2006

The Fractal Nature of Lightning: An Investigation of the Fractal Relationship of the Structure of Lightning to Terrain

Brian Clay Graham-Jones
THE FLORIDA STATE UNIVERSITY

COLLEGE OF ARTS AND SCIENCES

THE FRACTAL NATURE OF LIGHTNING: AN INVESTIGATION OF THE FRACTAL RELATIONSHIP OF THE STRUCTURE OF LIGHTNING TO TERRAIN

By

BRIAN CLAY GRAHAM-JONES

A Thesis submitted to the Department of Mathematics in partial fulfillment of the requirements for the degree of Master of Science

Degree Awarded:
Summer Semester, 2006
The members of the Committee approve the thesis of Brian Clay Graham-Jones defended on April 24, 2006.

Christopher Hunter
Professor Directing Thesis

James B. Elsner
Outside Committee Member

Steve Bellenot
Committee Member

The Office of Graduate Studies has verified and approved the above named committee members.
To Jean and Dorothy and Bob
ACKNOWLEDGEMENTS

First and foremost, thank you Jean. Thank you Jean for putting up with me for the thirteen-plus years I’ve taken to slowly get a Master’s while doing all of the other things one does in Tallahassee (and now New York).

Next, thank you Dr. Hunter. You were always and will always be my favorite professor at the Florida State University. I don’t know why, but I seemed to take half of my classes from you. They were always hard for me, I never really excelled in any of them, but I enjoyed them all. I enjoyed your clean, insightful, careful, precise teaching style. Thank you for allowing me to find my own way with this thesis, so that at this point I can take pride in my path and my product.

Next, thanks to Drs. Bellenot and Elsner for agreeing to be on my committee, and reading this document and serving. Thanks in particular to Dr. Bellenot, not only for this, but for all of your help over the years with my computer questions and quandaries, and for your fine sartorial style.

Thanks to Professor McWilliams, who gave me one of the highpoints of my academic life when he suggested I might have what it takes to be a pure mathematician.

Thanks to the two student librarians who went above and beyond duty that night at Dirac when I had lugged my books in to be renewed and you guys should have just gone home, but instead you reopened the library so that a difficult day ended on a very positive note.

Thanks to my homie from Hollywood who kept me company for the last several years while I was working, keeping me focused with his wry grin and entertained with his ever stylish zoot suit.

And thanks finally to Grace Brock and Esther Diaguila, who between them helped me navigate the Mathematics program at the Florida State University and live to tell the tale.
# TABLE OF CONTENTS

List of Figures ................................................................. vii

Abstract ................................................................. ix

1. STATEMENT OF PROBLEM ................................................. 1

2. LITERATURE REVIEW .................................................. 4
   2.1 Fractals and Lightning ............................................. 5

3. PRESENTATION OF SOLUTION: GENERAL FORMULATION .......... 12
   3.1 Electrostatic Formulation ........................................ 12
   3.2 Lightning and the Laplace Equation ............................ 14
   3.3 Seven Point Approximation ..................................... 15

4. PRESENTATION OF SOLUTION: CHOICE OF CALCULATION AND MODEL PARAMETERS ............................................ 20
   4.1 Iterative Generation of Streamers ............................... 20
   4.2 Downward Stepped Leaders ..................................... 21
   4.3 E-critical and streamer initiation ................................ 25
   4.4 Selection of stepped leader candidate mesh points ........... 28
   4.5 Selection of stepped leader point ................................ 29
   4.6 Varying Terrains .................................................. 30

5. DISCUSSION OF RESULTS ................................................... 32
   5.1 Overview of cases .................................................. 32
   5.1.1 Calculation of fractal number ................................. 33
   5.1.2 General Discussion of Calculation of Lightning Fractal Number ........................................ 34
   5.2 Discussion of the Data ............................................. 40
   5.2.1 Flat Terrain Data Results .................................... 42
   5.2.2 Hilly Terrain Data Results ................................... 42
   5.2.3 Empire State Building Data Results ........................ 43
   5.2.4 Megalopolis Data Results .................................... 44
   5.3 General Comments on the Results ................................ 44

6. SUMMARY ................................................................. 46
   6.1 Summary of what was original and significant in the work .......... 46
   6.2 Suggestions for Future Work ..................................... 47
APPENDIX A: Appendix A- Lightning Data Plots ....................... 49

APPENDIX B: FORTRAN Codes .......................................... 73

APPENDIX C: Suggested New Personal Lightning Safety Guidelines .... 117

REFERENCES ................................................................. 120

BIOGRAPHICAL SKETCH .................................................. 122
### LIST OF FIGURES

5.1 Bsln12: D=1.46, Viewing Angle - azimuth=0 deg, elevation=0 deg. ............ 35  
5.2 Bsln12: D=1.25, Viewing Angle - azimuth=45 deg, elevation=0 deg. ............ 36  
5.3 Bsln12: D=1.29, Viewing Angle - azimuth=90 deg, elevation=0 deg. ............ 37  
5.4 Bsln12: D=1.3, Viewing Angle - azimuth=300 deg, elevation=0 deg. ............ 38  
5.5 Lightning Summary Table ........................................................................... 41  
A.1 Flat Terrain Lower Boundary ........................................................................ 49  
A.2 Set 1:Flat Terrain, 373 Segments (Bsln12) ..................................................... 50  
A.3 Set 2:Flat Terrain, 340 Segments (Bsln13) ..................................................... 51  
A.4 Set 3:Flat Terrain, 284 Segments (Bsln14) ..................................................... 52  
A.5 Set 4:Flat Terrain, 344 Segments (Bsln15) ..................................................... 53  
A.6 Set 5:Flat Terrain, 274 Segments (Bsln16) ..................................................... 54  
A.7 Hilly Terrain Lower Boundary ...................................................................... 55  
A.8 Detail of Hilly Terrain Lower Boundary .......................................................... 55  
A.9 Set 1:Hilly Terrain, 263 Segments (Bsln17) ..................................................... 56  
A.10 Set 2:Hilly Terrain, 368 Segments (Bsln18) .................................................... 57  
A.11 Set 3:Hilly Terrain, 425 Segments (Bsln19) .................................................... 58  
A.12 Set 4:Hilly Terrain, 336 Segments (Bsln20) .................................................... 59  
A.13 Set 5:Hilly Terrain, 360 Segments (Bsln21) .................................................... 60  
A.14 The Empire State Building Lower Boundary .................................................. 61  
A.15 Set 1:The Empire State Building, 417 Segments (Bsln22) .............................. 62  
A.16 Set 2:The Empire State Building, 283 Segments (Bsln23) .............................. 63  
A.17 Set 3:The Empire State Building, 348 Segments (Bsln24) .............................. 64  
A.18 Set 4:The Empire State Building, 526 Segments (Bsln25) .............................. 65  
A.19 Set 5:The Empire State Building, 453 Segments (Bsln26) .............................. 66  
A.20 Megalopolis Lower Boundary ....................................................................... 67  
A.21 Detail of Megalopolis Lower Boundary .......................................................... 67  
A.22 Set 1:Megalopolis, 441 Segments (Bsln27) ..................................................... 68
A.23 Set 2: Megalopolis, 345 Segments (Bsln28) ........................................ 69
A.24 Set 3: Megalopolis, 414 Segments (Bsln29) ........................................ 70
A.25 Set 4: Megalopolis, 527 Segments (Bsln30) ........................................ 71
A.26 Set 5: Megalopolis, 382 Segments (Bsln31) ........................................ 72
ABSTRACT

This study focuses on the relationship between the structure of lightning and how it may or may not be related to the topography below it.
CHAPTER 1

STATEMENT OF PROBLEM

Lightning - A transient, high-current electric discharge
whose path length is measured in kilometers.

Lightning is one of Nature’s most destructive and lethal forces, some would say the most destructive and lethal. As documented by Rich Kithill of the National Lightning Safety Institute, [Kithill 1997], from the years 1940 to 1981 lightning killed more people (7,741) than tornadoes (5,268), floods (4,481) or hurricanes (1,923). And unlike the other disasters, lightning kills one by one, or in small groups, striking individuals and families with no warning and often fatal results. In addition to loss of life, lightning also results in substantial property loss. In 1995, the annual costs associated with lightning was $1 billion for homeowners alone, not counting the costs to businesses whose infrastructure is damaged or municipalities that have to battle lightning initiated wildfires. The average annual number of lightning strikes in the U.S. is 17,600,000, resulting in an average national lightning-related insurance claims of 307,000.

Lightning may strike when one least expects it. The saying, ”A bolt from the blue.” refers to lightning’s ability to travel over ten miles laterally before striking ground. Hence one could be standing with clear blue skies above and suffer a lightning strike. On a local level, Florida is the lightning capital of the nation, with more lightning strikes per year than any other state. Thus in Florida alone, we experience on average more than 352,000 lightning strikes per year, or just under 1,000 lightning strikes per day on average within the state.

The most common form of lightning is created when a threshold value is reached in a cell of negatively charged particles in the atmosphere and a stepped leader, or streamer, is initiated. A stepped leader is so-called due to its nature of progressing in discrete steps or sections, usually of a length of tens of meters. The leader is a channel of charged particles which
is trying to reach ground or a collection of positively charged particles. As the negatively charged stepped leader approaches ground positively charged upward streamers are often created due to the approaching concentration of charge. When a downward streamer connects with an upward streamer, a lightning discharge or lightning is created.

When modeling lightning for computer analysis, various difficulties arise. At the tip of a streamer, very fine modeling is required to begin to assess the plasma physics inherent in the tip phenomena appropriately. This typically requires very dense meshes and normally focuses primarily, if not solely, on the electromagnetic and plasma aspects of the tip, while ignoring the more meteorological aspects relating to air pressure, particulate matter, temperature, etc.

While the modeling of the tip phenomena requires very dense modeling, a downward or upward streamer, or the resulting lightning bolt, may be anywhere from one to twelve kilometers in length along its main trunk, ignoring the various branches. A mathematical model that allows for modeling of the plasma aspects, along with the more meteorological facets and including the electro-magnetism, as far as I have been able to assess, has not been developed to date.

It is my belief that creating a full model of a typically 2.5-20 kilometer lightning bolt is beyond current resources. Hence I, like others before me, made choices to allow work to move forward. Some researchers have focused on modeling the tip phenomena, and simplifying the rest of the streamer. Others have focused on the climatological aspects: temperature, barometric pressure, particulate matter, etc., and simplified the tip and electromagnetics, while still others have simplified the tip and climatological aspects to concentrate on the structure of the lightning bolt itself. That has been my choice and path.

The problem that I have chosen to investigate is a possible relationship between lightning’s shape and structure and the terrain below its development. To study the problem I have created a three-dimensional computer model to simulate various terrains and to create physically meaningful models of lightning.

To-date many papers have been written on the topic, often by authors based in organizations who try to mitigate the damage and loss of life caused by this large-scale electromagnetic discharge. From an applied mathematics angle, the problem is another
area where the techniques of the discipline may aid in furthering the understanding of an important phenomenon.

Finally, through my research into the mechanisms by which lightning is generated coupled with my background in movement and dance, I have arrived at what I believe is an improvement on the current nationally approved Personal Lightning Safety recommendation. I will lay that case out in an appendix. While my suggested revision is not directly related to my thesis work, if it is indeed found to be an improvement it could potentially save lives.
CHAPTER 2

LITERATURE REVIEW

My study focuses on two large areas – fractals and lightning. The former, of course, has its beginning with the work of Benoit B. Mandelbrot, the "father of fractal geometry" who famously created the field in part from a collection of curious physical and mathematical constructs. Since the publication of his 1975 French essay, "Les Objets Fractals: Forme, Hasard et Dimension" [Mandelbrot 1975], thousands of articles and books related to fractals have been produced. The class of objects known as fractals, to use Mandelbrot’s precise definition, is "... a set for which the Hausdorff-Besicovitch dimension strictly exceeds the topological dimension". More generally, many fractal entities share the quality of having similar levels of complexity of structure over a range of scales.

Lightning has a much older pedigree. As one of the most destructive forces in nature, it has long kept humanity’s attention, and has been written about and studied for as long as humankind has had the ability. However, for the past several decades, arguably one of the lead people investigating lightning would have to be Martin A. Uman, based at the University of Florida, whose many books, including "The Lighting Discharge" [Uman 1987] serve as primary sources for researchers in the field.

As I hope to show, and as many have done in the past that I will discuss below, it turns out that fractals provide a very useful and practical method of evaluating, discussing, and categorizing lightning. Lightning, given its dependence on an ever shifting combination of topographical, meteorological and electromagnetic variables, is almost definitely, as one says of snowflakes, unique, i.e. no two are alike. So how does one go about discussing lightning in a discrete, quantitative fashion. One may, as Uman and others do, speak of the average or range for various characteristics, but to evaluate discrete bolts, especially structurally, one is quite quickly led to fractals.
It is clear that lightning exhibits the self-similarity at varying scales that is a quality of a large group of fractals. The same root, -branching, -river delta structure is immediately evident after the most cursory visual inspection, excluding of course ball lightning which falls outside the scope of this thesis. But among its other features, using fractals allows us to discuss, evaluate and assign a unique fractal dimension to either images of lightning or to models of lightning, and by so doing adding another quantifiable, numeric characteristic that may discriminate a structure of a lightning bolt from that of say, a tumbleweed.

A selection from the first paragraph of Mandelbrot’s seminal work, "The Fractal Geometry of Nature" [Mandelbrot 1977]is often quoted, as follows "Clouds are not spheres, mountains are not cones, coastlines are not circles...", but the sentence continues with "and bark is not smooth, nor does lightning travel in a straight line." He goes on to say that these patterns "challenge us to study those forms that Euclid leaves aside as being "formless", to investigate the morphology of the "amorphous"." It is to this end that I decided to use fractal theory to probe and evaluate my computer models of lightning, for it is with fractals that it is possible to assign discrete values to these formless, amorphous shapes. It is with fractals that one is able to speak more clearly of group features and attributes.

I of course, am not the first person to make this choice. As you will read below, researchers for all the years since fractals were defined have been using them to discuss lightning in a concrete way.

2.1 Fractals and Lightning

As I stated earlier, thousands of books and articles have been written on lightning and fractals individually. However, when one focuses on the combination of fractals and lightning, and furthermore the modeling of lightning considering fractal aspects, the field begins to narrow.

The first article that I consider to be in a direct line with my area of investigation was authored in 1986 by A. A. Tsonis and J. B. Elsner [Tsonis 1987]. I will discuss in some detail their approach as it is very similar to subsequent efforts, including my own. In their article they calculated the fractal number of lightning using two methods: the first by measuring the length of the lightning by varying length measures, and the second analogous method known as boxcounting. They also made two simplifying assumptions, that the thickness of
the lightning branches was zero and that the lightning exists on a plane. Applying these techniques they arrived at a fractal value for lightning of $1.34 \pm 0.05$. In their paper they note "Such a result provides for the first time a quantitative characterization of lightning."

In the rest of their paper they describe how they developed a two-dimensional computer model to simulate lightning. Their approach was modeled on Niemeyer, et al. [Niemeyer 1984] work on modeling the two-dimensional radial discharge known as Lichtenberg figures. Lichtenberg figures are created when an electrical discharge interacts with an appropriate medium, causing a dielectric breakdown. They may be found naturally as when lightning strikes a medium such as sand, or may be created in a laboratory setting. Tsonis and Elsner created their model on a two-dimensional $151 \times 251$ lattice in which the potential ($\phi$) of the top and bottom row is fixed at a value of $\phi = 0$ and $\phi = 1$ respectively. Periodic boundary conditions were assumed at the sides of the lattice and only the middle point was capable of growth.

They then used the technique of successive-over-relaxation to solve the Laplace equation $\nabla^2 \phi = 0.0$ for the potential $\phi$ over the grid. Successive-over-relaxation (SOR) is an iterative scheme where the inclusion of a weighting factor or relaxation parameter $\omega$ allows for, when chosen correctly, a faster convergence. All grid points adjacent to the current bolt were considered candidates. The possible candidates are associated with a probability $P$ defined by $P_i = \phi^2_i / \sum_{j=1}^{N} \phi^2_j$ which depends on the local field determined by the equipotential discharge pattern. They noted the exponent used for the potentials may range from $0 \rightarrow D$, with $D$ the dimension of the space being investigated. They also noted that the higher the value of the exponent, the more "linear" or less spreading the resulting model pattern. Using this weighted probability distribution, a point is then selected randomly and added to the pattern. This process is continued until the first point of the bottom row is added to the discharge pattern. The discharge pattern, i.e. the lightning bolt, was chosen as equipotential, i.e. $= 0.0$. Calculating the fractal number of their computer-generated lightning, they arrived at $D = 1.37 \pm 0.02$, a very reasonable comparison to their results with the photographs.

The next article I will discuss goes again along the vein of Niemeyer, et al. [Niemeyer 1984], focusing on spark discharges and long air gaps. The main significance of this paper by Kutsaenko, et al. [Kutsaenko 1989] is that it determines experimentally what power of field is required to have an electrical breakdown. This sort of investigation and others like it will
aid in future modeling of lightning by setting what electrical field conditions need to exist before a streamer may be initiated, which will be one of the developments in subsequent work.

In 1990 Charles Richman with the Naval Ocean Systems Center wrote a paper [Richman 1990] in which he studied dielectric breakdown within a laboratory setting and constructed a method of modeling the dielectric breakdown, and then went on to apply his methodology to the study of lightning. When he computed the fractal dimension using the boxcounting method, he arrived at values ranging from 1.05 to 1.4, with an average for ten events of 1.213. Rather than applying the modeling methodology of Tsonis and Elsner based on weighted probabilities in accordance with the electrical potential, he utilized a Diffusion Limited Aggregation (DLA) fractal growth method, more of a Brownian motion methodology, on a 200 x 200 grid for building its features. He goes on to compute the fractal dimension of the DLA arrays, which he calculates in the 1.7 range and notes that due to this discrepancy DLA-created arrays are not necessarily good models for lightning. However, he believes that as DLA may be computed more quickly than solving the Laplace equation at each step it may have a place in studying lightning. Furthermore, Richman notes that the 1.7 does correlate well with his analysis of the fractal number of Lichtenberg figures.

Richman goes on to analyze the electrical nature of lightning. He posits that lightning is not as self-similar as DLA due to smaller branches not being visible, an idea I will return to later. Richman notes that his technique would be appropriate for the creation of artificial atmospheric noise for communication testing, and if more 'lightning-like’ models are desired, more rigorous and time-consuming methodologies will need to be used. Toward the end of the article, however, Richman describes how, by restricting the growth of his pattern to one branch initially, and by setting the maximum number of subsequent branches to be allowed, and by controlling a variable he uses called the Branching Probability (a sort of control parameter which will show up elsewhere in other investigations), he is able to produce patterns whose fractal number falls within the previously noted range of between 1.17 and 1.43.

