4.2.1. Initial Coding

The pre- and post-conference papers for each student were stapled together and sorted by problem. All of the papers for a problem were laid out on a long conference table, pre-conference paper on top. I analyzed each of the papers, looking for ways to group the answer sheets. I grouped the papers according to the potential coding categories (see Appendix F), compiled from the pilot study. Some of the characteristics used for grouping were the presence of words, numerals, tally marks, and drawings. The drawings presented in two categories: drawings that related to the problem and drawings that did not. There were three levels of organization noted. It was also noted that some students used a tens frame, an organizational strategy that had been didactically taught in the classroom. The absence of any drawing was also noted in a few cases. Drawings were also sorted by use of symbol, sign, or both. The
different groups that emerged were marked with different colored sticky notes. In most cases there was no overlap in the characteristics for each group. However, in a few cases, a paper was set aside and marked for further consideration. All analyses were done for both sets of papers for each problem.

4.2.2. Second Round of Coding

Before the second analysis session, I reviewed the literature for information that might help with coding. The categories of correct answer and valid strategy utilized by Carpenter et al. (1993) were adapted for use in the analyses.

4.2.2.1. Correct or incorrect answer. The next phase of data analysis was to code for correct or incorrect answer. I marked the codes on the answer sheet and recorded the score on a spreadsheet. To be coded as correct, the correct answer had to be correct and written in numeral form on the pre-conference paper. Reversals (numerals that were turned backwards) and inversions (numerals in the wrong order, such as 31 for 13 with the student identifying it as 13) were accepted if the student explained them prior to any modifications in representation.

The example of AX given in Figure 1, Section 4.2.1. demonstrates a correct answer because AX wrote a numeral for the answer and simply made a common counting error which affected the numerical answer he wrote. If the answer sheet did not include the answer in written form, it was coded as an incorrect answer.

4.3.2.2. Validity of the answer strategy. Building upon the coding designations documented in the study by Carpenter et al. (1993), I began to analyze the data for answer strategy validity. However, as I grouped the answer sheets, three levels of validity emerged from the data, rather than the two utilized by Carpenter et al. (1993). Coding for no strategy or invalid strategy was clear. However, almost one-third of the pre-conference responses (28 of 90) included a valid strategy that the students applied to the problem situation, but did not follow through to a correct answer. In some cases the student stopped representing the problem situation before representing the complete problem situation. For example, a student drew the faces and mittens for the children they remembered. Even though this indicated they did not pay attention to the significance of number in the problem, it did indicate a developing understanding of an effective problem solving strategy. In other cases the students carried the representation through to a correct representation of an answer, but did not answer the question. Again, this was important because it indicated these students had a capacity for problem solving, but did not
necessarily understand that the problem situation led to an answer, even though they documented
the entire problem situation. In an effort to give the fullest view of the range of thinking
represented in the students’ answers, I included one intermediate level which was characterized
by the use of a reasonable strategy that was applied to the problem situation, but did not include
a correct answer.

Three levels of validity of problem solving strategy emerged. The highest level, level 3,
include a strategy that, when applied, resulted in calculating a correct answer that was written
out (in number or word form). Level 2 included a reasonable strategy that was only partially
carried out to a correct answer. There was no answer written. At level 1, there was no
appearance of a strategy or the strategy would not have resulted in a correct answer if applied to
the problem situation. There was no answer written. Numerals that were written were either
numbers from the problem or numerals with no relationship to the problem.

As I applied the criteria for the validity of the problem solving strategy to the word
problem sheets and began to code the validity of the strategy, a relationship between the
representation of the problem situation and validity of the strategy became apparent. The
representations used in the valid strategies were more organized and included more concise
drawings than the level 2 strategies. The level 2 strategies included some representations, but
had qualitative detail that was not essential to solving the problem. The drawings in level 1
strategies were disorganized and often just looked like a “bunch of objects” as described by Kato
et al. (2002).

4.2.3. Hierarchical Inclusion

At this point in analyzing the data, I noticed a pattern in the answer sheets for problem 1
(You have 5 grapes for snack. The teacher gives you 8 more. How many grapes will you have
then?). Students often drew five grapes for the first number in the problem, then three more
grapes to make the second number, eight, as seen in Figure 4.
Alternatively, students would draw five grapes and then draw three more before adding them together (Figure 5). The literature was searched for an answer for why this may occur, relative to young children’s problem solving.

Kamii (2002) stated that this way of representing the problem indicated a student’s level of hierarchical inclusion, and thus provided the context for the next round of coding. The recording sheet was marked “yes,” for hierarchical inclusion if the student represented the two numbers in the problem as discontinuous numbers; they drew five objects and then drew eight more. Sometimes the students included a space between the numbers in the drawing, but not always. Other times the student represented their understanding that the two numbers were discontinuous by counting five, then counting eight as they were explaining this to me during the conference. It was notable that not all of the students who demonstrated hierarchical inclusion
got the correct answer or used a valid strategy. “No” was indicated for students who drew five circles, then added three more or who did not draw the numbers indicated in the problem situation.

4. 2.3.4. Levels of representation. As I reviewed the problems for this information, I was again confronted with the wide variation in the types of representation across the problems and within the entire class. I began to search for relationships between the level of thinking and the level of representation while solving problems. Table 2 presents examples of eight types of representation found. These are listed from highest level of sophistication to lowest.

Table 2
Range of Representation (highest – lowest)

<table>
<thead>
<tr>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example 8 ZY used a number sentence, and a tens frame. (Didactic strategy that the students were taught.) Includes a written answer.</td>
<td><img src="image.png" alt="Image of a student's work showing a number sentence and a tens frame with a written answer." /></td>
</tr>
<tr>
<td>Example 7 AV attempted to use the tens frame that she was taught in the class. She abandoned it after filling two and drew her own representation. Wrote 36 to represent the answer she calculated.</td>
<td><img src="image.png" alt="Image of a student's work showing a tens frame with numbers and an answer." /></td>
</tr>
<tr>
<td>Description</td>
<td>Example</td>
</tr>
<tr>
<td>-------------</td>
<td>---------</td>
</tr>
<tr>
<td>Example 6. CH drew 22 circles to represent each student, then two tally marks to represent the two mittens. Student wrote a correct answer in numeral form.</td>
<td><img src="image1" alt="Example 6" /></td>
</tr>
<tr>
<td>Example 5 CH drew 22 faces, with two eyes with limited qualitative details. The student was attending to the number not only the object.</td>
<td><img src="image2" alt="Example 5" /></td>
</tr>
<tr>
<td>Example 4. NA drew pairs of eyes eyelashes and pupils. This indicates that he attended to the number of objects in the problem, but not the number of sets of objects. He included a few qualitative details that were not relevant to the problem situation.</td>
<td><img src="image3" alt="Example 4" /></td>
</tr>
<tr>
<td>Example 3. AM drew a few children from the class and their eyes. Also included many qualitative details, hair, fingers, legs. Student had rudimentary understanding of the problem situation, applied a strategy (drawing each person and his two eyes), but did not represent entire problem solving situation.</td>
<td><img src="image4" alt="Example 3" /></td>
</tr>
</tbody>
</table>
Table 2 – continued

<table>
<thead>
<tr>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example 2. Student drew a pair of eyes, labeled it with a 2. Also drew 14 circles that he labeled “22.” JR read the problem and realized that the numerals were a part of the problem, but, beyond drawing one pair of eyes with a 2 by them, did not demonstrate an understanding of how to use these numbers to solve the problem.</td>
<td><img src="image" alt="Example Image" /></td>
</tr>
<tr>
<td>Example 1. MA drew circles to represent eyes. Only drew 18. He represented eyes, but did not pay attention to number or operation.</td>
<td><img src="image" alt="Example Image" /></td>
</tr>
</tbody>
</table>

I realized that the isolated task of measuring students’ representation of number of objects might not be sufficient to answer the research questions focused on representation within problem solving. At this point, I reviewed the literature and conclusions from the pilot study for information to help me identify and categorize levels of representation within the problem solving situations.

Although Kamii (2002) presents the progress children made over the year in her study, her conclusions also apply to the present research. Kamii presented information about students’ logico-mathematization and representation. Regarding logico-mathematization, she stated that there is a range in the logic used to solve problems, but that all students become able to solve problems throughout the year. The difference is not their logic, but their numerical thinking. Some students begin to reason by fives and tens, while others are still reasoning by ones. This indicated to me that some students would be able to solve problems and some would not and that there would be varying levels of numerical thinking.
Regarding representation, Kamii (2002) reminded the readers that students represent problem solving situations by evoking a mental image of the problem situation. She noted that many children progress from representing objects in the problem to representing number because of constructive abstraction. Students start off including details that are irrelevant to the problem situation - Some students focus on the entire object while others focus on only the parts that are relevant to the problem. Students then progress to using circles and tally marks, and then move on to using numbers. The appearance of numerals can be to indicate the action of counting or as a sign, then in equations. She indicates that children often go back and forth between tally marks and pictures and numerals. She states that students seem to write numerals for numbers that are easy for them. She sums up by saying that, “children seem able to choose for themselves the tools that work best for them…they give up tally marks when they decide that numerals work better for them” (Kamii, 2002, p. 142).

Kamii does not specifically mention the organization of the drawings, but this is something noted when data from the pilot study was analyzed. The students’ drawings varied in organization. The answer sheets that had a correct answer were more organized than the papers that had incorrect answers.

At this point I completed an initial sorting of the papers by approximate level of representation. This process revealed a wide range of representation types represented within this class of kindergarten students.

I reviewed the literature to search for a rationale for arranging these examples into a more manageable number of codes that would also accurately represent the range of problem solving. Kato et al. (2002) provided some clarification. In that study, they identified three levels. The levels and their various sub-levels and response types are described below and presented in Appendix C. They also identified sub-levels within the levels as well as different response types. In other words, within each level there may be sub-levels, variations in the level of thinking within a level, but insufficient to necessitate the creation of another level. Within each level there could also be different response types, different ways to draw the same level of thinking. I then analyzed the nine initial levels of problem solving that I had marked with the sticky notes to determine whether there were different response types and/or sub-levels.

