An Investigation of How African American Community College Students with Different Levels of Mathematics Anxiety Engage in Problem Solving Tasks

Calandra Moorman Walker
THE FLORIDA STATE UNIVERSITY

COLLEGE OF EDUCATION

AN INVESTIGATION OF HOW AFRICAN AMERICAN COMMUNITY COLLEGE STUDENTS WITH DIFFERENT LEVELS OF MATHEMATICS ANXIETY ENGAGE IN PROBLEM SOLVING TASKS

BY

CALANDRA MOORMAN WALKER

A Dissertation submitted to the Department of Middle and Secondary Education in partial fulfillment of the requirements for the degree of Doctor of Philosophy

Degree Awarded:
Fall Semester, 2007
The members of the Committee approve the dissertation of Calandra Moorman Walker defended on October 18, 2007.

________________________(signed)
Leslie Aspinwall
Professor Directing Dissertation

________________________(signed)
Ithel Jones
Outside Committee Member

________________________(signed)
Kathy Clark
Committee Member

________________________(signed)
Kenneth Shaw
Committee Member

Approved:

________________________
Pamela Carroll, Chairperson, Middle and Secondary Education

The Office of Graduate Studies has verified and approved the above named committee members.
ACKNOWLEDGEMENTS

I would like to thank God for allowing me complete this dissertation. All of this is possible only through Him!!!!

I would like to thank my committee: Dr. Leslie Aspinwall, Dr. Kathy Clark, Dr. Ken Shaw, and Dr. Ithel Jones. Thanks for the constant feedback and encouragement while writing my dissertation. A special thanks to Dr. Maria Fernandez for helping me to shape my dissertation when it was in its beginning stages. A special thanks to Dr. Kathy Clark for agreeing to come aboard my committee. I really appreciate your much needed wisdom and recommendations. Thanks for challenging me to do more and to think harder.

To my husband, Keenan, I appreciate your much needed support and encouragement. You always knew I could make it even when I was ready to quit. You supported me on my long nights in front of the computer and my long nights at the library. You listened to me when I was ready to give up, but your late night talks helped me to see things clearly. I am glad that you were by my side through this whole process. “That’s What Friends Are For!”

To my parents, Dianne and Larry Moorman, I appreciate your many years of financial and emotional support along with encouragement during my many years of school and life.

To my sisters, Latosha and Sondra, you guys thought I was crazy for spending so much time in school, but I’m finally finished and now it’s your turn.

To my immediate and extended family and church family, thanks for your many prayers! Thanks Pastor C. Watkins for your special prayers and your encouraging words.

Mr. Wilbert Butler, Jr., it was nice to have someone to go step by step with. Thanks for hanging in there with me.

A special thanks to E. W. Stringer III for giving me the encouragement to complete my work. Thanks for giving me inspiration to continue my work in the future and beyond…
TABLE OF CONTENTS

LIST OF TABLES ................................................................................................................ viii
LIST OF FIGURES ................................................................................................................. ix
ABSTRACT ............................................................................................................................. xi
INTRODUCTION ..................................................................................................................... 1
  Characteristics of the Community College ................................................................. 3
  Mathematical Problem Solving Standards for the Community College ....................... 3
  Mathematical Anxiety in the Community College ....................................................... 4
  Rationale .............................................................................................................................. 4
  Research Questions .......................................................................................................... 6
REVIEW OF THE LITERATURE .......................................................................................... 7
  Mathematics Anxiety ....................................................................................................... 7
  Effects of Mathematics Anxiety .................................................................................. 8
  Mathematics Anxiety and College Students ............................................................ 8
  Problem Solving ............................................................................................................. 10
  Metacognition ................................................................................................................ 13
  African American Students’ Approaches to Learning ................................................ 16
  Summary and Researcher’s Thoughts ...................................................................... 18
METHODOLOGY .................................................................................................................. 19
  Overview of the Study ................................................................................................. 19
  Setting ............................................................................................................................ 20
  Participant Selection .................................................................................................. 20
  Data ............................................................................................................................... 22
  Mathematics Anxiety Rating Scale – Shorten Version ............................................. 22
  Task-Based Interviews ............................................................................................... 23
  Problem Solving Tasks ............................................................................................... 24
  Analysis ......................................................................................................................... 25
HIGH MATHEMATICS ANXIETY RESULTS .................................................................... 27
  The Case of the High Mathematics Anxiety Students ............................................. 29
  The Students’ Strategies ............................................................................................ 30
<table>
<thead>
<tr>
<th>Problem 1: <em>Penny’s Dimes</em></th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students’ Work</td>
<td>30</td>
</tr>
<tr>
<td>Researcher’s Observations</td>
<td>31</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 2: <em>Night of the Howling Dogs</em></th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students’ Work</td>
<td>34</td>
</tr>
<tr>
<td>Researcher’s Observations</td>
<td>35</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 3: <em>Cascades State Park</em></th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students’ Work</td>
<td>37</td>
</tr>
<tr>
<td>Researcher’s Observations</td>
<td>39</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 4: <em>Divisors and Reciprocals</em></th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students’ Work</td>
<td>41</td>
</tr>
<tr>
<td>Researcher’s Observations</td>
<td>43</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 5: <em>Dad’s Wallet</em></th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students’ Work</td>
<td>44</td>
</tr>
<tr>
<td>Researcher’s Observations</td>
<td>46</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 6: <em>The Pool Deck</em></th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students’ Work</td>
<td>47</td>
</tr>
<tr>
<td>Researcher’s Observations</td>
<td>49</td>
</tr>
</tbody>
</table>

Strategies and Thinking Processes Common to Both Kate and Sharon ..................51

Mathematics Anxiety Survey .................................................................52

Emerging Themes ...................................................................................52

LOW MATHEMATICS ANXIETY RESULTS ..................................................54

The Case of the Low Mathematics Anxiety Students .........................54

The Students’ Strategies .......................................................................55

<table>
<thead>
<tr>
<th>Problem 1: <em>Penny’s Dimes</em></th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students’ Work</td>
<td>56</td>
</tr>
<tr>
<td>Researcher’s Observations</td>
<td>56</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 2: <em>Night of the Howling Dogs</em></th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students’ Work</td>
<td>60</td>
</tr>
<tr>
<td>Researcher’s Observations</td>
<td>60</td>
</tr>
</tbody>
</table>
Problem 3: Cascades State Park ................................................................. 63
   Students’ Work .................................................................................. 63
   Researcher’s Observations ................................................................. 65
Problem 4: Divisors and Reciprocals ...................................................... 66
   Students’ Work .................................................................................. 66
   Researcher’s Observations ................................................................. 68
Problem 5: Dad’s Wallet ........................................................................ 69
   Students’ Work .................................................................................. 69
   Researcher’s Observations ................................................................. 71
Problem 6: The Pool Deck ................................................................. 72
   Students’ Work .................................................................................. 72
   Researcher’s Observations ................................................................. 73
Strategies and Thinking Processes Common to Both Adam and Hank .... 75
Mathematics Anxiety Survey ............................................................... 75
Emerging Themes .................................................................................. 76
CONCLUSIONS ....................................................................................... 78
   Students’ Strategies ............................................................................ 78
      Problem 1: Penny’s Dimes ............................................................... 79
      Problem 2: Night of the Howling Dogs ........................................... 80
      Problem 3: Cascades State Park ..................................................... 81
      Problem 4: Divisors and Reciprocals ............................................. 81
      Problem 5: Dad’s Wallet ............................................................... 82
      Problem 6: The Pool Deck ............................................................. 83
Synopsis of Themes ............................................................................... 84
Implications ......................................................................................... 85
   Exhibiting Mathematics Anxiety Symptoms .................................... 85
   Drawing Pictures and Diagrams ....................................................... 86
   Inability to Recall Previously Studied Material ............................... 86
   Failure to Use All of Montague’s Thinking Processes ....................... 87
   Difficulty with Reading Skills ......................................................... 88
   Checking Calculations and Solution Process .................................. 89
LIST OF TABLES

Table 4.1 – Strategies and Thinking Processes Common to Both Kate and Sharon ..........51
Table 5.1 – Strategies and Thinking Processes Common to Both Adam and Hank ..........75
Table 6.1 – Penny’s Dimes Strategies and Thinking Processes Common to Each Case ......79
Table 6.2 – Night of the Howling Dogs Strategies and Thinking Processes Common to Each Case ....................................................................................................................... 80
Table 6.3 – Cascades State Park and Thinking Processes Common to Each Case ..........81
Table 6.4 – Divisors and Reciprocals Strategies and Thinking Processes Common to Each Case .....................................................................................................................82
Table 6.5 – Dad’s Wallet Strategies and Thinking Processes Common to Each Case .......83
Table 6.6 – The Pool Deck Strategies and Thinking Processes Common to Each Case ......84
## LIST OF FIGURES

| Figure 2.1 | (Wilson, Fernandez, & Hadaway, 1993) | 12 |
| Figure 4.1 | Problem 1: *Penny’s Dimes* | 30 |
| Figure 4.2 | Kate’s *Penny’s Dimes* | 31 |
| Figure 4.3 | Sharon’s *Penny’s Dimes* | 32 |
| Figure 4.4 | Problem 2: *Night of the Howling Dogs* | 34 |
| Figure 4.5 | Kate’s *Night of the Howling Dogs* | 35 |
| Figure 4.6 | Sharon’s *Night of the Howling Dogs* | 35 |
| Figure 4.7 | Problem 3: *Cascades State Park* | 37 |
| Figure 4.8 | Kate’s *Cascades State Park* | 38 |
| Figure 4.9 | Sharon’s *Cascades State Park* | 39 |
| Figure 4.10 | Problem 4: *Divisors and Reciprocals* | 41 |
| Figure 4.11 | Kate’s *Divisors and Reciprocals* | 42 |
| Figure 4.12 | Sharon’s *Divisors and Reciprocals* | 42 |
| Figure 4.13 | Problem 5: *Dad’s Wallet* | 44 |
| Figure 4.14 | Kate’s *Dad’s Wallet* | 45 |
| Figure 4.15 | Sharon’s *Dad’s Wallet* | 46 |
| Figure 4.16 | Problem 6: *The Pool Deck* | 47 |
| Figure 4.17 | Kate’s *The Pool Deck* | 48 |
| Figure 4.18 | Sharon’s *The Pool Deck* | 49 |
| Figure 5.1 | Problem 1: *Penny’s Dimes* | 56 |
| Figure 5.2 | Adam’s *Penny’s Dimes* | 57 |
| Figure 5.3 | Hank’s *Penny’s Dimes* | 58 |
| Figure 5.4 | Problem 2: *Night of the Howling Dogs* | 60 |
| Figure 5.5 | Adam’s *Night of the Howling Dogs* | 60 |
| Figure 5.6 | Hank’s *Night of the Howling Dogs* | 61 |
| Figure 5.7 | Problem 3: *Cascades State Park* | 63 |
| Figure 5.8 | Adam’s *Cascades State Park* | 64 |
| Figure 5.9 | Hank’s *Cascades State Park* | 65 |
| Figure 5.10 | Problem 4: *Divisors and Reciprocals* | 66 |
| Figure 5.11 | Adam’s *Divisors and Reciprocals* | 67 |
ABSTRACT

This qualitative case study examined the strategies and thinking processes used during problem solving tasks with African American community college students with varying levels of mathematics anxiety. Two students had high mathematics anxiety, while the other two students had low mathematics anxiety. These students were administered the Mathematics Anxiety Rating Scale – Shortened Version (MARS). They participated in task-based interviews that included six problem solving tasks while thinking aloud. After completion of the problem solving tasks the students completed a mathematics anxiety symptom survey and participated in a short interview. The study was completed in order to answer the following questions: (1) What strategies and thinking processes do students with low mathematical anxiety use when they engage in problem solving tasks? (2) What strategies and thinking processes do students with high mathematical anxiety use when they engage in problem solving tasks? (3) What differences and similarities are there, if any, in the strategies and thinking processes of students with high mathematical anxiety and low mathematical anxiety when they engage in problem solving tasks?

Using the frameworks of Malloy (1994) and Montague (2003), I was able to make the following recommendations. (1) Exhibiting mathematics anxiety symptoms may not completely hinder student performance. (2) Drawing pictures or diagrams maybe important for high mathematics anxiety students when solving problems, but not necessarily meaningful. (3) Mathematics anxiety may lead to an inability to recall previously studied material which hinders student performance. (4) Failure to use all of the thinking processes described by Montague (2003) does not hinder the student’s ability to successfully solve mathematical problems. (5) Difficulty with reading skills may hinder the problem solving process. (6) Students must be able to check their calculations and check for the use of correct procedures. (7) Mathematical anxiety symptoms can be visibly present even if a student does not notice the symptoms himself.
CHAPTER 1

INTRODUCTION

America is confronting a crisis as a world leader in science, math, engineering, and technology (Fullilove & Triesman, 1990). There is a decline in the number of students preparing for careers in these fields. Despite the increase in opportunities to enter such fields, African Americans continue to be significantly underrepresented among undergraduate and graduate degrees in science and math. According to Malloy (1994), the 1991 Sandia Report stated that African American high school students were more successful in mathematics than they had ever been in the past. Malloy also found that the national educational testing results showed that the achievement of African American students, as a whole, in mathematics was not reaching expected levels.

