Deep Circulation in the Eastern South Pacific Ocean

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DEEP CIRCULATION IN THE EASTERN SOUTH PACIFIC OCEAN

By

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ABSTRACT

It has been suggested in older and recent literature (e.g. Shaffer et al., 2004) that a broad deep eastern boundary flow is linking the equatorial Pacific Ocean to the Southern Ocean. The depth range of this flow corresponds to the Pacific Deep Water (2500m) where it is suggested by the distribution of primordial 3He originating from the East Pacific Rise. In this study, we propose to use a large set of data along with inverse techniques to estimate the stationary advection and diffusion of water properties and tracer concentrations (potential vorticity, potential temperature, salinity, dissolved oxygen and silica concentration) in the area of interest. The various data are combined into a new inverse model first developed by McKeague et al. (2005), Herbei et al. (2006). The model uses a forward advection-diffusion model and Markov Chain Monte-Carlo techniques to give estimates of velocities along and across surfaces of neutral density as well as isopycnal diffusivities. It is composed of 9 layers between the 27.4 and 28 neutral densities. The circulation in the upper layers of the model compares well with direct independent estimates of velocities from subsurface float trajectories (WOCE and Argo). We investigate the water exchange in the deeper layers in regards to the 3He distribution along the Eastern boundary and its mixing with the Antarctic Circumpolar Current. The model is able to estimate vertically dependent rates of oxygen utilization as well as lateral eddy diffusivities of tracers.
CHAPTER 1

INTRODUCTION

1.1 Objective of the study

The Eastern South Pacific Ocean is an area where old North Pacific deep water masses flow towards the Southern Ocean and meet with the Antarctic Circumpolar Current (ACC) upstream of the Drake Passage. It is believed that up to half of the deep outflow of the Pacific Ocean transits through this area, hence its relevance in the understanding of the global circulation and climate system. Recent studies pointed to the characteristically high mantle helium content of its deep waters and used this fact to infer some aspects of the circulation in the Southern Ocean and in particular advection and diffusion rates in the ACC (Naveira Garabato et al., 2007). Whereas these studies focus downstream of Drake Passage, less is known about the circulation of the high helium waters upstream and especially in the area of the helium sources over the South East Pacific Rise (EPR) and other active ridges.

The present study focuses on the pathways of the loosely defined Pacific Deep Water in the southeastern quadrant of the Pacific Ocean. Since helium is injected in a specific depth range (between 2000 and 2500m) just above the rise, we derive the isopycnal ocean circulation within the depth of the helium maximum and find significant zonal flow feeding a deep eastern boundary current. The balance of potential vorticity is found to depart from the classical Stommel-Arons equilibrium within depths of about 2500 m.

The inverse model implemented for this study seeks to compute a circulation that is consistent with the stationary advection and diffusion of ocean tracers along and across isopycnals. Thus, in order to achieve statistical significance, many tracers are used for the calculation. Conservative tracers of the deep ocean are the most obvious ones (temperature, salinity, silica, potential vorticity) but at least another nonconservative one is also needed for solving the clock rate problem (Wunsch, 1996) and that is dissolved oxygen. In the
deep ocean, oxygen is consumed at a rate that is difficult to know precisely. The oxygen consumption is parameterized through a constant decay coefficient (i.e. the inverse of a time scale). In principal, the choice of its amplitude is a challenge that needs to be answered prior to carrying on the inverse calculation. Here we use the technique to argue for a realistic one.

After a short introduction to the major features of the topography, we present the dominant water masses along with their neutral density (noted as $\gamma$) range. Then we present the state of knowledge of the eastern pathway of the Pacific Deep Water along the South American continent. The second chapter contains an overview of the dataset. The third chapter describes the inverse model. The fourth chapter presents the results the calculation.

## 1.2 Study Area and Water Masses

The dominant topographic feature of the eastern South Pacific is the East Pacific Rise (EPR) (Fig. 5.1). This active spreading center is north-south oriented between 120°W and 110°W. Its crest reaches depths of about 2500 m and roughly coincides with the neutral density surface $\gamma = 27.98$ (Fig. 5.4). The EPR volcanic activity is favorable for the generation of hydrothermal plumes rich in $^3$He (Lupton, 1998) as well as Mn, Fe and methane (Lupton, 1995). East of the EPR, the Roggeveen Basin and Chile Basin are surrounded to the north by the Sala y Gomez Ridge (25°S) prolonged by the Nazca Ridge to the coast of Chile, and to the south by the Chile Rise. Gaps are found through each ridge: 35°S and 55°S through the EPR, 42°S and 45°S through the Chile Rise. Multiple gaps are found between the Sala y Gomez and Nazca Ridges at 90°W.

The main water masses above the thermocline in the South Pacific Ocean are the following: the Subantarctic Mode Water (SAMW) formed north of the Subantarctic Front and the Antarctic Intermediate Water (AAIW) formed in the southeastern Pacific. The SAMW is a local minimum of potential vorticity magnitude and an oxygen maximum near 600 m (Fig. 5.4, $\gamma = 27.1 - 27.2$). AAIW is identified as a minimum in salinity around the depth of 800 m ($\gamma = 27.2 - 27.4$). These two water masses are responsible for ventilating the thermocline from high southern latitudes.

Below the AAIW, oxygen minima within the range $\gamma = 27.4 - 28$ denote the southward flowing Pacific Deep Water (PDW) to be described later. South of 30°S, PDW mixes with waters originating from the Antarctic Circumpolar Current. Circumpolar Deep Water (CDW) flows enter the South Pacific in the Tasman Sea, the Southwest Pacific Basin and
the southeast Basin (Reid, 1997). CDW is separated into Upper Circumpolar Deep Water (UCDW, oxygen minimum and nutrient maximum) and Lower Circumpolar Deep Water (LCDW, salinity maximum, nutrient minimum). In the South Pacific Ocean, UCDW mixes with PDW as these two water masses are located over a similar depth range (1000-2500db). LCDW is found below the PDW and is composed of two water masses: a local maximum of Salinity (and minimum of dissolved silica) at depth of 4000 m west of the EPR denoting water of North Atlantic origin that was carried from the Indian Ocean (Sloyan and Rintoul, 2001) and Antarctic Bottom Water (AABW), seen with a cold and fresh signature. The LCDW inflow to the Chile Basin occurs through gaps in the Chile Rise (Warren, 1973) and is seen as a local maximum of oxygen along the bottom of the Chile Basin (Fig. 5.4).

Main routes of water masses can be seen on the distribution of dissolved oxygen along the neutral surface $\gamma = 27.98$ (close to the boundary between PDW and LCDW) over the whole Pacific Ocean (Fig. 5.17). In the northern hemisphere and in the tropical zone, oxygen concentration is dominated by a north-south gradient and little can be said about the meridionnal circulation. However, in the southern hemisphere, as PDW encounters UCDW in the Southern Ocean and LCDW from below, more pronounced meridional structures are visible. In particular, two dominant export routes of PDW are apparent as tongues of low oxygen concentration. Looking at a WOCE section of oxygen concentration at 32°S, PDW is characterized by two cores of low oxygen (Warren, 1973; Wijffels et al., 2001), one is found to the east along the coast of Chile and the other west of the EPR.

The eastern core has a minimum value of 90 $\mu$mol kg$^{-1}$ and is centered on the neutral density layer $\gamma = 27.7$ (Fig. 5.4). A map of oxygen concentration along $\gamma = 27.7$ (Fig. 5.6) shows that this minimum corresponds to a broad tongue extending from 20°S to the tip of South America. The width of this tongue extends zonally from the coast of Chile to well over the EPR at 110°W along 32°S. At 54°S, the oxygen minimum is still visible with a concentration of about 150 $\mu$mol kg$^{-1}$ but the tongue is reduced in width as it extends only to 85°W (Fig. 5.8). The oxygen minimum deepens toward the South. At 54°S it reaches 1500 m is located between the layer $\gamma = 27.8$ and $\gamma = 27.9$. This deepening of the oxygen minimum is also seen on a meridional section at 88°W (Fig. 5.7) and is not easily interpretable. Oxygen is a non-conservative tracer and is consumed at a nearly constant rate in the deep ocean (e.g. Jahnke, 1996; del Giorgio and Duarte, 2002). Thus, consumption alone does not explain the minimum and is rather due to local variations in advection and
diffusion. The deepening is likely an indication of vertical mixing with the surrounding higher oxygen waters. According to the map of absolute flow (adjusted steric height) of Reid (1997) at 2000 db, UCDW flows eastward between 30°S and 40°S into the Chile Basin so its relatively high oxygen concentration could be responsible for the erosion of the oxygen minimum through isopycnal mixing. In addition, the minimum is presumably eroded from above by the far higher oxygen concentrations of AAIW through diapycnal mixing.

Another indication of mixing is seen from the variation of oxygen and silica along neutral surfaces in the Chile Basin on the 32°S section. Along the surfaces $\gamma = (27.4, 27.5, 27.6, 27.7)$, going eastward from the EPR, silica and oxygen concentrations vary little with an increase of 3-4 $\mu$ moles/kg over 35000 km. However, on the deeper layers $\gamma = (27.8 - 28)$ the isopycnal concentrations shift from a small increase between the EPR and 88°W (2 $\mu$ moles/kg) to a larger gradient between 88°W and the coast. This observation suggests that the isopycnal concentrations in the upper range of PDW are influenced by diapycnal mixing of oxygen poor/silicate rich waters. Whereas, in the lower portion of PDW, the strong isopycnal gradients are maintained by southward isopycnal advection of nutrient rich waters east of 88°W.

