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Exploration of Metacognition and Non-Routine Problem Based Mathematics Instruction on Undergraduate Student Problem Solving Success

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EXPLORATION OF METACOGNITION AND NON-Routine PROBLEM BASED
MATHEMATICS INSTRUCTION ON UNDERGRADUATE STUDENT
PROBLEM SOLVING SUCCESS

By

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This dissertation is dedicated to my loving wife, Susan. Not only did she encourage and support me during my protracted tenure as a student, she endured over twenty years of frequent and prolonged separations during my career in the Navy. As I pursued my dreams, she unselfishly put her career on hold and nearly single-handedly raised our children who also suffered greatly from my recurring absences.

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ABSTRACT

The purpose of this research was to examine and attempt to influence the problem solving processes used by typical undergraduate mathematics students. A structured problem solving methodology designed to foster a heuristic way of reasoning was introduced and a study of how this treatment affected the participants' non-routine problem solving styles and abilities was performed.

An approximately equal mix of male and female student participants self-selected into three sections of precalculus algebra at a private southeastern institution. Each section consisted of approximately 15 students. The researcher and one other professor instructed the three sections. All sections were instructed using the same set of notes, were given the same assignments, and covered the same mathematical topics in the same order.

The research model consisted of the following elements.
1. Testing students in treatment and control sections for routine algebra skills necessary to successfully find solutions to specific non-routine problems.
2. Testing students in treatment and control sections for ability to solve non-routine problems that can be handled using the routine algebra skills on which they had previously been tested.
3. Providing metacognitive control practice and instruction in the treatment section using a combination of weekly homework and in-class assignments.
4. Conducting pre- and post-treatment videotaped interviews of four treatment section students engaged in non-routine problem solving.
5. Analyzing interview records in an attempt to determine if any changes in metacognitive control occurred.
6. Examining statistical evidence gathered from the pre- and post-treatment examinations.
Results of the study were mixed. A significant increase in the statistical correlation between resources (mathematical facts and procedures) and non-routine problem solving success indicated that students may have benefited from the treatment by improving their metacognitive control of resources. However, analysis of the videotaped interviews did not reveal any significant change in the way students approached non-routine problems. Group statistical evidence comparing treatment to control sections seemed to substantiate most of what was observed during the interviews.
A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. (Polya, 1957, p.v)

Learning how to help students at every level to become successful problem solvers has emerged as one of the most important contemporary research issues in mathematics education. According to the Mathematical Association of America (MAA) "every college graduate should be able to apply simple mathematical methods to the solution of real-world problems" and "be expected to go beyond routine problem solving to handle situations of greater complexity and diversity, and to connect ideas and procedures more readily with other topics both within and outside mathematics" (Mathematical Association of America, 1998, p. 1). These expectations have evolved as a logical progression of the necessity for school students to achieve mathematical problem solving competency. In their guiding document, Principles and Standards for School Mathematics, the National Council of Teachers of Mathematics (2002) states, "a major goal of mathematics is to equip students with knowledge and tools that enable them to formulate, approach, and solve problems beyond those that they have studied" (p. 335).

Educators and researchers throughout the country are engaged in a myriad of curriculum development, implementation, and assessment activities in an effort to bolster lackluster student performance, especially as compared to contemporaries in other developed countries (Third International Mathematics and Science Study, 1995, and
Trends in Mathematical Science and Mathematics Study, 1999, for example). Because the development of students' problem solving abilities has been identified as such a fruitful source of improvement it is imperative that we strive to more fully understand the complex underlying cognitive, affective, and social mechanisms that successful problem solvers employ.

**Rationale**

Researchers have found that a large number of students are unable to solve moderately difficult non-routine problems even though they seem to possess the requisite factual and procedural knowledge necessary to do so (Fitzpatrick, 1994; Marshall, 1988; Schoenfeld, 1985; Selden, A., Selden, J., Hauk, S., & Mason, A., 2000). This disconnect between knowledge and effective application of knowledge in novel situations has been linked to absent or ineffective metacognitive control. These studies clearly indicate that students who exercise higher metacognitive control are better able to solve non-routine problems, "yet, there have been few studies of the effect of metacognition on mathematical performance" (Crawford, 1998, p. 4). For the most part, previous work in the field, at the undergraduate level, lacks continuity. Replication of results and employment of significantly similar methods of analysis are scarce.

The purpose of this research is to enhance the body of knowledge linking undergraduate mathematics student metacognition to successful problem solving by exploring connections between factual knowledge, procedural knowledge, metacognitive control and non-routine problems.

**Definitions**

**Factual and procedural knowledge.** As resources available to the problem solver, mathematical facts and procedures consist of specific information relevant to a given problem. These resources associated with competency on routine tasks are normally developed during routine exercises and mathematical drill. For example, most college algebra students can use their factual and procedural knowledge to find the slope of a line.
between two points. They know the fact that the algorithm \( \frac{y_2 - y_1}{x_2 - x_1} \) will yield the desired result when the procedure for using it is carried out properly.

**Metacognitive control.** Metacognition is "the act of thinking about one's own thinking . . . choosing and planning what to do and monitoring what is being done" (Crawford, 1998, p. 4). According to Schoenfeld (1985), control deals with "how one selects and deploys the [mathematical] resources at one's disposal" (p. 13). Accordingly, metacognitive control deals with the regulation of cognitive activities and is the mechanism students use while deciding when, how, and if they will use the mathematical facts and procedures at their disposal for planning, monitoring, and checking activities (Schoenfeld, 1985).

**Non-routine problems.** A non-routine problem is defined as "a cognitively non-trivial task; that is, the solver does not already know a method of solution" (Selden, A., Selden, J., Hauk, S., & Mason, A., 2000, p. 129). Non-routine problems require solvers to use facts and procedures in unfamiliar ways. Any one specific problem is classified as routine or non-routine not by the structure or content of the problem, but rather, by the previous experiences of the solver. For instance, a problem that can be solved mechanically by a person who has past experience working with exactly the same or very similar situations is more appropriately defined as a routine exercise.

**Questions**

This research explores how college precalculus algebra students who have practiced strategies designed to foster metacognitive control access, manage, employ, and monitor the mathematical resources available to them while attempting to solve non-routine problems. Specific questions addressed by this research are:

1. Will a method of helping college precalculus algebra students develop their metacognitive control result in significantly better problem solving ability as compared to students who do not learn the strategies?
2. How does a college precalculus algebra student's problem solving style change as a result of learning and practicing metacognitive control strategies?
3. Do students who practice the metacognitive control strategies when directed by class assignments throughout the semester retain them in other settings?

**Dissertation Format**

Chapter 2 discusses the theoretical framework within which the study was conducted and summarizes literature pertinent to the study. Chapter 3 describes the participants, instructional (treatment) materials, instruments, and analysis procedures. Chapter 4 contains data collected, analysis and interpretation of those data. Chapter 5 presents conclusions, implications for practice, limitations, and directions for further study.
CHAPTER 2
THEORETICAL FRAMEWORK

This chapter describes the theoretical framework for this study as well as significant previous work relevant to undergraduate problem solving, metacognition, and how the two are related.

Although fostering better non-routine problem solving ability has been convincingly identified as a worthwhile goal for contemporary mathematics education, previous work addressing the connection between metacognition and non-routine problem solving at the undergraduate level is fairly limited in scope and depth. A review of the literature applicable to this study, coupled with the personal experiences of the researcher during the analysis phase, revealed a glaring need for developing improved ways of interpreting verbal transcripts as data.

Constructivism

This study assumes that learning occurs according to the tenets of the constructivist viewpoint. Constructivism asserts that knowledge, no matter how defined, is in the heads of persons. According to this framework, knowledge is constructed by an individual's interactions with her/his environment. George Booker (1996) provides clear insight by saying: "An essential feature of this view is that existing conceptions, whether gained from everyday experiences or previous learning, guide the understanding and interpretation of any new information or situation that is met" (p. 381).

According to the constructivist paradigm, mathematical concepts are human inventions rather than objectively situated truths. Teachers and learners are meaning-makers who construct knowledge based on previous experiences and interactions with each other. Meanings do not reside in the "words, actions, and objects independently of the interpreter" (Booker, 1996, p. 382), rather they are a product of how teachers and
learners make sense of them. From a constructivist perspective, "learning mathematics results from the students' thinking, not from the training of behaviors" (Stigler, Fernandez, & Yoshida, 1996, p. 150).

Booker (1996), highlights the difference between the objectivity of traditional behaviorism teaching methodology and the subjectivity of constructivism stating, No longer is the object to be one of implanting an ideal structure in each student's mind, but to understand the student's conception for what it is. [Also], . . . it draws attention to the need for activity to be in the minds of the students and not merely in their hands if concrete experiences are to be of any help in the process of acquiring mathematical knowledge. (p. 385)

One struggle that must be dealt with is that of trying to get students to realize that mathematics is not a bag of tricks or a bunch of shortcuts that need to be memorized. Unfortunately, typical mathematics students have been conditioned to believe teachers are the ultimate authority, responsible for stating the correct answer and dismissing incorrect or cumbersome procedures. Students have traditionally expected their teachers to tell the class specifically when to carry out procedures, the exact steps to be followed, and what form the acceptable solution to a problem must take. In contrast, constructivism holds that students must learn to use their own metacognitive processes to solve problems and verify the validity of their solutions. In short, they must learn to think for themselves.

**Problem Solving**

Most researchers generally, at least in principle, accept frameworks for mathematical problem solving similar to what Schoenfeld (1985) has suggested. That is, a student's ability to solve non-routine problems is a function of how well they employ and regulate relevant cognitive and affective characteristics. The four components of knowledge and performance identified as fundamental to the problem solving process are resources, heuristics, metacognitive control, and belief systems.

*Resources* have been described earlier as factual and procedural knowledge. These are the basic mathematical tools students have available to them. According to Schoenfeld, resources including algorithmic procedures, routine non-algorithmic
procedures, facts, intuitions, and "understandings about the agreed upon rules for working in the domain" (1985, p. 15) are the foundation upon which all the other categories depend. To illustrate the point, suppose a student who does not know how to find derivatives is attempting to solve a problem that requires knowing the slope of a tangent to a curve. Regardless of how clever or persistent the attempt, success is virtually impossible because the student does not possess the tools essential for success.

*Heuristics* are basically strategies used to explore, analyze, and probe aspects of non-routine problems in an attempt to formulate pathways to a solution. In his seminal work, *How to Solve It*, Polya (1957) describes the tools and techniques of heuristics that have generally been credited as being the fundamental framework for the development of most efforts to improve student problem solving ability (National Council of Teachers of Mathematics, 2000; Schoenfeld, 1985). Examples of heuristic strategies include working backwards, exploiting similar or related problems, drawing figures, restating the problem, examining special cases, and conjecturing.

As mentioned earlier, *metacognition* is loosely defined as thinking about your own thinking. More formally

Metacognition refers to higher order thinking which involves active control over the cognitive processes engaged in learning. Activities such as planning how to approach a given learning task, monitoring comprehension, and evaluating progress toward the completion of a task are metacognitive in nature. (Livingston, 1997, p. 1)

According to Schoenfeld (1985), Lester, Garofalo, & Kroll, (1989), and Flavell (1987) as cited in Livingston (1997), the broad domain of metacognition can be divided into two basic components: awareness of one's cognition (metacognitive knowledge), and regulation of cognition (metacognitive control or regulation). *Metacognitive control* involves decisions that problem solvers make regarding if, when, and how they will use their factual knowledge, procedural knowledge, (resources) and heuristics while attempting to cope with non-routine problems.

*Beliefs* about mathematics can be subdivided into two categories. One kind of belief involves how students see mathematics as a discipline. Such things as what it means to think mathematically, how unfamiliar problems should be viewed, and
perceived usefulness of mathematics are issues related to this first kind of belief. The second component of belief deals with how students feel about themselves. Confidence in mathematical ability, assignment of reason for success or failure, and determination of reasonable effort are examples of components measured in this domain (McLeod, 1989).

Schoenfeld's 1985 book, *Mathematical Problem Solving*, offers an extensive overview of his theoretical framework and methodological approach to the exploration of undergraduate mathematical problem solving as well as comprehensive descriptions of experimental and observational studies he has conducted. His theoretical overview describes how students' problem solving performances are determined by how well they manage the four domains of knowledge and behavior briefly described earlier in this manuscript: resources, heuristics, metacognitive control, and beliefs.

From his studies Schoenfeld observed that:

a) "Students in a problem solving course can learn to employ a variety of problem-solving strategies" (p. 240).

b) "Explicit heuristics instruction does (or can) make a difference with regard to problem-solving performance" (p. 214).

c) "A problem-solving course that focuses on understanding and analysis can have a significant impact both on perception and performance" (p. 265).

d) "Courses in which there is a significant emphasis on metacognitive concerns...can have a significant effect on students' behavior at the control level" (p. 317).

e) "The ways we teach mathematics, and the lessons that students abstract from their experiences in doing mathematics, are equally important in shaping their mathematical behavior" (p. 375).

His studies included explicit instruction on heuristics and an unverified assumption that his students possessed the necessary requisite procedural and factual knowledge resources to successfully find solutions to specific non-routine problems.

Schoenfeld introduces and demonstrates a framework for the analysis of problem-solving protocols (transcripts) that first requires parsing (separating) attempts into episodes (stages) then transcribing the episodes into a graphical representation called a
timeline. Episodes are each parsed into one of six categories depending on the students' statements or actions: read, analyze, explore, plan, implement, and verify. Metacognitive control is analyzed by examining each episode and the transitions between episodes using a set of predetermined questions assigned to each stage or transition.

Selden et al. investigated the inability of differential equations course students to solve non-routine calculus problems. Their approach included testing each student's cognitive resources on a "routine test" (p. 129) designed to measure factual and procedural knowledge, then comparing the results to how well the students performed on related non-routine problems. They found there was a positive correlation (coefficient $r = 0.68$) between their students' ability to solve non-routine problems and their knowledge of facts and procedures (resources). However, many of the students who presumably possessed the necessary factual and procedural knowledge necessary to successfully solve a given non-routine problem were frequently unsuccessful in their attempts. Selden noted that, "The inability of many otherwise successful students to access and effectively use their factual knowledge of calculus in non-routine problem solving is perhaps the most striking feature of our data" (p. 144).

One recommendation on how to improve student performance that the investigators offer is to scatter throughout a course a considerable number of problems for students to solve without first seeing very similar worked examples. We mean to suggest a collection of problems that cover the course well and that most of the students really can eventually solve, albeit with some difficulty. The idea is that the students would struggle with these problems and reflect on their solutions more than they would with traditional exercises mimicking worked examples. (p. 150)

Metacognition

The terms self-regulation, monitoring, control, and executive decision are frequently used interchangeably throughout the literature to describe the concept of metacognition. Metacognition can be thought of as having two distinct components: metacognitive knowledge which is a personal awareness of how one thinks and
metacognitive control which consists of planning, evaluating, monitoring, and verifying
cognitive activities (Schoenfeld, 1985, 1992; Livingston, 1997; Lester, Garofalo, & Kroll,
1989). The metacognitive control component is central to this study.

can be characterized as follows.