The range of representation was consolidated to create a scale for leveling the representation types. The following scale was created to describe the levels of the
representation noted in this study for this set of problems. Examples 5, 6, 7, and 8 were consolidated to be representation types of level 3. Examples 2, 3, and 4 were consolidated into Level 2. Example 1 became level 1. These are presented in Table 3.
Table 3  
Levels of Representation Scale

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Levels of Representation (1 lowest - 3 highest)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Relationship to problem situation</td>
<td>No relationship to problem situation. No relationship to number of objects.</td>
</tr>
<tr>
<td>Use of symbols and signs to represent number and operation of word problem.</td>
<td>No numerals or operations represented.</td>
</tr>
<tr>
<td>Organization</td>
<td>No organization. &quot;Bunch of objects.&quot;</td>
</tr>
<tr>
<td>Qualitative detail</td>
<td>Very few details or an overabundance of details that are unrelated to the problem situation.</td>
</tr>
<tr>
<td>Examples</td>
<td><img src="image1.png" alt="Example Image 1" /></td>
</tr>
</tbody>
</table>
4.2.4. Symbol and Sign

In the review of the literature, Kamii’s (2000) discussion of Piaget’s interpretation of the use of a symbol or sign was revealed. Piaget defines symbols as drawings that have a relationship to the object. Signs can be invented by a child and do not require teaching by social convention. Examples of these are pictures or tally marks. Signs are abstract and cannot be invented by the child. Signs have to be taught by social convention. Examples are numerals or number sentences. It is possible for a child to learn to write a sign and not be using it to represent a number or mathematical operation. The numeral writing task in this study demonstrates this. Students can write, “5,” in response to the command to, “write five” without making the connection to the number five (five objects). Any conventional sign that appeared was coded as a sign. However, further studies exploring this topic are implicated.

The word problems were coded for use of symbol and sign at this point in the data analysis. The results of this coding were recorded on the excel sheet and not marked on the problem solving sheet.

4.2.5. Third Round of Coding: Problem Solving Levels

At this point in the data analysis, I had reviewed all 180 word problem answer sheets (90 pre-conference and 90 post-conference) several times. I had implicit knowledge of the range of problem solving strategies that could be found there, my hypothesis was that the levels for representation would be related to the levels of problem solving. I recorded the representation levels on the spreadsheet not on the papers so that I could code them blindly. I took each set of problem solution papers and grouped them according to trends or patterns noted in the problem solutions. I first grouped them according to correct and incorrect answer, then divided those groups into valid strategy and invalid strategy. Within those groups I placed the papers in order using the representation criteria – most organized to the least – directly representing the problem situation to partially representing the problem situation, to not representing the problem situation at all. In other words, the final coding for level of problem solving strategy was a combination of all previous coding.

This blind grouping resulted in three levels with sub levels. The study by Kato et al. (2002) once again provided the rationale for analyzing for sub-levels and representation types. However, in this round of analysis, variations in representation type were not present. The three
levels and sub-levels of problem solving are shown in Table 4, Problem Solving Levels and Sub-Levels.

### Table 4
Problem Solving Levels and Sub-Levels

<table>
<thead>
<tr>
<th>Level</th>
<th>Sub-Levels (a – highest to c – lowest)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td><img src="image1.png" alt="Image" /></td>
</tr>
<tr>
<td>2</td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td>1</td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
</tbody>
</table>

When the levels of representation were compared to the levels of problem solving, the primary levels were the same. Although the sub-levels and potentially the presence of variations in representation type were found, reporting the sub-levels and carrying out the analysis on the sub-levels was beyond the scope of this exploratory study. As a result, the problem solving scale only included three levels and no sub-levels or representation types. The coding matrix
that documents the levels follows as Table 5 Levels of Problem Solving Scale. The following
descriptive scale is offered as a starting point for the creation point of a scale that could be
applied across a range of problems. The descriptions offered apply only to the current study, but
future studies could confirm their usefulness across various settings and using another set of
problems.

Table 5
Levels of Problem Solving Scale

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Levels of Problem Solving Scale (1 lowest - 3 highest)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relationship to problem situation</td>
<td>No relationship to problem situation. No relationship to number of objects.</td>
</tr>
<tr>
<td>Use of symbols and signs to represent number and operation of word problem.</td>
<td>No numerals or operations represented. Numerals written were copied from the problem or random. Not used to solve problem.</td>
</tr>
</tbody>
</table>
Table 5 - continued

<table>
<thead>
<tr>
<th>Criteria</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qualitative detail</td>
<td>Very few details or an overabundance of details that are unrelated to the problem situation.</td>
<td>Limited number, focus on counted part of object. Number of object rather than the object.</td>
<td>Few, if any qualitative details. Use conventional signs or symbols, tally marks</td>
</tr>
<tr>
<td>Valid Strategy</td>
<td>strategy could not be determined</td>
<td>valid strategy partially carried out</td>
<td>strategy completely carried out</td>
</tr>
<tr>
<td>Correct Answer</td>
<td>No.</td>
<td>No. Answer may have been drawn but no numeral.</td>
<td>Yes. Written out.</td>
</tr>
</tbody>
</table>

The purpose of this exploratory study was to explore the feasibility of creating a taxonomy for rating problem solving strategies for kindergarten students. As an exploratory study, the scale presented applies only to the current study. Table 6 illustrates how the scale was applied to the set of problems given in this study.
Table 6
Examples of Problem Solving Level by Problem

<table>
<thead>
<tr>
<th></th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem 1</strong></td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
<td><img src="image3" alt="Image" /></td>
</tr>
<tr>
<td><strong>Problem 2</strong></td>
<td><img src="image4" alt="Image" /></td>
<td><img src="image5" alt="Image" /></td>
<td><img src="image6" alt="Image" /></td>
</tr>
<tr>
<td>Level 1</td>
<td>Level 2</td>
<td>Level 3</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td></td>
</tr>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td></td>
</tr>
<tr>
<td><strong>Problem 3</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
<tr>
<td><strong>Problem 4</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6 - continued
Table 6 - continued

<table>
<thead>
<tr>
<th>Problem 5</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image 1" /></td>
</tr>
</tbody>
</table>

4.3. Summary

Because no scale for rating problem solving existed in the literature, a scale was created as a part of this study. This chapter explained the procedures used for creating the scale for problem solving levels. This chapter explained the rationale for the categories that were coded and provided descriptions of each level as well as examples of each category that were found as a part of this study.
CHAPTER FIVE

RESULTS

Imagine more than 20 kindergarteners sitting cross-legged on a multicolored rug, rapt with attention, energy crackling in the room. Their eyes are sparkling as they search for shapes in famous works of art. Many raise their hands and hope for their chance to show their shape to the class. Imagine students reading along and making circles and spheres with their hands as these shape words appear. Turn the page and count how many circles are used to make the wheels of the train. Everyone is counting. Imagine children clapping their hands as the book is closed and the word problem sheet is held up. They eagerly ask, “What is the question today?” and hold out their hands to get their paper. The children sit down and get to work solving the word problem. They exchange a few quiet comments with other students or a teacher, but quickly settle into writing their own solution, Figure 6.

Figure 6
BC’s Word Problem Solution Sheet

As the children finish a few minutes later, they give their paper to one of the teachers and proceed to choosing a work station. (You might know this time as free-choice center time, where they get to choose their own activity from blocks, reading, art, etc.) They each get engaged in an activity of their own choosing. They usually don’t want to be interrupted during
this time of day, but when Ms. Nickey calls one of them to come to the work room, he jumps up and hurries enthusiastically with her. He will spend a few minutes telling her all about his solution to the word problem. There are no lessons and no wrong answers, but just questions that will help him think. When his turn is over, he will call the next student and then return to his activity. Later in the day, when Ms. Nickey returns, the students call out her name and ask for hugs and whether it is time to do another problem. The group returns to the carpet where they review the word problem solutions. Even though he and the students are tired, they are excited to see other ways to solve the problem and whether other students got the answer in the same way. They only want to leave the group time because the next activity is lunch!

That was the scene for the data collection for the current study. The data collection times fit into the natural flow to the day seamlessly. The students were excited to participate in solving the word problems and tasks. There was only one case where the student did not attempt an answer and that was a day when the student was having a very bad morning! However, this student left the conference with a detailed drawing – and correct – answer! The conferences, while quick, most lasting two to five minutes, were dynamic times of the sharing of ideas between a student and a researcher. Children were asked to share about their work and clarify their drawings. They were given support when needed, such as the next number in the series or a list of the students in the classroom. However, this was a time of student thinking being highlighted, student voices being heard.

It is the position of this study that problem solving in kindergarten is a complex process composed of interrelated strands relating to the student, the problem, and the problem solving context, rather than a lockstep process that can be taught step-by-step. This study explores the relationships between these related strands. The student’s level of constructive abstraction, level of representation and problem solving strategy were analyzed. Different types of problems of varying levels of difficulty were given to the students to solve independently. These problems were presented in authentic, engaging classroom contexts. The students drew their solutions using pencil and paper independently. Then they were provided an opportunity to explain their solutions in a conference situation with the teacher.

As expected, the results of the study show some typical relationships between any two of these factors. The kindergarten students, for example, represent at or below their level of abstraction. In other words these students represent numbers that they understand, but they do
not represent at a higher level than their understanding. Also, those students who get the correct answer demonstrate hierarchical understanding of the numbers in that problem. However, when all of the aspects were compared together, there were no clear patterns that could be traced. In this case there was not a set of clear-cut steps that composed a formula for problem solving. Based on the findings, neither a student’s level of conservation nor representation can be used to predict the child's level of problem solving. Furthermore, it seems that the student’s level of conservation cannot be used to determine whether a he or she will calculate a correct answer to the problem, nor whether he or she will formulate a valid solution strategy to the problem. Similarly, a student’s ability to formulate a valid strategy does not seem to predict whether he or she will apply the strategy to the problem situation, and thereby arrive at the correct answer. However, a student who calculates a correct answer must formulate a valid solution strategy, and represent accurately. The following section presents the data that was collected to support these broad conclusions.

5.1. General Data

Documenting what the kindergarten students did as they solved word problems in their natural setting of the classroom was an integral part of the design of this study. The 19 kindergarten student participants were from one kindergarten classroom. This classroom was a typical kindergarten class. The class roster was created prior the commencement of the current study, and so it reflects the enrollment policies of CLC. The students participating in the study ranged from five- to six-years of age, came from various social, economic, cultural and ethnic backgrounds. There were nine male and ten female participants. There were students who were classified as gifted, regular education and special needs: each one with individual strengths and weaknesses. As many students as possible were included in the data collection – even those students whose special needs kept them from participating fully on that task – because this represents the naturalistic situation within a typical kindergarten classroom.

All 19 students participated in the representation task, the conservation task, and the numeral writing task. If a student was absent on the day the task was given, then task was administered when the child returned to school. All students who were present in the classroom at the time of the word problem tasks and conferences participated in them. Six students were absent during the word problem tasks, one each for problems two and three. Four students were absent for problem four. The word problems and conferences were not made up. This
produced a total of 90 codable word problem answer sheets, each one a lively record of that student’s voice and thinking.

5.2. Research Question 1

To answer the first research question, “How do kindergarten students represent their mathematical thinking when solving word problems?” students were given different types of word problems to solve using pencil and paper. These papers were copied prior to the conferences to record the students’ independent responses. The researcher and student spent time together talking for a few minutes about the student’s written solution. These conferences were more like conversations than interviews, and were designed to ensure that the student’s thinking was accurately reflected on the problem solving sheet, and that the researcher could understand what was written on the paper. However, in many instances the student added to or changed his paper during the conference. The researcher was careful not to give explicit directions to the students, but to ask questions to clarify, make suggestions or ask the student to explain. The conference protocol is attached as Appendix B.