The second oxygen core to the west of the EPR is seen on the 32°S section at a depth of 2500m and is centered on the neutral layer $\gamma = 28$. On the map of oxygen concentration on the neutral surface $\gamma = 27.98$ (Fig. 5.17), it appears as a southeast oriented tongue extending from 180°W, 10°S to 135°W, 42°S and is visible on a meridional section along 135°W (Fig. 5.9) with a core value of 110 $\mu$ moles/kg. The map of dissolved silica concentration on the neutral surface $\gamma = 27.95$ shows similar structures as the oxygen (Fig. 5.18).

### 1.3 Deep eastern boundary flow

Several studies have been pointing to the existence of a deep poleward boundary flow along the coast of South America. Indications of a deep southward flow along the coast of Chile were first given by Reid (1997). His maps of adjusted steric height (showing flow paths) show that most of the flow enters the Peru and Chile basin zonally between 30°S and 35°S and that the feature extends from 800m down to the depth of the Chile Rise (2500m). The typical tracer signature of this flow is low oxygen, high nutrient concentration, low radiocarbon (Key et al., 2004). Shaffer et al. (2004) found a significant poleward flow at 2500m and below
from a current meter moored at 30°S, 150km away from the coast. Wijffels et al. (2001) analysis based on an inverse model along 32°S also found a significant southward flow of 18 Sv, a figure similar to the 12 Sv estimated by Sloyan and Rintoul (2001). The total PDW export is estimated to be 16 - 20 (Wijffels et al., 2001), 25 Sv (Sloyan and Rintoul, 2001), such that the eastern boundary flow would represent about half of the total PDW export from the Pacific Ocean (Shaffer et al., 2004).

A Southward flow is also suggested on the WHP hydrographic section along 52°S (Tsuchiya and Talley, 1998) and in agreement with values of the oxygen concentration. They also note from the distribution of tracers and geopotential the possible existence of an eastward flow across the meridional section at the latitude of the Sala y Gomez Ridge (25°S). This flow is locally rich in oxygen and poor in nutrients.

1.4 Zonal flow and mantle Helium

Submarine volcanos eject hydrothermal fluids rich in helium isotope ³He concentration relative to the atmosphere because of the natural enrichment of the Earth mantle in ³He. The amount of this isotope present in oceanic water is quantified with a practical unit, δ³He defined by \[ \delta^{3}\text{He} = 100\left(\frac{R_{\text{sample}}}{R_{\text{air}}} - 1\right) \] where \( R \) is the isotopic ratio (\( [^{3}\text{He}]/[^{4}\text{He}] \)). ³He is a conservative tracer because it is stable, inert and not utilized (by living organisms). Also, the oceanwide δ³He content is thought to be constant as degassing to the atmosphere in the surface mixed layer balances the hydrothermal input. The dominant source locations and rates are not precisely known, however they are confined to geologically active features of the ocean floor. In the southeastern Pacific, this includes the crest of the EPR, Galapagos Rift, Nazca Ridge and Chile Rise. At the level of \( \gamma = 27.98 \), where mantle helium concentration is the strongest, large plumes are seen to emanate from the EPR and spread zonally along 10°N and 15°S westward. In contrast, very low concentrations along the equator are interpreted as an eastward flow (Lupton, 1998). Talley and Johnson (1994) proposed that this zonal plume structure symmetric with respect to the equator is a general circulation pattern found in both the Atlantic and the Pacific Oceans.

In the southeastern Pacific Ocean, δ³He reveals a eastward plume extending from the EPR at 28°S (Lupton, 1998). A meridional section of δ³He along 88°W (that is, east of the EPR) shows the core to be centered on the neutral layer \( \gamma = 27.95 \) (Fig. 1.1) and that the
plume coincides with the Sala y Gomez Ridge. A zonal section along 32°S (Fig. 1.2) also confirms the extent of the plume and strong zonal gradient of δ³He toward lower values west of the EPR is reminiscent of an eastward flow over a source point. Eastward flow over the EPR at 28°S is also thought to advect δ³He rich waters toward the eastern boundary (Well et al., 2003).

Figure 1.1: δ³He (expressed in %, see text for a definition of δ³He) along the WOCE Hydrographic Program meridional section P19 (88°W). The white lines show the 9 neutral surfaces used to define the inverse model’s layers. The neutral density values range from 28 to 27.4.

1.5 Circulation study from Davis

Davis (2005) used ALACE floats in the Pacific Ocean and applied an objective analysis consistent with the continental boundary ‘no through flow’ condition and no horizontal divergence. The horizontal velocity estimates are based on spatial averaging the floats displacements at depth. He obtained circulation patterns at mid-depth (900m) which
Figure 1.2: $\delta^3$He (expressed in %, see text for a definition of $\delta^3$He) along the WOCE Hydrographic Program meridional section P06 (32°S).

correspond in the SE Pacific to the deep extension of the subtropical gyre, i.e. a broad equatorward return flow feeding the East Australian Current (EAC). The EAC bifurcates eastward to join the eastern shore of New Zealand. This southward current merges with the ACC at around 45°S. At this merging location, eddy diffusivity (computed from the float dispersion) is stronger than the surrounding values and exceeds 8000 m$^2$/s. The eastward flowing branch of the subtropical gyre lies between 42°S and, virtually, the subpolar front where it is indistinguishable from the ACC. The area chosen for our calculations (east of 150°W, south of 10°S) encompasses the broad equatorward flow (which is located along 100°W - 90°W) and both the eastward and westward branchings of the gyre (see Davis, 2005, Fig. 17) (the results of Russell and Dickson (2003) also argue for such an equatorward
flow along 110°W). The results of Davis (2005) brought new insight on the circulation of the southeast Pacific Ocean. However it is important to note that the float coverage in this area was limited (see Fig. 5.14 for the distribution of the Davis WOCE floats) and the direct velocity analysis has low statistical significance locally. Our study uses hydrographic data (both from profiling floats and hydrographic sections) and tracer data for the estimate of the mean circulation. Velocity estimates from spatially averaged float displacements are purposely not incorporated in the inverse calculation and are used to validate the resulting velocities. We claim that this approach helps to achieve a better statistical significance of the estimated circulation.
CHAPTER 2

DATA

2.1 Hydrography

All available hydrographic data available in the southeastern Pacific were gathered for this study and a quality check was carried out by systematically plotting histograms for each water property as a way to visually isolate and remove any spurious measurement. (Fig. 5.2, 5.3). Water properties used are temperature, salinity, dissolved silica, oxygen concentration and helium ratio ($\delta^{3}He$). South of 25°S all data were taken from the Southern Ocean Data Base (Orsi and Whitworth III, 2005). North of 25°S, data were taken directly from the NODC repository. All the hydrographic sections obtained during the World Ocean Circulation Experiment (WOCE) were cross-over adjusted (calibrated) using the Johnson et al. (2001) coefficients. Hydrography and tracer data obtained during the HELIOS program are also included (P. Froelich, personal communication). Table 2.1 summarizes all the data used.

Table 2.1: Summary of all hydrographic data. **WHP**: World Ocean Circulation Experiment - Hydrographic Program; **NODC**: National Oceanographic Data Center.

<table>
<thead>
<tr>
<th>name of dataset</th>
<th>WOCE dates</th>
<th>NODC (excluding WOCE) dates</th>
<th>Argo profile dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of profiles</td>
<td>975</td>
<td>2281</td>
<td>1200</td>
</tr>
</tbody>
</table>

All data are linearly interpolated on predefined neutral density surfaces (Jackett and McDougall, 1997) chosen for their vertical spacing of approximately 100 meters.
2.2 Floats

Float Profiles

Temperature and salinity profiles from all Argo profiling floats available in the area were obtained from the Argo Global Data Assembly Center. These data were collected and made freely available by the International Argo Project and the national programmes that contribute to it (www.argo.ucsd.edu, argo.jcommops.org). Argo is a pilot programme of the Global Ocean Observing System. Most Argo floats profile from 1500/2000m up. Only profiles passing the standard quality check were kept (passing for T,S and P simultaneously). Measurements adjusted by the experiment investigators (delayed mode data) were used in place of the original ones when available. Spurious large spikes in the profiles were detected and visually removed. This study being principally focused on the deep circulation below the density surface $\gamma = 27.4$ (1000 meters deep in the subtropics) and above $\gamma = 28$ (2000 meters deep in the subtropics), the contribution of the profiling floats is limited to the upper layers of the inversion (Fig. 5.2) in all the southeast Pacific except in the ACC region. The profiling float data is combined with the hydrographic data. Even though the hydrographic sections were predominantly conducted during austral summers, we kept all float profiles in an attempt to maximize the data sampling coverage.