These processes include planning activities prior to understanding a
problem (predicting outcomes, scheduling strategies, various forms of
vicarious trial and error, etc.), monitoring activities during learning
(testing, revising, rescheduling one's strategies for learning), and checking
outcomes (evaluating the outcome of any strategic actions against criteria
of efficiency and effectiveness). (p. 183)

Previous work indicates that student problem solving performance is
directly linked to how actively and efficiently students employ their
metacognitive control mechanisms. Students who self-regulate their cognitive
processes are unquestionably better at solving non-routine problems. (Examples
include Schoenfeld, 1985, 1988; 1992, Crawford, 1998; and Fitzpatrick, 1994.)

Unfortunately, even though it is widely recognized that mastery of metacognitive
skills is fundamental to competency as a problem solver, traditional instruction practices
rarely address the issue explicitly or implicitly. Most mathematics courses emphasize
facts and procedures (resources) while overlooking or only superficially covering topics
and concepts requiring more desirable higher-level thinking (metacognition).
Furthermore, even when such instruction is present it has been observed that most
students do "not grasp the significance of the skills they had been taught and
subsequently [apply] them only when prompted to do so" (Campione, Brown, and
Connell, 1988, p. 94).

Campione, Brown, and Connell, in their 1988 article, describe a simple method of
helping students develop their metacognitive skills that resulted in impressive gains using
what they call the "planning-drawing-doing" routine (p. 105). Each phase of the routine
consists of general heuristic techniques, in three fundamental steps, recorded by each
student separately and in succession. This strategy helps them to organize their problem-
solving approach while providing a record that helps them see and reflect on what they have done.

In the planning phase, students write all relevant facts that are given or can be inferred by the problem. The drawing phase prompts them to sketch figures, diagrams, graphs or other visual images based on what is known. The final phase, doing, is where computations based on the other two phases occur. This process employs an uncomplicated scheme to foster many of the heuristic strategies suggested by Polya (1957) and others.

Unfortunately, it seems that even though it is considered vital to student success, mathematical problem solving research efforts have been declining in numbers over the past few years. (Lester, 1994). A review of the literature yields a fairly shallow variety of studies related to metacognition and problem solving, particularly at the undergraduate level.
CHAPTER 3
RESEARCH DESIGN

Introduction

This research is exploratory in nature, relying on a mixed-methodology design. Qualitative and quantitative techniques were used in a complementary manner in an effort to best understand the research questions. Mixing the two paradigms widens the breadth of available information and serves to strengthen findings by providing multiple perspectives (Creswell, 1994). The methods employed are best termed "quasi-experimental" because even though treatment and control subjects are used they were not randomly assigned (Brown & Dowling, 1998).

The focus of this study is on (a) fostering metacognitive control in college precalculus algebra students by introducing and practicing a structured problem solving methodology designed to foster a heuristic way of reasoning, and (b) how this treatment effects the participant's non-routine problem solving style and ability. The research model consisted of the following elements.

1. Testing students in treatment and control sections for routine algebra skills necessary to successfully find solutions to specific non-routine problems.
2. Testing students in treatment and control sections for ability to solve non-routine problems that can be handled with the routine algebra skills on which they had previously been tested.
3. Providing metacognitive control practice and instruction in the treatment section using a combination of weekly homework and in-class assignments.
4. Analyzing taped interviews of treatment section students to detect changes in metacognitive control.

The results build on the previous work of Schoenfeld (1985) and Selden, et al. (2000) by describing how precalculus algebra students' metacognitive control effects their problem solving and if control can be significantly influenced in a treatment population using a technique inspired by the Campione, Brown, and Connell (1988) planning-drawing-doing routine. The technique is explained later in this chapter.

Participants

During the spring 2003 semester 59 participants self-selected into three sections of precalculus algebra at a private coeducational southeastern university. A diverse population of approximately 2000 undergraduates attend the university. Many students who take precalculus algebra at the university do so in order to satisfy the general education requirement for mathematics. Others enroll to prepare for the sequence of calculus courses. The 2002 average Scholastic Assessment Test (SAT) score for entering freshman was 1090, slightly above the national mean of 1020 and state mean of 995.

The researcher and one other professor who agreed to participate in the study taught the three sections. All sections covered the same material in the same order. The researcher taught students in the treatment section (n = 23) that met in the morning and one of the control sections (n = 19) that met in the afternoon. The assisting professor taught a control section (n = 17) that met during the same time as the treatment section. Students who did not complete all written examinations and at least seven of nine non-routine treatment assignments were excluded from the study. There were 14 treatment section students and 16 control section students in the researcher's two sections and 13 students in the assisting professor's section who met the criteria for inclusion in the study. Characteristics of the sections are shown in Table 3.1.

All sections met on Monday, Wednesday and Friday for 14 weeks in 55-minute periods. The researcher and assisting professor met weekly to guard against obvious differences in instruction, discuss class progress, and ensure all sections were working at roughly the same pace. Both instructors conducted each meeting using a lecture/discussion format, followed the same set of notes, and assigned the same sets of periodic
Table 3.1. Characteristics of the Sections.

<table>
<thead>
<tr>
<th>Section</th>
<th>Gender</th>
<th>Year in College*</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Male</td>
<td>Female</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control</td>
<td>15</td>
<td>14</td>
<td>1.79</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>(52%)</td>
<td>(48%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treatment</td>
<td>5</td>
<td>9</td>
<td>1.29</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>(36%)</td>
<td>(64%)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(*) 1 = Freshman, 2 = Sophomore, 3 = Junior, 4 = Senior

**Study Questions**

The research was conducted to explore how college precalculus algebra students' non-routine problem solving style and ability is influenced by explicit problem solving instruction. The specific questions addressed follow.

1. Will a method of helping college precalculus algebra students develop their metacognitive control result in significantly better problem solving ability as compared to students who do not learn the strategies?
2. How does a college precalculus algebra student's problem solving style change as a result of learning and practicing metacognitive control strategies?
3. Do students who practice the metacognitive control strategies when directed by class assignments throughout the semester retain them in other settings?

**Non-Routine Examinations**

Treatment and control section problem solving proficiency was measured at the beginning and end of the semester using non-routine problem examinations. When developing the examinations, care was taken to choose problems requiring more than recall of mechanical methods or common algorithms. The problems were applied in
nature, demanding that students synthesize their knowledge of mathematical facts and procedures (resources) in unfamiliar ways. Both resource examinations were piloted during the fall 2002 semester.

The examination used at the beginning of the semester included questions requiring mathematical proficiency considered prerequisite for enrolling in the course. In an effort to motivate participants to conscientiously attempt to solve the problems, they were advised that regardless of how well they did, credit would be awarded for sincere attempts as evidenced by the thoughtfulness of work that they submitted. Correct solutions were rewarded with slightly higher scores. Students were given 50 minutes to complete the examination. Scores were reported to the students but they were not factored into their final grade. An example of the pretreatment non-routine problems is given in Figure 3.1. The complete Pretreatment Non-Routine Problem Examination is located in Appendix A.

Mr. Lee takes his wife and two children to an amusement park. If the price of a child's ticket is \( \frac{1}{2} \) the price of an adult ticket and Mr. Lee pays a total of $12.60, find the price of a child's ticket (Kaprov, & Bronk, 1999, p. 158).

Figure 3.1. Sample Item from Pretreatment Non-Routine Problem Examination.

Success on the Post-Treatment Non-Routine Problem Examination required an understanding of topics and concepts covered throughout the course. Objectives for the course included (a) understanding the concept of function, (b) understanding the use of functions to model motion and change in the real world, (c) being familiar with linear, exponential and trigonometric functions, (d) being able to communicate mathematics orally and in writing, and (e) being able to use a computer and graphing calculator to solve problems.

Students were motivated to conscientiously attempt to solve the problems on the Post-Treatment Non-Routine Problem Examination by weighting their score on the examination as 5% of the course average. Similar to the Pretreatment Non-Routine
Problem Examination, they were advised that regardless of how well they did, credit would be awarded for sincere attempts as evidenced by the thoughtfulness of work submitted. Correct solutions were rewarded with slightly higher credit. Students were given 50 minutes to complete the examination. An example of the post-treatment non-routine problems is given in Figure 3.2. The complete Post-Treatment Non-Routine Problem Examination is located in Appendix B.

Suppose that tides vary from 5 feet above to 5 feet below sea level and that successive low tides occur every 20 hours. Assuming that on a particular day low tide occurs at 6:00 am, determine the water level at noon.

Figure 3.2. Sample Item from Post-Treatment Non-Routine Problem Examination.

Table 3.2. Non-Routine Problem Examination Grading Rubric

<table>
<thead>
<tr>
<th>Points Awarded</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>No errors</td>
</tr>
<tr>
<td>2</td>
<td>Minor error on setup, calculations correct or</td>
</tr>
<tr>
<td></td>
<td>No errors on setup, minor calculation error</td>
</tr>
<tr>
<td>1</td>
<td>Wrong, some reasonable work shown</td>
</tr>
<tr>
<td>0</td>
<td>Completely wrong, nothing makes sense</td>
</tr>
</tbody>
</table>

Following administration of each non-routine examination the grading reliability was established by comparing the coding of two independent graders, the researcher and the assisting professor (Brown & Dowling, 1998). The inter-rater reliability for the pre-test was 95%. The inter-rater reliability for the post-test was 94%. Consistency of
scoring for the Pre- and Post-Treatment Non-Routine Examinations was achieved by using the rubric given in Table 3.2.

**Resource Examinations**

Treatment and control section student procedural and factual knowledge was measured at the beginning and end of the semester immediately following the non-routine tests using resource examinations. As with the non-routine problem solving tests, the examination used at the beginning of the semester included questions requiring proficiency considered prerequisite for enrolling in the class. The resource examinations were designed to measure recall of facts and procedures that applied directly to specific problems on the non-routine examinations.

The Pretreatment Resource Examination consisted of 25 questions. Groups of five questions on the examination corresponded to one of five problems on the Pretreatment Non-Routine Examination. Students who could successfully answer each of the five resource questions associated with a non-routine problem demonstrated that, presumably, they possessed the resources necessary to successfully solve the problem. Students were given 30 minutes to complete the Pretreatment Resource Examination. Pretreatment Resource Examinations were graded using the rubric given in Table 3.3. An example question from the Pretreatment Resource Examination is given in Figure 3.3. The entire Pretreatment Resource Examination is located in Appendix C.

<table>
<thead>
<tr>
<th>Points Awarded</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Correct solution</td>
</tr>
<tr>
<td>0</td>
<td>Incorrect Solution</td>
</tr>
</tbody>
</table>

The entire Pretreatment Resource Examination is located in Appendix C.

Table 3.3. Pretreatment Resources Examination Grading Rubric

The end of course resource examination required familiarity with general knowledge and computational skills associated with linear, exponential, and sinusoidal modeling. The examination was designed to assess how well students achieved the
objectives of the course. Each of the questions on the Post-Treatment Resource Examination correspond to one or more of three problems on the Post-Treatment Non-Routine Examination administered at the end of the semester. The examination served as the course final examination and counted for 15% of a student's overall average. Students were given 180 minutes to complete the examination. An example question from the Post-Treatment Resource Examination is given in Figure 3.4. The entire Post-Treatment Resource Examination is located in Appendix D.

Write an expression representing the situation, "5 times x is 7 more than y."

Figure 3.3. Sample Item from Pretreatment Resource Examination.

Find a possible formula for the trigonometric function shown below. State your answer in the form

\[ y = A \sin(B(t - h)) + k \text{ or } y = A\cos(B(t - h)) + k. \]

Figure 3.4. Sample Item from Post-Treatment Resource Examination.

Consistency of scoring for the Pre- and Post-Treatment Resource Examinations was achieved by using the rubric given in Table 3.4. As with the non-routine examinations, grading reliability was established by comparing the coding of two independent graders, the researcher and assisting professor (Brown & Dowling, 1998).
The inter-rater reliability for the pre-test was determined to be 100%. The inter-rater reliability for the post-test was 94%.

A pilot study of resource and non-routine problem examinations was conducted during the fall 2002 semester. Feedback from students concerning how they interpreted the examination questions was used to construct the final version of each instrument (Brown & Dowling, 1998). Piloting and collaboration with the assisting professor during question development enhanced construct validity for all pre- and post-treatment examinations. Both the researcher and assisting professor have over five years of experience teaching precalculus college algebra. The assisting professor has considerable experience with similar measures in a research setting.

Results of the four examinations were analyzed and compared to qualitative evidence garnered during videotaped interviews to explore the first research question, "Will a method of helping college precalculus algebra students develop their metacognitive control result in significantly better problem solving ability as compared to students who do not learn the strategies?" Chapter 4 outlines the results of the analysis.

Table 3.4. Post-Treatment Resources/Final Examination Grading Rubric

<table>
<thead>
<tr>
<th>Points Available</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 6 4 2</td>
<td>No errors, correct solution</td>
</tr>
<tr>
<td>8 6 4 2</td>
<td>Minor error on setup, calculations correct or</td>
</tr>
<tr>
<td>7 5 3 1</td>
<td>No errors on setup, minor calculation error</td>
</tr>
<tr>
<td>4 3 2 1</td>
<td>Correct setup, major calculation error</td>
</tr>
<tr>
<td>2 2 1 1</td>
<td>Wrong setup, some reasonable work shown</td>
</tr>
<tr>
<td>0 0 0 0</td>
<td>Completely wrong, nothing makes sense</td>
</tr>
</tbody>
</table>

The Treatment

Inspiration for the study treatment was fostered by Campione, Brown, and
Connell (1988), who describe a method of helping students improve their non-routine problem solving ability. The technique provides instruction and experiences designed to help them access and employ mathematical resources appropriately.

The specific aim of the treatment was to help students develop their metacognitive control using a method similar to what Campione, Brown, and Connell call the "planning-drawing-doing" routine (p. 105).

<table>
<thead>
<tr>
<th>PLANNING</th>
<th>DRAWING</th>
<th>DOING</th>
</tr>
</thead>
<tbody>
<tr>
<td>GIVEN</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30-pound weight at end of 8-foot lever</td>
<td></td>
<td>D = 8 - 3</td>
</tr>
<tr>
<td>30 pound weight 3-feet from fulcrum</td>
<td></td>
<td>D = 5</td>
</tr>
<tr>
<td>How much weight at other end needed to balance?</td>
<td></td>
<td>5W = (3)(30)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>How far is unknown weight from fulcrum?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>INTRODUCED</th>
</tr>
</thead>
<tbody>
<tr>
<td>(weight)(arm) = moment</td>
</tr>
<tr>
<td>Let unknown distance = D</td>
</tr>
<tr>
<td>Let unknown weight = W</td>
</tr>
</tbody>
</table>

5W = 90/5 |
W = 18

Figure 3.5. Elementary Example of Planning-Drawing-Doing Process.
The strategy helps students organize their problem-solving approach while providing a record that enables them to see and reflect on what they have done. In the planning phase, students write all relevant facts that are given or can be inferred by the problem. The drawing phase prompts them to sketch figures, diagrams, graphs or other visual images based on what is known. The final phase, doing, is where computations based on the other two phases occur. This process employs an uncomplicated scheme to foster many of the heuristic strategies suggested by Polya (1957) and others. Figure 3.5 illustrates an elementary example of the planning-drawing-doing method as described by Campione, Brown, and Connell.

