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Design and Modeling of Very High-Efficiency Multijunction Solar Cells

Indranil Bhattacharya
DESIGN AND MODELING OF VERY HIGH-EFFICIENCY MULTIJUNCTION SOLAR CELLS

By

INDRANIL BHATTACHARYA

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Indranil Bhattacharya defended this dissertation on June 20, 2013.
The members of the supervisory committee were:

Simon Y. Foo
Professor Directing Dissertation

Anke Meyer-Baese
University Representative

Jim P. Zheng
Committee Member

Petru Andrei
Committee Member

Hui Li
Committee Member

The Graduate School has verified and approved the above-named committee members, and certifies that the dissertation has been approved in accordance with university requirements.
Dedicated to my loving parents (Mrs. Sipra Bhattacharya and Mr. Benoy Bhattacharya), brother (Mr. Rudranil Bhattacharya), professors & teachers, friends & well wishers and all who have inspired me to get to where I am today. Thank you all.
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ABSTRACT

The main challenge in the solar cell industry is making the solar cells more cost effective. Mono and poly-crystalline Si, CdTe, CIGS, Quantum dot, Organic and Dye-sensitized solar cell technologies do not produce high efficiencies. A low bandgap semiconductor generates larger current due to photon absorption over broader spectral region but do not produce high open circuit voltage because it is limited by the dark current of the low bandgap material. This limits them within the Shockley efficiency limit of 30%. The relevant solutions are to increase the efficiency of solar cells, for example by effective spectral splitting by different bandgap semiconductor subcell layers, implementation of III-V direct bandgap optically sensitive and high carrier mobility semiconductors, form better matching (lattice, optical and electrical) between subcell layers, usage of concentrator Fresnel lenses and most importantly reduce the fabrication cost of the epitaxial layers. In this work we introduced two novel quadruple junction solar cell designs, each having four semiconductor subcell layers. We have simulated the quantum efficiency vs. wavelength, current density vs. voltage, power density vs. voltage and compared the photon absorption of our two novel designs with state of art single junction and multijunction solar cells. We showed that antimony based subcell layers help in higher photon absorption in the infrared (IR) region. Photonic modeling is implemented by the transfer-matrix method of wave propagation through multilayer structures. The first quadruple junction solar cell design comprises of AlGaInP (2.3eV) / InGaAs (1.1eV) / GaSb (0.7eV) / InGaSb (0.5eV) and the second design comprises of AlGaInP (2.3eV) / InGaP (1.93eV) / InGaAs (1.1eV) / InGaSb (0.5eV) III-V direct-bandgap semiconductor materials. The quadruple-junction subcell layers capture photons of ultraviolet (UV), visible, and near- and far infrared (IR) regions of the electromagnetic spectrum. The combination of our subcell layers yield favorable photon absorption results in comparison to the state of art solar cells.
CHAPTER ONE

PHOTOVOLTAICS: AN INTRODUCTION

1.1 Photovoltaics: Mechanism, Advantages, Challenges and Different Generations

A close observation of natural phenomena has unfolded mysteries leading to major scientific discoveries. If we observe the photosynthesis mechanism in plants we find that plants acquire packets of energy (quanta) or photons from sunlight (electromagnetic radiation) and with the help of other substrates like carbon dioxide and water, metabolizes into carbohydrates such as sucrose, glucose or starch. This is a perfect example of light energy converted into chemical energy. This analogy is very similar to the conversion of solar energy into electrical energy by the photovoltaic cells. Photovoltaic is the technology of generating DC power in watts (W) or kilo-watts (kW) from the semiconductors in the event of illumination of semiconductor subcell layer by photons of energy equal to or higher than the band gap of the specified semiconductor material. Photovoltaic effect was first experimentally demonstrated by French Physicist A. E Becquerel in 1839.

1.1.1 Mechanism

Solar cells are made up of semiconductor materials having weakly bonded electrons occupying a band of energy named as valence band (VB). In the incident of energy exceeding a threshold value, referred to as the bandgap energy is applied to valence electrons, the covalent bonds are broken. In case of covalent bond being broken the freed electrons jump in a new energy band called the conduction band (CB) where they conduct electricity through the material. The free electrons in conduction band are now separated from the valence band by the bandgap energy measured in electron-volts (eV). This threshold energy is being supplied by the particles of light named as photons. When a solar cell is exposed to solar radiation of sufficient energy the incident photons are absorbed by atoms breaking the bonds of valence electrons and exciting them to the higher energy level in the conduction band. A specially made selective contact collects the conduction band electrons and drives them towards the external circuit. These electrons lose energy by performing work in the external circuit lighting an electric bulb, spinning an electric fan, powering a computer etc. The freed electrons are restored again in the
solar cell by the return loop of the circuit via a second contact. The electrons will return in the valence band with the same energy before illumination. Movement of the free electrons in the external circuit and the contacts is the electric current. This process is explained in figure 1.1 (a). The potential of the electrons is less than the threshold energy exciting the electrons. It is independent of the energy of photon as long as the energy is higher than the threshold. The electrical power generated is the product of current and voltage or number of free electrons times their electric charge time voltage [1]. Increased solar radiation will free more electrons contributing to higher power generation.

![Figure 1.1: (a) Mechanism of electron excitation from the valence band to the conduction band by incident photons, Courtesy of Handbook of Photovoltaic Science and Engineering, (b) Structure, generation and movement of free carriers in a PV cell, Courtesy of Clean Electricity from Photovoltaics](image)

A photovoltaic cell is normally a semiconductor diode containing a $p$-$n$ junction between two different materials across which there is a ‘built-in’ electric field as mentioned in figure 1.1 (b). The $n$-type emitter layer is thinner than the $p$-type base layer. The work function of the $p$ material is greater than that of $n$ material and the two layers attain electronic equilibrium by transferring some electrons from the $n$ to the $p$ side. The cell structure remains electrically neutral at open circuit condition but the junction region contains an electric double layer made by two space charge or depletion regions. At the event of cell illumination energized photons excite the electrons to jump from the valence band to the conduction band breaking the covalent bonds and create electron-hole pairs. This incident depends upon the band-gap of the semiconductor material and the wavelength of the incident light spectrum. The free carriers (electron and holes)
approaching the junction comes under the influence of built-in electric field and the electrons are swept from the \( p \) side to the \( n \) side and the holes from the \( n \) side to the \( p \) side [2]. The rate of generation of electric charge carriers depend on the flux of incident light and photon absorption efficiency of the semiconductor material. The photon absorption efficiency depends on solar irradiance, band-gap of semiconductor material, reflectance of the top surface, doping and carrier concentration, mobility of charge carriers, recombination rate etc.

1.1.2 Advantages of Photovoltaic Technology

Photovoltaic cells are the most remarkable photo-conversion devices and the most emerging renewable energy resource for the 21\textsuperscript{st} century.

- Clean technology; do not contribute in climate change or any kind of pollution.
- Fuel source is enormous, easily accessible and infinite.
- Generated anywhere, tropical, rural, urban, distributed or grid-feeding mode.
- No emissions, combustion or nuclear waste disposal.
- High reliability of solar cell modules (manufacturers guarantee over 25 years).
- Uniquely scalable, can supply power in milliwatts to megawatts.
- Proximity to demand, no transmission cost, can be used for stand-alone and grid-connected applications.
- Non toxic (except CdTe), made up of thin semiconductor wafers or films.
- No moving parts (wear and tear) and silent operation
- Easily integrated in new or existing building structures.
- Rapid installation

1.1.3 Challenges of Photovoltaic Technology

Photovoltaic technology has some major challenges to overcome.

- Diffuse fuel source or relatively low energy density.
- Intermittence and seasonality of sunlight.
- Inefficient and inexpensive storage devices.
- Solar Cells generated electricity ($1/kWh) vs. grid electricity (15¢/kWh).
- High installation costs.
- Solar conversion efficiency very low (8% - 18%).

1.1.4 Different Generations of Solar Cell (Photovoltaic) Technology

The development of solar cell technologies has been driven by the objective of making the solar cells cost effective for stand-alone and grid-connected applications. Solar cell technologies are classified into three generations depending upon the emergence. Continuous research is going on each of the three generations however first generation of solar cells is mostly in production representing > 85% of total solar cell production.

**First Generation: Mono and Poly-crystalline Silicon:** The first generation of solar cells is mostly comprised of high quality mono-crystalline silicon wafers having a semiconductor p-n junction. This technology is already matured and slowly approaching the Shockley theoretical efficiency of 31%. Growing large crystals of pure silicon is a very difficult and energy-intensive process, which is a major challenge in the reduction of production cost. Another challenge of mono-crystalline silicon solar cells is that the efficiency decreases with increasing temperature. Poly-crystalline silicon solar cells are an alternative to mono-crystalline formation. They are formed from multiple silicon crystals in mold reducing the cost along with a reduction in efficiency. The first generation devices attain cost parity with fossil fuel energy generation after a payback period of 5-7 years; however the cost of first generation technology is very unlikely to go down to $1/kWh.

**Second Generation: Thin-film Solar Cell:** Second generation solar cells have been researched to meet the challenges of expense and energy requirements of first generation solar cells. Reducing the amount of material needed will decrease the cost of first generation solar cells. Wafers of crystalline silicon are usually 150-300 μm thick. Advancement in technologies like chemical vapor deposition, metal organic chemical vapor deposition, molecular beam epitaxy, chemical phase epitaxy, electroplating, pulsed laser deposition etc. has helped in the development of thin film technology. The main difference between first and second generation devices is that the second generation solar cells are formed on thin films of semiconductor formed by any deposition techniques unlike the formation of first generation solar cells from the bulk crystalline silicon material. The deposited layers are quite thin in comparison to first generation solar cells reducing the fabrication cost. There are mainly three categories of thin film
solar cells - amorphous silicon, cadmium telluride or cadmium sulfide (CdTe/CdS) and the chalcopyrite family alloys like copper indium gallium (di) selenide (CIGS). First Solar is a pioneering manufacturer of thin film solar cells made up of CdTe material having the cost of 1$/kWh. This is the least expensive solar cell in the market; however there are some concerns about the technology due to the toxicity of the material. CIGS solar cells are yet to become a commercial success. The researchers have been able to produce 20% efficiency in the laboratory but mass production of CIGS solar cells is a big challenge. Organic materials also fall in this category as the processing is relatively simple and less expensive; however very low efficiency (1% - 5%) is the major shortcoming of this technology [3, 4].

Third Generation: Full Spectrum Utilization: First and second generation solar cells has two fundamental limitations. First the photons of energy less than the bandgap of the semiconductor material are not absorbed by the solar cell and secondly even if a photon has energy greater than the bandgap, actually the photons of equal energy of the bandgap are effective for conversion into electricity. These limitations have encouraged the researchers all over to design multijunction PV or tandem PV cells utilizing the entire solar spectrum with different bandgap energy semiconductors. The multijunction device is basically a stack of cells each capturing a different portion of the spectrum. The materials of which the stack of cells are made have different bandgap energies with the topmost cell having the highest bandgap and the bottom cell having the lowest bandgap energy. Higher energy photons are absorbed in the top cell layers and lower energy photons are passed to the bottom cell layers. The most efficient PV devices are made up of this multijunction technology. This technology is currently used for terrestrial purposes only but will be available for commercial applications soon. This technology will address the shortcomings of first and second generation PV technologies [3, 5].

1.2 Motivation of High-Efficiency Multijunction Research

Sunlight is the most abundant renewable energy resource having the average intensity of approximately 0.1W/cm² and over 1.5×10²²J (15,000 Exajoules) impinging earth’s surface every day. This enormous energy is 11,000 times greater than the daily consumption of 1.3EJ of the world [6, 7]. Earth’s ultimate recoverable resource of fossil fuel reserve is about 5,735 billions of barrels (BOE) according to the Congressional Research Service, US Fossil Fuel Resources [8].
This 5,735 BOE of fossil fuel yields $3.25 \times 10^{22}$ J of energy that is delivered to our planet by sun in less than 2 and 1/2 days. The main challenge in the photovoltaic industry is making the solar cells more cost effective. The PV generated electricity $1/kWh$ is still unfavorable to the grid electricity of $15\,\text{¢}/kWh$. These data definitely shows that there is a lot of space in research to make the solar cells more cost effective. Increasing the efficiency of the existing solar cells from 15% to 40% will reduce the material usage and land acreage the cost can be reduced significantly. The relevant solutions are increase the efficiency of solar cells, effective spectral splitting by different bandgap semiconductor subcell layers, implementation of III-V direct bandgap optically sensitive and high carrier mobility semiconductors, form better matching (lattice, optical and electrical) between subcell layers, usage of concentrator Fresnel lenses and most importantly reduce the fabrication cost of the epitaxial layers [9]. Single junction solar cells can only absorb a certain wavelength of the solar spectrum, hence produce less efficiency. In contrary multijunction solar cells direct sunlight towards matched spectral sensitivity by splitting the spectrum into smaller slices.

The high efficiency multijunction photovoltaics made up of III-V semiconductor material alloys with high optical sensitivity and ideal combinations of band-gaps increase absorption of photons,
helps overcome the dark current limitation, creates more electron-hole pairs and increase the efficiency of the solar cell. The multijunction solar cells have the highest promise of increasing the efficiency as evidenced by the efficiency vs. year chart data mentioned by National Renewable Energy Lab (NREL) and shown in figure 1.2. National Renewable Energy Laboratory (NREL), US Department of Energy (DOE) and many leading research organizations all over the world are investing money in the design of III-V multijunction solar cell projects.

1.3 Organization of Work

The dissertation provides an elaborated description about the device physics of solar cells, fundamental properties of semiconductors, different kinds of losses and reduction, III-V semiconductors and working of high efficiency multijunction solar cells, photonic modeling through transfer-matrix method of two novel quadruple-junction designs and electrical modeling of single and multijunction solar cells. The current density vs. voltage, power density vs. voltage and quantum efficiency of the novel designs has been discussed.

Chapter 2 discusses about device physics of solar cells, fundamental semiconductor properties, crystal and bandgap structure, particles in valence and conduction band at equilibrium and after excitation. Different recombination mechanisms like trap, radiative and Auger has been discussed. Carrier transport, semiconductor equations, minority carrier diffusion mechanism and particle movement in p-n junction has been described. Solar cell fundamentals along with generation rate, solar cell characteristic and properties of efficient solar cells have been discussed.

In Chapter 3 we will discuss about some other important aspects of solar cell design. The topics of interest ranges from optical design like light trapping in solar cell, surface recombination losses and their reduction, bulk recombination losses and their reduction etc. Light trapping structures, recombination losses in several interconnects, bulk recombination losses, grain boundary and losses and methods of loss reduction by graded base design, p-i-n junction and parallel multijunction processes are discussed.

Chapter 4 provides detailed description about design physics of high efficiency multijunction solar cells, wavelength dependence of photon conversion and effective spectrum splitting.
Quantum efficiency and current density calculation and formulas, cell construction and important considerations like current matching and top cell thinning are discussed. Effects of concentrator solar cells and anti-reflective coatings are explained. Different fabrication methods like lattice-matching, lattice-mismatching and inverted metamorphic designs of several combinations of semiconductors are discussed. The effects of tunnel junction, window layers and back surface field layers for good quality multijunction designs are discussed. Lastly different cell configuration and comparative analysis of fabrication technologies have been discussed.

Chapter 5 discusses about two novel quadruple-junction solar cell designs comprised of AlGaInP/InGaAs/GaSb/InGaSb and AlGaInP/InGaP/InGaAs/InGaSb subcell layers. The subcell layers implemented are all direct bandgap semiconductors, have very high absorption coefficient, good optical and electrical properties and all have zinc-blende lattice structures. The transfer-matrix method of wave propagation through multi-layer structures has been discussed. The matrices are formed for the intersection between two layers and wave propagation through each layer. The product of all the transfer matrices forms the actual transfer-matrix for each solar cell designs. The photon absorption of these two solar cell designs has been favorably compared with the state-of-art solar cell designs. The combination of subcell layers has yielded very high photon absorption through the entire solar radiation spectrum.

Chapter 6 describes the electrical modeling of single-junction and multijunction solar cell designs. The band structure and different current and voltages under equilibrium as well as in the condition of illumination are described. The band banding and the respective generation and recombination currents have been demonstrated. The current to voltage characteristics of simple solar cell model is demonstrated defining the voltage source and current source regions. Several important parameters like open-circuit voltage ($V_{OC}$), short-circuit current ($I_{SC}$), fill-factor (FF), maximum power-point (MPP) and cell efficiency has been discussed. Simple solar cell model, two diode model along with series and shunt resistance effects has been discussed. The temperature effects of solar cells have been executed. Current and voltage equations have been formed for all the different kind of modeling. Multijunction solar cell design for quadruple junction has been demonstrated. This designing takes into consideration the same current for all the subcell layers (current matching) and the voltage effect is the summation of the individual voltages generated in each subcell layers. Different loss mechanisms like the dark current
(saturation current), recombination losses in each subcell layers have been considered along with series and shunt losses. The two diode model representation of quadruple junction design is simulated. The quantum-efficiency or the photon conversion to electricity formulas as well as the simulation for quadruple junction designs have been described. The novel designs execute very high quantum efficiency. Lastly the current density vs. voltage simulations and power density vs. voltage simulations are demonstrated.
A solar cell is a semiconductor device designed and fabricated to efficiently absorb the light energy and convert it into electrical energy. Sunlight impinges through the top surface of the solar cell. A front metal grid forms one of the electrical contacts of the solar cell and allows light to fall on the device between the grid lines, gets absorbed and converted into electricity. Anti-reflective coating (ARC) is used to minimize the reflection on the top surface. The semiconductor diode is formed when an $n$-type and $p$-type semiconductor are brought together forming a metallurgical junction. This is achieved through diffusion or ion implantation of specific impurities or by deposition process. When the cell absorbs light, mobile electrons and holes are created. The electrons are holes move in opposite directions as shown in figure 2.1 and contribute in the generation of DC power from the illuminated cell.

Any electromagnetic radiation including solar radiation is composed of packets of energy or quanta which are made up of photons. Photons have wavelike properties having wavelength $\lambda$ related to photon energy $E_p = \frac{hc}{\lambda}$ where $h$ is Plank’s constant and $c$ is the velocity of light. Photons having energy equal or higher than the bandgap ($E_g$) of the semiconductor are capable of generating electricity. Spectral composition and utilization of solar radiation spectrum is very important in designing efficient solar cells.

Figure 2.1: Cross section of a simple conventional solar cell
Sunlight is considered equivalent to blackbody radiation at 5762K temperature and the radiation is isotropic in nature. The photons directly emitted towards the earth are only utilized due to huge distance between the sun and earth, hence the light falling on earth is considered as the parallel streams of photons. Above the earth’s atmosphere the radiation intensity is 1.353 kW/m² and the spectral distribution is considered as air mass zero (AM0). Air mass is a measure of how absorption in atmosphere affects spectral content and intensity of solar radiation. The air mass is derived by AM (Air Mass) = 1 / \cos\theta, where \theta is the angle of incidence [1, 10]. The air mass is greater than 1 always because of the refraction of sunlight in the medium before reaching the earth. The standard considered realistic by the American Society for Testing and Materials (ASTM) is AM 1.5 with the zenith angle of 48.19° and average solar irradiance of 1kW/m². Earth’s surface also consists of diffuse light component due to the scattering and reflection in the atmosphere; hence AM 1.5g standard is introduced to include the diffuse component.

2.2 Fundamental Properties and Physics of Semiconductors

Solar cells are formed by a number of simple or compound semiconductor materials most commonly mono- or poly-crystalline silicon (Si), gallium-arsenide (GaAs), cadmium-telluride (CdTe), copper-indium-gallium-diselenide (Cu(InGa)Se₂), gallium-indium-phosphide (GaInP) etc. The materials are chosen depending upon how well absorption characteristics matches solar radiation spectrum and fabrication cost. Silicon has been the common choice because it is the second most abundant element on earth’s crust and fabrication technology is well matured as a result of its importance in semiconductor electronics industry.

2.2.1 Semiconductor Crystal and Energy Band Structure

Electronic semiconductors are crystalline structures having atoms aligned in a periodic array. Atomic properties and periodicity give semiconductors their important electronic properties. A silicon atom has four valence electrons in the outermost shell to form covalent bonds with the next atom and orient itself in a diamond lattice structure. This tetrahedral diamond lattice structure has an angle of 109.5° between the bonds. The arrangement is represented by two face-centered-cubic (fcc) unit cells in which the second one is shifted ¼ th of the distance along the diagonal of the first unit cell. Lattice constant \( l \) is the length between the edges of the cubic unit cell [1]. One similar structure named as zinc-blende lattice formation occurs in III-V and II-VI
semiconductors such as GaAs, GaInP, GaInAs (III-V compound) and CdTe (II-VI compound). In case of GaAs one interpenetrating fcc unit cell comprises of Ga atoms and the other entirely of As atoms. Average valency is four for each compound; hence there are four bonds to and from each atom with each covalent bond of two valence electrons.

An electron moving in a semiconductor is similar to a particle confined in a three-dimensional box. The dynamic behavior of the electron can be stated by the electron wave function $\Psi$ solving the time-independent Schrödinger equation

$$\nabla^2 \Psi + \frac{2m}{\hbar^2} [E - U(r)] \Psi = 0 \quad (2.1)$$

Here $m$ is electron mass, $\hbar$ is reduced Planck constant, $E$ is the energy of the electron and $U(r)$ is the periodic potential energy inside the semiconductor. Solving the quantum mechanical equations is beyond the scope of the chapter; however the solution defines the band structure (electron energies and relation of electron energy with momentum) of the semiconductor and conveys information about the quantum mechanically computed motion of the electron in the crystal. If the mass of electron in free space is $m$ then it is replaced by effective mass $m^*$ in Newton’s second law of motion. Newton’s second law of motion from classical mechanics is given by

$$F = m^* a \quad (2.2)$$

Here $F$ is the applied force and $a$ is the acceleration of the electron. The electron effective mass $m^*$ is represented by the equation

$$m^* = \left[ \frac{d^2E}{dp^2} \right]^{-1} = \left[ \frac{1}{\hbar^2} \frac{d^2E}{dk^2} \right]^{-1} \quad (2.3)$$

Here $p = \hbar k$ is the crystal momentum of the electron and $k$ is the wave vector. Near the top of the valence band, the effective mass is negative. Electrons fill the states from bottom to top and the states near the top of the valence band are empty due to some electrons being thermally excited into the conduction band. The empty states can be considered as positively charged carriers or holes having positive effective mass. It is conceptually much easier dealing with a relatively few number of holes that have a positive effective mass since they behave like classical positively charged particles. The effective mass is not constant within each band. The top of the valence band and the bottom of the conduction band are parabolic in shape and therefore the electron effective mass $m_n^*$ near the bottom portion of conduction band is constant similar to the hole effective mass $m_p^*$ near the top of the valence band. This practical assumption greatly simplifies modeling of semiconductor devices like solar cells. When the minimum of the conduction band
occurs as same value of the maximum of the valence band then that type of semiconductor is called a direct bandgap semiconductor. In case the momentum of electrons in the valence band and the conduction band not aligned the type of semiconductor is called an indirect bandgap semiconductor. This is very important in consideration of light absorption by a semiconductor material. Amorphous materials exhibit a similar band structure. Over short distances, the atoms are arranged in a periodic manner and an electron wavefunction is defined. The wavefunctions from these small regions overlap in such a way that a mobility gap can be defined, with electrons above mobility gap denoting the conduction band and holes below the gap as the valence band. Unlike crystalline materials there is large number of localized energy states within the mobility gap.

### 2.2.2 Valence-Band and Conduction-Band Densities

Once the dynamics of the electron motion in a semiconductor is approximated by negatively charged particle having effective mass \( m_n^* \) in the conduction band and by positively charged particle having effective mass \( m_p^* \) in the valence band it is possible to calculate the densities of states in each band. This involved solving the time-independent Schrodinger equation for wavefunction of a particle in a box being the box empty. The complexities of the periodic potentials of the component atoms have been incorporated into the effective mass [1]. The density of states in conduction band and in the valence band is represented by [10]

\[
G_C (E) = m_n^* \sqrt{2} m_n^* (E - E_C) / \pi^2 h^3 \ cm^{-3} \ eV^{-1} \quad (2.4)
\]

\[
G_V (E) = m_p^* \sqrt{2} m_p^* (E_V - E) / \pi^2 h^3 \ cm^{-3} \ eV^{-1} \quad (2.5)
\]

### 2.2.3 Equilibrium Carrier Concentrations

In normal condition of the semiconductor the Fermi function determines the ratio of filled states to available states at each energy and is denoted by

\[
F (E) = 1 / (1 + e^{(E - E_F) / kT}) \quad (2.6)
\]

Here \( E_F \) is the Fermi energy, \( k \) is the Boltzmann’s constant and \( T \) is the temperature in degree Kelvin. The Fermi function is a strong function of temperature. At absolute zero it behaves like a step function and all the states below \( E_F \) are filled with electrons and the states above \( E_F \) are completely empty. With the increase in temperature thermal excitation will leave some states
below $E_F$ empty and the states above $E_F$ will be filled with excited electrons. The equilibrium electron and hole concentrations (number/cm$^3$) is mentioned by

$$n_0 = \int_{E_C}^{\infty} G_C(E) F(E) \, dE = (2 \, N_C / \sqrt{\pi}) \left[ F_{1/2} ((E_F - E_C) / kT) \right] \quad \ldots(2.7)$$

$$p_0 = \int_{E_C}^{\infty} G_V(E) [1 - F(E)] \, dE = (2 \, N_V / \sqrt{\pi}) \left[ F_{1/2} ((E_V - E_F) / kT) \right] \quad \ldots(2.8)$$

Here $F_{1/2}$ is the Fermi-Dirac integral of order 1/2.

The conduction-band and valence-band effective densities of state $N_C$ and $N_V$ are expressed as

$$N_C = 2 \left( \frac{2\pi m_n^* kT}{\hbar^2} \right)^{3/2} \quad \ldots(2.9)$$

$$N_V = 2 \left( \frac{2\pi m_p^* kT}{\hbar^2} \right)^{3/2} \quad \ldots(2.10)$$

In case of Fermi energy $E_F$ sufficiently distant from each band edge it is called as non-degenerate semiconductors, the carrier concentrations can be approximated as

$$n_0 = N_C e^{(E_F - E_C) / kT} \quad \ldots(2.11)$$

$$p_0 = N_V e^{(E_V - E_F) / kT} \quad \ldots(2.12)$$

In non-degenerate semiconductors, the product of the electron and hole concentrations is independent of the location of the Fermi energy and is given by

$$n_0 \, p_0 = n_i^2 = N_C N_V e^{(E_V - E_C) / kT} = N_C N_V e^{-E_G / 2kT} \quad \ldots(2.13)$$

In case of intrinsic (undoped) semiconductor in thermal equilibrium, the number of electrons in the conduction band and the number of holes in the valence band are equal $n_0 = p_0 = n_i$ where $n_i$ is the intrinsic carrier concentration. The intrinsic carrier concentration is denoted by

$$n_i = \sqrt{(N_C N_V) e^{(E_V - E_C) / 2kT}} = \sqrt{(N_C N_V) e^{-E_G / 2kT}} \quad \ldots(2.14)$$

The Fermi energy in an intrinsic semiconductor $E_i = E_F$ is expressed as

$$E_i = [(E_V + E_C) / 2] + [kT/2 (N_V / N_C)] \quad \ldots(2.15)$$

This is typically very close to the middle of the bandgap. The intrinsic carrier concentration is very small compared to the densities of states and typical doping densities and intrinsic semiconductors behave much like insulators. The number of electrons and holes in their respective bands can be regulated through the introduction of specific impurities or dopants called donors and acceptors. All impurities introduce additional localized electronic states in the
band structure, sometimes within the forbidden bandgap between $E_C$ and $E_V$ as shown in figure 2.2.