Float displacements

A measure of current velocity can be made from the subsurface displacement of a profiling float by dividing the distance travelled by the time spent at depth. Float displacements were computed from location of surfacing position using the method of Davis (1998, 2005). Standard velocity errors were also estimated (Davis, 1998).
CHAPTER 3

MODEL

3.1 Inverting a tracer field

We want to study the ocean circulation inferred from the distribution of ocean properties. Common approaches used to solve this inverse problem are the inverse box models (Wunsch, 1996) which typically estimate transport across hydrographic sections consistent with the adjusted thermal wind field. Inversion of a gridded tracer field can be achieved with the \( \beta \)-spiral technique (e.g. Hautala and Riser, 1993) which makes use of PV conservation. The inversion of St Laurent et al. (2001) uses the latter approach combined with direct microstructure measurements of diapycnal mixing. The method of Hogg (1987) uses least-square techniques to fit the advection-diffusion equation to climatological tracer data (where uncertainties are assumed on the equations). Methods for the inversion of climatological data were also developed with the use of forward models and assimilation techniques (see Dobrindt and Schroter, 2003; Stammer et al., 2002).

In this section we present a variant of the statistical inverse model employed by McKeague et al. (2005) for the study of the assumed steady state ocean circulation in the South Atlantic Ocean. The model first inverted a 2-dimensional tracer field using a forward advection-diffusion solver along a unique neutral density layer. Herbei et al. (2008) then extended the approach to include vertical components of advection and diffusion in a 'quasi 3D' setup. It was then augmented by adding a thermal wind constraint. In what follows we present a modified version of the Herbei et al. (2008) model. First, it is extended to 9 layers with realistic eastern boundary (Herbei et al. used a rectangular domain). Second, geostrophy is imposed locally, as opposed to integrating the mass equation zonally. Third, a no-flow condition is imposed at the eastern boundary where there is a continent.
3.2 Statistical method

The inverse model used for this study follows the approach developed by McKeague et al. (2005) and Herbei et al. (2008) for the inversion of ocean tracer distributions. It uses a fundamental result of probability theory, the Bayes' theorem (e.g. D'Agostini, 2003). In this approach, model parameters are connected to experimental observations through conditional probabilities. A physical model (see next section) relates the model parameters (the velocities and diffusivities) with the observations. In a generalized sense, the possible values of the model parameters (the parameter’s posterior probability density function, pdf) is given by:

\[ \Pi(\theta|x, I) \propto \Pi(x|\theta, I) \cdot \Pi(\theta|I) \]  

where \( \theta \) represents the model parameters, \( x \) represents the observations and \( I \) is the 'background knowledge' (i.e. the physical model). \( \Pi(.) \) denotes any probability density function. The conditional pdf \( \Pi(\theta|I) \) is the 'prior' and denotes the previous knowledge of the possible values of the parameters. The term \( \Pi(x|\theta, I) \) is the likelihood (that is, the likelihood of the parameters given the observations\(^1\)). The proportionality relation implies that a normalization constant on the right hand side is unknown. The evaluation of this distribution is achieved by sampling using a Markov Chain Monte Carlo (MCMC) method. Random parameter values are tested against a cost function \( K \) subject to reversibility (or detailed balance) defined by the following relation:

\[ K = -\log(\Pi(x|\theta, I)) \]  

Each iteration by which the cost function is tested is called a MCMC move. The chain of parameters produced by the method is guaranteed to converge under weak conditions, but Herbei et al. has shown that this chain converges geometrically making the method more adequate. (The average and variance (and possibly modes) of the Markov Chain, once it is stationary, constitute the solution). Once the Markov Chain becomes stationary, the sampling process is kept on in order to obtain a large enough number of parameter values to construct the pdf of the Markov Chain. A chain being produced for each parameter, the average and variance (and possibly modes) of the Markov Chain, once it is stationary, constitute the solution.

\(^1\)...or the probability of the observations given the parameters
The unknown model parameters are the three components of velocities, horizontal and vertical diffusion coefficients and boundary values of the chemical tracers. The observations are the non-synoptic tracer measurements (potential temperature, salinity, dissolved silica, dissolved oxygen, $^3$He concentration and potential vorticity). This makes 36,036 unknown parameters and 26,403 data on the model grid points.

The choice of a Bayesian approach over a more traditional least-square method is discussed by McKeague et al. (2005). The method is more adapted to a large non-synoptic dataset. No temporal average nor seasonal fit is carried out before the inverse calculation at the exception of a spatial averaging conducted on the model grid that is required to compute property gradients needed by the physical model. The method deals with data distributions rather than unique values so that the variability due to eddy activity (as well as due to seasons in our case) is inherently taken into account. Nonetheless, the model does not account for inter-annual variability. A growing number of studies are improving our knowledge on the deep ocean variability. Shaffer et al. (2000) report warming and an increase of the oxygen concentration around the depth of 1000m at 28°S in the Chile Basin. Johnson et al. (2007) also found significant warming in deep waters over the past twenty years. Roemmich et al. (2007) observed a 20% spinup of the subtropical gyre between 1993 and 2004 from Argo floats and satellite data. The data used here span a period of roughly forty years (Table 2.1) and the model’s posterior mean velocities and diffusivities are meant to be representative of this period. Thus, the present model large-scale tracer field is assumed to be at steady-state. As such, we want to find steady velocities and diffusivities consistent with the tracer field. In the following section we present the physical model in detail.

### 3.3 Physical model

We propose to compute a three dimensional velocity field consistent with the three dimensional advection and diffusion of ocean tracer properties along neutral density surfaces. First, in a locally orthogonal neutral density coordinate, geostrophy is written as

$$fu = -P_y, \quad fv = P_x \quad (3.3)$$

The continuity equation is written as

$$h_t + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) + w^T - w^B = 0 \quad (3.4)$$
with $w^T$ and $w^B$ as the diapycnal velocities across the top and bottom layers.

We relate the diapycnal advection to the diapycnal diffusivity using the vertical advective-diffusive balance for density:

$$ w^* = \gamma_z^{-1} \gamma_{zz} \kappa_z $$.  \(3.5\)

The lateral advection and diffusion of neutral density vanishes on isopycnal surfaces to a very
good approximation. The ratio $\frac{\gamma_{zz}}{\gamma_z}$ is related to potential vorticity, and could be diagnosed
from PV (for example, from one MCMC run to another). However, to simplify the model
and avoid the high sensitivity of diapycnal exchanges to noise in inverse models in general
(Wunsch, 1996), we take the ratio from a climatology.

Combining equations (3.3), (3.4) and (3.5), we can write the vorticity equation for the
instantaneous flow as:

$$ \frac{D}{Dt} \left( \frac{f}{h} \right) + \gamma_z w^* \left( \frac{f}{h} \right)_z = \gamma_z \kappa_z \frac{\partial \left( \frac{f}{h} \right)}{\partial \gamma} \left( \frac{\gamma_z}{\gamma_z} \frac{\partial \left( \frac{f}{h} \right)}{\partial \gamma} \right) $$ $$ \text{(3.6)} $$

A time average of (3.6) gives for a steady state (O’Dwyer et al., 2000)

$$ u \nabla \left( \frac{f}{h} \right) + \gamma_z w^* \left( \frac{f}{h} \right)_z = \kappa_x \left( \frac{f}{h} \right)_{xx} + \kappa_y \left( \frac{f}{h} \right)_{yy} + \gamma_z \kappa_z \frac{\partial \left( \frac{f}{h} \right)}{\partial \gamma} \left( \gamma_z \frac{\partial \left( \frac{f}{h} \right)}{\partial \gamma} \right) $$ $$ \text{(3.7)} $$

where PV appears as an advected and diffused tracer. For all passive tracers, in a locally
orthogonal neutral density coordinate, the model is:

$$ u \nabla C + \gamma_z w^* C_z = \nabla \cdot \kappa_h \nabla C + \gamma_z \kappa_z \frac{\partial C}{\partial \gamma} \left( \gamma_z \frac{\partial \left( \frac{f}{h} \right)}{\partial \gamma} \right) $$ $$ \text{− \lambda C} $$ $$ \text{3.8} $$

where $u$ is the lateral (horizontal) velocity, $w^*$ is the diapycnal velocity due to adiabatic
processes, $\kappa_z$ is the diapycnal eddy diffusivity and $\kappa_h$ is the isopycnal diffusivity arising from
the mesoscale eddy activity. The components of the velocity field of the large scale flow
along neutral surfaces are in geostrophic equilibrium and the horizontal divergence of $fu$
vanishes locally (McDougall, 1988) ($f$ is the Coriolis parameter):

$$ \nabla (fu) = 0 $$ $$ \text{(3.9)} $$

We will use this property of $u$ to constrain approximate vorticity balance in the calculation.

Furthermore, and keeping with the framework of two neutral surfaces separated by a
thickness $h$, the conservation of potential vorticity is written in the following form:

$$ u \nabla \left( \frac{f}{h} \right) + w^* \left( \frac{f}{h} \right)_z = \nabla \cdot \kappa \nabla \left( \frac{f}{h} \right) + \gamma_z \kappa_z \frac{\partial \left( \frac{f}{h} \right)}{\partial \gamma} \left( \gamma_z \frac{\partial \left( \frac{f}{h} \right)}{\partial \gamma} \right) $$ $$ \text{3.10} $$
where isopycnal and diapycnal diffusion of potential vorticity is considered in the mean field to allow the geostrophic flow to cross PV contours. We chose to define potential vorticity as the ratio of the planetary vorticity over the layer thickness defined by two neutral density surfaces (McDougall, 1988) rather than computing it from potential density surfaces. Differences in magnitude between the two definitions of PV in the deep ocean are up to 10% (O’Dwyer and Williams, 1997) when computed from climatological data.