In 1992, African American students, as a whole, did not demonstrate that they had an understanding of mathematical problem solving as evidenced by the scores of African American students in fourth and eighth grades on the extended responses section of the National Assessment of Educational Progress (NAEP) Assessment of Mathematics (Malloy, 1994). According to the 2005 Nation’s Report Card for Mathematics (Perie, Grigg, & Dion, 2005), African Americans had the lowest average scale score for fourth graders and eighth graders when compared with Asians, Whites, American Indians, and Hispanics. Even though there was an increase in the average scale scores from 1990 to 2005, African Americans scored lower than Whites.

Another assessment where African Americans scored low in mathematics is the Program for International Student Assessment (PISA). PISA is a system of international assessments that measures 15 year olds’ capabilities in reading literacy, mathematics literacy, and science literacy every three years (Organization for Economic Cooperation and Development, 2005). PISA is carried out by the Organization for Economic Cooperation and Development (OECD), an intergovernmental organization of industrialized countries. In the United States, the PISA 2003 average scores for African Americans were lower than for Whites, Asians, and students of more than one race in mathematics literacy and problem solving. The average scores on problem solving for African Americans were also below the OECD average scores while Whites scored above the OECD average scores.
Explanations for the poor mathematics performance of African Americans are plentiful. Frequently cited are a lack of appropriate role models, poor quality of mathematics instruction in elementary schools, and little exposure to careers in science that might motivate African Americans to exert themselves more in mathematics (Fullilove & Treisman, 1990). I believe that another possible explanation for poor mathematics performance of African Americans is mathematical anxiety. Recent research (Ashcraft & Kirk, 2001; Beilock, Kulp, Holt, & Carr, 2004) has found support for the idea that anxiety induced worries disrupt mathematical problem solving by consuming working memory capacity. The researchers suggested that increased mathematics anxiety is associated with decreased working memory (Ashcraft & Kirk). Ashcraft and Kirk studied people during problem solving to evaluate how mathematics anxiety affected increased mathematical cognition. Individuals with high mathematics anxiety showed a significant decline in problem solving performance. They had less working memory space to effectively deal with mathematics problems because their mathematics anxiety was using working memory space that could be used to solve mathematics problems.

As an African American mathematics educator, I believe that an African American student’s ability and confidence to do mathematics is critical for their future success in our high-tech globally competitive world. African American students who are anxious, bored, or fearful of mathematics are likely to avoid the study of mathematics. Mathematics is the gateway to engineering, scientific, and technical fields. However, in my experience as a mathematics educator, African Americans avoid mathematics, an action that could be due to mathematical anxiety. Students need to be confident in their ability to solve problems and understand mathematical concepts.

Numerous researchers (e.g. Ballew & Cunningham, 1982; Hembree, 1992; Malloy, 1994) have investigated students’ mathematical problem solving knowledge and skills. However, the mathematics educational community has little knowledge of how African American students approach mathematical problem solving. Traditionally, the studies have also not reported specific data for African American community college students’ ability to do mathematical problem solving in relation to mathematical anxiety. I believe that an investigation of mathematical anxiety, problem solving, and African American students could provide useful information to educators that may lead to an increase in the number of African Americans being successful in mathematics and entering mathematical disciplines.
**Characteristics of the Community College Student**

Since its inception, the community college has grown to offer a wide range of transfer, technical, and career-specific courses and programs to a diverse student population. The average community college student is 29 years old (American Mathematical Association of Two Year Colleges [AMATYC], 2006). Fifty-eight percent of the community college students are women and 30% are minority students. Twenty-eight percent of the minority students are African Americans. Eighty to ninety percent of the students are employed and 50% are employed full time. Many community college students are involved in a career change, have not attended school in several years, and are commuters.

**Mathematical Problem Solving Standards for the Community College**

In 1995, AMATYC in cooperation with many other national mathematics education organizations created a document to address the needs of college students in the introductory mathematics courses before calculus (AMATYC, 1995). The document was called *Crossroads in Mathematics: Standards for Introductory College Mathematics*. It stated, “This document takes the position that knowing mathematics means being able to do mathematics and that problem solving is at the heart of mathematics” (p. 12). Some of the recommendations in this document that are concerned with problem solving are listed below:

1. Basic skills, general principles, algorithms, and problem solving strategies should be introduced to the students in the context of real, understandable problem-solving situations so that students gain an appreciation for mathematics as a discipline, are able to use it as a base for further study, and can transfer this knowledge to problem-solving situations at work or in everyday life. Intuitive justifications for mathematical principles and procedures should be emphasized (p.6).

2. Students will use problem-solving strategies that require persistence, the ability to recognize inappropriate assumptions, and intellectual risk taking rather than simple procedural approaches. These strategies should include posing questions; organizing information; drawing diagrams; analyzing situations through trial and error, graphing, and modeling; and drawing conclusions by translating, illustrating, and verifying results. The students should be able to communicate and interpret their results (p. 9).

3. The conceptual framework of discrete mathematics should be integrated throughout the introductory mathematics curriculum in order to improve students’ problem solving skills and prepare them for the study of higher levels of mathematics as well as for their careers. Topics in discrete mathematics include sequence, series, permutations, combinations, recursion, difference equations, linear programming, finite graphs, voting systems, and matrices (p. 14).
The study that I will be conducting will analyze the thinking of community college students as they devise strategies for mathematical problem solving.

In 2006, AMATYC released their second standards document, *Beyond Crossroads*. The purpose of this second document was to stimulate faculty, departments, and institutions to examine, assess, and improve every component of mathematics education in the first two years of college. According to this document, becoming a competent, independent problem solver should be a goal of every mathematics student. “Expert problem solvers have access to rich, well-connected knowledge of mathematical concepts and possess confidence following a long history of successful problem solving” (p. 23). According to AMATYC, most community college students often have difficulties planning an advanced solution. They also lack the ability to monitor their progress toward the desired goal. These students also lack the ability to switch to an alternative strategy when their initial strategy is not successful.

**Mathematical anxiety in the community college**

Mathematical anxiety is also a major concern for many college students (AMATYC, 2006). Mathematical anxiety is a feeling of dread that is experienced when a person attempts to understand and solve mathematical problems. Previous experiences in mathematics play an important role in mathematical anxiety. The students’ mathematical anxiety can develop into the “learned helplessness” which is the belief that a person is unable to do mathematics at all. Pries and Biggs (2001) stated that the people who fear mathematics are “African American, Asian, Caucasian, Hispanic, Native American, individuals not in school, nontraditional college students, public school students, traditional college students, females, males, minorities” (p. 6). Tobias (1991) believed that “all people have some math anxiety, but it disables women and minorities more than others” (p. 91).

**Rationale for the study**

Mathematics anxiety can affect many people in different ways. Literature has shown many relationships between mathematics anxiety and other variables such as academic performance, self-efficacy, and self concept (Bandura, 1977; Hackett & Betz, 1989), but little is known about problem solving together with mathematics anxiety. Based on NAEP data, Tate (1997) reported that African American students have increased achievement in mathematics basic skills over time and have begun to close the gap with White students. At the same time, the NAEP extended-response assessment, measuring problem-solving achievement, has revealed
that African American students perform at low levels of proficiency, below that of White and Hispanic students (Tate, 1997). Such results support the merits of research on African American students’ mathematics problem solving in an effort to inform the teaching and learning of mathematics among these students.

As a mathematics educator at a community college, I have noticed that many African American students lack mathematical problem solving abilities. My observations are in accord with results from mathematics assessments completed by African Americans attending the community college at which I teach. Many of the African American students attending the community college graduated from high schools in the same state as the community college. The students in the intended study attend a community college in Florida, a state that requires high school students to take the Florida Comprehensive Assessment Test (FCAT). The FCAT measures selected benchmarks in Mathematics, Reading, Science, and Writing. It also measures individual student performance against national norms (Florida Department of Education, 2006). The 2005 results from the Mathematics portion of the FCAT for high school students showed that the African Americans average scaled score was lower than that of Whites, Hispanics, Asians, and American Indians. I have observed that African American students also typically score low on the Mathematics College Placement Test which places them into developmental mathematics courses. Data indicate that African American students score extremely low on the problem solving section of the placement test (Tallahassee Community College, 2006).

Additionally, some of these students have been tested for mathematics anxiety and placed in a mathematics anxiety course for a semester. No data has been collected to determine the outcome of the course or the students’ success.

There is little research available on African American learning and less on African American community college students’ learning in mathematics or mathematical problem solving. A knowledge base of the actions of African American community college students in mathematical problem solving must be established in order to address the problem solving achievement of these students.

The purpose of this study is to create an understanding of how African American community college students with high and low mathematical anxiety solve mathematical problems. This research will contribute to the body of knowledge of African American
undergraduate mathematics students in order to develop strategies to help those students increase their mathematics problem solving skills.

**Research Questions**

This research will investigate how African American community college students with different levels of mathematical anxiety solve mathematical problems. Specific questions addressed by this research are:

1. What strategies and thinking processes do students with low mathematical anxiety use when they engage in problem solving tasks?
2. What strategies and thinking processes do students with high mathematical anxiety use when they engage in problem solving tasks?
3. What differences and similarities are there, if any, in the strategies and thinking processes of students with high mathematical anxiety and low mathematical anxiety when they engage in problem solving tasks?
CHAPTER 2
REVIEW OF THE LITERATURE

The purpose of this chapter is to illuminate and define the range of this study and a lens through which the data will be observed and examined. Another purpose is to position the study within the body of research in this area. This section presents an overview of the studies that address metacognition, African Americans, college students, problem solving, and mathematical anxiety. This section will conclude with a brief synopsis of the purpose of my research and what it will add to the literature discussed in this section.

Mathematics Anxiety

Mathematics anxiety is the lack of comfort that a person might experience when required to perform mathematically (Ma, 1999). A person might feel tense, helpless, and mentally disorganized when required to manipulate numbers and shapes. Some researchers (Tobias, 1978; Kogelman & Warren, 1978) describe mathematical anxiety as the intense negative emotional reaction many people have when encountering mathematics. Bessant (1995) suggested that mathematics anxiety “has become a euphemism for debilitating test stress, low self-confidence, fear of failure, and negative attitudes toward mathematics learning” (p. 18). Cemen (1987) defines mathematics anxiety as an anxious state that is triggered by mathematics-related anxiety situations that are perceived as threatening to self-esteem. Fennema and Sherman (1977) believed that math anxiety is psychologically equivalent to the lack of confidence in a person’s ability to learn mathematics. Kitchens (1995) described mathematics anxiety as “an uneasy feeling accompanied by thoughts or fears that keep you from doing your best when working at math” (p. 6). According to Richardson and Woolfolk (1980), mathematics anxiety is a general dread of mathematics and tests in particular. Buxton (1981) concluded that math anxiety is a state of panic that takes control of a person’s thoughts when presented with a mathematics problem.

A common theme throughout all of the descriptions of mathematics anxiety is that it is a self-perceived idea of an emotional fear and tension which prevents the student from successfully performing in a mathematics course. The fears are based on years and years of painful experiences with mathematics (Miller & Mitchell, 1994).
Effects of Mathematics Anxiety

Many researchers have reported that the consequences to being anxious toward mathematics include the inability to do mathematics and the decline in mathematics achievement (Ma, 1999). Based on a meta-analysis of 26 studies, Ma reported that mathematics anxiety is usually associated with mathematics achievement but not necessarily collectively. Hembree (1992) conducted a study to determine the effects of mathematics anxiety. He found that mathematics anxiety depresses performance. Higher achievement consistently accompanies reduction in mathematics anxiety. However, he found no compelling evidence that poor performance causes mathematics anxiety. According to Hembree, mathematics anxiety appears to be a learned condition that is more behavioral than cognitive in nature.

According to Ma (1999), two theoretical models have been influential in the research on mathematics anxiety and mathematics achievement. The first model is the interference model that is based on the work of Liebert and Morris (1985), Mandler and Sarason (1952), and Wine (1971). This model shows that a high level of anxiety causes a low level of achievement due to the mathematics anxiety causing a disturbance of the recall of prior mathematics knowledge and experience. The second model is the deficits model based on the work by Tobias (1985). A student’s low level of mathematics achievement is credited to poor study habits and deficient test taking skills instead of mathematics anxiety. Hembree (1992) found that mathematics anxiety increases during junior high school and reaches its peak in the freshman and sophomore years. It also levels off during senior high school which implies that mathematics anxiety is related to grade level. Mathematics anxiety prevents students from learning because it creates a diminished self-esteem, frustration, and anger (Fenneman & Sherman, 1986).