5.2.1. Students Represented Their Thinking

The first finding confirmed that the students represented their thinking using pencil and paper both before and during the conferences. Even though the students sometimes changed their answer during the conference, this was a result of their thinking being challenged rather than because they were told that their answer was wrong. The data suggested that the students approached the conferences as conversations rather than a place to receive a didactic lesson. The students sometimes accepted and other times rejected the researcher’s suggestions made during the conferences. For example, in one conference, I thought it might have been helpful for RH to use the list of students to ensure he had included all 22 students in his drawing. I told him, “I have a class list here, would you like to see it?” He answered, “No, I’m just going to do the boys.” I accepted this answer because the written (or drawn) representations were to be representations of the student’s thinking. Another example of the student’s responses to the suggestions being accepted was JN’s answer to question 3 (There were a lot of shapes at the circus, 4 triangles, 4 squares and 4 rectangles, how many shapes were there all together?), seen in figure 7. JN represented the problem with four circles, four triangles, four squares and four rectangles. When I asked him what the answer was he said simply, “16.” He wrote, “16.” I
asked, “16 what?” He replied, “Shapes.” I asked, “Can you write that here?” He proceeded to write shapes. He drew a circle, a triangle and a square.

Figure 7
JN’s “shapes”

5.2.2. Types of Representation

The kindergarten participants in this study represented their thinking while solving word problems in one of three ways: by drawing symbols (pictures that had a relationship to the object), signs (abstract signs, numerals or number sentences), or both symbols and signs as seen in Figure 3.

Figure 8
Use of Symbol and Sign
All drawings that were related to the problem were coded as a symbol, no matter how peripheral to the problem. For example, when a student drew only two animals for problem 5, it was still coded as a symbol even though the student did not include any apples up on top. In the pilot study, unrelated drawings were found, a picture of “my brother and me” in a problem about dogs in their dog houses. In this study no totally unrelated drawings were found.

Any use of a conventional numeral or number sentence was coded as a sign. It was beyond the scope of the present study to examine whether the use of sign was a rote use of a conventional sign or an indication of more mature logical thinking as differentiated by Piaget so this determination was not made in the coding. No distinction was made in the coding whether the conventional sign represented a quantity in the problem or was a random numeral. The following example, seen in figure 9, was notable because it was the only use of a conventional number sentence.

![Figure 9](image.png)

Use of Symbol, Sign, and Number Sentence

In 37% of the cases students used only a symbol. In 58 percent of the total cases students used both a symbol and a sign. There were only two instances (2%) where the student did not draw either a symbol or a sign. In one instance the student wrote a letter string, see figure 10. She explained that she was re-writing the problem in invented spelling. In the second instance the student came into the conference with a blank paper. He explained that he
did not draw anything because he was having a hard morning and could not decide where to start on the word problem on that particular day.

Figure 10
Letter Stringing

5.2.3. Pre- and Post-Conference

There were slight changes in the frequency of representation type pre- and post-conferences. To illustrate this, frequency tables pre- and post-conference are presented below. Table 7 represents the frequency of representation types by problem before the conferences. Table 8 represents frequency post-conference. Both are broken down by problem number.

Table 7 shows that 94% of the time students drew a symbol to represent the problem pre-conference. In 58% of the cases both a symbol and a sign were used. In 37% of the pre-conference cases the student drew only a symbol and no sign to represent the problem. It was only in three percent of the cases that only a sign, and no symbol, was used to represent the problem. Table 8 shows the frequency of representation type post-conference, listed by problem. There are no major shifts in frequency documented in this table. Three students account for most of the post-conference changes. The student who wrote the letter string, AM, did not change her answer post-conference. However, KL, who came into the conference with a blank paper did change his paper during the conference. In fact, he left the conference with a full representation of the problem and a correct answer on his paper.

One student, CH, accounted for all three of the cases where a student only used a sign to represent his thinking before the conferences. However, in all cases he added a symbol or symbols to his paper during the conference. This meant that post-conference in 89 of 90 cases, students used a mathematical symbol to represent their thinking when solving word problems.
Table 7
Frequency of Representation Type by Problem Pre-Conference

<table>
<thead>
<tr>
<th>Problem Number</th>
<th>Other</th>
<th>Symbol</th>
<th>Only Symbol</th>
<th>Sign</th>
<th>Only Sign</th>
<th>Symbol and Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1 (N=19)</td>
<td>1 (5%)</td>
<td>17 (89%)</td>
<td>10 (53%)</td>
<td>8 (42%)</td>
<td>1 (5%)</td>
<td>7 (37%)</td>
</tr>
<tr>
<td>Problem 2 (N=18)</td>
<td>0</td>
<td>17 (100%)</td>
<td>5 (28%)</td>
<td>13 (72%)</td>
<td>1 (5%)</td>
<td>12 (67%)</td>
</tr>
<tr>
<td>Problem 3 (N=18)</td>
<td>0</td>
<td>18 (100%)</td>
<td>8 (44%)</td>
<td>10 (56%)</td>
<td>0</td>
<td>10 (61%)</td>
</tr>
<tr>
<td>Problem 4 (N=15)</td>
<td>1 (5%)</td>
<td>14 (93%)</td>
<td>3 (2%)</td>
<td>11 (73%)</td>
<td>0</td>
<td>10 (67%)</td>
</tr>
<tr>
<td>Problem 5 (N=19)</td>
<td>0</td>
<td>18 (95%)</td>
<td>7 (37%)</td>
<td>12 (63%)</td>
<td>1 (5%)</td>
<td>11 (58%)</td>
</tr>
<tr>
<td>Totals (N=90)</td>
<td>2 (2%)</td>
<td>85 (94%)</td>
<td>33 (37%)</td>
<td>55 (61%)</td>
<td>3 (3%)</td>
<td>52 (58%)</td>
</tr>
</tbody>
</table>

Table 8
Frequency of Representation Type by Problem After Conference

<table>
<thead>
<tr>
<th>Problem Number</th>
<th>Other</th>
<th>Symbol</th>
<th>Only Symbol</th>
<th>Sign</th>
<th>Only Sign</th>
<th>Symbol and Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1 (N=19)</td>
<td>1 (5%)</td>
<td>18 (95%)</td>
<td>8 (42%)</td>
<td>10 (53%)</td>
<td>0</td>
<td>10 (53%)</td>
</tr>
<tr>
<td>Problem 2 (N=19)</td>
<td>0</td>
<td>19 (100%)</td>
<td>5 (26%)</td>
<td>14 (74%)</td>
<td>0</td>
<td>14 (74%)</td>
</tr>
<tr>
<td>Problem 3 (N=18)</td>
<td>0</td>
<td>18 (100%)</td>
<td>3 (16%)</td>
<td>15 (83%)</td>
<td>0</td>
<td>15 (83%)</td>
</tr>
<tr>
<td>Problem 4 (N=15)</td>
<td>0</td>
<td>15 (100%)</td>
<td>1 (2%)</td>
<td>14 (93%)</td>
<td>0</td>
<td>14 (93%)</td>
</tr>
<tr>
<td>Problem 5 (N=19)</td>
<td>0</td>
<td>19 (100%)</td>
<td>5 (37%)</td>
<td>14 (74%)</td>
<td>0</td>
<td>14 (74%)</td>
</tr>
<tr>
<td>Totals (N=90)</td>
<td>1 (1%)</td>
<td>89 (99%)</td>
<td>22 (24%)</td>
<td>67 (74%)</td>
<td>0</td>
<td>67 (74%)</td>
</tr>
</tbody>
</table>

5.2.4. Other Relationships

There were no notable differences in symbol and sign use by conserver level or representation level. In other words, conservers were no more or less likely to use symbols or
signs than non-conservers. There were no patterns when the data were sorted by representation level. Similarly, there were no differences in symbol and sign use when sorted by problem type. Students were neither more or less likely to use a symbol or sign for any particular type of problem.

5.3. Research Question 2

The pre- and post-conference answer sheets were analyzed to determine the answer to the second research question, “What levels of representation do kindergarten students demonstrate when solving word problems?” The sheets were coded according to the levels of representation scale created as a part of this story and included as Table 3 in section 4.2.3.4.

5.3.1. Range of Representation Levels

The data revealed that students demonstrated a wide range of representation levels during problem solving. Table 2 presented in Section 4.2.3.4. gives examples of eight different representation types. These were consolidated into a scale denoting three levels of representation. The scale is presented in section 4.2.3.4.. The levels of representation were marked by at least four factors the relationship to the problem situation, the use of symbols and signs to represent number and operation, the organization of the drawing, and the amount of qualitative detail present in the drawing.

There were several notable facts about the representation levels. The first of which is the scales that were created as a part of this study ended up being the same for representation and problem solving. In other words, when the papers were coded for representation during problem solving and for problem solving level, the codes were the same. As such, further information discussing representation and problem solving levels will be presented in the section describing Research Question 4. This is because in both cases the information is the same. This information is presented in section 5.5.

5.3.2. Hierarchical Inclusion

While creating this taxonomy of levels of representation, one particular feature of problem one was noted (You have 5 grapes for snack. The teacher gives you 8 more. How many grapes will you have then?). Several students drew five circles for the five grapes noted in the problem, then they drew three more to make eight for the second number of grapes in the problem. Kamii (2002) identifies this as lack of hierarchical inclusion. She further explains that the students who represent this way do not think of the two numbers as being discontinuous.
sets which indicates a lack of hierarchical inclusion. Students who demonstrated hierarchical inclusion drew two separate quantities for $5 + 8$. Sometimes these separate quantities were separated by a physical space. Other times the researcher determined that the student intended two separate quantities because the student counted that way during the conference. This counted as pre-conference information.

Even though there were four instances ($n=19$) where the level of representation/problem solving improved, the students’ code for hierarchical inclusion did not change from pre-conference to post-conference. In other words, the students’ level of hierarchical inclusion was stable. The only changes in representation/problem solving were level two to level three. No changes were noted in students who demonstrated representation/problem solving at level one.

Eleven students demonstrated hierarchical inclusion in their answers, leaving 8 students who do not demonstrate hierarchical inclusion. There were some patterns that emerged in the nature of hierarchical inclusion. Table 9 shows the relationship of Hierarchical inclusion and the students’ levels of conservation.