3.4 Solving the system

The posterior density of the (control) parameters $\Phi = (u, v, w^*, K^{(x)}, K^{(y)}, C_{\partial s})$ is written following the proportionality relation (3.1), as

$$
\pi(\Phi|C_D) \propto \pi(C_D|\Phi)\pi(u)\pi(K^{(x)})\pi(K^{(y)})\pi(C_{\partial s})\pi(w^*) \tag{3.11}
$$

Following McKeague et al. (2005), the likelihood is written as

$$
\pi(C_D|\Phi) \propto \prod_{s} \prod_{i=1}^{n_C} \prod_{j=1}^{n_D} \exp \left\{ -\frac{1}{2} \delta^2_u (C_{D,i}^{(j)} - C_i^{(j)})^2 \right\} \tag{3.12}
$$

where the multiplication runs over all gridpoints (the grid has $n_D$ gridpoints) and all $n_C$ tracers. $C_{D,i}^{(j)}$ is the tracer concentration measured at the $i^{th}$ gridpoint. $C_i^{(j)}$ is the tracer concentration computed by a finite differences advection-diffusion solver. Note that $C_i^{(j)}$ is a function of the control parameters.

The prior density of the isopycnal velocities are modeled by a Gauss Markov Random Field (GMRF) (Winkler, 1995) in geostrophic equilibrium. It is written as the following

$$
\pi(u) \propto \prod_{s \sim s'} \exp \left\{ -\frac{1}{2\delta_u^2} (u_s - u_{s'})^2 - \frac{1}{2\delta_v^2} (v_s - v_{s'})^2 \right\} \prod_{j=1}^{\text{gridpoints}} \exp \left\{ -\frac{1}{2\epsilon^2_{(j)}} \left[ \nabla(fu_{(j)}) \right]^2 \right\} \tag{3.13}
$$

The first term is a product over adjacent grid points and imposes a degree of smoothness controlled by the 'smoothing' parameters $\delta_u$ and $\delta_v$. The second term is introduced to impose geostrophy on the lateral velocity. Such a constraint differs from Herbei et al. (2008) approach where the zonal component of the velocity is diagnosed from the meridional velocity component and a horizontal integration of the mass and linear vorticity equations from the eastern boundary. The advantage of Herbei’s approach is to enforce geostrophy exactly (in the framework of a quasi horizontal flow). Error propagation might also play a role in
increasing the solution uncertainty westward. Comparatively, the ‘weaker’ constraint used in (3.13), allows for small departures from geostrophy in the velocity field. This is beneficial for the MCMC to explore various model states. An attempt was made to replace the parameters \( u \) and \( v \) with a streamfunction (Appendix B) but was unsuccessful.

### 3.5 Model settings

The model grid has 33×33 grid points horizontally and 9 neutral surfaces with \( \gamma \) values consistent with the water masses presented in the introduction (UCDW and PDW). The values are (from deepest to shallowest) \( \gamma = [28, 27.98, 27.95, 27.9, 27.8, 27.7, 27.6, 27.5, 27.4] \) and are represented on Fig. 1.1 and subsequent figures. All the measurements are averaged to the nearest grid points. The model grid is curvilinear and fits to the coastline along the South American continent (Appendix A) to the east. The grid point spacing is adapted for the study of the large scale circulation, hence chosen to be larger than the eddy deformation scale. Consequently the contribution of the flow’s relative vorticity to the potential vorticity balance is negligible. The advection-diffusion equation is solved with a multi-grid solver (Mudpack, J. C. Adams) at each step in order to compute the concentrations \( C_{i}^{(j)} \).

### 3.6 Initial Model State

At the start of the calculation, the initial values of iso-neutral velocities are set using geostrophic profiles with a level of no motion at 3500m or topography (whichever is shallower). Such an initial state was used instead of initial no motion state as in Herbei et al. (2008). Indeed, after initial experimental tests, we found that the convergence rate of the MCMC of the present version of the model was considerably slower than in Herbei (2006). In our approach, each MCMC move focuses on one single grid point (or the whole model layers at one geographical location for the depth independent move). Only small deviations are allowed from the lateral divergence at each move. The error on the divergence is controlled by a tuning parameter and stays in a range judged acceptable. The tuning parameter (noted as \( \epsilon_{(j)} \) in Eq. 3.13) is a function of latitude in order to account for the high magnitude of the velocities in the ACC relative to the subtropics. The most efficient remedy for improving the convergence rate and the computation time was to use the prescribed initial state.
Initial tracer values along all boundaries are set to climatological values (we used the climatology of Gouretski and Jancke, 1998) to provide the Dirichlet boundary condition of the advection-diffusion solver. Tracer boundary values are then updated at each step of the calculation as in McKeague et al. (2005) and are defined as GMRF.
CHAPTER 4

RESULTS

4.1 Preferred solution: choice of the oxygen consumption rate $\lambda$

The magnitude of the oxygen consumption coefficient $\lambda$ sets the model’s ‘clock rate’ because the oxygen consumption term makes the conservation equation for oxygen non-homogeneous (Eq. 3.8). $\lambda$ is considered spatially constant and preset to a chosen realistic value (eventhough it could be considered as a model parameter). Herbei et al. (2008) has shown that in the South Atlantic Ocean the magnitude of $\lambda$ influences strongly the bulk Peclet number of the posterior mean model state and that a bad choice of this poorly known quantity will degrade the model solution. We carried out several simulations with various values of $\lambda$ to explore its influence on the posterior mean diffusivities and Peclet number (Table 5.2). In the southeast Pacific Ocean, values larger than $10^{-10}\, s^{-1}$ lead to a basin average Pe of order 0.1. In this diffusive regime, the MCMC moves on velocity have little influence on the likelihood. The posterior model tracer distributions are overly smooth and do not resemble the data. On the other hand, for values smaller than $\lambda = 10^{-11}\, s^{-1}$, the model tends to give small diffusivities. An optimum value of $\lambda$ is one corresponding to a minimum in the posterior mean of the cost function defined in relation (3.2), where the model posterior tracer field closely resemble the data. The effect of $\lambda$ on the cost function is illustrated by Fig. (5.11). For high values of $\lambda$, the degree of smoothness of the model tracer field is such that the structures of the modeled field are larger than on the measured one. For small magnitudes of $\lambda$, the consumption term becomes negligible and the model looses its clock rate. Following this reasoning, we chose $\lambda = 3 \times 10^{-11}\, s^{-1}$ as a preferred value and kept it fixed for all simulations.

It is of interest to know how the above choice of the oxygen consumption coefficient
λ compares with values estimated from biological observations in the ocean. Oxygen consumption, due to the remineralization of organic material, is measured in the ocean by Oxygen Utilization Rate (OUR). OUR is defined by the ratio of the apparent oxygen utilization divided by the age of the water mass. According to Feely et al. (2004), the OUR reaches a maximum just below the euphotic zone and decreases to smaller and nearly constant values below about 1000 m (level of the Antarctic Intermediate Water). OUR is expressed in $\mu\text{mol/kg/yr}$ and cannot be compared directly with $\lambda$ ($\lambda$ is the inverse of a decay time scale). OUR is given by the source term of Eq. 3.8 for oxygen concentration, $\lambda C$. Fig. 5.12 shows the distribution of OUR as a function of depth as obtained by the inverse model. In order to compute these values, we first computed a mean oxygen profile for the model grid and multiplied by the constant $\lambda$ value. A confidence interval was estimated by taking the range of values of $\lambda$ corresponding to the minimum of the cost function seen on Fig. 5.11 combined with the standard deviation of the posterior oxygen concentration. For comparison, OUR estimated by Feely et al. (2004) and Craig (1971) of respectively 0.10 and 0.18 $\mu\text{mol/kg/yr}$ in the South Pacific below 1000 dbar fall within our confidence interval. OUR obtained by this study are also consistent with the range of values of $(0.03 – 0.19 \, \mu\text{mol/kg/yr})$ given by older studies (synthesis of Chen, 1990).

### 4.2 Diffusivity coefficients

The clock rate set by $\lambda$ implies a control on the magnitude of both the velocities (i.e. the strength of the circulation) and the diffusivities. A look at the sensitivity of the bulk velocities to variations in $\lambda$ showed no significance. On the other hand, diffusivities are controlled by $\lambda$. This behavior of the model is due to the diffusivities being spatially constant. Diffusivities Markov Chains become stationary early on in the iterative process compared to the velocities Markov Chains because the velocity field has more degrees of freedom. The dependence of the diffusivities $K_x$ and $K_y$ on the oxygen consumption rate $\lambda$ is shown on figure (5.13). They are extremely anisotropic for large and small $\lambda$ that are far from realistic values. We defined in the previous section a cost function minimum confidence interval for the magnitude of $\lambda$ as $(2 - 4) \times 10^{-11} \text{s}^{-1}$. Within this interval $K_y$ increases with increasing $\lambda$ from 200 to just below 1000 $m^2\text{s}^{-1}$ whereas $K_x$ stays constant at 1000 $m^2\text{s}^{-1}$. In the lower bound, the diffusive regime is anisotropic and it is nearly isotropic in the upper bound. The ‘preferred’
diffusivity values set by the confidence interval defined above are comparable to the ones calculated by Davis (2005) in the southeastern Pacific from float data at 900dbar where the diffusivities are of the order of 1000 $m^2s^{-1}$ in the Ocean interior and 3000$m^2s^{-1}$ in the ACC and diffusion is isotropic away from the equator. Naveira Garabato et al. (2007) find isopycnal diffusivities of $1840 \pm 400m^2s^{-1}$ in the ACC region (Scotia Sea) in the meridional direction. Higher diffusivities are expected in the ACC and their estimate nearly matches our high oxygen consumption bound of $K_x$. There is a tendency for the magnitude of $K_y$ to be smaller than $K_x$ as found by Herbei et al. (2008) in the South Atlantic Ocean.