For this study the researcher modified the method by adding a "verifying" component. During this phase students were asked to look back at their work and check their solution or at least make sure the results make sense in the context of the problem.

Students in the treatment section were given specific instructions and a demonstration on how to use the planning-drawing-doing-verifying routine during the first week of class, prior to being assigned any graded problems. The demonstration problem is given in Figure 3.6.

The area of the floor of a rectangular room is 315 ft². The area of one wall is 120 ft² and the area of another is 168 ft². The floor and ceiling are parallel. What is the volume of the room?

Figure 3.6. Planning-Drawing-Doing-Verifying Demonstration Problem.
1. **Getting started:**
Begin by labeling four separate pieces of paper Planning, Drawing, Doing, and Verifying. Arrange the paper on a table in front of you so that you can see all four pieces at once.

2. **Use the four areas:**
   Planning Area - Write down all that seems relevant about the problem. Also, write down what you are looking for in the form of a question. As you work through the problem, return to this area to record any new information that you recognize or discover. If stuck, returning to this area may help you find alternate paths to try.

   Drawing Area - Sketch any diagrams, graphs, tables, pictures, etc that you suspect may prove useful. This area may help you to visualize what you are looking for and/or help reveal information you hadn't previously considered or recognized. If stuck, returning to this area may help you find alternate paths to try.

   Doing Area - Introduce suitable notation. Use the facts, procedures, sketches, and other information you recorded in the other areas to work toward a solution. If you get stuck, return to the Planning and/or Drawing Areas to help you think of new ways to approach the problem.

   Verification Area - Once you have found a solution to the problem you should use this area to test that it is correct or seems reasonable. Solutions to most problems can be checked in some way. If you can't verify your solution directly, you should at least be able to make sure that it seems reasonable.

3. **Getting stuck:**
   Getting stuck is a natural part of solving non-routine problems. Also, it's natural to require trying more than one approach until many problems can be solved. If you get stuck, try looking for obvious patterns, exploring simpler or related problems, working backwards, examining special cases, conjecturing, or setting the problem aside and returning to try again later. There is not one perfect way to solve a problem - You may use any method that works as long as you show what you did.

4. **Grading:**
   Showing all work is important - Part of the assignment is to show how the problem is being approached. Even if some attempts are fruitless please show what you tried to do. Simply providing an answer without a record of your attempt will result in a lower grade.

Figure 3.7 Non-Routine Problem Assignment Guidance.
The demonstration was conducted in such a manner so as to emphasize that there is not an established recipe for solving non-routine problems and that non-routine problems are usually not solvable by accomplishing any predetermined steps in a linear fashion. The demonstration was conducted discussion style, making every effort to engage the students by asking for their opinion each step of the way. During the demonstration, treatment section students were supplied with a sheet containing general instructions on how to employ the method. Throughout the remainder of the semester they were given extra copies of the sheet and frequently reminded to refer to it during all non-routine problem attempts. Control section students were not given any instructions nor did they see any demonstrations. Figure 3.7 lists the instructions.

A total of nine special problems were assigned throughout the course of the semester. Problems were carefully selected to coincide with the resources being covered in class. Three of the nine assignments were completed during class with students working in teams of three or four. Approximately 30 minutes were allotted for each in-class problem. The remaining six problems were assigned as homework and were completed by each student independently. Students were given three to five days to complete the take-home problems. After grading, each problem was thoroughly discussed in the treatment section. When multiple ways of approaching the problem were used by students each way was verbally revealed or demonstrated on the board. When control section students' papers were returned, questions were answered but solution methods were not covered in any detail.

According to one rule of thumb relating weight to height among adult males, if a man is 2 inches taller than another, then we expect him to be heavier by 10 pounds. A related rule of thumb is that a typical man who is 70 inches tall weighs 170 pounds. On the basis of these two rules of thumb, find a formula to express the weight of a man as a linear function of height. Be sure to identify the meaning of the letters you use (Crauder, Evans, & Noell, 2003, p. 203).

Figure 3.8 Periodic Non-Routine Problem Assignment Example.
The assignments were graded and counted as part of the lab grade component for the course. Students presumably were motivated to perform well on the problems because they knew the problems accounted for 25% of their overall course average. Control sections were given the same problems, on the same days, but were not given any instruction on using the planing-drawing-doing-verifying technique. An example of the non-routine problems is given in Figure 3.8 Appendix E contains the complete list of non-routine problems.

**Qualitative Measures**

Analysis of taped interviews was primarily used to address the second and third research questions, (2) How does a college precalculus algebra student's problem solving style change as a result of learning and practicing metacognitive control strategies? and, (3) Do students who practice the metacognitive control strategies when directed by class assignments throughout the semester retain them in other settings?

Six treatment section students who possessed the requisite factual and procedural knowledge but had difficulty accessing it in the context of non-routine problems as evidenced by their performance on the examinations at the beginning of the semester were selected for pre- and post-treatment videotaped interviews. The non-routine problem examination administered prior to the treatment consisted of five problems. Each of the five problems on the non-routine examination had five corresponding questions on the resource examination. Students who were chosen for interviews failed to find a solution to one or more of the non-routine problems even though they had demonstrated they possessed the necessary fact and procedure knowledge by correctly answering all of the corresponding questions on the resource examination.

Interviews were analyzed utilizing a technique designed to identify and characterize faulty metacognitive decisions and their consequences. Elements of the scheme and how they were used in this study are described below.

Six students were asked to work in teams of two on two timed non-routine problems at the beginning of the semester while explaining to each other what they were doing. Sessions were videotaped. Recording teams of two students rather than one seems to result in greater access to what they are thinking as decisions are made and
strategies to solve problems are planned (Schoenfeld, 1985). Apparently, students are more likely to verbalize what they are thinking because of awareness that partners expect to be kept informed.

Near the end of the semester, the same teams of students worked on two timed non-routine problems while explaining to each other what they were doing. These sessions were also videotaped. Twenty minutes were allotted for each problem. Matching pre- and post-treatment student attempts were transcribed for analysis. Because one of the original six students interviewed failed to complete at least seven of nine treatment non-routine problems, his team was excluded from further study. Appendix F contains the pre- and post-treatment videotaped interview non-routine problems.

The videotaped interviews were transcribed and partitioned into one-minute intervals. The resulting intervals were categorized or parsed into what Schoenfeld called "episodes" which are basically segments of the session divided into different domains based on the predominant activity occurring during the one-minute segment. Schoenfeld's protocol parsing consisted of six episodes or stages that students were classified in according to what they were doing or discussing during problem solving attempts. The episodes he used included read, analyze, explore, plan, implement, and verify. The original intent of the researcher was to employ the same categories. However, after experiencing considerable difficulty reliably distinguishing between some of them it was decided to combine several of the categories. The categories analyze and explore were combined into an aggregate category named survey. Plan and implement were also combined into a new category simply referred to as plan-implement. This slightly modified version of the scheme resulted in the four categories (read, survey, plan-implement, and verify) used in this study. Students' attempts were categorized during analysis using the indicators listed in Table 3.5.

Following parsing into the four categories, timeline representations of each attempt were constructed. Timeline representations are graphical illustrations of parsed problem-solving episode protocols. At a glance, these timelines show relative time spent in each domain as well as when failures in metacognition that negatively impact the students' problem solving attempt occur. Timeline representations were created for each
of the pre- and post-treatment non-routine problem protocols. Figure 3.9 is an example from this study.

Table 3.5. Problem Solving Attempt Domain Indicators.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Indicator</th>
</tr>
</thead>
</table>
| Read            | • Reads problem statement  
                  | • Rereads problem statement                                               |
| Survey          | • Explores problem space  
                  | • Attempts to understand problem  
                  | • Analyzes problem  
                  | • Trial-and-error calculations                                            |
| Plan-Implement  | • Synthesizes strategy to solve problem  
                  | • Systematic series of purposeful calculations                            |
| Verify          | • Checks calculations  
                  | • Checks reasonableness of solution  
                  | • Checks reasonableness of steps toward solution  
                  | • Explains solution or steps toward solution                              |

Figure 3.9. Sample Timeline Representation.
By studying timeline representations and the frequency and nature of students' metacognitive control failures before and after treatment the researcher explored if and how their problem solving style had changed. The timelines were used to provide a visual tool for understanding how each problem-solving attempt progressed and track the frequency and position of executive decisions that were not reasonable, not appropriate, or not helpful. Executive decisions (metacognitive control) can occur at shifts between the various domains, when any new information is considered, or when subjects decide to abandon their current approach and try something new. According to Schoenfeld (1985), executive decisions or managerial behaviors include selecting perspectives and frameworks for working a problem; deciding at branch points which direction a solution should take; deciding in the light of new information whether a path already embarked upon should be abandoned; deciding what (if anything) should be salvaged from attempts that are abandoned or adopted from approaches that were considered but not taken; monitoring and assessing implementation "online" and looking for signs that executive intervention might be appropriate; and much, much more. (p. 295)

For the purposes of this study, the terms faulty control and faulty metacognition are used to indicate instances where unreasonable, inappropriate, or unhelpful actions, statements, or questions had the potential to hinder the progress of a solution. Overt indicators of faulty metacognition considered in this study are illustrated in Table 3.6.

Protocol parsing reliability was established by comparing the coding of two independent graders, the researcher and the assisting professor (Brown & Dowling, 1998). The percentage of one-minute time intervals that the two graders coded the interview students in the same domain was established using ten-minute samples from two different protocols. The inter-rater reliability for timeline parsing was determined to be 75%. The percentage of one-minute intervals where failures were identified by both coders was established using ten-minute samples from two different protocols. The inter-rater reliability for the identification of metacognitive control failures was determined to be 90%.
Table 3.6. Overt Metacognitive Failure Indicators

<table>
<thead>
<tr>
<th>Category</th>
<th>Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planning</td>
<td>Faulty comments, questions, or actions regarding synthesis of approach to be used.</td>
</tr>
<tr>
<td>Monitoring</td>
<td>Faulty comments, questions, or actions regarding:</td>
</tr>
<tr>
<td></td>
<td>• Appropriateness of actions that have been or will be used.</td>
</tr>
<tr>
<td></td>
<td>• Assessment of progress.</td>
</tr>
<tr>
<td></td>
<td>• Appropriateness of planning.</td>
</tr>
<tr>
<td></td>
<td>• Reasonableness of steps toward solution.</td>
</tr>
<tr>
<td>Checking</td>
<td>Faulty comments, questions, or actions regarding:</td>
</tr>
<tr>
<td></td>
<td>• Verification of steps toward solution.</td>
</tr>
<tr>
<td></td>
<td>• Verification of final solution.</td>
</tr>
<tr>
<td></td>
<td>• Reasonableness of solution.</td>
</tr>
</tbody>
</table>

Quantitative Measures

Numerical data considered for this study consisted of the overall scores from the treatment and control section's four pre- and post-treatment resource and non-routine problem examinations.

The first research question, (1) Does a method of helping college precalculus algebra students develop their metacognitive control result in significantly better problem solving ability as compared to students who do not learn the strategies, was investigated by (a) testing for statistical significance between treatment and control sections on examination score means, and (b) comparing correlation coefficients between the students' performance on the pre- and post-treatment facts and procedures based resources examination and their scores on the associated non-routine problem solving examinations.

The mean scores between all sections on pre-treatment examinations were tested for statistical significance in order to achieve some verification that it was reasonable to assume that treatment and control section students' resource knowledge and non-routine problem solving ability were not significantly different prior to the treatment.

Following the treatment, mean scores between all sections on post-treatment examinations were tested for statistical significance in order to determine whether it was
tenable to believe that treatment section students had benefited from practicing the planning-drawing-doing-verifying routine. Specifically, the null hypotheses tested were:

**Hypothesis I:** There is no statistically significant difference between the change in pre- and post-treatment examination scores for the treatment and control groups.

**Hypothesis II:** There is no statistically significant difference between the treatment and control section Pretreatment Resource Examination score means.

**Hypothesis III:** There is no statistically significant difference between treatment and control section Pretreatment Non-Routine Problem Examination score means.

**Hypothesis IV:** There is no statistically significant difference between treatment and control section Post-Treatment Resource Examination score means.

**Hypothesis V:** There is no statistically significant difference between treatment and control section Post-Treatment Non-Routine Problem Examination score means.

To test these hypotheses two-tailed two-sample t-tests were performed. Differences were tested at the $\alpha = 0.05$ level of significance. Results of these quantitative tests were interpreted in concert with qualitative findings.

Also, the correlation between each section's Pretreatment Resource Examination scores versus Pretreatment Non-Routine Examination score were compared to corresponding post-treatment examination correlations. This comparison was made in order to detect any improvement and possible differences in the magnitude of each section's students' ability to access and employ resources during non-routine problem solving attempts. A significant increase in correlation would be interpreted by the researcher as evidence that students had improved their metacognitive control.
CHAPTER 4
DATA AND ANALYSIS

Introduction

This chapter begins with an explanation of how qualitative data were categorized and processed. Following that, comments regarding each of the two teams studied are presented one at a time. First, the team members are briefly described. Then, portions of each interview protocol associated with faulty executive control are presented along with comments and analysis by the researcher. Each protocol analysis is followed by the associated timeline representation of the attempt. The qualitative portion of this chapter concludes with the researcher’s observations about each student’s problem solving style and whether or not it has changed as evidenced by the data collected.

Quantitative data and analysis are then discussed. Treatment and control section descriptive statistics are presented and analyzed for significant differences between the mean scores on pre- and post-treatment resource and non-routine problem examinations using two-tailed t-tests. The correlation between treatment and control group scores for resource versus non-routine problem solving examinations are also presented and interpreted. The chapter concludes with some remarks concerning the quantitative results.

Qualitative Analysis

Pre- and post-treatment interviews were analyzed utilizing the protocol parsing and analysis scheme described in Chapter 3. Once parsing was completed, each episode was analyzed by identifying instances where inappropriate or missing action at the control level occurred. Timeline representations of each attempt were then constructed. This framework provided a mechanism for tracing the frequency, position, and consequences of metacognitive control failures. By studying timeline representations and
the nature of students' metacognitive control failures before and after treatment the researcher investigated if and how their problem solving style had changed.

**Significant Episodes**

Selected portions of each videotaped interview protocol that presented evidence of faulty executive control are presented next along with comments and analysis by the researcher. Each protocol statement is numbered according to its location on the complete transcript. Immediately prior to the protocol is the associated non-routine problem. Following each protocol is a timeline representation of the session.

**Description of Study Team 1 Subjects**

Team 1 students are referred to as A1 and B1 throughout this study. A1 was in the second semester of her sophomore year and B1 was in the second semester of her freshman year during the time of data collection.

Prior to the study, A1 indicated that she could use a Texas Instruments TI-83 graphing calculator with facility and could graph functions and find intersections using the calculator. She had earned an A grade in College Algebra during the previous semester at the study site. Prior to the study she described herself as a student who could learn mathematics relatively easy if it is explained well.

B1 also had considerable experience using a graphing calculator and declared that she could graph functions and find intersections using a Texas Instruments TI-83 model. She had earned B, C, and D grades in her high school mathematics courses and had earned an A grade in Intermediate Algebra at another institution and a B grade in College Algebra during the previous semester at the study site. She described herself as a student that struggles with math but thinks she can be successful.