<table>
<thead>
<tr>
<th>Table 9</th>
<th>Hierarchical Inclusion by Conservation Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hierarchical Inclusion</td>
<td>Conservation Level</td>
</tr>
<tr>
<td>yes ($n=11$)</td>
<td>1</td>
</tr>
<tr>
<td>no ($n=8$)</td>
<td>3</td>
</tr>
</tbody>
</table>

Of the 8 students who did not display hierarchical inclusion, only one of these is a conserver, AV. However, this student drew five, then eight more, then erased the 8 and drew three, demonstrating that she is at a transitional stage in representing. This conclusion is further corroborated when you add in her level of representation. She is a Level 1 on the representation task, so it could be that she understands hierarchical inclusion, but her lower level of representation keeps her from documenting it. All the remaining conservers demonstrate hierarchical inclusion. In this case a plausible, yet tentative, conclusion is that hierarchical inclusion comes before representation. No changes in hierarchical inclusion were noted after
the conferences thus suggesting that stability in hierarchical inclusion comes before stability in representation.

5.3.3. Hierarchical Inclusion and Level of Representation

Another interesting feature is the level of representation that the students who did not demonstrate hierarchical inclusion. Of the 8 who did not demonstrate hierarchical inclusion, 7 were at a level 1 for representation and the other student at level 2. However, the levels for the remaining students were distributed across the range, with 4 at the lowest level of representation, 5 in the middle and 2 at the highest level of representation. Table 10 shows the frequencies for hierarchical inclusion by representation level. Again, no clear cut relationship between a student’s hierarchical inclusion and level of representation was noted. A logical conclusion here is that hierarchical inclusion seems to come before a valid solution strategy can be accurately represented for the entire problem situation, thus calculating a correct answer. In light of a Piagetian perspective; students have to understand the nature of number before they can use numbers and numerals to quantify.

Table 10
Hierarchical Inclusion by Representation Level

<table>
<thead>
<tr>
<th>Hierarchical Inclusion</th>
<th>Representation Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>yes (n=11)</td>
<td>4</td>
</tr>
<tr>
<td>no (n=8)</td>
<td>7</td>
</tr>
</tbody>
</table>

5.3.4. Numeral Writing by Conservation and Representation

In the numeral writing task numbers were called to the students and they wrote the corresponding numeral of their paper. The numerals were the same numerals that were used in the word problems they were asked to solve as a part of the problem solving task. The numbers were 4, 3, 6, 8, 10, 21, 41, 30, 12, and 16.

Table 11 presents the number of times a student correctly wrote the corresponding numeral for the number that was called out. These results are categorized by the students’ levels
of conservation. Ten numerals were called out to the students in the same, non-consecutive order.

Overall, students were proficient at writing the numerals from the problems by rote. Sixteen of the students wrote the correct numeral for half or more of the numbers. Fifty-seven percent of the students who wrote all of the numerals correctly were conservers (level 3).

Table 11

<table>
<thead>
<tr>
<th>Numerals Written Correctly (N=10)</th>
<th>Conservation Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level 1 (N=5)</td>
</tr>
<tr>
<td>0 (N=1)</td>
<td>1 (20%)</td>
</tr>
<tr>
<td>4 (N=2)</td>
<td>0</td>
</tr>
<tr>
<td>5 (N=1)</td>
<td>1 (20%)</td>
</tr>
<tr>
<td>6 (N=1)</td>
<td>1 (20%)</td>
</tr>
<tr>
<td>7 (N=3)</td>
<td>1 (20%)</td>
</tr>
<tr>
<td>8 (N=2)</td>
<td>0</td>
</tr>
<tr>
<td>9 (N=2)</td>
<td>0</td>
</tr>
<tr>
<td>A (N=7)</td>
<td>1 (20%)</td>
</tr>
</tbody>
</table>

Table 12 shows the student’s levels of representation and numeral writing. Fifty-seven percent of the students who wrote all of the numerals correctly were at a representation level two, but interestingly, no students at the highest representation level wrote all numerals correctly and forty three percent of the students who wrote all of the numerals correctly were at level one for representation.
Table 12
Numeral writing by Representation Level

<table>
<thead>
<tr>
<th>Numerals written correctly (N=10)</th>
<th>Representation Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level 1a (N=8)</td>
</tr>
<tr>
<td>0 (N=1)</td>
<td>1</td>
</tr>
<tr>
<td>4 (N=2)</td>
<td>2</td>
</tr>
<tr>
<td>5 (N=1)</td>
<td>0</td>
</tr>
<tr>
<td>6 (N=1)</td>
<td>1</td>
</tr>
<tr>
<td>7 (N=3)</td>
<td>3</td>
</tr>
<tr>
<td>8 (N=2)</td>
<td>1</td>
</tr>
<tr>
<td>9 (N=2)</td>
<td>0</td>
</tr>
<tr>
<td>A (N=7)</td>
<td>3</td>
</tr>
</tbody>
</table>

5.3.5. Numeral Writing in Representation Task

Consistent with what Kato et al. (2002) and Sinclair et al. (1983) found, the students could easily write 4, 2, 6 & 8, the numbers from the representation task, when they were called at random. Yet, these same students did not use them to represent objects in the representation of objects task. Out of 76 numerals written, 75 were written correctly. However, only two students used these numerals in the representation task for a total of 8 numerals out of the 76 representations.

5.4. Research Question 3

To answer the third research question, “What strategies are used by kindergarten students to solve mathematical problems?” the written solutions for the word problems and notes from the conferences were analyzed. In a few cases the videos of the conferences were viewed to confirm or clarify conclusions. The data revealed that the participants in this study adopted one strategy, but utilized that strategy at varying levels. This resulted in the creation of a taxonomy of problem solving levels. This taxonomy is reported in Table 5 on page 71 of Chapter 4.

5.4.1. Direct Modeling

The students used one strategy, direct modeling, but applied it at different levels. This resulted in the creation of a problem solving levels scale. Examples of each level and sub-level are presented in Table 4 on page 70. A description of the levels is presented in Table 5 on page
There were only a few cases where the students did not model the problem at all. These students either drew, what Kato et al. (2002) called, “a bunch of objects” (p. 43) or wrote a numeral only. Only one student, CH, wrote only a numeral. On two occasions his answer sheets contained only the answer, written as a numeral. When asked how he got his answer he said he counted on his fingers or in his head. His answers were incorrect because he miscounted even though his strategy was valid. However, as a result of the conference, he went back and drew the problem. When he used his drawing to calculate the answer, his answers were correct. However, for one problem he chose not to use his drawing to calculate his answer, thus leaving the incorrect answer even though he had drawn an accurate representation.

5.4.2. Word Problem Scale Levels and Descriptions

Three levels of direct modeling strategies were observed. Consistent with previous research, students strategies ranged from no discernible strategy (level 1) to a direct modeling strategy where they modeled the action in the problem and used counting to solve the problem (level 3). Between these strategies was a level where the student directly modeled the action in the problem, but did not use that model to solve the problem (level 2). Following Kato et al. (2002), different representation types and sub-levels were described within the levels. The levels and their various sub-levels and response types are described below and presented in Table 5 presented in Section 4.2.5..

5.5. Research Question 4

What is the relationship between kindergarten students’ levels of constructive abstraction, levels of representation and level of problem solving? The answers to this final research question were obtained by configuring and reconfiguring data displays to search for relationships between the factors. The following section is a report of these findings.

5.5.1. Conservation Level

The conservation task data was consistent with previous research findings. There were five students who were designated level one, non-conservers without one-to-one correspondence whose ages ranged from 65 months to 74 months. There were nine at level two, non-conservers with one-to-one correspondence, whose ages ranged from 64 months to 75 months. There were five at level 3, conservers, ranging from 69 months to 74 months. There were no obvious data clusters around age or conservation level. This information is presented in Table 13.
Table 13

<table>
<thead>
<tr>
<th>months</th>
<th>yrs. months</th>
<th>Conservation</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>5.5</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>67</td>
<td>5.7</td>
<td>Level 1</td>
<td>1</td>
</tr>
<tr>
<td>71</td>
<td>5.11</td>
<td>(N=5)</td>
<td>1</td>
</tr>
<tr>
<td>74</td>
<td>6.2</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>64</td>
<td>5.4</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>66</td>
<td>5.6</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>67</td>
<td>5.7</td>
<td>Level 2</td>
<td>2</td>
</tr>
<tr>
<td>69</td>
<td>5.9</td>
<td>(N=9)</td>
<td>1</td>
</tr>
<tr>
<td>70</td>
<td>5.10</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>74</td>
<td>6.2</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>75</td>
<td>6.3</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>69</td>
<td>5.9</td>
<td>Level 3</td>
<td>1</td>
</tr>
<tr>
<td>73</td>
<td>6.1</td>
<td>(N=5)</td>
<td>3</td>
</tr>
<tr>
<td>74</td>
<td>6.2</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

5.5.2. Representation Level

Most of the students (58%, n=11) scored at level one on the representation task. Six scored at level 2 (32%). Only two scored at level three (11%). There was no discernible pattern between representation level and age.

5.5.3. Conservation and Representation

When level of conservation was compared with level of representation, the data revealed an expected result, as reported in Table 14. There were five students at Level 1 of conservation (non-conservers), nine at Level 2 (intermediate) and three Level 3 or conservers. Students represented at or below their level of conservation with the exception of four students (21% of the total of the students). Two of these exceptions were level 1 representation, the other two were level 2 representation. Their ages were 65, 66, 67 and 71 months, showing no pattern of age in the exceptions. There was no other discernible relationships except that the only “all”
statement that could be made was that all five of the Level 3 conservers were at Level 1a representation. This also meant that no conservers were in any other group.

Table 14
Relationship Between Level of Representation and Level of Conservation

<table>
<thead>
<tr>
<th>Level of Conservation</th>
<th>Level of Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level 1</td>
</tr>
<tr>
<td></td>
<td>(N=11) (58%)</td>
</tr>
<tr>
<td>Level 1</td>
<td>3 (16%)</td>
</tr>
<tr>
<td>(N=5) (26%)</td>
<td></td>
</tr>
<tr>
<td>Level 2</td>
<td>5 (26%)</td>
</tr>
<tr>
<td>(N=9) (47%)</td>
<td></td>
</tr>
<tr>
<td>Level 3</td>
<td>3 (16%)</td>
</tr>
<tr>
<td>(N=5) (47%)</td>
<td></td>
</tr>
</tbody>
</table>

5.5.4. Hierarchical Inclusion and Conservation

Of the 11 students who demonstrated hierarchical inclusion, two were at Level 1 for conservation, five at Level 2 and four at Level 3. Of the eight students who did not demonstrate hierarchical inclusion, there were three at Level 1, four at Level 2 and only one at Level 3 conservation. It appears that it might be more likely for a conserver to demonstrate hierarchical inclusion, however, given the small sample size such a conclusion can only be tentative. Table 7 on page 83 shows the frequency of hierarchical inclusion by conservation level.