4.3 Upper layer circulation: Comparison with float subsurface displacement data

In order to test the velocity field obtained by the inversion, we compare it with the velocities measured by the floats. First, velocities are computed from the Argo and WOCE (Davis, 2005) floats subsurface displacements, 27,308 point values are obtained in the area (Fig. 5.14). The nominal depth of the majority of the floats is close to 1000db. Each measured velocity vector is adjusted to the depth of the model’s upper layer ($\gamma_n = 27.4$) using the geostrophic velocity profiles computed with the SAC climatology. Despite the large number of float data values, there remain large areas of poor to no sampling. Float velocities are therefore averaged in boxes large enough so values are significant. The boxes are elongated in the zonal direction ($4^\circ$ by $20^\circ$) in order to capture the three main large scale features of the area’s circulation shown on Fig. 5.15. North of 30°S is the westward flowing branch of the subtropical gyre. Near 40°S, the flow in the southern branch is oriented to the north and east. Between the two zonal branches of the gyre, a lower spatial sampling (about 300 measured velocities per box versus 760 in average) renders the values difficult to interpret. South of 50°S, the ACC is apparent, centered at 54°S near 130°W, shifting to 60°S across 110°– 80°W.

A comparison of the box-average float velocity field with the model’s upper layer velocities averaged in the same boxes is shown in Fig. 5.16 (zonal components only). The amplitude of both fields is quite similar and the model reproduces the observed subtropical gyre structure with maximum velocities of $5 \times 10^{-3}ms^{-1}$. A few differences are apparent however. At 14°S, the model exhibits a eastward flow ($5 \times 10^{-3}ms^{-1}$) on average over all longitudes while floats imply westward flow. These velocities are at the boundaries of the model grid
and are not constrained by the tracer field in three dimensions. Thus, we do not trust the model solution at this latitude. This remark does not apply to the eastern boundary where a no-through-flow condition is enforced.

At 46°S, 130°W, the floats show a narrow westward flow which may be the westward signature of a wind-driven anticyclonic gyre recirculating from the Polar Front (located at 55°S), centered along 47°S (Iudicone et al., 2007). Such recirculation does not exist in the model solution (Fig. 5.16-a). The zonal velocity component at 46°S reaches a minimum (1 \times 10^{-3} ms^{-1}) at this latitude but does not attain significant negative (westward) values. This may be a sign that the model did not capture the barotropic part of the flow. We allowed for large vertical flow scales by introducing the full column MCMC velocity move but did not succeed reaching agreement in the particular situation shown.

At 26°S, 130°W, the floats show a narrow eastward flow (7 \times 10^{-3} ms^{-1}). This area is not sampled by the floats as well as the surrounding (from Table 5.1, 523 float displacements are used, as opposed to about a thousand at lower latitudes), the bin average velocity nevertheless rises above the 95% significance level. From figure (5.15), this velocity vector is ESE oriented, parallel to the westward extension of the Sala y Gomez Ridge. The model solution (posterior mean velocity) deviates significantly from the floats and shows opposite flow direction at the depth of the comparison. However, the model solution does show a corresponding eastward flow at levels below the $\gamma = 27.4$ surface used for the comparison (see Fig. (5.20) and (5.21) at 26°S, 130°W. Fig. (5.16) also shows the zonal component of velocity on layer 27.7 for comparison). At the deeper levels, the ridge exerts a stronger control on the flow which is opposite to the upper wind-driven circulation. Since the flow reversal occurs between 1000m and 1500m (the sampling depth of the floats is 1000m), we advance the hypothesis that the float velocities show control by the ridge extending higher into the water column.

East of EPR (Fig. 5.16-c) the float-derived velocities show northward flow south of 32°S and westward flow north of 32°S (consistent with the subtropical wind driven gyre centered on the latitude of maximum wind stress curl). The circulation map of Reid (1997) at 1000db shows poleward flow south of 35°S, between 90°W and the South American coast. North of 32°S, Reid’s map shows a southeastward flow, opposite to the model’s tracer based result. We will see in the following section that the posterior mean model circulation agrees with the float estimate.
In the two previous paragraphs, we have presented the differences arising from the comparison of float bin average velocities, adjusted to the neutral surface $\gamma = 27.4$, and the model solution along the same surface. We do not attribute these discrepancies to spurious data in the floats but rather to the fact that layer 27.4 is situated at the transition between a wind driven gyre regime and deep flow. Hence, by a relatively small vertical shift one can interpret the SGR flow or Reid’s result as pertaining to the deeper flow only, where the agreement is better.

4.4 Isopycnal circulation: upper, middle and lower layers

The velocity field for 3 selected layers is shown on figures (5.20), (5.21) and (5.22) for the respective $\gamma$ values 27.98, 27.8 and 27.5; and respective nominal depths of 2500db, 2000db and 1000db. Along the neutral surface $\gamma = 27.4$ (boundary between AAIW and PDW), the circulation is dominated by the subtropical gyre. A comparison with subsurface float measurements (previous section) shows that the model circulation is realistic in the upper levels. Looking deeper, the signature of the gyre progressively disappears on the eastern side of the EPR. At $\gamma = 27.8$ (Fig. 5.21), a southward flow appears over the eastern flank of the EPR (110°W) whereas a broad westward flow is still seen between 40°S and 20°S. Looking at the second deepest model layer, $\gamma = 27.98$ (Fig. 5.20), the flow is now dominated by zonal flows. At 35°S and west of the EPR, zonal velocities reach 0.28 cm/s at 125°W and up to 0.42 cm/s over the EPR crest. After passing over the EPR, the flow bifurcates into a branch over the southern flank of the Sala y Gomez Ridge (30°S) with similar magnitudes. Comparatively weak poleward flow is seen along the South American coast on the neutral layers 27.8 to 28. Very weak northward flow is seen at 50°S at all levels on the grid points closer to the coast and is opposite to the dominant southward flow at the surface $\gamma = 28$. This small scale recirculation results from the maintaining of the low oxygen concentrations along the coast through a balance between consumption and local advection (see below).

A comparison with the map of distribution of $\delta^3$He along the layer coinciding with the $\delta^3$He vertical maximum (eastward oriented plume, 25°S), $\gamma = 27.98$, is shown on figure 5.23. However the distribution is globally in a good agreement with the velocities. The eastward flow does nearly coincide with the eastward plume but is shifted to the south. The origin of the plume is not well documented and one could argue that helium emanates from the
latitude of the Sala y Gomez ridge over the EPR, explaining this way that the $\delta^3$He plume could be a signature of the SG ridge rather than a zonal flow. The SGR is a passive ridge but is thought to result from an ancient volcanically active fracture zone (so called leaky fracture zone, Faure, 2001) and is active at the junction with the EPR (Easter microplate).

Flow along the SGR slope is supported by the observations of Tsuchiya and Talley (1998) along a hydrographic section along 88°W. They note that a slight oxygen maximum (and silica minimum) at 25-26°S (the latitude of the SGR is 25°S) and at depths of 1200-1800m might be indicative of eastward flow (over the southern flank of the ridge) at mid depth (see Fig. 5.6). Our circulation suggests that the primary eastward flow is centered some 300 km south of the oxygen maximum and is aligned with the meridional oxygen gradient extremum rather than the local tracer extrema. On the northern flank of the SGR, Tsuchiya and Talley (1998) observed a high oxygen core (and low phosphate and nitrate) in the near bottom waters (2800 m, at 88°W) indicative of Chile Basin origin through gaps in the SGR. The deepest layers of the model have a flow across the ridge and show westward flow north of the SGR at depth of 2700 m ($\gamma = 27.98$, Fig. 5.20).

Figure (5.23) also exhibits the tendency of the flow to be parallel to tracer contours in areas of strong gradients rather than property tongues that is, a tendency to conserve tracer (e.g. Hogg, 1987; Zhang and Hogg, 1992). Sharp meridional gradients are associated with stronger advection as is the case in the Southern Ocean. The degree of tracer conservation compared to its diffusion and consumption can be quantified by the Peclet number. Maps of Peclet number were computed from the angle between the velocity vector and the lateral gradient of oxygen concentration (O’Dwyer et al., 2000; Rhines and Schopp, 1982) rather than using $\frac{u \nabla C}{\kappa \nabla^2 C}$ or the ratio of scaling numbers $\frac{UL}{\kappa}$. With this approach, the deviation of the velocity vector from the oxygen isopleths is seen to be due to oxygen consumption and mixing (both isopycnal and diapycnal). Large values of Pe are found in the Southern Ocean (Fig. 5.24) around the polar front. In the Pacific Ocean, locally large values are found along 30°S in the Chile Basin. East of 85°W, in a region of weak velocities, Pe is small and indicates a greater effect of mixing and oxygen consumption.