**Team 1 Pretreatment Problem 1 Protocol Analysis**

The first non-routine problem given to students interviewed prior to the treatment is given in Figure 4.1. Following the non-routine problem, portions of the interview protocol associated with instances of faulty executive control are presented along with comments and analysis by the researcher and a timeline representation of the attempt.
The surface of Clear Lake is 35 feet above the surface of Blue Lake. Clear Lake is twice as deep as Blue Lake. The bottom of Clear Lake is 12 feet above the bottom of Blue Lake. How deep is Blue Lake?

Figure 4.1. Pretreatment Interview Non-Routine Problem 1.

10. B1: We know the equation here is two-something, right? Because it's as deep as Blue Lake. Rather than trying to understand the relationships between what is known, student immediately attempted to think of an algorithm to solve the problem.

27. B1: They're not two different lakes are they? Student clearly did not understand the basic structure of the problem.

29. B1: I mean, could you make an equation out of this?

30. A1: I'm trying to.

31. B1: Would thirty-five equal two-x plus twelve?

32. A1: I don't think that would work.

33. B1: Minus twelve? The equation, out of three numbers, two numbers and a variable.

Student B1 continued to suggest using some kind of equation. The equation mentioned in line 31 is nearly correct but seemed to be recommended without reason. A1's intervention prevented a "wild goose chase." During the ninth minute A1 constructed a drawing that did not accurately reflect the spatial relationships given in the problem. Figure 4.2 shows A1's drawing.

43. A1: I was thinking we could use the Pythagorean Theorem for that.

Student suggested exploring the problem space using a method that did not make sense and most likely could not possibly be helpful.

50. B1: Would it be like thirty-five equals something-something? And then solve for x?

Student continued to search for a formula without regard to clear reasoning.
Figure 4.2. Student A1's Faulty Sketch of the Relationship Between Lakes.

45. A1: I was just thinking of Pythagorean's Theorem but then that would be...nope. That wouldn't work. I don't think so.

Student recognized that previous suggestion is inappropriate.

52. B1: So obviously Clear Lake isn't as deep as Blue Lake.

Student made a strikingly incorrect assumption. The problem statement clearly indicated that Clear Lake is twice as deep as Blue Lake.

61. B1: Actually, I think it's like, okay, thirty-five, I don't know why, I'm thinking thirty-five equals like two times a variable, then something else. Just like what it says. It's like Clear Lake is twice as deep as Blue Lake. So we don't know how deep Blue Lake is so it's two times something. And then thirty-five, where else would you put it besides the equal, because I mean...

65. B1: The bottom of Clear Lake is twelve feet above the bottom of Blue Lake. So I don't know if you like find the answer then maybe add twelve or subtract twelve? I don't know.

Student persisted with efforts to invent a formula.

68. B1: I think what is confusing me is if I do it this way, then it'll come out in a fraction. I think that will screw up the whole problem.
Student exhibited flawed reasoning by stating an unwillingness to accept any effort that resulted in a fraction.

73. A1: I want a better picture because then I could see is there. I'm stuck on this Pythagorean's Theorem here because it's this picture with...because we have two unknowns here. This was a triangle and the diagonal will give us another...I mean a value.

Student attempted to find a reason to somehow use the Pythagorean Theorem even though she had correctly decided against it on a previous attempt.

B1 persisted in her attempts to simply rush through some calculations. She suggested subtracting twelve from a number that she could not even define. A1 showed some positive control by asking questions about the proposed calculation.

89. B1: So, would you subtract the twelve from the number or...?

90. A1: From what number?

91. B1: Whatever number we get.

Students finally had a reasonable diagram of the lakes but failed to understand the significance of what they had done. They were unable to solve the problem before they ran out of time. Figure 4.3 is their diagram.
The block has the following properties.

- The height is $\frac{1}{2}$ of the width.
- The length is 3 times the height.
- The volume is 750 cubic inches.

What is the surface area of the block?

Figure 4.5. Pretreatment Interview Non-Routine Problem 2.
36. B1: Would it be seven-hundred-fifty equals?

37: A1: Why do you always do that?

38. B1: I don't know.

A1 made a comment about how B1 frequently tried to write an equation before understanding how the parameters of a problem are related.

81. B1: Could you do this? Three-h equals seven-hundred-fifty, solve and then plug it back in and all that? Take each one separately or would you have to put it all together?

Student suggested using an incorrect procedure, probably just to satisfy her constant urge to plug numbers into various formulas.

87. B1: I don't know. I'm just, I guess I'm taking every equation separately, every variable separately. But I don't know if it's going to work.

Student tries guess-and-check method using dimensions for width (250") and height (125") that are obviously much too large.

95. A1: Why are you doing that?

96. B1: I don't know. I'm just taking every piece, like every article and equal it to seven-fifty.

B1 demonstrated a lack of control by setting the expression for height in terms of width equal to 750 and solving for width \( \frac{1}{2}w = h \) somehow becomes \( \frac{1}{2}w = 750 \) even though she knew that the volume, not the height was equal to 750.

120. A1: It's like it says, it's just...like saying seven-fifty is equal to three-y plus half-x. But I can't do anything with that.

Even though A1 had shown on the Pre-Treatment Resources Examination that she knew exactly how to find the volume of this solid, she suggested using an incorrect procedure in this case. Assuming y is defined as height and x as width, A1 disregarded width and suggested adding factors instead of multiplying.

121. B1: You could...umm...you could get...you mean like that y-intercept problem. You have an x and you have a y but it's got nothing to do with it.

122. A1: Well maybe we could graph it. I don't know if it's going to help.

Students discussed the merits of graphing some kind of linear function but did not discuss
why or if such a strategy would even be useful. Eventually they found a formula for height as a function of width but did not ever indicate what they could do with it. They were unable to solve the problem before the allotted time elapsed.

Twenty horizontal feet east of a 50-foot building is a 35-foot wall. A man 6 feet tall wishes to view the top of the building from the east side of the wall. How far east of the wall must he stand in order to view the top of the building?

Figure 4.6. Team 1 Pretreatment Interview Problem 2 Timeline.

**Team 1 Post-Treatment Problem 1 Protocol Analysis**

The first non-routine problem given to students interviewed following the treatment is given in Figure 4.7.

Following the non-routine problem, portions of the interview protocol associated with instances of faulty executive control are presented along with comments and
analysis by the researcher and a timeline representation of the attempt.

6. B1: Is he like, I don't get the wall part. Is he like on the wall, or...? During the second minute the student was confused about the spatial relationship described by the question. She could not see how the man's position relates to the wall and building. During the third minute this initial confusion was alleviated after she reevaluated the situation and stated, "Oh, how far east of the wall. So the man is on the other side of the wall." She also sketched an appropriate diagram representative of the given scenario.

20. B1: I think the twenty feet has some kind of purpose. Maybe, you know, twenty feet away from the building and thirty-five feet high, you can see the top of that building, right?

This statement indicated some confusion about what was being asked or perhaps an attempt to find and justify using a simple formula or procedure.

22. B1: Well, he has to be far away, to at least see over that. And if he sees over that he'll see the building, right on top of the building. So maybe, like, double twenty feet? I don't know. Forty feet away?

Student further demonstrated poor control by expressing a willingness to forgo a structured approach while attempting to pursue a solution with unsubstantiated "stabs in the dark."

27. B1: View the top of the building, like over the building or just see the top of the building? Does it matter? As long as he can see over thirty-five feet, he can see the top of the fifty-foot building.

Student was confused about simple spatial relationship. She did not realize that the man would never be able to look down on the roof of a building if the building's top is above the man's eye-level.

34. A1: I'm just trying to figure out if I see any shapes here. A reasonable and appropriate pathway not successfully utilized. By looking for familiar shapes, the student had a chance to find a linear model and subsequently solve the problem. Though both students considered this a viable option, they were unable to capitalize.
During the twelfth minute A1 decided that because there were triangles involved she should try to make sense of the problem using the Pythagorean Theorem. B1 agreed and continued to offer advice throughout the effort, which lasted for about five minutes. The method was not reasonable. A diagram created during the ill-fated attempt is shown in Figure 4.8.

82. B1: Maybe we can plug this in our equation but just using this side.

Student continues to look for ways to solve the problem by simply using some kind of formula rather than synthesizing an approach based on a sound understanding of mathematical concepts. This comment exemplified the student’s usual style of problem solving.

85. B1: Because if you’re, I don’t know what this is, if you’re twenty feet away. Okay, if there’s a wall that is thirty-five feet, if you’re twenty feet away you can see the top of the fifty-foot building, right?

Again, student demonstrates an inability to understand the spatial relationships involved. Even though the Pythagorean Theorem was not studied nor emphasized as a modeling tool during the 14-week course, these students decided to see if they could somehow employ it to find the solution. Approximately one-third of the course was spent learning
how to find, interpret and employ linear models. This non-routine problem could have easily been handled using a linear model.

Figure 4.9. Team 1 Post-Treatment Interview Problem 1 Timeline.

Team 1 Post-Treatment Problem 2 Protocol Analysis

The second non-routine problem given to students interviewed following the treatment is given in Figure 4.10. Following the non-routine problem, portions of the interview protocol associated with instances of faulty executive control are presented along with comments and analysis by the researcher and a timeline representation of the attempt.

2. B1: So is it going to be less than five grams or more than five grams? Student clearly demonstrated a fundamental lack of understanding for the question scenario. Student did not realize that because the U$^{239}$ decays, more than five grams would need to be present after ten minutes of decay if five grams were required after one hour.

6. B1: So all you really, really need to figure out is how many grams are present after ten minutes, right?
8. B1: So do we do it in the calculator where we locate, like, trace where x?

Statement 6 is true, however, it is obvious that B1 still did not understand the relationship between what was given and what was being asked. A1 was probably not convinced but

Uranium 239 is an unstable isotope of uranium that decays rapidly. In order to determine the rate of decay, 1 gram of U\textsuperscript{239} was placed in a container and the amount remaining was measured at 1-minute intervals and recorded in the table below.

<table>
<thead>
<tr>
<th>Time in minutes</th>
<th>Grams remaining</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0.971</td>
</tr>
<tr>
<td>2</td>
<td>0.943</td>
</tr>
<tr>
<td>3</td>
<td>0.916</td>
</tr>
<tr>
<td>4</td>
<td>0.889</td>
</tr>
<tr>
<td>5</td>
<td>0.863</td>
</tr>
</tbody>
</table>

Suppose you are advising a research scientist who is conducting an experiment requiring the use of U\textsuperscript{239}. How many grams of U\textsuperscript{239} must be present after 10 minutes if the experiment requires 5 grams remaining after 1 hour?

Figure 4.10. Post-Treatment Interview Non-Routine Problem 2.

agreed with B1’s assessment, possibly in order to continue her exploration of the problem space without expending thought or effort on B1’s persistent unreasonableness.

19. B1: Exponential function. So do we write down the equation and plug in something?

As in all previous problem solving interviews, B1 had difficulty moving beyond simply searching for some kind of routine formula to solve the problem.
35. B1: Oh, I don’t know. For some reason I’m just thinking like you just put ten in the x-spot and then whatever that is, that’ll be y and that’ll be (mumble) grams are remaining.

B1 persisted in her attempts to solve the problem without understanding the scenario or even what was really being asked. By this time the students had constructed a valid exponential model for what was given in the table. However, they did not realize that the initial value for their model would need to be different than what was given in the table. Figure 4.11 shows their model.

59. A1: Let me see if you divide (mumbles). Yeah, it's almost the same thing. It's exponential. See? I think so, yep. We're calling it quits.

Curiously, A1 decided that she had satisfactorily finished the problem because she had constructed a formula that accurately modeled the data given in the table. Although the team had announced how many grams needed to be present after one hour, A1 decided that the session was over. Only after asking specific probing questions following the videotaped session did the researcher determine that the students had utilized the previously mentioned exponential model to determine that about 0.745 grams was their answer. Even though the correct solution would obviously require that more than 5 grams remain after ten minutes, neither of the students realized how unreasonable was their solution.

Figure 4.11. Team 1’s Model of Data Given in the Problem Statement.
**Description of Study Team 2 Subjects**

Team 2 students are referred to as A2 and B2 throughout this study. Both students were in the second semester of their freshman year during the time of data collection.

Prior to the study, A2 indicated that she did not own a graphing calculator and that she did not know how to graph functions or find intersections using a graphing calculator. She had earned A and B grades in her high school mathematics classes and was taking her first mathematics course at the undergraduate level. Prior to the study she stated that she enjoys math a lot but was somewhat concerned that she may have problems because she had not taken a mathematics class in over a year and a half.

B2 had experience using a graphing calculator and declared that she could graph functions and find intersections using a Texas Instruments TI-83 model. She had earned A and B grades in her high school mathematics courses and had earned an A grade in College Algebra during the previous semester at the study site. She described herself as hard-working and determined to learn the material.

---

**Figure 4.12. Team 1 Post-Treatment Interview Problem 2 Timeline.**

```plaintext
<table>
<thead>
<tr>
<th>Activity</th>
<th>Team 1</th>
<th>Problem: Post-Treatment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Read</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>Survey</td>
<td>A1</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>X X X X</td>
</tr>
<tr>
<td>Plan</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>Implement</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>Verify</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td></td>
</tr>
</tbody>
</table>

- Statement or action not reasonable, not appropriate, or not helpful. Had potential to hinder solution progress.
```

Elapsed Time (minutes)
The first non-routine problem given to students interviewed prior to the treatment is given in Figure 4.13. Following the non-routine problem, portions of the interview protocol associated with instances of faulty executive control are presented along with comments and analysis by the researcher and a timeline representation of the attempt.

The surface of Clear Lake is 35 feet above the surface of Blue Lake. Clear Lake is twice as deep as Blue Lake. The bottom of Clear Lake is 12 feet above the bottom of Blue Lake. How deep is Blue Lake?

Figure 4.13. Pretreatment Interview Non-Routine Problem 1.

29. A2: Hmmm. You think it's thirty-five plus twelve?

Student A2 either lost track of what was being asked or was trying to find a quick and easy way to answer the question prior to understanding the spatial relationship between the two lakes. B2 seemed equally unsure of how to proceed. Rather than finding how deep Blue Lake is, what A2 suggested would actually determine the total distance from the top of Clear Lake to the bottom of Blue Lake. It may be possible that she forgot the question she was supposed to answer.

33. A2: I don't know what they're asking.

Student demonstrated that almost six minutes into the session she still had not even figured out what was being asked.

36. A2: Okay, isn't it thirty-five plus twelve? Wouldn't that seem right?

Student seemed unwilling to move beyond her previous false assumptions and continued to exhibit evidence that she did not understand the problem.

48. A2: Well, because we only need to know the height of Blue Lake, if we add another thirty-five, then we have the equivalent of the Clear Lake and the Blue Lake. That's the only reason I'm wondering if we only need half, not both. You see what I'm saying?
Student A2 showed that she eventually understood what was being asked though the spatial relationship between the lakes still eluded her.

52. A2: Does that seem right to you, or does it seem right?

When student A2 asked B2 for her opinion about the validity of her approach, B2 admitted that she normally can not or will not verify mathematical results.