Mathematics Anxiety and College Students

Mathematics anxiety has limited college students’ selection of majors and sometimes the selection of career choices. Many times, students have not been able to rid themselves of mathematics anxiety as they matriculate through their academic careers (Melancon, Thompson, & Becnel, 1993). At the community college level, Hendershot (2000) observed that most math-anxious students could recall a classroom event that seemed to have triggered their mathematics anxiety. Community college students who worried about how well they were doing displayed mathematics anxiety (Bisse, 1994). Bisse also found that community college students displayed
mathematics anxiety when they were not able to memorize material and when they felt that they did not have enough time to complete assigned tasks.

Several researchers (Bisse, 1994; Norwood, 1994; Tominc, 1983; Willis, 1992) have found that mathematics anxiety exists at the community college level. Tominc (1983) found that as the level of mathematics anxiety increased, the students’ scores on standardized skills tests decreased. The students were less likely to be successful in solving problems. Norwood (1994) discovered that the students who had low mathematical anxiety levels performed at a higher mathematics achievement level. Mathematics anxiety affected how the students perceived the usefulness of individual courses as well as their ability to successfully solve problems. According to Willis (1992), students with high mathematical anxiety perceived courses to be less beneficial than students with low mathematical anxiety.

Bisse (1994) conducted a study of 14 students enrolled in a developmental mathematics course during the summer at a community college. The purpose of the study was to determine if exposure to a developmental mathematics course during the summer session would have any influence on a student’s mathematical anxiety. The purpose was also to determine the factors that influenced math anxiety. Bisse tested the students for math anxiety at the beginning of the course and at the end of the course. He found that there was a decrease in mathematical anxiety during the duration of the summer developmental course. Bisse also found that exams, homework, other students, and instructors greatly influenced math-anxious community college students. The researcher did not determine the reason for the decrease in mathematical anxiety in relation to being in a summer developmental course.

Betz (1978) conducted a study of 652 participants to investigate the factors related to the intensity of mathematics anxiety among college students. Betz investigated the relationships among mathematics anxiety and ability, general anxiety, and test anxiety. The study concluded that some of the students who reported test anxiety could be primarily mathematics-anxious students who experience their greatest difficulties with anxiety during mathematics tests. Betz also found that math anxiety occurs frequently among college students and that it is more likely to occur among students with inadequate high school mathematics backgrounds.

Green (1990) conducted a study of 132 students in the remedial mathematics program at Howard University. The purpose of the study was to examine the relationships among test anxiety, mathematics anxiety, teacher feedback, and achievement of undergraduate students in a
remedial mathematics course. Green’s study concluded that test anxiety had a greater effect on the mathematics achievement of remedial mathematics students than either mathematics anxiety or teacher comments.

Clute (1984) conducted a study of 81 students enrolled in a mathematics survey course at two different California state colleges. The survey course was designed to teach logical, problem-solving, and critical-thinking aspects of various mathematical topics. The results of the study showed that students with high mathematics anxiety tended to score lower on mathematics achievement tests than the students with low mathematics anxiety. According to Clute, the results of the study suggested the importance of considering anxiety level in planning the program to be used in teaching mathematics. This study also showed that there was a significant interaction between the instructional method and the level of anxiety. The students with the low mathematics anxiety tended to do better under a direct instruction discovery method, whereas students with high mathematics anxiety tended to do better under a direct instruction expository method.

**Problem Solving**

When people talk about problem solving they may not always be talking about the same thing (Wilson, Fernandez, & Hadaway, 1993). According to the National Council of Teachers of Mathematics ([NCTM], 2000), problem solving is defined as “engaging in a task for which the solution is not known in advance” (p. 52). Polya (1945) defines problem solving as the process used to solve a problem that does not have an obvious solution. Lester (1980) described problem solving as “a situation in which an individual or group is called upon to perform a task for which there is no readily accessible algorithm which determines completely the method of solution” (p. 287). Schoenfeld (1992) suggests that problem solving is a skill and an art that all students must construct. He believes that it is necessary to allow students to be part of the mathematics culture in order to engage students in problem solving. Schoenfeld also supports the view that learning to think mathematically means developing a mathematical point of view.

Polya (1945) suggested that problem solving was “a way out of a difficulty, a way around an obstacle, attaining an aim which is not immediately attainable” (p. vii). Polya also established a four-stage heuristic process: understanding, planning, carrying out the plan, and looking back. The first stage of the problem solving process is understanding the problem. In this stage, the student must see clearly what is required. Questions that the student should ask are: What is the
unknown? What are the data? What is the condition? The student should also consider drawing a figure and introducing a suitable notation. The student may have to separate the various parts of the condition and write them down.

The second stage of the heuristic process consists of devising a plan (Polya, 1945). Questions that the student should consider are: Have you seen it before? Do you know a related problem? The student should look at the unknown and think of a familiar problem having the same or similar unknown. If the student is successful at recalling a formerly solved problem which is closely related to the present problem, then he should determine how he can use the formerly solved problem.

The third stage of the heuristic process consists of carrying out the plan (Polya, 1945). Students should carry out their plan and check each step. Questions the student should ask are: Can you see clearly that the step is correct? Can you prove that it is correct? The student has to convince himself that the details fit into the devised plan. Each detail has to be examined until everything is clear.

The final stage of the heuristic process is looking back (Polya, 1945). Questions that the student should ask include: Can you check the result? Can you check the argument? Can you derive the result differently? Can you see it at a glance? Can you use the result, or the method, for some other problem? The student should examine the solution obtained very carefully. He should reconsider and reexamine the result and the path that led to the result. This helps the student consolidate his knowledge and develop his ability to solve problems.

Polya’s stages, Figure 2.1, are not linear as found in textbooks (Wilson, Fernandez, & Hadaway, 1993). The stages are dynamic and cyclic. The problem solving process in mathematics cannot be depicted as one-directional. There are times when the first stage may reoccur during the second and third stage. For example, a student may begin with a problem and engage in thought to understand it. The student may then develop a plan attempt to solve the problem but become unable to do so. Thus, the student makes a new plan or goes back to develop a new understanding of the problem. The following model shows the dynamic and cyclic nature of problem solving.
Lester, Garofalo, and Kroll (1989) developed an expansion of Polya’s heuristic process to include how students approach, organize, execute, and verify solutions to problems. Artzt and Armour-Thomas (1992) extended Lester et al.’s framework by enabling students to revisit stages at any time. Ballew and Cunningham (1982) used a framework that determined the difficulties in problem solving that were related to the ability to correctly use computation, reading, problem interpretation, and the integration of these three.

According to Montague (2003), good problem solvers use a variety of processes and strategies as they read and plan to solve a problem. Montague developed a research-based program that makes mathematical problem solving easy to teach. The program provides students with the processes and strategies that make mathematical problem solving easy to learn. Students also gain a better attitude toward problem solving when they are successful. Montague’s processes and strategies are the following: read, paraphrase, visualize, hypothesize, estimate, compute, and check.

Reading consists of reading for understanding (Montague, 2003). Students should read the problem and read the problem again if they do not understand it. Paraphrasing consists of putting the information in your own words. Students should underline important information and put the important information in their own words. Students should also make sure that the information goes with the question. Visualizing consists of drawing a picture or diagram to
show the relationships among the problem parts. Students should ask if the picture fits the problem. Hypothesizing consists of a plan to solve the problem. Students should decide how many steps and operations are needed and list the operation symbols if the mathematical problem calls for steps and certain operations. Students should also check that the plan makes sense. Estimation consists of predicting the answer. Students should round the numbers, do the problem in their heads, and write the estimate. They should make sure that they used the important information. Computing consists of doing the arithmetic. The students should make sure the operations are in the right order and make sure that the answer is similar to the estimation. Checking consists of making sure that everything is done right. The students should check the plan to make sure that each step is correct. The computation should also be checked to ensure that the answer is correct.

A study conducted by Malloy (1994) involved African American students’ mathematical problem solving skills. The researchers investigated how 24 African American eighth grade students solved mathematical problems and their use of problem solving strategies in relation to their success in problem solving. The researchers used data collected from talk-aloud problem solving sessions and interviews. Malloy found that the use of certain kinds of strategies was highly related to problem solving success and the use of verification was moderately related to problem solving success. The strategies that students used to solve the problems included drawing a picture or diagram, looking for patterns, making a list or chart, guess-and-checking, working backward, using logical deduction, and disregarding unnecessary data. The researcher also found that the students’ problem solving actions were a function of the word problem instead of the students’ achievement level. Students’ use of more than one strategy or verification action resulted in greater success.

Metacognition

Flavell (1979) defined metacognition as the knowledge about cognitive objects. Metacognition is involved in choosing, planning, and monitoring what is to be done. According to Garofalo and Lester (1985), learners must have knowledge about strategies in order to carry out certain tasks. Many researchers (Lesh & Akerstrom, 1982; Schoenfeld, 1992; Silver, 1982; Wilson, Fernandez, & Hadaway, 1993) consider metacognitive actions to be important in problem solving at all phases of problem solving. According to Davidson, Deuser, and Sternberg (1994), the use of metacognitive techniques is believed to allow students to keep track
of what they did and what they will do. This includes making connections between what they know and the problem situation. Wilson and Clarke (2002) believe that more emphasis should be given to metacognition in mathematical problem solving if problem solving is regarded to be a deliberate alternation between cognitive and metacognitive activity.

Schoenfeld (1985) identified four categories of knowledge and behavior important for problem solving include resources (mathematical knowledge), heuristics (problem-solving techniques), control (metacognition), and belief systems (attitudes). Schoenfeld believed that even when specific problem-solving knowledge fails, control or metacognition can guide a student to successful implementation of strategies. Metacognition plays an important role during the initial stages of mathematical problem solving where students build representations. It also plays an important role in the final stages of interpretation and evaluation of the results and the solution processes.

Goos (2002) believed that failure in mathematical problem solving and mathematical thinking is guaranteed by poor metacognitive decisions. Problem solving is a cognitive task that requires continuous thinking and reflection. It requires an ability to keep track, monitor, regulate, and be flexible in the task at hand. Metacognition is a quality that mathematical problem solvers need to develop in order to access the knowledge and skills they have so they can be less rigid in their problem solving plan. The characteristics of metacognition include organizing information from one’s knowledge store, devising a plan, and evaluating all processes.

Artz and Armour-Thomas (1992) conducted a study of 27 seventh grade students of varying abilities to monitor students’ behavior and regulate their cognitive processes while solving problems. Their research involved the problem-solving interactions of students in small groups to investigate whether or not the interactions would foster monitoring and regulation of behaviors. The researchers found that the interactions within the groups allowed the attitudes of all students to affect the problem-solving behaviors of fellow group members. Artz and Armour-Thomas concluded that an interplay of cognitive and metacognitive behaviors were necessary for successful problem solving. The students in the study repeatedly returned to reading, understanding, exploring, analyzing, planning, implementing, and verifying. The results of this study was very similar to the study conducted by Montague (2003).
Chamot, Dale, O’Malley, and Spanos (1992) conducted a study of the effect of language-based curriculum designed to help English as a Second Language students use metacognition to solve mathematical problems. The purpose of this study was to identify learning and problem solving strategies of students at high, average and low mathematics achievement levels, and to compare strategic approaches of students in high implementation (extensive involvement in staff development and project activities) and low implementation (limited involvement in project activities) classrooms. The study used the think-aloud protocol during problem solving and a metacognitive analysis of student actions. The researchers found that the students in the high implementation classrooms where the curriculum was taught did not use the problem solving steps more often than students in the control group. Students rated high in math performance also performed significantly better on finding the correct problem solution.

Swanson (1993) conducted a study of 63 children from grades four and five for the differences in mental processes of learning disabled, gifted, and regular education students during problem solving. The researcher observed whether or not processing differences were influenced by metacognition. Swanson discovered that learning disabled children relied on different mental processes for problem solving more often than gifted or regular educated children. The learning disabled children were less aware of certain metacognitive variables in task performance. They relied on a different set of strategies for task solution when compared to the gifted and regular education students. Swanson’s conclusion was that metacognition plays an important role in problem solving. He also concluded that the links between metacognition, mental processing, and solution finding are more likely to emerge in gifted and regular educated students than learning disabled children.

Yimer (2004) conducted a study with 17 college students enrolled in a problem-solving course. The purpose of the study was to identify and characterize metacognitive behaviors students exhibit during problem solving. The study revealed metacognitive behaviors such as implementation, evaluation, internalization, and transformation-formulation. The study also revealed that students viewed problem solving as a challenge but as essential in other disciplines and in life in general.