### 4.5 Deep flow in the East Pacific Rise region

Hautala and Riser (1993) used a non-conservative $\beta$-spiral inverse model applied to the Helios data east of the EPR equatorward of 25°S in an area encompassing the westward
δ³He plume (centered along 11°S). They found deep northward flow over the western flank of the EPR, westward flow north of 15°S and possible eastward flow south of 15°S. They find that such circulation is consistent with an anticyclonic gyre forced by a hydrothermal source on the EPR. It also agrees with the β-plume theory of Stommel (1982) (although, they discuss why a cyclonic gyre underlying the anticyclonic one is not observed). A layered model forced by strong and localized vertical velocities over the ridge in a background Stommel-Arons circulation also points to the observed gyre and eastward flow prolonging the southern branch of the β-plume. Here, localised forcing would be represented by significant diapycnal velocities over the EPR indicating strong hydrothermal activity.

Herbei et al. (2008) found a weak correlation of diapycnal fluxes with ridges in the South Atlantic Ocean. Our study does not lead to significant positive diapycnal velocities over ridges, and we find no significant correlation of diapycnal velocities with ridges. This finding is possibly due to the lack of data over the EPR area. Despite the absence of a diapycnal forcing, the westward plume is visible in the posterior mean velocity field along 12°S and in the vicinity of the ridge. The plume agrees with velocity estimates from Rafos floats (Hautala and Riser, 1993, Fig. 14) launched at 2500dbars showing northward velocities over the western flank at 12°S. However, the sampling period of the floats was limited and more measurements are needed to confirm the existence of a poleward flow over the western flank of the ridge.

The circulation obtained by the inverse model arises from the isopycnal and crosspycnal advection-diffusion of tracers including helium. The model does not specified explicit point source terms accounting for the input of Helium (the exact locations of hydrothermal inputs are not precisely known). The Helium input is controled by the boundary condition at the lowest surface (γ = 28) and the distribution is updated throughout the MCMC calculation. This layer corresponds to the depth of the vertical He maximum. The fact that the circulation is consistent with a β-plume type of circulation does not relate to a β-plume diapycnal forcing. The overall vertical transport of He is due to the distribution of diapycnal velocities (see appendix D).
4.6 The deep eastern boundary flow

The existence of a poleward flow along the coast of South America was noted in a previous section. It is apparent from high concentration of nutrient, low oxygen and low salinity water in a band 100 Km wide along the eastern boundary. For the purpose of comparison with previous results, we present the model solution along 32°S. This latitude corresponds to the basin wide P6 hydrographic WOCE section. The model does show a generally poleward flow east of 90°W in the depth range of PDW (Fig. 5.20). Along 32°S, it is consistent with several previous results (Tsimpilis et al., 1998; Sloyan and Rintoul, 2001; Wijffels et al., 2001; Shaffer et al., 2004). This flow is more intense near 85°W where it reaches magnitudes of 0.4 cm/s at layer $\gamma = 27.8$ (Fig. 5.21) and 0.1 cm/s at layer $\gamma = 27.98$ (Fig. 5.20). The poleward flow is detached from the coast and coincides with zonal gradient of tracers. The oxygen minimum found east of 90°W in the depth of PDW corresponds to an area of weak poleward flow. Figure (5.19) shows the transport across a section taken at 32°S. We find a flow of 2.2 Sv poleward between the EPR (110°W) and the coast of Chile. This figure falls within the estimate of Shaffer et al. (2004) for this depth range.

Currentmeter measurements at 30°S off the coast of Chile (150 km from the coast) show a poleward flow of $0.6 \pm 0.3 \text{ cm.s}^{-1}$ averaged over a 5 year period from observations obtained between 1993 and 2001 (Shaffer et al., 2004). The model solution shows mean poleward flow of about $0.1 \text{ cm/s}$ at 30°S for the same longitude as the currentmeter. This attenuated value compared to the direct measurements can be attributed to resolution. First, the tracer data is sparse in time. The poleward flow measured by the currentmeter has an important interannual variability (Shaffer et al., 2004) and even reversed equatorward for a period of 7 months in 1994. Second, the measured flow is unresolved by the model grid. A boundary current apparently confined within 200-400 km of the coast where the eastward shoaling of isopycnals (Fig. 5.4) seen on the hydrography is consistent with a southward geostrophic flow referenced to a deep level of no motion but has a scale just below the model spatial resolution.

Wijffels et al. (2001) analysis of the 1992 WOCE hydrographic section along 32°S finds a broad poleward flow of PDW between the EPR an the continent with a strong core centered at a depth 3000 m within roughly 300 km of the continental slope. This core accounts for most of their estimated 10 Sv.of poleward transport. The two cited analysis point to a
stronger flow along the coast and stronger bulk southward flow between the EPR and the South American continental slope. The present study gives an estimate of 2.2Sv.

The eastward flowing Antarctic circumpolar current (ACC) is reproduced by the model in all layers and matches well the estimates calculated with the floats. The ACC is deflected northward when passing over a fracture zone between 140°W and 125°W. It is also deflected at 115°W when crossing over the EPR as would be predicted by a simple statement of potential vorticity conservation of a barotropic flow over a ridge.

4.7 PV balance regimes

We want to exploit the model solution for the advection-diffusion of PV to gain insight to the abyssal PV balance. In the deep ocean, circulation away from direct forcing on PV, the flow mainly follows contours of PV. Here we will discuss the PV distribution and flow and show in which areas PV is more conservative and in which areas it is not.

In the upper layers of the model, PV contours are mostly zonal for latitudes lower than 30°S and are grounded on the eastern boundary (Fig. 4.2). South of 30°S and north of the polar front, the contours tend to close on themselves and the meridional gradient is weaker. In the deepest layers of the model (Fig. 4.1), PV contours are north/south oriented east of 100°W which suggests advection of PV by the poleward flow.

When the contours are zonal, and where variations of $h$ are small, variations of PV are dominated by the planetary vorticity $f$ and a meridional flow is expected to be in the Stommel and Arons (1960) equilibrium. It corresponds to the balance where the rate of change of the planetary vorticity is balanced by diapycnal stretching. In the following, we want to examine each term of the PV equation and find where diapycnal stretching or isopycnal stretching balances the planetary vorticity. The PV equation can be rewritten as

$$\frac{\beta \nu_f}{f} + u \frac{\nabla h}{h} = \frac{w^*_T - w^*_B}{h} + \frac{h}{f} \kappa \nabla^2 \left( \frac{f}{h} \right)$$

(4.1)

The first term represents the meridional advection of planetary vorticity. The second term (recall $h$ is layer thickness) is vortex stretching due to the isopycnal component of the velocity (conservative stretching). The third term represents the diapycnal (nonconservative) stretching due to diapycnal diffusion and the fourth term is the isopycnal diffusion of PV.

By looking at the maps of Peclet number on three neutral layers ($\gamma = 27.98, 27.8, 27.5$, respectively Fig. 5.24, 5.25, 5.26), we can obtain useful information on the PV balance. Areas
of large Peclet number (dominated by PV advection) indicate that advection of planetary vorticity is balanced by conservative stretching. This is the case in the southwest corner of the study area where latitudinal excursions of the ACC (when crossing the EPR) provide a high magnitude of the $v$ component. Also in the southeast basin (Fig. 5.24), the southeastward flow also follows to the same balance.

There is significant diffusion of PV in the deep layer north of 30°S where Pe is smaller than one (Fig. 5.24). On the upper layers (Fig. 5.26) the Peclet number has a minimum along the latitude 35°S - 45°S and over the Chile Basin. In this band, the flow is essentially meridional and PV contours are zonal (weakly) as shown on fig. 4.2. Hence, in this area the PV balance is nonconservative, diapycnal and isopycnal diffusion are important. The case where the isopycnal stretching dominates over isopycnal diffusion would indicate that a Stommel-Arons equilibrium is possible. Fig. 5.27 shows the dependence of the ratio of the latter terms as a function of Pe. It appears that the PV balance of eq. 4.1 is largely dominated by the conservation of PV (large value of Pe). When PV is diffused, it is explained by mixing along isopycnals (when small Pe and small dia/iso ratio occur at the same location). The fact that isopycnal diffusion dominates the diffusion of PV suggests that the SA balance is not the main PV regime in the southeast Pacific Ocean.