58. A2: Want it to stay forty-seven?
59. B2: Uh...
60. A2: Does that seem right? Try. Try to do what I was thinking and see if it makes sense.

A2 returned to her flawed initial reaction to the problem by offering the sum of thirty-five and twelve as a solution. B2 did not seem willing to commit one way or the other to A2's suggestion.

![Student B2's Sketch of the Relationship Between Lakes.](image)

Figure 4.14. Student B2's Sketch of the Relationship Between Lakes.

68. B2: I'm just um, I made a little Blue Lake in there for myself, and then I just, um, cause the Blue Lake, well the Clear Lake is thirty-five feet above the Blue Lake so I just drew a little arrow telling myself that. And then, uh...
B2 drew an accurate representation of the given scenario but A2 failed to recognize the value of the new diagram. Student A2's somewhat dominate personality resulted in this potential pathway to success being mostly ignored. B2's diagram is illustrated in Figure 4.14.

71. B2: Um. Okay, I got a quick question for you. Would you double the thirty-five or would you take half of thirty-five?

Even though student B2 had recently demonstrated an understanding of how the lakes were related, she asked an unreasonable question.

83. A2: What if it's just thirty-five?
84. B2: It could be.
85. A2: Where did it...I think it, now that, that makes much more sense doesn't it?
86. B2: Yeah. So we're going with thirty-five.
87. A2: Yeah. We're going with thirty-five, finally.

Figure 4.15. Team 2 Pretreatment Interview Problem 1 Timeline.
Student A2 declared that the solution may be thirty-five then convinced herself and B2 that it was reasonable and probably correct. Even though student B2 could simply glance at her diagram and see that this could not possibly be true, she readily agreed with her dominant partner. A2 persisted in saying "the" Blue Lake and "the" Clear Lake when reading the problem even though that is not how the problem was worded. Also, although B2 seemed to have some good ideas and an understanding of what needed to be done, the dominant personality of A2 worked to overshadow or depress her attempts. Even though B2 probably knew that her understanding of the spatial relationship was much better than her teammate's, she did not try to lead the attempt.

**Team 2 Pretreatment Problem 2 Protocol Analysis**

The second non-routine problem given to students interviewed prior to the treatment is given in Figure 4.16. Following the non-routine problem, portions of the interview protocol associated with instances of faulty executive control are presented along with comments and analysis by the researcher and a timeline representation of the attempt.

---

The block has the following properties.

- The height is \( \frac{1}{2} \) of the width.
- The length is 3 times the height.
- The volume is 750 cubic inches.

What is the surface area of the block?

---

Figure 4.16. Pretreatment Interview Non-Routine Problem 2.

5.  
   A2: All right. This one's a little easier. I can do this. One half times one, one half times...(mumble)...I'll just use a calculator, it'll be faster. (mumble) one point...Oh, um, I'm solving for the width...

Student constructed a correct equation to solve for width but the failed to multiply the variable \( w \). The mistake resulted in a first- rather than third-degree equation as would be
expected when dealing with three dimensions. This false representation caused significant confusion throughout the entire attempt. Figure 4.17 shows her error.

13. A2: The surface area, what is the surface area? I know what volume is based on...wait, what is the volume? Is that base times width times height, that's volume right?


Although both students on this team clearly demonstrated on the Pre-Treatment Resource Examination that they understand how to find the area of a rectangle and the volume of a rectangular solid, they became confused and unsure of how to handle the same situation during the early stages of the interview.

Figure 4.17. Student A2's Initial Calculation of Width.

44. A2: Okay, half of w is five hundred...(whisper)...five hundred times three...surface area. And I'm pretty sure that the (mumble)...

Student had mistakenly decided that width was equal to 1000 inches even though that would be completely unreasonable given the fact that the volume was only 750 cubic inches. Based on her answer for width she solved for height and length. Subsequently she used an incorrect procedure to calculate the surface area by adding instead of multiplying associated dimensions. Figure 4.18 illustrates her decision and subsequent erroneous calculation of surface area.
50. A2: These, um, for how long the length is, it seems huge that it's going to be a huge number. The surface area. And the surface area is smaller than volume, so I'm doing something wrong, I know it. Because there's more volume than surface. Do you have any idea what I'm doing? Student recognized the unreasonableness of her solution attempt.

Figure 4.18. Student A2's Unreasonable Calculation of Surface Area.

66. A2: This isn't right, because the volume is seven hundred and fifty and when you plug in a thousand it's way higher than seven hundred and fifty. Even though A2 had realized her previous assumptions concerning the magnitude of dimensions were unreasonable, she persisted in using them in an attempt to find a solution.

87. A2: This is wrong. That's why we're messing up somewhere.


During the last minute of the attempt both students finally realize that their decision to use an unreasonably high value for width had guaranteed failure. Had they decided to more thoroughly explore their early work they would have most likely had a much better chance of being successful.
Twenty horizontal feet east of a 50-foot building is a 35-foot wall. A man 6 feet tall wishes to view the top of the building from the east side of the wall. How far east of the wall must he stand in order to view the top of the building.

Figure 4.20. Post-Treatment Interview Non-Routine Problem 1.

7. A2: How far east on top of this building?
Both students demonstrated a misunderstanding of the problem statement. The researcher immediately corrected this erroneous understanding of spatial relationships in order to eliminate wasted time.

11. A2: We could probably do this. Um, thirty-five equals fifty over x, plus x plus twenty, solve for x first, this distance.

12. A2: Seven hundred divided by fifteen is forty-six point six. Is that two-thirds? Yeah. Forty-six and two-thirds. So, we know that this whole distance...He's six feet tall.

14. A2: So it's not just...Forty-six...seven hundred divided by fifteen plus twenty plus sixty-six. Um.

16. A2: I got it. Wait, I know I can find this distance. Okay. So six feet over equals fifty feet. Six over x is fifty over the whole distance which is. Wait. Right. Sixty-six point six?

18. A2: So fifty x. Sixty-six point six times six is equal to four hundred.

20. A2: Okay, four hundred divided by fifty...x equals eight. So we just found out...

Figure 4.21. Student A2's Calculations Using Similar Triangles.
By the sixth minute A2 had enough information to easily answer the question. Figure 4.21 shows her work. Using similar triangles she had determined each of the dimensions illustrated in Figure 4.22 but failed to recognize that the distance she needed to solve the problem was readily at hand. Interestingly, both students mistakenly drew sketches with the man standing to the west rather than east side of the wall.

27. B2: Okay. So we got that, that and now we solve for the top, $y$.
28. A2: Okay. So then we can do...oh. Point six and sixty-six point six and fifty (mumbles).
29. B2: Can you use the Pythagorean Theorem for that? The bigger side? You go like that...

These statements show that the students had actually decided that what they needed to solve for was the slant range between the person and the top of the wall or building rather than the horizontal distance between the person and the wall. This was probably the reason why they could not find a solution based on the dimensions they had already determined.

Figure 4.22. Summary of Dimensions Team 2 Calculated.
38. A2: Ten plus x. Sixty plus six x. Five hundred minus sixty. That's four forty plus six x. Four forty divided by six is seventy-three point three.

Student A2 solved for the slant range between the person and the top of the building, a dimension that was not called for or needed to solve the problem. It would have been more reasonable had they determined the slant range between the person and the top of the wall considering how the problem was worded. Figure 4.23 illustrates their work.

![Figure 4.23. Correct Calculation of Unneeded Dimension.](image)

45. A2: Now, should we check?

Student A2 demonstrated good judgement by showing a willingness to verify their answer.

56. A2: We're done. Seventy-three point three is how far east of the wall he must be standing in order to view the top of the building.

Students finished verification of their result without reviewing what they had actually been asked, a precaution that seems advisable given the fact that they usually had difficulty understanding what was being asked in non-routine problems. Following this interview the researcher pointed out that they had not answered what was being asked. The students were then able to provide the correct solution within approximately four minutes.
Team 2 Post-Treatment Problem 2 Protocol Analysis

The second non-routine problem given to students interviewed following the treatment is given in Figure 4.25. Following the non-routine problem, portions of the interview protocol associated with instances of faulty executive control are presented along with comments and analysis by the researcher and a timeline representation of the attempt.

9. A2: Well, we can figure out the difference between here and here and here and see if there's a like term.


Even though these students had explored uranium decay models and studied several exponential decay problems during the course they decided to check for linearity.

17. A2: How many grams of U must be present after ten minutes? It has to be after ten. It's not just ten.

Student did not understand the problem statement. Rather than keying on how much uranium should be present when time equals 10 minutes, she believed that time is greater
than ten minutes was important to specify. She clearly did not understand the meaning of the related portions of the problem statement.

Uranium 239 is an unstable isotope of uranium that decays rapidly. In order to determine the rate of decay, 1 gram of $\text{U}^{239}$ was placed in a container and the amount remaining was measured at 1-minute intervals and recorded in the table below.

<table>
<thead>
<tr>
<th>Time in minutes</th>
<th>Grams remaining</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0.971</td>
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<tr>
<td>2</td>
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<td>3</td>
<td>0.916</td>
</tr>
<tr>
<td>4</td>
<td>0.889</td>
</tr>
<tr>
<td>5</td>
<td>0.863</td>
</tr>
</tbody>
</table>

Suppose you are advising a research scientist who is conducting an experiment requiring the use of $\text{U}^{239}$. How many grams of $\text{U}^{239}$ must be present after 10 minutes if the experiment requires 5 grams remaining after 1 hour?

Figure 4.25. Post-Treatment Interview Non-Routine Problem 2.

18. A2: Um, if the experimenter has five grams remaining after one hour. So does it have to be higher than an hour too?
19. B2: It could be, because it says it’s after one hour.
20. A2: So we should do sixty minutes at least, then, right, and see if we're close?

Students further demonstrated they misunderstood how the word "after" was being used in the problem statement.
22. A2: Sixty minutes over x is equal to, let's do zero. Let's do one minute (mumbles).

Student decided to treat the problem as a linear relationship by constructing a ratio with one unknown. The setup seemed completely unreasonable. Figure 4.26 shows A2's work.

![Figure 4.26. A2's Work Using Ratios in an Unreasonable Way.](image)

23. A2: Fifty-eight minus. That seems so high. How many grams must be present after ten minutes? I think it's, I don't think it's asking that. How many grams of U²³⁹ must be present after ten minutes? That's what we're solving, after ten minutes. If the experiment requires five grams remaining after an hour.


25. A2: That's all it's asking. That just seems too, that's it?

26. B2: No, it can't be.

After nearly five minutes, the students continued to demonstrate that what was being asked was unclear and thought that 5 grams may be the solution. Student B2 who normally agreed with A2 without much thought helped to refocus their efforts.

35. A2: Yeah. Five grams remaining? I don't think the number; it should be going down. 'Cause this is going down, it's not going up, so how are we getting fifty-eight?
Students did not realize that the table would allow them to determine the decay rate but could not be used after that. A2 questioned how the grams remaining could increase if the grams remaining are decreasing on the table.

A2: Let me see what you're doing and see if I can get on the same page. What are you doing?

47. B2: I'm just; I found the slope and then I put it back in the equation. Now, we still have to look for, right there. So, I want to put ten there.

48. A2: But did you know that the slope decreases? It goes two-nine, two-eight, two-seven, two-six, it's not just the same slope.


50. A2: So, how did you determine two-eight?

51. B2: I just picked two coordinates. I picked these two.

52. A2: Oh, I got ya'. So...

53. B2: We have to just go back and I'm going to put ten in there because we're looking for the ten minutes.

54. A2: Okay.

Even though the team had determined that the data from the table did not exhibit a constant rate of change, B2 constructed a linear formula to model the data. Figure 4.27 illustrates her efforts.

![Figure 4.27: Team 2's Linear Formula to Model Exponential Data.](image-url)
64. B2: Maybe it does come back up very slowly? At the ten minute mark. B2 considered the faulty notion that perhaps eventually the uranium stops decaying and begins to grow.

72. B2: Well, we don't know if it's going to come back up because we don't have six through nine. I can verify and put that all; try that. See what it does.

B2 unreasonably believed that if the table contained data points for minutes six through nine they may see growth of the uranium remaining.

94. A2: Let's go back to what we were trying to do before. We were trying to say that in...the experiment requires five grams remaining after a...How is it increasing? How would it increase? What are the possibilities of it increasing?
95. B2: I don't; well.
96. A2: There's got to be something. Uranium is an UN-stable isotope of uranium.
97. B2: So that means it could go up, it's unstable.
98. A2: Right. In order to determine the rate of decay, one gram U-...

During the final minute of the attempt both students continued to believe that decay could somehow become growth. A2 bolstered their false assumption by noting that uranium is "unstable."

These students were extremely confused throughout the entire attempt. As time wore on they seemed to move further and further from what they had studied throughout the semester. They tried using ratios, modeling non-linear data with linear formulas, and even attempted to change a decreasing function into something more appropriately termed periodic. It seemed their basic misunderstanding of what was being asked coupled with their failure to grasp the relationship between what was given in the problem statement resulted in an increasingly unreasonable series of unfruitful experiments.
Interpretations of Qualitative Data

In this section, the researcher's interpretation of what the protocol analysis revealed is summarized.

During Team 1's pretreatment problem solving sessions the researcher quickly noticed significant differences in the way A1 and B1 responded to unfamiliar problems. B1 was much more likely to suggest simply trying to recall and use a previously memorized formula or to experiment with solution methods without first understanding why it would be an appropriate response. B1 was constantly suggesting different formulas or computations, seemingly without reason. Though A1 also occasionally tried using unreasonable, inappropriate, or unhelpful strategies, B1 was much more likely to do so. A tally of metacognitive failures identified in the pretreatment timelines shows that B1 was four times more likely to exhibit a metacognitive failure than A1. Timelines also illustrate that Team 1 rarely moved outside of the Read and Survey domains during the pre- and post-treatment sessions.

An examination of Team 1’s post-treatment problem solving transcripts and timelines reveal that very little seemed to have changed regarding the frequency of
metacognitive failures. Also, as in the pretreatment sessions, they spent very little time outside of the Read and Survey domains. Furthermore, Team 1 students never even mentioned using the Planning-Drawing-Doing-Verifying technique that had been stressed throughout the entire course. B1 persisted in her overt displays of poor executive decision making. A1 continued to show better control, however, her overall performance gave the researcher no reason to believe that she had changed her problem solving style in any significant way following the treatment.

Even though the entire 14-week course had been spent studying, interpreting, and learning how to create linear, exponential and sinusoidal models, Team 1 students did not make the connection between the usefulness of those skills and application to the solutions for the post-treatment interview problems.

![Relative Percentage of Behaviors Identified - Team 1](image)

Figure 4.29. Relative Percentage of Time Spent on Each Behavior for Team 1.

Team 2 student A2 was much more proactive than B2 in her approach during both pre- and post-treatment interviews. Her personality, though not overbearing or controlling, conveyed an air of confidence that fostered a leadership roll within the team for A2. During both the pre- and post-treatment problem solving interview attempts A2 exhibited considerably more instances of overt metacognitive control failures. Though on the surface this appears as an indicator that A2 is a less skilled problem solver, the leadership role she assumed guaranteed that she would overtly exhibit more mistakes.
Both Team 2 students had considerable difficulty understanding spatial relationships, what was being asked, and how various components of what was given in the problem statements were related, particularly during the post-treatment interviews. Even though they were not completely successful on any of the four problems, at some point during each attempt, Team 2 clearly demonstrated that they possessed the mathematical resources to solve each problem.