Jones (2006) conducted a study involving two middle school students to examine the cognitive processes used during problem-solving tasks. The students had different levels of mathematics anxiety and self-esteem. Jones recommended that students having issues with
mathematics anxiety and self-esteem can be taught meaningful strategies to reduce mathematics anxiety and self-esteem. She also recommended that problem-solving should be more of a priority in the curriculum in order to give teachers more opportunities to teach and enforce application of the cognitive processes in the classroom.

Watson (1980) investigated the cognitive levels and problem solving abilities of seventh grade males in New York City. His study concluded that there are differences between the problem solving abilities of students from Piaget’s concrete stages and formal stages. The students who were in the concrete stages were at a disadvantage when completing the problem solving task. Therefore when students are being assessed, they should be on the same cognitive level in order to ensure fairness with the assessment.

**African American Students’ Approaches to Learning**

According to Malloy (1994), the approaches that African American students use in learning may indicate the general preferences of African American students during mathematical problem solving. Stiff (1990) believes that African American students’ use of verbal expressions, holistic or relational approaches, and field dependency may influence how they solve mathematical problems. Willis (1992) believes that the influence of African heritage and culture results in preferences for student interaction with the environment and this influence affects cognition and attitude.

Smith and Drumming (1992) conducted a study of African American college students. The researchers used Wason’s Four Card Selections Task to look at how African American college students reasoned deductively. This study was the first study to have African Americans as subjects. The researchers found that the African American college students of privileged socioeconomic backgrounds were similar to their white counterparts. There was no evidence of a deficit found in the achievement of the students.

Shade (1992) conducted a study of the African American cognitive style. The researcher used 178 ninth grade students categorized by race, sex, and achievement level. The students were administered three cognitive style tasks. The tasks included embedded figures, object sorting, and the Myers-Briggs indicator. The study revealed a significant difference between African Americans and European-Americans in their perceptual orientation to the environment.

Willis (1992) believes that African Americans learn in ways characterized by factors of social/affective emphasis, harmony, holistic perspectives, expressive creativity and nonverbal
communication. These approaches have an effect on cognition, attitude, behavior, and personality and are summarized below:

1. **Social/affective emphasis.** Shade (1992) suggests that African American students rely on personalistic stimulation in learning instead of inanimate or object stimulation. In other words, the students are people-oriented and they rely heavily on affective interactions. According to Willis (1992), social interaction is important for African Americans because social learning is common in the African American community.

2. **Respect for harmony.** According to Willis (1992), African Americans are inclined to live in harmony with nature instead of trying to control it. Knowledge is sought for practical, utilitarian, and relevant purposes.

3. **Holistic Reasoning.** African Americans tend to respond in terms of the whole picture instead of its parts (Willis, 1992). The pieces derive their meaning from the pattern of the whole instead of using an analytical approach where the whole is revealed through the parts.

4. **Field dependence.** Shade (1992) found that African Americans view the world as a unified environment with natural order where relationships among the parts are the focus.

5. **Expressive creativity.** Willis (1992) suggests that African Americans are creative, adaptive, variable, novel, stylistic, and intuitive. They prefer simultaneous stimulation that contains animated oral expression.

6. **Nonverbal communication.** According to Willis (1992), movement and rhythm are components that are vital to the information transmitted. According to Hale-Benson (1986), African Americans are generally not word dependent. They tend to be very talented in nonverbal communication using tone and body language.

The current presentation of mathematics is in conflict with the learning modes of African Americans (Stiff, 1990). African American students learn from direct contact with teachers and peers. They take a holistic approach in a subject that is generally taught using an expository method in a linear, meticulous and analytical manner.
Summary and Researcher’s Thoughts

In summary, this literature review points to the importance of problem solving in the learning of mathematics, the general orientation of African American student learning and the impact that these orientations have on the learning of mathematics. A review of the literature yields a shallow variety of studies related to metacognition, problem solving, and mathematics anxiety at the community college level with African Americans.

I am motivated by the lack of empirical research available about how African American community college students solve mathematics problems. Research exists on undergraduates and problem solving, but there is little to none specifically on African American community college students. This study will add to the body of knowledge that exists about African American community college students concerning problem solving and math anxiety.
CHAPTER 3

METHODOLOGY

This chapter outlines the methodology for the research project designed to investigate the problem solving abilities of African American community college students. Qualitative methods were used to determine how African American community college students with low mathematical anxiety and high mathematical anxiety engage in problem solving tasks including their strategies and thinking processes.

Overview of the Study

According to Wilson, Fernandez, and Hadaway (1993) various research methodologies are used in mathematics education research including a clinical approach that is frequently used in problem solving. Usually, mathematical tasks are created, and students are studied as they perform the tasks. Often they are asked to talk aloud while working or they are interviewed and asked to reflect on their experience, especially their thinking processes.

Multiple methods, including observations, interviews, and analysis of documents were required to gather data. This provided a window for investigating the thought processes involved during students’ problem solving tasks. According to Rossman and Rallis (2003), qualitative research provides insights into the phenomena being studied that could be difficult to obtain by other means.

Within the qualitative design, case studies were used with task-based interviews as the primary means of data collection. Goldin (2000) described structured, task-based interviews as those involving a participant who is the problem solver and an interviewer. Goldin also stated that task-based interviews focus on complex behaviors and patterns such as the problem solver’s spoken words, movements, drawings, gestures, facial expressions, and so forth.

Pugalee (2001) believed that students exercise metacognitive behaviors during mathematical problem solving at any stage while solving the actual problem individually, while being silent, and in between tasks. This implies that there is no one specific method that can serve in capturing the processes involved in problem solving. Therefore, a qualitative design with task-based interviews, observations, and document analysis was deemed appropriate for the purpose of this study.
According to Rossman and Rallis (2003), a case study offers engaging and detailed description of a particular occurrence, and allows the researcher to gain understanding of a larger happening through investigating specific participants. A qualitative case study proves useful for those “interested in insight, discovery, and interpretation” by focusing on “holistic description and explanation” (Merriam, 1998 pp. 28-29). Two cases were investigated and data were collected and analyzed for this research study qualifying it as a multiple case study, which involved using data collected from more than one participant (Merriam).

The first case involved the students with high mathematics anxiety. The second case involved the students with low mathematics anxiety. A multi-case study was used in order to compare and contrast the students with low math anxiety against the students with high math anxiety. Using two cases provided a more convincing interpretation of the findings. Four students were used in order to gain more insight into the study. I was able to spend more time with the participants individually therefore being able to understand how they engage in the problem solving tasks, the strategies they used, and their thinking processes.

**Setting**

The research was conducted in the HP-MESA (Hewlett-Packard Mathematics, Engineering, and Science Achievement) Diversity Center at a community college in Florida. This center is an area where students involved in the HP-MESA program spend a minimum of five hours per week studying and participating in mathematics and professional development workshops. The study took place in this environment in order to provide a comfortable environment that the students have been experiencing all year. The students were interviewed in a small conference room located within the HP-MESA Diversity Center. The students were very familiar with the conference room because they use the room for various activities.

**Participant Selection**

Six African American students were recruited to participate in the study, but only four students were used. Three students had high math anxiety and three students had low math anxiety. If one participant from each category had left prior to the completion of the study, then the additional students would participate in the study. All students were enrolled in the community college’s HP-MESA Program. This program is designed to recruit and retain African Americans who plan to transfer to a four-year program and major in Science, Technology, Engineering, or Mathematics (STEM). Students are invited to join the program if
they are an African American and if they express interest in majoring in one of the STEM disciplines. HP-MESA students are required to spend a minimum of five hours per week studying individually or in groups in the HP-MESA Diversity Center.

The students were chosen using purposeful sampling. According to Merriam (1998), purposeful sampling is a method through which participants are chosen based on the idea of studying those who can provide the most insight into what the researcher is attempting to gain information about. In order to determine who would provide the most insight into the research, certain criteria must first be established.

The first criterion was high mathematics anxiety. By using the rating scale of the scores provided by Suinn (2003) on the Mathematics Anxiety Rating Scale – Short Version (see Appendix A), the students who scored a 78 or higher were considered as having high mathematics anxiety for this research study.

The second criterion was low mathematics anxiety. By using the rating scale of the scores provided by Suinn (2003) on the Mathematics Anxiety Rating Scale – Short Version (see Appendix A), the students who scored a 46 or lower were considered as having low mathematics anxiety for this research study.

The third criterion was verbal communication. Students must be able to verbally express themselves in order to participate in this study. The study required the participants to “talk-aloud” while they performed their problem solving tasks and during interviews. As the director of the program HP-MESA, I met with the students individually on a regular basis. Some of the students are very verbal and others are very quite. I used this information when deciding which students met the verbal communication criterion.

The fourth criterion was the students’ willingness to participate. As the director of the HP-MESA program, I asked the HP-MESA students to participate. Students’ willingness is important because I want them to want to participate in the activities of the research study. According to Mayer (1998), a students’ willingness may assist in them being more successful at problem solving.

The fifth criterion was the mathematics courses completed by the participant. All participants have completed similar mathematics courses. This information was determined by examining the students’ academic records.
The data that to be used for analyses were

- The Mathematics Anxiety Rating Scale – Short Version (see Appendix A) (Suinn, 2003)
- Task-based interview
  - Problem solving activity (see Appendix B)
  - Coding form for strategies (see Appendix C)
  - Physical and Psychological Survey (see Appendix D)
  - Problem-related interview questions (see Appendix E)

Mathematics Anxiety Rating Scale – Short Version

The HP-MESA program consists of 80 students. All 80 students were asked to take the Mathematics Anxiety Rating Scale (Suinn, 2003) assessment and students who met the criteria were selected. Each student was required to take the assessment in order to continue to participate in the HP-MESA program. I administered the assessment for four days at different times each day for one week. Students were assigned a specific time and day that coincided with their school schedule. The students completed the assessment in groups of twenty in the conference room in the HP-MESA Diversity Center.

The Mathematics Anxiety Rating Scale – Short Version (MARS-S) is a 30 item self-rating scale, which may be administered either individually or to groups (see Appendix A). The assessment is a shortened version of the original 98 item self-rating scale. Each question can be responded to by checking one of the categories, namely: Not at all, A little, A fair amount, Much, Very Much, with points ranging from 1 to 5, respectively. A test-retest reliability coefficient for the MARS-S was calculated from the scores of college students retested one week later (Suinn, 2003). The reliability coefficient of .90 compared favorably with the reliability for the longer MARS of .90 for the same period. According to Suinn, as an index of internal consistency, a coefficient alpha was computed and found to be .96. In effect, this shows that the test is highly reliable and indicates that the test items are heavily dominated by a single, homogeneous factor, presumably mathematics anxiety.
Task-based Interview

Task-based interviews were completed for each participant. This study focused on the students’ spoken words and drawings which were recorded through audio and video tapes, transcribed, and recorded.

There were a series of approximately three task-based interviews for each participant. Each task-based interview consisted of the participant solving a mathematical problem as they revealed their thought processes aloud. The participant solved 2 problems (see Appendix B), completed a survey (see Appendix D), and an interview (see Appendix E) which lasted approximately 1.5 hours. The purpose of the study was not only to determine if the student could reach a correct solution, but also to determine the strategies and the thinking processes used in an attempt to reach the solution. Therefore, the time allotted for completing the problems was revised depending upon the students’ ability to work at fast or slower paces.

The students were asked to record all work. They were asked not to erase anything regardless of whether or not the solution or information was correct. The participants were asked to draw a single line through any work that was incorrect or not usable. The unused or incorrect information let me know what strategy a student was using at that time.

As the students completed the activities, I used a coding form (see Appendix C) in order to document strategies used by the participants. The coding form was based on a form used by Watson (1980). The form enabled me to keep an accurate count of the strategies used by the participants during the activity. Since all of the students used in the study had taken the FCAT, then they were exposed to the FCAT Problem Solving Strategies which were included on the coding form (see Appendix C).

Next, the student completed a form to determine how the student felt during the solving of the problem (see Appendix D). The form listed some of the physical and psychological symptoms commonly displayed during mathematical anxiety. The form was based upon information by Haralson (2003) which was presented at the National Council of Teachers of Mathematics Central Regional Conference in Kentucky.

Finally, the student participated in a follow-up interview (see Appendix E). The purpose of the follow-up interview was to clarify the strategies and thinking processes used by the participants. This follow-up interview was also an opportunity to ask the participants why they used the strategies and thinking processes that they used. This process also helped clarify why a
participant could not solve a problem. This follow-up interview also allowed me to gather information on how the participant felt while solving the problem.