![Figure 2.2: Donor and acceptor levels in a semiconductor, Courtesy of Handbook of Photovoltaic Science and Engineering, Page 90](image)

If the energy of the state $E_D$ introduced by a donor atom is very close to the conduction band (within few kT) there will be sufficient energy to energize extra electron to occupy a state in the conduction band. The donor stage will then be positively charged. In case of acceptor atom there will be a negatively charged state $E_A$. Controlled introduction of donor and acceptor impurities in a semiconductor will allow creation of $n$-type (electrons as primary carriers of electric current) and $p$-type (holes as primary carriers of electric current) semiconductors. This is the basic principle of creation of semiconductor devices including solar cells. The number of ionized donor and acceptors are depicted as

\[
N_D^+ = N_D / [1 + G_D e^{(E_F - E_D') / kT}] = N_D / [1 + e^{(E_F - E_D') / kT}] \quad \text{...(2.16)}
\]

\[
N_A^- = N_A / [1 + G_A e^{(E_A - E_F') / kT}] = N_A / [1 + e^{(E_A - E_F') / kT}] \quad \text{...(2.17)}
\]

Here $G_D$ and $G_A$ are the donor and acceptor site degeneracy factors. These factors are normally combined into the donor and acceptor energies so that $E_D' = E_D - kT \ln G_D$ and $E_A' = E_A + kT \ln G_A$. The donors and acceptors are assumed to be completely ionized so that $n_0 \approx N_D n_0 \approx N_D$ in $n$-type material and $p_0 \approx N_A$ in $p$-type material. The Fermi energy in $n$-type material and then in $p$-type material can be written respectively as

\[
E_F = E_i + kT \ln N_D / n_i \quad \text{...(2.18)}
\]

\[
E_F = E_i - kT \ln N_A / n_i \quad \text{...(2.19)}
\]
If a large concentration of dopants is introduced into semiconductor then they have significant effect on the band structure. This heavy doping effect reduces the bandgap $E_G$ and increases the intrinsic carrier concentration. This bandgap narrowing is detrimental to solar cell performance and solar cells are designed to avoid this kind of effects. Only near the solar cell contacts heavy doping is a factor [1, 11].

2.2.4 Light Absorption

The generation of electron-hole pairs by the absorption of sunlight is the main operation of solar cells. The excitation of electron from the valence band to the conduction band is denoted as fundamental absorption. In case of electron excitation a hole is left behind in the valence band. The total energy and the momentum of all particles involved in the absorption process must be conserved. The photon momentum $p_\lambda = h / \lambda$ is very small compared to the crystal momentum $p = h / l$, the photon absorption process effectively conserves the momentum of the electron. Absorption coefficient of a semiconductor for a given photon energy is proportional to the probability $P_{1, 2}$ of the transition of an electron having the initial state energy $E_1$ to the excited state energy $E_2$. The density of electrons in the initial state $G_V (E_1)$ and density of electrons for available final states is the summation over all possible transition states where $E_2 - E_1 = h\nu$.

$$\alpha (h\nu) \propto \Sigma P_{1, 2} G_V (E_1) G_C (E_2) \ldots (2.20)$$

Assuming that all valence band states are full and all the states in the conduction band empty, photon absorption results in creation of electron-hole pairs because an excited electron from the valence band to the conduction band will leave behind a free hole in the valence band.

2.2.4.1 Direct and indirect bandgap semiconductors

Direct bandgap semiconductors such as GaAs, GaInP, GaInAs, CdTe, GaInSb, Cu(InGa)Se$_2$ have both energy and momentum conserved during the excitation and transition phases. Every electron with initial state energy $E_1$ and crystal momentum $p_1$ in the valence band is associated with final state in the conduction band at energy $E_2$ and crystal momentum $p_2$. As the momentum of electron is conserved, the crystal momentum of the final state is the same as the momentum of the initial state i.e. $p_1 \approx p_2 = p$. Conservation of energy implies that the energy of the absorbed photon is
\[ \hbar \nu = E_2 - E_1 \ldots (2.21) \]

The valence and conduction bands have been assumed parabolic band as shown in figure 2.3; hence

\[ E_V - E_1 = \frac{p^2}{2m_p^*} \ldots (2.22) \]
\[ E_2 - E_C = \frac{p^2}{2m_n^*} \ldots (2.23) \]

![Figure 2.3: Photon absorption in direct bandgap semiconductor for an incident photon with energy \( \hbar \nu = E_2 - E_1 > E_G \). Courtesy of Handbook of Photovoltaic Science and Engineering, Page 91](image)

Combining equations (2.22) and (2.23) results in

\[ \hbar \nu - E_G = \left( \frac{p^2}{2} \right) \left[ \frac{1}{m_n^*} + \frac{1}{m_p^*} \right] \ldots (2.24) \]

The absorption coefficient for direct transition is represented by

\[ \alpha (\hbar \nu) \approx A^* \sqrt{(\hbar \nu - E_G)} \ldots (2.25) \]

Here \( A^* \) is a constant. In some semiconductors quantum selection rules do not allow transitions at \( p = 0 \) but allow them for \( p \neq 0 \). For that condition [12]

\[ \alpha (\hbar \nu) \approx \left( \frac{B^*}{\hbar \nu} \right) \sqrt{(\hbar \nu - E_G)^3} \ldots (2.26) \]

\( B^* \) is also a constant.
In case of indirect bandgap semiconductors such as Si and Ge, the valence band maximum occurs at a different crystal momentum than the conduction band minimum. Conservation of electron momentum implies that this difference in momentum in electrons in the two bands generate an intermediate process during the photon absorption for the energy transfer. The energy transfer process creates an additional low-energy and very-high momentum particle named phonon that represents the lattice vibrations in the semiconductor. This incident is depicted in figure 2.4.

Figure 2.4: Photon absorption in direct bandgap semiconductor for a photon with energy \( h\nu < E_2 - E_1 \) and a photon with energy \( h\nu > E_2 - E_1 \). Energy and momentum are conserved by the absorption and emission of a phonon particle respectively. Courtesy of Handbook of Photovoltaic Science and Engineering, Page 92

The light absorption is facilitated by either phonon absorption or phonon emission. The absorption coefficient in case of phonon absorption and phonon emission is expressed as

\[
\alpha_a (h\nu) = A \left( h\nu - E_G + E_{ph} \right)^2 / \left( e^{E_{ph}/kT} - 1 \right) \quad (2.27)
\]

\[
\alpha_e (h\nu) = A \left( h\nu - E_G - E_{ph} \right)^2 / \left( 1 - e^{-E_{ph}/kT} \right) \quad (2.28)
\]

Both the absorption and emission processes are feasible and therefore

\[
\alpha (h\nu) = \alpha_a (h\nu) + \alpha_e (h\nu) \quad (2.29)
\]

As both electron and phonon are needed for the indirect gap absorption process the absorption coefficient not only depends on the density of full initial electron states and empty final electron
states but also on the availability of both emitted and absorbed phonons with the required momentum. For this reason in comparison to direct bandgap semiconductors the indirect bandgap semiconductors has a lower absorption coefficient. The light penetrates more deeply into indirect bandgap semiconductors than direct bandgap semiconductors. In both direct and indirect bandgap semiconductors a number of photon absorption processes are involved, however the mechanisms mentioned are more dominant. A direct transition without the phonon assistance is possible in indirect bandgap semiconductors if the photon has a very high energy. Similarly in direct bandgap semiconductors phonon-assisted absorption is a possibility. There are several other mechanisms in determining the optical absorption in semiconductors. These include absorption in presence of an electric field, absorption supported by localized states in the forbidden gap, degeneracy effects when a significant number of states in the conduction band are not empty or a significant number of states in the valence band are not full (happen in heavily doped semiconductors). The cumulative absorption coefficient is then the sum of absorption coefficients involved in the absorption process.

\[ \alpha (h\nu) = \Sigma \alpha_i (h\nu) \ldots (2.30) \]

The rate of creation of electron-hole pairs per cm\(^3\) as a function of position within a solar cell is

\[ G (x) = (1 - s) \int [1 - r(\lambda)] f(\lambda) \alpha(\lambda) e^{-\alpha x} d\lambda \ldots (2.31) \]

Here \( s \) is the grid-shadowing factor, \( r(\lambda) \) is the reflectance, \( \alpha(\lambda) \) the absorption coefficient and \( f(\lambda) \) is the incident photon flux (number of photons incident per unit area per second per wavelength). The absorption coefficient is expressed in terms of light’s wavelength by the relationship \( h\nu = hc/\lambda \). The photon flux \( f(\lambda) \) is obtained by dividing incident power density by photon energy at each wavelength. Free carrier absorption in which electrons in the conduction band absorb energy of a photon and move to an empty state higher in the conduction band (same for holes in the valence band) is only significant for photons with \( E < E_G \) because the free carrier absorption coefficient increases with increasing wavelength. In single-junction solar cells it does not affect the creation of electron-hole pairs and is ignored; however free carrier absorption needs to be considered in tandem solar cell systems in which a higher bandgap (\( E_{G1} \)) subcell layer is stacked on top of a lower bandgap subcell (\( E_{G2} < E_{G1} \)). Photons with energy lower than the bandgap of the top subcell (\( h\nu < E_{G1} \)) will be transmitted to the next subcell layers in the stack for absorption (if \( h\nu > E_{G2} \)). There can be more subcell layers as long as \( E_{G1} > E_{G2} > \)
E_{G3}… and so on. The number of photons transmitted to the next cell in the stack will be reduced by whatever amount of free carrier absorption occurs. This loss can be reduced by splitting the incident spectrum and directing the matched portion of the spectrum to each subcell layer of the multijunction system [1].

### 2.2.5 Recombination

In case of semiconductors perturbed of thermal equilibrium by illumination and/or injection of current the concentrations of electrons and holes try to come back towards the equilibrium condition through a process referred to as recombination in which the electrons fall from the conduction band back to the valence band. There are several recombination mechanisms important to the operation of solar cells: i) recombination through traps (defects) in the forbidden gap, ii) radiative (band-to-band) recombination and iii) auger recombination. The three recombination processes are shown in figure 2.5.

![Recombination process in semiconductors](image)

**Figure 2.5:** Recombination process in semiconductors, Courtesy of Handbook of Photovoltaic Science and Engineering, Page 95

The net recombination rate per unit volume per second through a single level trap (SLT) located at energy $E = E_T$ within forbidden gap is referred to as Shockley-Read-Hall recombination and is given by
\[ R_{\text{SLT}} = (pn - n_i^2) / \left[ \tau_{\text{SLT},n} (p + n_i e^{(E_i - E_T)/kT}) + \tau_{\text{SLT},p} (n + n_i e^{(E_T - E_i)/kT}) \right] \] …(2.32)

The carrier lifetime is expressed as

\[ \tau_{\text{SLT}} = 1 / (\sigma \nu_{\text{th}} N_T) \] …(2.33)

Here \( \sigma \) is the capture cross-section (\( \sigma_n \) for electrons and \( \sigma_p \) for holes), \( \nu_{\text{th}} \) is the thermal velocity of carriers and \( N_T \) is the concentration of traps. If a trap has large target for the carrier the recombination rate will be high with low carrier lifetime. In case the carrier has a very high velocity it has a higher chance within a given time to encounter trap. The probability of interaction with trap increases as the concentration of trap increases. The carrier lifetime is inversely proportional to trap concentration. If the material is p-type (\( p \approx p_0 >> n_0 \)) in low-injection (\( n_0 \leq n << p_0 \)) and the trap energy is near the middle of forbidden gap (\( E_T \approx E_i \)) the recombination rate can be expressed as

\[ R_{\text{SLT}} \approx (n - n_0) / \tau_{\text{SLT},n} \] …(2.34)

The recombination rate is solely dependent on minority carriers since there are fewer minority carriers in comparison to majority carriers and one of each is necessary for the recombination process. If the injection conditions prevail (\( p \approx n >> p_0, n_0 \)) then

\[ R_{\text{SLT}} \approx n / (\tau_{\text{SLT},n} + \tau_{\text{SLT},p}) \approx p / (\tau_{\text{SLT},n} + \tau_{\text{SLT},p}) \] …(2.35)

The effective combination lifetime in this case is the summation of two carrier lifetimes. The recombination rate is high due to large number of electron and holes; the carrier lifetime is actually longer than in the case of low injection semiconductors. This can have significant effect in the base region of solar cells, specifically for concentrator solar cells as the base layer is the least doped layer.

Radiative (band-to-band) recombination is exactly the opposite of optical generation process and is much more evident in direct bandgap semiconductors than indirect bandgap semiconductors. In case of radiative recombination the energy of electron is given to an emitted photon (this is the principle of working of semiconductor lasers and light emitting diodes (LEDs)). In indirect bandgap semiconductors some of that energy is shared with a phonon. The recombination rate due to radiative recombination is given by

\[ R_{\lambda} = B (pn - n_i^2) \] …(2.36)
If we have an $n$-type ($n \approx n_0 >> p_0$) semiconductor in low injection ($p_0 \leq p << n_0$) the net radiative recombination is expressed in terms of effective lifetime.

$$R_{\lambda} \approx (p - p_0) / \tau_{\lambda, p} \ldots (2.37)$$

Here

$$\tau_{\lambda, p} = 1 / (n_0 B) \ldots (2.38)$$

A similar expression can be written for $p$-type semiconductors if the injection conditions prevail ($p \approx n >> p_0, n_0$)

$$R_{\lambda} \approx B_p \approx B_n \ldots (2.39)$$

As photons with energies near the bandgap are emitted during the recombination process, it is possible for these photons to be reabsorbed before exciting the semiconductors. A well designed direct bandgap semiconductor takes advantage of this photon recycling and increase effective lifetime.

Auger recombination is similar to the radiative recombination with the only difference that the energy of transition is given to another carrier as shown in figure 2.5. The electron or hole releases the excess energy to a particle called phonon. As radiative recombination is the inverse process of optical excitation, Auger recombination is the inverse process of impact ionization. An energetic electron collides with a crystal atom, breaks the bond and creates an electron-hole pair. The net recombination due to Auger process is expressed as

$$R_{\text{Auger}} = (C_n n + C_p p) (p n - n_i) \ldots (2.40)$$

In an $n$-type material in low injection the net Auger recombination rate becomes

$$R_{\text{Auger}} \approx ((p - p_0) / \tau_{\text{Auger}, p} \ldots (2.41)$$

$$\tau_{\text{Auger}, p} = 1 / C_n n_0 \ldots (2.42)$$

A similar expression can also be derived for minority electron lifetime in $p$-type material. If the injection conditions prevail ($p \approx n >> p_0, n_0$)

$$R_{\text{Auger}} = (C_n + C_p) p^3 \approx (C_n + C_p) n^3 \ldots (2.43)$$

The single level trap recombination rate can be minimized by reducing the density of single level traps, the radiative recombination can be reduced by photon recycling and the Auger
recombination rate is a fundamental property of the semiconductor. Each of the three recombination processes happen in parallel. The total recombination rate is the summation of all the three processes

$$R = \left[ \sum R_{SLT, i} \right] + R_\lambda + R_{Auger} \ldots (2.44)$$

An effective minority carrier lifetime for a doped material in low level injection is expressed as

$$\frac{1}{\tau} = \left[ \sum \left( \frac{1}{\tau_{SLT, i}} \right) \right] + \left( \frac{1}{\tau_\lambda} \right) + \left( \frac{1}{\tau_{Auger}} \right) \ldots (2.45)$$

The distribution of traps in the energy gap for semiconductor materials can be influenced by specific growth process, impurities and crystal defects. An interface between two dissimilar materials similar to the one that happens in the front surface of a solar cell contributes to a high concentration of defects due to the abrupt termination of crystal lattice. The surface recombination rate for $p$-type and $n$-type material is generally expressed for simplicity as

$$R_s = S_p (p - p_0) \ldots (2.46)$$
$$R_s = S_n (n - n_0) \ldots (2.47)$$

$S_p$ and $S_n$ are effective surface recombination velocities. It is to be noted that these effective recombination velocities are not necessarily constants independent of carrier concentration even though sometime they are treated as such [1, 13].

### 2.2.6 Carrier Transport

Electrons and holes in a semiconductor behave much like a free particle having same electronic charge with effective masses $m_n^*$ and $m_p^*$ respectively. They are subjected to classical processes of drift and diffusion. Drift is the charged particle’s movement for an applied electric field. When an electric field is applied across uniformly doped semiconductor, the bands bend upward in the direction of the applied electric field. Electrons in the conduction band travel in the opposite direction of the applied electric field due to the negative charge, whereas the holes move in the same direction of the applied field. The electrons sink and holes float. This is the conceptual method to understand the movement of electrons and holes in semiconductor devices.
Without any objection the electrons and holes continue to accelerate without bound. The semiconductor crystal is full of objects with which the carriers collide and gets scattered. These objects include component atoms of the crystal, dopant ions, crystal defects and other electrons and holes. The electric field can be created by the internal build-in potential of the junction or by an externally applied bias voltage. The carriers moves in the direction prescribed by the applied electric field, $E = -\nabla \Phi$. Here $\Phi$ is the electrostatic potential. Carriers appear to move on a microscopic scale at a constant velocity or the drift velocity $v_d$. The drift velocity is directly proportional to the electric field

$$|v_d| = |\mu E| = |\mu \nabla \Phi| \quad (2.48)$$

Here $\mu$ is the carrier mobility. The carrier mobility is generally independent of electric field strength unless the field is extremely strong, a situation not common in solar cells. The drift current densities for holes and electrons can be written as

$$J_p^{\text{drift}} = q_p \nu_{d,p} = q \mu_p p E = - q \mu_p p \nabla \Phi \quad (2.49)$$

$$J_n^{\text{drift}} = - q_n \nu_{d,n} = q \mu_n n E = - q \mu_n n \nabla \Phi \quad (2.50)$$

The most significant scattering mechanisms in solar cells are lattice (phonon) and ionized impurity scattering. These component mobilities are expressed for lattice scattering and ionized impurity scattering as
\[ \mu_L = C_L T^{-3/2} \quad \text{(2.51)} \]
\[ \mu_I = C_I T^{3/2} / (N_D^+ + N_A^-) \quad \text{(2.51)} \]

These two equations can be combined together to constitute the carrier mobility according to Mathiessen’s rule

\[ (1 / \mu) = (1 / \mu_L) + (1 / \mu_I) \quad \text{(2.52)} \]

This is the first order approximation neglecting the velocity dependency of scattering mechanisms. At low impurity levels the mobility of carriers is governed by intrinsic lattice scattering and at high levels the mobility the mobility is governed by ionized impurity scattering. Electrons and holes as a result of their random thermal motion tend to move from regions of high concentration to regions of low concentration. This process is called diffusion and the diffusion current densities are expressed as

\[ J_p^\text{diff} = - q D_p \nabla p \quad \text{(2.53)} \]
\[ J_n^\text{diff} = q D_n \nabla n \quad \text{(2.53)} \]

Here \( D_p \) and \( D_n \) are hole and electron diffusion coefficients respectively. The currents are driven by the gradient of the carrier densities. In thermal equilibrium there is no electron or hole current hence the drift and diffusion currents are balanced. In non-degenerate materials this is given by

\[ D / \mu = kT / q \quad \text{(2.54)} \]

The total hole and electron currents are the summation of their drift and diffusion components

\[ J_p = J_p^\text{drift} + J_p^\text{diff} = q \mu_p p \vec{E} - q D_p \nabla p = - q \mu_p p \nabla \Phi - q D_p \nabla p \quad \text{(2.55)} \]
\[ J_n = J_n^\text{drift} + J_n^\text{diff} = q \mu_n n \vec{E} + q D_n \nabla n = - q \mu_n n \nabla \Phi + q D_n \nabla n \quad \text{(2.56)} \]

The total current is

\[ J = J_p + J_n + J_{\text{disp}} \quad \text{(2.57)} \]

\( J_{\text{disp}} \) is the displacement current given by

\[ J_{\text{disp}} = \partial \vec{D} / \partial t \quad \text{(2.58)} \]

\( \vec{D} = \varepsilon \vec{E} \) is the dielectric displacement field having \( \varepsilon \) as the permittivity of semiconductor. The displacement current is neglected in solar cells because they are static or direct current (dc) devices [1].
2.2.7 Semiconductor Equations

The operation and working of most semiconductor devices including solar cells can be expressed in the generalized form as

$$\nabla \varepsilon \mathbf{E} = q (p - n + N) \ldots (2.59)$$

This is a form of Poisson’s equation where N is the net charge due to dopants and other trapped charges. The electron and hole continuity equations are

$$\nabla J_n = q (R_n - G + (\partial n / \partial t)) \ldots (2.60)$$

$$\nabla J_p = q (G - R_p - (\partial p / \partial t)) \ldots (2.61)$$

Here G is the optical generation rate of electron-hole pairs. Thermal generation is included in $R_n$ and $R_p$. The electron and hole current densities are mentioned by

$$J_p = - q \mu_p p \nabla (\Phi - \Phi_p) - k T \mu_p \nabla p \ldots (2.62)$$

$$J_n = - q \mu_n n \nabla (\Phi + \Phi_n) + k T \mu_n \nabla n \ldots (2.63)$$

The two new terms $\Phi_p$ and $\Phi_n$ has been introduced in the equations to account for degeneracy and spatially varying bandgap for hetero-junction solar cells and electron affinity. They are ignored in single-junction solar cells.

2.2.8 Minority Carrier Diffusion Equation

In a uniformly doped semiconductor bandgap and permittivity are independent of position. We are mainly interested in the steady-state operation of the solar cell; hence the semiconductor equations reduce to

$$\frac{d\mathbf{E}}{dx} = \left(\frac{q}{\varepsilon}\right) (p - n + N_D - N_A) \ldots (2.64)$$

$$(q \mu_p) \frac{d}{dx} (p \mathbf{E}) - q D_p \frac{d^2 p}{dx^2} = q (G - R) \ldots (2.65)$$

$$(q \mu_n) \frac{d}{dx} (n \mathbf{E}) + q D_n \frac{d^2 n}{dx^2} = q (R - G) \ldots (2.66)$$

Regions sufficiently distant from the $pn$-junction of the solar cell (quasi-neutral regions), the electric field is very small. Considering the minority carrier (holes in the $n$-type material and electrons in the $p$-type material) and low-level injection the drift current can be ignored with respect to diffusion current. In low-level injection $R$ simplifies to
\[ R = (n_P - n_{P0}) / \tau_n = \Delta n_P / \tau_n \quad (2.67) \]
\[ R = (p_N - p_{N0}) / \tau_p = \Delta p_N / \tau_p \quad (2.68) \]

Here \( \Delta p_N \) and \( \Delta n_P \) are excess minority-carrier concentrations and \( \tau_n \) and \( \tau_p \) are minority carrier lifetimes. For distinction capital P and N subscripts are used to indicate quantities in \( p \)-type and \( n \)-type regions respectively. Lower-case \( p \) and \( n \) subscripts are for minority carriers. Minority carrier diffusion equations can be written as

\[
(D_p) \left( \frac{d^2 \Delta p_N}{dx^2} \right) - \frac{\Delta p_N}{\tau_p} = - G(x) \quad (2.69)
\]
\[
(D_n) \left( \frac{d^2 \Delta n_P}{dx^2} \right) - \frac{\Delta n_P}{\tau_n} = - G(x) \quad (2.70)
\]

### 2.2.9 Diode Electrostatics \( pn \)-junction

When an \( n \)-type semiconductor comes into contact with a \( p \)-type semiconductor then a \( pn \)-junction is formed. In case of thermal equilibrium there is no net current flow and Fermi energy is independent of position. Because of the concentration difference of holes and electrons between the two types of semiconductors, holes diffuse from the \( p \)-type region into the \( n \)-type region and electrons from the \( n \)-type region to the \( p \)-type region. As the carriers diffuse the charged impurities are no longer screened by minority carrier. In this case an electric field is produced that counteracts further diffusion of holes and electrons. In thermal equilibrium condition the diffusion and drift currents for each carrier type are balanced resulting in no net current flow. The transition region between the \( n \)-type and \( p \)-type semiconductors is referred to as space-charge region also called as depletion region because it is effectively depleted of both holes and electrons. Assuming the \( p \)-type and \( n \)-type regions are of sufficient width, the regions on either sides of the depletion region are charge-neutral or quasi-neutral. The electrostatic potential difference resulting from the junction formation is termed as built-in voltage. It arises from positive and negative space charge in depletion region [1]. The electrostatics of this situation (assuming single acceptor and single donor level) is depicted by Poisson’s equation

\[
\nabla^2 \Phi = \left( \frac{q}{\varepsilon} \right) (n_0 - p_0 + N_{A^-} - N_{D^+}) \quad (2.71)
\]

Here \( \Phi \) is the electrostatic potential, \( q \) is the charge of the electron, \( \varepsilon \) is the permittivity, \( p_0 \) is the equilibrium hole concentration, \( n_0 \) equilibrium electron concentration, \( N_{A^-} \) is the ionized acceptor concentration and \( N_{D^+} \) is the ionized donor concentration. Figure 2.7 shows a simple
one-dimensional $p$-$n$ junction uniformly doped solar cell having doping density of $N_D$ on the $n$-type, $p$-type density of $N_A$ and the metallurgical junction at $x = 0$.