HR93 used a layered $\beta$-spiral method, where diapycnal and isopycnal stretching is estimated in each discrete layer. The layers are approximately 100 m thick which is comparable to our model. They found that at the depth of the helium maximum, isopycnal stretching has a large effect south of 15°S. North of 15°S, PV is essentially in a Stommel Arons balance. They hypothesize that south of 15°S the topography may have a greater effect on the PV balance (as the $\beta$ effect diminishes southward). In the present study, there is a tendency for low Peclet number north of about 20°S (Fig. 5.24) indicative of a non-conservative PV balance where isopycnal diffusion dominates diapycnal diffusion. South of about 20°S, Peclet number values are larger and the PV balance is conservative. This finding is consistent with the result of HR93. However we find no clear evidence of conservative stretching over ridges (although unsmoothed maps of Pe suggests some localized correlation). Rather, conservation of PV seems to occur when the flow is zonal in the following locations: eastern Chile Basin in the deep layers, within the zonal branching of the subtropical gyre, in the southeastern basin where the deep poleward flow joins the southern ocean (north of the SAF) and in the ACC where it flows over the EPR.
Figure 4.1: Contours of potential vorticity on $\gamma = 28$. 
Figure 4.2: Contours of potential vorticity on $\gamma = 27.4$. 
CHAPTER 5

DISCUSSION

The Well et al. (2003) study of the mixing of water masses at Drake Passage identified the water properties of the eastern boundary flow as low oxygen, high nutrient, high helium and found a contribution of 50% of this water mass in the depth range of the helium maximum ($\gamma = 27.98$), north of the Polar Front. They hypothesize that the water characteristics of the flow are strongly influenced by an eastward flow at 30°S responsible for the high concentration of mantle helium along the South American continent (hence their definition of South Pacific Deep Slope Water). This view of an eastward zonal flow at 30°S is consistent with Reid’s (1997) analysis (e.g. Reid, 1997, Fig. 5-h) which also suggests weak eastward flow over the EPR south of 30°S (velocities are dominantly meridional). The present study shows that zonal flow over the EPR is present at all latitudes greater than 30°S (Fig. 5.20), advecting helium-poor water eastward over the EPR to maintain the strong meridional helium concentration gradient seen on section P18 (103°W) at 40°S (Fig. 5.10 and 5.23). At the southeast corner, as the flow turns southeastern and southward around 85°W, waters of western origin continue to mix with waters of northern origin, eroding the isopycnal property extrema (minimum in oxygen and salinity, maximum in nutrient) seen along the coast. Helium concentration stays relatively high as it may be replenished by the active Chile Rise at 45°S, east of the EPR (Well et al., 2003). Only few Helium measurements have been made over the Chile Rise and it is not possible to be definite about a possible source there. The southeastward flow is dynamically explained by an equilibrium between the southward advection of planetary vorticity and conservative vortex stretching (section 4.7).

The inversion conducted here uses six ocean tracers that are S, $\theta$, oxygen, helium, PV and silica. From a trace plot of each term of the costfunction (Eq. 3.12) (MCMC trace plot, Herbei et al., 2008) associated with each tracer, it is possible to see how the
posterior agrees with the data. Fig. 5.28 shows that oxygen and potential vorticity have the greatest agreement. $\delta^3$He comes in third position and shows better agreement than the remaining tracers (temperature, salinity and silica). $\delta^3$He was significant contributor to the flow calculation and the fact that $\delta^3$He is one of the more constraining tracers of the model is an interesting finding that shows its interest for the study of ocean circulation.

5.1 Summary

We have used an inverse method to compute the deep circulation in the eastern South Pacific Ocean. The method gives probability density functions of the model unknowns on a grid fitted to the eastern boundary of the South Pacific Ocean. The model unknowns are isopycnal and diapycnal velocities and diffusivities. We found that the oxygen consumption rate parameter (assumed spatially constant) is an important variable controlling the effectiveness of the inverse method (i.e. the minimum value of the model cost function) and the magnitude of the diffusivities. The isopycnal circulation is in comparison less dependent on the consumption rate. The model circulation agrees broadly with the float trajectory data at the 1000db level when they are spatially averaged. When it does not agree we showed that it is because the floats sample a deeper circulation than that within the layer. The isopycnal circulation below 1000db is dominated by zonal flow controlled in part by the topography (flow eastward over the Sala y Gomes Ridge). The westward plume at 15°S seen on $\delta^3$He maps is reproduced by the model but is less intense than expected by the $\beta$-plume theory of Hautala (1993) and the model did not find correlations between the diapycnal velocity (forcing mechanism of the $\beta$-plume) and the known active part of the EPR. The eastern boundary flow along the coast of South America appears as a main exit route for PDW but flow is not restricted to the vicinity of the coast as suggested in previous studies. We showed also that the deep southeastward flow out of the southeastern Pacific (the deep outflow of the Eastern South Pacific) is not controlled by a Stommel-Arons PV balance but instead by isopycnal PV stretching.

The results of this study could be completed with a study of oxygen consumption. Here, it is parameterized by a spatially constant decay time scale. The parameter could be depth dependent, the bulk oxygen utilization rate could be a constant term or set as an unknown parameter. The role of a source term along the coastal boundaries might also be investigated.
Eventually, a more complex advection-diffusion solver on a grid fitted to the ocean bottom would allow the circulation study to be extended to the bottom flow.
Table 5.1: Number of float velocity measurements data per box.

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<tbody>
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<td>1095</td>
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<tr>
<td>368</td>
<td>365</td>
<td>832</td>
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Table 5.2: Posterior mean diffusivities for various oxygen consumption rates. The diffusivities are spatially constant. Those were produced with runs where the diapycnal diffusivity is held constant. The Peclet number Pe is averaged over the model domain.

<table>
<thead>
<tr>
<th></th>
<th>$\lambda$ (s$^{-1}$)</th>
<th>$\kappa_x$ (error) (m$^2$s$^{-1}$)</th>
<th>$\kappa_y$ (error) (m$^2$s$^{-1}$)</th>
<th>Pe –</th>
</tr>
</thead>
<tbody>
<tr>
<td>10$^{-11}$</td>
<td>840 (50)</td>
<td>0.2 (0.2)</td>
<td>(1-10$^4$)</td>
<td></td>
</tr>
<tr>
<td>3 $\times$ 10$^{-11}$</td>
<td>1600(100)</td>
<td>1300(100)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5 $\times$ 10$^{-11}$</td>
<td>4500 (270)</td>
<td>885 (55)</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>10$^{-10}$</td>
<td></td>
<td></td>
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</tbody>
</table>
Figure 5.1: Bathymetry of the south eastern Pacific Ocean and its main features.
Figure 5.2: Position of all hydrographic profiles along the neutral surface $\gamma = 27.4$: WOCE and pre-WOCE (dots) ARGO profiling floats (diamonds).
Figure 5.3: Same as figure 5.2 for $\gamma = 28$. 
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Figure 5.12: Oxygen Utilization Rate (OUR) as a function of depth. The thick blue curve is the preferred values. The two thin blue curves define a confidence interval where $\lambda$ is optimal (corresponds to a minimum of the model costfunction). The vertical red and black lines show the estimates of Feely et al. (2004) and Craig (1971) respectively (no confidence intervals provided). These studies assume a constant value of OUR in the depth range 1000-4000 dbar.
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Figure 5.15: Box averaged velocities from subsurface floats displacements (data from Argo and WOCE experiments). The boxes dimensions are 4° in latitude and are delimited by three meridians (140°W, 120°W, 100°W) and the coast line. A different scale is used for velocities larger than 1cm/s.
Figure 5.16: Zonal components of box average velocities from the inverse model’s upper layer ($\gamma_n = 27.4$, blue line) and from the float data adjusted to the same layer depth as a function of latitude for three longitudes: (a) 130°W, (b) 110°W and (c) 80°W. The region of the ACC is excluded. Positive values are eastward. The grey shadings represent uncertainties.
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Figure 5.19: Upper panel: meridional transport across 32°S, the upper part of the EPR is also shown. Lower panel: cumulative meridional transport, from east to west, in the depth range of the model (between the neutral layer $\gamma = 28$ and $\gamma = 27.4$).
Figure 5.20: Posterior mean velocities on isopycnal $\gamma = 27.98$. Two scales are used (black and gray) and the arrows on the right indicate 1cm/s.
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Figure 5.22: Posterior mean velocities on isopycnal $\gamma = 27.5$. Two scales are used (black and gray) and the arrows on the right indicate 1 cm/s.
Figure 5.23: Map of $\delta^3$He (posterior mean) overlaid with the posterior mean velocity field along the layer $\gamma = 27.98$. 
Figure 5.24: Base-10 logarithm of Peclet number computed from the zonal and meridional components of the posterior mean velocities and diffusivities for the neutral layer $\gamma = 27.98$. The 2700db isobath is shown in 3.2 to locate the EPR and Sala y Gomez ridge. The Subantarctic Front (dash line), Polar Front (continous line) and Southern ACC Front (dash) are also shown.
Figure 5.25: Same as figure 5.24 for the neutral layer $\gamma = 27.8$. 
Figure 5.26: Same as figure 5.24 for the neutral layer $\gamma = 27.5$. 