The task based interviews were video and audio taped. The purpose of the video tapes was to record and analyze any non-verbal behavior. The video and audio tapes were also used to compare what students claim that they did during the tasks with the things they were observed doing during the tasks, each of the data were important for triangulation.

**Problem Solving Tasks**

Since the tasks were the main means of collecting data to observe the occurrences of metacognitive behaviors, actions, and/or processes, care was taken in selecting the appropriate mathematical tasks. Hatfield (1978) believed that problems for research into the problem solving process should be non-trivial mathematical problems of the sort that students might meet in the classroom…that should emphasize commonly used as well as non-routine settings which utilize appropriate mathematical concepts, principles, and skills either known or readily learned by subjects (p. 35).

Three criteria were used to choose problems for the task-based interviews. The first criterion was that the solution to the problems should be accessible to the students, and should not require them to apply a mathematical concept or principle with which they are not familiar. The second criterion was that the problems should be challenging. The problem solvers should experience a blockage because they will have no readily available algorithm to use. The third criterion was that the problems be non-routine. Non-routine problems are described as those which “require students to move out of their automatic mode and focus on trying out other possible approaches and strategies” (Ho, Teong, & Hedberg, 2005, p.2). They will not all require algebraic manipulations nor will they be trivial. The problems will challenge the students’ thought processes.

Six problems (see Appendix B) based on the appropriate level of mathematics were selected from those used by Johnson, Kerr, & Kysh (2003). These problems were chosen with the assistance of a university mathematics educator who had experience teaching community college students.
Analysis

During the analysis process, the taped interviews, both audio and video, were transcribed verbatim. The interviews, observations, and coding forms were examined repeatedly. Common strategies and thinking processes were underlined in the text during the examination process. The strategies and thinking processes identified were based upon the literature. The literature revealed terms such as drawing diagrams, guessing, estimation, looking back, drawing conclusions, etc. After common strategies and thinking processes were identified, they were clustered according to patterns.

According to Creswell (2003), validity is seen as the strength of qualitative research. Creswell offers eight primary strategies that can be used to check the accuracy of the findings in order to ensure internal validity. I used four of the eight strategies to analyze data collected for the study. The four methods to enhance the internal validity that were used in the study are detailed here.

*Triangulation* is defined as using multiple data sources that could be used to draw from which confirms my findings by examining evidence from the sources and using it to build a coherent justification for themes (Creswell, 2003). The multiple data sources that were used for this research study improved the opportunity for triangulation of the data. I looked for similar patterns or occurrences in more than one follow-up interview, problem solving activity, or survey in order to make a compelling argument regarding observations made during data collection.

*Member checking* occurs when the investigator determines the accuracy of the qualitative findings through taking the final report or specific descriptions or themes back to participants and determining whether these participants feel that they are accurate (Creswell, 2003). A summary of the each participant’s interviews was given to each participant after all the data had been collected and transcribed. The students did not make any comments, nor did they find any discrepancies among the summaries.

The third way to ensure internal validity is *prolonged time* in the field (Creswell, 2003). This allows me to develop an in-depth understanding of the phenomenon under study. I spent approximately one or two months interviewing students. Each task-based interview lasted approximately 1.5 hours depending upon the participant’s ability to solve the word problems...
quickly or slowly. The large number of interviews and problem solving tasks allowed me to collect and analyze data over a period of time.

The fourth way to ensure internal validity is peer debriefing (Creswell, 2003). This method is used to enhance the accuracy of the account. This process involves locating a person who reviews and asks questions about the qualitative study so that the account will resonate with people other than me. During the analysis, I conferred with a fellow colleague on the findings. The fellow colleague had qualitative research and mathematics expertise. The colleague provided me with feedback on interpretations.

According to Goldin (2000), reliability of any study “includes measuring the consistency with which a task-based interview is conducted, observations are taken, and inferences are made from the observations using defined criteria. The criteria that was used for choosing the participants, the coding forms used for observing the students’ problem solving activities, the survey the students completed, and the follow up interviews all provided uniformity across the interviews I conducted.

Creswell (2003) believes that reliability can be used to check for consistent patterns of theme development. It also refers to the extent to which research findings can be replicated. This research method was clearly defined and detailed so that another researcher can replicate and reproduce it if necessary. All participants were administered the same assessment test, the same problem solving tasks, and similar semi-structured interview questions to ensure consistency.
CHAPTER 4

HIGH MATHEMATICS ANXIETY RESULTS

In this chapter I discuss the data collected from the interviews of the two student participants with low mathematics anxiety. The interviews consisted of the students completing mathematical problem solving activities. The problems came from *Crossing the River with Dogs: Problem Solving for College Students* (Johnson, Herr, Kysh, 2003). The problem solving tasks were used to investigate the students’ solution strategies and results as well as their thinking processes.

I used the frameworks of Malloy (1994) and Montague (2003) in order to analyze the data. Malloy (1994) found that the use of certain kinds of strategies was highly related to problem solving success. According to Malloy, the strategies most used by African-American students included:

1. Drawing a picture or diagram.
2. Looking for patterns.
3. Making a list or chart.
5. Working backward.
6. Using logical deduction.
7. Disregarding unnecessary data.

Montague (2003) stated seven thinking processes that successful problem solving involved:

1. Comprehending linguistic and numerical information in the problem. (Students read the problem more than once or reread parts of the problem as they progress and think through the problem. Use self-regulation strategies by asking if they understood the problem.)
2. Translating and transforming that information into mathematical notations, algorithms and equations. (Students paraphrase the problem by putting it into their
own words. They underline parts of the problem. They ask themselves what the question is and what are they looking for.)

3. Observing relationships among the elements of the problem. (Students visualize the problem by drawing a picture or diagram. The use of visual representation is a guide toward understanding the problem and developing a plan to solve the problem.)

4. Formulating a plan to solve the problem. (Students think about the appropriate solution path and the algorithms they need to carry out the plan.)

5. Predicting the outcome. (Students estimate or predict the answer to get a “ballpark” idea.)

6. Regulating the solution path as it is executed. (Students compute their arithmetic and algorithms. They ask themselves if their answer makes sense.)

7. Detecting and correcting errors during problem solution. (Students check their calculations and check for the use of correct procedures.)

There were four students in total: two students with low mathematics anxiety and two students with high mathematics anxiety. Kate and Sharon experienced high mathematics anxiety, and Adam and Hank experienced low mathematics anxiety. (The names are pseudonyms chosen by the students themselves.) Their results are presented in two separate cases; one of the high mathematics anxiety students and one of the low mathematics anxiety students.

This chapter presents observations made about the data collected. First, I will describe the strategies and results of the students work on the problem solving activity and any similarities or differences I found between the two high mathematics anxiety students. Secondly, I will describe the thinking processes used by the high mathematics anxiety students. Third, I will discuss what I found when looking at the students’ answers to the math anxiety survey (see Appendix D) adapted from Haralson (2003).

This chapter presents the investigation of the strategies and thought processes of students with high mathematics anxiety while performing mathematics problem solving tasks. With regard to the data presented in this chapter, I address the following research question:

What strategies and thinking processes do students with high mathematics anxiety use when they engage in problem solving tasks?
The Case of the High Mathematics Anxiety Students

Kate and Sharon

Kate is an African American female student. She graduated from high school in May of 2005 and entered the community college in August 2005. She is excited about recently finishing her Associate of Arts Degree. She will begin her studies at the local university in August of 2007. Kate plans to pursue a degree in Pharmacy. She has been a part of the HP-MESA program for one year. She is a well rounded student who is determined to be successful in her academic career.

Kate has one brother and a step-sister. She is the oldest of her siblings. Kate is the first one in her family to go to college. She is a high school graduate, but Kate had to take the graduation exam twice. The first time she took the exam, she did not pass it. She completed several preparation classes to learn strategies and skills necessary to pass the exam. When Kate took the graduation exam a second time, she passed.

Upon entering the community college, Kate had to take the college placement test (CPT). She took the math portion of the CPT twice. The first time she completed the exam, Kate placed into the remedial math courses. The second time Kate placed into the College Algebra course. Kate took the reading portion of the CPT three times. She placed in the remedial reading course each time. Even though Kate had to take a remedial reading course, she was able to complete community college with a GPA of 3.8. Kate took College Algebra, Pre-calculus, Trigonometry, and Calculus I as her math courses. She received As and Bs as her grades in the math courses.

Sharon is also an African American female student. She was born in Jamaica and came to the United States when she was 10 years old. She graduated from high school in June of 2003 and entered the community college in August 2003. She completed her Associate of Arts Degree in December 2006, but she decided to stay an extra semester because she loves the community college and the support programs that are available only at the community college. She will begin her studies at the local university in August 2007. Sharon plans to pursue a degree in Agricultural Engineering. She has been a part of the HP-MESA program for two years. She is a well rounded student who is determined to be successful in her academic career.

Sharon is an only child. She is the first person in her family to go to college. Sharon took her graduation exit exam only once and passed the exam the first time she took it. She participated in sports in high school including track and field. She was also a part of the student
government and several other service clubs while in high school such as the school decoration club and the prom planning club.

Upon entering the community college, Sharon used her scores on the SAT. She took the SAT in November 2002 and again in April 2003. She placed in the same courses using scores from both exams. Sharon did not have to take any remedial courses. She completed community college with a GPA of 2.92. Sharon took College Algebra, Pre-calculus, Trigonometry, and Calculus I as her math courses. She received As, Bs, and Cs as her grades in the math courses.

These two students were chosen based on the criterion that was presented in a previous chapter. They each experienced high mathematics anxiety based on MARS (Appendix A), were able to verbally express themselves well, were willing to participate in the study, and they completed similar math courses during their course of study at the community college.

**The Students’ Strategies**

The students completed six mathematical problem solving tasks during the course of this research study. After the students completed the problem solving tasks, I analyzed the strategies and thinking processes which the students utilized to complete the problems. I was looking for similarities between the students with high mathematics anxiety. In this section, I will give each student’s strategy and thinking processes for the six problems and my observations of them.

**Problem 1: Penny’s Dimes**

Nick’s daughter Penny has 25 dimes. She likes to arrange them into three piles, putting an odd number of dimes into each pile. In how many ways could she do this?

Figure 4.1 – Johnson, Herr, Kysh (2003, p. 30)

The problem, *Penny’s Dimes* (Figure 4.1) can be solved by using a systematic list (Johnson, Herr, & Kysh, 2003). A system can be generated to organize the information in a methodical way. The systematic list that is used should be understandable and clear so that the person making the list can verify its accuracy quickly. Many systematic lists are in the form of a table whose columns are labeled with information given in a problem. The rows of the table are used to indicate possible combinations.
Students’ Work:

Kate and Sharon both used a similar method to solve the problem. Kate had to work the problem several times before getting a solution. Kate read the problem aloud, and she read the problem again. The first time she worked the problem, Kate divided 25 by 3 (Figure 4.2). She came up with 8.3. She then drew a picture of 8 piles. Kate read the problem again. She realized that she could not use the number 8 because it was even.

Figure 4.2 – Kate’s Penny’s Dimes

Kate read the problem again and realized that she could only have 3 piles, so she started working the problem again. She then drew a picture of 3 piles and she put 8 dimes in each pile. Kate checked her answer by reading the problem again. Again, she realized that she could not use the number 8 because it was even. She read the problem again. Kate decided that 25 was divisible by 5 and 5 was odd, so she drew a picture of 5 piles with 5 dimes in each pile. Kate read the problem again to check her answer. She once again realized that she could only have 3 piles.
Kate then drew a picture of 3 piles. She put 5, 11, and 9 dimes respectively in each pile. She drew another picture of 3 piles and she put 3, 15, and 7 dimes respectively in each pile. Kate listed two more possibilities in her list before she realized that she could use all the odd multiples up to 25. Kate decided that the answer was 25 piles.

Sharon read the problem aloud, and she read the problem again. She drew a picture of 25 dimes using only 3 columns (Figure 4.3). She drew 9 dimes in one column, 9 dimes in the next column, and 7 dimes in the last column. She then added 9, 9, and 7 and got 25.

![Figure 4.3 – Sharon’s Penny's Dimes](image)

Sharon drew another picture of 25 dimes using only 3 columns. This time she put 11, 11, and 3 dimes respectively in each column. She added 11, 11, and 3 and got 25. Sharon repeated this method of drawing 3 columns of 25 dimes with different amounts of dimes in each column 3 more times. She made piles that consisted of 7, 7, 11 and 5, 5, 15 and 3, 3, 19. Sharon determined that only the numbers 1, 3, 5, 7, 9, and 11 could be used. She gave 6 piles as her final answer.
Researcher’s Observations:

One important observation is that both students used a list to find the answer. However, neither student organized the information in a methodical way in order to develop a system. Both students listed possibilities which were inefficient because it could take a long time to figure out all the possibilities and they could never be sure that they thought of all the ways. Both students also drew pictures while solving the problem. It appears that the students have been taught to draw pictures as a strategy to solve problems. I will continue to see if the pattern persists in future problem solving tasks before making any conclusions.