5.5.5. Hierarchical Inclusion and Representation

Of the 11 students who demonstrated hierarchical inclusion, there were four at Level 1 of representation, five at Level 2 and two at Level 3. This information is presented in Table 8 on page 83. Of the students who did not demonstrate hierarchical inclusion, only one was at Level 1. It appears that hierarchical inclusion comes before the ability to represent the problem solution accurately.
5.5.6. Correct answer, Valid Strategy and Hierarchical Inclusion

Table 15 shows the link between hierarchical inclusion, using a valid strategy and arriving at a correct answer. This table shows that the same number of students demonstrated a valid strategy as demonstrated hierarchical inclusion (11). However, only nine of these arrived at a correct answer. It appears that hierarchical inclusion is necessary to arrive at a valid strategy, and a valid strategy is necessary to arrive at a correct answer. However, it also demonstrates that neither demonstrating hierarchical inclusion nor using a valid strategy will guarantee that a student can arrive at the correct answer.

Table 15
Relationship of Correct Answer, Valid Strategy & Hierarchical Inclusion

<table>
<thead>
<tr>
<th></th>
<th>Correct Answer</th>
<th>Valid Strategy</th>
<th>Hierarchical Inclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>9</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>no</td>
<td>10</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

5.5.7. Level of Problem Solving and Representation

Tables 16 and 17 document the levels of problem solving when categorized by level of representation pre-conference and post-conference respectively. Table 2 on page 63 describes the levels of representation. The following tables demonstrate that the problem solving levels improved an all categories after the conferences.
Table 16
Relationship of Problem Solving Level and Representation Level Pre-Conference

<table>
<thead>
<tr>
<th>Problem Solving Level</th>
<th>Level of Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level 1</td>
</tr>
<tr>
<td>1</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 17
Relationship of Problem Solving Level and Representation Level Post-Conference

<table>
<thead>
<tr>
<th>Problem Solving Level</th>
<th>Level of Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level 1</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

5.6. Summary

The purpose of this study was to explore problem solving with kindergarten students. This line of inquiry is highly significant given that CCSS emphasizes problem solving with kindergarten students in the current educational climate, however, there is little research on problem solving with kindergarten students. This study is one of a few to explore problem solving with kindergarten students. Chapter 5 discussed findings relevant to the research questions that guided the inquiry. The data suggests that the instructional method solving a wide variety of authentic and relevant mathematical word problems using pencil and paper utilized in this study is a viable means of improving students’ problem solving proficiency. It also suggests that there are levels of representation and problem solving through which students progress. However, problem solving is not a lockstep process through which students progress, rather, it is a complex individual process that combines factors related to the student, the problem and the problem context. These implications are discussed in Chapter 6.
CHAPTER SIX
DISCUSSION

This study was undertaken to examine how kindergarten students solve mathematical word problems, an understudied area that is currently being highlighted by the adoption of the CCSS (2010). Naturalistic observational research was adopted as an appropriate methodology in order to study how an intact class of kindergarten students set about the task of solving several word problems. Guiding the study were four broad questions as follows:

1. How do kindergarten students represent their mathematical thinking when solving word problems?
2. What levels of representation do kindergarten students demonstrate when solving word problems?
3. What strategies are used by kindergarten students to solve mathematical problems?
4. What is the relationship between kindergarten students’ levels of constructive abstraction, levels of representation and level of problem solving?

This final chapter includes a summary of the findings as well as general, yet tentative conclusions. Then, drawing on the findings, implications for both research and practice are discussed. The discussion of the findings is organized around several broad themes that emerged both from the review of literature as well as from the findings.

6.1. Main Findings

The study's findings seem to confirm Kamii’s (2000) proposition that kindergarten students are able to solve mathematical word problems using pencil and paper. In this study, students primarily used drawn symbols as a written representation of their mental representation. Their use of pencil and paper was not confined to drawing symbols, however. The students also included conventional signs. A related key finding is that although the students represent their thinking about the problem situation, it is not necessarily the actual problem situation. In other words, students represent what they understand about number and operations. This was evident when the students solved the first problem. In this problem, those students who had an understanding of the hierarchical nature of number usually represented the simple addition problem as two distinct quantities (xxxxx + xxxxxxxx = 13). They then showed the operation of addition. However, those students who did not understand the hierarchical nature of number,
represented differently. These students represented the first number with five shapes (xxxxx), then added three more (XXX) to make the second quantity (xxxxxXXX). In other words, how children represent word problems on paper is limited by their understanding of number and operations.

Because children's mental representations cannot be studied outside of some type of documentation, the pencil and paper solutions were used as a record of the students’ mental representations. There was a wide range of representation within this typical class of kindergarteners. Students’ written representations ranged from drawing a "bunch of objects", to the use of a conventional number sentence. Students’ written representations directly modeled the problem situation at varying levels. The written solutions also documented varying levels of solution strategies, application of those strategies, and success in finding the correct answer.

There were no apparent direct relationships between students’ levels of representation, conservation, and problem solving. In other words, a high level in one area did not seem to predict a high level in another level. This led to the plausible conclusion that problem solving is a complex and individual process. It seems that there could be patterns of development within each of these areas. Such potential patterns are discussed in the following sections. This discussion is organized around several general themes that emerged in answering each of the research questions.

6.2. Range of Problem Solving Levels

The data revealed a range of problem solving levels within the kindergarten class studied. This range suggests two possibilities. First, is that there are patterns of development in children's mathematical problem solving. Then, second is that mathematical problem solving is individual in nature. The data reported in this study suggest that there are patterns of development, but these patterns do not serve as predictors of problem solving proficiency. A correct answer to a word problem includes the application of a valid solution strategy. In turn, this solution strategy has to be applied to the entire problem situation. It seems that for a student to accurately represent the problem situation in writing, he or she has to accurately represent it mentally. Yet, the ability to form an accurate mental representation requires an understanding of the hierarchical nature of problem solving. However, the presence of hierarchical inclusion does not necessarily predict an accurate written representation. Relatedly, the production of an
accurate written representation does not predict the use of a valid strategy. Then, even if a child knows a valid solution strategy it does not follow that he or she will apply it to the entire problem situation. Similarly, applying an accurate solution strategy does not mean that a student will calculate the correct answer. For example, a student can start with a valid solution strategy, but fail to apply it to the entire problem situation, resulting in a wrong answer. Moreover, a student can represent the entire problem situation with a reasonable solution strategy, and yet fail to arrive at the correct answer.

The data seems to confirm Carpenter et al.'s (1993) conclusion that young children directly model the problem situation when they attempt to solve mathematical word problems. In the current study the students directly modeled the problem situation primarily using symbols, and also some signs. The students written products provide evidence that there may be a wide range of problem solving levels within a class of typical kindergarteners. Such a range was evident across all of the word problems administered in the current study. Surprisingly, however, individual children did not remain on the same level across all of the mathematical word problems. The presence of such a range of problem solving levels suggests that an individual student may progress through several stages within this range when attempting to solve different types of word problems. The results of the study suggest that this progression will not be in a neat lockstep or linear fashion, but that the student will oscillate from one level to another according to the maturity of his mental structures, the type of problem presented, and the context in which the problem solving activity is embedded.

It is recommended that future research studies should examine the ranges of kindergarten students' problem solving levels. First, studies researchers should seek to verify the initial findings reported in the current study by adopting microgenetic designs. Then, complementing such studies, longitudinal designs should be conducted to examine the development of young children's mathematical problem solving abilities.

6.3. Conferences Are Integral

The position of this study is that children become problem solvers through a complex interaction of the student, the problem, and the problem context while solving mathematical word problems. This is because children do not engage in mathematics tasks in a vacuum. They each bring their own unique abilities and expertise to the situation, the problem that is posed elicits a specific type of response, and in turn, the children's mental and written
representations are shaped by the social context of the classroom. Simply put, kindergarten students do not become problem solvers by applying rote skills they have previously learned. The results of this study seem to suggest that the conference with the teacher and student is an integral part of that process. Essentially the teacher-child conference serves to create and shape a context that can improve students’ representation. Furthermore, the conference can confirm for the child the validity of his or her strategy, acknowledge a correct answer, or advance the child's problem solving level.

While students did remarkably well in problem solving prior to the conferences, in almost 30% of the cases the conferences resulted in a change from a lower level to a higher level of problem solving. At no time did a student’s level of problem solving decrease a level after the conference. In some cases the students did not make changes to their problem sheets even when incorrect answers or faulty reasoning were revealed. This can be explained by the Piagetian framework that maintains that students learn when they are faced with cognitive disequilibrium. Arguably, the conference with the teacher created cognitive dissonance for the child. Thus, when students explained their ideas, their faulty or incorrect reasoning was exposed. With an appropriate and timely prompt or probe from the teacher, a child who is at the mental level to understand that false reasoning, they will work to resolve the cognitive conflict. It seems that the interaction with the teacher encourages children to reflect on the meaning of their responses, then faced with the cognitive conflict, they subsequently strive to resolve the conflict and maintain equilibrium. This was evident in the current study, in particular those cases where the students revised their written representations or answers during or immediately after the conference.

The conferences did not seem to benefit any group of students over another. This implies that the conferences are influential for all age groups, and students at all levels of representation, levels of conservation, and regardless of whether they are categorized as special needs or general education students. In sum, it seems that the post problem solving teacher-child conference is an important pedagogical strategy for increasing kindergarten students problem solving proficiency.

6.4. Levels Across Problems

There were no marked differences in problem solving levels across the problems. In every one of the word problems administered to the kindergarten students, they performed at all
three levels. As was evident in earlier studies, the students were more successful at solving some problems than others. In this case, it seems that the type of problem presented to the students influences or shapes the type of strategy that they use, and the level at which they engage with the task of solving the problem.

There was a marked improvement in the number of changes and number of correct answers in the two problems that were of the same type. It is possible that presenting students with variations of the same problem type leads to improvement in the extent to which kindergarten students adopt an appropriate strategy. Indeed the repetition of similar mathematical word problems with young children seems worthy of further research. Such research could not only validate these preliminary findings, but also extend our understanding the pedagogical value of repeated practice with similar word problems.

In the current study, it seems that the students improved the second time a specific problem situation was presented (e.g., as in problems 2. There are 22 children in our class. Everyone has two eyes. How many eyes do we have in our class? and 4. There are 22 children in our class. Yesterday each child wore two gloves. How many gloves were there all together?). In the first instance 17% of the students were successful in calculating a correct answer after the teacher-child conference. Five of the students’ problem solving level improved after the conference, which was a 28% improvement rate. Then when the students were presented with the second variation of the same problem, 33% of the students calculated a correct answer after the conference. This time, a total of eight of the students problem solving improved, which was a 53% improvement rate. Clearly, since the study was conducted with a small sample of students, it is not possible to attribute these improvements directly to the repetition of the problem type, numbers and operation. Yet, based on what was observed with this group of students it is plausible that the type of problem, and repetition of problem type matters when teaching kindergarten students how to use pencil and paper to solve mathematical word problems. Further research is therefore recommended to examine how the type of word problem that is administered relates to students understanding of problem solving and use of an appropriate strategy.