Timelines offered little evidence that Team 2 had changed their problem solving style. Just as with Team 1, they did not use or even mention the Planning-Drawing-Doing-Verifying routine during any of the post-treatment problem solving interview attempts. The overall performance of Team 2 gave the researcher no reason to believe that following the treatment they had changed their problem solving style in any significant way. However, unlike the other team, Team 2 successfully made some connection between the usefulness of the modeling skills studied throughout the semester and at least attempted to use them to their advantage during the post-treatment interview problem solving sessions.

![Relative Percentage of Behaviors Identified - Team 2](image)

Figure 4.30. Relative Percentage of Time Spent on Each Behavior for Team 2.

**Quantitative Analysis**

Pre- and post-test statistical evidence for control and treatment sections was explored in an effort to substantiate qualitative results. The researcher also studied the
quantitative evidence to search for any other possible affects of the treatment and to gain further insight into the complex issue of how students attempt to solve non-routine problems.

**Descriptive Statistics**

The first step taken in the quantitative data analysis phase was to determine the mean scores for each section on the four examinations. These statistics were then used to check for significant differences using two-tailed t-tests. Treatment and control section performance on pre- and post-treatment examinations are illustrated in Figures 4.31 through 4.35. Results of the t-tests are presented and discussed in the section following the figures.

![Examination Means](image)

Figure 4.31. Examination Mean Scores.
<table>
<thead>
<tr>
<th>Pretreatment Resource Examination Scores</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Pretreatment Resource Examination</td>
<td>29</td>
<td>71.7241</td>
<td>15.88327</td>
</tr>
<tr>
<td>Treatment Pretreatment Resource Examination</td>
<td>14</td>
<td>74.6429</td>
<td>13.79301</td>
</tr>
</tbody>
</table>

Figure 4.32. Pretreatment Resource Examination Scores.

<table>
<thead>
<tr>
<th>Pretreatment Non-Routine Problem Examination Scores</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Pretreatment Non-Routine Problem Examination</td>
<td>29</td>
<td>45.0690</td>
<td>26.79943</td>
</tr>
<tr>
<td>Treatment Pretreatment Non-Routine Problem Examination</td>
<td>14</td>
<td>46.5714</td>
<td>28.33512</td>
</tr>
</tbody>
</table>

Figure 4.33. Pretreatment Non-Routine Problem Examination Scores.
Figure 4.34. Post-Treatment Resource Examination Scores.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Post-Treatment Resource Examination</td>
<td>29</td>
<td>69.3448</td>
<td>19.15962</td>
</tr>
<tr>
<td>Treatment Post-Treatment Resource Examination</td>
<td>14</td>
<td>83.7857</td>
<td>15.36319</td>
</tr>
</tbody>
</table>

Figure 4.35. Post-Treatment Non-Routine Problem Examination Scores.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Post-Treatment Non-Routine Problem Examination</td>
<td>29</td>
<td>47.8276</td>
<td>30.29034</td>
</tr>
<tr>
<td>Treatment Post-Treatment Non-Routine Problem Examination</td>
<td>14</td>
<td>39.6429</td>
<td>35.67720</td>
</tr>
</tbody>
</table>
T-Tests

Independent-sample t-tests can be used to compare the mean scores for two groups. The validity of t-tests is dependent upon three assumptions (a) normal distribution, (b) equal variance, and (c) independence of samples. If the sample size is greater than 5, the assumption of normality is robust (can safely be violated) providing the distribution is not extremely skewed. The assumption of equal variance is robust providing the ratio of sample sizes and variances are less than 1:2 and 1:3 respectively or Levene's test for equality of variance is satisfied (Glass & Hopkins, 1996). For t-tests, subjects should be randomly assigned to groups, so that any difference in the results are due to the treatment and not to other factors. Unlike the other assumptions associated with t-tests, the assumption of independent samples can not be mitigated if violated. In this case, subjects were not strictly randomly selected but self-selected into the treatment and control groups during registration for classes. Because subjects were not strictly selected for the two groups at random, the results of the statistical tests in this study were viewed more as antidotal evidence rather than scientific proof.

The researcher calculated the difference between pre- and post-test scores for matching pairs in the treatment and control group. The resulting data were analyzed to identify any statistically significant differences. In the tables of statistics in this section Group 0 represents the control students and Group 1 represents the treatment students.

Table 4.1. Examination Score Mean Differences Between Pre- and Post-Tests.

<table>
<thead>
<tr>
<th>Group Statistics</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resource Examination Score Difference</td>
<td>0</td>
<td>29</td>
<td>-2.4828</td>
<td>22.41431</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>14</td>
<td>9.1429</td>
<td>18.73646</td>
</tr>
<tr>
<td>Non-Routine Examination Score Difference</td>
<td>0</td>
<td>29</td>
<td>2.5862</td>
<td>30.59639</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>14</td>
<td>-6.9288</td>
<td>30.29715</td>
</tr>
</tbody>
</table>

The software package SPSS was used to conduct two tests of the change in score means to determine if either group improved significantly better than the other between
pre- and post-treatment examinations. One test assumes that the variances of the two groups are equal. The Levene statistic tests this assumption. In this case, the significance values of the statistic are 0.244, and 0.918 as highlighted in Table 4.2. Because these values are greater than 0.05, it was assumed that the groups have equal variances so the second test, which assumes unequal variances, was ignored. For each test, the assumption of normality was safely violated because the sample sizes are greater than 5 and the distributions are not extremely skewed. Table 4.2 shows the results of testing for significant differences between the treatment and control group examination score differences described in Table 4.1. If the significance values of the t-tests were less than 0.05, it would be safe to conclude that the differences in mean scores were probably not due to chance alone. In this case, the specific null hypothesis tested was:

**Hypothesis I**: There is no statistically significant difference between the change in pre- and post-treatment examination scores for the treatment and control groups.

In this instance, because each of the significance values are greater than 0.05, the researcher concluded that the differences in change of scores between treatment and control sections due to chance alone remains tenable.

Table 4.2. Test for Equality of Mean Differences.

<table>
<thead>
<tr>
<th>Independent Samples Test</th>
<th>Levene’s Test for Equality of Variances</th>
<th>t-test for Equality of Means</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F</td>
<td>Sig</td>
</tr>
<tr>
<td>Resource Examination Score Difference</td>
<td>1.387</td>
<td>.244</td>
</tr>
<tr>
<td>Non-Routine Examination Score Difference</td>
<td>0.11</td>
<td>.919</td>
</tr>
</tbody>
</table>

Next, the researcher analyzed the mean scores between control and treatment groups on the four pre- and post-treatment examinations to identify any statistically significant differences. Two-tailed t-tests were performed using the software package
SPSS. Levene's test was again used to test the assumption that the variances between the two groups are equal. Because statistical significance values are greater than 0.05, (0.462, 0.513, 0.313, and 0.389 as highlighted in Table 4.3) it was assumed that the variances of the two groups are equal. For each test, the assumption of normality was safely violated because the sample sizes are greater than 5 and the distributions are not extremely skewed.

Table 4.3. Significance of Differences Between Means

<table>
<thead>
<tr>
<th></th>
<th>Levene's Test for Equalities of Variances</th>
<th>Test for Equality of Means</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F</td>
<td>Sig.</td>
<td>t</td>
</tr>
<tr>
<td>Pretreatment Resource Examination</td>
<td>.918</td>
<td>.41</td>
<td>20.381</td>
</tr>
<tr>
<td>Pretreatment Non-Routine Problem Examination</td>
<td>.436</td>
<td>.513</td>
<td>.189</td>
</tr>
<tr>
<td>Pretreatment Non-Routine Problem Examination</td>
<td>.166</td>
<td>24.526</td>
<td>.570</td>
</tr>
<tr>
<td>Post-Treatment Resource Examination</td>
<td>1.044</td>
<td>.313</td>
<td>-2.459</td>
</tr>
<tr>
<td>Post-Treatment Non-Routine Problem Examination</td>
<td>.759</td>
<td>.383</td>
<td>7.704</td>
</tr>
<tr>
<td>Post-Treatment Non-Routine Problem Examination</td>
<td>.739</td>
<td>22.384</td>
<td>.467</td>
</tr>
</tbody>
</table>

If the resulting significance values of the t-tests were less than 0.05, the researcher would conclude that the differences in average scores between treatment and control groups were probably not due to chance alone. In this case, the specific null hypotheses tested were:

Hypothesis II: There is no statistically significant difference between the treatment and control section Pretreatment Resource Examination score means.

Hypothesis III: There is no statistically significant difference between treatment and control section Pretreatment Non-Routine Problem Examination score means.
**Hypothesis IV:** There is no statistically significant difference between treatment and control section Post-Treatment Resource Examination score means.

**Hypothesis V:** There is no statistically significant difference between treatment and control section Post-Treatment Non-Routine Problem Examination score means.

In this case, only the Post-Treatment Resource Examination means were significantly different at the 0.05 level of significance. Accordingly, it was assumed that it is tenable to believe that all other differences in means could be explained by chance alone. Table 4.3 shows the results of the test.

The results obtained using t-tests for hypotheses I through V were corroborated by considering the treatment and each control sections’ mean scores separately and conducting one-way analysis of variance (ANOVA) tests. As with the t-tests, only the Post-Treatment Resource Examination means were significantly different at the 0.05 level of significance.

**Correlations**

To further investigate the quantitative data, correlations between Resource Examination scores and Non-Routine Examination scores were determined to see if there was any improvement in the students' abilities to access and appropriately employ mathematical facts and procedures during problem solving attempts.

Following the treatment, the control group's correlation remained about the same. The treatment group's correlation improved dramatically indicating that the treatment may have been beneficial. This large shift in correlation provides some evidence that the treatment could have helped students to improve their metacognitive control thereby allowing better use of mathematical resources when faced with unfamiliar problems. Table 4.4 illustrates this point.

Scatterplots of Resource verses Non-Routine Problem Examinations for treatment and control section pre- and post-treatment scores and the associated correlation coefficients were generated using SPSS. The resulting relationships are illustrated in Figures 4.36 through 4.39 and Tables 4.4 and 4.5.
Correlation is significant at the 0.01 level (2-tailed).

Figure 4.36. Control Group Pretreatment Examination Score Scatterplot.

Correlation is not significant at the 0.05 level (2-tailed).

Figure 4.37. Treatment Group Pretreatment Examination Score Scatterplot.
Correlation is significant at the 0.01 level (2-tailed).

Figure 4.38. Control Group Post-Treatment Examination Score Scatterplot.

Correlation is significant at the 0.05 level (2-tailed).

Figure 4.39. Treatment Group Post-Treatment Examination Score Scatterplot.
Table 4.4. Correlations Between Resource and Non-Routine Problem Examinations.

<table>
<thead>
<tr>
<th></th>
<th>Pretreatment</th>
<th>Post-Treatment</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Group</td>
<td>0.629</td>
<td>0.697</td>
<td>+ 0.068</td>
</tr>
<tr>
<td>Treatment Group</td>
<td>-0.141</td>
<td>0.578</td>
<td>+ 0.719</td>
</tr>
</tbody>
</table>

Computing the correlation for each treatment section's scores separately supports the results obtained using pooled scores from both groups. Table 4.5 lists the results.

Table 4.5. Correlation by Section.

<table>
<thead>
<tr>
<th></th>
<th>Pretreatment</th>
<th>Post-Treatment</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM Control Section</td>
<td>0.576</td>
<td>0.778</td>
<td>+ 0.202</td>
</tr>
<tr>
<td>PM Control Section</td>
<td>0.661</td>
<td>0.502</td>
<td>-0.159</td>
</tr>
<tr>
<td>Treatment Section</td>
<td>-0.141</td>
<td>0.578</td>
<td>+ 0.719</td>
</tr>
</tbody>
</table>

In summary, group statistical evidence substantiated most of what was observed on individual performances during problem solving interview sessions. That is, students in the treatment section, for the most part, did not seem to benefit in any obvious way from the treatment. However, two possible interesting exceptions were suggested by the data.

First, the correlation between resource examinations and non-routine problem solving examinations for the treatment group increased from -0.141 to 0.578 - a shift of 0.719. The same measures for the control group increased only slightly from 0.629 to 0.697. This large increase for the treatment group, as compared to the control group,
implies the possibility that the treatment may have resulted in some degree of enhanced executive control manifested by an improved ability for students to access and manage their mathematical resources during non-routine problem solving attempts.

Secondly, the Post-Treatment Resource Examination mean score was statistically better for the treatment group when tested at the 0.05 level of significance. The researcher interpreted this to be further evidence that the treatment may have actually helped some students in the group by somehow improving their ability to accumulate and access mathematical resources. This improvement may have been a byproduct of improved study skills or some other kind of general improvement in metacognitive thought processes brought about by the treatment. Although the principle aim of the treatment was to improve students' abilities to solve non-routine problems in the local domain of mathematics, their entire way of thinking may have benefited in a more global way.
CHAPTER 5
CONCLUSIONS

Introduction
The purpose of this study was to explore connections between factual knowledge, procedural knowledge, metacognitive control and the non-routine problem solving ability of undergraduate mathematics students. Using a mixed methodology approach, qualitative evidence garnered from interviews was examined in concert with quantitative data collected on pre- and post-treatment examinations.

Chapter 1 established the need and rationale for the study, defined key concepts, and declared the specific questions explored within the study. Chapter 2 advanced the researcher's constructivist perspective on teaching and learning and positioned the study within the current research by summarizing previous work and noting voids in the literature. Chapter 3 described the participants, treatment, data collection, and analysis procedures. Chapter 4 provided the data collected, the researcher's interpretations of the non-routine problem solving interviews, pre- and post-treatment examination scores, and statistical analysis. This chapter will delineate conclusions based on the data to address the research questions outlined in Chapter 1.

Summary of Findings
The subjects involved in this study were students who self-selected into three sections of precalculus algebra at a small university. The researcher and one other professor instructed the three sections. All sections covered the same material in roughly the same order. Students who did not complete all examinations and at least seven of nine non-routine treatment problems were excluded from the study.

Prior to beginning the treatment, all students were given examinations designed to measure their routine algebra competence and how well they could apply their skills on
associated non-routine mathematical problems. Throughout the semester, treatment section students practiced a problem solving method designed to improve their metacognitive control using a combination of weekly homework and in-class assignments. Pre- and post-treatment videotaped interviews of four treatment section students engaged in non-routine problem solving were analyzed in an effort to uncover any changes in their metacognitive control. Statistical evidence gathered from four pre- and post-treatment examinations was also examined. In the next section, each research question and related hypotheses are restated. Then the researcher's interpretations of how evidence from the data answers each question are presented.

**Question 1**
Will a method of helping college precalculus algebra students develop their metacognitive control result in significantly better problem solving ability as compared to students who do not learn the strategies?