Another observation is the time that it took each student to complete the problem. It took Kate approximately 12 minutes to submit an answer whereas it took Sharon approximately 5 minutes to submit an answer.

Kate used only five of Montague’s (2003) seven cognitive processes. Kate comprehended linguistic and numerical information in the problem by reading the problem more than once as she progressed and thought through the problem. Kate observed relationship among the elements of the problem. She demonstrated this process by drawing a diagram. She had drawn different diagrams each time she decided to start the problem again. Kate formulated a plan to solve the problem. After she read the problem a few times, Kate discussed putting different amounts of dimes in each pile in order to reach the solution. She also stated that it would take a long time to try a different number of dimes in each pile several times in order to determine the correct solution. Kate regulated the solution path as it was executed because she did the arithmetic and computed an answer. Kate also detected and corrected errors during the problem solution. This led Kate to make several attempts to solve the problem before submitting a solution.

Sharon used four of Montague’s (2003) seven cognitive processes. Sharon translated and transformed the information by stating what she was looking for in the problem. She said that she needed to find a way to arrange the 25 dimes so that she could have an odd number of dimes in each pile and she can only have 3 piles. Sharon also observed relationships among the elements of the problem by drawing a diagram to represent the information given in the problem. Sharon formulated a plan to solve the problem. She stated that she would put different odd amounts of dimes in each pile that totaled to 25 in order to come up with the final answer. She regulated the solution path by computing her answer.
Both students’ solutions were incorrect. There were 16 ways to form three piles of 25 dimes. Both students formed lists to solve the problem. The lists were not systematic which caused them to fail to list all of the possible outcomes. The common strategies used by Kate and Sharon for this problem includes making a list and drawing a picture. The common thinking processes included observing relationships among the elements of the problem, formulating a plan to solve the problem, and regulated the solution path.

**Problem 2: Night of the Howling Dogs**

Shawna liked to jog in the late afternoon. One day she noticed an unusual phenomenon. As she jogged, dogs would hear her and bark. After the first dog barked for about 15 seconds, two other dogs would join in and bark. In about another 15 seconds, it seemed that each barking dog would “inspire” two more dogs to start barking. Of course, long after Shawna passed the first dog, it continued to bark, as dogs are inclined to do. After about 3 minutes, how many dogs were barking?

Figure 4.4 – Johnson, Herr, Kysh (2003, p. 120)

The problem, *Night of the Howling Dogs* (Figure 4.4), can be solved by looking for a pattern (Johnson, Herr, & Kysh, 2003). The study of mathematics if often called the study of patterns. The patterns can repeat and extend indefinitely. A student’s ability to recognize and extend patterns is a very valuable problem-solving skill. Recognizing patterns is extremely useful for real-world problems. For example, detectives look for patterns of behavior in to determine the characteristics of the person who committed a crime. As a problem solving strategy, recognizing patterns enables a student to reduce a complex problem to a pattern and then use the pattern to find a solution. The key to finding a pattern is organizing information.

**Students’ Work:**

Kate read the problem aloud. She recalled that there are 60 seconds in 1 minute and 180 seconds in 3 minutes. Kate drew a diagram showing 15 second intervals and the number of dogs in each interval (Figure 4.5). She showed that there was 1 dog barking, then 3 dogs, and then 5 dogs. She read the problem again for understanding. Kate stated that the ending number would be odd even though the dogs were increasing by 2 because the first dog has to be included in the count. She took 180 seconds and divided it by 15 seconds and got 12. She added 1 to 12
and got a final answer of 13 dogs barking. She read the problem again to check her answer. She then checked her calculations and submitted her answer.

![Figure 4.5 – Kate’s Night of the Howling Dogs](image)

Sharon read the problem aloud. She stated, “I like pictures.” She drew a diagram showing that each dog caused two more to bark (Figure 4.6). Sharon determined that there was 180 seconds in 3 minutes. She read the problem again. She read the problem one more time. Sharon took 180 seconds and divided it by 15 seconds. She got 12 and determined that the final answer was 12 dogs. She checked her calculations in the calculator submitted her final answer.

![Figure 4.6 – Sharon’s Night of the Howling Dogs](image)

**Researcher’s Observations:**

One observation is that both students drew diagrams to demonstrate the number of dogs barking in each interval. This is the second problem where both students began the problem solving process by drawing a diagram. However, it is still too early to conclude if the drawing a diagram pattern will continue with the remainder of the problem solving tasks.
Another observation is that both students divided 180 seconds by 15 seconds and got 12 dogs. However, Kate added 1 dog to her 12 dogs for a final answer of 13. It appears that neither student understood the problem. During the follow-up interview, the students were asked if there was anything that they did not understand about the problem. Kate replied that she understood the problem, but she had to keep reading it. Sharon also replied that she understood the problem. However, their methods for solving the problem show that they did not fully understand the problem. It appears that the students had trouble identifying the important information in the problem that revealed that the number of dogs increased exponentially.

Kate also struggled with pronunciation and comprehension when reading the problem aloud. She mispronounced several words. Sharon read the problem aloud without any pronunciation and comprehension errors. It took Kate approximately 9 minutes to submit an answer whereas it took Sharon approximately 3 minutes to submit an answer.

Kate used six of Montague’s (2003) seven thinking processes. She translated and transformed the information by reading the problem several times as she thought and progressed through the problem. Kate observed the relationships among the elements of the problem by drawing a diagram that showed that two dogs would bark every 15 seconds. She also predicted the outcome by stating that the answer would definitely be an odd number even though the number of dogs increased by a factor of two. Kate formulated a plan to solve the problem by stating that the answer should be derived by dividing 180 seconds by 15 seconds and adding 1 to the answer. She also regulated the solution path as it was executed by computing the answer. Kate detected and corrected errors when she checked her computations.

Sharon used five of Montague’s (2003) thinking processes. She translated and transformed information by stating what she was looking for in the problem. She said that she knew she was looking for the number of dogs if 2 dogs started barking each time another dog started barking. Sharon also observed relationships among the elements of the problem by drawing a diagram to show the dogs increasing every 15 seconds. Sharon formulated a plan to solve the problem by stating that the answer could be derived by dividing 180 seconds by 15 seconds. She also regulated the solution path by computing the arithmetic to get an answer. Sharon detected and corrected errors by checking her calculations before she submitted her answer.
Both students’ solutions were incorrect. There were 531,441 dogs barking at the end of 3 minutes, given by the model $3^x$ where $x$ is the number of 15 minute intervals in 3 minutes. Both students used drawing a diagram to solve the problem. However, the diagram was not very useful. Sharon verbally expressed that her diagram would be too long and confusing. The students should have organized the information into a table and looked for a pattern. The common strategy used by Kate and Sharon for this problem included drawing a picture. The common thinking processes included comprehending linguistic and numerical information, observing relationships among the elements, formulating a plan, regulating the solution path, and detecting and correcting errors.

Problem 3: Cascades State Park

![Figure 4.7 – Johnson, Herr, Kysh (2003, p. 145)](image)

The problem, Cascades State Park, (Figure 4.7) can be solved by guess and check (Johnson, Herr, & Kysh, 2003). Guess and check helps a student understand the problem and helps to set up an algebraic equation. This strategy involves organizing the problem’s information into a useful form and guessing an answer. The student should evaluate each guess in a systematic way that enables a student to advance to more refined guesses. When guessing and checking, the student must believe that they can solve the problem, even if they do not understand it well. Through organization and persistence, the student will work towards a solution. The key to guess and check is the chart that is built to organize the guesses and the operations that are performed to check them.

Students’ Work:

Kate read the problem aloud, and she read the problem again. She drew a picture of a tree and put the measurement of 20 feet next to it (Figure 4.8). She also drew a picture of a waterfall and put the measurement 80 feet next to it. She also drew a picture of a person
standing at the foot of the waterfall. Kate read the problem again and wrote an equation for the tree. The equation read 20 ft x 3 = 60 ft + 20 ft = 80 ft. She named this equation Emi.

Kate read the problem again and wrote another equation for the tree. The second equation read 20 ft x 4 = 80 ft – 50 ft = 30 ft. She named this equation Margit. Kate believed that the redwood tree had to be a very tall tree because it was being compared to a waterfall. She immediately thought the waterfall had to be 80 feet. She also believed that Margit was exaggerating about the height because there was a 50 feet difference between Emi’s guess and Margit’s guess. Kate concluded that the waterfall was 50 feet and the redwood was 20 feet. She read the problem again and checked her answer.

Sharon read the problem aloud. She drew a picture of a tree and she drew a picture of a waterfall with two people standing at the base of the waterfall (Figure 4.9). Sharon read the problem again and wrote an expression to represent Emi. The expression was 20 + 3(h). She also wrote an expression to represent Margit which was 4(h) – 50. Sharon let the two expressions equal each other and solved for “h”. She found h = 70 which was the height of the
redwood. She substituted 70 into the expression for Emi and got an answer of 230 feet. She also substituted 70 into the expression for Margit and got an answer of 230 feet. She concluded that height of the waterfall was 230 feet.

![Diagram of redwood tree and waterfall]

Figure 4.9 - Sharon’s Cascades State Park

**Researcher’s observations**

One observation is that both students drew diagrams to show the tree and the waterfall. Their drawings also included stick figures which represented Emi and Margit. Both students stated that the diagrams were not much help, so they resorted to other methods. This is the third problem where both students began the problem solving process by drawing a diagram. However, I will wait to conclude that the drawing a picture pattern will continue with the remainder of the problem solving tasks.

Another observation is Kate’s lack of understanding of the information given in the problem. She experienced a large amount of trouble reading this problem solving task aloud.
had expected her to mispronounce several words. However, I did not expect her to point to the word ‘cascades’ and say, “What is this word?” I was very shocked. Kate also kept referring the word ‘redwood’ as ‘red’. She also asked, “What is a red?” I quickly pronounced the word as redwood and told her that a redwood was a type of tree. I gathered from this performance that Kate did not have a clear understanding of the problem. She read the problem aloud several times, but she struggled each time. On the other hand, Sharon had no problems with understanding the problem, nor the pronunciation. It took Kate approximately 15 minutes to submit an answer and it took Sharon approximately 5 minutes to submit an answer.

A third observation is the strategies that the students used. Both students drew a picture, and both students also formed equations. Kate did not understand how to set up her equations which caused her to get confused. She also incorrectly used the equal sign in her equation (Figure 4.8). Sharon immediately formed equations and set the equations equal to each other in order to solve for the height of the tree. Neither students used the guess and check strategy, even though this problem was specifically designed for the guess and check method. The guess and check method leads to using algebraic equations. Kate and Sharon decided not the waste time guessing and checking, but to directly use algebraic equations instead.

Kate used five of Montague’s (2003) thinking processes. She made an attempt of comprehend the linguistic and numerical information by reading the problem several times. She observed relationships among the elements of the problem by drawing a diagram of the tree and waterfall and placing the measurements on the diagram. She also formulated a plan to solve the problem. Kate stated that needed to form equations in order to solve the problem. She formed equations to represent the height of both the tree and the waterfall. Kate regulated the solution path by completing the numerical algorithms. She detected and corrected errors by checking her answer.

Sharon also used four of Montague’s (2003) thinking processes. She observed relationships among the elements of the problem by drawing a diagram of the tree and waterfall. Sharon formulated a plan to solve the problem. She stated that the she needed to write equations and set the equations equal to each other in order to solve for the unknown height. She formed equations to represent the Emi’s guess and Margit’s guess. Sharon regulated the solution path by computing her answer.
Kate’s solution was incorrect, and Sharon’s solution was correct. The height of the redwood tree was 70 ft and the height of the waterfall was 230 ft. Both students drew a diagram to begin solving the problem. However, the diagram was not very useful. The common strategies used by Kate and Sharon for this problem included drawing a picture and using algebraic equations. The common thinking processes included observing relationships among the elements, formulating a plan, and regulating the solution path.

**Problem 4: Divisors and Reciprocals**

The divisors of 360 add up to 1170. What is the sum of the reciprocals of the divisors of 360?

Figure 4.10 (Johnson, Herr, & Kysh, 2003 p. 229)

The problem, *Divisors and Reciprocals* (Figure 4.10), can be solved by solving an easier related problem (Johnson, Herr, & Kysh, 2003). Solving an easier version of a problem usually reveals a useful method of solution. There are many ways to make a problem easier and more manageable. Students can use a number instead of a variable. Sometimes it is necessary to use a smaller or easier number in place of a more difficult one in order to develop the process for solving the problem. Students can also do the following: do a set of specific easier examples and look for a pattern; do a specific easier example and figure out an easier process that will work to solve the problem; change, fix, or get rid of some of the conditions; and eliminate unnecessary information. Solving an easier related problem is a strategy that uses some of the elements of organizing information; the main purpose is to move the student’s focus away from a difficult original problem to an easier related problem.