Finally, it was interesting that for one of the problems none of the students changed their responses after the conference. This was when children solved the fifth problem: The lion, tiger and dog each have 10 apples on top. How many apples are up on top all together? In this case
the students were fairly successful at calculating the correct answer. The 32% ranking was second from the lowest. The other rankings were 17%, 33%, 42% and 83%. Further studies might be able to shed light on why this problem resulted in few changes despite there being room for improvement.

These findings suggest that giving a variety of problems that vary in type, difficulty, numbers and operations used is beneficial in helping students become proficient problem solvers. The data suggests that giving student different problems that use the same number and operations repeatedly is helpful in increasing their problem solving level.

There were two main types of problems in the study, routine and non-routine. However, the findings suggest that there are different levels of difficulty within these categories. The taxonomy for routine problems is presented in Cognitively Guided Instruction. Although Kamii (2002), mentioned this approach, she did not but outline a taxonomy for problem difficulty when dealing with non-routine problems. The results of this study suggest that there are varying levels of problem difficulty in non-routine problems, (e.g., problem 2. There are 22 children in our class. Everyone has two eyes. How many eyes do we have in our class? 4. There are 22 children in our class. Yesterday each child wore two gloves. How many gloves were there all together? And problem 5. The lion, tiger and dog each have 10 apples on top. How many apples are up on top all together?). A related possibly is that there is a number threshold above which students cannot calculate – a point where what should be a routine problem becomes a non-routine problem because of the number of the addends. While this proposition seems plausible in light of the findings presented in this study, further research is recommended. Direct observations of children solving word problems involving different numbers of addends could be instrumental in identifying the possible number threshold.

6.5. Assessment

The findings suggest that it is possible to assess kindergarten students problem solving and representation. Indeed, such assessment is necessary in order to determine whether a student has developed deep conceptual understanding. The scale that was developed offers a means for assessing students’ problem solving development. Potentially, the scale could be refined and validated by other researchers and used as an assessment tool by teachers.

The results of this study document the wide range of problem solving levels that are present in one typical kindergarten class when students represent their solutions with pencil and
The presence of the various levels suggests that students’ problem solving does have some patterns or sequence in its development. For example students’ drawings become more organized, representational of the problem situation and move from qualitative to quantitative in nature. The problem solving scale is important because it provides a means for documenting student progress in problem solving. Future studies will be needed that trace students’ problem solving development over time. This study provides a starting point for analyzing the problem solving development of a student. Without a scale, such as the one created for this study, the relationship between factors relating to the student and the problem have not been possible because there were no scales that would allow such comparisons. Thus, in this exploratory study, the creation of the problem solving scale allowed for the comparison of the relationships between constructive abstraction, representation and problem solving.

The problem solving scale that was developed is particularly useful and practical because it allows for the inclusion of pre- and post- conference data. That is, the problem solving scale allowed some of the contextual factors of problem solving to be included in the picture. The creation of the scale within the current study provides a tool to measure and, thus relate these other factors. Further studies will be needed to replicate the effectiveness of the scales and the methodology presented in this study.

6.6. Representation in Context

Representation in context of word problems is more accurate than isolated representation. To some extent, the results of this study replicated findings of Kato et al. (2002) and Sinclair et al. (1983) demonstrating that students represent at or below their levels of constructive abstraction. However, there were notable exceptions that support the idea that students can be taught representational strategies without understanding or without the ability to use this information in problem solving situations. The prime example is ZK who wrote the number sentence on problem one. It appeared that she understood how to use this abstract conventional representation. However, for the remaining problems, her representation level and problem solving level were much less sophisticated, and even disorganized. In the numeral writing task she represented all of the numerals, but did not use these numerals in the representation task or in the word problems after problem one.

Further support for the idea that writing a number does not indicate understanding of number is that in 75 of the 76 cases students could represent the numerals from the
representation task, but there were only 8 instances from two students were they used these numerals during the representation task. These findings do support Piaget’s theory that students represent their thinking about reality – but also support the idea that they can be taught to memorize certain sums. When solving non-routine word problems students can no longer use the simple algorithms that they have memorized, and their representation and solutions represent their thinking. When writing a list, students are demonstrating their ability to memorize – when solving word problems and using number, they do not use these numbers. We need to be careful what we spend time teaching students to do – think about how they are solving word problems or memorizing numerals that they can’t use. As such, this study provides limited support for using strategies other than rote algorithms for teaching problem solving and mathematics in general.

6.7. Instructional Method

It is beyond the scope of the current study to establish any type of causal relationships or to attribute the superiority of one type of problem solving task over another. Yet, the success with which the students accomplished their problem solving tasks that approaching mathematical problem solving in the way described in this study is a viable option for kindergarten classes. The students were engaged in all aspects of the tasks and seemed to enjoy the process. All students participated in solving the problems and in discussing their solutions in the conferences. The students calculated the correct answer 22% of the time pre-conference and 41% of the time post-conference. Almost one-third of the conferences resulted in a change to a higher level of cognitive understanding. Additionally, the written record proved useful for assessment and documentation of student levels. It appears that solving word problems with paper and pencil and participating in conferences with a teacher is both an effective and manageable instructional method that can be used in school settings. The effectiveness of using mathematical word problems with kindergarten students should be further studied in order to validate its effectiveness as an instructional approach.

6.8. In light of CCSS

In addition to answering the research questions posed, the findings from this study chapter presents an alternative way to conceptualize mathematical problem solving. This chapter presents problem solving as interwoven strands and provides support for the
praxeological perspective, an emerging paradigm for research in early childhood settings. These findings are particularly relevant to the current educational climate as articulated by CCSS.

CCSS (2012) is influencing the current educational climate in the states that have adopted and are implementing them. CCSS emphasizes deep conceptual understanding while acknowledging the importance of practical knowledge and skills. The complexity of the written solutions produced by the kindergarten students reveal this method of approaching problem solving to be useful for developing deep understanding rather than simply rote procedural skills. Also woven throughout the standards is the idea that learning is individual; there is a wide range of abilities, needs, learning rates, and achievement levels within a classroom (CCSS, 2012). CCSS articulates content and practice goals for kindergarten mathematics as well as acknowledging that assessment is necessary to ensure students are developing understanding. The findings from this study provide a vehicle for delivering the content and procedural goals as well as proposing a means for assessing students’ understanding as articulated by CCSS. However, it is important to note that the findings of this study are also situated within a solid foundation of decades of early childhood research. It is my hope that this research will not simply be current or perhaps trendy, but another step upon which future research can be built.

6.9. Interwoven Strands

Mathematical problem solving is often presented as a set of steps that can be followed. This study offers a new way to conceptualize problem solving as a complex interwoven relationship between the student, the problem and the problem context. Each of these strands is made up of different components. Table 18 shows some of the factors included in each strand.

Table 18

<table>
<thead>
<tr>
<th>The Student</th>
<th>The Problem</th>
<th>The Problem Solving Context</th>
</tr>
</thead>
<tbody>
<tr>
<td>constructive abstraction</td>
<td>representation</td>
<td>authentic level of difficulty</td>
</tr>
<tr>
<td>problem solving level</td>
<td>type</td>
<td>problem situation</td>
</tr>
</tbody>
</table>
Conceptualizing problem solving as the interweaving of many different strands could provide a helpful framework for approaching problem solving in the early childhood years because it focuses on the individual and dynamic nature of problem solving. Kindergarten students’ problem solving does not follow a lockstep progression. Similarly, a student’s progress in becoming a problem solver is not linear. A student does not master problem solving once and for all. Hearing that news, educators might become disheartened and wonder if it is worth it to try to teach problem solving. The study’s findings provide some good news on that front; there is a viable way to teach problem solving in the typical kindergarten public school setting. There are different levels of representation and problem solving that follow a distinct pattern. Moreover, this study offers the idea that there is a way to monitor a student’s progress using this pattern.

6.10. Research Methodology

The methodological issues faced by this study highlight the complexity of conducting research with young children. In the present study, merely quantifying the problem solving success of kindergarten children would not have captured the information needed to answer the research questions. This should lead early childhood researchers to reconsider the ways we undertake research with young children. The new praxeological paradigm being discussed in the European journals for young children appears to also be taking this stance. Further research should be undertaken based on this alternative perspective. In doing so problem solving in mathematics should be seen as interwoven strands and any research methods should seek to capture and preserve children’s voices and spaces.

6.11. Implications for Teaching

The conclusions from this study offer many implications for teaching problem solving, and to a greater extent mathematics, in kindergarten. The first is for teachers to approach the mathematics as an opportunity to build students’ logico-mathematical knowledge rather than a place to teach a didactic set of skills. This will affect the entire process from situating the problem in an authentic and relevant context, to the amount of freedom the students have to represent their own thinking, to the emphasis that is placed on getting a correct answer, to how the conference is approached and students’ progress in problem solving is assessed.
Another implication is obvious – give kindergarten students word problems to solve using pencil and paper. The following related implications involve the actual procedures of the instructional process:

1. Situate the word problems in an authentic and relevant context. When an authentic context is not present, create a context that is relevant for students by using books.

2. Give varying levels of mathematical word problems with different operations.

3. Also, give problems with the same numbers and operations, but different problem contexts.

4. Teach students how to solve the problem rather than how to get a correct answer. Teach them to re-read the problem and explain their answers.

5. Conference with all students, not just the ones who get the answer wrong. Focus on understanding students’ thinking. Use open-ended questions and offer suggestions.

6. Use multiple means of assessment of problem solving level. Assess students’ trajectory toward more sophisticated problem solving, rather than problem solving on one day with one problem.

7. Adjust expectations based on students’ mental processes rather than on age or previous answers.

Further implications relate to the praxeological research method. Research done by an insider deeply embedded in the research context is valid and effective. This type of research should be encouraged and supported.

6.12. Limitations of the Study

This study was undertaken as a preliminary venture into a lightly explored area of kindergarten problem solving in a naturalistic, whole class setting. The small size and the fact that the study was not a comparative study mean that conclusions cannot be applied to a broader population. As this is a first attempt at documenting the taxonomy of representation levels and
problem solving strategies, these scales are rudimentary. There is no assertion that these scales are leveled across the tasks.

6.13. Summary

The findings reported in this study shed some light on the complexity of children’s mathematical thinking in kindergarten. There is a wide range of representation and problem solving within one typical kindergarten class. However, there is evidence that students mathematical thinking improved by being involved in the problem solving process presented in this study. Students were successful with solving a wide range of problems. When they were not successful at calculating a correct answer, they often presented reasonable strategies for solving the problem. Those who did not present reasonable strategies for solving often demonstrated an understanding of the problem situation by representing a part of the process. All students participated in solving the problems and conferences. Clearly the problem solving method presented in this study offered manageable instructional strategy as well as a way to assess students’ problem solving levels.