Figure 2.7: Simple solar cell structure, free carriers have diffused across the junction ($x = 0$) leaving a space-charge or depletion region devoid of any free or mobile charges, Courtesy of Handbook of Photovoltaic Science and Engineering, Page 104

In this example it is assumed that each side is non-degenerately doped and the dopants are fully ionized. The $n$-type side is assumed to be more heavily doped ($n^+$) than the $p$-type side. Within the depletion region, defined by $-x_N < x < x_P$ it is assumed that $p_0$ and $n_0$ both are negligible compared to $|N_A - N_D|$, hence equation 2.71 is simplified to

$$\nabla^2 \Phi = - \frac{q}{\varepsilon} N_D, \text{ for } -x_N < x < 0 \text{ and}$$

$$\nabla^2 \Phi = \frac{q}{\varepsilon} N_A, \text{ for } 0 < x < x_P \ldots (2.72)$$

Outside the depletion region, charge neutrality is assumed

$$\nabla^2 \Phi = 0, \text{ for } x \leq -x_N \text{ and } x \geq x_P \ldots (2.73)$$

This is commonly referred to as the depletion approximation and the regions on either side of the depletion region are called as quasi-neutral regions. The electrostatic potential difference across the junction is named as built-in voltage. It is obtained by integrating the electric field, $E = - \nabla \Phi$

$$\int_{-x_N}^{x_P} E \, dx = - \int_{-x_N}^{x_P} (d \Phi / dx) \, dx = \int_{V(-x_N)}^{V(x_P)} d\Phi = \Phi (-x_N) - \Phi (x_P) = V_{bi} \ldots (2.74)$$
Applying \( \Phi(x_P) = 0 \) gives

\[
\Phi(x) = V_{bi} \text{, for } x \leq -x_N
\]

\[
= V_{bi} - (qN_D/2\varepsilon)(x + x_N^2), \text{ for } -x_N < x \leq 0
\]

\[
= (qN_A/2\varepsilon)(x - x_P)^2, \text{ for } 0 \leq x < x_P
\]

\[
= 0, \text{ for } x \geq x_P \ldots (2.75)
\]

The electrostatic potential must be continuous at \( X = 0 \); hence

\[
V_{bi} - (qN_D/2\varepsilon)x_N^2 = (qN_A/2\varepsilon)x_P^2 \ldots (2.76)
\]

In the absence of any interface charge at the metallurgical junction, the electrostatic field is continuous and

\[
x_N N_D = x_P N_A \ldots (2.77)
\]

The net charge on either side of the depletion region exactly balance each other and therefore the depletion region extends furthest into the more lightly doped side as shown in figure 2.7. The depletion width \( W_D \) is given by

\[
W_D = x_N + x_P = \sqrt{2\varepsilon/q} [(N_A + N_D) / N_A N_D] (V_{bi}) \ldots (2.78)
\]

For non-equilibrium conditions the electrostatic potential difference across the junction is modified by applied voltage \( V \) which is zero in thermal equilibrium. The depletion width in this case is dependent on applied voltage.

\[
W_D(V) = x_N + x_P = \sqrt{2\varepsilon/q} [(N_A + N_D) / N_A N_D] (V_{bi} - V) \ldots (2.79)
\]

The built-in voltage \( V_{bi} \) can be calculated by noting that under thermal equilibrium the net electron and hole currents are zero. The hole current density is

\[
\vec{J}_p = q\mu_p p_0 \vec{E} - q D_p \nabla p = 0 \ldots (2.80)
\]

The electric field can be written as

\[
\vec{E} = (kT/q) (1/p_0) (dp_0/dx) \ldots (2.81)
\]

The built-in field in this case is

\[
V_{bi} = (kT/q) \ln [N_D N_A / n_i^2] \ldots (2.82)
\]
Built-in electric field is responsible for electron and holes moving in opposite direction.

2.3 Solar Cell Fundamentals

2.3.1 Solar Cell Boundary Conditions

At $x = -W_N$ in figure 2.7 the normal assumption is that the front contact can be treated as an ideal ohmic contact, i.e.

$$\Delta p (-W_N) = 0 \ldots (2.83)$$

Since the front contact is usually a grid having metal contacting semiconductor on only a little portion of the front surface, modeling the front surface with an effective surface recombination velocity is more realistic. This effective recombination velocity takes into consideration the combined effects of the ohmic contact and the anti reflective coating layer. In this case the boundary condition at $X = -W_N$ is

$$\frac{d \Delta p}{dx} = \left(\frac{S_{F,\text{eff}}}{D_P}\right) \Delta p (-W_N) \ldots (2.84)$$

Here $S_{F,\text{eff}}$ is the effective front surface recombination velocity. As $S_{F,\text{eff}} \rightarrow \infty$, $\Delta p \rightarrow 0$ and the boundary condition reduces to an ideal ohmic contact. In reality $S_{F,\text{eff}}$ depends upon a number of parameters and is bias dependent. The back contact can also be treated as an ideal ohmic contact.

$$\Delta n (W_P) = 0 \ldots (2.85)$$

The solar cells are often fabricated with a back-surface field (BSF) layer, heavily doped region at the back of the base region to avoid recombination losses. The BSF layer keeps minority carriers away from the back ohmic contact and increases their chances of being collected and it can be modeled by an effective and relatively low surface recombination velocity. The boundary condition is having the surface recombination velocity $S_{BSF}$ for the BSF layer.

$$\left(\frac{d \Delta n}{dx}\right) |_{x=W_P} = - \left(\frac{S_{BSF}}{D_n}\right) \Delta n (W_P) \ldots (2.86)$$

We just have to find the suitable boundary conditions at $x = -x_N$ and $x = x_P$. These boundary conditions are referred to as the law of junction. Under equilibrium conditions, zero applied voltage and without any illumination, the Fermi energy $E_F$ is constant with position. If a bias voltage is applied concept of quasi-Fermi energies come into effect. The equilibrium carrier
concentrations could be related to the Fermi energy. Assuming the semiconductor is non-degenerate

\[ p = n_i e^{(E_i - F_p)/kT} \quad \ldots (2.87) \]

\[ n = n_i e^{(F_N - E_i)/kT} \quad \ldots (2.87) \]

Under equilibrium conditions \( F_p = F_N = E_F \). In case of non-equilibrium assuming that the majority carrier concentrations at the contacts retain their equilibrium values the applied voltage can be expressed as

\[ qV = F_N (-W_N) - F_P (W_P) \quad \ldots (2.88) \]

Since, in low-level injection, the majority carrier concentrations are essentially constant through the quasi-neutral region i.e. \( p_P (x_P \leq x \leq W_P) = N_A \) and \( n_N (-W_N \leq x \leq -x_N) = N_D, F_N (-W_N) = F_N (-x_N) \) and \( F_P (W_P) = F_P (x_P) \). Assuming both the quasi-Fermi energies remain constant inside depletion region

\[ qV = F_N (x) - F_P (x) \quad \ldots (2.89) \]

For \(-x_N \leq x \leq x_P\) or everywhere inside the depletion region, the boundary conditions used at the edges of the depletion region

\[ p_N (-x_N) = (n_i^2 / N_D) e^{(qV/kT)} \quad \ldots (2.90) \]

\[ n_P (x_P) = (n_i^2 / N_A) e^{(qV/kT)} \quad \ldots (2.91) \]

### 2.3.2 Generation Rate

For light impinging at the front surface of the solar cell, \( x = -W_N \), the optical generation rate is given by

\[ G (x) = (1 - s) \int [1 - r (\lambda)] f (\lambda) \alpha (\lambda) e^{-\alpha (x + W_N)} d\lambda \quad \ldots (2.92) \]

Here \( s \) is the grid-shadowing factor, \( r (\lambda) \) is the reflectance, \( \alpha (\lambda) \) the absorption coefficient and \( f (\lambda) \) is the incident photon flux (number of photons incident per unit area per second per wavelength). Only photons having \( \lambda \leq (hc / E_G) \) contribute to the generation rate.

### 2.3.3 Interpreting Solar Cell Characteristic

The general expression for the current produced by a single-junction solar cell is given by
\[ I = I_{SC} - I_{D1} \left( e^{\frac{qV}{kT}} - 1 \right) - I_{D2} \left( e^{\frac{qV}{2kT}} - 1 \right) \] (2.93)

Here \( I_{SC} \) is the short-circuit current and \( I_{D1} \) and \( I_{D2} \) are dark currents of a two-diode model representation of the solar cell. The topics will be discussed in detail in chapter 6. The currents depend on the solar cell structure, material properties, and operating conditions. From a circuit perspective, a solar cell can be modeled by an ideal current source \( I_{SC} \) in parallel with two diodes – one having the ideality factor of 1 and the other with the ideality factor of 2. The direction of the current source is such that it serves to forward bias the diodes. The minority carrier properties determine the behavior of the solar cell. For simplicity, the dark current due to the depletion region is being ignored. At small applied voltage, the diode current is negligible, and the current is just the short-circuit current \( I_{SC} \) as can be seen when \( V \) is set to zero. In case when the applied voltage is high enough so that the diode current or the recombination current becomes significant, the solar cell current drops drastically. The \( n \)-type emitter is \( \sim 1000 \) times thinner, \( 10,000 \) times heavily doped and its diffusion length is \( \sim 100 \) times shorter than the \( p \)-type base.

At open-circuit voltage all the current is flowing through the diode 1 so the open-circuit voltage can be expressed as for \( I_{SC} \gg I_{D1} \)

\[ V_{OC} = \left( \frac{kT}{q} \right) \ln \left( \frac{I_{SC} + I_{D1}}{I_{D1}} \right) \approx \left( \frac{kT}{q} \right) \ln \left( \frac{I_{SC}}{I_{D1}} \right) \] (2.94)

Maximum power point \( V = V_{MP} \), and \( I = I_{MP} \) defines a rectangle having the area \( P_{MP} = V_{MP} \times I_{MP} \) is the largest rectangle for any \( I-V \) curve of the solar cell. The maximum power point is found by solving for \( V = V_{MP} \).

\[ (\partial P / \partial V)_{V = V_{MP}} = (\partial IV / \partial V)_{V = V_{MP}} = [I + V(\partial I / \partial V)]_{V = V_{MP}} = 0 \] (2.95)

The current at maximum power point, \( I_{MP} \) is found by putting \( V = V_{MP} \) in equation 2.93. Fill factor (FF) is a measurement of squareness of the \( I-V \) curve and is an important parameter for judging the performance of solar cell. It has a value less than one.

\[ FF = \frac{V_{MP} \times I_{MP}}{V_{OC} \times I_{SC}} = \frac{P_{MP}}{P_{MP} \times I_{SC}} \] (2.96)

The most important figure of merit of a solar cell is its power conversion efficiency \( \eta \)

\[ \eta = \frac{P_{MP}}{P_{in}} = FF \left( \frac{V_{OC} \times I_{SC}}{P_{in}} \right) \] (2.97)

The incident power is dependent on properties of light spectrum incident upon solar cell. Another important figure of merit is the collection efficiency. It is determined relative to both optical and recombination losses as an external collection efficiency.
\[\eta_{C}^{\text{ext}} = \frac{I_{\text{SC}}}{I_{\text{inc}}} \ldots(2.98)\]

\[I_{\text{inc}} = q A \int f(\lambda) \, d\lambda \ldots(2.99)\]

\(I_{\text{inc}}\) is the maximum photo current in case if all photons with \(E > E_{G}\) \((\lambda < \lambda_{G} = \frac{hc}{E_{G}})\) created electron-hole pairs that were collected. The collection efficiency can also be defined with respect to recombination losses as the internal collection efficiency

\[\eta_{C}^{\text{int}} = \frac{I_{\text{SC}}}{I_{\text{gen}}} \ldots(2.100)\]

\[I_{\text{gen}} = q A (1 - s) \int [1 - r(\lambda)] f(\lambda) \left( 1 - e^{-\alpha(W_{N} + W_{P})} \right) \, d\lambda \ldots(2.101)\]

\(I_{\text{gen}}\) is the light-generated current.

### 2.3.4 Properties of Efficient Solar Cells

An efficient solar cell will have high short-circuit current \(I_{\text{SC}}\), high open-circuit voltage \(V_{\text{OC}}\) and a fill factor close to 1. A more detailed understanding of solar cell efficiency can be obtained by \([1, 14]\)

\[\eta = \frac{P_{\text{max}}}{P_{\text{in}}} = \eta_{\text{ideal}} \eta_{\text{photon}} \eta_{\text{FF}} \eta_{\text{V}} \eta_{C}^{\text{int}} \ldots(2.102)\]

\(\eta_{\text{FF}}\) and \(\eta_{C}^{\text{int}}\) has been mentioned earlier. Assuming the maximum energy that can be extracted from an absorbed photon is \(E_{G}\) the ideal efficiency can be expressed as

\[\eta_{\text{ideal}}(E_{G}) = \frac{(E_{G} I_{\text{inc}} / q)}{P_{\text{in}}} = \frac{E_{G}}{(P_{\text{in}} / A)} \int f(\lambda) \, d\lambda \ldots(2.103)\]

Photons having energy \(h\nu > E_{G}\) only can create electron-hole pairs and contribute to the output power of the solar cell. It is clear that the bandgap determines how good a solar cell is matched with the solar radiation spectrum. The photon efficiency \(\eta_{\text{photon}}\) accounts for photons that are reflected transmitted or otherwise not absorbed in the solar cell and can be written as

\[\eta_{\text{photon}} = \frac{I_{\text{gen}}}{I_{\text{inc}}} = \frac{\eta_{C}^{\text{ext}}}{\eta_{C}^{\text{int}}} \ldots(2.104)\]

To maximize \(\eta_{\text{photon}}\) \((\eta_{\text{photon}} \rightarrow 1\) when \(I_{\text{gen}} \rightarrow I_{\text{inc}}\) or \(\eta_{C}^{\text{ext}} \rightarrow \eta_{C}^{\text{int}}\)) the solar cell should be designed with minimum grid shadowing \(s\), minimum reflectance \(r(\lambda)\) and optically thick enough such that all the photons having \(E > E_{G}\) are absorbed. The voltage efficiency \(\eta_{V}\) is the ratio of the open-circuit voltage to the bandgap voltage.

\[\eta_{V} = \frac{V_{\text{OC}}}{(E_{G} / q)} \ldots(2.105)\]
Designing efficient solar cells is very important. There are several key criterions in designing high quality solar cells. Semiconductor materials should be chosen targeting certain wavelength of the solar radiation spectrum. Minimizing optical losses and maximizing photon absorption, reducing recombination losses in the quasi-neutral and depletion region. This will increase the fill factor and efficiency of the solar cells.
CHAPTER THREE
LOSS REDUCTION MECHANISMS

3.1 Introduction

In this chapter we will discuss about some other important aspects of solar cell design. The topics of interest ranges from optical design like light trapping in solar cell, surface recombination losses and their reduction, bulk recombination losses and their reduction etc.

3.2 Optical Design of Solar Cells

The main challenges of solar cells are how can light be coupled efficiently into the cell and how can it be retained in the cell to maximize absorption.

3.2.1 Light Trapping

Light trapping is a very important aspect of solar cell design. In this chapter it has been discussed with respect to crystalline silicon solar cells; however it is true for all other types of solar cell design including multijunction. The indirect band-gap of Si results in absorption coefficient that is less than $10^4$/cm in the wavelength region of 450-750nm, which has maximum energy. In case of multijunction each of the subcell layers has different absorption coefficients with the top subcell layer having the highest absorption coefficient. Most of the experimental work on light trapping has been performed over Si solar cells. Two basically different types of light-trapping schemes are mentioned: schemes based on lambertian surfaces and those based on geometrical light-trapping schemes.

A lambertian surface is one that scatters light uniformly in all directions and lambertian light-trapping schemes are based on full internal randomization of the light ray direction by lambertian surface [2]. Two possible implementations are expressed in figure 3.1 (a) and (b). The first is based on a perfect lambertian rear reflector and the front surface with zero reflectance for incoming light. The second figure is equivalent as far as the final result is concerned whereas relies on a randomizing front surface and perfect rear reflector. A detailed analysis of randomizing schemes was performed by Jablonovitch and Cody [15]. The result of their analysis is the intensity enhancement of $2n^2$ inside a sheet with refractive index n and a perfect white
reflector at back. Under equilibrium conditions, the in and outgoing radiation fluxes have to be equal. Since the fully randomized light inside the cell can only escape over a limited solid angle determined by critical angle from an optically dense material to a material of lower refractive index the intensity inside the cell has to increase. Only under these circumstances the in and outgoing radiation fluxes remain equal.

Jablonovitch and Cody showed that this results in an optical path length enhancement of $4n^2$ for weakly absorbed light. The spectral absorbance $a(\lambda)$ for a material with refractive index $n$, thickness $W$ and absorption coefficient $\alpha(\lambda)$ is given by

$$a(\lambda) = \alpha(\lambda) / [\alpha(\lambda) + (1 / 4n^2W)] \cdots (3.1)$$

If the back side of the solar cell can not reflect the light perfectly then the expression of path-length enhancement $A$ should be modified to

$$A = 2(1 + R_{\text{back}}) / [1 - R_{\text{back}}(1 - 1 / n^2)] \cdots (3.2)$$

Here $R_{\text{back}}$ is the back side reflectance. Light-trapping structures with lambertian surfaces represent the limiting case for cells having an isotropic response i.e. cells whose response in independent of the incident angle. The implementation of surface structures with dimensions larger than the wavelength of light (geometrical light-trapping schemes) can increase the cell absorption above the limits of the lambertian schemes due to the directionality of sunlight. In
contrast to the lambertian schemes it is not possible to describe the schemes by an analytical approach alone; ray tracing programs are needed to perform a detailed analysis of these structures. Two basic types of geometrical light-trapping are there: 2 dimensional and 3 dimensional geometries. The two-dimensional structures can consist simply of a number of parallel grooves at the surface. The number of passes can be increased by tilting the grooves as shown in figure 3.2.

![Cross-section of cell with parallel tilted grooves](image)

Figure 3.2: Cross-section of cell with parallel tilted grooves, Courtesy of Clean Electricity from Photovoltaics, page 103

A further reduction of probability of light escape is accomplished by three-dimensional geometries. In this case instead of two-dimensional grooves, three-dimensional pyramids are formed. For regular pyramids the light-trapping efficiency strongly depends on details of the layout specifically the ration of pyramid size to the wafer thickness [2]. Further improvement in light-trapping is gained by structuring the backside of the cell. A very efficient method is making backside grooves perpendicular to the front-side grooves.

### 3.2.2 Reduction of First Reflection: Anti Reflective Coating

Reflection on the front surface of the solar cell causes a less amount of light energy to impinge through the solar cells. The light trapping schemes substantially reduce the reflectance and using of grooves instead of perfectly smooth surface increases the probability of the reflected ray striking back the surface again. A further reduction of front surface reflectance is obtained by the use of anti-reflective coatings (ARC). The ARC layer which is interposed between the
photovoltaic cell material and the surrounding environment (air) acts as a quarter wavelength impedance matching element between the characteristic impedance of the environment and the photovoltaic material. The ray reflected at the interface between the semiconductor (refractive index $n_2$) and the ARC layer interferes destructively with the ray reflected at the interface between ARC and surrounding medium with refractive index $n_0$. The conditions for zero reflectance at wavelength $\lambda$ is mentioned as

$$d_1 = \frac{\lambda}{4n_1} \cdots (3.3)$$

$$n_1 = \sqrt{n_0 n_2} \cdots (3.4)$$

Here $d_1$ and $n_1$ are thickness and refractive index of intermediate anti-reflective layer. These equations are for a certain wavelength; however the solar spectrum is over a wavelength range so ARC thickness is chosen to give minimal integrated reflectance weighted by the energy content at each wavelength. On a structured surface the minimum reflectance is no longer zero because the angle at which ray travels in the ARC layer is not unique. However the better light trapping properties for the light coupled in the spectral reflectance curve in case of geometrical light trapping structure, largely compensate for this non-zero minimum reflectance. Practical examples of single layer ARCs for Si is SiN$_x$H$_y$, TiO$_2$ and Ta$_2$O$_5$. The reflectance can further be reduced by using double layer ARCs like ZnS + MgF$_2$, MgO$_2$+TiO$_2$, TiO$_2$+Al$_2$O$_3$ etc.

### 3.3 Surface Recombination Loss and Reduction

#### 3.3.1 Surface Recombination

Surface recombination occurs at the extremities of the cell and its effect is described in terms of surface recombination velocity ($S$). In a p-type semiconductor $S$ is the relation between recombination current density $i_{\text{rec}}$ and the excess minority carrier concentration $\Delta n$ at the surface

$$i_{\text{rec}} = q S \Delta n \cdots (3.5)$$

The relation between $S$ and the properties of the uncharged surface in presence of continuous distribution of states $D_{it}(U)$ and cross-sections $\sigma_i(U)$ is expressed as

$$S = v_{\text{th}} \int_{U_c}^{U_v} \frac{D_{it}(n_0 + p_0 + 2\Delta n)}{[n_0 + \Delta n + n_i(U)/\sigma_p(U)] + [p_0 + \Delta n + p_i(U)/\sigma_n(U)]} dU \cdots (3.6)$$
Here $v_{th}$ refers to thermal velocity of the charge carriers. When the capture cross-sections and their energy dependences are known it is possible to calculate the value of $S$ from capacitance-voltage and deep-level transient spectroscopic measurements. Surface passivation often puts strong constraint on cell efficiency. This can be explained by the following quantitative comparison of the contribution to the dark current of the Auger recombination term [2]. For a cell in low-level injection the dark-current contribution from Auger recombination in a bulk region of doping $N_b$ and width $W_b$ is expressed as

$$i_b = q \frac{n_i^2}{N_b} C N_b^2 W_b e^{\left[\frac{3q}{2kT}\right]} \quad \text{...(3.7)}$$

Here $C$ is the Auger coefficient. The surface recombination current is created by a surface recombination velocity $S_{eff}$ is

$$i_s = q \frac{n_i^2}{N_b} S_{eff} e^{\left[\frac{3q}{2kT}\right]} \quad \text{...(3.8)}$$

The condition for ensuring that the surface recombination does not determine the open-circuit voltage is

$$S_{eff} < C N_b^2 W_b \quad \text{...(3.9)}$$

In the high-injection regime, the Auger-limited bulk contribution to the recombination is given by

$$i_b = q W_b n_i^2 (C_n + C_p) e^{\left[\frac{3q}{2kT}\right]} \quad \text{...(3.10)}$$

The surface recombination term is given by

$$i_s = q n_i S_{eff} e^{\left[\frac{3q}{2kT}\right]} \quad \text{...(3.11)}$$

Comparing the two equations of surface recombination current and using the fact that under open-circuit conditions the photogenerated current is balanced by total recombination current we get to know that

$$S_{eff} < (i_{sc} / q)^{2/3} \left[ W_b (C_n + C_p) \right]^{1/3} \quad \text{...(3.12)}$$

The $S_{eff}$ ensures that bulk recombination dominates the dark current. Under concentration the requirements are relaxed in comparison to the low-injection case.
3.3.2 Surface Passivation of Non-Contacted Areas

The ability to passivate a semiconductor surface strongly depends upon the material the semiconductor is made up of. The surface of a material is a major disturbance of the crystal and is characterized by high surface-state density (of the order $10^{12} - 10^{13}$/cm²). By growing or depositing thin dielectric layers on the surface reduces the surface state density. The surface of most II-VI and III-V semiconductors and Ge is very difficult to passivate. The methods are known for Si surfaces. The oxidation of a Si surface results in a low and stable surface density of $10^{10}$/cm². In n-type material the surface recombination velocity is lower than p-type material by two orders of magnitude. In the case of crystalline silicon solar cells this has been achieved by introducing low-cost "solar-grade" silicon, with poorer electronic properties than the high quality silicon employed in the microelectronic industry, accompanied by a wafer thickness reduction [16]. In industrial Si solar cells the use of nitride layers, grown by the plasma-enhanced chemical vapor deposition technique combines the desired optical properties ($n = 2$) with surface and bulk passivation features. High efficiencies $> 17\%$ have been reported for large area screen-printed Si solar cells [2]. A relatively new approach to passivation of the cell is by using floating junctions. The figure 3.3 depicts the basic structure. The structure consists of an additional non-contacted diffused region at the backside and this approach allows very low values of surface recombination velocity. When the light-induced forward bias over floating junction electrons are injected towards the $n^+$ contact the surface effective recombination velocity given by

$$S_{\text{eff}} = N_b \frac{i_{\text{op}}}{q n_i^2} \ldots(3.13)$$

Here $i_{\text{op}}$ is the saturation current density of hole injection into the $n^+$ region and $N_b$ is the base doping. The backside of Si solar cells resulted in an open-circuit voltage of 717mV, the highest ever achieved for a Si cell under 1-sun illumination. The derivation of $S_{\text{eff}}$ floating junction takes into account the recombination in the quasi-neutral part of the diffused region. The formula for effective surface recombination velocity is

$$S_{\text{eff}} = \{i_{\text{op}} \left[ e^{\left( q V / k T \right)} - 1 \right] + i_{\text{o}2} \left[ e^{\left( q V / 2 k T \right)} - 1 \right] + \frac{V}{R_{\text{shunt}}} \} N_b / q n_i^2 \left[ e^{\left( q V / k T \right)} - 1 \right] \ldots(3.14)$$

This expression takes the shunt resistance into account but neglects generation at cell backside. To avoid the dominance of depletion-layer recombination the minority carrier concentration at the backside of the cell should be high. This suggests the importance of floating junction
passivation for bifacial cells, where light can enter from the backside also. The shunt resistance should also be kept high.

![Figure 3.3: Cross-section of cell with n-type floating junctions for passivation of the cell backside, Courtesy of Clean Electricity from Photovoltaics, page 112](image)

### 3.3.3 Surface Passivation of Metal-Contacted Areas

The region under ohmic contact is characterized by a large surface recombination velocity which is often assumed to be infinitely large. This strongly enhances the dark current injected into the base. The expressions of the recombination current in the case of a base with a well passivated surface or when the surface recombination velocity is infinite and for the short base is given by the following equations

\[
i_b (S_{\text{back}} = \infty) = q D_b n_i^2 / N_b W_b \left[ e^{(q V / k T)} - 1 \right] \ldots (3.15)
\]

\[
i_b (S_{\text{back}} = 0) = q n_i^2 W_b / N_b \tau_b \left[ e^{(q V / k T)} - 1 \right] \ldots (3.16)
\]

Here \( D_b \) is the doping in bulk region, \( W_b \) is the width of the bulk region and \( \tau_b \) is the base lifetime. The use of dielectric passivation layers may result in well passivated surfaces in the non-contacted areas but if no actions are taken to reduce the recombination velocity near the ohmic contacts, no significant reduction of recombination current is to be expected. Assuming metal contact coverage of 1% and a ratio of \( 10^5 \) between the surface recombination velocity between the metalized and non-metalized areas, the contribution of the recombination at the contacts to the total recombination is still a factor of \( 10^3 \) larger than the contribution of non-contacted area [2]. The method of reducing the recombination current in the metalized areas is to
increase the doping level near the contact. In $n^+ - p$ Si solar cells this technique is widely used to passivate the backside. If the doping concentration is high then the heavy-doping-induced band gap narrowing reduces the effect of surface field. This effect will increase the effective intrinsic career concentration in the heavily doped part thus increasing the recombination current. In case of too high doping ratio the low electric field in the accumulation layer results in an enhanced recombination. An additional reduction of the recombination at the contacts can be obtained by incorporating a thin layer of material with larger band-gap near the contact. The more general expression for recombination current in a short base with high backside surface recombination velocity is given by

$$i_b = \frac{q}{w_b} \int_0^{N_b} \frac{D_b n_i^2}{D_b n_i^2} dx \ldots (3.17)$$

The integral is called the base Gummel number. The value of this number can be increased by decreasing the value of $n_i^2$ in part of the short base by including a higher band-gap region in the heavily doped region near the contact [17].

### 3.3.4 Front Surface Passivation: Homojunction vs. Heterojunction Design

Most p-n homojunctions and heterojunction used in photovoltaic cell design are one sided strongly asymmetric junctions in which the base layer of the cell has a lower doping level. Such design is suggested by the strongly peaked character of the carrier generation when the cell is illuminated by the sunlight. To keep the photocurrent loss in the top region within a reasonable limit, the top semiconductor has to remain thin. In most of the cells there is an additional requirement of reducing resistive losses caused by horizontal carrier flow in the top layer there should be high doping levels in the top layer and Auger recombination is the dominant recombination mechanism. The lower doping level in the base is required to obtain reasonable diffusion lengths in the base of the cell to ensure efficient collection of carriers generated in the bulk of the cell. The practicality of a homojunction approach depends on the degree of the achievable surface passivation of the front surface. The ratio of junction depth to absorption depth should be sufficiently small. The latter condition is satisfied in Si cells but in CdTe the optical absorption is very high and therefore the homojunction approach is being ruled out. The
The success of Si homojunction approach is largely based on the ability to passivate its surfaces effectively, even in a high doping level. Because of poor surface passivation most III-V compounds (except InP) and II-VI compounds the homojunction approach is not suited in these types of materials. A high surface recombination velocity results in significant recombination losses in the emitter. One way to suppress the dark current contribution of the emitter is to use heterostructure with the emitter as the wide-band-gap material. Because of the high band-gap in window layer the cell is less sensitive to actual surface. Assuming interfacial recombination low the wide-band-gap emitter to the total recombination current is negligible and hence the ratio of dark currents between base and emitter of a heterojunction structure is given by
\[
\frac{i_{oc}}{i_{ob}} \propto n_i^2 \text{ (emitter)} / n_i^2 \text{ (base)} \quad \cdots (3.18)
\]

The top layer with the larger band-gap is called the window layer and the underlying base layer is called as the absorber. Table 3 depicts the band-gap and electron affinities of most common wide band-gap window layers. The lattice mismatch between two materials of a heterostructure should be low to minimize large recombination losses at the defective interface.