**Students’ Work:**

Kate read the problem aloud. She laughed out loud after reading the problem. She read the problem again and said that she had no idea of how to solve the problem, but she said that she was willing to try it. She read the problem again and tried to determine the definition of the word ‘divisor’. She read the problem again and realized that she also needed the definition of the word ‘reciprocal’. She said that she did not know where to begin.
Kate read the problem again and started listing numbers that would divide 360 evenly such as 1, 3, 9, and 15 (Figure 4.11). She thought that all the odd numbers would work, but they did not. She continued to list numbers that were divisible by 360. Kate added the number together and realized that they did not add up to 1170. She gave up on solving the problem.

Sharon read the problem aloud. She realized that she did not remember the definition of the word ‘divisor’. She began to think aloud about the difference between a divisor and a dividend. She read the problem again and concluded that the divisors were the numbers that divided 360 evenly. Sharon began to list the divisors of 360 (Figure 4.12). She added them together and realized that they did not add up to 1170.

Figure 4.11 – Kate’s Divisors & Reciprocals

Figure 4.12 – Sharon’s Divisors & Reciprocals
Sharon read the problem again. She said, “I bet this is an easy problem. There is probably some weird step that I am leaving out that is keeping me from getting to the answer.” She read the problem again and wrote $\frac{1}{1170}$ as her answer. She said she knew that her answer was incorrect, but she wanted to make a guess.

**Researcher’s Observations:**

The first observation is that neither student drew a picture for this problem. This was the fourth problem solving task and the only one thus far where the students did not draw a picture or diagram. When I asked the students why they decided not to use a diagram, they both replied that this problem did not require a picture or diagram. Several of the problems did not require drawing a diagram, but they felt comfortable drawing the diagrams. Sharon verbally expressed that she liked drawing pictures whenever she was given a word problem.

The second observation is that both students tried to list all of the divisors of 360. They also both wanted to see if the divisors would add up to 1170. When I inquired about this procedure, both students stated that they wanted to check their divisors that they had listed to see if they had left any out. Kate and Sharon were determined to find all the divisors of 360, list their reciprocals, and find the sum of the reciprocals. Sharon believed that the problem was easier than what she was making it, but she could not determine the easier way to solve the problem.

The third observation is that neither student remembered the definitions of the word ‘divisors’. It took both students a couple of minutes to determine the meaning of the word. Kate did not mispronounce any words during this task. The problem solving task consisted of two sentences and she read it aloud without any mispronunciations. Kate spent approximately 12 minutes working on the problem. Sharon only spent approximately 5 minutes working on the problem.

Kate used one of Montague’s (2003) thinking processes. She attempted to comprehend the linguistic and numerical information in the problem by reading the problem several times. She also asked herself the meaning of important words in the problem.

Sharon also used only one of Montague’s (2003) thinking processes. She attempted to comprehend the linguistic and numerical information in the problem by reading the problem several times. She also asked herself the meaning of important words in the problem.
Both students submitted incorrect solutions. The correct solution was $\frac{1170}{360}$. Neither of the students used a strategy for this problem solving task. The students could have found the answer by working an easier problem such as finding the divisors of 10 and finding the sum of the reciprocals of the divisors. They could have also used the number 16. The students would have realized that the method to solve the problem was the sum of the divisors divided by the number. However, the students could have taken more time and solved the problem based on their current method, but they would have had to list all the correct divisors of 360 and then add all the reciprocals which would have been several fractions. The students did not have any common strategies used for this problem. The common thinking process included comprehending linguistic and numerical information.

Problem 5: Dad’s Wallet

<table>
<thead>
<tr>
<th>Dad went to the ATM on Wednesday of spring break and withdrew some money. On Thursday morning my brother borrowed half of Dad’s money to open a checking account because he was always short of money. On Friday I needed some money for a date, so I borrowed half of what remained. My sister came along next and borrowed half of the remaining money. Dad then went to gas up the car and used half of the rest of his money, and he wondered why he had only $15 left. How much money did he start with in his wallet?</th>
</tr>
</thead>
</table>

Figure 4.13 (Johnson, Herr, & Kysh, 2003 p. 283)

The problem, Dad’s Wallet (Figure 4.13), can be solved by working backwards (Johnson, Herr, & Kysh, 2003). With most strategies, students work forwards through the information in a problem. In order to work successfully backwards, the student needs to change their focus and consider the whole problem in reverse. Much of algebra is based on working backwards and is very useful for planning schedules or agendas. A difficult aspect of working backwards is keeping track of a problem’s information and organizing it in a meaningful way.

Students’ Work:

Kate read the problem aloud. She read it again and she made a schedule of the days and what fraction of the money was used on each day (Figure 4.14). She asked how much money was needed to open an account at the bank. She then realized that her question was irrelevant. Kate read the problem again. She decided that she would start with the number 100 and check to
see if she would be left with 15 after following the information given in the problem. Kate realized that 100 did not work. She then tried 120, 140, 160, 200, and 220. She tried the numbers on her calculator, but none of the numbers worked.

Kate read the problem again and for a second time inquired about the amount of money to needed to open an account at the bank. She said that she had to have at least 200 to open up her account at the bank, so she assumed that Dad had to give at least $200 for the brother to open the account. So, she tried the numbers 400, 450, and 460, but none of the numbers worked.

Kate read the problem again and decided to start with the number 15 and multiply it by 2. She kept going with the process until she reached the number 480. She decided that 480 was her answer. She checked her answer by dividing 480 by the specified amounts of money that Dad gave away and she was left with 15. Kate submitted her answer of $480.

Sharon read the problem aloud. She read the problem again, and she underlined the phrases in the paragraph that specified how much money was borrowed (Figure 4.15). Sharon circled $15. She then wrote 15 and multiplied it by 2. She kept multiplying her answers by 2 until she reached 240. She then checked 240 by dividing by 2 according the amounts that she underlined in the problem. She was left with $15. She submitted her answer of $240.
Figure 4.15 – Sharon’s Dad’s Wallet

Researcher’s Observations:

The first observation is that neither student drew a picture for this problem. This was the fifth problem solving task and the second problem thus far where the students did not draw a picture or diagram. When I asked the students why they decided not to use a diagram, they both replied that this problem did not require a picture or diagram. As stated in an earlier observation, several of the problems did not require a picture or a diagram.

The second observation is that Kate had returned to her difficulties with reading the problem solving tasks. She mispronounced several words. Kate made a mistake when she made a schedule to show the amount that Dad gave away each day. She included an extra amount in her schedule which caused her to arrive at the incorrect answer. Even though Kate checked her answer, she misread the problem and included the additional amount when she checked her answer.

The third observation is that Sharon underlined each phrase in the problem that stated the amount of Dad gave away which allowed her to use the correct number of deductions. She used
the strategy of working backwards to solve the problem. Kate used the working backwards strategy after her guessing and checking strategy did not prove to be successful. Kate spent approximately 21 minutes solving the problem whereas Sharon only spent approximately 3 minutes.

Kate used five of Montague’s (2003) thinking processes. Kate comprehended linguistic and numerical information by reading the problem several times. She observed relationships among the elements of the problem by forming a schedule to show the amount of money that was given away on certain days. She also formulated a plan to solve the problem. Kate stated that there had to be a large number that she could divide by 2 until she reached 15. She regulated the solution path by computing her algorithm. Kate also detected and corrected errors by checking her calculations. This did not lead Kate to the correct answer.

Sharon used four of Montague’s (2003) thinking processes. She translated and transformed information by underlining key terms while reading the problem. Sharon formulated a plan to solve the problem by deciding to use an algorithm of multiplying 15 by 2 for each amount Dad gave away. She also stated that this problem required working backwards. She also regulated the solution path by computing her algorithm. Sharon detected and corrected errors by checking her answer.

Kate submitted an incorrect answer, and Sharon submitted a correct answer. Dad started with $240 in his wallet. Both students used the working backwards strategy. One student used the guess and check method along with the working backwards strategy. Kate checked her answer, but she did see her mistake when she checked her answer which resulted in the incorrect answer. The common strategy used was working backwards. The common thinking strategy used was formulating a plan, regulating the solution path, and detecting and correcting errors.

**Problem 6: The Pool Deck**

Curly used a shovel to dig his own swimming pool. He figured he needed a pool because digging it was hard work and he could use it to cool off after working on it all day. He also planned to build a rectangular concrete deck around the pool that would be 6 feet wide at all points. The pool is rectangular and measures 14 ft by 40 ft. What is the area of the deck?

![Figure 4.16](Johnson, Herr, & Kysh, 2003 p. 15)
The problem, *The Pool Deck* (Figure 4.16), can be solved by drawing a diagram (Johnson, Herr, & Kysh, 2003). Drawing diagrams are often the key to getting started on a problem because they can clarify relationships that appear complicated in written. Problem solving often revolves around how information is organized. A diagram helps organize information spatially, which then allows the visual part of the brain to become more involved in the problem-solving process. A diagram can help a student understand and correctly interpret the information contained in a problem.

**Students’ Work:**

Kate read the problem aloud, and she read the problem again. She read the problem once more and drew a diagram (Figure 4.17). The diagram consisted of a rectangle, square, triangle, and rectangle. Kate read the problem again and drew a larger rectangle that she labeled as the pool. She wrote the measurements next to the width and length of the pool. She drew another rectangle around the pool and labeled it as the deck. She labeled the space between the pool and the deck as 6 feet on the side of the width and again on the side of the length.

![Diagram of the pool and deck](image)

**Figure 4.17 – Kate’s The Pool Deck**
Kate wrote the formula for finding the area of a rectangle as \( A = W \times H \). She added 6, 14, and 6 and got 26 as the height on the rectangle. She added 40, 6, and 6 as and got 52 as the width of the rectangle. She substituted 26 and 52 into the area formula. She found the area of the pool to be 1352. She checked her calculations on the calculator and submitted her answer.

Sharon read the problem aloud. She read the problem again and drew a diagram (Figure 4.18). She drew a rectangle and labeled the dimensions as 40 by 14. She drew another rectangle around the existing rectangle using a dashed line and labeled the figure as the deck. She labeled the space between the two rectangles as 6 feet on the side for the length and again on the side for the width.

![Figure 4.18 – Sharon’s The Pool Deck](image)

Sharon read the problem again and wrote the formula for finding the area of a rectangle as \( A = l \times w \). She substituted 14 for the length and 40 for the width and solved the formula. She found the area to be 560. She took 560 and multiplied it by 6. Sharon got an answer of 3360. She submitted her answer.

**Researcher’s Observations:**

The first observation is that both students drew a diagram. This problem was designed to be solved by drawing a diagram and both students used that strategy to solve the problem. Both students drew diagrams on four out of the six of the problem solving tasks. According to the
students, they were taught to always draw a diagram, if possible. However, both students drew diagrams in cases where diagrams were not necessarily needed or useful.

The second observation is that Sharon surprisingly did not arrive at the correct answer even though her drawing appeared as though she understood the problem (Figure 4.18). Sharon started by finding the area of the pool alone which she calculated correctly. She then took that amount and multiplied it by 6. I asked her why she multiplied by 6 and she said that her mind went blank after she found the area of the pool. She said that she remembered working a problem similar to this one. However, she said, “I spaced out towards the end of the problem. I couldn’t remember anything, so I just multiplied my answer by 6 and submitted my answer.”

The third observation is that Kate did not remember her shapes. She drew a picture of a rectangle, but she kept referring the shape as a triangle. Kate also mispronounced several words when reading the problem solving task aloud. She read the word ‘rectangle’ as ‘triangle’. She read the problem several times and each time she read the word ‘rectangle’ as ‘triangle’. When she first drew her pictures, she became confused with her shapes. She said that she could not remember how rectangle looked. She did not make the connection that most swimming pool are in the shape of a rectangle. Kate spent approximately 8 minutes completing the problem solving task. Sharon spent approximately 4 minutes completing the problem solving task.

A fourth observation is that Kate used different variables for the formula for area. The formula is defined as \( A = \text{length} \times \text{width} \). Kate used \( A = \text{width} \times \text{height} \). I asked Kate why she used height instead of width. She pointed to her diagram and said that she was measuring how wide and how tall the pool and deck were. Kate said the she was measuring how tall the water was in the pool so she needed to use height. This is interesting because the problem did not mention the amount of water in the pool.