In [Femia 1993], Femia, Niemeyer, and Tucci make rigorous the studies previously conducted. They note that the general topological characteristics of branched electrical discharges have been shown to have fractal qualities and that computer models may
be created which replicate the general aspects of the discharges. They then go on to model a particular Lichtenberg figure, one created in $SF_6$, sulfur hexafluoride, a gas used extensively for insulating. They create the figures experimentally, discuss the underlying physical principles, and develop those principles in numerical simulations. They find good agreement between their experiment and simulations and conclude that the fractal scaling is a consequence of the underlying physical discharge propagation mechanism.

They note that three aspects of their study that improve the simulation’s utility are: (1) the three-dimensionality of the electric field associated with the two-dimensional discharge pattern; (2) a threshold field below which the discharge is no longer able to propagate; and (3) a finite voltage drop along the discharge channels. All three of these notes, in particular the latter two, seem very on point and will show up as fundamental aspects of future investigations.

Another point of particular interest is their discussion of the streamer tip corona, the leading point of a stepped leader, and the ”randomness” inherent in the charge distribution from the corona plasma structure. It is this randomness that shows up in previous and subsequent studies, an acknowledgement of the extreme difficulty of computing all of the parameters which come into play to determine the direction of charge propagation from a corona tip.

Another innovation introduced by the researchers is the ability they have given their simulation to build multiple branches simultaneously. One of the main motivating factors for this addition is a perceived improvement on the mimicking of the actual natural phenomena. They also note that the computing time is greatly reduced (I believe due to their obtaining potentially multiple step branches per phase of figure creation). However, by making this choice, they also need to introduce various restrictions to keep physically inappropriate results from propagating.

The resulting calculated figure matched the experimentally derived Lichtenberg figure after two final modifications were implemented. The first was to widen the analytic figures channel in proportion to the charge being carried, the second was to delete the ultimate step along each of their branches, which they note is reasonable as the last step represents streamers that do not receive a final charge pulse and therefore remain invisible. With the
adjustments made, the experimental and analytic Lichtenberg figures match qualitatively and quantitatively very well, each with fractal dimension of 1.7.

The next paper, by the team of Petrov and Petrova [Petrov 1992], pulls together many of the most advantageous aspects of the previous studies. In this paper they study intra-cloud lightning, or lightning discharges which travel between clouds, rather than between a cloud and the ground. They begin by noting the fractal nature of the lightning. They use a probability of breakdown factor $\rho$, which is related to the electric field by $\rho \sim E^n$, $\eta > 0$, where $E$ stands for the local electric field. Propagation will only be possible if some critical electric field value, $E_{\text{crit}}$, is met. At each step of the simulation, the potentials $U_{ij}$ will be computed using a center difference methodology. This process will be repeated until no point is able to exceed the required threshold for further propagation. Here you see many of the aspects from the earlier investigations: a critical electric field value to be exceeded; choosing the next direction influenced by the potential difference between the existing and potential grid points; then the fractal number of the model is evaluated.

They go on to define the various other parameters, including $E_k$, the electric field strength in the channel, $l$, the step length; $\eta$, the exponent characterizing the sensitivity of the probability $\rho$ to the field strength; $U_o$, the potential of the cloud; and $R$, the cloud radius. Related to the electric field strength in the channel, they also set up their model so that points of the grid crossed by the discharge are assumed to have a potential $U$ equal to the potential of the cloud $U_o$ minus the voltage drop along the channel. The magnitude of the voltage drop is determined by the length of the channel and $E_k$.

They go on to discuss the need for a separate electric field threshold for positive and negative streamers, setting $E_+$ at 5 kV/cm and $E_-$ at 10 kV/cm, based on experimental results. They continue with a discussion of the need for a reduction of the internal field channel strength as the leader channel steps increase. They run a series of cases between their circular charge center and a plane, which they use to represent positive and negative charge centers within a cloud system. As mentioned earlier they run their cases until all of the charge within the circular region has been allocated to the point where the critical propagation value is no longer achievable. Thus they have multiple bolts represented in their models.
Later in their investigation they add a terrain with a building located on it. The range of distances between their dipole or between their dipole and the terrain surface tends to be on the order of 150 to 1500 meters. Another significant aspect of their work is that they model not just the downward streamer but the upward streamers that are created in response to the charge carried down by the downward streamer(s).

Vecchi, Labate, and Canavero [Vecchi 1994], focus on the electromagnetic field radiated by the lightning, pointing out that the field too is fractal in nature and may be usefully evaluated from this perspective. Their approach includes another of the main threads of articles written on the topic, one using transmission line theory to model the lightning. For instance, they assume the lightning channel comprises N straight, lossless segments. They note that their radiation is calculated without taking into account the possible lumped admittances at junctions between sections. They then go on to develop their model for the electrical field, utilizing these and other assumptions to allow for a closed form solution to their stated problem of finding the transient field radiated by a pulse traveling along a fractal channel and subsequently analyze the relationship between the fractality of the path and the transient wave form. As they are focused on evaluating the field, they do not place too much emphasis on the actual creation of the model lightning bolt. The create it by starting at a point \((x, y, z)\) and then varying \(x(z)\) and \(y(z)\) using two statistically independent fractal random processes, with an ability to build the bolt to have a fractal dimension of \(1.2 \pm 0.02\).

We now return to Petrov and Petrova, who in 1994 published their second joint paper on the topic [Petrov 1995]. In this paper they take the same methodology they developed in their previous paper but focus more directly on lightning strikes to earth, in particular to structures, either with or without modeled lightning rods nearby. They then discuss the efficacy of the lightning rods protecting the structures based on their results.

In 1998, Petrova [Petrova 1998] presented a paper at the International Conference on Lightning Protection, including various cases similar to the 1994 study and discussing the protective zone afforded by lightning conductors.

Finally in 1999, Dul’zon, et al. [Dol’zon 1999] investigate the development of the stepped leader of the lightning discharge. Unlike Petrov and Petrova, they initiate their downward streamer within a band of negatively charged cells. They also discuss the related temporal aspects of their downward streamers.
This was the extent of the pertinent literature that I was aware of when I began my thesis in earnest. I have occasionally surveyed recent articles to see if any of particular importance to my effort were published. To date I have not discovered any with a new, direct bearing on my work. Over the past several years sprites, blue or red, which are essentially plasma jets launching into the upper atmosphere, have been of interest to investigators, and so there have been articles written about them with fractal qualities. The last article I can find from Petrov, et al. [Petrov 2003] extends their earlier work to more elaborate ground structures in combination with more elaborate series of lightning rods. So it would seem that my particular focus may still provide a contribution to the field.
CHAPTER 3

PRESENTATION OF SOLUTION: GENERAL FORMULATION

My approach builds upon the work completed by the investigators noted in chapter two. I of course made decisions based on my own belief of how best to model the underlying physical phenomena, coupled with my particular area of focus. In this chapter I will lay out my particular approach, including explanations and definitions as appropriate. I will begin with the argument which allows for Maxwell’s equations to be reduced to the Laplace equation for this study. I will then discuss my solution for the Laplace equation, including my particular choices for the various aspects of the model.

3.1 Electrostatic Formulation

Lightning generation or more directly stepped leader development, is dependant on a range of variables, including atmospheric pressure, particulate matter, humidity, wind, temperature, electromagnetic potentials, and as I hope to show, terrain. However, for this study, as with the majority of the articles cited in the previous chapter, the problem is generally reduced to one of electrostatic interactions. The electrostatic formulation is valid due to certain aspects of the problem which allows for this simplification from the more general electromagnetic formulation. In this section I will look at how one justifies moving from the electro-magnetic to -static. The elements of the following discussion may be found in a range of texts. My primary sources were the books of Uman, [Uman 1987] and Griffiths "Introduction to Electrodynamics", [Griffiths 1981].

Electromagnetics in free space may be summed up famously by Maxwell’s Equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$
\[ \nabla \cdot \mathbf{B} = 0 \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \]

As the other researchers have done in the previously noted literature review, I am simplifying the problem from electromagnetics to electrostatics. For this simplification to be valid, one of the most important restrictions is that one must assume that the associated lightning charges and currents are relatively slowly varying. By relatively slowly varying we mean that the significant wavelengths of the relevant electric and magnetic fields must be much larger than the overall system being studied. The wavelengths may be determined roughly by multiplying the speed of light by the characteristic time during which the sources are changing.

In my formulation I am using a grid with a maximal dimension of 7,500 meters. Hence, for this assumption to be valid, and using the approximation for the speed of light of \( c = 3 \times 10^8 \text{ m/sec} \), we must satisfy \( 3 \times 10^8 \text{ m/sec} \times \text{x sec} \gg 7.5 \times 10^3 \text{ m} \). Thus, the time during which the sources are changing must be much greater than \( 2.5 \times 10^{-5} \text{ sec} \). Hence, as the system becomes more turbulent, our assumptions become less valid. With the restriction of turbidity in place, Maxwell’s equations then become those more associated with electrostatics:

\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \]
\[ \nabla \cdot \mathbf{B} = 0 \]
\[ \nabla \times \mathbf{E} = 0 \]
\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \]

The transformation is clear as we are limiting our turbidity so that \( \partial \mathbf{B}/\partial t \) and \( \partial \mathbf{E}/\partial t \), the change of the magnetic and electric fields, respectively, are negligible.

Now, because \( \mathbf{E} \) has zero curl, it may be defined as the gradient of a scalar potential, \(-\nabla V\). Using this formulation for \( \mathbf{E} \), the above electrostatic equations related to \( \mathbf{E} \) may be rewritten as,
\[ \nabla \cdot \mathbf{E} = \nabla \cdot (-\nabla V) = -\nabla^2 V \]

so the divergence of \( \mathbf{E} \) becomes the Laplacian of \( V \). Thus the first equation, known as Gauss’s law, becomes

\[ \nabla^2 V = -\frac{\rho}{\epsilon_0} \]

However, in our formulation, we will have no contained space charges, as I will explain below, so setting \( \rho = 0 \), we arrive at Laplace’s equation,

\[ \nabla^2 V = 0 \]

As for the other two equations dealing with \( \mathbf{B} \), the magnetostatic equations, as we are assuming negligible charge movement, no significant magnetic fields are produced. Also, given our assumption that the charges are slow moving, the electric fields will be of a much larger magnitude than the magnetic fields. For the magnetic fields to be of comparable strength, the particles would need to be traveling at near the speed of light, which we are clearly not allowing. Thus, we see that in our case, we may reduce Maxwell’s equation to solving the Laplace equation, an elliptic equation.

### 3.2 Lightning and the Laplace Equation

Having arrived at the Laplace equation from Maxwell’s equations, let me now expand on the particular aspects of the formulation as it relates to lightning and to my investigation. As I noted in the first chapter, lightning is created when a threshold value is reached in a cell of typically negatively charged particles in the atmosphere and a stepped leader, or streamer, is initiated. The most common arrangement is for a region of negatively charged particles to congregate closest to the earth’s surface, with a related positively charged region above it, creating an effective dipole. Variations of course have been noted, with multiple alternating layers of negative/positive clusters, or positive charges being the closest to the earth, but the dipole described is the most common, based on field measurements, and it is this configuration that I study.
In the papers discussed in Chapter 2, the lower region of negative charge, the so-called N-region, was modeled as a flat plane, or a band of charge, or a sphere. The surface of the earth is then modeled as a flat plane some distance beneath it, with a potential set to zero.

Between these two planes, various systems are used to investigate the electrical discharge commonly known as lightning. But before lightning is initiated, stepped leaders form to make the connection between the charge centers in the atmosphere and the ground. A stepped leader is so-called due to its nature of progressing in discrete steps or sections, usually of a length of tens of meters. The leader is a channel of charged particles which is trying to reach ground or a collection of oppositely charged particles. As the negatively charged stepped leader approaches ground, positively charged upward streamers are often created due to the approaching concentration of charge. When a downward streamer connects with an upward streamer or directly to the ground, a lightning discharge or flash or stroke is created. A lightning discharge or flash usually consists of a number of strokes, or high-current pulses traveling along a path between two charge centers. Positive flashes or discharges probably originate from upper (+) areas or P-regions, and are more common in winter storms, and are more common at the end of storms. This investigation will focus solely on negative flashes.

In the next section I will describe the mathematical formulation I chose to study this phenomenon.

### 3.3 Seven Point Approximation

To solve the Laplace equation I chose to use an iterative technique known as a seven point approximation. The following description of the approach may be found in numerous texts, including "Numerical Analysis" by Burden and Faires [Burden 1993]. The following description closely follows theirs with the main difference that I have expanded it to three-dimensional from their two-dimensional discussion.

The elliptic partial-differential equation considered is the Laplace equation,

\[
\nabla^2 u(x, y, z) = \frac{\partial^2 u}{\partial x^2}(x, y, z) + \frac{\partial^2 u}{\partial y^2}(x, y, z) + \frac{\partial^2 u}{\partial z^2}(x, y, z) = 0 (3.1)
\]
for \((x, y, z) \in R\) and \(u(x, y, z) = g(x, y, z)\) for \((x, y, z) \in S\), where \(R = \{(x, y, z) | a < x < b, c < y < d, e < z < f\}\), and \(S\) denotes the boundary of \(R\). We assume that \(g\) is continuous on its domain.

The first step is to set the integers \(NX\), \(NY\), and \(NZ\) and to define step sizes \(x_h\), \(y_j\), and \(z_k\) by
\[
x_h = \frac{(b - a)}{NX}, \quad y_j = \frac{(d - c)}{NY}, \quad z_k = \frac{(f - e)}{NZ}.
\]
Partitioning the intervals \([a, b]\), \([c, d]\), and \([e, f]\) into \(NX\), \(NY\), and \(NZ\) equal parts, respectively, provides a grid within the parallelepiped \(R\) by drawing vertical and horizontal lines through the points with coordinates \((x_i, y_j, z_k)\) where
\[
\begin{align*}
x_i &= a + i \times x_h, \text{ for each } i = 0, 1, \ldots, NX, \\
y_j &= c + j \times y_j, \text{ for each } j = 0, 1, \ldots, NY, \\
z_k &= e + k \times z_k, \text{ for each } k = 0, 1, \ldots, NZ.
\end{align*}
\]
The lines \(x = x_i, y = y_j,\) and \(z = z_k\) are called the grid lines, and their intersections are the mesh points of the grid.

Let me expand at this time on the boundary value \(g\). For this class of problems, three different types of boundary conditions are commonly used, the so-called Dirichlet, Neumann, or periodic conditions. The three boundary conditions may be briefly defined as follows. For the Dirichlet condition, a value is defined for the variable of interest at the boundary. For a Neumann boundary condition, the gradient or derivative of the variable would be specified at the boundary. Finally, as the name applies, a periodic boundary condition sets a periodicity to the boundary, where for example, in the one-dimensional case on the interval \([0, 1]\), \(u(0) = u(1)\), or more generally on the real number line, \(u(x) = u(x+l)\) for any integer \(l\). Of course, this final boundary condition is not really a boundary at all, but only a formulation for the problem.

In the articles discussed in the previous chapter which used a similar geometry, the Dirichlet condition was chosen by all for the upper and lower boundaries, a selection that I continued. Hence, in my formulation, for the upper and lower boundary conditions we have
\[
\begin{align*}
u(x_i, y_j, 0) &= g(x_i, y_j, 0), \quad \text{for each } i=0, 1, \ldots, NX, \text{and } j=0, 1, \ldots, NY, \\
u(x_i, y_j, NZ) &= g(x_i, y_j, NZ), \quad \text{for each } i=0, 1, \ldots, NX, \text{and } j=0, 1, \ldots, NY.
\end{align*}
\]

In the studies noted in the previous chapter, a range of choices were made of Dirichlet, Neumann, or periodic for the sides. I did not want to use the Dirichlet condition as I
thought the setting of any predetermined charge structure along the sides, while allowed by the formulation of the problem, not the most ideal model of the phenomenon in nature. While a charge distribution may be spread laterally at a particular altitude, the development of lightning seldom includes a rectangular set charge structure along vertical planes kilometers in height and breadth. A similar concern kept me from applying the Neumann conditions. However, I felt that the periodic condition could be constructed in a manner appropriate to the problems formulation and more naturally modeling nature.

To discuss my choice of the periodic condition, let me explain more fully than before, what my understanding of a periodic condition is and how I implemented it in my study. My implementation is based on inferences from the previously noted papers and descriptions found elsewhere, in particular in John C. Strikwerda’s textbook, *Finite Difference Schemes and Partial Differential Equations* [Strikwerda 1989].

To understand the periodic condition, let us consider 1-D case, with grid points $x(i)$, $i = 0, 1, ..., NX$. By choosing a periodic structure, we are essentially saying that $x(0) = x(NX)$. Furthermore, as I describe below, the technique I have chosen to solve the Laplace equation uses those grid points adjacent to the grid point of interest. Thus, for a periodic condition, to evaluate $x(1)$ we use $x(0)$ and $x(2)$, the grid points found to the immediate right and left of the point being evaluated. And in exactly the same manner, to evaluate $x(0)$ we use $x(NX-1)$ and $x(1)$, as $x(0)$ is equivalent to $x(NX)$ and so the point to the left is $x(NX-1)$.

It is this formulation for the side boundaries that I implemented. However instead of being in a one-dimensional formulation, it was a three-dimensional one. I did make, the slightly odd in hindsight, decision to evaluate the endpoints redundantly, i.e. in my iterative code for the analogous 1-D case I calculated the value for both $x(0)$ and $x(NX)$. At the time I chose to do this to keep from favoring one "side" versus the other, to ensure symmetry. Upon reflection I probably did not need to do that, but given the iterative formulation I do not believe it adversely affected my results.

Hence, for the side boundaries we have,

$$u(0, y_j, z_k) = u(NX, y_j, z_k), \text{ for each } j=0,1,\ldots,NY, \text{and } z=0,1,\ldots,NZ,$$

$$u(x_i, 0, z_k) = u(x_i, NY, z_k), \text{ for each } i=0,1,\ldots,NX, \text{and } z=0,1,\ldots,NZ,$$
With a firm understanding of the boundaries, let me now discuss the remaining points. At each mesh point to be evaluated of the grid, \((x_i, y_j, z_k)\), for \(i = 0, 1, 2, ..., NX\), and \(j = 0, 1, 2, ..., NY\), and \(k = 1, 2, ..., NZ - 1\) we use the Taylor series, first in the variable \(x\) about \(x_i\) to generate the central difference formula

\[
\frac{\partial^2 u(x_i, y_j, z_k)}{\partial x^2} = \frac{u(x_{i+1}, y_j, z_k) - 2u(x_i, y_j, z_k) + u(x_{i-1}, y_j, z_k)}{h^2} - \frac{h^2}{12} \frac{\partial^4 u(\xi, y_j, z_k)}{\partial x^4} (3.2)
\]

where \(\xi \in (x_{i-1}, x_{i+1})\)

We apply similar formulations for the other two derivatives in \(y\) and \(z\).