### 3.3.5 Homojunction Emitter Design and its Relation to Surface Passivation

Thermal diffusion is not the only way of realizing a homojunction emitter; however this technique has been effectively implemented in Si solar cells and other homojunctions. The minority carriers in a thermally diffused emitter are not only moving under the influence of minority carrier concentration gradients. Due to the high and rapidly changing doping level two other forces contribute to the transport of the charge carriers: the first one is the electric field resulting from doping gradient and the second one is due to the field in the emitter from heavy doping induced effects. As long as the excess carrier concentration is much lower than the doping level the strength of the field is given by
\[
E(x) = \frac{(kT)}{q} \left( \frac{d N_d}{dx} \right) \frac{1}{N_d} \quad \cdots (3.19)
\]

Here \( N_d(x) \) is the dopant concentration at depth \( x \) in the emitter. For diffused profiles with the maximum dopant concentration at the surface, the electric field will allow the minority carriers to move in the direction of the junction promoting the collection of photogenerated carriers. The term front surface field is used to describe this effect. In the second contribution the high doping level in the emitter reduces the semiconductor energy gap. This is result of the breaking down of
one electron approximation at high doping levels. The charge carriers are no longer considered as non-interacting; hence correlation and exchange contributions must be included in the energy of the carrier. Short-range variations of the electrostatic potential caused by doping density fluctuations form energy band tails near the conduction and the valence band edges. Together with the shift of Fermi level these changes result in apparent heavy-doping induced bandgap narrowing. The extent of narrowing depends on doping level and dielectric constant of the material [2]. The bandgap and the position of the band edges become position dependent leading to a quasi-electrical field pointing in the same direction for minority and majority carriers. The quasi-electrical field is given by

$$E(Q) = (kT/q) \left( \frac{dn_{ic}^2}{dx} \right) \left( 1/n_{ic}^2 \right) \ldots (3.20)$$

$$n_{ic}^2 = n_i^2 e \left( \frac{\Delta E_g}{kT} \right) \ldots (3.21)$$

Here $\Delta E_g$ is the apparent narrowing of bandgap, $n_i$ is the intrinsic carrier concentration at low doping levels and $n_{ic}$ is the effective intrinsic carrier concentration. The quasi-field points in the direction of the higher doping level objects the movement of minority carriers towards the junction. The total emitter field is the summation of the two contributions. The flux $j$ for minority carriers in an emitter is

$$j = \text{normal diffusion} + \text{drift by dopant gradient} + \text{drift by bandgap variation} \ldots (3.22)$$

Minority carriers injected in the emitter under forward biased condition are capable of reaching the emitter surface. The emitter behaves like transparent and the forward dark current injected in the emitter is sensitive to surface recombination velocity at the emitter surface. There will be a strong increase in surface recombination velocity with doping level. The contact resistance between metal contact and emitter layer should also be taken into consideration. In certain cases a high surface concentration is important for a low contact resistance. The design of a homogeneous emitter is a trade-off between avoiding dead layers at the surface, reduction of contact resistance and screening of high surface recombination velocity at contacts. This can be avoided by implementing a selective emitter.

3.4 Bulk Recombination Losses and Reduction

Bulk of the cell is the lightly doped part of the cell. In contrast to the homojunction emitter or window layer, in the heterjunction case with high doping levels the sheet resistance is a major
concern, the bulk is characterized by lower doping level to maintain high minority carrier diffusion lengths. Most of the photogenerated current in Si solar cell is generated by the absorption of photons in the base and collection of photogenerated carriers by the emitter-base junction. Collection efficiency strongly depends on the minority carrier lifetime. This is true in indirect bandgap semiconductors where the base is sufficiently wide to absorb the collectable photons. Efficient carrier collection by the junction needs sufficiently high values of minority-carrier lifetime as determined by various recombination processes. In indirect bandgap semiconductors radiative recombination is not important and Auger recombination is the major recombination process. This is mostly important in the heavily doped emitter or back surface field (BSF) regions. Near to Auger limited diffusion lengths can be achieved by using high quality material with very low defect densities. High quality material with low defect density is expensive. A short description and classification of the effect of grain boundaries and methods to avoid the detrimental effects of grain boundaries is mentioned.

### 3.4.1 Electronic Description of Grain Boundaries

Grain boundaries are transition regions between adjacent grains of crystalline material having different orientation. In principle it is possible to calculate the electronic structure of the material adjacent to the grain boundaries, knowing the properties of the grain boundary. The problem is complicated by the chemical orientation of the boundary regions by impurities that diffuse preferentially towards these regions. Two different classes of grain boundary effects can be distinguished. In the first model in figure 3.4 (a), the basic band structure of the semiconductor is conserved in the grain boundary region. This is the more popular model. The deviations from perfect crystal structure are incorporated as electronic states inside the forbidden energy gap. These states can have acceptor or donor character and are distributed throughout the bandgap. Depending upon the occupancy, a charge sheet is formed at the grain boundary surface which electro-statically induces a depletion or accumulation charge around the grain boundary region. The second model shown in figure 3.4 (b), depicts the grain boundary region as a thin layer of material with different properties.
The crucial element is the formation of interface states near the grain boundary region [2].

### 3.4.2 Effect of Grain Boundaries on Majority Carrier Transport

In the two models mentioned above the effect of grain boundaries on majority-carrier transport is understood. Grain boundary usually makes a depletion layer around itself as shown in figure 3.4. Depending upon the grain size, different situations may arise. As long as the grain size is such that the grain is partially depleted, the depletion layer acts as an energy barrier for majority carriers. The dominant charge carrier transport mechanism taking majority carriers across the barrier is termed as thermionic emission. At high levels of doping, tunneling phenomenon creates an additional transport mechanism. Thermally activated conduction is indeed found in multi and poly crystalline semiconductor solar cells. The height of this barrier is a function of the level of illumination. Under illumination the excess minority carrier charge leads to reduction of charge associated with the barrier and as a result the barrier height decreases. In case of grain sizes becoming smaller than the depletion layer width associated with the grain boundary charge sheet, the grain gets fully depleted accounting in the reduction of barrier height. The mobility can be high again in that case, although scattering at the numerous grain boundaries reduces the mobility in most instances below the value of single-crystal material.
3.4.3 Effect of Grain Boundaries on Carrier Recombination

Effect of grain boundaries on minority carrier transport can be easily determined. In the case of a depletion layer around the grain boundary, the minority carriers are attracted to the grain boundary. In case of accumulation layer the minority carriers are repelled similar to the incident as in high-low doping transitions. Effect of grain boundary on minority carrier recombination is usually described by grain boundary recombination velocity. This is similar in concept as the surface recombination velocity and can be calculated if the capture cross section and state distribution of grain boundary states in the forbidden gap are known. Due to the dependence of the band bending on the injection level near the grain boundary the recombination velocity is can be defined only for a grain boundary running parallel to the junction. For a vertical grain boundary the excess carrier concentration along the grain boundary is no longer constant, resulting in recombination velocity dependent on depth. In case of low surface recombination velocities at a horizontal grain boundary the excess minority carrier profile will differ only slightly from the minority carrier profile without the grain boundary. For this ‘small perturbation’ case the effect of grain boundary can be easily constituted by increasing the dark current contribution of base without the grain boundary by a factor $K$ expressed as

$$K = [(1 / \tau_b) + (S / W_b)] / (1 / \tau_b) \ldots (3.23)$$

Here $\tau_b$ is the bulk lifetime, $S$ is the grain boundary recombination velocity and $W_b$ is the base width. The recombination current of the region containing the horizontal grain boundary is
increased by a factor of $K$ in comparison to the recombination current of the same region without the grain boundary

$$i_b = q \int_0^{w_b} \Delta n(x) \, dx / \tau_b \quad \text{(3.24)}$$

In a quasi-neutral region, for high grain boundary recombination velocities at horizontal grain boundary, the grain boundary competes with actual junctions for the photo-generated carriers. The carriers closer to the grain boundary have more chance of recombining at the grain than being collected by the junction. The effects are minor as long as the electrical field of the junction depletion layer is much larger than the field associated with the grain boundary. For vertical grain boundaries things are more complex. The reduction of the field in the junction depletion layer by interaction with the field in the grain boundary depletion layer can significantly increase recombination current. In case of cell design the doping level near junction should be sufficiently high to have strong electric field in the depletion layer.

### 3.4.4 Designs to Overcome Low Diffusion Lengths

Presence of grain boundaries and other deviations from a perfect crystalline structure results in less efficient collection of the photo-generated carriers. There are a number of design options. The three important approaches are: i) graded base design, ii) $p-i-n$ cell design and iii) parallel multijunction cells.

#### 3.4.4.1 Graded base design

In a graded base design the properties (composition, doping) of the base region are varied over the base to provide an additional force besides diffusion to promote collection of photo-generated carriers. Expression for photo-generated and dark current for a constant electrical field in $p$-type base under the assumption of constant carrier mobility and lifetime is given by

$$L_{\text{eff}} = L_e [(E(x) / E_c) + 1] \quad \text{(3.25)}$$

Here $L_{\text{eff}}$ is effective diffusion length, $L_e$ is normal diffusion length, $E(x)$ is the local electric field and $E_c$ is the critical electric field.

$$E_c = k T / q L_e \quad \text{(3.26)}$$
In the most straightforward implementation of the graded base concept the electric field is realized by means of graded dopant profile. The advantage of this approach is somehow diminished because of the dependence of transport parameters like mobility and lifetime, on the doping level. The degradation happens at higher doping levels. A better approach is grading the bandgap of the base so that the lowest bandgap is reached at depletion layer. Intentionally changing the growth conditions during base growth help in achieving such a bandgap profile.

### 3.4.4.2 p-i-n cell design

The second approach for overcoming the low diffusion lengths lies in broadening the zone where electrical field of the junction is effective. This is achieved by incorporating an intrinsic (i) part in the transition zone between p-type and n-type regions. The resulting band scheme is shown in figure 3.6.

![Schematic bandgap representation of p-i-n design](image)

**Figure 3.6**: Schematic bandgap representation of p-i-n design, electric field in the intrinsic region separates the electron-hole pairs, Courtesy of Clean Electricity from Photovoltaics, page 129

The photo-generated carriers in the depleted zone are immediately separated and accelerated away from each other by the electric field. This design concept is very effective when the diffusion constant and mobility are low and absorption coefficient is high. The design criterion to choose between a p-n junction and a p-i-n design can be expressed as

\[ M \tau E_{dep} > \sqrt{\mu (kT/q) \tau} \ldots (3.27) \]

Here \( E_{dep} \) is the field in the extended depletion layer between the \( p^+ \) and \( n^+ \) region and \( \tau \) is the lifetime of the excess carriers. If this condition is satisfied a p-i-n solar cell will deliver higher output current in comparison to its p-n counterpart. A limitation of the p-i-n design is that superposition principle is no longer valid. Under forward biased condition the internal field
decreases resulting in lower photo-current; hence the fill factor will be lower than the $p$-$n$ solar cell design. Diffusion and drift components of the current in the $p$-$i$-$n$ solar cell design are yet to be understood.

### 3.4.4.3 Parallel multijunctions

The third method of ensuring efficient collection of the photo-generated carriers in a material having low minority carrier lifetime and diffusion length is the parallel multijunction approach. The requirements on surface passivation become less stringent in the multijunction design. The most important criterion in designing these cells is to make the thickness of individual subcell layers smaller than the diffusion length in order to keep the collection efficiency close to unity. Thickness of each layers play a role in reducing the series resistance. Increase in the number of depletion regions in the multijunction design raises a concern about recombination in the junction region which can have some effects in the dark current behavior. Careful design of the junctions by including of an intrinsic region in the depletion layer and optimization of field strength help overcome this challenges \[2\].

The design of an efficient solar cell has several key goals

i) Selection of semiconductor material with bandgap well matched to the solar radiation spectrum to maximize $\eta_{\text{ideal}}$.

ii) Minimizing optical losses like grid shadowing, reflectance, maximizing the photon absorption in sub cell layers and maximizing the optical thickness of the solar cell to have maximum $\eta_{\text{photon}}$.

iii) Minimizing series and shunt resistances in the cell and its connections to maximize the fill factor (FF).

iv) Minimizing bulk & surface recombination rates, maximize $\eta_V$ and the open-circuit voltage ($V_{\text{OC}}$).

v) Minimize internal collection efficiency $\eta_{C\text{int}}$ and the short-circuit current ($I_{\text{SC}}$).
CHAPTER FOUR

HIGH-EFFICIENCY III-V COMPOUND SEMICONDUCTOR
MULTIJUNCTION SOLAR CELLS

4.1 Introduction

The biggest challenge in the Photovoltaics industry is making the solar cells more cost effective and competent to grid electricity of 10-15¢/kWh in USA and Europe [2]. The relevant solutions are-increase the efficiency of solar cells, effective spectral splitting by different bandgap semiconductor subcell layers, implementation of III-V direct bandgap optically sensitive and high carrier mobility semiconductors, form better matching (lattice, optical and electrical) between subcell layers, usage of concentrator Fresnel lenses and most importantly reduce the fabrication cost of the epitaxial layers. Increasing the efficiency of the solar cell and implementation of the concentrator lenses will reduce the real land acreage.

Single junction solar cell can capture only a certain portion of the solar radiation spectrum and being a low bandgap material will produce higher current but the open circuit voltage is limited by the dark current of the semiconductor. In such solar cells photons having energy higher than the bandgap loses excess energy as heat while photons having energy lower than the bandgap are not absorbed at all. Mono and poly crystalline silicon is an indirect bandgap semiconductor and the momentum of electrons in the valence band and conduction band is not same so it loses some energy in the intermediate stage. Direct bandgap III-V compound semiconductor materials provide significant advantages over silicon solar cells due to better optical property and high electron mobility. Multijunction III-V solar cells have generated significant interest among researchers in recent years for their very high conversion efficiencies and significant reduction in balance of system cost for PV electricity generation [18]. They use multiple subcell bandgaps to split solar radiation spectrum into smaller slices leading to efficient photon absorption and electricity generation, easily surpassing the Shockley-Quiesser theoretical limit of 31% for single-junction cells [19]. Multijunction solar cell with 37% efficiency cost between $8 and $10/cm² but research efforts and the exponential decrease in the fabrication cost has the potential of generating electricity at 7¢/kWh [1, 18].
4.2 Design Physics of III-V Multijunction Solar Cells

4.2.1 Wavelength Dependence of Photon Conversion Efficiency

In a single junction solar cell having a bandgap of $E_g$ eV, a photon of energy $\hbar \nu > E_g$ will be absorbed and converted into electricity. Excess energy photons $\hbar \nu - E_g$ are lost as heat energy. The photons having energy $\hbar \nu < E_g$ will not get absorbed and will not contribute in the generation of electricity. Hence, the efficiency of photon conversion is maximum at $\hbar \nu = E_g$. The solar radiation spectrum is broad, containing photons of energies ranging from near 0 to 4 eV. Out of the average solar irradiance of 1 kW/m$^2$, 445 Watts come from the visible range, 527 Watts from the infrared range and 28-30 Watts from the UV range. Single junction solar cell efficiency without concentration is limited to 31% by Shockley-Queisser theoretical limit. The best solution to this problem is dividing up the solar radiation spectrum into several spectral regions and covert each with a subcell layer tuned for that spectral region. In case of the spectrum being divided into three regions $\infty - \hbar \nu_1$, $\hbar \nu_1 - \hbar \nu_2$ and $\hbar \nu_2 - \hbar \nu_3$ where $\hbar \nu_1 > \hbar \nu_2 > \hbar \nu_3$, the photons from these spectral regions will be converted by the subcells with bandgaps $E_{g1} = \hbar \nu_1$, $E_{g2} = \hbar \nu_2$ and $E_{g3} = \hbar \nu_3$ respectively [1]. In theory infinite number of subcells can be constructed, however more than five is not feasible due to the complexity in matching and fabrication cost.

4.2.2 Effective Spectrum Splitting

The conceptually simplest explanation of a multijunction solar cell will be splitting the solar radiation spectrum using an optically dispersive element like prism as shown in figure 4.1. The real design has many optical, electrical and mechanical complexities associated with it. Subcell 1 layer has the highest bandgap. Photons with energy equal and greater than the bandgap ($E_{g1}$) will get absorbed in subcell 1 layer. Photons with energy less than $E_{g1}$ but greater than $E_{g2}$ ($E_{g1} > \hbar \nu > E_{g2}$) will be absorbed in the subcell layer 2. Lower energy photons will be channeled to the next subcell layers in the stack for absorption. Subcell layer 3 will absorb the photons of energy less than $E_{g2}$ but greater than $E_{g3}$ ($E_{g2} > \hbar \nu > E_{g3}$) and the less energized photons will be absorbed by the subcell layer 4 in the stack. The top subcell layer should have the highest bandgap energy and the bottom layer the least bandgap energy. The bandgap energies will reduce in the design from the top subcell layer to the bottom layer. This method makes use of the
fact that the junctions act like low-pass photon energy filters distributing the spectrum to appropriate junction for multijunction photo-conversion. Even though the subcell layers are physically separate from each other they can be mechanically brought together by mechanical stacking. The best approach to this problem is to fabricate all the subcell layers monolithically on a single substrate. Nowadays inverted metamorphic approach has gained popularity because of its advantages over other fabrication techniques.

![Image of spectrum splitting by subcell layers of multijunction solar cell]

**Figure 4.1: Spectrum splitting by subcell layers of multijunction solar cell**

### 4.2.3 Cell Quantum Efficiency (QE) and Current Density (J_{sc})

Short-circuit current density ($J_{sc}$) of any subcell layer is determined by the quantum efficiency (QE) of the subcell and spectrum of light incident $\Phi_{in}(\lambda)$ on that particular subcell layer.

$$J_{sc} = e \int QE(\lambda) \Phi_{in}(\lambda) \, d\lambda \quad ... (4.1)$$

QE for an ideal subcell layer having the base thickness $T_b$, emitter thickness $T_e$ and depletion width $W_d$ for a total thickness of $T = T_e + W_d + T_b$ is given by

$$QE = QE_{emit} + QE_{depl} + \exp \left[ - \alpha (T_e + W_d) \right] QE_{base} \quad ... (4.2)$$

Here $\alpha$ is the absorption coefficient of the material used to form the subcell layer. The short-circuit current density for a particular subcell layer, say nth layer is given by

$$J_{sc,n} = e \int (1 - \exp \left[ - \alpha_n (\lambda) x_n \right]) \Phi_n(\lambda) \, d\lambda \quad ... (4.3)$$

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Here \( \lambda_n = \frac{hc}{E_{gn}} \) is the wavelength corresponding to bandgap of the \( n \)th subcell. To understand the dependence of current density on bandgaps of multiple junctions, it is helpful to consider the simple case of a double-junction solar cell. The top-junction subcell layer will filter the photons and the remaining ones having the bandgap equal to or higher than the bandgap of the bottom cell will be absorbed by the bottom subcell. The bottom-junction current density \( J_{sc, 2} \) depends on both bandgaps of both the subcell layers \( E_{g1} \) and \( E_{g2} \). The \( J_{sc, 1} \) depends only on \( E_{g1} \).

\[
J_{sc, 1} = e \int_{0}^{\lambda_{1}} \Phi s (\lambda) \, d\lambda, \quad J_{sc, 2} = e \int_{\lambda_{1}}^{\lambda_{2}} \Phi s (\lambda) \, d\lambda \ldots (4.4)
\]

### 4.2.4 Current Matching and Top-subcell Thinning

Relative magnitudes of top and bottom subcell current densities \( J_{sc, top} \) and \( J_{sc, bot} \) depend upon the bandgaps of the subcell layers \( E_{gtop} \) and \( E_{gbot} \). As the bandgap of top subcell \( E_{gtop} \) decreases the current density \( J_{sc, top} \) increases however the current density of bottom subcell layer \( J_{sc, bot} \) decreases. The total current density \( J_{sc} \) for series combination of subcell layers will be less than the summation of \( J_{sc, top} \) and \( J_{sc, bot} \). The current density is highest when the top and bottom subcells are matched.

The absorption coefficient \( \alpha (hv) \) of a solar cell subcell layer is not infinite; hence a cell of finite thickness (not optically thick) is unable to absorb all the incident photons having energies higher than the bandgap. A small portion of the light will be transmitted at photon energies near the bandgap having a small \( \alpha \), the thinner the cell, the greater will be the transmission. Thinning the top subcell layer will increase the current of the bottom subcell at the expense of top subcell current. In case \( J_{sc, top} > J_{sc, bot} \) before thinning the top subcell layer then after thinning \( J_{sc, top} = J_{sc, bot} \). As the series multijunction cell current \( J_{sc} \) is limited to lesser of \( J_{sc, top} \) and \( J_{sc, bot} \), hence the cell efficiency will be maximum when the current matching is done by thinning the subcell layers \( J_{sc, top} \) and \( J_{sc, bot} \). This criterion does not significantly increase the overall efficiency of the solar cell because cell’s maximum power point is not at short circuit current; however it is a very good approximation for high-quality and high-bandgap solar cells. The ability of the thin top subcell layer of a multijunction solar cell to absorb a high fraction of the incoming light is due to a very high absorption coefficient of the top subcell \([1]\). The cell thinning also help in reducing the amount of material used making the solar cells more cost effective.
4.2.5 Current Matching Effect on Fill Factor and Open-Circuit Voltage

The fill factor (FF) of a multijunction tandem cell depends upon the photocurrents of the subcell layers. In case of two-junction it depends upon the photocurrent of the top and bottom subcell. The fill factor is at minimum for current matched condition. This effect slightly undermines the efficiency gain obtained by increased current density ($J_{sc}$) at current-matched condition. The decrease in fill factor at current-matching is roughly half the gain in current density by current-matching. This dependence of FF on ratio of subcell currents is important. Open-circuit voltage ($V_{oc}$) also depends on cell thickness. For a subcell layer having a well-passivated base thinning the upper cells contributes in meaningful increase in open-circuit voltage. For poorly passivated subcell layers thinning the subcell layers will lower the $V_{oc}$. For tandem structures thinning the top subcell layer for current matching, the passivation of the base of the subcell layer is a very important for overall device efficiency [1].

4.2.6 Spectral Effects

Amount of light distributed in each subcell layers and photocurrent generation in each subcell layers depend upon the spectrum of the incident light. The optimal bandgaps and optimal top-subcell thinning depends upon incident spectrum. For a particular bottom subcell bandgap the optimal top subcell bandgap $E_{gtop}$ is higher for AM0 spectrum in comparison to AM1.5 global spectrum. The difference occurs because the AM0 spectrum has higher intensity of blue light component than the AM1.5 global spectrum resulting in greater $J_{sctop}/J_{scbot}$. Increasing the bandgap of the top subcell layer compensates for this by directing more light to the top subcell layer. Similarly for a particular $E_{gtop}$ and $E_{gbot}$ the optimal top-subcell thickness is lower for AM0 spectrum than the global spectrum. AM0 spectrum is a blue rich spectrum in comparison to global spectrum; hence the top subcell thickness is about 0.5 μm less. No one spectrum precisely represents the actual spectrum seen by terrestrial solar cell. Constantly changing position of the sun in the sky and changing and changing atmospheric conditions are also significant in the variation of the spectrum. It is generally considered that series connected tandem solar cells are sensitive to the fluctuations in the air-mass. The good thing is that the efficiency at high air-mass is trivial as the net power output is small under these conditions. In all cases a well designed multijunction solar cell will outperform a single-junction solar cell by a reasonable margin even
considering the spectral fluctuations. Another issue of concern in series connected multijunction solar cells is the chromatic aberration of the spectrum caused by concentrating optics in concentrator solar cells, particularly for Fresnel concentrator lenses [1, 20]. Chromatic aberration results in a position dependent variation in the spectrum across the cell. In case of low concentrator lenses the adverse effect of spatial variations can be limited by making the emitter of the bottom subcells highly conductive.

4.2.7 Anti Reflective Coating (ARC) Effects

Applying antireflective coating in III-V multijunction solar cells reflectance in the order of 30% without ARC can be mitigated. Double layer antireflective coating (DLAR) like (MgO₂+TiO₂ or ZnS+MgF₂ or TiO₂+Al₂O₃) reduces the reflection loss to ~1% [21]. Implementation of antireflective coating is therefore very important. The dependence of reflectance on layer thickness can be broken into two parts: ratio of two thicknesses and total thickness of each layer. Proper choice of ratio of layer thicknesses produces a reflectance having flat, low and notch-shaped minimum. If the ratio is held constant the total thickness of ARC determines the position of the minimum, increasing thickness shifts the notch position to lower photon energy. The width of the notch is less than solar radiation spectrum range so irrespective of the position of the notch, the photocurrents of subcell layers will be less than ideal case of zero reflectance. In case of shifting the notch to higher photon energy will send more light to the top subcell layer at the expense of the bottom subcell and vice versa. The antireflective coating affects the current matching in the subcell layers. If the top subcell thickness increases, the optimal ARC thickness increases to compensate by directing more light to the bottom subcell [1].

4.2.8 Concentrator Cells

Solar cells are normally expensive. A potential method of making the solar cell modules more cost effective is by casting on them concentrator lenses that can focus higher light intensity than the available solar intensity. Concentrators are optical elements collecting solar energy in a certain area and redirect them onto the solar cells. The collecting optical element must be cheaper per unit area than the solar cell. Solar cells can convert additional luminous power impinging without significant loss of efficiency. Concentrating systems normally needs tracking mechanism to keep sunlight always focused on the cell surface. Static concentrators are also
available but they normally yield moderate concentration. The level of solar irradiance (luminous power flux) at which concentrating cells operates is variable. The average solar irradiance is 1kW/m². The level of irradiance in static concentrator cells is in the range of 1.5 - 5kW/m². Silicon tracking concentrators range from 10 to 500kW/m² whereas for GaAs it is 100 - 1500kW/m². It is a very common to mention irradiation level in ‘suns’ meaning the number of times the actual irradiance is increased by the application of concentrator lenses. At higher temperatures semiconductors with larger bandgap energy are advantageous than their low bandgap counterparts. Tandem cells comprised of III-V semiconductors have become more cost effective due to the implementation of concentrator lenses [2, 22].