Clearly, more research is needed to validate these initial findings. Like most studies, the current study raises more questions than gives answers. It is hoped that the findings will inform further research on this important topic and provide a research-based springboard for the creation and implementation of instructional practices in the modern classroom.
APPENDIX A
PILOT STUDY

The pilot study was conducted in one Pre-kindergarten class and one kindergarten class May 7-9, 2012. Conducting the study over two grade levels was done to ensure that data was collected from students across the typical age and ability range of kindergarten students (4 years 10 months - 6 years 8 months) and would yield an accurate representation of the outcomes expected from the actual data collection.

On the first day of the pilot study, the students were brought to the group area and were introduced to the researcher who sang songs with them and shared information about the research study. The word problem was read to them and it was determined that the students understood the problem. They were instructed to think about how they would get an answer and then directed to show that thinking on the paper so that someone else could know what they had done. The teachers walked around the room and helped the students with the problems, despite being told not to give any assistance. They were reminded not to give any assistance, but to direct the students with comments like, “What do you think?” or “How could you figure that out?”

As the students completed their papers, they were called over to a table and interviewed. Each interview was videotaped with a small hand-held video camera. Many of the students continued to work, however, other students began to line up to be interviewed. The students who had lined up to be interviewed were directed to go work at their centers and the teacher collected their papers. As the conferences were completed, the remaining students were called up in the order in which their papers were turned in.
The teachers and the researcher discussed the process and made a few adjustments for the second day of interviewing. It was decided that the students should be informed that they would not be getting any help from the teachers. The children would be told to do their best work by themselves and that we would talk about their questions at the interview. They would also be reminded that it was fine if they did not get an answer. It was decided that the conferences would be less distracting to the other students if they were done outside in the hallway. The video tapes were reviewed and it was discovered that the audio was distorted by background classroom noise. These aspects of the procedure were modified for the second round of conferences.

On the second day with the Pre-kindergarten class, the revised procedures were put into place. The conferences were moved into the hallway, with the teacher directing the students when and where to go to be interviewed. As students finished, the teachers collected the papers and sent them to centers. As a child finished his interview, he was sent inside to get the next student, according to the order in which the papers were collected. After review, this procedure satisfied both the teachers and researchers. The videos were more clear and audible. However, after reviewing the videos, it was determined that they did not yield significantly more information than the interviewer notes and data, so their review may not be necessary. It was decided that the videos would be recorded, but analyzed only if the need arose, for example, because of missing data or for member checking.

On the third day of data collection with Pre-kindergarten and the fourth day with kindergarten, the revised data collection procedure ran smoothly, with the teachers and students moving through the steps easily. However, the kindergarten students spent more time adding to or correcting their problems than the Pre-kindergarten students. The actual data collection for
the problem solving piece took about one hour in total. The teachers suggested that an alternative schedule for the conferences should be created in the event that it was not possible for the conferences to take place at one time. Some suggestions were to do the conferences during rest time or the following day.

The students’ papers were reviewed to determine whether there were any problems. With the exception of some minor organizational items (e.g., the papers should be dated), there were no problems with the data collected. The data yielded predictable outcomes considering the wider age range of the pilot study participants (four years seven months to six years seven months) as compared to the study participants (five years one month – six years seven months).

The teachers were asked to offer suggestions for improving the process. Several suggestions were made. First, the data collection time should be right before center or other free movement time in the classroom for best results. Next, a supply of erasers and pencils should be kept on the interview table in the case the students want to edit their work. Finally, a script should be created for the teachers to use if a student requests help. In summary, the findings of the pilot study verified the practicality of the research design and the procedures.
APPENDIX B

WORD PROBLEM CONFERENCE PROTOCOL

1. Restate the problem question. “How many dogs are at the park?”
2. Tell me about what you have drawn/written.
3. Ask for explanation of any formal or informal symbol or sign. Pointing to the symbol or sign asking, “What is this?” or “What does this mean?”
4. Ask why the student drew what they did.
5. Ask how they knew to use that quantity or symbol. “How did you decide to draw (use or write) that?”
6. Ask them to re-state the problem.
7. “Would it be helpful to draw a picture?” or “Would it be helpful to count?”
8. Ask them the question, “So, how many cookies will you have for snack?”
9. “How could you show me?”
10. “How could you make sure?”
11. “Would you like to use the class roster list?”
APPENDIX C

REPRESENTATION LEVELS KATO ET AL.

Level 1: Global, prenumerical representation
Sublevel 1a. Absence of a one-to-one correspondence even when the number of objects is smaller than five
Example: Child draws 5 circles to represent four dishes.

Sublevel 1b. Absence of a one-to-one correspondence when the number of objects is greater than five
Example: Child draws 4 circles to represent four dishes, but draws 12 shapes to represent eight blocks.

eight blocks

four dishes

Level 2: Representation with one-to-one correspondence
Response type 2a: One-to-one correspondence, with pictures
Example: Child draws 6 pencils to represent six pencils.

Response type 2b: One-to-one correspondence, with numerals
Example: Child writes 4 numerals to represent four dishes.

Level 3: Representation with one numeral indicating the total quantity as a composite whole
Response type 3a: Writing numeral only
Example: Child writes “6” for six pencils.

Response type 3b: Writing numeral and name of object or a complete sentence that includes both
Example: Child writes “There are 4 dishes” in Japanese.

お皿が4にあります
APPENDIX D
PROCEDURAL TIMELINE

<table>
<thead>
<tr>
<th>Timeline</th>
<th>Procedures</th>
</tr>
</thead>
<tbody>
<tr>
<td>March 17, 2012</td>
<td>IRB Approval</td>
</tr>
<tr>
<td>November 2012 – January 2013</td>
<td>Consent Forms Home (11/6/2012)</td>
</tr>
<tr>
<td></td>
<td>collection of demographic information about participants 1/11/2013</td>
</tr>
<tr>
<td>January 14, 2013</td>
<td>Desensitization Session</td>
</tr>
<tr>
<td></td>
<td>Introduce researcher to class. Participate in morning group time. Read book on current topic of study (shapes). Book Title: I Spy Shapes I Art by Lucy Micklethwait.</td>
</tr>
<tr>
<td></td>
<td>Participate in center/work/small group time.</td>
</tr>
<tr>
<td>January 15, 2013</td>
<td>Participate in morning group time. Read So Many Circles So Many Squares by------- Hoban. Introduce word problem task. Hand out word problem sheets. You have 5 grapes and I give you 8 more. How many grapes will you have then?</td>
</tr>
<tr>
<td></td>
<td>Facilitate conferences. Word Problem Task 1.</td>
</tr>
<tr>
<td></td>
<td>Teacher and Researcher review of process; preliminary data analysis</td>
</tr>
<tr>
<td>1/16/2013 Wednesday</td>
<td>Representation of Number Task; Member checking with data analysis</td>
</tr>
<tr>
<td>1/17/2013 Thursday</td>
<td>Read The Greedy Triangle</td>
</tr>
<tr>
<td></td>
<td>Continue Representation of Number Task</td>
</tr>
<tr>
<td>1/18/2013 Friday</td>
<td>Read Circles: Seeing Circles All Around Us by Schuette. Word Problem 2 –</td>
</tr>
<tr>
<td></td>
<td>There are 22 children in our class. Everyone has 2 eyes. How many eyes do we have all together?</td>
</tr>
<tr>
<td>1/21/2013 Monday</td>
<td>Martin Luther King Holiday</td>
</tr>
<tr>
<td>1/22/2013 Tuesday</td>
<td>Read: Circle City</td>
</tr>
<tr>
<td></td>
<td>Conservation of Number Task; member checking with data analysis</td>
</tr>
<tr>
<td>1/23/2013 Wednesday</td>
<td>Read: Solid Shapes</td>
</tr>
<tr>
<td></td>
<td>Continue Conservation of Number Task</td>
</tr>
<tr>
<td>1/24/2013 Thursday</td>
<td>Read: Circus Big Book - - - check name</td>
</tr>
<tr>
<td></td>
<td>Word Problem Task 3 – How many shapes were at the circus?</td>
</tr>
<tr>
<td>Date</td>
<td>Activity</td>
</tr>
<tr>
<td>------------</td>
<td>--------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1/25/2013</td>
<td>Read: The Shape of Things&lt;br&gt;Word problem #4. How many feet are in your house?</td>
</tr>
<tr>
<td>1/28/2013</td>
<td>Read Ten Apples Up on Top.&lt;br&gt;Word Problem #5: How many apples up on top?</td>
</tr>
<tr>
<td>1/29/2013</td>
<td>Wrap up Session and celebration with students; Teacher and Researcher; copy and return word problems to teacher.</td>
</tr>
<tr>
<td>Post Study</td>
<td>Send Home Thank You Notes; Picture Books; Participation Photographs</td>
</tr>
</tbody>
</table>
## APPENDIX E
### RELATION OF RESEARCH QUESTIONS TO METHODS

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Data Collection Method and Source</th>
<th>Data Type</th>
</tr>
</thead>
</table>
| How do kindergarten students represent their mathematical thinking when solving word problems? | Word Problem Solving Task - Students solve standard and non-standard word problems with paper and pencil. Kamii, 2000 | • word problem task recording sheets  
• word problem conference recording sheets  
• video tapes of word problem conferences |
| What levels of numerical representation do kindergarten students demonstrate? | Levels of representation Task. (Sinclair et. al, XXX; Kato, et, al XXX)  
Word Problem Solving Task, Kamii, 2000 | • word problem task recording sheets  
• word problem conference recording sheets  
• video tapes of word problem conferences  
• representation task recording sheets |
| What strategies are used by kindergarten students to solve mathematical problems? | Word Problem Solving Task | • word problem task recording sheets  
• word problem conference recording sheets  
• video tapes of word problem conferences |
| What is the relationship between kindergarten students’ levels of constructive abstraction, levels of representation and level of problem solving? | Word Problem Solving Task  
Conservation Task  
Representation Task | • word problem task recording sheets  
• word problem conference recording sheets  
• video tapes of word problem conferences  
• representation task recording sheets  
• conservation of number recording sheets. |
## APPENDIX F