Using these formulas we are able to express the Laplace equation at the point \((x_i, y_j, z_k)\) as:

\[
u(x_{i+1}, y_j, z_k) - 2u(x_i, y_j, z_k) + u(x_{i-1}, y_j, z_k) + u(x_i, y_{j+1}, z_k) - 2u(x_i, y_j, z_k) + u(x_i, y_{j-1}, z_k) = \frac{h^2}{12} \frac{\partial^4 u(\xi_i, y_j, z_k)}{\partial x^4} + \frac{j^2}{12} \frac{\partial^4 u(i, \eta_j, z_k)}{\partial y^4} + \frac{k^2}{12} \frac{\partial^4 u(i, j, \zeta_k)}{\partial z^4},
\]

(3.3)

for each \(i = 0, 1, 2, ..., NX\) and \(j = 0, 1, 2, ..., NY\) and \(k = 1, 2, ..., (NZ - 1)\).

Having developed the formulation of the problem, we will transform it into a system compatible with computer implementation. We have been using \(u(x_i, y_j, z_k)\) to represent the exact solution to the problem, we will use \(w_{i,j,k}\) to represent the centered difference approximation to the exact formulation. In difference-equation form, this results in the Central-Difference method, with local truncation error of order \(O(h^2 + j^2 + k^2)\):

\[
2 \left[ \left( \frac{h}{j} \right)^2 + \left( \frac{h}{k} \right)^2 + 1 \right] w_{i,j,k} - (w_{i+1,j,k} + w_{i-1,j,k}) - (h/j)^2 (w_{i,j+1,k} + w_{i,j-1,k}) - (h/k)^2 (w_{i,j,k+1} + w_{i,j,k-1}) = 0 \quad (3.4)
\]

for each \(i=0,1,...,NX\), and \(j=0,1,...,NY\), and \(k=1,2,...,NZ-1\), and
\[ \begin{align*}
w_{i,j,0} &= g(x_i, y_j, 0), \quad \text{for each } i=0,1,\ldots,N_X, \text{and } j=0,1,\ldots,N_Y, \\
w_{i,j,N_Z} &= g(x_i, y_j, N_Z), \quad \text{for each } i=0,1,\ldots,N_X, \text{and } j=0,1,\ldots,N_Y,
\end{align*} \]

where \( w_{i,j,k} \) approximates \( u(x_i, y_j, z_k) \).

The typical equation involves approximating \( u(x, y, z) \) at the points
\[
(x_{i-1}, y_j, z_k), (x_{i+1}, y_j, z_k), (x_i, y_j, z_k), (x_i, y_{j+1}, z_k), (x_{i-1}, y_{j+1}, z_k), (x_i, y_{j-1}, z_{k+1}), \text{ and } (x_i, y_j, z_{k-1}).
\]

As the approximation for each point requires the use of itself and the six adjacent points, we arrive at the name of the estimation, the seven point approximation.
CHAPTER 4

PRESENTATION OF SOLUTION: CHOICE OF CALCULATION AND MODEL PARAMETERS

4.1 Iterative Generation of Streamers

For my particular three-dimensional model, I chose to use a $90 \times 90 \times 30$ grid, where $\text{NX}=\text{NY}=90$, and $\text{NZ}=30$. This grid represented a 7.5 kilometer by 7.5 kilometer by 2.5 kilometer space. The upper and lower boundaries correspond to the charged cloud and the earth, respectively. As noted in Uman [Uman 1987] 2.5 kilometers is a very reasonable height for a cloud cell charge distribution, as a negatively charged lightning discharge may originate from altitudes ranging from one to eight kilometers. Also in [Bazelyan 2000] it is noted that the average altitude of lightning origination is 3 km.

At $z = 2.5$ km, a value of $-10^9$ Volts was assigned to the central 60% of the plane, with the remaining 40% set to 0 Volts. The value of $-10^9$ Volts was selected based upon the charge found in a typical cloud cell, representing a thundercloud N-region charge distribution. For the earth’s potential, by standard convention I set it to 0.0 Volts.

Let me discuss the 60/40 coverage of the upper boundary and the $90 \times 90 \times 30$ grid array. As I said earlier, the Dirichlet boundary seemed to be a very straightforward choice for the cloud charge distribution and the earth, while the choice for the side boundaries took more thought. By choosing a 60% charge coverage I am modeling a charge distribution with an edge length of 4.5 kilometers, not an unreasonable size as noted by field measurements. Moreover, in conjunction with the periodic structure, I am also saying that I have a 4.5 kilometer charge center separated by a 6 kilometer space with no initial charge. So I can imagine a series of charge centers spread over a plane. If I had chosen to have the entire upper surface carry the charge, then in conjunction with the periodic condition, I would
have been modeling an infinite charged upper plane, which while reasonable for modeling purposes I do not find as physically satisfying.

4.2 Downward Stepped Leaders

The crux of my investigation includes the generation of a downward and multiple upward stepped leaders, or streamers. As was mentioned earlier, a stepped leader initiates the first return stroke in a lightning flash. It is a stream of charged particles originating from a charge center traveling in an attempt to connect to an effective ground allowing for a discharge. A stepped leader, as the name implies, comprises a series of "steps", of connected but discrete instances of progression.

Berger [Berger 1978] categorized lightning between a cloud and the earth using its direction of motion (upward, downward) and its charge (positive, negative). Further self-explanatory classifications used include intra-cloud, cloud-to-cloud, and cloud-to-ground or ground-to-cloud. These designations are used by the majority of investigators and will be used in this investigation. The same directional categories are and will be applied to the stepped leaders/streamers. However, as I will explain that my investigation only allows for negatively charged predominantly downward moving stepped leaders and positively charged predominantly upward moving stepped leaders, both charge and direction are not needed to identify my streamers uniquely.

Some aspects of a stepped leader from Uman [Uman 1987] include that typically a step of a stepped leader is: 1 \( \mu \) sec in creation duration and from 10 to 200 meters in length; a typical pause of 50 \( \mu \) sec is noted between steps; .8 - 26 \( \times 10^5 \) m/sec speeds with the majority 1-2 \( \times 10^5 \) m/sec; that the reason for glow may be the transition from a glow to arc due to charge buildup. Average leader currents are in the 100-1000 Ampere range. Steps have pulse currents in excess of 1 kA. The charge lowered by the stepped leader is in the range of 5 Coulombs. We may note from Uman that the best expression relating peak current \( I \) to charge transfer \( Q \) is 

\[
I = 10.6Q^{0.7},
\]

with \( I \) in kA and \( Q \) in Coulombs, which implies a typical peak current of 25 kA, with typical leader charge of 3.3 C.

From [Bazelyan 2000] we have: positive downward charged leaders are continuous, rather than stepped, but on average have the same values, i.e. moving with an average speed of 3 \( \times 10^5 \) meters/sec. Also, the differences in propagation behavior between (+) and (-)
streamers are noted, such as positive streamers being dominated by ambient E-field while negative ones advance in a diffuse and self-dissipating manner. They go on to note that while positive streamers propagate in $E_{\text{crit}}$ fields $\approx 500\text{kV/m}$, such a level of $E_{\text{crit}}$ is not required, and observations of $E$ at the ground are commonly 10-15 kV/m. They further note that downward traveling negatively charged lightning, $l_{\downarrow}^-$, is over 90% of worldwide C-G lightning and that upward moving negatively charged lightning, $l_{\uparrow}^-$, are considerably longer than $l_{\downarrow}^+$. The upward moving (-) attachments have been observed to be kilometers in length. Note that many of the facts included in this paragraph are not applicable to my investigation as my study is restricted to only predominantly downward moving negative stepped leaders and predominantly upward moving positive streamers, but I wanted to give a bit of an overview to put my particular streamers in perspective.

In the papers discussed in the previous chapter, a range of choices were made for the initial downward moving stepped leader. Tsonis and Elsner [Tsonis 1987] initiated their downward streamer with an initial segment set in the middle of their upper boundary. Richman [Richman 1990] initiates his DLA pattern with a seed, whose placement he doesn’t specify but would seem to be centered in his 2-D series in both the X- and Y-directions, based on his images, as does Femia et al. [Femia 1993].

Petrov and Petrova [Petrov 1992] vary their configurations over the various papers. In their 1992 paper, they model a circle of charge with a plane near above it simulating the charged dipole cell to study intra-cloud discharges, and then another series with a second lower plane to allow for both intra-cloud and cloud to ground discharges. In subsequent papers they use variations of these geometries. In all of these configurations, they allow their streamers to initiate seemingly from any point of the surface, once the electric field has surpassed the previously noted critical magnitude.

An aspect of Petrov and Petrova’s work, which I considered emulating, is their allowance for multiple streamers to be initiated from the 2-D circle, and later 3-D sphere. I considered allowing multiple downward streamers being initiated, but decided that for my investigation, I really only needed one downward streamer, and that I lost little generality by imposing such a restriction. So I made the choice, along the lines of Tsonis and Elsner and others, to stipulate that my downward streamer would be initiated from a line segment which was centered on the upper plane and initially descended one cell.
Where I followed Petrov and Petrova’s lead, rather than Tsonis and others, was in the stipulation of the charge or potential value at the end of the initial downward streamer segment. As mentioned in Chapter 2, Tsonis and Elsner set the top and bottom of their grid equal to a potential of zero and one, respectively. Then, as their streamer/bolt developed from the top, each segment of the bolt was also set to the same potential as the upper boundary, zero, so that as the bolt developed it had an equipotential charge of zero throughout. While this served their purposes admirably, for my study another choice was made.

Petrov and Petrova, for their models, based on field measurements and an understanding of the evolution of downward streamers, chose to vary their charge distribution from one segment of their streamer to the next. This choice is based on the current which is running through the downward streamer, which they denote using the variable $E_k$, for the critical field strength in the leader channel. $E_k$ is inversely dependent on the step length and varies from 5.0 to $2.0 \times 10^7$ V/km for a step length range of 10 to 100 meters. With my step length between grid cells set at 83.3 meters, (given my height of 2.5 kilometers and 30 grid cells of height), I chose the value of $1.7 \times 10^7$ V/km for my $E_k$, which I designated as BEF for Bolt Electromagnetic Force. I chose this based both on the data and experimental results noted by Petrov and Petrova, but also on the more current data provided in Bazelyan and Raizer’s book *Lightning Physics and Lightning Protection* [Bazelyan 2000] where they note a value in Table 2.3 of $1.7 \times 10^7$ V/km for a 100 meter step.

Having made this choice of the field strength with the stepped leader, the methodology and rationale is as follows. As the leader descends from the cloud charge cell, the charge at each subsequent step is decreased based on the distance traveled and the internal field strength. As my span between my grid cells is equidistant in the x, y, and z directions, at 83.3 meters, at each step the charge found at the next associated step will drop by $1.7 \times 10^7$ V/km times 83.3 meters, or $1.416 \times 10^6$ Volts. But actually, as my cloud cell is set as a negative charge, each step will actually increase or be stepped up by this value.

Using this methodology, each grid point which becomes part of my downward stepped leader (in a method to be described shortly) will be maintained at a charge level unique to any other stepped leader grid point. And actually, one of the methods by which my program
would stop iterating was if all of the cloud charge had been used up, i.e. if the charge at the next grid point tip reached 0.0 volts.

At this point, let me refer back to an earlier statement I made that we would be assuming that there is no space charge in our formulation. That is true, in that I consider the downward stepped leader, whose first segment I just described above, to be part of the boundary, in particular the upper boundary. In this I am following the methodology of Tsonis and Elsner, Petrov and Petrova and others, and maintaining the consistency of my computational formulation. In a directly analogous manner, when I discuss the generation of the upper streamers, they will be considered part of the lower boundary.

Let me now address the 1:3 ratio I set between the height of my computational space and its width and breadth. As I have noted above, 2.5 kilometers is a very reasonable height for a stepped leader to initiate propagation. Given the expected spread of the bolt I thought it very likely that the 1:3 ratio would allow the streamer to reach ground before crossing one of the side boundaries and re-emerging on the opposite side. In this way I aided in the downward streamer propagating in a more straightforward manner. Also, as I touched on earlier, this ratio, when coupled with the 60:40 charge coverage of the upper surface, allowed for a more true to nature charge configuration, with a cloud cell separated from other cells by 6 kilometers.

With the various topics covered so far, I am now able to move on to the methodology for adding the first non-predetermined segment to my streamer(s). Within my 90 × 90 × 30 grid, with the top two-thirds of the upper boundary set to -10⁹ Volts, the bottom of the grid set to 0.0 Volts, and periodic boundary conditions along the sides, and my initial downward stepped leader extending down from the center of the upper boundary, with a charge on the NZ-1 point of -10⁹ Volts minus 1.416 ×10⁶ Volts, I can now begin to use the seven point approximation to arrive at the charge distribution at the other mesh points within the model.

With the various parameters set, the seven point approximation is initiated, with each point having a set, initial value. Besides the values set at the upper and lower limits, all of the other mesh points, excepting the just noted initiator streamer segment, is set to 0.0 Volts.

To decide when enough iterations have been performed for a particular cycle, two criteria are specified. The first is a failsafe, to ensure a problem with the code or formulation doesn’t
cause the application to run indefinitely. That failsafe is that the code will iterate no more
than the set upper limit, which in my case was 40,000. The second more useful method of
terminating the iterations was that if the absolute value difference between a mesh point’s
current and just previous charge changed less than by a predesignated tolerance, TOL, then
the iteration had converged sufficiently for my purposes, and the arrived at charge pattern
could be used for the next step.

To put it more precisely, if at iteration K, the value at \(w_{i,j,k}\) was A, and at iteration K+1,
the value at \(w_{i,j,k}\) was A ±δ, with \(∥\delta∥\leq\) Tol, then the iteration had converged sufficiently
and could stop. For my tolerance, I used the cubic of the distance between two grid points
\((2.5/30)^3\), or approximately \(5.79 \times 10^{-4}\). Once the iteration had converged, I was then ready
to select the next stepped leader segment.

4.3 E-critical and streamer initiation

At this point I need to return to the topic of downward streamers and add to the discussion
upward streamers. In the work of Petrov and Petrova and others, a value \(E_{critical}\) is defined.
This is the field strength that has been determined, often through discharge gap laboratory
experiments, as being the field strength necessary for ionic breakdown to occur, i.e. for a
stepped leader to become initiated.

In some texts, this effect is known as the breakdown voltage, i.e. the potential difference
required under certain set conditions for a discharge to be initiated. For example, the
dry-air atmospheric-pressure breakdown voltage is \(3 \times 10^6\) V/m. This is closely related to
the preliminary breakdown required for lightning initiation. Uman notes that the breakdown
voltage of a non-uniform gap is always less than the breakdown voltage of a uniform gap
with the same spacing.

In “The Electrical Nature of Storms” [MacGorman 1998], it is noted that the threshold
\(E_{be}=\pm 167 \rho_A(z)\) and that \(\rho_A(z) = 1.208 e^{-z/8.4}\) Later on in this book they use the term
onset electric field to discuss the breakdown voltage, where the electric field is in relation
to zero ground. The onset electric field, \(E_{on}\), to create lightning, is a function of pressure.
\(E_{on} \propto P_a^{1.65}\), where \(P_a\) is the pressure of dry air and \(E_{on}\) is measured at the initiation of the flash.
They go on to note that some sources point to corona initiation at values as low as 1 kV/m (no height reported) with other sources providing 6 kV/m at 18 meters (tall tree) or 4 kV/m at treeless plot of ground on a mountain ridge. They also note that lightning initiation in E \(
abla \times 300 \text{kV/m},\) that they don’t really know how but expect it to have atmospheric pressure dependence, and that a sustaining E-field is an order of magnitude lower than initiation. Finally, in the Electrical Nature of Storms [MacGorman 1998], they note the discharge threshold: 500 kV/m.

Based on experimental data, Petrov and Petrova set a value for \(E_{\text{crit}}\) of 10 kV/cm for negatively charged leaders, and half of that 5 kV/cm for positively charged leaders. They require in their models for this threshold to be surpassed before any of their streamers may be initiated or extended. It is by using this condition that they are able to allow multiple streamers to be developed.

For the downward streamer I made a different choice. This choice was based on the data and discussion contained within the literature of Uman, Bazelyan and Raizer, and others noted above. The main points were the following. First, that no such strong fields have been measured consistently in charge cells. And in fact stepped leaders have initiated in areas of the atmosphere showing orders of magnitude lower fields. Some of the ideas put forward, which I found meritorious, were that other conditions: difference in atmospheric pressure, particulate matter, humidity, even cosmic rays, could cause the initial breakdown or initial discharge pattern. And once the initial breakdown occurred, the plasma tip, the front portion of a leader with dimensions at the molecular scale, had a field strength significant enough to allow for further progression. This latter point and others are discussed at some length by Femia et al [Femia 1993] in their excellent article *Fractal characteristics of electrical discharges: experiments and simulation*. So given this evidence and arguments I chose not to require a particular field strength for my downward stepped leaders. Thus in my formulation, any mesh point adjacent to a currently included downward stepped leader mesh point is a candidate to join the streamer. To be more specific, given my initial starting mesh point at position \((\text{NX}/2, \text{NY}/2, \text{NZ}-1)\), the candidates for being added are those at \(((\text{NX}/2)-1, \text{NY}/2, \text{NZ}-1), ((\text{NX}/2)+1, \text{NY}/2, \text{NZ}-1), (\text{NX}/2, (\text{NY}/2)-1, \text{NZ}-1), (\text{NX}/2, (\text{NY}/2)+1, \text{NZ}-1), \) and \((\text{NX}/2, \text{NY}/2, \text{NZ}-2)\). I will address how the choice is made between these points momentarily.
However, let me now discuss my upward stepped leaders. The discussion is based once again primarily on Uman [Uman 1987], though similar material is available in a range of other sources. Physically, these streamers are thought to come into being in the following manner. As a say negatively charged downward stepped leader descends from a cloud cell charge center and begins to approach the earth’s surface, the charge being carried down by the streamer excites oppositely charged particles from the surface. The particles then can form upward moving streamers, which are being attracted by the downward streamer(s). And, as has been mentioned earlier, when one of these downward streamers manages to make contact with an upward streamer, the circuit is completed between the cloud cell and ground and an electrical discharge, or lightning bolt, or stroke is created.

As noted previously, Petrov and Petrova, among others, had experimentally arrived at a critical value for positive streamer initiation, namely 5 kV/cm. And while I found another rationale more compelling for the negatively charged downward stepped leader, I thought that this value for the initiation of the upward stepped leader appropriate, for various reasons. First, similar sized fields had been observed below developing cloud cells. Second, the atmospheric and other conditions, i.e. the pressure, particulate, moisture, and cosmic ray levels at the earth’s surface were more in line with the conditions during the spark gap investigations. So for a positive, upward stepped leader to be initiated, that critical value had to be exceeded.

I differed between the downward and upward stepped leaders in another important manner. As I had described earlier, a new downward stepped leader mesh point is assigned a charge based on the charge of its "parent", i.e. the charge of the mesh point that was already part of streamer minus the afore calculated charge difference. From my reading of the experimental results as found in Uman and elsewhere, and the description of the upward streamers, such an arrangement did not seem appropriate. So the decision I made for the newly added upward streamer mesh points was that I set the charge at whatever the charge had been when the mesh point transitioned from a candidate to part of an upward streamer.
4.4 Selection of stepped leader candidate mesh points

Having now laid out the criteria required for an addition to the stepped leader, let me be more specific to the actual process.