4.3 III-V Multijunction Solar Cells: Lattice-Matched & Lattice-Mismatched

4.3.1 Lattice-Matched Systems

Major advantages of monolithic solar cells are that they are composed of closely lattice-matched semiconductors. The fabrication of these monolithic structures is done by MBE (Molecular Beam Epitaxy), MOCVD (Metal Organic Chemical Vapor Deposition), CPE (Chemical Phase Epitaxy) etc. Lattice-matching makes hetero-epitaxial fabrication easier for chemically similar materials like AlGaAs or GaInP on GaAs substrate. To acquire optical transparency and maximum current conductivity in monolithic multijunction solar cell it is very important to choose subcell layer materials having very similar crystal structure. The subcell layers are normally grown on top of each other over the same substrate. In designing III-V high efficiency solar cell it is therefore very important to consider the relationship between the bandgap and lattice constant. Lattice constant is the spacing of atom locations in a crystal structure. Mismatch in the lattice constant in the crystal structure of different subcell layers can significantly deteriorate the overall efficiency of the solar cell [2, 23]. Some lattice-matched multijunction examples are mentioned below.

AlGaAs/GaAs two-junction cell: The Al$_{0.4}$Ga$_{0.6}$As/GaAs 2-junction cell having a theoretical AM1.5 efficiency of 34% and was favored in the past because the device fabrication technologies for this system were well developed. A 1.93 eV AlGaAs/GaAs metal interconnected two terminal two-junction cell with an efficiency of 27.6% under 1 Sun AM1.5G and 23% at 1 Sun (Chung et al.,1989) has been reported. More recently an AlGaAs/GaAs two-
terminal two-junction (having a tunnel-junction) solar cell has been reported by Takahashi et al. (1998) yielding 28.8% efficiency under 1 Sun AM1.5. There are still some challenges with the growth of high-quality, oxygen free AlGaAs and the fabrication of a high-conductance subcell interconnection [2].

**InGaP/GaAs two-junction cell:** A tandem combination of an In$_{0.5}$Ga$_{0.5}$P top cell with a direct bandgap of $\sim$1.9 eV and GaAs in the bottom cell with a band-gap of 1.43 eV has a theoretical efficiency of 34% but the lattice parameters and current of these cells should be matched. The absence of oxygen related defects has enabled the development of high-efficiency InGaP/GaAs two-terminal, double-junction solar cell with AM1.5 efficiencies of 29.5% (Bertness et al., 1994) and later 30.3% (Takamoto et al., 1997). Tunnel junction is being implemented in between the subcell layers. This high efficiency was achieved by increasing the InGaP top cell efficiency with the progress in epitaxial growth process. The InGaP/GaAs cells have great potential for future space applications and have shown superior radiation resistant properties in comparison to single-junction GaAs or Si cells (Yamaguchi et al., 1997).

**InP/InGaAs two-junction cell:** The combination of InP having bandgap energy of 1.35 eV and In$_{0.53}$Ga$_{0.47}$As with band-gap energy of 0.75 eV is a lattice matched double-junction solar cell and has a theoretical conversion efficiency of 37% under 500 Suns illumination at AM1.5G and 33% at AM0 at 80°C (Wanlass et al., 1989). This system also has a great potential for several space applications. The InP cell is more radiation-resistant than other popular cells like Si or GaAs. Efficiency of 31.8% has been obtained InP/InGaAs metal-interconnected three-terminal monolithic two junction cell under 50 suns AM1.5 at 25°C (Wanlass et al., 1991) [2].

**GaAs/Ge two-junction cell:** The solar cell having the combination of GaAs with band-gap energy of 1.43 eV and Ge with band-gap energy of 0.73 eV is a lattice-matched system that produces 41% efficiency under 500 Suns illumination AM1.5 at 50°C. A GaAs/Ge two-junction cell having efficiency of cell having efficiency of 24.1% at 1 Sun AM1.5 and 21.3% at 1 Sun AM0 was fabricated by MOCVD (Vermon et al., 1989). The p-n junction in the Ge bottom cell was developed by in-diffusion of gallium and arsenic into the Ge from growing GaAs cell structure through MOCVD and the cell-interconnect formed by highly doped GaAs-Ge hetero-junction.
**InGaP/GaAs/Ge three-junction cell:** A three-junction cell made up of In$_{0.5}$Ga$_{0.5}$P top cell with a direct band-gap of about 1.9 eV, a GaAs middle cell with band-gap energy of 1.43 eV and a Ge bottom cell with band-gap of 0.73 eV is lattice-matched and has a theoretical efficiency of 38% under 1 Sun AM1.5 and 42% under 400 Suns AM1.5 at 25ºC (MacMillan et al., 1989). An InGaP/GaAs/Ge two-terminal three-junction solar cell with the efficiency of 26.5% under 1 sun AM0 was demonstrated by Chiang et al. in 1994.

**GaInP/GaInAs/Ge three-junction cell:** A lattice-matched three-junction solar cell comprised of GaInP/GaInAs/Ge subcell layers was demonstrated by the researchers in Boeing Spectrolab having the efficiency of 40.1% measured at 135 suns (AM1.5D, low-AOD, 13.5 W/cm$^2$, 25ºC). This is the record efficiency produced by any lattice-matched multijunction solar cell [18].

### 4.3.2 Lattice-Mismatched Systems

The attractive feature of lattice-mismatched materials is the wide range of semiconductors available having precisely the bandgap energies needed to obtain maximum efficiency. The only concern is reducing the stress-strain effects and misfit dislocations in the subcell layers. Hetero-epitaxy of lattice-mismatched materials is relatively challenging. Lattice mismatch is accommodated by nucleation and propagation of dislocations in concentrations that depend on mismatch and thickness of individual layers. The dislocations are often centers for recombination limiting minority-carrier lifetime, diffusion length and efficiency of the device. Improvement in deposition technologies and implementation of buffer layers has helped reducing the adverse effects in lattice-mismatched systems. Some lattice-mismatched multijunction examples are mentioned below.

**AlGaAs/Si monolithic two-junction cell:** An attractive combination of materials to obtain high-efficiency double-junction cells is either Al$_x$Ga$_{1-x}$As or GaAs$_{1-y}$Py with x~0.3, y~0.27 (band-gap energy ~1.75eV) for a top cell and a Si bottom cell. Apart from the ideal combination of band-gaps the Si substrate technologies are very well developed and inexpensive. An AlGaAs/Si two-junction solar cell with an AM0 efficiency of 21.2% has been reported (Umeno et al., 1998). The cell was developed by MOCVD technique.

**InGaP/InGaAs monolithic cell:** An InGaP ($E_g = 1.7$eV) top cell with InGaAs ($E_g = 1.1$eV) bottom cell solar cell developed on GaAs surface has also an ideal combination of band-gaps and
produces theoretical efficiency of 33.1% under 1 sun AM0. There have been some approaches to
develop InGaP/InGaAs monolithic two-junction solar cells; however the problem of dislocations
by the lattice-mismatch needs to be overcome.

**GaAs/Si mechanically stacked cell:** The GaAs/Si cell does not offer an ideal combination of
band-gaps but produces a theoretical efficiency of 38% under 500 Suns AM1.5 at 50ºC. In
addition both the cells are very well developed and the Si is also quite inexpensive. A GaAs/Si
mechanically stacked cell with an efficiency of 31% has been reported by Gee and Virshup
(1998) under 347 suns AM 1.5 [2].

**GaAs/GaSb mechanically stacked cell:** The GaAs/GaSb solar cell has nearly-ideal combination
of band gaps (1.43/0.73eV) and has theoretical efficiency of 41% under 500 suns AM1.5 at 50ºC.
A GaAs/GaSb mechanically stacked cell with efficiency of 34.2% under 100 suns AM1.5 and
32.2% under 100 Suns AM0 has been demonstrated by Fraas et al. (1990).

**GaInP/GaInAs/Ge triple-junction solar cell:** A metamorphic triple-junction solar cell
comprised of Ga\textsubscript{0.35}In\textsubscript{0.65}P/Ga\textsubscript{0.83}In\textsubscript{0.17}As/Ge subcell layers having virtually ideal band gap
combination is a good example of high lattice mismatched solar cell. Improvement in material
quality of the lattice-mismatched layers along with the implementation of a highly relaxed Ga
\textsubscript{1-y}In\textsubscript{y}As buffer structure between the Ge substrate and the middle cell has led to a very high
efficiency. Metamorphic growth with low dislocation densities below 10\textsuperscript{6}/cm\textsuperscript{2} has been reported.
This solar cell has a conversion efficiency of 41.1% at 454 suns AM1.5d ASTM G173–03 [24].

**4.3.2.1 Inverted metamorphic multijunction solar cells**

The recent introduction of inverted lattice-mismatched or inverted metamorphic device structure
has demonstrated significant promises in the development of next generation multijunction solar
cells. The basic idea is to obtain high performance subcells having near optimal bandgaps is by
growing the subcell layers in increasing order of lattice-mismatch to the substrate, thus
minimizing the propagation of stress-strain induced defects in the device structure. The lattice
matched top subcell layer is developed first, leaving an essential an essential stress-strain and
defect free growth surface. The subcell layers with highest mismatch is grown last limiting the
misfit dislocations. Fabricating the subcell layers in this order, which is the inverse of the growth
direction of conventional multijunction solar cells requires removing the substrate in order to

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allow the light enter the highest bandgap subcell first similar to the stacked subcell multijunction configuration. Growth substrates are currently removed destructively by wet chemical etching. There are several methods of substrate removal technique in development to reuse the substrate reducing the fabrication cost. This possibility is more significant with GaAs substrate. Before the substrate removal a handle-mount foreign substrate is bonded to the other side of the cell for mechanical support. The handle is likely to be less expensive than the substrate. Step-graded buffer layers are being used in between the mismatched subcell layers to relieve strain and misfit dislocations away from active region of the subcell layers. The compositions of the buffer layers are chosen to be transparent to light. Careful control of growth conditions allows relaxation of the grown layers in the buffer to the new lattice constant with minimal propagation of growth defects to the subsequent subcell layers [1, 25]. To design solar cells having higher levels of lattice-mismatch the national renewable energy lab (NREL) researchers has successfully demonstrated an inverted metamorphic structure [26].

4.4 Cell Interconnects, Window and Back Surface Field Layers

4.4.1 Tunnel-junction Interconnects

Tunnel junctions act like diodes in reverse polarity to the original configuration in the stack of subcells and are used to make better optical, electrical and mechanical matching between the subcell layers [27]. The tunnel-junction interconnects between subcell layers is to provide a low resistance connection between the p-type back surface field (BSF) layer of a subcell and the n-type window layer of the subcell beneath it. In absence of tunnel-junctions this p-n junction has a polarity of forward turn-on voltage that is opposite to that of the top or bottom subcells. If tunnel junction is not present in a design the p-n junction could produce a photo-voltage that could roughly negate the photo-voltage generate by the top subcell in case of illumination. Tunnel-junction is made up of a $p^{++}n^{++}$ junction where $p^{++}$ and $n^{++}$ represents degenerately doped material. The space-charge region for a $p^{++}n^{++}$ junction should be very narrow ~ 10 nm. In case of small forward bias and any reverse bias the normal thermal characteristic of a $p^{++}n^{++}$ junction is shorted by tunneling through the narrow space-charge region and the tunnel-junction behave like a resistor. In forward bias condition current density greater than critical value of peak tunneling current $J_p$ this resistor like behavior disappears and the behavior of tunnel-junction
switches to the one dominated by usual thermionic emission. Voltage drop across the tunnel-junction increases to that of a typical $p$-$n$ junction. The functional form of $J_p$ is described by

$$J_p \propto \exp \left( -\frac{E_g^{3/2}}{\sqrt{N^*}} \right) \ldots (4.5)$$

Here $E_g$ is the bandgap and $N^* = N_A N_D / (N_A + N_D)$ is the effective doping concentration [28]. The value of $J_p$ must be higher than the value of photocurrent of the tandem cell. For a concentrator cell operating at 1000 sun concentration $J_{sc}$ is $\sim 14$A/cm². The best tunnel-junctions for very high-efficiency multijunction solar cells are normally defect free. Lifetime-limiting, midgap defects usually add to the excess current. No evidence has been found that extended defects add to $J_p$ or increase conductivity in the tunneling portion of the I-V curve. High excess currents can mask a low $J_p$ but the junction conductivity is unacceptably low. It is possible that high concentrations of extended defects can compensate donors or acceptors in the junction leading to increased depletion width and lower tunneling currents. Defects can also reduce the thermal stability of tunnel-junction and the quality of overlying layers. It is good to grow tunnel-junction interconnects free of extended defects [1].

### 4.4.2 Window Layers

The function of an emitter window layer is to passivate the surface associated with emitter surface. The states are minority carrier traps. Their effect is characterized by surface recombination velocity $S$. In any solar cell a high surface recombination velocity will reduce the photoresponse of the solar cell, mostly in the blue portion of the solar radiation spectrum. The conditions to design an effective window layer for an n-on-p cell the materials should have: 1) Lattice constant close to the material used for the subcell layer, 2) Bandgap $E_g$ much higher than that of emitter, 3) Large valence band offset with respect to emitter to provide a potential barrier for minority holes, 4) High electron concentration ($n \geq 10^{18}/\text{cm}^3$) and 5) Material quality good to produce low interface-recombination velocity. AlInP is a well known window layer. It provides a reasonable confinement in the emitter of an n-on-p device. AlInP can be easily doped with either Si or Se. Due to the strong affinity of Al with oxygen if the reactor chamber or the source materials are contaminated with water vapor or other oxygenated compounds then the performance including the conductivity will suffer. Poor quality AlInP will degrade blue response and the fill factor of the solar cell [1].
4.4.3 Back-Surface Field (BSF)

After every diode layer, a high quality back surface field/passivation layer (BSF), typically with higher band gap is introduced to provide confinement to the photo generated minority carriers keeping them within the reach of p/n junctions to be efficiently collected. The high recombination velocity at the interface will affect photoresponse (particularly the red portion of the solar radiation spectrum) and open-circuit voltage ($V_{oc}$). Magnitude of this effect can be quite large and is also affected by the base minority-carrier diffusion length and thickness. The requirements of a good back-surface field layer are similar to the window layers. They should have the following characteristics:

1) Lattice constant close to the next subcell layer material.
2) Bandgap $E_g$ higher than that of the next subcell layer.
3) Large conduction-band offset.
4) Relatively high hole concentration (of the order of $p = 10^{18}/cm^3$)
5) High transparency to photons.

AlGaInP and AlInP are popularly used as BSF layers. The back-surface field layers are a very important integral part of multijunction solar cell design.

4.5 Cell Configuration

4.5.1 Four-Terminal

There are several ways of connecting power leads to the junctions comprising a multijunction stack. In case of four-terminal configuration each subcell has its own two terminals and is electrically isolated from other subcells. The main advantage of this configuration is that it does not set any constraint on the polarities ($p/n$ vs. $n/p$) of the subcells or on their current and voltages. The terminals and the electrical isolation between subcells in the four-terminal configuration are difficult to accomplish monolithically requiring a more complex cell structure and processing. Generally four-terminal device is implemented as mechanical stack, whose complexities of fabrication and assembly make it less attractive structure than their monolithic counterparts. In addition to that unique power controls are required to keep each cell at its maximum power point and then to sum the power off chip [1].
4.5.2 Three-Terminal

In three-terminal configuration the subcells are not electrically isolated; the bottom of each cell is electrically connected to the top of the next subcell layer in the stack. The fabrication of a monolithic three-terminal device is relatively straightforward. The fabrication is more complex than a two-terminal device though. The semiconductor structure must be designed to provide a layer for contact to the intermediate terminal and to provide processing steps required to put the intermediate terminal in place. With this intermediate terminal, the different subcells in the stack do not need to have the same photo generated currents. Furthermore in the three-terminal configuration the different subcells in the stack may have different polarities for example p/n configuration for the top cell and n/p configuration for the bottom cell.

![Diagram of three-terminal cell configuration](image)

Figure 4.2: (a) Four-terminal, (b) Three-terminal and (c) Two-terminal cell configuration, Courtesy of Clean Electricity from Photovoltaics

4.5.3 Two-Terminal Series Connected and Current-Matched

The two-terminal series connected configuration provides fewest possibilities for interconnection of devices. The fundamental design rule is that the subcells should have the same polarity and
photo generated currents of the subcell layers should be closely matched. This current matching criterion puts relatively tight constraints on selection of bandgaps for various subcells in the solar cell structure. There are several advantages of this design. The availability of high-quality monolithic tunnel junction subcell interconnects helps making these stacks as a monolithic subcell structures with metallization on the top and bottom of the entire stack. This in turn means that such solar cells can be integrated into modules with same simplicity of that of the single-junction solar cells. The four-terminal, three-terminal and two-terminal cell configurations are shown in figure 4.2.

4.6 Epitaxial Technologies for Growing III-V Compounds

Epitaxial technologies has evolved and improved a lot since 1972. The merits and demerits of different epitaxial technologies are mentioned in Table 4.1.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>LPE</th>
<th>MOCVD</th>
<th>MBE</th>
<th>CBE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality</td>
<td>Excellent</td>
<td>Very Good</td>
<td>Fairly Good</td>
<td>Very Good</td>
</tr>
<tr>
<td>MQW</td>
<td>Bad</td>
<td>Very Good</td>
<td>Excellent</td>
<td>Excellent</td>
</tr>
<tr>
<td>Abrupt Interface</td>
<td>Bad</td>
<td>Very Good</td>
<td>Excellent</td>
<td>Excellent</td>
</tr>
<tr>
<td>Heavy Doping</td>
<td>Fairly Good</td>
<td>Fairly Good</td>
<td>Very Good</td>
<td>Excellent</td>
</tr>
<tr>
<td>Large Area</td>
<td>Bad</td>
<td>Excellent</td>
<td>Fairly Good</td>
<td>Very Good</td>
</tr>
<tr>
<td>Throughput</td>
<td>Very Good</td>
<td>Excellent</td>
<td>Fairly Good</td>
<td>Very Good</td>
</tr>
<tr>
<td>Efficient use of Source Materials</td>
<td>Very Good</td>
<td>Fairly Good</td>
<td>Fairly Good</td>
<td>Excellent</td>
</tr>
<tr>
<td>Equipment Cost</td>
<td>Very Good</td>
<td>Fairly Good</td>
<td>Fairly Good</td>
<td>Bad</td>
</tr>
</tbody>
</table>

Liquid phase epitaxy (LPE) was used to fabricate GaAs solar cells in 1972 because it produces high quality epitaxial film and has a simple growth system but it has some disadvantages also. LPE is good for devices with multilayer construction because of the difficulty of control over layer thickness, doping, composition and speed of throughput. Since 1977, metal-organic-chemical-vapor-deposition (MOCVD) is being used to fabricate large-area GaAs solar cells.
because it is capable of large-scale-large-area production with a good reproducibility and controllability. Using large MOCVD systems (example AIXTRON AIX-3000 or EMCORE Enterprise 400) we can process up to 25 wafers, each of diameters of 4 inch, two-junction InGaP/GaAs cells and three-junction InGaP/GaAs/Ge cells are being commercially produced by TECSTAR (Yeh et. al., 1997) and Spectrolab (Chiang et al., 1996).
CHAPTER FIVE
PHOTONIC MODELING OF NOVEL MULTIJUNCTION (QUADRUPLE-JUNCTION) SOLAR CELLS

5.1 Introduction

This chapter discusses about two novel quadruple-junction solar cell designs comprised of AlGaInP/InGaAs/GaSb/InGaSb and AlGaInP/InGaP/InGaAs/InGaSb subcell layers. The subcell layers implemented are all direct bandgap semiconductors, have very high absorption coefficient, good optical and electrical properties and all have zinc-blende lattice structure. These qualities along with effective spectrum splitting make the designs very interesting. The transfer-matrix method of wave propagation through multi-layer structures has been implemented to find the photon absorption of the solar cell designs. The matrices are formed for the intersection between two layers and wave propagation through each layer. The product of all the transfer matrices forms the actual transfer-matrix for each solar cell designs. The photon absorption of these two solar cell designs has been favorably compared with the state-of-art solar cell designs. The combination of subcell layers has yielded very high photon absorption through the entire solar radiation spectrum. The novelty of this designs lie in introduction of antimony (Sb) based subcell layers increase photon collection in the near and far infrared regions of the solar radiation spectrum. This combination of subcell layers will generate higher voltage because of the summation of the voltages in the subcell layers, promoting to higher efficiency of the solar cell. This makes the solar cells cost more cost effective and help reduce the material and land acreage cost.

5.2 Two Quadruple-Junction Structures

I have designed two novel quadruple junction solar cells comprising of AlGaInP (2.3eV) / InGaAs (1.1eV) / GaSb (0.7eV) / InGaSb (0.5eV) and AlGaInP (2.3eV) / InGaP (1.93eV) / InGaAs (1.1eV) / InGaSb (0.5eV) III-V semiconductor material subcell layers covering ultraviolet (UV), visible, and near- and far infrared (IR) regions of the electromagnetic spectrum. The quadruple junction designs are shown in figure 5.1 (a) and (b).
AlGaInP along with InGaP/InGaAs combination of solar cells has been proposed by several researchers. GaSb solar cell has also been proposed to increase the efficiency [29, 30]. InGaSb is a novel layer being implemented as the bottom subcell layer along with the bandgap combinations mentioned above. In case of the first design mentioned in figure 5.1 (a) the sunlight strikes the AlGaInP subcell layer having the highest bandgap first. This layer will absorb photons of energy 2.3eV and higher and will transmit the less energetic photons to the next subcell layer (InGaAs) in the stack. InGaAs subcell layer will capture photons having energy less than 2.3eV but equals to or higher than 1.1eV. The GaSb subcell layer will absorb photons of energy equals to and higher than 0.7 eV but less than 1.1eV and InGaSb layer will capture photons having energy equals to or higher than 0.5eV respectively. In case of the second design mentioned in figure 5.1 (b) AlGaInP layer will absorb photons of energy 2.3eV and higher and will transmit the less energetic photons to the next subcell layer (InGaP) in the stack. InGaP subcell layer will capture photons having energy less than 2.3eV but equals to or higher than 1.93 eV. The InGaAs subcell layer will absorb photons of energy equals to and higher than 1.1 eV but less than 1.93eV and InGaSb layer will capture photons having energy equals to or higher than 0.5eV respectively. This arrangement makes use of the fact that junctions behave like low-pass photon energy filters transmitting only sub-bandgap light and acting like optical
elements splitting spectrum to appropriate junctions for multijunction photo conversion. The quadruple junction solar cell will trade a lower current density for higher voltage and will divide the spectrum more efficiently. The lower current density in the subcells will significantly reduce resistive power loss compared to triple, double or single junction solar cells [11]. The device is normally made on the same substrate. In this case the substrate is InGaSb as shown in figure 2. High material quality with good precision in thickness is important for the efficient collection of minority carriers (electrons and holes). After every diode layer, a high quality back surface field/passivation layer (BSF), typically with higher band gap is introduced to provide confinement to the photo generated minority carriers keeping them within the reach of p/n junctions to be efficiently collected. The confinement should not increase the series resistance of the device [12]. AlGaInP, InGaP/InGaAs combination of solar cells has been proposed by several researchers. GaSb solar cell has also been proposed to increase the efficiency [9, 10]. InGaSb is a novel layer being implemented as the bottom subcell layer along with the bandgap combinations mentioned above. In case of the first design mentioned in figure 2 a) the sunlight strikes the AlGaInP subcell layer having the highest bandgap first. This layer will absorb photons of energy 2.3eV and higher and will transmit the less energetic photons to the next subcell layer (InGaAs) in the stack. InGaAs subcell layer will capture photons having energy less than 2.3eV but equals to or higher than 1.1eV. The GaSb subcell layer will absorb photons of energy equals to and higher than 0.7 eV but less than 1.1eV and InGaSb layer will capture photons having energy equals to or higher than 0.5eV respectively. In case of the second design mentioned in figure 2 b) AlGaInP layer will absorb photons of energy 2.3eV and higher and will transmit the less energetic photons to the next subcell layer (InGaP) in the stack. InGaP subcell layer will capture photons having energy less than 2.3eV but equals to or higher than 1.93eV. The InGaAs subcell layer will absorb photons of energy equals to and higher than 1.1 eV but less than 1.93eV and InGaSb layer will capture photons having energy equals to or higher than 0.5eV respectively. This arrangement makes use of the fact that junctions behave like low-pass photon energy filters transmitting only sub-bandgap light and acting like optical elements splitting spectrum to appropriate junctions for multijunction photo conversion. The quadruple junction solar cells will trade a lower current density for higher voltage and will divide the spectrum more efficiently. The lower current density in the subcells will significantly reduce resistive power loss compared to triple, double or single junction solar cells [31]. The device is normally made on
the same substrate. In this case the substrate is InGaSb as shown in figure 5.1. The alternative substrate that can be used in the inverted metamorphic designs is GaAs. High material quality with good precision in thickness is important for the efficient collection of minority carriers (electrons and holes). After every diode layer, a high quality back surface field/passivation layer (BSF), typically with higher band gap is introduced to provide confinement to the photogenerated minority carriers keeping them within the reach of p/n junctions to be efficiently collected. The confinement should not increase the series resistance of the device [32].

5.3 Wave Propagation through Multilayer Structures (Stratified Medium): Transfer Matrix Method

Transfer-matrix method is being used to analyze the electromagnetic (light) wave propagation through multilayer structures. It is based on the fact that according to Maxwell’s equations there is continuity in the electric or magnetic field across boundaries from one medium to the next [33]. The stratified medium can be considered as an optical system with one input and one output port. Transfer matrix is formed for each subcell layer depending upon the Fresnel equations and optical properties of the material. The complete t-matrix is formed by multiplying all the individual subcell later matrices [34, 35, 36, 37].

Figure 5.2: Optical wave propagation in stratified medium, reflection and transmission at interface

If we consider a layered medium stack of layers with thickness $d_i$, complex refractive indices $n_i$ and a plane wave with wave vector $k$ incident on the stack. The normal to the interface plane (x) and the wave vector is called incident plane. The plane orthogonal to the x axis is the z axis as a
result the wave vector lies entirely in the x, z plane with y component \( k_y = 0 \). The problem is solved for both TE and TM polarizations. The system is shown in figure 5.2. In each layer forward propagating wave (towards increasing x) has the same direction. It is also appropriate for the backward propagating wave (towards decreasing x). The total field in layer i depicted in figure 5.2 is

\[
E(x, y, z) = A_F e^{-j(k_{x,i} x + k_z z)} + A_B e^{-j(-k_{x,i} x + k_z z)}
\]

\[
= A_F e^{-j k_{x,i} x} e^{-j k_z z} + A_B e^{+j k_{x,i} x} e^{-j k_z z}
\]

\[
= E_F(x) e^{-j k_z z} + E_B(x) e^{-j k_z z} \quad (5.1)
\]

Here \( E(x, y, z) \) is the total electric field and \( E_F(x) \) and \( E_B(x) \) are forward and backward electric fields respectively. The x component of the wave vector \( k_i \) in layer i is \( k_{x,i} \) given by

\[
k_{x,i} = [(n_i k_0)^2 - k_z^2]^{1/2} \quad \text{for } i = 0, 1, 2, \ldots, N \quad (5.2)
\]

Here \( k_0 = \omega/c \). The value of \( k_z \) remains constant because of boundary conditions relating to field amplitudes of incident reflected and transmitted wave. In case of lossless media and barring total internal reflection \( k_{x,i} \) is related to incident angle \( \theta_i \) according to Fresnel formula

\[
k_{x,i} = n_i k_0 \cos(\theta_i) \quad (5.3)
\]

The idea is to build transfer matrices for wave propagation through interface between two layers and then transfer matrix for the propagating layer. The complete transfer matrix is formed by multiplying each individual transfer matrices.