Question 1 was explored using both statistical analysis and interoperation of qualitative evidence gathered during non-routine problem solving interviews. Analysis of transcripts revealed that none of the students who participated in the videotaped interviews appeared to have significantly changed or improved their problem solving ability. The researcher noted that students who exhibited a particular metacognitive weakness during the pretreatment interviews generally persisted with the same kinds of faulty control during the post-treatment sessions. For example, B1 never wavered in her chronic efforts to simply try different formulas in an attempt to somehow stumble upon a solution.

Statistical results for Question 1 were somewhat mixed. Group statistical evidence comparing treatment to control sections seemed to substantiate most of what was observed during the interviews with one obvious exception. Although analysis of the videotaped interviews did not reveal any significant change in the way students approached non-routine problems, a significant increase in the correlation between the Resource Examination and Non-Routine Problem Examination scores indicate that students may have benefited from the treatment by improving their metacognitive control of resources. Specifically, for the control group, pretreatment $r = 0.639$ and post-
treatment $r = +0.697$, for a total change of $+0.068$. For the treatment group, pretreatment $r = -0.141$ and post-treatment $r = 0.578$ for a total change of $+0.719$. Results of null hypothesis tests applicable to Question 1 are now presented.

**Hypothesis I**

*There is no statistically significant difference between the change in pre- and post-treatment examination scores for the treatment and control groups*, was not rejected at the $\alpha = 0.05$ level of significance. This result was interpreted as evidence that the treatment students may not have benefited significantly from the treatment.

**Hypothesis II**

*There is no statistically significant difference between the treatment and control section Pretreatment Resource Examination score means* was not rejected at the $\alpha = 0.05$ level of significance. This statistic indicated that the treatment and control sections represented samples of the same population before the treatment because it is reasonable to believe that both groups possessed similar mathematical resources germane to the study prior to the treatment.

**Hypothesis III**

*There is no statistically significant difference between treatment and control section Pretreatment Non-Routine Problem Examination score means* was not rejected at the $\alpha = 0.05$ level of significance. This statistic further indicated that the treatment and control sections represented samples of the same population before the treatment because it is reasonable to believe that both groups possessed similar non-routine mathematical problem solving abilities prior to the treatment.

**Hypothesis IV**

*There is no statistically significant difference between treatment and control section Post-Treatment Resource Examination score means* was rejected at the $\alpha = 0.05$ level of significance. After demonstrating that pretreatment resource score means were not significantly different, the researcher interpreted the significant difference in post-treatment resource score means as evidence that the treatment students’ ability to accumulate and access mathematical resources may have been influenced significantly from the treatment.
Hypothesis V

There is no statistically significant difference between treatment and control section Post-Treatment Non-Routine Problem Examination score means was not rejected at the $\alpha = 0.05$ level of significance. After demonstrating that pretreatment score means were not significantly different, the researcher interpreted the difference in post-treatment score means as evidence that treatment students' non-routine problem solving ability may not have benefited significantly from the treatment.

In summary, students who exhibited poor executive decision making during interviews before the treatment persisted with the same kinds of difficulties following the treatment. Group statistical evidence comparing treatment to control sections seemed to substantiate what was observed during the interviews. Based on the correlation statistic, the researcher concluded that it is reasonable to believe that the treatment method used in an attempt to help undergraduate precalculus algebra students to develop their metacognitive control may have helped them to gain improved access to their mathematical resources. However, because of the contradictory nature of the other evidence gathered, it is difficult to posit with any real degree of certainty.

Question 2
How does a college precalculus algebra student's problem solving style change as a result of learning and practicing metacognitive control strategies?

Question 2 was investigated using qualitative evidence secured during videotaped interviews. The videotaped interviews did not reveal any obvious change in the way each student approached problems. It was disappointing that following treatment they did not effectively employ the modeling skills they had studied in class. Timelines did not reveal any significant change in the students' approach to the non-routine problems either. The percentage of time spent in each of the four domains remained relatively similar, as did the nature and frequency of overt metacognitive failures.

The researcher had previously noticed that mathematics students at this level frequently lose sight of what is being asked during problems that require several steps to solve. For example, students looking for functions to model linear data in order to answer a question requiring a prediction based on extrapolation may accurately determine
the equation parameters but then fail to write them in a usable formula or use their formula to actually answer what had been asked. They seem to understand portions of the process but have difficulty considering the problem in its entirety. The students in this study were no exception.

It was also observed that students in this study very frequently did not understand spatial relationships. For example, Team 2 showed significant difficulty understanding and visualizing how the two lakes were related in the first pretreatment interview problem. Both teams were confused by the post-treatment interview problem involving the distance a six-foot man would need to be away from the wall in order to see over the building. Not only did they demonstrate a fundamental lack of understanding regarding the spatial relationship between the building, wall, and man, they appeared to not even be sure of what was being asked during the first several minutes of the interview.

It was further observed that many students have great difficulty understanding how various components of a problem statement are related. For example, it seemed that none of the students ever really understood the uranium problem in the second post-treatment interview. Team 2 thought the word unstable implied that growth was possible following decay even though half-life, doubling time, exponential growth and decay had been studied at length during the course. Also, Team 2 tried to use a linear model even though they had previously demonstrated an ability to correctly identify and model exponential data such as was required in this instance. The researcher concluded that, in this case, the study subjects' problem solving style did not change significantly as a result of learning and practicing the treatment metacognitive control strategies. In short, based on available evidence, the treatment most likely did not make a significant difference.

**Question 3**

Do students who practice the metacognitive control strategies when directed by class assignments throughout the semester retain them in other settings?

Question 3 was investigated using qualitative evidence secured during videotaped interviews. Students did not use or even mention the Planning-Drawing-Doing-Verifying routine on any of the post-treatment non-routine problems. Although throughout the course of the semester they were given nine graded non-routine problems requiring use of
the method, the fact that they never used it unless they knew it was required for a grade was interpreted by the researcher as evidence that they probably considered the method nothing more than a burden. Because the four videotaped interview subjects who practiced the metacognitive control treatment strategies when directed by class assignments throughout the semester did not choose to employ them when use was optional, the researcher concluded that most of the treatment students will probably not retain and use the method.

**Implications for Practice**

The goal of mathematics education must be to help prepare students for a rapidly changing world by equipping them with the ability to solve complex problems and communicate and reason mathematically. Secondary school and undergraduate mathematics classes must stress problem solving as a vital skill or primary learning objective rather than an optional topic explored only as time permits.

Students in this study consistently demonstrated confusion about spatial relationships and what basic information presented in a problem actually meant. This phenomenon may be indicative of a fundamental uncertainty of what it means to truly understand and effectively employ mathematics. Because the students in this study were a representative sample from the United States undergraduate student population at large, it is reasonable to assume that their past experiences in the mathematics classroom have included very little in the way of solving application problems requiring the use of facts and procedures in unfamiliar ways. The challenge for mathematics educators is to find better ways of helping them to realize that there is much more to learning mathematics than simply memorizing knowledge that is passed down from so-called experts. Students who are conditioned by years of working through application problems via a boilerplate approach, never really needing or trying to understand the concepts involved, generally have difficulty with non-routine problems (Schoenfeld, 1985). Subjects in this study were no exception.

Educators who are mostly interested in teaching their students a bag of tricks, applicable to specific routine kinds of application problems, can never hope to also somehow foster an understanding of underlying concepts necessary to be a successful
problem solver. Students enrolled in courses conducted in this manner are missing a great deal. Instructors should provide assignments and experiences that stress the underlying key concepts of mathematics while striving to help their students realize that learning mathematics is not simply a matter of rote memorization. The mechanics of number crunching are necessary skills but should not be interpreted as what is most significant in the curriculum.

**Limitations of Study**

Assignment of subjects to treatment and control groups was dictated not by strict random selection but rather by convenience. Students self selected into the three sections of precalculus algebra included in this study. Although it is difficult to imagine circumstances which would allow for true random selection during this kind of inquiry, such a case would, of course, enhance the level of confidence with which the researcher could report the statistical results.

Because there is not currently a widely accepted, reliable and authoritative convention for coding and interpreting mathematical problem solving interview transcripts, and the pool of similar research is so sparse, it is difficult to evaluate how the observations obtained from this study correspond and compare to the results obtained by other education researchers. Only by repetition and refinement will more generalizable results emerge.

All of the videotape interview subjects were female. The initial pool of three teams included a team of two males, however, because one of the males only completed five of nine assigned treatment problems, his team was excluded from transcript analysis. An examination of both male and female videotaped interview transcripts would be a more representative sample of the student population.

**Future Research**

Much more research is needed to understand the complex relationship between factual knowledge, procedural knowledge, metacognitive control, and the non-routine mathematical problem solving ability of undergraduate students. The results of this study suggest the following paths for future research.
1. Metacognitive studies concentrating on students' understanding of what is being asked may be fruitful. Because they seem generally ill-equipped to grasp how the elements of non-routine problems are related, it may be better to first focus on helping them find ways to understand the problem statement and visualize what is given rather than trying to help them learn a global system for dealing with all unfamiliar problems.

2. Extending the treatment period in increments to determine if there is a critical period of time after which most students become habituated by the treatment and show clearly obvious signs of significantly improved metacognitive control would be instructive.

3. Following the progress of study subjects as they work their way through other math and science courses may be illuminating. Perhaps benefits from the treatment would emerge in other settings and as students mature academically.

4. Studies that utilize methods of protocol analysis similar to those used here could be beneficial. Refining and firmly establishing a standardized method of protocol coding and analysis, generally accepted and utilized in similar studies, would help to generate more continuity in the field.

5. Studying the result of a slightly modified version of the treatment may be useful. Students could be assigned to small groups consisting of members who exhibit characteristics of weak executive control and at least one student whom has demonstrated relatively strong control. Then, analysis of videotaped interviews to determine if and how the weaker students benefited from the experience could follow a treatment consisting of non-routine problems completed as group assignments during class.

6. Follow-up interviews conducted with students who participated in the non-routine problem solving interview sessions could help to define the kinds of influences that caused them to make the executive decisions classified as faulty during transcript analysis.

Because students who exercise higher metacognitive control are better able to solve non-routine problems it is important that voids in the literature be filled. It is difficult to find continuity and replication among past studies addressing the connection between
metacognition and non-routine problem solving at the undergraduate level. The suggestions for future research presented here provide insight into several possible directions that may prove useful.
APPENDIX A

PRETREATMENT NON-ROUTINE PROBLEM SOLVING EXAMINATION
Pretreatment Non-Routine Problem Solving Examination

1. There are two numbers whose sum is 50. Three times the first is 5 more than twice the second. What are the numbers?

2. If the area of each small circle in the figure below is $9\pi$, what is the total area of the two shaded squares?

3. Two ships depart from the same port at 11:30 A.M. If one sails due east at 15 miles per hour and the other due south at 20 miles per hour, how many miles apart are the ships at 1:30 P.M.?

4. Mr. Lee takes his wife and two children to an amusement park. If the price of a child's ticket is $\frac{1}{2}$ the price of an adult ticket and Mr. Lee pays a total of $12.60, find the price of a child's ticket.

5. The base of a rectangular tank is 6 feet by 5 feet and its height is 16 inches. Find the number of cubic feet of water in the tank when it is $\frac{3}{4}$ full.
APPENDIX B

POST-TREATMENT NON-ROUTINE PROBLEM SOLVING EXAMINATION
Post-Treatment Non-Routine Examination

1. Suppose that tides vary from 5 feet above to 5 feet below sea level and that successive low tides occur every 20 hours. Assuming that on a particular day low tide occurs at 6:00 am, determine the water level at noon.

2. The amount of income tax $T$, in dollars, owed to the state of Oklahoma is a linear function of the taxable income $I$, in dollars, at least over a suitably restricted range of incomes. According to the year 2002 Oklahoma Income Tax tables, a single Oklahoma resident taxpayer with an income of $15,000 owes $780 in Oklahoma income tax. If the taxable income is $15,500, then the tables show a tax liability of $825. How much does the taxpayer owe if the taxable income is $15,350?

3. Radioactive substances decay over time, and the rate of decay depends on the element. If, for example, there are $G$ grams of heavy hydrogen $H$ in a container, then as a result of radioactive decay, 1 year later there will be $0.783G$ of heavy hydrogen left. Supposing we begin an experiment with 50 grams of heavy hydrogen, how much will be left after 5 years?
APPENDIX C

PRETREATMENT RESOURCE EXAMINATION
Pretreatment Resource Examination

1. What is the area of the given circle?

\[ A = \pi r^2 = \pi \times 1^2 = \pi \] square units

2. What is the radius of the given circle?

\[ r = \frac{d}{2} = \frac{25}{2} \] inches

3. What is the diameter of the given circle?

\[ d = 2r = 2 \times 4 = 8 \] inches

4. What is the area of the rectangle?

\[ A = \text{Length} \times \text{Width} = 8 \times 5 = 40 \] square inches
5. Given that the circle inside the square has a 10-inch diameter, what is the total area enclosed inside the square?

![Circle inside a square]

6. What is the volume of a rectangular solid that is 5 feet long by 8 feet wide by 2 feet tall?

7. Simplify. \( y = 5 \times 3 \times \frac{1}{3} \)

8. Simplify. \( y = 16 \times \frac{3}{4} \)

9. What are the dimensions of one cubic foot of water?
10. How deep, in inches, is the water in a tank that is 4 feet tall when it is \( \frac{3}{4} \) full?

11. Label South, East, and West on the figure.

12. The Pythagorean Theorem is related to the lengths of the three sides of a right triangle. State the theorem as it relates to the given right triangle.

13. How far does a car traveling at a constant rate of 55 miles per hour travel in 3 hours?
14. Simplify. \( y = 2^3 \)

15. Solve for \( y \) and simplify. \( y^2 = 100 \)

16. Simplify. \( y = \frac{14.96}{4} \)

17. Simplify by combining like terms. \( 3x + \frac{1}{2} x + \frac{1}{2} x \)

18. Solve for \( x \) and simplify. \( 6x = 21.60 \)

19. What is \( \frac{1}{2} \) of $3.40?
20. If three tickets cost a total of $15.30, how much is each ticket?

21. Solve for \( x \). \( 5x = 3 + 75 - 2x \)

22. Solve for \( x \). \( 4x = 6 + 3(60 - x) \)

23. Write an expression representing the situation, "5 times \( x \) is 7 more than \( y \)."

24. Write an expression representing the situation, "\( y \) is twice as big as \( x \)."

25. Write an expression representing the situation, "The sum of two numbers, \( x \) and \( y \), is 22."
APPENDIX D

POST-TREATMENT RESOURCE EXAMINATION
Post-Treatment Resource Examination

1. Use the table to decide if \( W \) is a function of \( R \). Explain your reasoning.

<table>
<thead>
<tr>
<th>( R )</th>
<th>3</th>
<th>12</th>
<th>11</th>
<th>12</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W )</td>
<td>103</td>
<td>97</td>
<td>88</td>
<td>97</td>
<td>91</td>
</tr>
</tbody>
</table>

2. If \( f(x) = 3x^3 - 10x + 2 \), what is the average rate of change of \( f \) on the interval from \( x = 0 \) to \( x = 2 \)?