Assuming a successful series of iterations have been completed, i.e. convergence below the set tolerance, a set of approximate charge values have been determined for each mesh point within the model which is not part of the upper or lower boundary. Using those charges a set of candidate mesh point pairs are evaluated. For the model discussed above, this first candidate set would include sets of the five points surrounding the initial downward streamer segment, and all of the mesh points at Z=1, i.e. all of the grid points found in the plane parallel to the bottom surface one cell above.

For the five points surrounding the initial starter, each set comprises one of the five adjacent points and the initial point. For the mesh points just above the bottom plane, the mesh point pairs are the point at Z=1 and the point directly beneath it, with the same X and Y coordinate, but Z=0. Now for each of these sets, the potential difference is computed, by subtracting the charge from one of the points from the other, we will denote this, as Tsonis and others have, by \( \phi \), with the jth occurrence noted as \( \phi_j \). As may be inferred by my previous discussion, all of the five pairs surrounding the downward moving streamer are included, but only those potential upward moving pairs where the potential difference surpassed the \( E_{crit} \) value are included. This potential difference is then squared and used to arrive at a probability, often called a growth probability, and notated as

\[
P_i = \frac{\phi_i^2}{\sum_{j=1}^{N} \phi_j^2}
\]

What we can see to have arrived at then is a weighted probability for a segment to be added as an upper or downward streamer, with the weighting based on the potential difference between the set boundary (including the streamers) and the candidate mesh points. This general approach is found in Tsonis, Petrov, and most of the other papers referenced in Chapter 2.

The exponent 2 used on \( \phi \) is sometimes referred to, by Petrov and Petrova and others, as \( \eta \), the sensitivity of the probability \( P \) to the field strength. Tsonis and Elsner use an \( \eta \) equal to 2. Many of the other papers discuss how the value of \( \eta \) may range anywhere from
greater than zero to the Euclidean dimension of the target space to infinity. As it might be imagined the value of $\eta$ may have a marked effect on the generation of the streamers. As mentioned, Tsonis and Elsner uses a value of 2, Femia et al discuss a range of values but focus primarily on $\eta \approx 1$. Petrov and Petrova use an $\eta$ ranging from .125 to .25. The latter groups note how as the value of $\eta$ increases, the resultant discharge patterns "narrow", or become more compact. Thus, they note how a particular value of $\eta$ may be used to refine a pattern for a particular series. Petrov and Petrova discuss this in various points in their 1995 article, [Petrov 1995].

For my investigation I chose to set $\eta$ at 2. I chose this for a range of reasons. First, I did not want to "tune" my results. It is my belief that if I set my parameters correctly and develop the code appropriately, discharge patterns will develop that resemble lightning. Second, Tsonis and Elsner used 2. Third, Femia et al settled on 1 in their excellent paper and that was for basically a 2-D effect, so for a 3-D effect I thought 2 was reasonable. Fourth and finally, I consider the use of $\eta$ to be conceptually akin to the $L^2$ norm, and so again thought that the value should be set at 2. And as no arguments had been made to contradict such a decision, I chose 2.

4.5 Selection of stepped leader point

Now that a weighted probability had been arrived at for each of the candidate pairs, it was time to select one. This was accomplished in the following manner. Setting $\sum_{j=1}^{N} \phi_j^2$ equal to unity, then each $\phi_j^2$ was assigned its proportionate section between zero and one. So for instance, if I had four candidate pairs, and they each had an equivalent potential difference, the first point would be assigned the range from 0.0 to 0.25, the second from greater than .025 to 0.5, and likewise for the third and fourth. I then used a random number generator to produce a value between zero and one, and whichever pair was matched to the range that the randomly generated number fell within, that pair was the new downward or upward stepped leader segment. So for example, if the random number generator returned 0.333, then the second pair of my example would have been chosen.

Once this selection was made, the new segment was added to the appropriate streamer, all non-boundary points were re-set to 0.0 Volts, and the process was repeated until one of
three criteria were met. One possibility, already discussed was that the new segment of the
downward stepped leader had a set value of 0.0 Volts. That would have been equivalent to
the cloud cell expending all of its charge in generating the stepped leader without reaching
ground.

The second criterion was that if over 40,000 mesh points were part of the combined
upward and downward stepped leaders, or roughly 15 % of the available mesh points. That
was deemed a reasonable cutoff point for the run having a substantial error which was borne
out as the final cases tended to have 100 times fewer points in general involved.

The final completion criterion was if a downward and upward stepped leader shared
a mesh point, i.e. if a downward and upward streamer connected, or equivalently if the
downward stepped leader reached the bottom of the model, or ground. That also was noted
as a completion, and one that resulted in a lightning discharge, a lightning bolt.

4.6 Varying Terrains

So far, in all of my examples and discussions, I have stipulated a flat plane as the bottom
surface of my model. And if that was the only bottom surface my thesis would have no
conclusions, for my interest and focus is to investigate the possible relationship between the
structure of a lightning discharge and the terrain over which it manifests. I created four
different terrains to be used as my lower boundary and ran a series of five cases above each
of them. The five cases run were related in the following way. My aforementioned random
number generator uses a number to initiate it. If you use the same initiation number,
the same sequence of random numbers will be generated. This provided not only critical
reproducibility for my analysis purposes but a sort of consistency between my terrains.

In the first set of cases, the lower boundary is flat, a 2-D plane if you will. An image of
this terrain and the subsequent ones may be found in Appendix A.

For the second set of tests I created a lower terrain with a flat frame of width 10 cells, but
then within that frame I created an alternating series of hills, with a peak to value distance
of one grid cell, or 83.3 meters. So each peak has four valleys along each of its sides, or
alternatively, the peaks run along the diagonals of the grid.

The third set is once again primarily a flat plane, with one difference. Near the center
of the bottom plane, is a structure roughly the size of the Empire State building, or 5 cells
high, by 1 x 2 cells at the base, or roughly 1365’ tall and 273’ x 546’ at the base. I would have everyone know that I chose this configuration at least a year before learning we were moving to New York City.

The final set is what I called my megalopolis, or large metropolitan area. I created it using my random number generator. I stepped through each grid point of the x-y plane and assigned a value between 0 and 7 as a z-component, so that from one grid point to the next the height may vary by 560 meters, or 1,862’. Currently the world’s tallest building is the Taipei 101, which is 1,670’ tall, while the tallest tower is the Canadian National Tower at 1,815’. So while my maximal height is a slightly above currently existing structures, I can imagine a metropolis, fifteen or twenty years in the future, a megalopolis, with buildings going from very low heights to very high ones right next to each other. This is one possible image for my megalopolis.
CHAPTER 5

DISCUSSION OF RESULTS

5.1 Overview of cases

For my thesis I have generated and am focusing on twenty different cases of three-dimensional lightning that I created in the method set forth in the previous chapters. A plot of each of these bolts may be found in Appendix A. These bolts may be grouped most straightforwardly in either four or five sets, depending on your interest.

If you want to focus on the continuity of the terrain above which the lightning stepped leader was formed, then the sets would be broken down into four groups, one for each of the created terrains. Using my designators for the different cases, this grouping would be:

- Group 1 Flat Terrain: Baselines 12-16
- Group 2 Hilly Terrain: Baselines 17-21
- Group 3 The Empire State Building: Baselines 22-26
- Group 4 Megalopolis: Baselines 27-31

As one might surmise from the numbering of the cases, this ordering also preserves the sequence in which the cases were created, as I would run a set after creating each new terrain, varying the random number initiator value, ISEED, five times per terrain. This then leads to the second most natural way of grouping the cases, by the ISEED value used to initiate the run.

To review from the previous sections, the ISEED value is the first value fed into the random number generator subroutine. While all of the values are deemed for this purpose to be random in their sequence, they are also reproducible as a string of values, i.e. if a value 'a' is fed into the subroutine, 'b' will always be the returned value.
Thus, another way of grouping the cases, but not probably as theoretically interesting, is by the ISEED value used to start the case. Using this methodology we arrive at five sets, as follows:

Set 1 (ISEED = 12357): Baseline’s 12, 17, 22, 27
Set 2 (ISEED = 15357): Baseline’s 13, 18, 23, 28
Set 3 (ISEED = 25357): Baseline’s 14, 19, 24, 29
Set 4 (ISEED = 45357): Baseline’s 15, 20, 25, 30
Set 5 (ISEED = 15847): Baseline’s 16, 21, 26, 31

5.1.1 Calculation of fractal number

For each of the cases, a fractal value was arrived at, using the boxcounting method, defined as follows. For the planar case, given a series of boxes with varying edge length $\delta$, count the number of boxes required to cover the curve of interest. One may then use the equation $N(\delta) \sim 1/\delta^D$ to solve for $D$ and arrive at the fractal or box dimension of the curve.

From Tsonis and Elsner’s article we obtain the following:

For Euclidean structures, the amount of mass $M$, scales with some characteristic length $l$, as $M(l) \propto l^d$ with $\delta$ equivalent to the spatial or Euclidean dimension.

The method to evaluate a two-dimensional fractal may also be found in various sources, e.g. "Fractals" by J. Feder [Feder 1988], and may be summarized as follows:

1. Take a big square of side set to 1 which includes the object.
2. Then pave with subeddies of sides $r=1/2$ and find the number of squares they intersect.
3. Repeat with subeddies of side $2r$.

The number $N$ scales as a function of $r$ according to $N \propto r^{-D}$ where $D$ is an estimate of the fractal dimension of the object. For three-dimensional spaces, cubes of varying sizes are used rather than the boxes for the planar cases.

Using the boxcounting method, a count was arrived at for a range of scales for the side $r$ with $r=2, 4, 8, 16, 32$. For calculation purposes a volume of $2.75 \leq x \leq 5.25$, $2.75 \leq y \leq 5.25$, and $0.00 \leq z \leq 2.5$ was divided into cubes of edges of $2.5/r$ and then the number of cubes with a part of the final lightning bolt was tallied for a value of $N_r$. Using then $N_r$ and $r$, a fractal number could be assigned for each scale $r$. Using this method, a mean value with a standard deviation was arrived at using the formula, $D_{fractal} = ln(N_r)/ln(r)$. 

33
A variation of this was also calculated for the Megalopolis case. Due to structure of this terrain, the working surface was raised from $z = 0$km to roughly $z = 0.500$km. It was therefore thought prudent to calculate the fractal number for the affected lightning using a cube with dimensions $3.00 \leq x \leq 5.00$, $3.00 \leq y \leq 5.00$, and $0.50 \leq z \leq 2.5$. Both the original and modified spaces always included all of the created lightning bolts.

At this time let me discuss more generally the calculation of the three-dimensional fractal number and its relation to other calculations of related fractal numbers.

5.1.2 General Discussion of Calculation of Lightning Fractal Number

As was related in chapter 2, the first calculation of the fractal number of lightning was first calculated by Tsonis and Elsner, primarily using the boxcounting method and photographs of lightning. They arrived at a value of $1.34 \pm .05$ which compared very well with their similarly calculated value for their model of $1.37 \pm .02$.

As they discuss in their paper, an inherent difficulty is of course that the photographs of the lightning is reducing a three-dimensional phenomenon to a two-dimensional representation. This then raises the question of whether their two-dimensional computer model is modeling lightning if it were restricted to two dimensions or a two-dimensional view of a three-dimensional phenomenon.

To expand upon the topic I created a set of images of my first case, the flat terrain case I note as Bsln12. All of my images of my lightning discharges in Appendix A are shown at an azimuthal angle of 300 degrees and an elevation of 10 degrees, but the following images are shown at an elevation of 0 degrees and an azimuthal angle of 0, 45, 90 and 300 degrees, respectively.
Lightning Discharge, showing downward and upward streamers
Set 1: Flat Terrain, 373 segments (Bsln12)
View along X-axis at 90 degree

Figure 5.1. Bsln12: D=1.46, Viewing Angle - azimuth=0 deg, elevation=0 deg.
Lightning Discharge, showing downward and upward streamers

Set 1: Flat Terrain, 373 segments (Bshln12)
View along 45 degree angle between X and Y Axis

Figure 5.2. Bshln12: D=1.25, Viewing Angle - azimuth=45 deg, elevation=0 deg.
Lightning Discharge, showing downward and upward streamers
Set 1: Flat Terrain, 373 segments (Bsln12)
View along Y-axis at 90 degree

Figure 5.3. Bsln12: D=1.29, Viewing Angle - azimuth=90 deg, elevation=0 deg.
Figure 5.4. Bshn12: D=1.3, Viewing Angle - azimuth=300 deg, elevation=0 deg.
As one may see, visually the images are, while similar, clearly different. If one uses the boxcounting method as outlined by Tsonis and Elsner to calculate the fractal numbers for each of these two-dimensional images, the values range from 1.25 to 1.46. This illustrates that by rotating the three-dimensional model and then assessing it in a collapsed two-dimensional manner, a range of fractal numbers for the same configuration may be arrived at.

Returning to the work of the previous researchers, in Richman’s paper, he calculates a value of 1.7, which he notes as being more akin to the value for Lichtenberg figures rather than lightning, once again focusing on the two-dimensional lightning representation, and two-dimensional Lichtenberg figures. He goes on to note that if he allows only one branch of his model to try to model a lightning discharge, his fractal number drops to values near 1.17 to 1.43. Femia goes on to arrive at the same value for the Lichtenberg figures as Richman, of 1.7, both experimentally and with their computer model. In their model however, they choose to remove the last stage in each of their branches development, which allows them to arrive at models which more closely resemble the Lichtenberg figures.

This raises an issue that I have spent some time reflecting on. To-date, any attempt to assign a fractal number to actual lightning has depended on photographs. Aside from the aforementioned collapsing of a three-dimensional object to a two-dimensional representation, the question arises to the relative luminosity of the main trunk of the lightning and sub-branches.

In a photograph, the quality of the image will be one of the main governing factors of how much of the structure of the bolt is actually captured. The central trunk is almost always the brightest as it is the channel conducting the most charge and thus producing the largest amount of energy in the visible spectrum. As the charge drops off for each branch and sub-branch, the luminosity will also drop off. So the question arises, is the structure that is capable of being captured in a photograph a reasonable cutoff point for the actual structure of a lightning discharge?

I would say probably not. I would put forward that while the images are the best tool we currently have to discern the structure there are almost certainly further branches, coming off in a fractal fashion, from every visible branch, branches that are part of the charge distribution but just didn’t glow enough to register in the photo. If these non-luminescent branches were included, the fractal numbers would of course increase.
Petrov and Petrova’s reported fractal values tend to be lower than other researchers. This lower value is not that surprising as their models tend to display only the main trunk and sub-branches. In their 1993 paper, they arrive at a value of 1.1 for their co-axial discharges. In 1995, they discuss the various aspects which may affect the fractal number of the models, and note that the fractal value will vary from one to two for the two-dimensional cases, and between two and three for the three-dimensional cases. Finally, in Petrova’s single author paper she reports a value of 1.03 for her two-dimensional lightning and 1.06 for her three-dimensional discharges.

But one of the main issues that they raise is one that I discuss in a previous chapter, relating to the variable $\eta$. As has been mentioned, most of the researchers have the variable $\eta$ which is the exponent used to set the sensitivity or weighting of the selection of subsequent branches to the current potential configuration.

As many of the researchers noted, in general, as the value of $\eta$ increases, the structure of the lightning narrows, and thus the associated fractal number lowers. So, by varying $\eta$ one may to some degree adjust the model’s fractal number higher or lower. So each of these factors need to be considered and remembered when discussing the fractal number of lightning.

5.2 Discussion of the Data

The following table summarizes the results of the different runs. The table includes the baseline number I assigned, the type of terrain, the number of segments comprising the final bolt, the fractal number calculated using the above system, with its associated standard deviation. After each type of terrain I also show the average values for the lightning discharges associated with that terrain, with the average for the number of points and the fractal number being the average of the five cases, and the standard deviation being the standard deviation of the five fractal numbers being evaluated. The final row shows a similar set of averages for all of the bolts number of segments, fractal number and deviations, considering however all twenty cases.
<table>
<thead>
<tr>
<th>BSN</th>
<th>Terrain</th>
<th>No. of Pts</th>
<th>Fractal No.</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>Flat</td>
<td>350</td>
<td>2.11</td>
<td>0.516</td>
</tr>
<tr>
<td>13</td>
<td>Flat</td>
<td>320</td>
<td>2.00</td>
<td>0.461</td>
</tr>
<tr>
<td>14</td>
<td>Flat</td>
<td>266</td>
<td>1.76</td>
<td>0.144</td>
</tr>
<tr>
<td>15</td>
<td>Flat</td>
<td>325</td>
<td>1.99</td>
<td>0.364</td>
</tr>
<tr>
<td>16</td>
<td>Flat</td>
<td>254</td>
<td>1.89</td>
<td>0.397</td>
</tr>
<tr>
<td></td>
<td>Flat Terrain Avg.</td>
<td>303</td>
<td>1.95</td>
<td>0.133</td>
</tr>
<tr>
<td>17</td>
<td>Hilly</td>
<td>350</td>
<td>1.94</td>
<td>0.387</td>
</tr>
<tr>
<td>18</td>
<td>Hilly</td>
<td>338</td>
<td>2.01</td>
<td>0.653</td>
</tr>
<tr>
<td>19</td>
<td>Hilly</td>
<td>382</td>
<td>2.03</td>
<td>0.342</td>
</tr>
<tr>
<td>20</td>
<td>Hilly</td>
<td>315</td>
<td>1.99</td>
<td>0.356</td>
</tr>
<tr>
<td>21</td>
<td>Hilly</td>
<td>331</td>
<td>2.13</td>
<td>0.520</td>
</tr>
<tr>
<td></td>
<td>Hilly Terrain Avg.</td>
<td>343</td>
<td>2.02</td>
<td>0.067</td>
</tr>
<tr>
<td>22</td>
<td>Empire State Building</td>
<td>312</td>
<td>2.05</td>
<td>0.572</td>
</tr>
<tr>
<td>23</td>
<td>Empire State Building</td>
<td>237</td>
<td>1.81</td>
<td>0.295</td>
</tr>
<tr>
<td>24</td>
<td>Empire State Building</td>
<td>288</td>
<td>1.90</td>
<td>0.263</td>
</tr>
<tr>
<td>25</td>
<td>Empire State Building</td>
<td>429</td>
<td>2.14</td>
<td>0.410</td>
</tr>
<tr>
<td>26</td>
<td>Empire State Building</td>
<td>380</td>
<td>1.84</td>
<td>0.116</td>
</tr>
<tr>
<td></td>
<td>Emp. St. Bldg. Avg.</td>
<td>329</td>
<td>1.95</td>
<td>0.141</td>
</tr>
<tr>
<td>27</td>
<td>Megalopolis</td>
<td>210</td>
<td>2.00</td>
<td>0.491</td>
</tr>
<tr>
<td>28</td>
<td>Megalopolis</td>
<td>211</td>
<td>1.90</td>
<td>0.300</td>
</tr>
<tr>
<td>29</td>
<td>Megalopolis</td>
<td>214</td>
<td>1.98</td>
<td>0.397</td>
</tr>
<tr>
<td>30</td>
<td>Megalopolis</td>
<td>336</td>
<td>2.10</td>
<td>0.348</td>
</tr>
<tr>
<td>31</td>
<td>Megalopolis</td>
<td>183</td>
<td>1.85</td>
<td>0.266</td>
</tr>
<tr>
<td></td>
<td>Megalopolis Average</td>
<td>231</td>
<td>1.97</td>
<td>0.094</td>
</tr>
<tr>
<td></td>
<td>Combined Terrain Averages</td>
<td>302</td>
<td>1.97</td>
<td>0.106</td>
</tr>
</tbody>
</table>

**Figure 5.5. Lightning Summary Table**
Using this calculation, we note an average fractal number of 1.97 ± 0.105. You may note that this figure is higher than many of the figures cited from previous studies, but remember that this is the value for a three-dimensional model, not a two-dimensional representation of the lightning.