Figure 5.3: (a) Reflection and transmission at an interface between i and j layers, (b) Translation of light through a layer with thickness \( d_i \) and refractive index \( n_i \)
Figure 5.3 depicts interface between any two adjacent layers (i and j) and the amplitudes of forward and backward propagating waves before and after the interface. The t-matrix is formed from the complex reflection and transmission coefficients (Fresnel coefficients).

\[
\begin{bmatrix}
E_F(x_i^+) \\
E_B(x_i^-)
\end{bmatrix} =
\begin{bmatrix}
t_{ij} & r_{ji} \\
r_{ij} & t_{ji}
\end{bmatrix}
\begin{bmatrix}
E_F(x_i^-) \\
E_B(x_i^+)
\end{bmatrix} \quad \ldots \,(5.4)
\]

For TE polarization the Fresnel coefficients are

\[
r_{ij} = \frac{E_B(x_i^+)}{E_F(x_i^-)} = \frac{k^i_x - k^j_x}{k^i_x + k^j_x} \quad \ldots \,(5.5)
\]

\[
t_{ij} = \frac{E_F(x_i^+)}{E_B(x_i^-)} = 1 + r_{ij} = \frac{2k^i_x}{k^i_x + k^j_x} \quad \ldots \,(5.6)
\]

In case of TM polarization

\[
r_{ij} = \frac{E_B(x_i^+)}{E_F(x_i^-)} = \frac{n_j^2 k^j_x - n_i^2 k^i_x}{n_i^2 k^j_x + n_j^2 k^i_x} \quad \ldots \,(5.7)
\]

\[
t_{ij} = \frac{E_F(x_i^+)}{E_B(x_i^-)} = \frac{n_i}{n_j} (1 + r_{ij}) \quad \ldots \,(5.8)
\]

According to Fresnel formulae and a function of refractive indices \( n_i, n_j \) and incident, reflected and transmitted angle \( \theta_i, \theta_j \) for TE polarization equation (5.5) and (5.6) can be represented by

\[
r_{ij} = \frac{E_B(x_i^+)}{E_F(x_i^-)} = \frac{n_i \cos \theta_i - n_j \cos \theta_j}{n_i \cos \theta_i + n_j \cos \theta_j} \quad \ldots \,(5.9)
\]

\[
t_{ij} = \frac{E_F(x_i^+)}{E_B(x_i^-)} = 1 + r_{ij} = \frac{2n_i \cos \theta_i}{n_i \cos \theta_i + n_j \cos \theta_j} \quad \ldots \,(5.10)
\]

In case of TM polarization equation (5.7) and (5.8) can be represented as

\[
r_{ij} = \frac{E_B(x_i^+)}{E_F(x_i^-)} = \frac{n_i \cos \theta_i - n_j \cos \theta_j}{n_i \cos \theta_i + n_j \cos \theta_j} \quad \ldots \,(5.11)
\]

\[
t_{ij} = \frac{E_F(x_i^+)}{E_B(x_i^-)} = \frac{n_i}{n_j} (1 + r_{ij}) = \frac{2n_i \cos \theta_i}{n_j \cos \theta_j + n_i \cos \theta_j} \quad \ldots \,(5.12)
\]
Equation (5.4) is rewritten to find a relation between forward and backward propagating waves in layers i and j respectively.

\[
\begin{bmatrix}
E_F(x_i^-) \\
E_B(x_i^-)
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{t_{ij}} & -r_{ji} \\
\frac{r_{ij}}{t_{ij}} & -\frac{r_{ij}}{t_{ij}}
\end{bmatrix}
\begin{bmatrix}
E_F(x_i^+) \\
E_B(x_i^+)
\end{bmatrix}
\] 

…(5.13)

According to the symmetry of Fresnel coefficients

\[ r_{ij} = -r_{ji} \quad \text{and} \quad t_{ij} \times t_{ji} - r_{ij} \times r_{ji} = 1 \] 

…(5.14)

Simplifying equation (5.13) we get

\[
\begin{bmatrix}
E_F(x_i^-) \\
E_B(x_i^-)
\end{bmatrix} =
\frac{1}{t_{ij}}
\begin{bmatrix}
1 & r_{ij} \\
r_{ij} & 1
\end{bmatrix}
\begin{bmatrix}
E_F(x_i^+) \\
E_B(x_i^+)
\end{bmatrix}
\] 

…(5.15)

Hence the transfer matrix \( T_{ij} \) for wave propagation through the interface between two layers i and j is

\[
T_{ij} = \frac{1}{t_{ij}}
\begin{bmatrix}
1 & r_{ij} \\
r_{ij} & 1
\end{bmatrix}
\] 

…(5.16)

As we have formed the transfer matrix for the interface between two layers now we form the t-matrix for the wave propagating through a layer (i layer) as shown in figure 5.3 (b). For a given \( z \), amplitudes of forward and backward propagating waves just before and after the interface of adjacent layers are related as

\[
E_F(x_i^-) = E_F(x_{i-1}^+) e^{-j \frac{k}{\lambda} x_i} \\
E_B(x_{i-1}^+) = E_B(x_i^-) e^{-j \frac{k}{\lambda} x_i} 
\] 

…(5.17)

The transfer matrix \( T_i \) for wave propagation through a layer is

\[
T_i =
\begin{bmatrix}
e^{j\phi_i} & 0 \\
0 & e^{-j\phi_i}
\end{bmatrix}
\] 

…(5.18)
\( \Phi_i \) represents a phase change (distance between the two phase fronts) and is a complex quantity but in case of a lossless medium and in absence of total internal reflection

\[
\Phi_i = \frac{2\pi}{\lambda_0} n_i d_i \cos \theta_i \quad \ldots(5.19)
\]

The complete transfer matrix for wave propagation can be represented by multiplication of individual transfer matrices

\[
\begin{bmatrix}
E_F(x_0^-) \\
E_B(x_0^-)
\end{bmatrix}
= T_{0N}
\begin{bmatrix}
E_F(x_{N-1}^+) \\
E_B(x_{N-1}^+)
\end{bmatrix}
= \begin{bmatrix}
T_{11}^{0N} & T_{12}^{0N} \\
T_{21}^{0N} & T_{22}^{0N}
\end{bmatrix}
\begin{bmatrix}
E_F(x_{N-1}^+) \\
E_B(x_{N-1}^+)
\end{bmatrix}
\]

\( \ldots(5.20) \)

Here \( T_{0N} = T_{01} T_{11} T_{12} T_2 \ldots T_{(N-1)} T_{(N-1)N} \quad \ldots(5.21) \)

In case of solar cells the light enters the stack from one side only, hence \( E_B(x_{N-1}^+) = 0 \), leading to 2 equations and 2 unknowns as shown in equation (5.22).

\[
\begin{bmatrix}
E_F(x_0^-) \\
E_B(x_0^-)
\end{bmatrix}
= \begin{bmatrix}
T_{11}^{0N} & T_{12}^{0N} \\
T_{21}^{0N} & T_{22}^{0N}
\end{bmatrix}
\begin{bmatrix}
0 \\
E_F(x_{N-1}^+)
\end{bmatrix}
\]

\( \ldots(5.22) \)

The reflection \( (r) \) and transmission \( (t) \) coefficients are ratios

\[
r = \frac{E_B(x_0^-)}{E_F(x_0^-)} = \frac{T_{21}^{0N}}{T_{11}^{0N}}
\]

\[
t = \frac{E_F(x_{N-1}^+)}{E_F(x_0^-)} = \frac{1}{T_{11}^{0N}} \quad \ldots(5.23)
\]

The power density of a forward and backward plane wave in an arbitrary layer \( i \) is given by

\[
\langle P(x) \rangle = \text{Re} \left( S(x) \right) = \text{Re} \left( E(x) \times H(x) \right) = \text{Re} \left( n \frac{|E(x)|^2}{2Z_0} \right) \quad \ldots(5.24)
\]

Here \( Z_0 = 377 \) is the impedance of the vacuum. This represents power per unit area perpendicular to the wave propagation that varies from layer to layer. To determine the power
reflection and transmission of multilayer thin film the power density per unit area is expressed parallel to the layers.

\[
\langle P(x_i) \rangle = \cos(\theta_i) \langle P(x_i) \rangle = \text{Re}(n) \cos(\theta_i) \frac{|E(x)|^2}{2Z_0} \quad \ldots(5.25)
\]

The power reflectance (R), transmittance (T) and absorption (A) is hence calculated as

\[
R = \left| r \right|^2, \quad T = \frac{\text{Re}(n_N) \cos(\theta_N)}{\text{Re}(n_0) \cos(\theta_0)} \left| t \right|^2, \quad A = 1 - R - T \quad \ldots(5.26)
\]

### 5.4 Cell Design I: Quadruple-junction Solar Cell AlGaInP/InGaAs/GaSb/InGaSb and Comparative Analysis with State-of-Art Solar Cells

We have simulated and compared favorably the photon absorption of the novel quadruple junction solar cell comprised of AlGaInP (2.33 eV) / InGaAs (1.1 eV) / GaSb (0.7 eV) / InGaSb (0.5 eV), with state of art solar cells. \( \theta_i \) is chosen as 48.19º in all cases following the standard incident angle of AM1.5 solar radiation spectrum. 400nm - 850 nm wavelength has been considered for simulation because of the higher density of photons in that region of solar radiation spectrum. Higher photon absorption will generate more electron-hole pairs promoting to higher current and leading to higher efficiency of the solar cell. Photon absorption and solar cell efficiency are not same. Photon absorption is directly proportional to the solar cell efficiency but there are dark current and recombination losses in every subcell layers of the solar cell, resulting in lower cell efficiency than the photon absorption. The simulations have been performed with MATLAB and Photonics-RT [24]. Figure 5.4 depicts the photon absorption, reflection and transmission in the novel quadruple junction solar cell formed by AlGaInP/InGaAs/GaSb/InGaSb direct bandgap III-V semiconductor materials. The incident light enters the lamellar subcell (diode) layers of specific thickness and gets absorbed, reflected on the top surface and transmitted through the material layers. This ideal combination of bandgaps with great optical matching properties has resulted in very high photon absorption in the solar cell. The green line indicates the absorption, blue the reflection and red the transmission through the lamellar subcell structures. We observe that at 400 nm wavelength the photon absorption is
74.60%, at 500 nm 85.54%, at 600nm 84.08%, at 700nm 77.20% and at 800nm 79.93% respectively.

![Absorption, Reflectance and Transmittance of the novel AlGaInP/InGaAs/GaSb/InGaSb quadruple junction solar cell](image)

Figure 5.4: Photon absorption, reflection and transmission of the novel AlGaInP/InGaAs/GaSb/InGaSb quadruple junction solar cell

We can clearly see that the novel quadruple junction solar cell has very high absorption in the visible and near IR regions. The quadruple junction solar cell will deliver higher absorption all through the spectral distribution of solar radiation. The photon absorption of the quadruple junction solar cell is tabulated in table 5.1.

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Photon Absorption</th>
</tr>
</thead>
<tbody>
<tr>
<td>400nm</td>
<td>74.60%</td>
</tr>
<tr>
<td>500nm</td>
<td>85.54%</td>
</tr>
<tr>
<td>600nm</td>
<td>84.08%</td>
</tr>
<tr>
<td>700nm</td>
<td>77.20%</td>
</tr>
<tr>
<td>800nm</td>
<td>79.93%</td>
</tr>
</tbody>
</table>

Crystalline silicon is the most researched and widely used material in the solar cell industry. The c-Si PV modules have dominated the solar cell market since its inception and still favored with 85% of the world market share. Silicon is an indirect bandgap semiconductor material hence its optical properties and mobility of generated electrons through cell illumination are not as promising as direct bandgap semiconductor materials. The efficiency of Si solar cell modules has approached to its highest obtainable efficiency mentioned by Shockley theoretical limit. In figure
5.5 we have compared the photon absorption of our quadruple junction solar cell with c-Si solar cell. The dark green line indicates the photon absorption of our design and the light blue/aqua color indicates the photon absorption of the crystalline silicon solar cell. We can clearly see that the quadruple junction solar cell is favorable to c-Si solar cell in the solar radiation spectrum.

![Comparison of photon absorption efficiency of quadruple junction vs. crystalline silicon solar cell](image)

**Figure 5.5: Comparison of photon absorption of the quadruple junction solar cell and crystalline silicon solar cell**

The quadruple junction solar cell has photon absorption of 74.60% at 400nm wavelength, 85.54% at 500nm wavelength, 84.08% at 600nm wavelength, 77.20% at 700nm wavelength and 79.93% at 800 nm wavelength whereas the absorption are 50.8%, 11.54%, 4.83%, 3.2% and 1.25% for c-Si solar cell for the respective wavelengths. The comparative study is shown in table 5.2.

**Table 5.2: Comparison of AlGaInP/InGaAs/GaSb/InGaSb Four-Junction Solar Cell with Crystalline-Si Solar Cell**

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Photon Absorption of Quadruple junction</th>
<th>Photon Absorption of crystalline silicon</th>
</tr>
</thead>
<tbody>
<tr>
<td>400nm</td>
<td>74.60%</td>
<td>50.8%</td>
</tr>
<tr>
<td>500nm</td>
<td>85.54%</td>
<td>11.54%</td>
</tr>
<tr>
<td>600nm</td>
<td>84.08%</td>
<td>4.83%</td>
</tr>
<tr>
<td>700nm</td>
<td>77.20%</td>
<td>3.2%</td>
</tr>
<tr>
<td>800nm</td>
<td>79.93%</td>
<td>1.25%</td>
</tr>
</tbody>
</table>
Figure 5.6: Comparison of photon absorption of quadruple junction solar cell and thin film cadmium-telluride solar cell

The advancement in thin film technology has promoted second generation solar cell technologies like Cadmium Telluride (CdTe) or CIGS solar cells to compete with the silicon solar cells. CdTe solar cells having bandgap of 1.45eV has been commercialized in 2007. First Solar Inc. is the second largest PV module maker worldwide and produces the CdTe solar cells at $1/kWh making it the cheapest solar cell technology. CdTe is a toxic material. In figure 5.6 we have favorably compared our quadruple junction solar cell with thin film CdTe solar cell. The CdTe thin film solar cell has photon absorption of 77.39% at 400nm wavelength, 66.62% at 500nm wavelength, 55% at 600nm wavelength, 43.27% at 700nm wavelength and 34.45% at 800nm wavelength in comparison to the photon absorption of 74.60%, 85.54%, 84.08%, 77.20% and 79.93% for the same wavelengths respectively. This is mentioned in table 5.3 below.

Table 5.3: Comparison of AlGaInP/InGaAs/GaSb/InGaSb Four-Junction Solar Cell with CdTe Solar Cell

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Photon Absorption of Quadruple junction</th>
<th>Photon Absorption of CdTe</th>
</tr>
</thead>
<tbody>
<tr>
<td>400nm</td>
<td>74.60%</td>
<td>77.39%</td>
</tr>
<tr>
<td>500nm</td>
<td>85.54%</td>
<td>66.62%</td>
</tr>
<tr>
<td>600nm</td>
<td>84.08%</td>
<td>55%</td>
</tr>
<tr>
<td>700nm</td>
<td>77.20%</td>
<td>43.27%</td>
</tr>
<tr>
<td>800nm</td>
<td>79.93%</td>
<td>34.45%</td>
</tr>
</tbody>
</table>
Al\textsubscript{0.4}Ga\textsubscript{0.6}As/GaAs double junction solar cell having theoretical AM1.5 efficiency of 34% is a well researched double junction solar cell. AlGaAs/GaAs metal interconnected cell having two-terminal and two-junction has executed 27.6% efficiency under 1 sun AM 1.5G (Chung et al.) and 28.8% efficiency under 1 Sun AM 1.5G using a tunnel junction in between two junctions (Takahashi et al.) [2]. It can be clearly seen that the novel quadruple junction solar cell has better performance than the double junction cell in the solar radiation spectrum. The dark green line in figure 5.7 denotes the photon absorption of the novel quadruple junction solar cell and the aqua color mentions the photon absorption of the double junction solar cell for different wavelengths. The comparative photon absorption of my quadruple junction design and the double junction design is mentioned below in table 5.4.

Table 5.4: Comparison of AlGaInP/InGaAs/GaSb/InGaSb Four-Junction Solar Cell with AlGaAs/GaAs Two-Junction Solar Cell

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Photon Absorption of Quadruple junction</th>
<th>Photon Absorption AlGaAs/GaAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>400nm</td>
<td>74.60%</td>
<td>65.82%</td>
</tr>
<tr>
<td>500nm</td>
<td>85.54%</td>
<td>66.48%</td>
</tr>
<tr>
<td>600nm</td>
<td>84.08%</td>
<td>40.64%</td>
</tr>
<tr>
<td>700nm</td>
<td>77.20%</td>
<td>29.85%</td>
</tr>
<tr>
<td>800nm</td>
<td>79.93%</td>
<td>11.53%</td>
</tr>
</tbody>
</table>
Figure 5.8: Comparison of photon absorption of quadruple junction solar cell and GaInP/GaAs/GaInAs triple junction solar cell

Researchers at National Renewable Energy Laboratory (NREL) have developed an ultra-thin, ultra-high efficiency triple junction inverted metamorphic tandem solar cell comprised of Ga$_{0.5}$In$_{0.5}$P/GaAs/Ga$_{0.73}$In$_{0.27}$As materials. The triple junction solar cell has efficiency of 33% under 1 Sun AM0 spectrum. It has ~36% efficiency under 10 sun concentration [1, 39]. Figure 5.8 compares the photon absorption of our quadruple junction solar cell with tandem triple junction solar cell of NREL. The quadruple junction solar cell has absorption of 74.60% at 400nm wavelength, 85.54% at 500nm wavelength, 84.08% at 600nm wavelength, 77.20% at 700nm wavelength and 79.93% at 800 nm wavelength whereas the absorption of the triple junction cell are 70.38%, 80.98%, 70.40%, 56.71% and 32.99% for the respective wavelengths.

Table 5.5: Comparison of AlGaInP/InGaAs/GaSb/InGaSb Four-Junction Solar Cell with GaInP/GaAs/GaInAs Triple-Junction Solar Cell

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Photon Absorption of Quadruple junction</th>
<th>Photon Absorption GaInP/GaAs/GaInAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>400nm</td>
<td>74.60%</td>
<td>70.38%</td>
</tr>
<tr>
<td>500nm</td>
<td>85.54%</td>
<td>80.98%</td>
</tr>
<tr>
<td>600nm</td>
<td>84.08%</td>
<td>70.40%</td>
</tr>
<tr>
<td>700nm</td>
<td>77.20%</td>
<td>56.71%</td>
</tr>
<tr>
<td>800nm</td>
<td>79.93%</td>
<td>32.99%</td>
</tr>
</tbody>
</table>
The dark green line in figure 5.8 mentions the photon absorption of the quadruple junction solar cell and the aqua line for the triple junction tandem cell. The comparative photon absorption values for different wavelengths are mentioned in table 5.5.

![Comparison of photon absorption efficiency of quadruple junction vs. GaInP/GaAs/Ge triple junction solar cell](image)

Figure 5.9: Photon absorption comparison of quadruple junction solar cell with GaInP/GaAs/Ge triple junction solar cell

Triple junction solar cell comprised of $\text{Ga}_{0.5}\text{In}_{0.5}\text{P}/\text{GaAs/Ge}$ semiconductor materials are in production at Boeing Spectrolab and are appropriate for both terrestrial concentrator and non-concentrating space applications. Efficiency up to 32.3% have been achieved under 47 Sun AM 1.5 illumination [40]. This triple junction solar cell has photon absorption of 70.38% at 400nm wavelength, 82.81% at 500nm wavelength, 86.43% at 600nm wavelength, 45.77% at 700nm wavelength and 57.1% at 800nm wavelength of the solar spectrum as shown by the aqua line in figure 5.9.

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Photon Absorption of Quadruple junction</th>
<th>Photon Absorption GaInP/GaAs/Ge</th>
</tr>
</thead>
<tbody>
<tr>
<td>400nm</td>
<td>74.60%</td>
<td>70.38%</td>
</tr>
<tr>
<td>500nm</td>
<td>85.54%</td>
<td>82.81%</td>
</tr>
<tr>
<td>600nm</td>
<td>84.08%</td>
<td>86.43%</td>
</tr>
<tr>
<td>700nm</td>
<td>77.20%</td>
<td>45.77%</td>
</tr>
<tr>
<td>800nm</td>
<td>79.93%</td>
<td>57.1%</td>
</tr>
</tbody>
</table>
Our novel quadruple junction solar cell definitely shows favorable absorption for the entire solar spectrum than the triple junction solar cell as shown in table 5.6.

![Comparison of photon absorption efficiency of quadruple junction vs. GaInP/GaInAs/Ge triple junction solar cell](image)

**Figure 5.10: Quadruple junction solar cell with triple junction solar cell made up of GaInP/GaInAs/Ge subcell layers**

With the support US DOE through the NREL high performance PV program researchers at Boeing Spectrolab achieved record solar cell efficiency of 40.7% (AM 1.5D, low AOD, 240 suns, 25°C) for Ga\textsubscript{0.44}In\textsubscript{0.56}P/Ga\textsubscript{0.92}In\textsubscript{0.08}As/Ge metamorphic and 40.1% (AM 1.5D, low AOD, 135 suns, 25°C) for lattice matched triple junction solar cell comprised of Ga\textsubscript{0.5}In\textsubscript{0.5}P/Ga\textsubscript{0.99}In\textsubscript{0.01}/Ge subcell layers [18]. We have compared our quadruple junction solar cell photon absorption favorably with the record solar cell in figure 5.10. The dark green line indicates the photon absorption efficiencies of the novel quadruple junction solar cell and the aqua line for the record triple junction solar cell. We see that the quadruple junction solar cell performs better. The quadruple junction solar cell at 400nm wavelength has the photon absorption of 74.60%, at 500nm 85.54%, at 600nm 84.08%, at 700nm 77.20% and at 800nm 79.93% respectively in comparison to 70.35%, 81.34%, 84.53%, 53.59% and 65.51% for those respective wavelengths. The comparative photon absorption is shown in table 5.7.
Table 5.7: Comparison of AlGaInP/InGaAs/GaSb/InGaSb Four-Junction Solar Cell with GaInP/GaInAs/Ge Triple-Junction Solar Cell

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Photon Absorption of Quadruple junction</th>
<th>Photon Absorption of GaInP/GaInAs/Ge</th>
</tr>
</thead>
<tbody>
<tr>
<td>400nm</td>
<td>74.60%</td>
<td>70.35%</td>
</tr>
<tr>
<td>500nm</td>
<td>85.54%</td>
<td>81.34%</td>
</tr>
<tr>
<td>600nm</td>
<td>84.08%</td>
<td>84.53%</td>
</tr>
<tr>
<td>700nm</td>
<td>77.20%</td>
<td>53.59%</td>
</tr>
<tr>
<td>800nm</td>
<td>79.93%</td>
<td>65.51%</td>
</tr>
</tbody>
</table>

Figure 5.11: Photon absorption comparison of our novel solar cell with (Al)GaInP/AlGa(In)As/Ga(In)As/Ge quadruple junction solar cell

A next generation quadruple junction solar cell comprised of (Al)GaInP/AlGa(In)As/Ga(In)As/Ge has been proposed by the Spectrolab researchers [41, 42]. We have compared our novel quadruple junction solar cell to the proposed quadruple junction solar cell by Spectrolab in figure 5.11. We see that our quadruple junction solar cell has photon absorption of 74.60% at 500nm, 85.54% at 600nm, 84.08% at 700nm, 77.20% and 79.93% at 800 nm respectively whereas the other quadruple junction solar cell has photon absorption of 74.17% at 400nm, 81.23% at 500nm, 83.45% at 600nm, 62.20% at 700nm and 65.76% at 800nm wavelength respectively as shown in table 5.8. We can easily see that our novel quadruple junction solar cell comprised of direct bandgap compound semiconductor materials has better optical properties than the state of art solar cells.
Table 5.8: Comparison of AlGaInP/InGaAs/GaSb/InGaSb Four-Junction Solar Cell with -
(Al)GaInP/AlGa(In)As/Ga(In)As/Ge Four-Junction Solar Cell

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Photon Absorption of Quadruple junction</th>
<th>Photon Absorption of (Al)GaInP/AlGa(In)As/Ga(In)As/Ge</th>
</tr>
</thead>
<tbody>
<tr>
<td>400nm</td>
<td>74.60%</td>
<td>74.17%</td>
</tr>
<tr>
<td>500nm</td>
<td>85.54%</td>
<td>81.23%</td>
</tr>
<tr>
<td>600nm</td>
<td>84.08%</td>
<td>83.45%</td>
</tr>
<tr>
<td>700nm</td>
<td>77.20%</td>
<td>62.20%</td>
</tr>
<tr>
<td>800nm</td>
<td>79.93%</td>
<td>65.76%</td>
</tr>
</tbody>
</table>

5.5 Cell Design II: Quadruple-junction Solar Cell AlGaInP/InGaP/InGaAs/InGaSb and
Comparative Analysis with State-of-Art Solar Cells

The second design is another quadruple junction solar cell mentioned in figure 5.1 (b) composed
of AlGaInP (2.3 eV) / InGaP (1.93 eV) / InGaAs (1.1 eV)/ InGaSb (0.5 eV) subcell layers. As
the atomic structures of P, As and Sb are very similar they can be matched easily. This helps
reduce the difficulties in optical and electrical matching. The AlGaInP (2.3eV) top subcell layer
will capture photons for the UV and visible range up to 532 nm wavelength and pass the less
ergereved photons to the next subcell layer of the stack (InGaP). InGaP having the bandgap of
1.93eV will capture photons of energy between the wavelength 532 nm and 642 nm. Less
ergereved photons will then be transferred to the subsequent subcell layers. InGaAs (1.1eV) will
capture photons of energy between 642 nm-1126 nm wavelengths and InGaSb will capture
photons in the infrared (IR) region. The absorption of this novel quadruple junction has been
compared to the state-of-art solar cell designs.

Figure 5.12 shows the photon absorption, reflection and transmission of the novel quadruple
junction solar cell comprised of AlGaInP / InGaP / InGaAs / InGaSb direct bandgap III-V
semiconductor materials. This ideal combination of bandgaps with great optical matching
properties has resulted in very high photon absorption in the solar cell. The green line indicates
the absorption, blue the reflection and red the transmission through the lamellar subcell
structures. The photon absorption of this design is mentioned in table 5.9.
In table 5.9 we observe that at 400 nm wavelength the photon absorption is 74.18 %, at 500 nm 81.24%, at 600nm 82.69%, at 700nm 70.35% and at 800nm 69.27% respectively. We can depict easily that the novel quadruple junction solar cell has very high absorption in the visible and near infrared regions. The quadruple junction solar cell will deliver higher absorption all through the spectral distribution of solar radiation.