### WORD PROBLEM SOLVING TASK ELEMENTS FOR ANALYSIS

<table>
<thead>
<tr>
<th>Description</th>
<th>Example from Problem Solving</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>representation</strong></td>
<td></td>
</tr>
<tr>
<td>Did not draw</td>
<td></td>
</tr>
<tr>
<td>Drew only items not present in story problem</td>
<td>Drew rockets and doghouses for # of dogs problem.</td>
</tr>
<tr>
<td>Drew qualitative details with the wrong quantitative details</td>
<td>drew every detail of students in problem.</td>
</tr>
<tr>
<td>Drew qualitative details present with the correct quantitative details</td>
<td>drew entire person for foot problem, but drew the exact number of students</td>
</tr>
<tr>
<td>Drew only qualitative details that were relevant to problem with incorrect number</td>
<td>Drew circles, but miscounted</td>
</tr>
<tr>
<td>Drew only qualitative details that were relevant to problem with correct number</td>
<td>drew feet with five toes</td>
</tr>
<tr>
<td>Drew symbols in disorganized manner</td>
<td>drew lines for feet, but drew all together – IIIIIIIII</td>
</tr>
<tr>
<td>Drew symbols in an organized manner</td>
<td>II II II II II II</td>
</tr>
<tr>
<td>informal written signs for operations</td>
<td>(arrows, lines, drawing in groups)</td>
</tr>
<tr>
<td><strong>Writing</strong></td>
<td></td>
</tr>
<tr>
<td>Wrote letter-like and/or numeral-like shapes</td>
<td></td>
</tr>
<tr>
<td>wrote numerals from the problem</td>
<td>wrote one or more numeral from the problem</td>
</tr>
<tr>
<td>Action-indicating drawings</td>
<td>directional arrows, circles, motion lines</td>
</tr>
<tr>
<td>labeled drawings</td>
<td></td>
</tr>
<tr>
<td>formal written signs for operations used</td>
<td>+, -, =, etc.</td>
</tr>
<tr>
<td>Indication of formal algorithm</td>
<td>signs for operations, alignment</td>
</tr>
<tr>
<td>wrote addition sentences</td>
<td>5+8=13</td>
</tr>
<tr>
<td><strong>counting</strong></td>
<td></td>
</tr>
<tr>
<td>did not count – said an unrelated number</td>
<td>“I thinked it in my head”</td>
</tr>
<tr>
<td>said numbers from problem as answer</td>
<td>note whether said first number or second number</td>
</tr>
<tr>
<td>counted randomly</td>
<td>“1,2, 4, 10, 11…”</td>
</tr>
<tr>
<td>counted 1,2,3,4… without one-to-one correspondence</td>
<td></td>
</tr>
<tr>
<td>counted 1,2,3,4 using one-to-one correspondence</td>
<td>few mistakes, self-correcting when mistakes made.</td>
</tr>
<tr>
<td>counted by twos</td>
<td>2,4,6,8... or 1,2...3, 4...5,6...</td>
</tr>
<tr>
<td>-----------------</td>
<td>---------------------------------</td>
</tr>
<tr>
<td>counted on</td>
<td>5, 6, 7,8,9, 10</td>
</tr>
<tr>
<td>knew sums instantly</td>
<td>“I knew 5+5 was 10 and I added 2 more”</td>
</tr>
<tr>
<td>multiplicative thinking</td>
<td>draws units of 5</td>
</tr>
<tr>
<td><strong>Answers</strong></td>
<td></td>
</tr>
<tr>
<td>Does not give an answer verbally</td>
<td></td>
</tr>
<tr>
<td>Gives a random answer</td>
<td></td>
</tr>
<tr>
<td>Gives one of the numbers from the problem as an answer</td>
<td></td>
</tr>
<tr>
<td>Says and explains an answer, but did not write answer.</td>
<td></td>
</tr>
<tr>
<td>Wrote an answer in numeral form</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX G
HUMAN SUBJECTS APPROVAL

Office of the Vice President For Research
Human Subjects Committee
Tallahassee, Florida 32306-2742
(850) 644-8673, FAX (850) 644-4392

APPROVAL MEMORANDUM

Date: 3/16/2012

To: Nickey Johnson

Address:
Dept.: EDUCATION

From: Thomas L. Jacobson, Chair

Re: Use of Human Subjects in Research
Children Solving Mathematical Word Problems

The application that you submitted to this office in regard to the use of human subjects in the research proposal referenced above has been reviewed by the Human Subjects Committee at its meeting on 03/14/2012. Your project was approved by the Committee.

The Human Subjects Committee has not evaluated your proposal for scientific merit, except to weigh the risk to the human participants and the aspects of the proposal related to potential risk and benefit. This approval does not replace any departmental or other approvals, which may be required.

If you submitted a proposed consent form with your application, the approved stamped consent form is attached to this approval notice. Only the stamped version of the consent form may be used in
recruiting research subjects.

If the project has not been completed by 3/13/2013 you must request a renewal of approval for continuation of the project. As a courtesy, a renewal notice will be sent to you prior to your expiration date; however, it is your responsibility as the Principal Investigator to timely request renewal of your approval from the Committee.

You are advised that any change in protocol for this project must be reviewed and approved by the Committee prior to implementation of the proposed change in the protocol. A protocol change/amendment form is required to be submitted for approval by the Committee. In addition, federal regulations require that the Principal Investigator promptly report, in writing any unanticipated problems or adverse events involving risks to research subjects or others.

By copy of this memorandum, the Chair of your department and/or your major professor is reminded that he/she is responsible for being informed concerning research projects involving human subjects in the department, and should review protocols as often as needed to insure that the project is being conducted in compliance with our institution and with DHHS regulations.

This institution has an Assurance on file with the Office for Human Research Protection. The Assurance Number is FWA00000168/IRB number IRB00000446.

Cc: Ithel Jones, Advisor

HSC No. 2012.7752
REFERENCES


Isaacs, Carroll and Bell (2001)


**BIOGRAPHICAL SKETCH**

Nickey Owen Johnson

### Professional Preparation

**August 2013**  
**Doctor of Philosophy Curriculum and Instruction**  
The Florida State University  
Major: Early Childhood Education  
Dissertation: Kindergarten Children Solving Mathematical Word Problems  
Major Professor: Dr. Ithel Jones

**May 2002**  
**Educational Specialist**  
The Florida State University  
Major: Educational Administration and Leadership

**June 1997**  
**Master of Education**  
Auburn University  
Major: Early Childhood Education

**June 1994**  
**Bachelor of Science**  
Auburn University  
Major: Early Childhood Education

### Professional Experience

**9/08 – 6/2013**  
**Graduate Teaching Assistant and Research Assistant**  
Early Childhood and Elementary Education Departments  
School of Teacher Education, College of Education,  
The Florida State University  
Developed and delivered face-to-face and web-supported instruction to meet university proscribed objectives. Supervised pre-service teachers in their field experiences. Collaborated with School of Teacher Education staff and faculty to evaluate students and plan instruction.  
Courses Taught:  
- Supervision of Interns SP 2013  
- Teaching Math in the Primary Grades FA 2008, FA2012  
- Language Arts & Literature SP 2011  
- Early Childhood Curriculum SP 2010  
- Field Experience Supervisor SP 2010 & SP 2013  
- Differentiated Reading Instruction FA 2009 & FA 2010  
- Parents as Teachers SP 2009

**2007 – 2008**  
**School Administrator – Triple C School, Cayman Islands**  
Early Childhood Education Department Supervisor  
PK – 12 Curriculum Coordinator  
Supervisor for the day-to-day operations of the Early Childhood Department (PS – 3rd). Performed teacher evaluations and established professional improvement plans. Led monthly PS-12 staff meetings comprised of professional development

SP 2006 - 2007
School Administrator - First Baptist Christian School, Cayman Islands
Vice Principal/Curriculum Specialist
Responsible, with principal, for the day-to-day operation of PS-8 school. Reviewed and Revised School Policies. Revised Parent and Staff Handbooks. Carried out teacher evaluations and professional improvement plans. Planned and led weekly staff meetings including professional development component. Participated in Inspectorate (National Accreditation Program) Training for Administrators. Served as Inspectorate Coordinator, collecting documentation for Inspectorate site visit. Wrote Three-Year Strategic Plan. Coordinated the review and revision of curriculum to accommodate national standards. Acted as WEE Care Director and substitute teacher when needed. Handled discipline issues with students. Responsible for implementation of school-wide discipline plan and reporting.

9/2004
Classroom Teacher, Grace Christian Academy, Cayman Islands, Second Grade. School closed as a result of damage sustained in Hurricane Ivan 9/11/04.

2003-2004
Private Schools Association Substitute Teacher, Cayman Islands. Served as long-term substitute teacher at member schools while on maternity leave. Grace Christian Academy, Pre-School, 9/03 – 12/03. Triple C School, First Grade, 1/04-2/04.

2002-2003
Teaching/Administration, First Baptist Christian School, Grand Cayman
First Grade & Fifth Grade
Worked with principal in managing day-to-day operations of the school. Responsible for approving teacher newsletters and report cards. Created and submitted reports as requested. Responsible for teaching fifth grade self-contained class until certified teacher was hired mid-year. Responsible for teaching self-contained first grade class when teacher resigned mid-year.

2001-2002
Graduate Teaching Assistant, Early Childhood Department
The Florida State University

2000 - 2002
Graduate Assistant, Monroe County Distance Learning Project,
Educational Leadership Department, The Florida State University
Converted courses from face-to-face to online format. Worked with Professors to upload course documents and create useful discussion and sharing capabilities. Monitored online discussion boards. Trained professors in using Blackboard.

1999-2000  Teaching, Faulkner Academy, Cayman Islands, Kindergarten
Responsible for teaching Kindergarten class.

1998-1999  Teaching, Triple C School, Cayman Islands, Kindergarten

1995-1998  Teaching, Auburn Early Education Center, Auburn, Alabama
Kindergarten
Taught Kindergarten class. Served on Lee County Science Curriculum Committee. Revised Lee County science curriculum. Received Butterfly House Grant. Supervised university interns.

1994-1995  Graduate Teaching Assistant, Early Childhood Education Department
Lead Teacher, Child Development Center
West Georgia College (University of West Georgia), Carrollton, Georgia
Planned, organized and taught out lessons for the 25 pre-school students. Supervised undergraduate practicum students: scheduling, mentoring and evaluating students in their pre-school practicum.

Summer 1994  Teaching, Opelika City Schools, Summer School Second Grade
Opelika, Alabama
Provided remedial instruction for second grade students.

Professional Development Presentations


Johnson, N. O., (2009). What is so Great About Conscious Discipline? Top Ten Reasons We Use


Professional Service

District School Improvement Committee Parent Representative, Thomas County Schools, Thomasville, Georgia. 2008 - 2011

Wee Bee Boosters (PTO) Officer, Thomas County Schools, Thomasville, Georgia. 2010 - 2011

Wee Bee Booster (PTO) Volunteer, Thomas County Schools, Thomasville, Georgia. 2010 - 2011

Strategic Planning Committee Member. Triple C School, Cayman Islands 2007-2008


Elementary Teachers PTA Representative, Faulkner Academy, Cayman Islands 1999-2000.

Inspectorate (National School Accreditation) Committee, Coordinator, Faulkner Academy, Cayman Islands 1999-2000.

Inspectorate (National School Accreditation) Committee, Member, Triple C School, 1998-1999.