3. Write an equation for the horizontal line through the point (5, -8).

4. Find the slope of the line parallel to \( 9x - 3y = 18 \).

5. Find the point where the lines intersect.

\[
y = x - 3 \quad \text{and} \quad y = \frac{1}{3}x - 12
\]

6. Find a formula for the function that has the following graph.

7. Solve for \( x \).
   
   a) \( 25(1.35)^x = 50 \)
   
   b) \( x^3 - 2x + 17 = -1 + x \)
8. The percentages of women in state legislatures for past years are illustrated in the following table.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent - P</td>
<td>10.3</td>
<td>12.1</td>
<td>13.3</td>
<td>14.8</td>
<td>15.7</td>
<td>17.0</td>
<td>18.3</td>
<td>20.5</td>
<td>20.7</td>
</tr>
</tbody>
</table>

a) Use a calculator to find the equation of the linear regression that models these data.

b) Use the equation you found in part (a) to estimate the percentage of women in state legislatures in 2002.

9. A population of bacteria in a glass dish grows exponentially. Let \( P(t) \) be the population after \( t \) hours and suppose \( P(1) = 100 \) and \( P(2) = 500 \). Find a formula for \( P(t) \) in the form \( P(t) = a \cdot b^t \) or \( P(t) = a \cdot e^{kt} \).

10. A $5,000 initial deposit in a bank account grows according to the formula

\[
B(t) = 5000e^{0.03t}
\]

where \( B \) is the amount in the bank \( t \) years after 1995.

a) Evaluate \( B(5) \) and explain the meaning of your answer in practical terms.

b) How long will it take for the initial deposit to double in size?

11. State the domain for each function

a) \( g(x) = \frac{22}{2x - 6} \)

b) \( h(x) = \sqrt{x + 37} \)

12. Determine the radian measure of 220°.

13. What is the length of an arc which is cut off by an angle of 135° in a circle of radius 8 feet?

14. Solve for \( t \). \( 2 \sin \left( \frac{1}{2} t \right) = \frac{\sqrt{3}}{2} \)
15. Find an exact value for:
   a) \( \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \)
   b) \( \tan^{-1}(1) \)

16. The following tables contain values from an exponential or a linear function.

   a) Show if the function represented by each table is linear or exponential. 
      You must show appropriate calculations or explain your conclusions for credit.
   b) Find a possible formula for each function.

   Table A
   \[
   \begin{array}{c|c|c|c}
   x & 2 & 5 & 7 \\
   \hline
   f(x) & 0.90 & 4.50 & 6.90 \\
   \end{array}
   \]

   Table B
   \[
   \begin{array}{c|c|c|c}
   x & 1 & 2 & 4 \\
   \hline
   g(x) & 3.000 & 4.500 & 10.125 \\
   \end{array}
   \]

17. Mark the following angles on a unit circle and give the coordinates of the point.

   a) 45°
   b) 120°
18. Fill in the exact values for the sine, cosine, and radian measures in the following table.

<table>
<thead>
<tr>
<th>Degrees</th>
<th>0°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cosine</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radians</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Degrees</th>
<th>150°</th>
<th>210°</th>
<th>270°</th>
<th>300°</th>
<th>315°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cosine</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radians</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

19. Find a possible formula for the trigonometric function shown below. State your answer in the form

\[ y = A \sin(B(t - h)) + k \quad \text{or} \quad y = A \cos(B(t - h)) + k. \]
APPENDIX E

PLANNING-DRAWING-DOING-VERIFYING TREATMENT PROBLEMS
1. A cathedral ceiling as shown below is 8 feet high at the west wall of a room. As you go from the west wall toward the east wall, the ceiling slants upward. Three feet from the west wall, the ceiling is 10.5 feet high. The width of the room (the distance from the west wall to the east wall) is 17 feet.

You want to install a light in the ceiling as far away from the west wall as possible. You intend to change the bulb, when required, by standing at the top of your small stepladder. If you stand on your stepladder, you can reach 12 feet high. How far from the west wall should you install the light?

2. According to one rule of thumb relating weight to height among adult males, if a man is 2 inches taller than another, then we expect him to be heavier by 10 pounds. A related rule of thumb is that a typical man who is 70 inches tall weighs 170 pounds. On the basis of these two rules of thumb, find a formula to express the weight of a man as a linear function of height. Be sure to identify the meaning of any letters you use.

3. You want to choose a long distance company from the following options.

- Company A charges $0.37 per minute.
- Company B charges $13.95 per month plus $0.22 per minute.
- Company C charges a fixed rate of $50.00 per month.

Find the number of minutes for which company B is the cheapest.
4. In 1980, there were about 170 million vehicles (cars and trucks) and about 227 million people in the United States. If the number of vehicles has been growing at 4% a year, while the population has been growing at 1% a year, in what year will there be, on average, one vehicle per person?

5. At 3:00 P.M. a park ranger discovered a dead bald eagle that had been impaled by an arrow. Only two archers were found in the region. The first archer is able to establish that, between 11:00 A.M. and 1:00 P.M., he was in a nearby diner having lunch. The second archer can show that he was in camp with friends between 9:00 A.M. and 11:00 A.M. The air temperature in the park has remained at a constant 62 degree. Beginning at 3:00 P.M., the difference \( D = D(t) \) between the temperature of the dead eagle and that of the air was measured and recorded in the table below. (Here \( t \) is the time in hours since 3:00 P.M.) This table, together with the fact that the body temperature of a living bald eagle is 105 degrees, exonerates one of the archers, but the other main remain suspect. Show which archer is probably innocent.

<table>
<thead>
<tr>
<th>( t ) = hours since 3:00 P.M.</th>
<th>( D ) = temperature difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>26.83</td>
</tr>
<tr>
<td>1</td>
<td>24.42</td>
</tr>
<tr>
<td>2</td>
<td>22.22</td>
</tr>
<tr>
<td>3</td>
<td>20.22</td>
</tr>
<tr>
<td>4</td>
<td>18.40</td>
</tr>
<tr>
<td>5</td>
<td>16.74</td>
</tr>
</tbody>
</table>

6. A tank of water is contaminated with 60 pounds of salt. In order to bring the salt concentration down to a level consistent with EPA standards, clean water is being piped into the tank, and the well-mixed overflow is being collected for removal to a toxic-waste site. The result is that at the end of each hour there is 22% less salt in the tank than at the beginning of the hour. The cleanup procedure cost $8000 per hour to operate. In order to meet EPA standards there can be no more than 3 pounds of salt in the tank. If the cleanup procedure costs $8000 per hour, what is the total cost to reduce the salt to meet the EPA standard?
7. Scientists have studied the daily biorhythm of a certain amino acid in the liver of rats. The level varies from a low of 1.5 units to a high of 2.5 units over the period of one day. Assuming the level is lowest at 10:00 A.M. each morning, at what time of day will a level of 2.2 units first occur?

8. City A is a modest tourist town, which means that its population undergoes a seasonal variation. In January it dips down to 4,500 people, but by July, with warm weather, its population climbs to around 5,500 people. By the following January, however, the population has again fallen to 4,500 people. This trend repeats every year. City B on the other hand, is a small town not far from City A. Its population has been steadily growing ever since the arrival a new automobile assembly plant. There were only 4,000 people living there on January 1, 2000, but its population has grown by 2% every year thereafter. During what year and month did the population of City B surpass and always remain greater than the population of City A?

9. Human blood pressure varies as the heart beats. When the heart contracts, blood pressure increases. But as the heart relaxes in preparation for the next contraction, blood pressure decreases. The result is that a person's blood pressure changes in a periodic fashion, each period corresponding to a single heartbeat. Physicians refer to the maximum of this pressure as the systolic pressure and to the minimum pressure as the diastolic pressure. A certain individual has a pulse rate of 75 beats per minute, a systolic pressure of 130, and a diastolic pressure of 70. Here the blood pressure is measured in millimeters of mercury. For how long during each cycle is the blood pressure lower than 90 millimeters of mercury?
APPENDIX F

HUMAN SUBJECTS COMMITTEE MATERIAL
Office of the Vice President for Research  
Tallahassee, Florida 32306-2763  
(850) 644-8673 • FAX (850) 644-4392

APPROVAL MEMORANDUM  
from the Human Subjects Committee

Date: October 21, 2002
From: David Quadagno, Chair
To: Michael Nancarrow  
11770 Donato Drive  
Jacksonville, FL 32226
Dept: Middle & Secondary Education
Re: Use of Human subjects in Research  
Project entitled: Exploration of Metacognition and Non-Routine  
Problem Based Mathematics Instruction on Undergraduate Student  
Problem Solving Success

The forms that you submitted to this office in regard to the use of human subjects in the proposal referenced above have been reviewed by the Secretary, the Chair, and two members of the Human Subjects Committee. Your project is determined to be exempt per 45 CFR § 46.101(b)2 and has been approved by an accelerated review process.

The Human Subjects Committee has not evaluated your proposal for scientific merit, except to weigh the risk to the human participants and the aspects of the proposal related to potential risk and benefit. This approval does not replace any departmental or other approvals which may be required.

If the project has not been completed by October 20, 2003 you must request renewed approval for continuation of the project.

You are advised that any change in protocol in this project must be approved by resubmission of the project to the Committee for approval. Also, the principal investigator must promptly report, in writing, any unexpected problems causing risks to research subjects or others.

By copy of this memorandum, the chairman of your department and/or your major professor is reminded that he/she is responsible for being informed concerning research projects involving human subjects in the department, and should review protocols of such investigations as often as needed to insure that the project is being conducted in compliance with our institution and with DHHS regulations.

This institution has an Assurance on file with the Office for Protection from Research Risks. The Assurance Number is IRB00000446.

Cc: Dr. Elizabeth Jakubowski  
APPLICATION NO. 02.513
Office of the Vice President
For Research
Tallahassee, Florida 32306-2763
(850) 644-5280 · FAX (850) 644-4392

REAPPROVAL MEMORANDUM
from the Human Subjects Committee

Date: 1/7/2004

Michael Nancarrow
11770 Donato Dr.
Jacksonville FL 32226

From: David Quadagno, Chair

Dept.: Middle and Secondary Education

Re: Reapproval of Use of Human subjects in Research:
   Exploration of Metacognition and Non-Routine Problem Based Mathematics Instruction on
   Undergraduate Problem Solving Success

Your request to continue the research project listed above involving human subjects has been approved by
the Human Subjects Committee. If your project has not been completed by 10/20/2004 please request
renewed approval.

You are reminded that a change in protocol in this project must be approved by resubmission of the project
to the Committee for approval. Also, the principal investigator must report to the Chair promptly, and in
writing, any unanticipated problems involving risks to subjects or others.

By copy of this memorandum, the Chairman of your department and/or your major professor are reminded
of their responsibility for being informed concerning research projects involving human subjects in their
department. They are advised to review the protocols of such investigations as often as necessary to
insure that the project is being conducted in compliance with our institution and with DHHS regulations.

Cc: Dr. Elizabeth Jakubowski
HSC No. 2003.546-R
Florida State University

STATEMENT OF INFORMED CONSENT

I, _______________________________ freely and voluntarily and without element of force or coercion, consent to be a participant in the research project entitled

“EXPLORATION OF METACOGNITION AND NON-Routine PROBLEM BASED MATHEMATICS INSTRUCTION ON UNDERGRADUATE STUDENT PROBLEM SOLVING SUCCESS.”

This research is being conducted by Mr. Michael Nancarrow, who is an assistant professor of mathematics at Jacksonville University and a doctoral candidate at Florida State University. I understand the purpose of his research project is to better understand the problem solving ability of undergraduate mathematics students.

I understand that in order to participate in this study I must be at least 18 years old. I further understand that if I participate in the project I will be asked to solve several non-routine mathematical problems throughout the semester. Many of the problems will be weekly homework assignments. Others will be assigned during normal classroom hours. The total time commitment for homework will not be any greater than for students who chose not to participate in the study. I may also be asked to participate in one or two interviews with the researcher or his assistant. Interviews will occur outside of normal classroom hours. The total time commitment for each interview will be approximately one hour. I understand that I may be tape recorded or videotaped by the researcher during the interview. The researcher will keep these tapes in a locked filing cabinet. I understand that only the researcher will have access to these tapes and that they will be destroyed by December 31, 2004.

I understand my participation is totally voluntary and I may stop participation at any time. If I choose not to participate or to withdraw from the study at any time, it will not affect my grade. All my answers to the problems will be identified by a subject code number and kept confidential to the extent allowed by law. My name will not appear on any of the results. Individual responses may be reported using pseudonyms that will assure confidentiality. Group findings will also be reported.

I understand there is a minimal level of risk involved if I agree to participate in this study. I might experience anxiety while being closely monitored and/or videotaped as I attempt to solve non-routine mathematical problems. The researcher will be available to talk with me about any emotional discomfort I may experience while participating. I am also able to stop my participation at any time I wish.

I understand there are benefits for participating in this research project. First, my own awareness and understanding about how I approach non-routine mathematical problems may be increased. I may also improve my ability to solve mathematical problems. Furthermore, I will be providing mathematics educators with valuable insight into how students’ thought processes influence their problem solving behaviors. This knowledge can assist them in providing better instruction.

I understand that this consent may be withdrawn at any time without prejudice or penalty and that non-participation will not affect my grade. I have been given the right to ask and have answered any inquiry concerning the study. Questions, if any, have been answered to my satisfaction.

I understand that I may contact Mr. Michael Nancarrow, Jacksonville University, Department of Mathematics, MP-210, (904) 745-7315, or the professor supervising his research, Dr. Elizabeth Jakubowski at (850)-644-6885 or ejakubow@coe.fsu.edu for answers to questions about this research or my rights. Results will be sent to me upon my request.

I am at least 18 years old and I have read and understand this consent form.

(Subject) _______________________________ (Date) _______________________________

If you have any questions about your rights as a subject/participant in this research, or if you feel you have been placed at risk, you can contact the Chair of the Human Subjects Committee, Institutional Review Board, through the Vice President for the Office of Research at (850) 644-8633 or

HUMAN SUBJECTS COMMITTEE, Mail Code 2783, or
2035 E. Paul Dirac Drive, Box 15
100 Sliger Blvd., Innovation Park
Tallahassee, FL 32310

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BIBLIOGRAPHY


BIOGRAPHICAL SKETCH

Michael Jay Nancarrow was born in Richmond, Indiana in 1957 and was graduated from Delta High School near Muncie, Indiana in 1975. He served as an enlisted man in the United States Navy from 1977 to 1997 when he retired to pursue a second career as an educator. During his time in the Navy he advanced to the rank of Senior Chief and amassed over 5600 flight hours as a turboprop flight engineer in P3 Orion aircraft.

In 1991 he earned a Bachelor of Science in Liberal Studies with a concentration in sociology from Excelsior College in Albany, New York. He earned a Master of Arts in Teaching Mathematics from Jacksonville University in Jacksonville, Florida in 1997. Following graduation from Jacksonville University, he worked as an Adjunct Mathematics Instructor for Florida Community College until the fall semester of 1998 when he began teaching undergraduate mathematics classes fulltime in his present position as an Assistant Professor of Mathematics at Jacksonville University.

Michael is an avid aviation enthusiast, spending much of his free time flying small aircraft. He also enjoys kayaking, hiking and fishing. In 1978, while stationed in Key West, Florida, he met his wife, Susan. They have two sons, Michael and Lance.