The dark green line in figure 5.13 indicates the photon absorption of our novel quadruple junction design and the light blue/aqua color indicates the photon absorption of the crystalline silicon solar cell. We can clearly see that the quadruple junction solar cell is favorable to c-Si solar cell in the solar radiation spectrum. The quadruple junction solar cell has photon absorption of 74.18% at 400 nm wavelength, 81.24% at 500 nm wavelength, 82.69% at 600 nm wavelength, 70.35% at 700 nm wavelength and 69.27% at 800 nm wavelength respectively whereas for c-Si the photon absorption is 50.8% at 400 nm, 11.54% at 500 nm, 4.83% at 600 nm, 3.2% at 700 nm and 1.25% at 800 nm respectively as shown in table 5.10.
Figure 5.13: Comparison of photon absorption of the quadruple junction solar cell and crystalline silicon solar cell

Table 5.10: Comparison of AlGaInP/InGaP/InGaAs/InGaSb Four-Junction Solar Cell with Crystalline-Si Solar Cell

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Photon Absorption of Quadruple junction</th>
<th>Photon Absorption of crystalline silicon</th>
</tr>
</thead>
<tbody>
<tr>
<td>400nm</td>
<td>74.18%</td>
<td>50.8%</td>
</tr>
<tr>
<td>500nm</td>
<td>81.24%</td>
<td>11.54%</td>
</tr>
<tr>
<td>600nm</td>
<td>82.69%</td>
<td>4.83%</td>
</tr>
<tr>
<td>700nm</td>
<td>70.35%</td>
<td>3.2%</td>
</tr>
<tr>
<td>800nm</td>
<td>69.27%</td>
<td>1.25%</td>
</tr>
</tbody>
</table>

Cadmium Telluride (CdTe) is a popular thin film solar cell. First Solar Inc. is the second largest PV module maker worldwide producing the CdTe solar cells at $1/kWh making it the cheapest solar cell technology. In figure 5.14 we have favorably compared our quadruple junction solar cell with thin film CdTe solar cell. The CdTe thin film solar cell has photon absorption of 77.39% at 400nm wavelength, 66.62% at 500nm wavelength, 55% at 600nm wavelength, 43.27% at 700nm wavelength and 34.45% at 800nm wavelength in comparison to the photon absorption of 74.18%, 81.24%, 82.69%, 70.35% and 69.27% for the same wavelengths respectively.
Figure 5.14: Comparison of photon absorption of the quadruple junction solar cell and CdTe solar cell

Table 5.11: Comparison of AlGaInP/InGaP/InGaAs/InGaSb Four-Junction Solar Cell with CdTe Solar Cell

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Photon Absorption of Quadruple junction</th>
<th>Photon Absorption of CdTe Solar Cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>74.18%</td>
<td>77.39%</td>
</tr>
<tr>
<td>500</td>
<td>81.24%</td>
<td>66.62%</td>
</tr>
<tr>
<td>600</td>
<td>82.69%</td>
<td>55%</td>
</tr>
<tr>
<td>700</td>
<td>70.35%</td>
<td>43.27%</td>
</tr>
<tr>
<td>800</td>
<td>69.27%</td>
<td>34.45%</td>
</tr>
</tbody>
</table>

The Al_{0.4}Ga_{0.6}As/GaAs double junction solar cell having theoretical AM1.5 efficiency of 34% is a well researched double junction solar cell. Metal interconnected AlGaAs/GaAs solar cell having two-terminal and two-junction has executed 27.6% efficiency under 1 sun AM 1.5G (Chung et al.) and 28.8% efficiency under 1 Sun AM 1.5G using a tunnel junction in between two junctions (Takahashi et al.) [2]. It can be easily seen that the novel quadruple junction solar cell has better performance than the double junction cell. The dark green line in figure 5.15 denotes the photon absorption of the quadruple junction solar cell and the aqua color mentions the photon absorption of the double junction solar cell for different wavelengths. The comparative photon absorption of my quadruple junction design and the double junction design is mentioned below in table 5.12.
Table 5.12: Comparison of AlGaInP/InGaP/InGaAs/InGaSb Four-Junction Solar Cell with AlGaAs/GaAs Double-Junction Solar Cell

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Photon Absorption of Quadruple junction</th>
<th>Photon Absorption of AlGaAs/GaAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>400 nm</td>
<td>74.18%</td>
<td>65.82%</td>
</tr>
<tr>
<td>500 nm</td>
<td>81.24%</td>
<td>66.48%</td>
</tr>
<tr>
<td>600 nm</td>
<td>82.69%</td>
<td>40.64%</td>
</tr>
<tr>
<td>700 nm</td>
<td>70.35%</td>
<td>29.85%</td>
</tr>
<tr>
<td>800 nm</td>
<td>69.27%</td>
<td>11.53%</td>
</tr>
</tbody>
</table>

The photon absorption comparison of the novel quadruple junction solar cell with AlGaAs/GaAs/InGaAs triple-junction solar cell is shown in figure 5.16 and the photon absorption at different wavelengths has been executed in table 5.13. We can see that the photon absorption in triple-junction solar cell is 68.02% at 400 nm, 79.77% at 500 nm, 72.22% at 600 nm, 55.88% at 700 nm and 35.27% at 800 nm in comparison to the photon absorption of 74.18%, 81.24%, 82.69%, 70.35% and 69.27% for the same wavelengths respectively. The quadruple-junction solar cell design works better.
Figure 5.16: Comparison of photon absorption of the quadruple junction solar cell and AlGaAs/GaAs/InGaAs triple-junction solar cell

Table 5.13: Comparison of AlGaInP/InGaP/InGaAs/InGaSb Four-Junction Solar Cell with AlGaAs/GaAs/InGaAs Three-Junction Solar Cell

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Photon Absorption of Quadruple junction</th>
<th>Photon Absorption of AlGaAs/GaAs/InGaAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>400nm</td>
<td>74.18%</td>
<td>68.02%</td>
</tr>
<tr>
<td>500nm</td>
<td>81.24%</td>
<td>79.77%</td>
</tr>
<tr>
<td>600nm</td>
<td>82.69%</td>
<td>72.22%</td>
</tr>
<tr>
<td>700nm</td>
<td>70.35%</td>
<td>55.88%</td>
</tr>
<tr>
<td>800nm</td>
<td>69.27%</td>
<td>35.27%</td>
</tr>
</tbody>
</table>

In figure 5.17, I have compared the photon absorption of our novel quadruple junction solar cell made up of AlGaInP/InGaP/InGaAs/InGaSb subcell layers with the photon absorption of the record triple junction solar cell developed by the researchers in Boeing Spectrolab and tested by NREL. It shows efficiency of 40.7% (AM 1.5D, low AOD, 240 suns, 25°C) for Ga\textsubscript{0.44}In\textsubscript{0.56}P/Ga\textsubscript{0.92}In\textsubscript{0.08}As/Ge metamorphic and 40.1% (AM 1.5D, low AOD, 135 suns, 25°C) for lattice matched triple junction solar cell comprised of Ga\textsubscript{0.5}In\textsubscript{0.5}P/Ga\textsubscript{0.99}In\textsubscript{0.01}/Ge subcell layers [18]. We see that our quadruple junction design has higher photon absorption for the solar spectrum in comparison to the triple junction solar cell. The dark green line in figure 5.17 mentions the photon absorption of our design vs. the aqua color mentioning the photon absorption of the triple junction solar cell. The quadruple junction solar cell has photon absorption of 74.18% at 400 nm wavelength, 81.24% at 500 nm wavelength, 82.69% at 600 nm wavelength, 70.35% at 700 nm wavelength and 69.27% at 800 nm wavelength respectively.
whereas the triple junction solar cell has photon absorption of 70.35%, 81.34%, 84.53%, 53.59% and 65.51% for those wavelengths respectively as sown in table 5.14.

Figure 5.17: Comparison of photon absorption of the AlGaInP/InGaP/InGaAs/InGaSb quadruple junction with triple junction solar cell made up of GaInP/GaInAs/Ge subcell layers

Table 5.14: Comparison of AlGaInP/InGaP/InGaAs/InGaSb Four-Junction Solar Cell with GaInP/GaInAs/Ge Three-Junction Solar Cell

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Photon Absorption of Quadruple junction</th>
<th>Photon Absorption of GaInP/GaInAs/Ge</th>
</tr>
</thead>
<tbody>
<tr>
<td>400nm</td>
<td>74.18%</td>
<td>70.35%</td>
</tr>
<tr>
<td>500nm</td>
<td>81.24%</td>
<td>81.34%</td>
</tr>
<tr>
<td>600nm</td>
<td>82.69%</td>
<td>84.53%</td>
</tr>
<tr>
<td>700nm</td>
<td>70.35%</td>
<td>53.59%</td>
</tr>
<tr>
<td>800nm</td>
<td>69.27%</td>
<td>65.51%</td>
</tr>
</tbody>
</table>

Next generation quadruple junction solar cell comprised of (Al)GaInP/AlGa(In)As/Ga(In)As/Ge has been proposed by the Spectrolab researchers [41, 42]. We have compared our novel quadruple junction solar cell to the proposed quadruple junction solar cell by Spectrolab in figure 5.18. We see that our quadruple junction solar cell has photon absorption of 74.18% at 400 nm wavelength, 81.24% at 500 nm wavelength, 82.69% at 600 nm wavelength, 70.35% at 700 nm wavelength and 69.27% at 800 nm wavelength respectively whereas the other quadruple junction solar cell has photon absorption of 74.17% at 400nm, 81.23% at 500nm, 83.45% at 600nm, 62.20% at 700nm and 65.76% at 800nm wavelength respectively as shown in table 5.15. We can
easily see that our novel quadruple junction solar cell comprised of direct bandgap compound semiconductor materials has favorable results.

Figure 5.18: Comparison of photon absorption of the AlGaInP/InGaP/InGaAs/InGaSb quadruple junction with (Al)GaInP/AlGa(In)As/Ga(In)As/Ge four-junction solar cell

Table 5.15: Comparison of AlGaInP/InGaP/InGaAs/InGaSb Four-Junction Solar Cell with (Al)GaInP/AlGa(In)As/Ga(In)As/Ge Four-Junction Solar Cell

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Photon Absorption of Quadruple junction</th>
<th>Photon Absorption of (Al)GaInP/AlGa(In)As/Ga(In)As/Ge</th>
</tr>
</thead>
<tbody>
<tr>
<td>400nm</td>
<td>74.18%</td>
<td>74.17%</td>
</tr>
<tr>
<td>500nm</td>
<td>81.24%</td>
<td>81.23%</td>
</tr>
<tr>
<td>600nm</td>
<td>82.69%</td>
<td>83.45%</td>
</tr>
<tr>
<td>700nm</td>
<td>70.35%</td>
<td>62.20%</td>
</tr>
<tr>
<td>800nm</td>
<td>69.27%</td>
<td>65.76%</td>
</tr>
</tbody>
</table>

5.6 Conclusion

Single-junction semiconductors can generate higher current due to photon absorption over broader spectral region but will not produce high open circuit voltage because of the limitations of dark current of the low bandgap material. This limits them within the Shockley efficiency limit of 30%. Mono and poly crystalline Si solar cells are formed by indirect bandgap semiconductor material that cannot split the solar radiation spectrum effectively because of bandgap limitation, less optical sensitivity and mobility of carriers. Multijunction solar cells formed with several subcell layers effectively split the solar radiation spectrum and overcome the
limitation of single-junction cells producing higher voltage and current matched to the bottom subcell, promoting to higher efficiency. Several researchers around the world have designed multijunction solar cells by effective spectrum splitting. We have designed two novel quadruple junction solar cell designs mentioned above (AlGaInP/InGaAs/GaSb/InGaSb and AlGaInP/InGaP/InGaAs/InGaSb) with four direct bandgap subcell semiconductor layers each matched to photons of a spectral region. The photon absorption of these quadruple junction solar cells have been favorably compared with single-junction and multijunction state of art solar cells. Introduction of antimony (Sb) based subcell layers increase photon collection in the near and far infrared regions of the solar radiation spectrum. If the Ge (indirect bandgap) bottom subcell layer is substituted with InGaSb layer in the record solar cell then it will show significant increase in photon absorption. The novel quadruple junction solar cells execute very high photon absorption in the entire solar radiation spectrum as evidenced by the results discussed. This will generate higher voltage because of the summation of the voltages in the subcell layers, promoting to higher efficiency of the solar cell.
CHAPTER SIX

ELECTRICAL MODELING OF NOVEL MULTIJUNCTION SOLAR CELLS

6.1 Introduction

In this chapter the electrical modeling of single-junction and multijunction solar cell designs have been demonstrated. The band structure and different current and voltages under equilibrium as well as in the condition of illumination are described. The band banding and the respective generation and recombination currents have been demonstrated. The current to voltage characteristics of simple solar cell model is demonstrated defining the voltage source and current source regions. Several important parameters like open-circuit voltage ($V_{OC}$), short-circuit current ($I_{SC}$), fill-factor (FF), maximum power-point (MPP) and cell efficiency has been discussed. Simple solar cell model, two diode model along with series and shunt resistance effects has been discussed. The temperature effects of solar cells have been executed. Current and voltage equations have been formed for all the different kind of modeling. Multijunction solar cell design for quadruple junction has been demonstrated. This designing takes into consideration the same current for all the subcell layers (current matching) and the voltage effect is the summation of the individual voltages generated in each subcell layers. Different loss mechanisms like the dark current (saturation current), recombination losses in each subcell layers have been considered along with series and shunt losses. The two diode model representation of quadruple junction design is being executed. The quantum-efficiency or the photon conversion to electricity formulas as well as the simulation for quadruple junction designs have been described. The novel designs execute very high quantum efficiency. Lastly the current density vs. voltage simulations and power density vs. voltage simulations are demonstrated.

6.2 Junction Structure and Dark Current

The electric double layer at the $p$-$n$ junction has a very important effect on the energy levels of the semiconductor [2]. The uncharged phases have the same conduction band and valence band energies ($E_c$ and $E_v$). They are separated by the forbidden gap $E_g$ and have different work functions $\Phi_p$ and $\Phi_n$ and therefore different Fermi levels $\mu F$ for $p$ and $\mu F$ for $n$. 

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Figure 6.1: Energy band diagram of p-n junction in dark: (a) Uncharged blocks of p-type and n-type semiconductors before contact showing conduction band, valence band and Fermi level energies \( E_c, E_v, E_g \), (b) p-n junction after contact and equilibrium showing electrical double layer by transient charge transfer, depletion regions (DRs) and quasi-neutral regions (QNRs), Courtesy of M. D. Archer, Clean Electricity from Photovoltaics, page 17

In the equilibrium condition mentioned in figure 6.1 (b) the Fermi level is same throughout the device but the band-end energies changes due to the local electric field. The equilibrium bending energy \( qV^0 \) is the difference of the two work functions.

\[
qV^0 = \Phi_n - \Phi_p \quad \text{(6.1)}
\]

In a doped semiconductor the Fermi level normally lies within the forbidden gap but close to band-edge of majority carrier, hence \( qV^0 \) is slightly less than \( E_g \).

Figure 6.2: Dark p-n homojunction cell (a) at equilibrium, (b) under forward bias \( V_j \), (c) under reverse bias \( V_j \), Courtesy of M.D. Archer, Clean Electricity from Photovoltaics, page 18
Figure 6.2 explains how the band bending happens and current flows when a bias voltage $V_j$ is applied across the cell in dark. At equilibrium no net current flow in the cell, however small fluxes of electrons in conduction band and holes in valence band pass each way across the junction. It is referred to as generation and recombination of currents. The generation currents shown in figure 6.2 (a) comes from minority carriers (electrons in $p$-side and holes in $n$-side). Those minority carriers that reach the junction without recombining move in opposite directions due to the strong electric field. The recombination currents come from majority carriers (holes in $p$-side and electrons in $n$-side). At equilibrium generation and recombination currents balance each other. The sum of hole and electron thermal generated current is called saturation current density $i_0$ and is given by

$$i_0 = i_{h,\text{gen}} + i_{e,\text{gen}} = i_{h,\text{rec}}^0 + i_{e,\text{rec}}^0 \quad \ldots(6.2)$$

A forward bias $V_j$ applied across the junction reduces the barrier height to $qV_b = q(V_{b0} - V_j)$ as shown in figure 6.2 (b). This does not affect the generation current, however drastically increases recombination currents. The difference between recombination and generation current is called as dark current or junction current $i_j$.

$$i_j(V_j) = i_{h,\text{rec}}(V_j) + i_{e,\text{rec}}(V_j) - i_{h,\text{gen}} - i_{e,\text{gen}} = i_{h,\text{rec}}(V_j) + i_{e,\text{rec}}(V_j) - i_{h,\text{rec}}^0 - i_{e,\text{rec}}^0 \quad \ldots(6.3)$$

A reverse bias ($V_j < 0$) increases the barrier height $qV_b = q(V_{b0}^0 + |V_j|)$ as shown in figure 6.2 (c). The generation currents are still not altered, however recombination currents are now suppressed. In this case very small bias independent saturation current flows

$$i_j(V_j < 0) = -i_0 \quad \ldots(6.4)$$

The dependence of recombination currents $i_{h,\text{rec}}(V_j)$ and $i_{e,\text{rec}}(V_j)$ on $V_j$ is determined by dominant recombination mechanism of carriers injected in the junction [2]. In most solar cells the current-voltage characteristic abide by the empirical diode equation

$$i_j(V_j) = i_0 \left[ e^{(qV_j/βkT)} - 1 \right] \quad \ldots(6.5)$$

Here $β$ is called diode ideality factor and $β = 1$ for ideal junction without no injected carrier recombination. A non ideal junction in which some carriers recombine in the junction has $β$ in between 1 and 2 ($1 < β < 2$). For some cells, particularly in thin-film ones it is better represented by double diode equation.
The first term corresponds to the carriers moving across the junction without recombining and the second term with the carriers that recombine in the mid-gap.

### 6.3 Illuminated Solar Cell

When a solar cell is illuminated photocurrent and photovoltage are generated. In figure 6.3 this phenomenon is explained using the p-n homojunction cell. Absorption of photons of energy greater than the band-gap energy of the semiconductor generates excess minority carriers throughout the illuminated portion of the cell. The photo generated minority carriers in the illuminated cell behave like the minority carriers in the dark cell. They diffuse from the QNRs towards the junction where they are swept across by the junction field. These fluxes of photogenerated minority carriers contribute in photogeneration currents $i_{e,ph}$ and $i_{h,ph}$ shown in figure 6.3 consisting of photogenerated electrons from p to the n side of the junction and holes drifting the other way. The sum of these two currents is the overall photocurrent $i_{ph}$

$$i_{ph} = i_{h,ph} + i_{e,ph} \quad (6.7)$$

![Figure 6.3: Illuminated p-n homojunction cell (a) at open circuit (b) at short circuit, showing photo generation of hole-electron pairs and photo currents, Courtesy of M.D.Archer, Clean Electricity from Photovoltaics, page 20](image)

At open circuit (fig 6.3 (a)) no current is drawn from the cell and the photocurrent and recombination current balances each other. The junction self biases in forward direction by open circuit voltage $V_{oc}$ and recombination current opposes photocurrent.
\[ i_{ph} - i_j (V_{oc}) = 0 \quad \ldots(6.8) \]

In figure 6.3 (a) \( q \ V_{oc} \) is the difference of Fermi levels between the two sides of the junction. Figure 6.3 (b) explains what happens when the illuminated cell is short circuited. The cell delivers maximum current at 0 output voltage. The junction bias \( V_j \) is always zero. The short circuit current is expressed as

\[ i_{sc} = |i_{ph}| - i_0 \quad \ldots(6.9) \]

In closed circuit condition the band bending and junction current are intermediate between open-circuit and short-circuit cases. The cell current \( I \) at output voltage \( V = V_j \) is

\[ i = i_{ph} - i_j (V_j) \quad \ldots(6.10) \]

### 6.4 Solar Cell Current to Voltage Characteristics

In ideal PV it is assumed that there is no internal loss and no loss due to leakage in ground. The current-voltage characteristics of an ideal PV is given by

\[
I = I_{PV} - I_s \left[ e^{(q \ V_j) \beta / kT_c} - 1 \right] \quad \ldots(6.11) 
\]

Here \( I_{PV} \) = Light generated photocurrent, \( I_s \) = Diode saturation current or dark current, \( q = 1.6 \times 10^{-19} \) C, \( k = \) Boltzmann’s constant = \( 1.38 \times 10^{-23} \) J/K = \( 8.6173 \times 10^{-5} \) eV/K, \( V \) = Cell voltage, \( \beta \) = diode ideality factor and \( T_c \) = Cell’s working temperature. \( I_d \) is Shockley’s diode equation. The simple PV model forming equation 11 is denoted in figure 6.4.

![Simple solar cell circuit model](image)

Figure 6.4: Simple solar cell circuit model
Short Circuit Current: Highest amount of current generated by a cell. Short circuit current ($I_{sc}$) is achieved when $V = 0$. In case the applied voltage is high enough so that the diode recombination current becomes significant, solar cell current drops quickly.

Open Circuit Voltage: The voltage drop across the diode ($p-n$ junction) when the current is $I = 0$, is called open circuit voltage. The product of $I$ and $V$ is the output power and it is the area under the rectangle of the I-V curve. The power is zero for both open circuit and short circuit conditions. Maximum power is achieved when the area ($V_{mp}$, $I_{mp}$) under the curve is maximum.

Maximum Power Point (MPPT): It is the operating point at which power dissipated to the resistive load is maximum.

$$P_{\text{max}} = V_{mp} \times I_{mp} \; \cdots (6.12)$$

Fill factor (FF): It is the ratio of maximum power that can be delivered to the load and the product of $V_{oc}$ and $I_{sc}$. Fill factor denotes the squareness of the I-V characteristic curve.

$$FF = \frac{P_{\text{max}}}{(V_{oc} \times I_{sc})} = \frac{V_{mp} \times I_{mp}}{(V_{oc} \times I_{sc})} \; \cdots (6.13)$$

In efficient cells the fill factor is around 0.7 - 0.8 and in poor cells it is 0.5 or lower. By setting $I = 0$, $V = V_{oc}$ the open circuit voltage of the illuminated cell is

$$V_{oc} = \beta k T / q \ln (1 + I_{PV} / I_s) \approx \beta k T / q \ln (I_{PV}/I_s) \; \cdots (6.14)$$
For better performance of a cell $I_{PV}$ and $V_{oc}$ must be as large as possible. The maximum value of $I_{PV}$ is achieved when all photogenerated electron-hole pairs are collected as photocurrent and $I_{PV}$ can obtain 80-90% of this limit if the light absorption and minority carrier collection has a very high efficiency. $V_{oc}$ increases logarithmically with solar irradiation and $I_{sc}$ increases linearly.

**Cell Efficiency:** The maximum-power solar conversion efficiency of a solar cell is denoted by $\eta_{mp}$ and is expressed as

$$\eta_{mp} = \frac{V_{mp} I_{mp}}{E_0} = \frac{V_{mp} \times I_{mp}}{A G} \cdots (6.15)$$

Here $E_0$ is incident solar irradiance (watts per unit area). Since $I_{PV}$ increases in direct proportion to $E_0$ and $V_{oc}$ logarithmically, hence $\eta_{mp}$ should increase logarithmically with irradiance. This is observed for solar concentrations of up to several hundred Suns for some cell types but ultimately series resistance of the cell and increased operating temperature will limit increasing efficiency being obtained from concentrated sunlight [2]. Commercially available PV (solar) cells have (1Sun) efficiencies in the range of ~ 8 - 18%. The best laboratory cells execute higher efficiencies now up to ~ 24% for single-junction device. Solar cells are direct conversion devices hence they don’t obey Carnot limits that control efficiencies of heat engines. There are some constraints on PV cell efficiency. The major constraint is the poor match between the broadband spectral distribution of sunlight and single band gap ($E_g$) of the semiconductor. Solar photons having the energy $E < E_g$ are usually not absorbed in the semiconductor and if they are absorbed they don’t contribute in generation of hole-electron pairs. Photon energy of $E \geq E_g$ get absorbed and contribute in generation of hole-electron pairs but their initial ‘excess’ energy ($E - E_g$) is lost very fast by thermalization. The band gap of the photoactive semiconductor determines the upper bound on both the open circuit voltage $V_{oc}$ and short circuit current $I_{sc}$. A large band gap cell has larger $V_{oc}$ than a small band gap one but it absorbs fewer solar photons and hence less $I_{sc}$. The limiting efficiency of an ideal isotropic single junction cell of band gap $E_g \approx 1.4$ eV is ~ 32%. Higher efficiencies in the single junction cell lies in eliminating non-ideal losses as far as possible. This is in large measured achieved in c-Si and GaAs single junction solar cells. An alternative method is the stack two or three single-junction devices on top of each other so that each absorbs the slice of solar spectrum best suited to its band gap and the loss of energy from carrier thermalization is reduced. This is the approach of designing multijunction solar cells. If thermalization losses can be diminished then very high efficiencies like 80% can be achieved.

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6.5 Parasitic Resistance Effects

The simple PV model mentioned in figure 6.4 and equation 6.11 does not include the internal losses and leakage current. The generalized PV model in figure 6.6 represents the practical photovoltaic model.

![Generalized solar cell (PV) circuit model](image)

Internal losses and leakage current are mentioned by series resistance ($R_s$) and a parallel resistance ($R_p$) respectively. I-V characteristics equation of generalized model is

$$I = I_{PV} - I_s \left[ e^{\frac{q(V + IR_s)}{\beta kT_c}} - 1 \right] - \frac{V + IR_s}{R_p} \quad \text{(6.16)}$$

Single diode model does not take into account the recombination loss in the depletion region of the semiconductor. Several researchers have proposed a two diode model as mentioned in figure 6.6. Recombination of carriers represents substantial loss at low voltages. $D_2$ takes into consideration the loss in the depletion region.

![Double diode generalized solar cell (PV) circuit model](image)
The I-V characteristics equation of the two diode generalized PV or solar cell circuit model is given by

\[ I = I_{PV} - I_{s1} [e^{(V + IR_s)/\beta_1 V_T} - 1] - I_{s2} [e^{(V + IR_s)/\beta_2 V_T} - 1] - \frac{(V + IR_s)}{R_p} \ldots(6.17) \]

Here \( I_{s1} \) and \( I_{s2} \) are reverse saturation currents of diode 1 and diode 2, \( V_{T1} \) and \( V_{T2} \) are thermal voltages of diode 1 and diode 2 (\( V_T = kT_c / q \)), \( \beta_1 \) and \( \beta_2 \) are diode ideality factors of the two diodes and \( I_{s2} \) compensates the recombination loss in the depletion region. A PV array consists of several solar cells connected in series and parallel. Cells connected in parallel increase overall current and cells connected in series provide greater output voltage.

If the array is composed of \( N_p \) parallel connection of cells then

\[ I_{PV\text{total}} = N_p \times I_{PV} \quad \text{and} \quad I_{stotal} = N_p \times I_s \ldots(6.18) \]

In case of an array of solar cells equation (6.16) is modified as

\[ I = I_{PV\text{total}} - I_{stotal} [e^{(V + IR_s)/\beta V_T} - 1] - \frac{(V + IR_s)}{R_p} \ldots(6.19) \]

Here \( V_T = N_s kT_c / q \) and \( R_s \) is the equivalent series resistance of the array. In modeling of PV cells it is assumed that \( I_{sc} \approx I_{PV} \) as \( R_s \) is low and \( R_p \) is high.