The focus of this effort was to investigate the possible relationship between the structure of a lightning discharge and the terrain below which the lightning was created. We are now in a position to make some comments on this topic.

5.2.1 Flat Terrain Data Results

Looking at Table 5.5 we see that the cases run with this terrain result in an average fractal number of 1.95 ± 0.133. Comparing it to the other sets we see that it is on average the set tied for the lowest fractal number and with the second highest standard deviation. The large standard deviation highlights that the cases span the range of fractal values for the test cases, spaced fairly evenly throughout the twenty cases.

This case is the one of the flat plane, in nature analogous to anything from a flat field in Nebraska to a calm lake or sea. Qualitatively, we might draw some possible interpretations. With no particular feature to focus the charge accumulation on the ground, each case would be able to unfold as it saw fit, having a fairly even diffuse field beneath it would allow for paths to develop without being drawn by anomalous upward streamers.

Looking at the plots of the cases, Figures A.2 through A.6, we see that the upward streamers are fairly evenly distributed underneath the developed downward streamer, with few apparent double segment upward streamers. Hence the wide range of cases.

5.2.2 Hilly Terrain Data Results

Moving on to the hilly terrain, Figures A.9 to A.13, we see the other extreme from the flat plane. At an average fractal number of 2.02 ± 0.067, this terrain has the highest associated fractal number with the smallest standard deviation. Thus the bolts are consistently dense.

To remind ourselves, the terrain is periodically hilly, with the variance between the peak and value of one grid cell. An analogous example in nature might be a sea with a large periodic wave pattern induced by the wind or rolling hills in the plains.
Once again, if we wished to consider the matter qualitatively, we might arrive at the following thoughts. First, that the hilly, periodic structure would be prone to concentrate the upward streamer development and initiation on the peaks, with the charge and current developing in the valleys feeding the streamers rising from the peaks. In this way, the development of the downward streamers would be both more spread as the upward streamers would be pulling more broadly, and also denser, as the greater charge concentrations would develop finer streamer branches.

Actually, as it turns out, while working in aerospace for Northrop I oversaw a contractual research and development project Surface Waves and Gaps, which included studying a similar effect, both with computer models and laboratory experiments, and such an effect can be quite striking.

### 5.2.3 Empire State Building Data Results

As a terrain, this is probably the most singular one, but at the same time the one most like the terrains used in various other studies, particularly those of Petrov and Petrova. To review, this terrain is a flat plane with a rectangular shape in the middle of the plane with dimensions roughly those of the Empire State Building, or 5 cells high, by 1 x 2 cells at the base, or roughly 1365’ tall and 273’ x 546’ at the base, with the potential = 0.0. I would like to point out that I chose this terrain, according to my log, at least by May 2004, long before we had any indication of our subsequent move to the Empire State. So it goes.

With an average fractal value of $1.95 \pm 0.141$, this set, Figures A.15 to A.19, matches the flat terrain for the lowest fractal number but surpasses it with the largest standard deviation. One of the interesting aspects of this set is that even though the lightning starts at grid points (45, 45, 30) and my building is only offset by three cells with its top at (48, 48, 5), (48, 49, 5) and (48, 50, 5), only two of the five subsequent bolts connect either directly or with an upward streamer originating from the building. The other three cases all reach ground at points unrelated to the structure. While only two connect to the building, all five cases have streamers from the building very close to the downward streamer, the other three cases just don’t quite finish the connection.

This set also contains probably the most dramatic looking bolt, Fig. A.18 (Bsln25), which is spread out and very complex. A quantitative aspect of this is found in Table 5.5 where
we note this case has the highest number of points, 429, more than any of the other cases. Tied for the lowest group fractal value but the one with the highest standard deviation, heuristically this might be explained in the following manner.

The building standing in the middle of the plane created an anomalous situation, both disrupting and encouraging the creation of the lightning. Sometimes this manifested in a thin, dense charge distribution downward streamer, and sometimes in a broad distribution as the path that might have occurred without the building there gets pulled at by the spike of upward streamers.

5.2.4 Megalopolis Data Results

The average fractal number for this group, Figures A.22 to A.26 of 1.97 ± 0.094, puts it near the tied lower pair for a fractal value, but with a smaller standard deviation, grouped more compactly like the hilly terrain. Looking at the associated plots in Appendix A, we can see that this class of lightning might be qualitatively called the most diffuse, ethereal or wispish, having on average a broad span in the X and Y direction but not a very dense compactness of streamers within.

If we recall the structure of the terrain associated with the megalopolis, we might infer some qualitative associations. This terrain was created by stepping through the $z=0.0$ section of the cube and assigning a grid height of between zero and seven using the random number generator. Hence this surface is very disjointed, with heights changing dramatically from cell to cell. As noted before, a possible example would be a developing major metropolis, such as Taipei or Shanghai, in the not too distant future.

Given such a terrain, it is possible to imagine the upward streamers being very disparate and not of particular strength. Hence the downward streamer and associated bolt being broad and not very dense.

5.3 General Comments on the Results

I feel the results are of interest for a range of reasons. For example, the type of terrain modeled is unique - all of the other studies I am familiar with that have a similar structure have only dealt with variations of terrains 1 and 3, i.e. the flat plane or the flat plane with
rectangular structures or spikes/antennae/lightning rods. The study of surfaces such as the periodic hills and variated surfaces, the hilly terrain or the megalopolis respectively, open a new area of investigation that I believe leads to new insights.

Also, this study expands upon the general analysis of the parameters surrounding the shape and behavior of lightning charges. For example, by studying the cases we may also note that three of the cases have the downward streamer hitting ground, rather than making contact with an upward streamer. The paths of these connections offer additional insight into the parameters surrounding the lightning discharge.

I also feel that the results presented in Table 5.5 are very valuable as a comparison of the fractal values that may be assigned to the lightning. In many of the papers noted in the bibliography, a fractal number is derived for lightning, either through the use of photographs of actual lightning, for example in Tsonis and Elsner’s paper, [Tsonis 1987] or Richman’s, [Richman 1990], or through calculations based on their own models, for example in Petrova’s paper, [Petrova 1998].

In the former case, the evaluation of the fractal number is problematic in a manner that some of them speak to, in that they are attempting to assign a fractal number to a three-dimensional object using a two-dimensional data representation. This is a difficult hurdle to overcome as getting three-dimensional representation of natural or even artificially induced lightning is extremely difficult. And the evaluation of the lightning discharge models I have felt suffered from a lack of detail clearly evident in nature.

In the above-cited articles, the fractal numbers associated with a lightning discharge have tended to range from 1.03 [Petrova 1998] to between 1.13 to 1.43 in the earlier noted papers of Richman, Tsonis and Elsner. As may be noted in my Table 5.5 I arrive at fractal values from 1.76 to 2.14. I feel these range of values to be reasonable as my calculations were based on 3-D models. The adding of the third dimension would quite reasonably cause an increase in the computed fractal dimension.

I therefore believe my models and results do provide for the potential of new insights in the study of the structure of lightning and furthermore into the relationship between the structure of the lightning and the terrain beneath which it is created.
CHAPTER 6

SUMMARY

6.1 Summary of what was original and significant in the work

At this point I wish to discuss what was original and significant in my work. Though I may have touched on some of these points already, let me recap or expand on them here.

I believe that my choices of terrain constitute a new step for the area of investigation. As I had noted earlier, many of the studies investigate terrains of the sort of the flat plane or the Empire State Building, but I know of none with the surfaces akin to the Hilly Terrain or the Megalopolis. So these areas provide new fodder for analysis and development.

Next I think a unique aspect of my work is my focus on the relationship between the shape of the lightning with regards to the terrain situated directly below it. I believe, in my discussion of my results, I note some definite structural aspects of the lightning that would seemed to have been affected by the various terrains. While in other investigations they discuss whether or not a discharge will or will not strike an object, their focus is more on the discharge’s ground contact point and not on the shape of the lightning. So in this way I believe I have also expanded the discussion.

Also, due to my background, I chose to include qualitative descriptions of the lightning, which is a bit unusual. For seven and one half years I worked for Northrop in their Low Observables group, primarily for the group involved with the Radar Cross Section, or the electromagnetic interactions between waves and objects. While we developed prediction codes and ran many experiments, we also needed to develop a sense of how electromagnetic radiation would interact with complex geometries, both to guide the investigations and because some of the phenomena were beyond our abilities to model. I believe it was from this period that I discussed some of the qualitative, group aspects per terrain type.
As might have been discerned, I also have arrived at a unique formulation of the problem. As I noted in my previous chapters, my choices for the boundaries, the use of $E_{\text{crit}}$, the generation of the upward and downward streamers, all of these and other choices make this approach singular.

Finally, as mentioned briefly earlier, during my work it occurred to me that I believe I have a suggestion for a significant improvement on the current best practices of what to do if caught outside and believing a lightning strike to be imminent. However, as it is not part of my thesis focus, I will discuss that matter in Appendix C.

### 6.2 Suggested for Future Work

As for suggestions for future work, there are numerous possibilities. I will discuss just a few.

Rather than modeling the cloud as the negative part of a dipole, model the cloud charge distribution as a fractal.

In *Lightning Physics and Lightning Protection*, [Bazelyan 2000] the authors go on to discuss Stroke frequency, or how often an object is likely to be struck by lightning. They list a stroke frequency $N_l$ on the height $h$ of lumped objects (their height larger than other dimensions). $N_l \approx hl$ for extended objects (objects of length $l$). For particular terrain examples, they note that lightning intensity in Europe is $N_l < 1$ per $1km^2$ per year for the tundra, 2-5 for flat areas, and $\leq 10$ for mountainous areas such as the Caucasus. While these values are probably affected by factors not currently part of my application, e.g. the meteorological differences between tundra and flat areas, one might use my code to investigate these relationships further.

Of course one could also investigate more complicated terrains, using actual geographic data to begin to compare predicted lightning strike patterns with historical records.

As computing power allows, one could combine this model with other computer models already in existence which studies the lightning discharge by accounting for such aspects as particulate matter in the atmosphere, temperature gradients through the altitude, moisture content, wind velocities, other nearby charge centers (i.e. other clouds). Next one could begin adding in modeling of the tip plasma interactions at the molecular level to replace the
random number generator used for choosing the next segment’s direction. This will keep people busy for a while.
APPENDIX A

APPENDIX A - LIGHTNING DATA PLOTS

The following are figures showing the various terrains used in my study and the plots of the three-dimensional lightning that I created.

A.1 Plots of the 3-D lightning from a viewing angle of 80,300

A.1.1 Terrain One: Flat

![Flat Terrain Lower Boundary](image)

**Figure A.1.** Flat Terrain Lower Boundary
Lightning Discharge, showing downward and upward streamers

Figure A.2. Set 1: Flat Terrain, 373 Segments (Bsln12)
Figure A.3. Set 2: Flat Terrain, 340 Segments (Bsln13)
Lightning Discharge, showing downward and upward streamers

Set 3: Flat Terrain, 284 segments (Bsln14)

Figure A.4. Set 3: Flat Terrain, 284 Segments (Bsln14)
Lightning Discharge, showing downward and upward streamers

Set 4: Flat Terrain, 344 segments (Bsln15)

Figure A.5. Set 4: Flat Terrain, 344 Segments (Bsln15)
Lightning Discharge, showing downward and upward streamers
Set 5: Flat Terrain, 274 segments (B01n16)
A.1.2 Terrain Two: Hilly

**Figure A.7.** Hilly Terrain Lower Boundary

**Figure A.8.** Detail of Hilly Terrain Lower Boundary
Lightning Discharge, showing downward and upward streamers

Set 1: Hilly Terrain, 263 segments (Bsln17)

Figure A.9. Set 1: Hilly Terrain, 263 Segments (Bsln17)
Lightning Discharge, showing downward and upward streamers
Set 2: Hilly Terrain, 368 segments (Bsln18)
Figure A.11. Set 3: Hilly Terrain, 425 Segments (Bsln19)
Lightning Discharge, showing downward and upward streamers

Set 4: Hilly Terrain, 336 segments (Bsln20)

Figure A.12. Set 4: Hilly Terrain, 336 Segments (Bsln20)
Lightning Discharge, showing downward and upward streamers

Set 5: Hilly Terrain, 360 segments (Bsln21)
A.1.3 Terrain Three: The Empire State Building

![Empire State Building Lower Boundary](image)

**Figure A.14.** The Empire State Building Lower Boundary
Figure A.15. Set 1: The Empire State Building, 417 Segments (Bshn22)
Figure A.16. Set 2: The Empire State Building, 283 Segments (Bsln23)
Lightning Discharge, showing downward and upward streamers

Set 3: Empire State Building, 348 segments (Bsn24)

Figure A.17. Set 3: The Empire State Building, 348 Segments (Bsn24)
Figure A.18. Set 4: The Empire State Building, 526 Segments (Bsn25)
Figure A.19. Set 5: The Empire State Building, 453 Segments (Bshn26)
A.1.4 Terrain Four: Megalopolis

Figure A.20. Megalopolis Lower Boundary

Figure A.21. Detail of Megalopolis Lower Boundary
Figure A.22. Set 1: Megalopolis, 441 Segments (Bshn27)
Figure A.23. Set 2: Megalopolis, 345 Segments (Bsln28)
Lightning Discharge, showing downward and upward streamers

Figure A.24. Set 3: Megalopolis, 414 Segments (Bshn29)
Lightning Discharge, showing downward and upward streamers

Figure A.25. Set 4: Megalopolis, 527 Segments (Bshn30)
Lightning Discharge, showing downward and upward streamers

Set 5: Megalopolis, 382 segments (Bshn31)

Figure A.26. Set 5: Megalopolis, 382 Segments (Bshn31)
APPENDIX B

FORTRAN CODES

B.1 Description of Code Lightning Model 74 (lm74.f)

As the section’s title implies, my final version of my lightning modeling code is the seventy-fourth version that I created. When I would achieve a new milestone I would save that version and start a new one. I then started the habit of saving every fifth version just in case I found out that I had made a terrible error and needed to retrace my steps and go down a different path. Happily that only occurred occasionally, only costing me one or two versions at most.

My code contains the following routines:
- Main program
- Subroutine LECDS (Laplace Equation Computed Difference Solver)
- Subroutine RANDOM (Generates the random number sequence)
- Subroutine CBOLTCAND (Solves for the next part of the downward moving streamer)
- Subroutine GBOLTCAND (Solves for the next part of the upward moving streamer)
- Subroutine LowerBound (Generates the various terrains modeled to be investigated)

The code begins with the various required formatting and set of constants and variables. The next section lists some of the various previous versions and the particular improvements made in that version. After that is a description of some of the included constants, variables, and files associated with the running of the code.

The next section reads in set values for constants and then reads from file 9 other variables. Some constants are calculated and then values are written to file 19 to be used for the plots.

Next, a series of variables are generated and/or calculated, included the grid steps, values for DXBOLT, DYBOLT, and DZBOLT, lambda, mu, Tol, plotting variables, and ws.
In Do Loop 130 we clear the values for the *BOLT files and create the initial lightning originator, i.e. the first segment that originates at the center point of the top of the cube descending one grid step down. From this initial segment all of the subsequent lightning stepped leaders originate.

At this point, subroutine LowerBound is called to generate one of the following four terrains used in the study. Depending on the value of IGChoice, one of the following terrains is evaluated:

- if equals 1 ⇒ lower bdry. is flat, potential = 0.0.
- if equals 2 ⇒ lower bdry. is periodic hilly, with peak at 1 grid height, potential = 0.0.
- if equals 3 ⇒ lower bdry. is a flat plane with a building roughly the size of the Empire State building, or 5 cells high, by 1 x 2 cells at the base, or roughly 1365’ tall and 273’ x 546’ at the base, potential = 0.0.
- if equals 4 ⇒ lower bdry. is a ‘fractal’ surface I generated using my random number generator. I stepped through each grid point of the x-y plane and assigned a value between 0 and 7 as a z-component, Potential = 0.0.

In the next Do Loop 113, I assigned the potential value for the top of the cube, with the central 58% of the top plane being set to the potential of a thunder cell, aka CP, or $-1.0 \times 10^9$ Volts, and the rest of the top to 0 Volts.

In Do Loop 200 we begin the generation of the lightning. The first step is to call the subroutine CBOLTCAND which will generate a list of all of the candidate gridpoints. The candidate gridpoint set contains all of those gridpoints adjacent to the currently defined downward-moving bolt. So for example, for the first step there are five candidate gridpoints consisting of the four points at the same height as the bottom of the starter segment, and the point directly below, if they potential difference between the bolt and the grid point is greater than CBOLTINIT. In my lightning model, as in all of the others found in the literature survey, the lightning is restricted to 90 degree generation movement, no diagonal movement is allowed. As the four sides are set with a periodic boundary condition, the eight corner point potential values are set as the averages of the adjacent grid points. (Note: Sadly, I carried this practice over from a previous version which had different boundary conditions. Upon reflection I didn’t need to have done this averaging, but given the way that the iterative code was implemented, I also don’t think it caused any problems.)
call LECDS to solve the elliptical PDE for the cube, using the five point method outlined earlier.

Having solved for the potential at each of the grid points inside the cube, I now call GBOLTTCAND to generate which of the gridpoints are candidates to be added as an upward-moving streamer. For the first iteration, for IGCHOICE = 1, i.e. a flat plane, this would possibly include all of the grid points directly above at Z=1 whose potential is greater than GBOLTINIT, or the potential difference level for which streamer initiation may occur.

To choose the next streamer bolt point, I now add up all of the potentials for the candidate points for either the downward- or upward-moving streamers. I then create the weighted probability for each grid point by dividing the sum into the potential for each grid point, sequentially assigning the resulting fractional amount to each candidate.

I then call the subroutine RANDOM, which generates in the aforementioned manner a value between 0 and 1.0. Whichever candidate point contains the value returned by RANDOM is then chosen as the next part of either the downward or upward moving streamer.