Peclet number, $\gamma = 27.5$
Figure 5.27: Dependence of the ratio: diapycnal stretching over isopycnal diffusion of PV (y-axis) against the 'PV' Peclet number (x-axis) shown on Fig. 5.24 representing the balance of eq. 4.1. Area where Pe ≤ 1 and ratio ≥ 1 denotes possible Stommel-Arons regime.
Figure 5.28: MCMC trace plot of the cost function defined in Eq. 3.2 and 3.12. All terms associated with each tracer are shown. Each step corresponds to 100 iterations as described in the text. Only the last 20,000 steps of the calculation are plotted. These last steps were used to compute the posterior mean of the model parameters.
APPENDIX A
MODEL GRID

The model equations are solved on a curvilinear grid fitted to the eastern solid boundary (using the Matlab program Seagrid). The advection-diffusion solver used in the inverse model is based on finite differences formulas that are applicable on a rectangular cartesian coordinates. The equations to be solved are thus transformed from the curvilinear-physical domain to a rectangular-computational domain. Considering the 2-D physical domain \((x, y)\) and the computational domain \((\xi, \eta)\) the gradient operator in the computational domain is (from Chung, 2002):

\[
\begin{bmatrix}
\frac{\partial}{\partial \xi} \\
\frac{\partial}{\partial \eta}
\end{bmatrix}
= J
\begin{bmatrix}
\frac{\partial}{\partial x} \\
\frac{\partial}{\partial y}
\end{bmatrix}
\]

where \(J\) is the Jacobian matrix \([x_\xi, y_\xi; x_\eta, y_\eta]\), and also:

\[
\begin{bmatrix}
\frac{\partial}{\partial x} \\
\frac{\partial}{\partial y}
\end{bmatrix}
= J^{-1}
\begin{bmatrix}
\frac{\partial}{\partial \xi} \\
\frac{\partial}{\partial \eta}
\end{bmatrix}
\]

similarly, the laplacian operator \((\partial^2/\partial x^2, \partial^2/\partial y^2)\) can be expressed in term of the above relation. The advection-diffusion equation:

\[uT_x + vT_y + wT_z - \nu_x T_{xx} - \nu_y T_{yy} - \nu_z T_{zz} - f = 0\]

becomes,

\[
(\ddot{u} - (\nu_x a_1 + \nu_y a_2))T_\xi + (\ddot{v} - (\nu_x a_3 + \nu_y a_4))T_\eta + \ddot{w}T_z - \frac{1}{|J|^2}(\nu_x y_\eta^2 + \nu_y x_\eta^2)T_{\xi\eta} - \frac{1}{|J|^2}(\nu_x y_\xi^2 + \nu_y x_\xi^2)T_{\eta\eta} - \nu_z T_z - f = 0
\]

where, \(\ddot{u} = \frac{1}{|J|}(uy_\eta - vx_\eta)\); \(\ddot{v} = \frac{1}{|J|}(vx_\xi - uy_\xi)\) and the \(a_i\) are coefficients depending on derivatives of \((x,y)\) with respect to \((\xi, \eta)\). Cross derivative term of the form \(T_{\xi\eta}\) have been
 omitted from the final equation because the curvilinear grid is [close to] orthogonal. $T_{\xi\eta}$ can be resolved by MUDPACK for more accuracy.
Recall that the model parameters (unknowns of the inverse problem) are the isopycnal velocity components \((u, v)\), the diffusivities \((κ_x, κ_y, κ_z)\) and the diapycnal velocity \(w^\star\) is a function of \(κ_z\). For each iteration of the calculation, new model parameters values are selected randomly at one randomly selected grid point and is so called 'MCMC move'. The costfunction is evaluated for each proposed parameter value. In order to improve the convergence rate, we introduce a full depth move where one new velocity component value is selected without a depth dependence. This move would be analogous to a barotropic move if the model were to be extended over the whole water column.
APPENDIX C

USING A STREAMFUNCTION TO IMPOSE GEOSTROPHY

One practical way to impose the condition of geostrophy of the isopycnal velocity field along a neutral density layer is to define a geostrophic stream function $\phi$ such that the two components of velocity are defined by:

$$fu = -\phi_y, fv = \phi_x$$  \hspace{1cm} (C.1)

where $f$ is the Coriolis parameter dependent on latitude. In the inverse modeling framework of this study, the stream function $\phi$ could be set as a model parameter (model unknown) instead of the two component of velocity ($u$ and $v$ can be replaced by $\phi$). Then, the weak geostrophic constraint introduced in Eq. 3.13 can be dropped. We experimented this idea by replacing the velocity MCMC move on $(u,v)$ by a streamfunction move. The initial value of $\phi$ is set to an arbitrary constant corresponding to a state of no-flow and the streamfunction is kept constant along the eastern boundary in order to impose a no-through-flow condition. One MCMC move consist in the following steps. First, one grid point is randomly selected. Second, the streamfunction value at this grid point is set to a new value. Third, the costfunction is updated and tested for acceptance of the MCMC move.

Each move creates a small gyre and all the small gyres eventually merge to produce a large scale circulation. The method converges quickly to a solution, however, it always gives a strong recirculation gyre north of the ACC as shown on Fig. C.1 such that the southern branching of the gyre produces a strong west-to-east zonal circulation (the ACC) fed by the northern branching. Various constraints of smoothing were experimented in order to force the model away from the tendency to form an ACC gyre. None of them led to an acceptable result. Consequently, a calculation using MCMC move on $(u,v)$ was preferred over the move.
on $\phi$.

Figure C.1: Posterior streamfunction for a run using a streamfunction MCMC move.
The isopycnal velocity field and diffusion coefficients obtained by the inverse model are discussed in the main chapters of this thesis. The diapycnal component of the velocity is presented here mainly as a check of physical consistency. One possible interesting outcome of the model would be the correlation of diapycnal processes with the topography (Herbei et al., 2008) and we don’t clearly find such spatial structure.

A typical magnitude of the diapycnal velocities is given by the Stommel-Arons type of equilibrium and amount to about $2 \times 10^{-7} ms^{-1}$. A typical diapycnal flux divergence would amount to $10^{-10} s^{-1}$. Using scaling arguments Hautala and Riser (1993) suggest that the typical vertical velocity due to a hydrothermal source at the EPR is about $5 \times 10^{-7} ms^{-1}$. The following four figures show the distribution of the diapycnal stretching and diapycnal velocities given by the model (posterior mean) for the shallowest (1500m) and the deepest layers (2500m). On the shallowest layer (Fig. D.1) the diapycnal stretching is maximum in magnitude north of 25S and in the ACC with typical values in average. In the deepest layer (Fig. D.2), the distribution tends to show meridional structures over the ridge but also in deeper area. The same remarks apply to the distribution of the diapycnal velocities (Fig. D.3 and D.4). In the deepest layer, velocity magnitudes are in good agreement with canonical values. Note that positive values are seen over the crest of the EPR with a maximum around the latitude 30S consistent with a vertical input of mantle helium.
Figure D.1: Posterior mean of the diapycnal stretching $\frac{\partial w}{\partial z}$ in $s^{-1}$ along the shallowest layer (1000m).
Figure D.2: Posterior mean of the diapycnal stretching $\frac{\partial w}{\partial z}$ in s$^{-1}$ along the deepest layer (2500m).
Figure D.3: Posterior mean of the diapycnal velocities in $m s^{-1}$ along the shallowest layer (1000m) in m.
Figure D.4: Posterior mean of the diapycnal velocities in $ms^{-1}$ along the second deepest layer (2500m).
APPENDIX E

AN ATLAS

In the following, we present the isopycnal maps (along surface of neutral density) and sections of the hydrographic tracer data. Hydrographic data from the the WOCE and Helios programs. The mapping was done with a spline in tension interpolation using the method proposed by Wessel and Bercovici (1998). It solves the spline in tension problem using Green’s functions. The Green function approach is more appropriate for the mapping of closely spaced data in the presence of large gaps. This is preferable for the mapping of the high spatial resolution WOCE section on wide un-surveyed areas. The length scale of the mapping is 200km. Maps of δ³He (E.1), oxygen concentrations (E.2) and silica concentrations (E.3) are shown below. They are projected on 5 neutral density layers: 27.8, 27.9, 27.95, 27.98, 28]. Other similar maps for other tracers were published as part of the World Ocean Circulation Experiment Hydrographic Atlas Series (Talley, 2007).
delta helium (permil) $\gamma' = 27.8$, $l=10^\circ$, $R=200$
delta helium (percnt) \( \gamma = 27.9, l = 10^\circ, R = 200 \)
delta helium $\gamma$ = 27.95, $l=10^0$, $R=200$
delta helium $\gamma = 28$, l=10°, R=200
\( \delta_{\text{helium}_3} \text{ (percent)} \gamma^n = 27.8, \ l=10^\circ, \ R=200 \)
delta_ellium (percent) $\gamma^H = 27.95$, $l=10^\circ$, $R=200$
\( \Delta \text{helium} \) (percent) \( \gamma^\text{n} = 27.98, l=10^\circ, R=200 \)}
$\delta_{\text{helium}} \text{ (percent)} \gamma_{\text{n}} = 28$, $l=10^\circ$, $R=200$
The diagram shows the distribution of oxygen (umol/kg) in the ocean with the equation $\gamma = 27.8$, $l=10^6$, $R=200$. The color bar indicates oxygen concentration ranging from 100 to 250 umol/kg.
\( \gamma^n = 27.98, l=10^\circ, R=200 \)
silicate (umol/kg) $n = 27.9$, $l=10^\circ$, $R=200$
silicate (umol/kg)\(\gamma^n = 27.95\), \(l=10^\circ\), \(R=200\)
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BIOGRAPHICAL SKETCH

Vincent Faure

Vincent Faure was born in Bordeaux, France. He graduated from the University of Bordeaux I with a Maitrise de Physique in 1999. The next year he obtained a Diplome d’Etudes Approfondies in Physical Oceanography in Brest, France. In 2005 he obtained a Master of Science degree in Physical Oceanography and his thesis was about the circulation of the Labrador Sea Water.