Manufacturers of photovoltaic arrays provide only a limited experimental data about electrical and thermal characteristics. They don’t provide the I-V equation of a particular type of solar cell. Unfortunately some of the important parameters to adjust PV array models are not found in manufacturer’s datasheet like light-generated PV current, series and shunt resistances, diode ideality constant, diode reverse saturation current, carrier mobility, recombination in depletion region and bandgap energy of the semiconductor. All PV array datasheets basically convey following information-nominal open-circuit voltage (\( V_{oc, n} \)), short-circuit current (\( I_{sc, n} \)), maximum power-point voltage (\( V_{mp} \)), maximum power-point current (\( I_{mp} \)), open-circuit voltage temperature coefficient (\( K_V \)), short-circuit current temperature coefficient (\( K_I \)), maximum experimental peak power (\( P_{maxe} \)). This information is provided for standard test condition for temperature and solar irradiation. Some manufacturers provide I-V curves for several irradiation and temperatures. It makes the modeling and adjustment easier [43]. Electric generators are normally considered as current or voltage source, however practical PV devices behave in a hybrid manner (voltage or current source depending upon the operating point). The effect of series resistance \( R_s \) is stronger in the voltage source region as mentioned in figure 6.5 and
parallel resistance $R_p$ in current source region. Series resistance ($R_s$) is the sum of several structural resistances and $R_p$ depends on leakage current and fabrication method.

**6.6 Temperature Effects**

I-V characteristics of a PV device depend on internal characteristics and external influences such as solar irradiation and temperature. In modeling of PV cells it is assumed that $I_{sc} \approx I_{PV}$ as $R_s$ is low and $R_p$ is high. The light generated current is directly proportional to solar irradiation and also influenced by temperature.

$$I_{PV} = \left( I_{PVn} + K_1 \Delta T \right) \frac{G}{G_n} \ldots(6.20)$$

Here $I_{PVn} = \text{Light generated current at normal condition (25°C and 1000 W/m}^2\text{)}, \Delta T = T - T_n (T$ and $T_n$ are actual and normal temperatures in K), $G = \text{Actual irradiation and } G_n = \text{Average/Normal irradiation (1000 W/m}^2\text{)}$. The diode saturation current $I_s$ depends upon temperature by the following relation [43, 44].

$$I_s = I_{sn} \left( \frac{T_n}{T} \right)^3 e^{\left( \frac{q E_g}{\beta k} \right) \left( \frac{1}{T_n} - \frac{1}{T} \right)} \ldots(6.21)$$

Here $E_g = \text{Bandgap of the semiconductor and } I_{sn} = \text{Normal saturation current. Normal saturation current is denoted by the expression}$

$$I_{sn} = I_{scn} / e^{\left( \frac{V_{ocn}}{\beta V Tn} \right)} - 1 \ldots(6.22)$$

Here $I_{scn} = \text{Short circuit current under normal condition, } V_{ocn} = \text{Normal open circuit voltage, } V_{Tn} = \text{Thermal voltage of } N_s \text{ series connected cells at normal temperature } T_n$. Saturation current $I_s$ depends on saturation current density ($J_0$ in A/cm$^2$) and effective area of the cells. Current density $J_0$ depends on intrinsic characteristics of the PV cell which is not provided by the manufacturers. $I_{sn}$ value is found from experimental data which are obtained by putting $V = V_{ocn}$, $I = 0$ and $I_{PV} = I_{scn}$. The PV current-voltage equation in (6.21) can be further improved by

$$I_s = I_{sn} + K_1 \Delta T / e^{\left( \frac{(V_{ocn} + K V \Delta T)}{\beta V T} \right)} - 1 \ldots(6.23)$$

This equation aims to match open-circuit voltages with experimental data for variety of temperatures. Equation 6.23 is obtained taking into consideration current and voltage coefficients $K_1$ and $K_V$. Saturation current $I_s$ depends on temperature and equation 6.23 takes into
consideration voltage current coefficient to justify the net effect of temperature in linear variation of open-circuit voltage [43].

6.7 Multijunction Modeling

Recent solar cell designs of single-crystal multijunction solar cells have established the importance of developing electrical model for better understanding of current-voltage (I-V) characteristics and predict device performance of multijunction solar cells. The modeling should take into consideration dark current in individual \( p-n \) junctions as well as shunt and series resistance. The model should have information about tunneling, recombination and diffusion mechanism as well. The parameters determined from each of the subcell layers are combined together to find the overall cell model for the multijunction solar cell. The dark current in each subcell layer constitutes of three major components: diffusion-limited current, space-charge recombination and tunneling. To determine the model of multijunction design series connected junctions are derived using high voltage assumption. Saturation currents and ideality factors are calculated assuming each \( p-n \) junction behaving as single diode having one ideality factor resulting from combined effects of diffusion, recombination and tunneling. The overall expression for current as a function of device voltage is given by

\[
I_{\text{total}} = I_{\text{sub1}} I_{\text{sub2}} I_{\text{sub3}} I_{\text{sub4}} e^{(qV/\beta_{\text{total}}kT)} \quad \ldots(6.24)
\]

Here \( I_{\text{total}} \) is the total current, \( I_{\text{sub1}}, I_{\text{sub2}}, I_{\text{sub3}} \) and \( I_{\text{sub4}} \) are saturation currents of subcell layers 1, 2, 3 and 4 respectively and \( \beta_{\text{total}} \) is the total ideality factor expressed as

\[
\beta_{\text{total}} = \beta_{\text{sub1}} + \beta_{\text{sub2}} + \beta_{\text{sub3}} + \beta_{\text{sub4}} \quad \ldots(6.25)
\]

\( \beta_{\text{sub1}}, \beta_{\text{sub2}}, \beta_{\text{sub3}}, \beta_{\text{sub4}} \) are ideality factors of four subcell layers respectively. High ideality factor and high reverse saturation current indicate poor quality material and junction interface. The electrical modeling of multijunction solar cell having four subcell layers is shown below in figure 6.8. For the complete circuit model of the quadruple junction we have a series circuit (representing multiple subcell layers) of parallel circuits (representing multiple dark current mechanisms in each subcell) and added shunt resistance across each subcell layer [44]. A series resistance is implemented across the device. The current is matched to the current of the bottom subcell as the bottom subcell will generate the maximum amount of current due to the photon absorption for a broader spectrum.
The total voltage is the summation of individual voltages of the subcell layers connected in series. The current matching is done by thinning the subcell layers of the upper subcells. In figure 6.8 \( I_{PV} \) is the same current matched for each subcell layer. \( D_{1,1}, D_{2,1}, D_{3,1}, D_{4,1} \) are the saturation or dark current losses in subcell layer 1, subcell layer 2, subcell layer 3 and subcell layer 4 respectively. \( D_{1,2}, D_{2,2}, D_{3,2}, D_{4,2} \) takes into consideration the recombination losses is subcell layer 1, subcell layer 2, subcell layer 3 and subcell layer 4 respectively. Internal losses and leakage current are mentioned by series resistance \( R_s \) and a set of parallel resistances \( R_p \) respectively.

### 6.8 Quantum Efficiency (QE) and Current Density \( (J_{sc}) \) of Quadruple-Junction Solar Cell

Short-circuit current density \( (J_{sc}) \) of any subcell layer is determined by the quantum efficiency \( (QE) \) of the subcell and spectrum of light incident \( \Phi_{in} (\lambda) \) on that particular subcell layer.

\[
J_{sc} = e \int QE(\lambda) \Phi_{in} (\lambda) \, d\lambda \quad \ldots (6.26)
\]
QE for an ideal subcell layer having the base thickness $T_b$, emitter thickness $T_e$ and depletion width $W_d$ for a total thickness of $T = T_e + W_d + T_b$ is given by

$$QE = QE_{\text{emit}} + QE_{\text{depl}} + e^{-\alpha(T_e+W_d)}QE_{\text{base}} \quad \ldots(6.27)$$

Here $\alpha$ is the absorption coefficient of the material used to form the subcell layer. The equations for $QE_{\text{emit}}$, $QE_{\text{depl}}$ and $QE_{\text{base}}$ are given below

$$QE_{\text{emit}} = F_\alpha(L_e) \left[ (l_e + \alpha L_e) e^{-\alpha T_e} \left( l_e \cosh \left( \frac{T_e}{L_e} \right) + \sinh \left( \frac{T_e}{L_e} \right) + \cosh \left( \frac{T_e}{L_e} \right) - \alpha L_e e^{-\alpha T_e} \right) \right] \ldots(6.28)$$

$$QE_{\text{depl}} = e^{-\alpha T_e} \left[ 1 - e^{-\alpha W_d} \right] \ldots(6.29)$$

$$QE_{\text{base}} = F_\alpha(L_b) \left[ (\alpha L_b) - ((l_b \cosh \left( \frac{T_b}{L_b} \right) + \sinh \left( \frac{T_b}{L_b} \right)) + (\alpha L_b - l_b) e^{-\alpha T_b}) / [l_b \sinh \left( \frac{T_b}{L_b} \right) + \cosh \left( \frac{T_b}{L_b} \right)] \right] \ldots(6.30)$$

$$l_b = \left( \frac{S_b L_b}{D_b} \right) / D_e, \ l_e = \left( \frac{S_e L_e}{D_e} \right) / D_e, \ D_b = \left( \frac{kT \mu_b}{e} \right) / e, \ D_e = \left( \frac{kT \mu_e}{e} \right) / e \ldots(6.31)$$

$$F_\alpha(L) = \alpha L / [(\alpha L)^2 - 1] \ldots(6.32)$$

The photon wavelength dependence is related to absorption coefficient $\alpha(\lambda)$. Here $\mu_{b(e)}$, $L_{b(e)}$, and $S_{b(e)}$ are mobility, diffusion length and surface recombination velocity for minority carriers in the base (emitter), $T$ is the absolute temperature and $D_{b(e)}$ is the diffusivity of base (emitter). If we make the assumption that each absorbed photon has contributed in photocurrent, the QE will only depend on the thickness of the solar cell device, $T = T_e + W_d + T_b$ because a fraction of the $e^{-\alpha(\lambda)T}$ incident light is transmitted through the cell rather than being absorbed.

$$QE(\lambda) = 1 - e^{-\alpha(\lambda)T} \ldots(6.33)$$

This equation can also be achieved by setting $S = 0$, $L >> T$ and $L >> 1 / \alpha$ in equations 6.27 – 6.30. Let us consider the light absorbed and converted by each subcell layers in a multijunction design. Let us consider a multijunction solar cell having $n$ subcell layers numbered from top to bottom as $1 / 2 / \ldots / n$ having corresponding bandgaps of $E_{g1} / E_{g2} / \ldots / E_{gn}$. In case of sub-bandgap photons, $\alpha(\lambda) = 0$; hence $e^{-\alpha(\lambda)T} = 1$. The light $\Phi_{in}$ incident on the top subcell is simply the solar spectrum $\Phi_S$. In case of light entering the top subcell is the solar spectrum $\Phi_s$. Light entering the $m$-th subcell is filtered by subcell layers above in the stack so $m$-th subcell incident spectrum is

$$\Phi_m = \Phi_s(\lambda) e^{-\sum_{i=1}^{m-1} \alpha(\lambda)T_i} \ldots(6.34)$$
Here $T_i$ and $\alpha_i(\lambda)$ are thickness and absorption coefficient of the i-th subcell. The short-circuit current density of the m-th subcell is then given by

$$J_{sc,m} = e \int_0^{\lambda_m} (1-e^{-\alpha_m(\lambda) T_m}) \Phi_m(\lambda) \, d\lambda \quad \ldots (6.35)$$

Here $\lambda_m = hc/E_{gm}$ is the wavelength corresponding to bandgap of the m-th subcell. In the case that m-th subcell is optically thick to absorb essentially all above-bandgap light incident on it (i.e. $\alpha_m T_m \gg 1$) the exponential term goes to zero for photon energies above the bandgap $E_{gm}$.

To understand the dependence of current density on bandgaps of multiple junctions, it is helpful to consider the simple case of a double-junction solar cell. The top-junction subcell layer will filter the photons and the remaining ones having the bandgap equal to or higher than the bandgap of the bottom cell will be absorbed by the bottom subcell. The bottom-junction current density $J_{sc,2}$ depends on both bandgaps of both the subcell layers $E_{g1}$ and $E_{g2}$. The $J_{sc,1}$ depends only on $E_{g1}$ [1].

$$J_{sc,1} = e \int_0^{\lambda_1} \Phi_s(\lambda) \, d\lambda, \quad J_{sc,2} = e \int_{\lambda_1}^{\lambda_2} \Phi_s(\lambda) \, d\lambda \quad \ldots (6.36)$$

Figure 6.9: Internal quantum efficiency (IQE) of the novel quadruple junction solar cell

Figure 6.9 is the simulation of internal quantum efficiency of the novel design with respect to wavelength. The IQE is a measure of the minority carriers generated by the photons that are not reflected. We see that the novel solar cell has very high internal quantum efficiency. The quantum efficiency decreases with reduction in solar radiation intensity.
6.9 Multijunction J-V (Current Density to Voltage) Curves

Let us consider m series connected subcells having individual current-density-voltage curve described by \( V_i(J) \) for i-th subcell, the expression for J-V curve is given by

\[
V(J) = \sum_{i=1}^{m} V_i(J) \quad \ldots(6.37)
\]

The voltage at a given current is equal to the summation of all subcell voltages at that current. Each individual subcell will have its own maximum-power point \((V_{mpi}, J_{mpi})\) which maximizes \( J \times V_i(J) \). In case of series connected multijunction solar cell the currents through each of the subcell layer are designed to have the same value; hence each subcell will be able to operate at maximum-power point only if \( J_{mpi} \) is same for all subcell layers i.e. \( J_{mpi,1} = J_{mpi,2} = J_{mpi,3} = \ldots = J_{mpi,m} \). In this case the maximum power output of the combined multijunction device is the sum of maximum power outputs \( V_{mpi,1}J_{mpi,1} \) of the subcells. On the other hand if the subcells do not have the same value of \( J_{mpi} \) then in their series-connected multijunction combination some subcells must operate away from the maximum-power points. The consequences of the last point is essential for high quality III-V junctions, the subcells do not leak or quickly break down in reverse bias. The bottom subcell layer in the multijunction has the highest \( J_{sc} \). For any given value of current the tandem voltage is

\[
V_{tandem} = V_{\text{subcell}1} + V_{\text{subcell}2} + V_{\text{subcell}3} + V_{\text{subcell}4} + \ldots + V_{\text{subcell}m}
\]

To model multijunction devices we have to find expressions for the subcell J-V curves \( V_i(J) \). Implementing classical ideal-photodiode J-V (neglecting the depletion region) \([1, 45]\) is given by

\[
J = J_0\left[ e^{(eV/kT)} - 1 \right] - J_{sc} \quad \ldots(6.38)
\]

Here \( e \) is the charge of electron. Open-circuit voltage is given by

\[
V_{OC} \approx (kT / e) \ln \left( J_{sc} / J_0 \right) \quad \ldots(6.39)
\]

We know that \( J_{sc} / J_0 >> 1 \). The dark current density \( J_0 \) is given by

\[
J_0 = J_{0,\text{base}} + J_{0,\text{emitter}} \quad \ldots(6.40)
\]

Here

\[
J_{0,\text{base}} = e (D_b / L_b) (n_i^2 / N_b) \left[ (S_b L_b / D_b) + \tanh (T_b / L_b) \right] / (S_b L_b / D_b) \tanh (T_b / L_b) + 1 \]

\[
\ldots(6.41)
\]

A similar expression denotes \( J_{0,\text{emitter}} \). The intrinsic carrier concentration \( n_i \) is given by
\[ n_i^2 = 4M_c M_v (2\pi kT / h^2)^3 (m_{e*} m_{h*})^{3/2} e^{(-E_g / kT)} \ldots (6.42) \]

Here \( m_{e*} \) and \( m_{h*} \) are electron and hole effective masses and \( M_c \) and \( M_v \) are number of equivalent in the conduction and valence bands respectively. \( N_{b(e)} \) is the base (emitter) ionized-impurity density. These equations fully define the multijunction \( J-V \) curve. The maximum-power point \( (J_{mp}, V_{mp}) \) can be calculated numerically at the point where \( V \) \( (J) \) maximizes \( J \times V \) (\( J \)). The various solar cell performance parameters can be found by putting \( V_{OC} = V \) \( (0) \), \( FF = (J_{mp} V_{mp}) / (J_{SC} V_{OC}) \) [1].

The current density \( J \) (A/cm\(^2\)) of the novel design with respect to increasing voltage is shown in the figure 6.10. We clearly see that in the current source region (current value close to \( J_{SC} \)) current is maximum while the voltage value is minimum. The effect of parallel resistances is higher in the current source region. In the voltage source region (near to \( V_{OC} \)) the voltage value is maximum, however the current value is smaller. The effect of series resistance is prominent here.

![Figure 6.10: Current density vs. voltage of the novel quadruple junction solar cell](image)

Different parameters required for the simulations are described in table 6.1. The bottom subcell layer (InGaSb) has the highest thickness with gradually decreasing thickness for the upper subcell layers. The top subcell layer (AlGaInP) has the highest absorption coefficient. The refractive indices of the subcell layers gradually increase as the light travels from the uppermost subcell layers to the respective bottom subcell layers.
Table 6.1: Parameter Values Used in Simulation

<table>
<thead>
<tr>
<th>Refractive Index (n)</th>
<th>Extinction Coefficient (k)</th>
<th>Absorption Coefficient (α)</th>
<th>Thickness (nm)</th>
<th>Subcell Layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.475</td>
<td>3.8375</td>
<td>5.845e^4/cm</td>
<td>1600</td>
<td>InGaSb</td>
</tr>
<tr>
<td>4.39</td>
<td>3.46</td>
<td>5.2737</td>
<td>1100</td>
<td>GaSb</td>
</tr>
<tr>
<td>3.721</td>
<td>2.644</td>
<td>4.027</td>
<td>800</td>
<td>InGaAs</td>
</tr>
<tr>
<td>3.1365</td>
<td>7.53</td>
<td>9.47e^3/cm</td>
<td>600</td>
<td>AlGaInP</td>
</tr>
</tbody>
</table>

Figure 6.11 is a display of power density with respect to voltage. As the voltage value increases we observe that the power generated also increases. We achieve a maximum power value for an optimum voltage (V_m) and current (J_m) value denoting the maximum power point [46].

Figure 6.11: Power density vs. voltage of the novel quadruple junction solar cell
CHAPTER SEVEN

CONCLUSION AND FUTURE RESEARCH

7.1 Summary of Work

This dissertation addressed the device physics and design of multijunction solar cells, specifically the fundamental properties of semiconductors, losses occurring in electron-hole generation and through recombination, III-V semiconductors materials, lattice matching and spectral splitting. The photonic modeling of wave propagation through multilayer structures using transfer-matrix method and two novel quadruple-junction solar cell designs has been discussed. Comparative analysis of the two quadruple-junction designs with state-of-art solar cells has been simulated and photon absorption for different wavelengths has been favorably compared. The electrical modeling of single and multijunction solar cells is also explained. The current density vs. voltage, power density vs. voltage and quantum efficiency of the novel designs has been discussed in detail. Two quadruple-junction solar cell designs and their subcell configurations comprised of AlGaInP/InGaAs/GaSb/InGaSb and AlGaInP/InGaP/InGaAs/InGaSb subcell layers is shown. The subcell layers are direct bandgap semiconductors with very high absorption coefficient, good optical and electrical properties and all have zinc-blende lattice structures. The transfer-matrix method of wave propagation through multi-layer structures has been discussed. The matrices are formed for the intersection between two layers and wave propagation through each layer. The product of all the transfer matrices forms the actual transfer-matrix for each solar cell designs. The photon absorption of these two solar cell designs has compared favorably with other state-of-art solar cell designs. The combination of subcell layers has yielded very high photon absorption through the entire solar radiation spectrum. Electrical modeling of single-junction and multijunction solar cell designs is discussed. Several important parameters like open-circuit voltage ($V_{OC}$), short-circuit current ($I_{SC}$), fill-factor (FF), maximum power-point (MPP) and cell efficiency has been discussed. Solar cell model such as the two diode model along with series and shunt resistance effects has been explained. Multijunction solar cell design for quadruple junction has been demonstrated. These designs take into consideration the same current for all the subcell layers (current matching) and the voltage effect is the summation of the individual voltages generated in each subcell layers.
Different loss mechanisms such as the dark current (saturation current), recombination losses in each subcell layers, have been considered along with series (resistance in the quasi-neutral and depletion region) and shunt losses (leakage loss). The two diode model representation of quadruple junction design is presented. The quantum-efficiency of the quadruple junction designs has been described. The novel designs execute very high quantum efficiency. Current density and power density vs. voltage simulations are demonstrated for AM1.5, 1 sun concentration.

7.2 Specific Technical Contributions

Average solar irradiance is $1\text{ kW/m}^2$ for AM1.5 solar radiation spectrum. 445 Watts of power comes from the the visible range of the spectrum, 527 Watts come from the infrared (IR) region and 28 Watts from the ultraviolet (UV) range of the solar radiation spectrum. 85% of the solar cell industry is being dominated by the crystalline silicon solar cells. There are some limitations of crystalline silicon solar cells. Silicon is a low bandgap semiconductor so it can generate high current due to the photon absorption for a broader wavelength of the solar radiation spectrum but it can generate very small voltage due to the limitations of dark current. Silicon is an indirect bandgap semiconductor so some energy is being lost in the intermediate process because of the creation of phonon particles while electrons are being excited from the valence band to the conduction band. Silicon can only capture photons for a certain wavelength range but not the entire solar radiation spectrum so the photons of bandgap lower than 1.1 eV are not being absorbed the by silicon. The high energy photons having higher energy than the bandgap of silicon are being wasted as heat and not contribute in photo-current generation. The efficiency of crystalline silicon solar cells are normally in the range of 8% - 18%. It is relatively low. We have designed two quadruple junction solar cells comprised of AlGaInP/InGaAs/GaSb/InGaSb and AlGaInP/InGaP/InGaAs/InGaSb subcell layers respectively. Each of the subcell layers has been designed targeting photons for a specific spectral region by effectively splitting the solar radiation spectrum. The top subcell layer has the bandgap of 2.3 eV capturing the photons of energy equal to or higher than that bandgap, In case of InGaP being the second subcell layer having bandgap of 1.93 eV and InGaAs having the bandgap of 1.1 eV, it will capture the photons of energies lower than the 2.3 eV but higher than 1.93 eV or 1.1 eV respectively. The bottom subcell layer will capture photons of energies equal to or higher than 0.3 or 0.5 eV but less than
the bandgap of Gasb layer of 0.7 eV and InGaAs of bandgap 1.1 eV respectively. The effective spectral splitting helps overcome the limitations of dark current and each subcell layers generate a certain amount of voltage that is being added together because of series connection of the subcell layer, promoting a significantly higher value of voltage than single-junction designs. The current matching is done for each subcell layer to have the same current in every subcell layer. Cell thinning is performed in each subcell layer to match the current. Same current and significantly higher value of voltage results in higher efficiency of the solar cells. We have designed our quadruple-junction designs based on antimony based materials. Antimony based subcell layers are very effective in the lower and higher infrared regions helping the generation of electricity from the photons in the region, unlike other single and multi-junction designs. The subcell layers we have implemented are all direct bandgap materials with zinc blende lattice structure. This helps reduce the loss of energy in the intermediate process due to creation of phonon particles. The unique combination of subcell layers is capable of higher photon absorption all through the solar spectrum in comparison to the state-of-art designs. The photon absorption of our designs for different wavelengths in the solar radiation spectrum compares favorably with other single junction and multi-junction designs. The specific technical contributions are the introduction of InGaSb in the bottom subcell layer to capture IR photons, the combinations of subcell layers used in the designs to have better photon absorption, the similarities in the crystal lattice structures of the compound semiconductors, the electrical as well as optically sensitive properties and high absorption coefficient. The aforementioned properties make the two quadruple junction designs unique in comparison to the state-of-art solar cells. We also have simulated the quantum efficiency, current density and power density to voltage to show the advantages of our designs.

7.3 Future Research on Multi-Junction Solar Cells

Multijunction solar cell research has tremendous potential to meet the challenges of energy requirements in the 21st century and beyond. Cutting-edge research for improved designs on the optical and electrical aspects is needed to make them cost effective for household applications. The multijunction solar cells are currently used for terrestrial applications; however it is becoming more popular for commercial applications with the improvement in thin film technology and exponential reduction in fabrication cost. The multijunction solar cells system
balance cost is around $7/kWh currently with the potential of becoming 7-8¢/kWh by 2030. There is lot of room to find cheaper materials with high optical and electrical properties to replace the existing technologies. The fabrication systems such as metal-organic-chemical-vapor deposition (MOCVD), molecular-beam-epitaxy (MBE), chemical-phase-epitaxy (CPE), liquid-phase-epitaxy (LPE) etc. are very expensive and improvements are needed to bring down the cost of these fabrication devices. There is room for improvement in the storage technology to store the DC power generated by solar cells. Improvements in the converter technology as well as effective control mechanisms are also needed to incorporate the solar cells for micro-grid and large scale utility applications.

Each of the subcell layers of a multijunction solar cell is comprised of emitter, base and back surface field layers. There are also tunnel junctions connected for better optical and electrical matching. In a quadruple-junction design each of the subcell layers will have 3 semiconductor layers within and three tunnel junctions in between the subcell layers. There is scope for research to find the resistance and losses in each of the subcell layers. The resistance is due to the resistances in the quasi-neutral regions as well as the depletion regions in each subcell. It is important to determine the exact resistivity of a subcell layer for better electrical modeling and predict the performance of each subcell layer as well as the entire solar cell module. Research is also needed to minimize the trap, radiative and auger recombination losses in each of the subcell layers. Modeling of the diodes with proper ideality factor to accommodate the recombination losses will be another aspect of research. There is room for improvement in modeling the quadruple junction designs for high temperature or concentrated solar applications. The performance of multijunction designs for a very high temperature, losses due to heating and several hundreds of concentration needs to be accommodated in the modeling.
REFERENCES


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BIOGRAPHICAL SKETCH

EDUCATION

2013 Summer  Ph.D in Electrical Engineering from Florida State University, Tallahassee, Florida, USA.

2009 Spring  Masters in Electrical Engineering from Florida State University, Tallahassee, Florida, USA.

2001 - 2005  Bachelor of Engineering in Electronics and Communication Engineering, Bangalore, India.
Recognition: First Class with Distinction.

PROFESSIONAL EXPERIENCE

2011 - 2013  Adjunct Professor, Electrical and Computer Engineering, Florida State University, Tallahassee, Florida, USA.

2007 - 2011  Graduate Teaching Assistant, Electrical and Computer Engineering, Florida State University, Tallahassee, Florida, USA.

2008 - 2011  Graduate Research Assistant, Electrical and Computer Engineering, Florida State University, Tallahassee, Florida, USA.

2008  Project leader Boeing Project, Electrical and Computer Engineering, Florida State University, Tallahassee, Florida, USA.
AWARDS AND DISTINCTIONS

- **FSU Innovators Award**, December 5, 2011 presented by Florida State University of Research Division.
- **Grant Assistance Program (GAP) Funding Award** from Florida State University Council of Research and Creativity, December 2010.
- **2013 Statewide (Florida) Graduate Student Research Symposium, Award for Research Excellence (2nd Place) in the STEM Category**, April 19, 2013, Tampa, Florida.
- **Graduate Research Poster Presentation Award**, FAMU-FSU College of Engineering, Florida State University, March 22, 2013.
- Bachelor of Engineering in Electronics and Communication Engineering from India in "First Class with Distinction". This is the highest accolade given in India.
- **National Scholarship** from the state of West Bengal, India.

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