If the new point is part of the downward moving streamer, then a potential is assigned to it equal to the value of its parent bolt gridpoint, i.e. the already existing bolt gridpoint that it is joined to, minus the potential difference of the step, the previously calculated DXBOLT, DYBOLT or DZBOLT, depending on the new segments orientation.

If the new point is part of an upward moving streamer, its potential is set equal to the potential previously calculated for the gridpoint within the LECDS subroutine.

The series of events is then repeated until one of the following events occur:

1. A gridpoint is shared by a downward- and upward-moving streamer, i.e. the streamers met and a lightning discharge or lightning bolt was created.

2. A downward-moving streamer reaches the bottom surface or an upward-moving streamer reaches the upper surface.

3. NumPts is exceeded, i.e. 40,000 points were generated without either prior noted results 1 or 2 occurring, or

4. All the the potential of the cloud is expended with the downward-moving bolt without 1 or 2 occurring.
Results 1 or 2 would be considered desirable results, with 3 or 4 being considered errors. Once one of the conclusions is reached, all of the results are written out to various files to allow for data analysis and graphing.

The code lm74.f or comparable earlier versions was used to run five sets of cases, each set having a unique seed value to start the random number generator and consisting of four results, one for each of the different terrains. Hence the data to be considered consists of 20 sets of data or 20 different lightning bolts.

The final requirement then is to calculate the fractal number associated with each of the twenty lightning models. This was accomplished using the aforementioned boxcounting method. In particular, a number N was ascertained of how many cells within the grid held a part of the lightning, with the cell edge length varying over a range of r-values. The number N scales as a function of r according to $N \propto r^{-D}$ where D is an estimate of the fractal dimension of the object.

To accomplish this task for my three-dimensional lightning, I created a series of small programs.

First I used a program sm4163d.f to take a case’s file 41, which lists the elements of the lightning bolt as a series of pairs of x, y, z coordinates, where each segment is a segment of the downward or upward streamer. The program reads from file 41 and writes only the new lightning grid points to file 44, i.e. discard the XO, YO, ZO elements while also removing from the file those data points belonging to upward streamers that did not manage to connect to the downward streamer, i.e. that did not end up being a part of the lightning bolt, and write the remaining coordinates, i.e. the coordinates for the lightning bolt to file 44. We find which values we need to strip by inspecting file fort.42 to determine where the final segment was created. If the downward streamer reached ground, then we discard all of the upward streamers. If the downward streamer connected with an upward streamer, we then backtrack the upward streamer, mark each attached segment, and then remove the remaining upward streamers that weren’t attached to the upward streamer of interest.

Then I run bc3dv2.f which started with an r equal to one half the full height of the graph, 1.25 kilometers, and count how many of the eight cubes contain part of the lightning. r is then halved and the sequence is repeated and continued for five sequences, writing the results each time to file 46.
I then run mod7.f which puts log(r) and log(N) in fort.47. I then calculate the mean and standard deviation based on this output and arrive at my fractal number for each of the lightning discharges.

The following is FORTRAN code lm74.f, the final version of the main code developed to conduct my thesis investigation.

C Line 1 Set Initial Statements
C
C Lightning Model Program, v74
C
C
C IMPLICIT REAL (A-H,O-Z)
IMPLICIT INTEGER (I-N)

REAL a,b,c,d,el,g,lambda,mu,TOL,xh

1 FORMAT ('set label 1 "lm74.f, ',F4.2,'->',F5.2,'0 km, ',F4.2,'->',F5.2,'0 km, ',F4.2,'->',F5.2,'0 km, ',I3,',',I3,',', I3,',',F5 &.2,',' ECrit= ',E10.2,'V/km" at .1,.1,','F6.3)
2 FORMAT ('set label 2 "EP=',F10.2,'V, CP=',E10.2,'V, BEF= ',E10.4,'&V/km, IEND = ',I2,', IGc = ',I2,', ISEED=',I8,'" at .1,.1,','F6.3)
3 FORMAT ('set xrange [:',F5.2,']')
4 FORMAT ('set yrange [:',F5.2,']')
5 FORMAT ('set zrange [:',F5.2,']')
6 FORMAT ('set arrow 1 from ',F6.3,',',F6.3,',',F6.3,' to ',F6.3,',',F6.3,',',F6.3)
7 FORMAT ('set arrow ',I4,' from ',F6.3,',',F6.3,',',F6.3,' to ',F6 &.3,','F6.3,','F6.3)
8 FORMAT ('set arrow ',I4,' from ',F6.3,',',F6.3,',',F6.3,' to ',F6 &.3,','F6.3,','F6.3, ' ls 1')
9 FORMAT ('set arrow ',I4,' from ',F6.3,',',F6.3,',',F6.3,' to ',F6 &.3,','F6.3,','F6.3)

COMMON NX,NY,NZ,xh,yk,zj,ECRIT,GBOLTINIT
COMMON /CBOLT/ CBOLT(0:500,0:500,0:500)
COMMON /GBOLT/ GBOLT(0:500,0:500,0:500)
COMMON /g/ g(0:500,0:500,0:500)
COMMON /w/ w(0:500,0:500,0:500)
COMMON /x/ x(0:500)
COMMON /y/ y(0:500)
COMMON /z/ z(0:500)

C
C Lines 21 Comments
C For subsequent versions, see notes in bgjlog.
C lm45.f - Modifying with option of how to end run, whether lightning
C initiates from the ground, and random number generator seed
C set or tied to clock.
C lm44.f - Going to be substantially modifying the code to use
C the various insights from my latest round of research
C and analysis
C lm37.f - added the (NX/2,NY) initiation point to the CBOLT file
C to clean up the bolt creation, and not have it added later.
C lm36.f - to make the bolt charge held throughout
C lm35.f - In particular, made so could put in tip & bolt charges
C lm34.f - a money version. Cleaned up the error message which
C highlighted what was actually happening, copied to lmb.f
C lm30.f - going to clean up the code to make it more tractable
C lm28.f - going to clean things up and try the periodic BC
C Lm27.f - going to add the periodic boundary conditions
C lm26.f - added using the poetential difference, rather than the
C potential to choose the next step of lightning
C
C Some documentation on the currently used files
C
C w(500,500,500) The file holding the calculated values of the grid
points during the solving of the FD equation

and \( w(I,J) \) \( I \) from 0 -> NX, \( J \) from 1 -> NY-1

The file holding the boundary values

The file holding the lightning bolt coordinates running from 0 ->NX, 0 -> NY

\( x(I) \) from 0 -> NX

\( y(J) \) from 0 -> NY

\( z(K) \) from 0 -> NZ

Variable set to the cloud potential to track the reduction of the cloud potential based on the charge in the lightning CBOLT.

Contains the input parameters to be read for each case

The coordinates of the cloud host points and their candidates

The coordinates of the ground host points and their candidates

IBOLT/IGRND,IO,JO,KO,I,J,KO,APD,CandSum

- IBOLT/IGRND a flag for Cloud or ground, 1,2, resp.
- IO,JO,KO The host of the new candidate point
C - I,J,K The new candidate points
C - APD The absolute value of the potential difference
C between the host and the candidate points
C - CandSum The sum of the APD’s
C
C File 15 The lightning in Z,Y,X order
C
C File 16 The lightning as it’s being created
C
C File 19 Receives the output for plotting each run in gnuplot
C
C File 41 The lightning in the form needed for gnuplot
C to plot it in the form of lightning
C
C Constants
C
C ISEED     = 12357
C el        = 1.0
C NumIter   = 1
C Iter      = 5.E+05
C NumPts    = 40000
C
C
C-------------------------------------------------------------------
C
C ------------
C SETUP PROBLEM
C ------------
C Lines 106 Set initial values for ISEED,el,NumIter,Iter,NumPts
C
C ISEED     = 12357
el = 1.0
NumIter = 1
Iter = 5.E+05
NumPts = 40000
IGchoice = 1
IBOLT = 1
IGRDND = 2

C Line 90 Read the value of IRNGC,a,b,c,d,NX,NY,EXP,EP,CP,BEF,IEND,
C ISEED,IGchoice
C a,b,c,d in kilometers
C NX,NY as integers
C EP,CP in Volts
C BEF in Volts/kilometer
C ECrit= the Critical Voltage for an upward moving
C streamer, in Volts/km
C IEND= 1 for ground strike, 2 for Potential depletion
C ISEED = Traditionally, 12357. The initial seed for
C the random number generator
C IBOLT,IGRDN = These constants, set to 1,2 respectively,
C are flags for whether the candidate is a candidate
C for the Bolt or for the Grnd.
C
C Rewind all of the files used by the program
C
C
C OPEN(UNIT=9,FILE='fort.9',STATUS='OLD')
OPEN(UNIT=10,FILE='fort.10')
OPEN(UNIT=11,FILE='fort.11')
OPEN(UNIT=12,FILE='fort.12')
OPEN(UNIT=15,FILE='fort.15')
OPEN(UNIT=16,FILE='fort.16')
OPEN(UNIT=19,FILE='fort.19')
OPEN(UNIT=20,FILE='fort.20')
OPEN(UNIT=31,FILE='fort.31')
OPEN(UNIT=41,FILE='fort.41')
OPEN(UNIT=42,FILE='fort.42')
OPEN(UNIT=56,FILE='fort.56')

C
Rewind(9)
Read(9,*)a,b,c,d,NX,NY,EXP,EP,CP,BEF,ECrit,GBOLTINIT,IEND,ISEED,I &Gchoice

C
e = a
f = b
NZ = NX
POTEND = ABS(CP)
Frmtlbl1= d+.2
Frmtlbl2= d+.1
Write(19,1)a,b,e,f,c,d,NX,NZ,NY,EXP,ECrit,Frmtlbl1
Write(19,2)EP,CP,BEF,IEND,IGchoice,ISEED,Frmtlbl2
Write(19,3)b
Write(19,7)b
Write(19,8)d
Close (19)

C Write(6,*)'Here I am at line 176'

C Line 101 Calculate the grid steps in kilometers
C
xh = (b-a)/FLOAT(NX)
yk = (d-c)/FLOAT(NY)
zj = (f - e)/FLOAT(NZ)

C

C Calculate DXBOLT,DYBOLT in Volts
C

DXBOLT = xh*BEF
DYBOLT = yk*BEF
DZBOLT = zj*BEF
BOLTCHK= AMIN1(ABS(DXBOLT),ABS(DYBOLT),ABS(DZBOLT))

C

C Line 106 Calculate lambda and mu
C

lambda = xh**2/yk**2
mu = 3.0*(1.0 + lambda)

C

C Line 183 Calculate Tol
C

IF(xh.GE.yk)Then
  If(zj.GE.yk) Then
    TOL=yk**3
  else
    TOL=zj**3
  Endif
else
  If(zj.GE.xh) Then
    TOL=xh**3
  else
    TOL=zj**3
  Endif
ENDIF

C

C Line 119 Set the x-grid values for plotting

83
Do 100 I=0,NX
   x(I) = a + float(I)*xh
100 Continue

Write(6,*)'Here I am at line 219'

Line 126 Set the y-grid values for plotting

Do 101 J=0, NY
   y(J) = c + float(J)*yk
101 Continue

Line 214 Set the z-grid values for plotting

Do 111 K=0,NZ
   z(K) = e + float(K)*zj
111 Continue

Line 201 Set the w's

Do 102 I=0,NX
   Do 102 J=0, NY
      Do 102 K=0,NZ
         w(I,J,K) = 0.0
102 Continue

Line 163 Clear the bolt files & create the initial lightning starter
DO 130 K=NZ,0,-1
DO 130 J=NY,0,-1
DO 130 I=NX,0,-1
CBOLT(I,J,K) = 0.0
GBOLT(I,J,K) = GBOLTINIT
If((I.EQ.(NX/2)).AND.(J.EQ.(NY-1)).AND.(K.EQ.(NZ/2))) Then
   CBOLT(I,J,K) = CP-DYBOLT
   CBOLT(I,J+1,K) = CP
   w(I,J,K)= CP-DYBOLT
   Write(16,*)I,J,K
   Write(20,*)'Midpoint is',I,J,K,w(I,J,K)
   Close (20)
   Write(41,*)x(I),z(K),d
   Write(41,*)x(I),z(K),y(J)
   Write(42,4)x(I),z(K),d,x(I),z(K),y(J)
   IVector = 2
ENDIF
130 CONTINUE
C
C   Write(6,*)'Here I am at line 263'
C Line 173 Assign the values of the lower surface
C
C   Call LowerBound(NX,NY,NZ,IGchoice,EP)
C
C Assign the values of the upper surface
C Note: Here I am driving that x and z will be the same length
C
XStart = Float(NX)*0.21
ZStart = Float(NZ)*0.21
XEnd = Float(NX)*0.79
ZEnd = Float(NZ)*0.79

85
Do 113 I=0,NX
  Do 113 K=0,NZ
    FltX = FLOAT(I)
    FltZ = FLOAT(K)
    &AND.(FltZ.LE.ZEnd)) Then
      g(I,NY,K) = CP
    else
      g(I,NY,K) = 0.0
    Endif
  Enddo 113

C
C Line 156 Set the eight corner points as a straight forward average
C
  g(NX,0,0) = (g(NX-1,0,0)+w(NX,1,0)+g(NX,0,1))/3.0
  g(NX,0,NZ) = (g(NX-1,0,NZ) + w(NX,1,NZ)+g(NX,0,NZ-1)) /3.0
  g(NX,NY,NZ) = (w(NX,NY-1,NZ)+g(NX-1,NY,NZ)+g(NX,NY,NZ-1))/3.0
  g(0,0,0) = (w(0,1,0) + g(1,0,0)+g(0,0,1))/3.0
  g(0,NY,0) = (w(0,NY-1,0) + g(1,NY,0)+g(0,NY,1))/3.0
  g(NX,NY,0) = (w(NX,NY-1,0)+g(NX-1,NY,0)+g(NX,NY,1))/3.0
  g(0,0,NZ) = (w(0,1,NZ)+g(1,0,NZ)+g(0,0,NZ-1))/3.0
  g(0,NY,NZ) = (w(0,NY-1,NZ)+g(1,NY,NZ)+g(0,NY,NZ-1))/3.0

C
C Line 182 Initial Parameters Set - Begin the creation of the lightning
C
  Do 200 M=1,NumPts
    C
    If(NumIter.EQ.NumPts) Then
      Write(56,*) "NumIter equals NumPts"
      Go To 5000
    Endif
  Enddo 200
else

C Line 189  Create the array for the candidates for the bolt step
C
    Call CBOLTCAND(NX,NY,NZ)
C
C Line 249  Reset the eight corner points as a straightforward average
C
    g(NX,0,0) = (g(NX-1,0,0)+w(NX,1,0)+g(NX,0,1)) /3.0
    g(NX,0,NZ) = (g(NX-1,0,NZ) + w(NX,1,NZ)+g(NX,0,NZ-1)) /3.0
    g(NX, NY, NZ) = (w(NX, NY-1, NZ) + g(NX-1, NY, NZ)+g(NX, NY, NZ-1))/3.0
    g(0,0,0) = (w(0,1,0) + g(1,0,0)+g(0,0,1))/3.0
    g(0, NY, 0) = (w(0, NY-1, 0) + g(1, NY, 0)+g(0, NY, 1))/3.0
    g(NX, NY, 0) = (w(NX, NY-1, 0)+g(NX-1, NY, 0)+g(NX, NY, 1))/3.0
    g(0, 0, NZ) = (w(0, 1, NZ)+g(1, 0, NZ)+g(0, 0, NZ-1))/3.0
    g(0, NY, NZ) = (w(0, NY-1, NZ)+g(1, NY, NZ)+g(0, NY, NZ-1))/3.0
C
C Line 256  CALL LECDS and solve the elliptical PDE
C
C     Write(6,*)'Here I am at line 327'
    Call LECDS(el,Iter,lambda,mu,NX,NY,NZ,TOL,NoGood,M)
C
C     Write(6,*)'Here I am at line 329'
C
C Check and see if the case needs to be stopped.
C
If (NoGood.EQ.1) Then
    Write(6,*) 'Iteration number exceeded.  Oops.'
    Write(56,*) 'Iteration number exceeded.  Oops.'
    Go To 5000
Endif

If(POTEND.LE.CBOLTCHK) Then
Write(6,*), 'Cloud potential expended. Oops.'
Write(56,*), 'Cloud potential expended. Oops.'
Go To 5000
Endif
C
Write(6,*),'Here I am at line 343'
C
Write(6,*),'This is before the call',NX, NY, NZ, GBOLTINIT
C
C Line 245 Create the array for the candidates for the Gbolt step
C
Call GBOLTCAND
C
C Line 264 Add up the U-values for the candidate grid points
C
SUM = 0.0
Rewind(10)
139 Read(10,*,END=140)IO,JO,KO,I,J,K
If((J.EQ.0).OR.(J.EQ.NY)) Then
   PD = CBOLT(IO,JO,KO) - g(I,J,K)
else
   PD = CBOLT(IO,JO,KO) - w(I,J,K)
Endif
SUM = SUM+(ABS(PD))**EXP
Go to 139
140 Continue
C
Rewind(11)
239 Read(11,*,END=240)IO,JO,KO,I,J,K
If((J.EQ.0).OR.(J.EQ.NY)) Then
   PD = GBOLT(IO,JO,KO) - g(I,J,K)
else
   PD = GBOLT(IO,JO,KO) - w(I,J,K)
C Line 278 Set the probability increments for each of the candidates
C
CandSum = 0.0
Rewind(10)
Rewind(12)
149 Read(10,*,END=150)IO,JO,KO,I,J,K
If((J.EQ.0).OR.(J.EQ.NY)) Then
   PD = CBOLT(IO,JO,KO) - g(I,J,K)
else
   PD = CBOLT(IO,JO,KO) - w(I,J,K)
Endif
APD = (ABS(PD))**EXP
CandSum = CandSum+ (APD/SUM)

C Write(12,*)IBOLT,IO,JO,KO,I,J,K,APD,CandSum

C Go To 149
150 Continue
C
Rewind(11)
249 Read(11,*,END=250)IO,JO,KO,I,J,K
If((J.EQ.0).OR.(J.EQ.NY)) Then
   PD = GBOLT(IO,JO,KO) - g(I,J,K)
else
   PD = GBOLT(IO,JO,KO) - w(I,J,K)
Endif
Endif
APD = (ABS(PD))**EXP
CandSum = CandSum+ (APD/SUM)
C
Write(12,*)IGRND,IO,JO,KO,I,J,K,APD,CandSum
C
Go To 249
250 Continue
C
Call RANDOM(ISEED,RVAL)
C
Line 319 Use the new random value to select the next addition to CBOLT or GBOLT(s)
C
OldVal = 0.0
Rewind(12)
159 Read(12,*,END=160)ITYPE,IO,JO,KO,I,J,K,APD,CandSum
If((OldVal.LE.RVAL).and.(RVAL.LE.CandSum)) Then
C
Check if the candidate is for downward or upward moving streamer
C
If (ITYPE.EQ.1) Then
C
Set the appropriate bolt with the new value
C
If ((I.EQ.IO).OR.(K.EQ.KO)) Then
   CBOLT(I,J,K)= CBOLT(IO,JO,KO)-DYBOLT
   POTEND = POTEND-ABS(DYBOLT)
else
   CBOLT(I,J,K)= CBOLT(IO,JO,KO)-DXBOLT
   POTEND = POTEND-ABS(DXBOLT)
Endif
else

GBOLT(I,J,K) = w(I,J,K)

Endif

IN = I
JN = J
KN = K

Write(16,*) ITYPE, IN, JN, KN
Write(41,*) x(IO), z(KO), y(JO)
Write(41,*) x(IN), z(KN), y(JN)

If (ITYPE.EQ.1) Then

Write(42,5) IVector, x(IO), z(KO), y(JO), x(IN), z(KN), y(JN)
else

Write(42,6) IVector, x(IO), z(KO), y(JO), x(IN), z(KN), y(JN)

Endif

IVector = IVector + 1

If (IEND.EQ.1) Then

If ((ITYPE.EQ.1).AND.(JN.eq.0)) Then

Write(6,*) 'Jean, we made it!'
Write(56,*) 'Jean, we made it!'

Go To 5000
endif
Go To 160
else

If (POTEND.LE.BOLTCHK) Then

Write(6,*) 'Cloud potential expended'
Write(56,*) 'Cloud potential expended'

Go To 5000
endif
Go To 160
Endif
endif
OldVal=CandSum
Go to 159
160 Continue
Do 440 I=0,NX
   Do 440 J=0, NY
      Do 440 K=0, NZ
         If((ABS(CBOLT(I,J,K)).GE.1.E-06).AND.(ABS(GBOLT(I,J,K) - &GBOLTINIT).GT.1.E-06)) Then
            Write(6,*)'Streamers met. Jean, we made it.'
            Write(56,*)'Streamers met. Jean, we made it.'
            Go To 5000
         Endif
440 Continue
   NumIter = NumIter + 1
Endif
C
C Line 329 Reset the w’s
C
   Do 202 I=0, NX
      Do 202 J=1, NY-1
         Do 202 K=0, NZ
            w(I,J,K) = 0.0
202 Continue
C
C Line 339 Write the bolt to file 15
C
   Rewind(15)
Do 3000 K=0,NZ
   DO 3000 J=0,NY
      DO 3000 I=0,NX
         If(ABS(CBOLT(I,J,K)).GT.1.E-06) Then
            Write(15,*)I,J,K
         endif
   3000 Continue
C
C Close all of the files used
C
   CLOSE(9)
   Close(10)
   CLOSE(11)
   CLOSE(12)
   CLOSE(15)
   CLOSE(16)
   CLOSE(31)
   CLOSE(41)
   CLOSE(42)
   CLOSE(56)
   STOP
END

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C Line 351 Subroutine LECDS
C
SUBROUTINE LECDS(el,Iter,lambda,mu,NX,NY,NZ,TOL,NoGood,Main)
IMPLICIT REAL (A-H,O-Z)
IMPLICIT INTEGER (I-N)
REAL el,EP,g,lambda,mu,NORM,TOL,z
COMMON /CBOLT/ CBOLT(0:500,0:500,0:500)
COMMON /GBOLT/ GBOLT(0:500,0:500,0:500)
COMMON /g/ g(0:500,0:500,0:500)  
COMMON /w/ w(0:500,0:500,0:500)  
COMMON /x/ x(0:500)  
COMMON /y/ y(0:500)  
COMMON /z/ z(0:500)

C
C Line 362 Comments
C
C To approximate the solution to the Poisson Equation
C \( \frac{d^2u}{dx^2}(x,y) + \frac{d^2u}{dy^2}(x,y) = f(x,y), a \leq x \leq b, c \leq y \leq d \)
C subject to the boundary conditions
C \( u(x,y) = g(x,y), \) if \( x=a \) or \( x=b \) and \( c \leq y \leq d, \)
C \( u(x,y) = g(x,y), \) if \( y=c \) or \( y=d \) and \( a \leq x \leq b; \)
C
C INPUT endpoints xmin, xmax, ymin, ymax; integers m>=3, n>=3;
C maximum number of iterations N.
C
C OUTPUT approximations wi,j to u(xi,yj) for each I=1,n-1 and for
C each j=1,m-1 or a message that the maximum number of iterations
C was exceeded.
C
C VARIABLES TOL=tolerance
C
C Begin the main loop
C
C Line 382 Set the initial parameters NoGood,NORM,el
C
NoGood = 0
NORM = 0
el = 1.0
Do 10 M=1,Iter
If(EL.GT.FLOAT(ITER)) Go to 6000

C

C Line 390 Evaluate w(0,NY-1,0) Step 7

C

If(ABS(CBOLT(0,NY-1,0)).GT.1.E-06) Then
  else
    xnum1 = w(NX-1,NY-1,0)+LAMBDA*G(0,NY,0)+LAMBDA*w(0,NY-2,0)+w(1,NY-1,0)+w(0,NY-1,1)+w(0,NY-1,NZ-1)
    zz = xnum1/mu
    NORM = Abs(zz - w(0,NY-1,0))
    w(0,NY-1,0) = zz
  endif
ENDIF

C

C Evaluate w(0,NY-1,K) for K=1,NZ-1

C

Do 1105 K=1,NZ-1
  If(ABS(CBOLT(0,NY-1,K)).GT.1.E-06) Then
    else
      xnum1 = w(NX-1,NY-1,K)+LAMBDA*G(0,NY,K)+LAMBDA*w(0,NY-2,K)+w(1,NY-1,K)+w(0,NY-1,K-1)+w(0,NY-1,K+1)
      zz = xnum1/mu
      If (Abs(zz - w(0,NY-1,K)).GT.NORM) Then
        NORM = Abs(zz - w(0,NY-1,K))
      endif
      w(0,NY-1,K) = zz
  endif
1105 Continue

C

C Evaluate w(0,NY-1,NZ)

C

If(ABS(CBOLT(0,NY-1,NZ)).GT.1.E-06) Then
else
   xnum1 = w(NX-1,NY-1,NZ)+lambda*g(0,NY,NZ)+lambda*w(0,NY-2,NZ
   &)+w(1,NY-1,NZ)+w(0,NY-1,NZ-1)+w(0,NY-1,1)
   zz= xnum1/mu
   If (Abs(zz-w(0,NY-1,NZ)).GT.NORM) Then
      NORM = Abs(zz - w(0,NY-1,NZ))
   Endif
   w(0,NY-1,NZ) = zz
ENDIF

C
C Line 400 Evaluate w(I,NY-1,0) for I=1,NX-1 Step 8
C
Do 1015 I=1,NX-1
   If(ABS(CBOLT(I,NY-1,0)).GT.1.E-06) Then
else
   zz =(lambda*g(I,NY,0) + w(I-1,NY-1,0) + w(I+1,NY-1,0) +
   &lambda*w(I,NY-2,0)+w(I,NY-1,1)+w(I,NY-1,0))/mu
   If( ABS(w(I,NY-1,0) - zz).GT.NORM) Then
      NORM = ABS(w(I,NY-1,0) - zz)
   Endif
   w(I,NY-1,0) = zz
ENDIF
1015 Continue
C
C Line 400 Evaluate w(I,NY-1,K) for I=1,NX-1 and K=1,NZ-1
C
Do 1016 K=1,NZ-1
   Do 1016 I=1,NX-1
      If(ABS(CBOLT(I,NY-1,K)).GT.1.E-06) Then
else
      zz =(lambda*g(I,NY,K)+w(I-1,NY-1,K)+w(I+1,NY-1,K)+lambda*
&w(I,NY-2,K)+w(I,NY-1,K+1)+w(I,NY-1,K-1)/mu
    If( ABS(w(I,NY-1,K) - zz).GT.NORM) Then
        NORM = ABS(w(I,NY-1,K) - zz)
    Endif
    w(I,NY-1,K) = zz
ENDIF
1016 Continue
C
C Line 400 Evaluate w(I,NY-1,NZ) for I=1,NX-1 Step 8
C
Do 1017 I=1,NX-1
    If(ABS(CBOLT(I,NY-1,NZ)).GT.1.E-06) Then
        else
            zz =(lambda*g(I,NY,NZ)+w(I-1,NY-1,NZ)+w(I+1,NY-1,NZ)+lambda
            &*w(I,NY-2,NZ)+w(I,NY-1,NZ)+w(I,NY-1,NZ-1))/mu
            If( ABS(w(I,NY-1,NZ) - zz).GT.NORM) Then
                NORM = ABS(w(I,NY-1,NZ) - zz)
            Endif
            w(I,NY-1,NZ) = zz
        ENDF
    1017 Continue
C
C Line 414 Evaluate w(NX,NY-1,0) Step 9
C
    If(ABS(CBOLT(NX,NY-1,0)).GT.1.E-06) Then
    else
        zz=(w(1,NY-1,0)+lambda*g(NX,NY,0)+w(NX-1,NY-1,0)+lambda*w(NX,
        &NY-2,0)+w(NX,NY-1,1)+w(NX,NY-1,NZ-1))/mu
        IF( ABS(w(NX,NY-1,0) - zz).GT.NORM) Then
            NORM = ABS(w(NX,NY-1,0) - zz)
        Endif
w(NX,NY-1,0) = zz
Endif

C
C Line 414 Evaluate w(NX,NY-1,K) for K=1,NZ-1  Step 9
C

Do 1108 K=1,NZ-1
  If(ABS(CBOLT(NX,NY-1,K)).GT.1.E-06) Then
    else
      zz=(w(1,NY-1,K)+lambda*g(NX,NY,K)+w(NX-1,NY-1,K)+lambda*w(NX,NY-2,K)+w(NX,NY-1,K+1)+w(NX,NY-1,K-1))/mu
      IF( ABS(w(NX,NY-1,K) - zz).GT.NORM) Then
        NORM = ABS(w(NX,NY-1,K) - zz)
      Endif
      w(NX,NY-1,K) = zz
  Endif
1108 Continue

C
C Line 414 Evaluate w(NX,NY-1,NZ) Step 9
C

If(ABS(CBOLT(NX,NY-1,NZ)).GT.1.E-06) Then
  else
    zz=(w(1,NY-1,NZ)+lambda*g(NX,NY,NZ)+w(NX-1,NY-1,NZ)+lambda*w(NX,NY-2,NZ)+w(NX,NY-1,1)+w(NX,NY-1,NZ-1))/mu
    IF( ABS(w(NX,NY-1,NZ) - zz).GT.NORM) Then
      NORM = ABS(w(NX,NY-1,NZ) - zz)
    Endif
    w(NX,NY-1,NZ) = zz
  Endif

C
C Step 10
C
DO 106 K=NZ-2,2,-1
  DO 106 J=NY-2,2,-1
C
C Line 430 Evaluate w(0,J,K) for J=NY-2,2, K=NZ-2,2 Step 11
C
  If(ABS(CBOLT(0,J,K)).GT.1.E-06) Then
  else
    zz = (w(NX-1,J,K)+lambda*w(0,J+1,K)+lambda*w(0,J-1,K)+w(1,&J,K)+w(0,J,K-1)+w(0,J,K+1))/mu
    If ( Abs( w(0,J,K) - zz).GT.NORM) Then
      NORM = ABS(w(0,J,K) - zz)
    Endif
    w(0,J,K) = zz
  Endif
C
C Line 441 Evaluate w(I,J,K) for I=1,NX-1; J=NY-2,2; K=NZ-2,2 Step 12
C
  Do 116 I=1,NX-1
C
C So after much thought, this does take care of it. It looks at if
C either the bolt belongs to CBOLT, or GBOLT. If it belongs to CBOLT
C then w was set in the CBOLTAND subroutine before the evaluation.
C If rather, it belongs to GBOLT, then it was set to zero when the
C w's were reset, and so is fine. And this criteria just makes sure
C that it isn't evaluated if it is GBOLT as part of the ground, ratherC than GBOLT part
C calculated.
C
  If ( (ABS(CBOLT(I,J,K)).GT.1.E-06).OR.(ABS(GBOLT(I,J,K)-EP
  &).LE.1.E-06)) Then
  else
    zz = ( w(I-1,J,K) + lambda*w(I,J+1,K) + w(I+1,J,K) +

&lambda*w(I,J-1,K)+w(I,J,K-1)+w(I,J,K+1))/mu
   If ( Abs( w(I,J,K) - zz).GT.NORM) Then
      NORM = ABS( w(I,J,K) - zz)
      Endif
      w(I,J,K) = zz
   Endif
Continue

C
C Line 455 Evaluate w(NX,J,K) for J=NY-2,2, K=NZ-2,2 Step 13
C
   If(ABS(CBOLT(NX,J,K)).GT.1.E-06) Then
      else
      zz = ( w(1,J,K) + w(NX-1,J,K) + lambda*w(NX,J+1,K) +
        &lambda*w(NX,J-1,K)+w(NX,J,K+1)+w(NX,J,K-1))/mu
      If ( ABS( w(NX,J,K) - zz).GT.NORM) Then
         NORM = ABS( w(NX,J,K) - zz)
         Endif
         w(NX,J,K) = zz
      Endif
106 Continue

C
C Line 468 Evaluate w(0,1,0) Step 14
C
   If(ABS(CBOLT(0,1,0)).GT.1.E-06) Then
      else
      zz = ( w(NX-1,1,0) + lambda*g(0,0,0) + lambda*w(0,2,0) + w(1,
        &1,0)+w(0,1,1)+w(0,1,NZ-1))/mu
      If ( ABS( w(0,1,0) - zz).GT.NORM ) Then
         NORM = ABS( w(0,1,0) - zz)
         Endif
         w(0,1,0) = zz
   Endif
C Line 468 Evaluate $w(0,1,K)$ for $K=1,NZ-1$ Step 14

C

DO 1156 K=1,NZ-1
    If(ABS(CBOLT(0,1,K)).GT.1.E-06) Then
else
    zz = ( $w(NX-1,1,K) + \lambda g(0,0,K) + \lambda w(0,2,K) + w(1,1,K)+w(0,1,K+1)+w(0,1,K-1)$ )/mu
    If ( ABS( $w(0,1,K) - zz$ ).GT.NORM ) Then
        NORM = ABS( $w(0,1,K) - zz$ )
    Endif
    $w(0,1,K) = zz$
Endif
1156 Continue
C

C Line 468 Evaluate $w(0,1,NZ)$ Step 14

C

If(ABS(CBOLT(0,1,NZ)).GT.1.E-06) Then
else
    zz = ( $w(NX-1,1,NZ) + \lambda g(0,0,NZ) + \lambda w(0,2,NZ) + w(1,1,NZ)+w(0,1,NZ-1)+w(0,1,NZ-1)$ )/mu
    If ( ABS( $w(0,1,NZ) - zz$ ).GT.NORM ) Then
        NORM = ABS( $w(0,1,NZ) - zz$ )
    Endif
    $w(0,1,NZ) = zz$
Endif

C

C Line 744 Evaluate $w(I,1,0)$ for $I=1,NX-1$ Step 15

C

Do 107 I=1,NX-1
If ( ((ABS(CBOLT(I,1,0)).GT.1.E-06).OR.(ABS(GBOLT(I,1,0)-EP).LE.
&.1.E-06)) ) Then
else
zz = (lambda*g(I,0,0)+w(I-1,1,0)+lambda*w(I,2,0)+w(I+1,1,0)+
&w(I,1,1)+w(I,1,NZ-1))/mu
ENDIF
IF ( ABS( w(I,1,0) - zz).GT.NORM) Then
NORM = ABS ( w(I,1,0) - zz)
ENDIF
w(I,1,0) = zz
ENDIF
107 Continue
C
C Line 744 Evaluate w(I,1,K) for I=1,NX-1, K=1,NZ-1 Step 15
C
Do 1107 K=1,NZ-1
Do 1107 I=1,NX-1
If ( ((ABS(CBOLT(I,1,K)).GT.1.E-06).OR.(ABS(GBOLT(I,1,K)-EP).
&.1.E-06)) ) Then
else
zz = (lambda*g(I,0,K)+w(I-1,1,K)+lambda*w(I,2,K)+w(I+1,1,K)
&)+w(I,1,K+1)+w(I,1,K-1))/mu
ENDIF
IF ( ABS( w(I,1,K) - zz).GT.NORM) Then
NORM = ABS ( w(I,1,K) - zz)
ENDIF
w(I,1,K) = zz
ENDIF
1107 Continue
C
C Line 744 Evaluate w(I,1,NZ) for I=1,NX-1 Step 15
C
Do 1117 I=1,NX-1
If ((ABS(CBOLT(I,1,NZ)).GT.1.E-06).OR.(ABS(GBOLT(I,1,NZ)-EP).
&LE.1.E-06)) Then
  else
    zz =(lambda*g(I,0,NZ)+w(I-1,1,NZ)+lambda*w(I,2,NZ)+w(I+1,1,
    &NZ)+w(I,1,1)+w(I,1,NZ-1))/mu
    IF ( ABS( w(I,1,NZ) - zz).GT.NORM) Then
      NORM = ABS ( w(I,1,NZ) - zz)
    Endif
    w(I,1,NZ) = zz
  Endif
1117  Continue
C
C Line 790 Evaluate w(NX,1,0) Step 16
C
If(ABS(CBOLT(NX,1,0)).GT.1.E-06) Then
  else
    zz =(w(1,1,0)+lambda*g(NX,0,0)+w(NX-1,1,0)+lambda*w(NX,2,0)+w
    &(NX,1,1)+w(NX,1,NZ-1))/mu
    IF ( ABS( w(NX,1,0) - zz).GT.NORM) Then
      NORM = ABS ( w(NX,1,0) - zz)
    Endif
    w(NX,1,0) = zz
  Endif
C
C Line 790 Evaluate w(NX,1,K) for K=1,NZ-1 Step 16
C
Do 1118 K=1,NZ-1
  If(ABS(CBOLT(NX,1,K)).GT.1.E-06) Then
    else
      zz =(w(1,1,K)+lambda*g(NX,0,K)+w(NX-1,1,K)+lambda*w(NX,2,K)
      &+w(NX,1,K+1)+w(NX,1,K-1))/mu
      If ( ABS( w(NX,1,K) - zz).GT.NORM) Then
        NORM = ABS ( w(NX,1,K) - zz)
      Endif
      w(NX,1,K) = zz
    Endif
  Endif
  1118 Continue
If ( ABS( w(NX,1,K) - zz).GT.NORM) Then
    NORM = ABS( w(NX,1,K) - zz)
Endif
w(NX,1,K) = zz
Endif

1118 Continue

C
C Line 790 Evaluate w(NX,1,NZ) Step 16
C
If(ABS(CBOLT(NX,1,NZ)).GT.1.E-06) Then
    else
        zz = (w(1,1,NZ)+lambda*g(NX,0,NZ)+w(NX-1,1,NZ)+lambda*w(NX,2,N &Z)+w(NX,1,NZ-1)+w(NX,1,1))/mu
        If ( ABS( w(NX,1,NZ) - zz).GT.NORM) Then
            NORM = ABS( w(NX,1,NZ) - zz)
        Endif
        w(NX,1,NZ) = zz
    Endif
Endif

C
C Line 503 Check to see if we're done. Step 17
C
If (NORM.LE.TOL) Then
    Go to 5000
else
    el = el + 1.0
Endif

10 Continue

6000 Continue
Write(6,*)el,'Number of iterations exceeded. Run unsuccessful.'
Write(56,*)el,'Number of iterations exceeded. Run unsuccessful.'
NoGood = 1

104
Go To 5000

5000 Continue

Return

End

C Line 520 Subroutine NBC

C

C SUBROUTINE NBC(I,J)
C IMPLICIT REAL (A-H,O-Z)
C IMPLICIT INTEGER (I-N)
C REAL xh,g
C COMMON /g/ g(500,500)
C COMMON /w/ w(0:500,0:500)
C COMMON xh
C gtemp = g(I,J)
C IF(I.EQ.0) THEN
C g(I,J) = (2.0*w(I+1,J)+g(I,J-1)+g(I,J+1)-4.0*gtemp)/(2.0*xh)
C ELSE
C g(I,J) = -(2.0*w(I-1,J)+g(I,J-1)+g(I,J+1)-4.0*gtemp)/(2.0*xh)
C ENDIF
C RETURN
C END

C Line 540 Subroutine RANDOM

C

SUBROUTINE RANDOM(ISEED,RVAL)
Integer ISEED
Real RVAL
ISEED = 2045*ISEED + 1
ISEED = ISEED - (ISEED/1048576)*1048576
RVAL = REAL(ISEED +1)/1048577.0
RETURN
END

SUBROUTINE CBOLTCAND(NX,NY,NZ)
IMPLICIT REAL (A-H,O-Z)
IMPLICIT INTEGER (I-N)
REAL g
COMMON /CBOLT/ CBOLT(0:500,0:500,0:500)
COMMON /GBOLT/ GBOLT(0:500,0:500,0:500)
COMMON /g/ g(0:500,0:500,0:500)
COMMON /w/ w(0:500,0:500,0:500)
COMMON /x/ x(0:500)
COMMON /y/ y(0:500)
COMMON /z/ z(0:500)

C
C Line 189 Create the array for the candidates for the bolt step
C
Rewind(10)
DO 120 K=0,NZ
   DO 120 J=1,NY-1
      DO 120 I=0,NX
         ABSCBOLTIJ = ABS(CBOLT(I,J,K))
   120 CONTINUE
C
C Line 196 Reset the bolt values for the next evaluation
C
   IF(ABSCBOLTIJ.GT.(1.E-06)) w(I,J,K)=CBOLT(I,J,K)
C
C Line 200 Check for candidate above a current bolt gridpoint in the
C CBOLT file
If(J.LE.(NY-1)) Then
  P4=CBOLT(I,J+1,K)
  If( (ABS(P4).LE.(1.E-06) ).AND.
    & (ABSCBOLTIJ.GT.(1.E-06) ) ) Then
    Write(10,*)I,J,K,I,J+1,K
  ENDIF
ENDIF

C Line 211 Check for candidate to the left of a current bolt gridpoint
C

If(I.EQ.0) Then
  P1=CBOLT(NX-1,J,K)
else
  P1= CBOLT(I-1,J,K)
Endif
If( ( ABS(P1).LE.(1.E-06) ).AND.
    & ( ABSCBOLTIJ.GT.(1.E-06) ) ) Then
  Write(10,*)I,J,K,I-1,J,K
endif

C Line 221 Check for candidate to the right of a current bolt gridpoint
C

If(I.EQ.NX) Then
  P2=CBOLT(1,J,K)
else
  P2=CBOLT(I+1,J,K)
Endif
If( ( ABS(P2).LE.(1.E-06) ).AND.
    & ( ABSCBOLTIJ.GT.(1.E-06) ) ) Then
  Write(10,*)I,J,K,I+1,J,K

ENDDIF

C

C Line 211 Check for candidate in front of a current bolt gridpoint

C

If(K.EQ.0) Then
   P5=CBOLT(I,J,NZ-1)
else
   P5= CBOLT(I,J,K-1)
Endif
If( ( ABS(P5).LE.(1.E-06) ).AND. 
&( ABSCBOLTIJ.GT.(1.E-06) ) ) Then
   Write(10,*)I,J,K,I,J,K-1
endif

C

C Line 221 Check for candidate behind a current bolt gridpoint

C

If(K.EQ.NZ) Then
   P6=CBOLT(I,J,1)
else
   P6=CBOLT(I,J,K+1)
Endif
If( ( ABS(P6).LE.(1.E-06) ).AND. 
&( ABSCBOLTIJ.GT.(1.E-06) ) ) Then
   Write(10,*)I,J,K,I,J,K+1
ENDIF

C

C Line 231 Check for candidate below a current bolt gridpoint

C

If(J.GE.1) Then
   P3= CBOLT(I,J-1,K)
   If( ( ABS(P3).LE.(1.E-06) ).AND. 
   
108
& ( ABSCBOLTIJ.GT.(1.E-06) ) ) Then
  Write(10,*)I,J,K,I,J-1,K
endif

Endif

120 CONTINUE
C
RETURN
END

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

C Line 351 Subroutine GBOLTCAND
C
SUBROUTINE GBOLTCAND
IMPLICIT REAL (A-H,O-Z)
IMPLICIT INTEGER (I-N)
REAL CBOLT,GBOLT,g
COMMON NX,NY,NZ,xh,yk,zj,ECRIT,GBOLTINIT
COMMON /CBOLT/ CBOLT(0:500,0:500,0:500)
COMMON /GBOLT/ GBOLT(0:500,0:500,0:500)
COMMON /g/ g(0:500,0:500,0:500)
COMMON /w/ w(0:500,0:500,0:500)
COMMON /x/ x(0:500)
COMMON /y/ y(0:500)
COMMON /z/ z(0:500)
C
C Line 985 Create the array for the candidates for the bolt step
C
Rewind(11)
C
Write(6,*)’This is within GBOLTCAND’,NX,NY,NZ,GBOLTINIT
DO 120 K=0,NZ
  DO 120 J=0,NY-1
    DO 120 I=0,NX

If(ABS(GBOLT(I,J,K)-GBOLTINIT).GT.1.E-06)Then

C Line 200 Check for candidate above a current bolt gridpoint in the GBOLT file

If(ABS(GBOLT(I,J+1,K)-GBOLTINIT).LE.1.E-06) Then
    gridptdiff= w(I,J,K)-w(I,J+1,K)
    gpdf = gridptdiff/yk
    If (ABS(gpdf).GE.ABS(ECrit)) Then
        Write(11,*)I,J,K,I,J+1,K
    ENDIF
ENDIF

C

C Line 211 Check for candidate in front of a current bolt gridpoint

If(K.EQ.0) Then
    gridptdiff= w(I,J,K)-w(I,J,NZ-1)
    GBOLTCHK5=GBOLT(I,J,NZ-1)
else
    gridptdiff= w(I,J,K)-w(I,J,K-1)
    GBOLTCHK5=GBOLT(I,J,K-1)
Endif
If(ABS(GBOLTCHK5-GBOLTINIT).LE.1.E-06) Then
    gpdf = gridptdiff/xh
    If (ABS(gpdf).GE.ABS(ECrit)) Then
        Write(11,*)I,J,K,I,J,K-1
    ENDIF
ENDIF

C

C Line 221 Check for candidate behind a current bolt gridpoint
If(K.EQ.NZ) Then
  gridptdiff=w(I,J,K)-w(I,J,1)
  GBOLTCHK6=GBOLT(I,J,1)
else
  gridptdiff=w(I,J,K)-w(I,J,K+1)
  GBOLTCHK6=GBOLT(I,J,K+1)
Endif

gpdf = gridptdiff/xh
If(ABS(GBOLTCHK6-GBOLTINIT).LE.1.E-06) Then
  If (ABS(gpdf).GE.ABS(ECrit)) Then
    Write(11,*)I,J,K,I,J,K+1
  ENDIF
Endif

C Line 211 Check for candidate to the left of a current bolt gridpoint

If(I.EQ.0) Then
  gridptdiff=w(I,J,K)-w(NX-1,J,K)
  GBOLTCHK1=GBOLT(NX-1,J,K)
else
  gridptdiff=w(I,J,K)-w(I-1,J,K)
  GBOLTCHK1=GBOLT(I-1,J,K)
Endif
If(ABS(GBOLTCHK1-GBOLTINIT).LE.1.E-06) Then
  gpdf = gridptdiff/xh
  If (ABS(gpdf).GE.ABS(ECrit)) Then
    Write(11,*)I,J,K,I-1,J,K
  ENDIF
Endif
C Line 221 Check for candidate to the right of a current bolt gridpoint
C

If(I.EQ.NX) Then
  gridptdiff=w(I,J,K)-w(1,J,K)
  GBOLTCHK2=GBOLT(I,J,K)
else
  gridptdiff=w(I,J,K)-w(I+1,J,K)
  GBOLTCHK2=GBOLT(I+1,J,K)
Endif

gpdf = gridptdiff/xh
If(ABS(GBOLTCHK2-GBOLTINIT).LE.1.E-06) Then
  If (ABS(gpdf).GE.ABS(ECrit)) Then
    Write(11,*)I,J,K,I+1,J,K
  ENDIF
Endif

C

C Line 231 Check for candidate below a current bolt gridpoint
C

If(J.GE.1) Then
  GBOLTCHK3= GBOLT(I,J-1,K)
  IF(ABS(GBOLTCHK3-GBOLTINIT).LE.1.E-06) Then
    gridptdiff= w(I,J,K)-w(I,J-1,K)
    gpdf = gridptdiff/yk
    If (ABS(gpdf).GE.ABS(ECrit)) Then
      Write(11,*)I,J,K,I,J-1,K
    ENDIF
  endif
Endif

120 CONTINUE
RETURN
C Subroutine LowerBound

C

SUBROUTINE LowerBound(NX,NY,NZ,IGchoice,EP)
IMPLICIT REAL (A-H,O-Z)
IMPLICIT INTEGER (I-N)
REAL CBOLT,GBOLT,g
COMMON /CBOLT/ CBOLT(0:500,0:500,0:500)
COMMON /GBOLT/ GBOLT(0:500,0:500,0:500)
COMMON /g/ g(0:500,0:500,0:500)
COMMON /x/ x(0:500)
COMMON /y/ y(0:500)
COMMON /z/ z(0:500)

C Set the lower boundary, which represents the ground. If IGchoice
C equals= 1 -> lower bdry. is flat, potential = 0.0.
C
If (IGchoice.EQ.1) Then
113 Read(401,*,END=114)I,K,J
   g(I,0,K)=EP
   GBOLT(I,0,K)=EP
   Go to 113
114 Continue
Endif
C
C Set the lower boundary, which represents the ground. If IGchoice
C equals= 2 -> lower bdry. is periodic hilly, with peak at 1 grid
C height, potential = 0.0
C
If (IGchoice.EQ.2) Then
C Set the lower boundary, which represents the ground. If IGchoice C equals= 3 -> lower bdry. is a flat plane with a building roughly C the size of the Empire State building, or 5 cells high, by 1 x 2 C cells at the base, or roughly 1365' tall and 273’ x 546’ at the base C

If (IGchoice.EQ.3) Then

133  Read(403,*,END=134)I,K,J  
g(I,0,K)=EP  
Do 135 L=0,J  
    GBOLT(I,L,K)=EP  
135  Continue  
Go to 133  
134  Continue  
Endif

C Set the lower boundary, which represents the ground. If IGchoice C equals= 4 -> lower bdry. is a 'fractal' surface I generated using my C random number generator. I stepped through each grid point of the C x-y plane and assigned a value between 0 and 7 as a z-component. C

If (IGchoice.EQ.4) Then

143  Read(404,*,END=144)I,K,J  
g(I,0,K)=EP  
Do 145 L=0,J  

GBOLT(I,L,K)=EP

145 Continue
    Go to 143

144 Continue
Endif
Return
End

B.1.1 Random Number Generator

A random number generator is one that ideally provides a series of numbers, possibly
within a specified range, where each value in the sequence bears no particular relationship
to any other values within the sequence. More precisely, if one knows the previous values
generated, \( a_1 \) to \( a_{N-1} \), one has no way of estimating what value will be assigned to quantity
\( a_N \).

Various methodologies exist for creating an effective random number generator. Extensive
research continues to optimize computer routines to allow for true random number generated
streams, ones with no hidden structures, repetitions, or modes.

For my purposes I used the random number generator noted in my venerable copy of
Etters "Structured FORTRAN 77 for Engineers and Scientists" [Etter 1990], which makes
use of the difference between the real number system and the floating point number system
used in computers, and the related value of the machine epsilon. For a good discussion of
this effect, refer to the article by Khali Kalbasi in the April 1990 issue of "IEEE Potentials".

SUBROUTINE RANDOM(ISEED,RVAL)
    Integer ISEED
    Real RVAL
    ISEED = 2045*ISEED + 1
    ISEED = ISEED - (ISEED/1048576)*1048576
    RVAL = REAL(ISEED +1)/1048577.0
    RETURN
END
The subroutine takes advantage of the single precision capability of the UNIX system, generating values that are too large for the system to store. The result is the generation of a random number, between 0.0 and 1.0. This particular formulation of a random number generator requires a seed value to initiate the generation of a stream of random numbers. If the same seed value is input, the same stream of disassociated values is produced. This was an important aspect for my envisioned computer code. For details of the algorithm, one may refer to *A Portable Random Number Generator for Use in Signal Processing*, by S. D. Stearns from Sandia National Laboratories Technical Reports.
APPENDIX C

SUGGESTED NEW GUIDELINES

As I have mentioned in my thesis, through my thesis research together with my background in movement and dance I have arrived at what I believe to be an improvement on the current nationally approved recommendation for personal lightning protection. While my suggested revision is not directly related to my thesis work, if it is indeed found to be an improvement, it could potentially save lives.

C.1 Current Guidelines

A review of the current literature or web-sites yields these examples of the current guidelines for what to do if caught for instance in a field and you believe a lightning strike is imminent.

From the National Lightning Safety Institute’s website, we find for instance, at http://www.lightningsafety.com/nlsi_pls/lst.html

“2. IF OUTDOORS...Avoid water. Avoid the high ground. Avoid open spaces. Avoid all metal objects including electric wires, fences, machinery, motors, power tools, etc. Unsafe places include underneath canopies, small picnic or rain shelters, or near trees. Where possible, find shelter in a substantial building or in a fully enclosed metal vehicle such as a car, truck or a van with the windows completely shut. If lightning is striking nearby when you are outside, you should:

A. Crouch down. Put feet together. Place hands over ears to minimize hearing damage from thunder.

B. Avoid proximity (minimum of 15 ft.) to other people.”

Similarly, from the Department of Commerce, National Oceanic and Atmospheric Administration, Office of Oceanic and Atmospheric Research, National Severe Storms
Laboratory, we find these guidelines from Bill Roeder with the National Weather Association (http://www.nwas.org):

"Level-5: USE THIS ONLY AS A DESPERATE LAST RESORT! If you are outside and far away from a safer place, proceed to the safest location. If lightning is imminent, it will often give a few seconds of warning: hair standing up, tingling skin, light metal objects vibrating, seeing corona discharge, and/or hearing a crackling or "kee-kee" sound. If you are in a group, spread out so there are several body lengths between each person. Once spread out, use the lightning crouch - put your feet together, squat down, tuck your head, and cover your ears.

When the immediate threat of lightning has passed, continue heading to the safest place possible. Remember, this is a desperate last resort; you are much safer following the previous guidance and avoiding this high-risk situation."

In the above two examples and elsewhere the suggestion for what to do if you are caught out in the open and believe a lightning strike is probable is that you should crouch and have your feet close together and cover your ears. The reasoning is as follows: crouching makes you a lower target and less likely to be struck.

Having your feet together, or as I have read in some instances balancing on one foot, comes from the fact that many people, I believe the great majority, are harmed not by a direct lightning strike, but by the lightning striking near them and then the current traveling through the ground to potentially harm them. The level of harm has a relationship to the distance between your two feet along the direction of travel of the lightning current. Thus, if you were standing with a distance of .5 meters between your two feet and a bolt struck twenty meters to your right or left, you would have a greater chance of suffering adversely than if the bolt struck twenty meters directly in front or behind you. Or if the distance between your two feet were 1 meter rather than .5 meters and the bolt struck to your right or left, you would have a greater chance of harm occurring to you. Covering your ears is of course to protect you from the thunder.
C.2 Proposed Modification to the Guidelines

I would suggest that rather than crouching with your feet together or on one foot, you do what our ancient ancestors would probably want us to do and that is to run. If you can run crouching over, all the better, but run.

Along with a bachelors degree in physics I also have an extensive movement and dance background. I believe in fact that it was Judy Massee, my Dance 110 instructor at Reed College, who taught me initially that when you are running, for 50% of the time you are on one foot and the other 50% of the time you are in the air. Hence, by running, you not only minimize your contact to the ground but you may also attempt to move to safer environs.

By running you have the chance that the lethal part of the wave front might pass underneath you while you are airborne, or at the worst it hits with one foot on the ground, and your momentum might allow you to break contact for a critical instant. As I noted before, if you can run and crouch, that would be ideal, but if you can’t, I would assert the benefits of running outweigh your being approximately one meter higher in relation to a lightning discharge whose length is of the order of one to twenty kilometers.
REFERENCES


121
BIOGRAPHICAL SKETCH

Brian Clay Graham-Jones

Well, Brian...he’s just this guy, you know?

Brian Graham-Jones was born Brian Clay Jones on July 31st, 1956 in Atwater, California. Subsequently Brian has lived in Louisiana, Alabama, Nebraska, Hawaii, Long Island New York, Missouri, Oregon, Los Angeles, Tallahassee, and Manhattan. Brian received his Bachelor of Arts degree in Physics from Reed College in 1981 and has worked as a service station attendant, pineapple picker, sorter/strapper, waiter, floor manager, Zellerbach Paper Company salesman, Griffith Park Observatory guide, Northrop Low Observables aerospace engineer, relational database developer/manager, GIS programmer, assistant dean, institutional researcher, and analyst. In 1993, upon moving to Tallahassee, Brian began taking classes at the Florida State University Math Department, which has led to this thesis and the Master of Science degree in Applied Math.

Brian also started, with two other friends, Chiaroscu - A Performance Gallery, in Portland, Oregon; appeared in 'Hawaii 5-0' (fencing) and 'Moonlighting' (Bruce Willis look-alike); worked for nine years with the Mickee Faust Club, creating original social-political-cultural cabaret/theater and film; performed with the Florida State University Flying High Circus; performed and choreographed dance and stage combat; acted, written, and directed plays; built sets, costumes and props; taught ballroom; played the ice block; created the annual tradition of ”Punk’n Chuck’n, The Pentathlon Plus of Halloween”; and is currently very happily residing in Manhattan, New York.