Kate used five of Montague’s (2003) thinking processes. She attempted to comprehend linguistic and numerical information by reading the problem several times. She observed relationships among the elements of the problem by drawing a diagram and placing the given measurements on the diagram. She formulated a plan to solve the problem by using an algorithm. Kate stated that she would have to add the measurement of the deck to the measurement of the pool in order to find the dimensions to compute the area. She also regulated the solution path by computing her formula. Kate detected and checked errors by checking her calculations.
Sharon used four of Montague’s (2003) thinking processes. She comprehended linguistic and numerical information by reading the problem several times. She observed relationships among the elements of the problem by drawing a diagram. Sharon formulated a plan to solve the problem by deciding to use an algorithm that involved using the formula for area to solve the problem. She regulated her solution path by computing the formula.

Both students submitted incorrect answers. The area of the deck was 792 square feet. Kate found the correct area of the pool combined with the deck, but she did subtract the area of the pool from the total area in order to obtain the area of the deck. Sharon found the correct area of the pool, but she did not find the area of the pool and the deck. The common strategy used in this problem solving task included drawing a picture. The common thinking processes used were comprehending linguistic and numerical information, observing relationships among the elements, formulating a plan to solve the problem, and regulating the solution path.

Strategies and Thinking Processes Common to Both Kate and Sharon:

Table 4.1 summarizes the common strategies and thinking processes that both high mathematics anxiety students used to solve the six problems presented to them.

Table 4.1

<table>
<thead>
<tr>
<th>Problem</th>
<th>Strategy</th>
<th>Thinking Processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penny’s Dimes</td>
<td>Drawing a picture</td>
<td>Observing relationships</td>
</tr>
<tr>
<td></td>
<td>Making a list</td>
<td>Formulating plan to solve</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Regulating the solution path</td>
</tr>
<tr>
<td>Night of the Howling</td>
<td>Drawing a picture</td>
<td>Comprehending linguistic &amp; numerical info</td>
</tr>
<tr>
<td>Dogs</td>
<td></td>
<td>Observing relationships</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Formulating plan to solve</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Regulating the solution path</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Detecting &amp; correcting errors</td>
</tr>
<tr>
<td>Cascades State Park</td>
<td>Drawing a picture</td>
<td>Observing relationships</td>
</tr>
<tr>
<td></td>
<td>Algebraic Equations</td>
<td>Formulating plan to solve</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Regulating the solution path</td>
</tr>
</tbody>
</table>
Table 4.1 Continued

<table>
<thead>
<tr>
<th>Problem</th>
<th>Strategy</th>
<th>Thinking Processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Divisors and Reciprocals</td>
<td>None</td>
<td>Comprehending linguistic &amp; numerical info</td>
</tr>
<tr>
<td>Dad’s Wallet</td>
<td>Working Backward</td>
<td>Formulating plan to solve</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Regulating the solution path</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Detecting &amp; correcting errors</td>
</tr>
<tr>
<td>The Pool Deck</td>
<td>Drawing a picture</td>
<td>Comprehending linguistic &amp; numerical info</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Observing relationships</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Formulating plan to solve</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Regulating the solution path</td>
</tr>
</tbody>
</table>

Mathematics Anxiety Survey

All of the students completed a math anxiety survey (Appendix D) after completing the problem solving tasks. The survey asked students to respond according to their feelings. The results are shown in the following table.

Kate experienced some symptoms only a couple of times such as queasy stomach and clammy hands and feet. Kate did experience increased or irregular heartbeat four out of six times. She experienced shakiness, panic or fear, and extreme tension and nervousness six times. The shakiness was very apparent during the task-based interview. Her leg was visibly shaking while she was solving the problems. Kate also experienced an inability to recall material studied four out of six times. This could have contributed to her inability to get any problems correct.

Sharon consistently experienced a queasy stomach, increased or irregular heartbeat, panic or fear, and extreme tension and nervousness during each problem solving activity. Sharon responded that she had an inability to recall material studied for problems 2, 4, 5, 6. She answered questions 2, 4, and 6 incorrectly.

Emerging Themes

Looking at the students’ problem solving tasks and their surveys, I discovered several themes that emerged through analyses. The themes were:

1. Exhibiting mathematics anxiety symptoms may not completely hinder student performance. Even though Sharon exhibited mathematics anxiety symptoms, she was able to successfully complete two problems. It appears that Sharon could control her
mathematics anxiety better than Kate which allowed Sharon to successfully solve two of the problems.

2. Difficulty with reading skills may hinder the problem solving process. Kate had several difficulties with reading the problem solving tasks. This led to errors in her problem solving tasks.

3. Mathematics anxiety may lead to an inability to recall previously studied material which hinders student performance. Kate and Sharon admitted that they could not recall information that they had previously studied. Kate did not answer any of the questions correctly. She visibly displayed mathematics anxiety symptoms during the problem solving tasks. Sharon also incorrectly answered questions when she admitted that she could not recall information that she had previously studied.

4. Students must be able to check their calculations and check for the use of correct procedures. Kate and Sharon checked their calculations, but they still had the incorrect answer because they did not check for the use of correct procedures.

5. Failure to use all of the thinking processes described by Montague (2003) does not hinder the student’s ability to successfully solve mathematical problems. For the problems that Sharon answered correctly, she did not use all of the thinking processes described by Montague.

6. Drawing pictures or diagrams maybe important for high mathematics anxiety students when solving problems, but not necessarily meaningful.

I analyzed the problem solving abilities and surveys responses of the high mathematics anxiety students in order to determine what emerged from their work. The next chapter will analyze the work of the low mathematics anxiety students.
CHAPTER 5

LOW MATHEMATICS ANXIETY RESULTS

In this chapter I discuss the data collected from the interviews of the two student participants with low mathematics anxiety. The interviews conducted consisted of the students completing mathematical problem solving activities that were identical to the tasks completed by the high mathematics anxiety as discussed in Chapter 4.

I used the frameworks of Malloy (1994) and Montague (2003) in order to analyze the data. The frameworks were discussed in Chapter 4.

This chapter presents the investigation of the strategies and thought processes of students with low mathematics anxiety while performing mathematics problem solving tasks. With regard to the data presented in this chapter, I address the following research question:

What strategies and thinking processes do students with low mathematics anxiety use when they engage in problem solving tasks?

The Case of the Low Mathematics Anxiety Students

Adam and Hank

Adam is an African American male student. He graduated from high school in May 2003 and entered a university in August 2003. After three years, he left the university and started taking courses at the community college in August 2006. He has completed his Associate of Arts degree and is looking forward to returning to the university. Adam plans to pursue a degree in Engineering. He has been a part of the HP-MESA program for one year. He is a well rounded student who is very talented academically.

Adam is an only child. His parents are from West India. He is a high school graduate, and had to take the graduation exam once. Adam took the SAT and ACT tests while in high school. He did not have to take the college placement test (CPT) upon entering the community college because he had already taken several classes at the university.

According to Adam’s transcript, his grades at the university were not very good. He earned several Fs and Ws in his math and science courses. When Adam entered the HP-MESA program, he had to complete an interview. During the interview, Adam admitted that he did not
take school seriously which caused his grades to be low. He felt that we would do better in school if he was back at home with his parents. Adam completed community college with a GPA of 3.42, even though he had a 2.3 while at the university. He took College Algebra, Pre-calculus, Trigonometry, and Calculus I as his math courses. He received As and Bs as his grades in the mathematics courses.

Hank is also an African American male student. He graduated from high school in June 2004 and entered the community college in August 2004. He completed his Associate of Arts degree in April 2007, and plans to begin the university in Fall 2007. Hank plans to pursue a degree in Mathematics. He has been a part of the HP-MESA program for one year. He is a good student who has defeated the odds in order to be successful. Hank lived in a drug infested community. He continued to attend high school and graduate despite the fact that most of his friends and family members were high school drop-outs. Hank admitted that he did not have a support system at home and it was hard to stay in school when his friends were dealing drugs and making fast money as high school drop-outs.

Hank has several siblings. He is the first person in his family to go to college. Hank took his graduation exit exam only once and passed. He struggled in high school and during the beginning of his program at the community college. Hank faced several negative influences in throughout his life. However, he had a high school teacher who took an interest in his well-being and made sure that Hank stayed on the right path to success.

Upon entering the community college, Hank used his scores on the CPT. He placed in the college level Math and English courses. However, Hank placed in a remedial reading course. He completed community college with a GPA of 2.39. Hank took College Algebra, Pre-calculus, Trigonometry, and Calculus I as his math courses. He received As, Bs, and Cs as his grades in the mathematics courses.

These two students were chosen based on the criterion presented in an earlier chapter. They each experienced low mathematics anxiety based on MARS (Appendix A), were able to verbally express themselves well, were willing to participate in the study, and they completed similar math courses during their course of study at the community college.

The Students’ Strategies

The students completed six mathematical problem solving tasks during the course of this research study. After the students completed the problem solving tasks, I analyzed the strategies
and thinking processes which the students utilized to complete the problem solving activities. I was looking for similarities between the students with low mathematics anxiety. In this section, I will give each student’s strategy and thinking processes for the six problems and her observations of them. Chapter 6 will address the similarities and differences between the students with high mathematics anxiety and low mathematics anxiety.

The same mathematical problems were used with Adam and Hank as with Kate and Sharon.

**Problem 1: Penny’s Dimes**

Nick’s daughter penny has 25 dimes. She likes to arrange them into three piles, putting an odd number of dimes into each pile. In how many ways could she do this?

<table>
<thead>
<tr>
<th>Penny’s Dimes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nick’s daughter penny has 25 dimes. She likes to arrange them into three piles, putting an odd number of dimes into each pile. In how many ways could she do this?</td>
</tr>
</tbody>
</table>

Figure 5.1 – Johnson, Herr, Kysh (2003, p. 30)

**Students’ Work:**

Adam read the problem aloud. He read the problem again and asked if he needed to assume that order does not matter. I told him to make an assumption based on his understanding but write his assumption at the top of the page. Adam read the problem again wrote his assumption that order does not matter. He then listed the odd numbers up to 25 (Figure 5.2).

Adam read the problem again and decided to use 23, 1, 1 and as possible pile. He then said that he would look for an equation to determine the number of piles. He said
that he could start with n + 1, but he had forgotten the methods to use. Adam said that the only sure way to solve the problem was to write out all the possible outcomes. He went through his list of odd numbers and crossed out the number 25 because he knew he could not have a pile with 25 dimes.

Adam formed another pile with 21, 3, and 1. He read the problem again and began making his systematic list. He used piles with 19, 3, 3 and 17, 7, 1. Adam continued his list until he got to 7, 17, and 1. He decided that there was another way to solve the problem. He decided to take 25 and subtract every odd number going backwards in order to determine the number of possibilities for each result. For example, 25 – 21 = 4; and there is only one possible way to make a pile of 25, 21, and ___. Adam noticed that every two odd numbers would get a new possible number of arrangements. So, Adam added the numbers 1, 2, 3, 4, 5, and 6 and multiplied the result by 2 in order to get an answer of 42.

Hank read the problem aloud. He read the problem again and wrote the phrase “penny has 25 dimes” (Figure 5.3). He said that he had to write important information. He said that this problem was solvable. He said that he knew that there was a shorter way to solve the problem but he could not think of it.
Hank made three columns and labeled each column $P_1$, $P_2$, and $P_3$. He wrote the combination 3, 11, 11. He then used 3, 9, 13. Hank continued to make piles until he piles started to repeat. He crossed out the piles that repeated and counted 16 piles. Hank drew boxes to show that he was looking for a pattern, but he could not determine the pattern. He read through his piles again to make sure that he had not repeated any combinations. He counted his piles again and submitted his answer.

Researcher’s Observations:

One important observation is that both students used a systematic list to find the answer. Each student made an organized list of the possible combinations of ways to form the piles. One student started the list with 3, 11, and 11 while the other student started with 23, 1, and 1.

A second observation is that Adam used statistics. He asked if order mattered or not when forming the piles. He spent a couple of minutes trying to determine if the answers would be different if order mattered. Adam assumed that order did not matter when he formed his
systematic list which meant that 23, 1, 1 was the same as 1, 23, 1. Hank silently assumed that order did not matter and crossed out the combinations that repeated.

Another observation is that both students believed that there was a shorter way to solve the problem. Adam stopped with his systematic list because he thought he had found the shorter method. He even checked his shorter method to make sure it worked. Hank also believed that there was a shorter method, but he could not understand what the pattern would be in order to find the shorter method. Hank was satisfied with using the long method. It took Hank approximately 25 minutes to complete the problem. It took Adam approximately 26 minutes to complete the problem.

Adam used three of Montague’s (2003) thinking processes. He comprehended linguistic and numerical information in the problem by reading the problem several times. He formulated a plan to solve the problem by stating that he was looking for an equation or pattern to determine the answer. He decided upon a plan to solve the problem. Adam regulated the solution path by computing his algorithm.

Hank used five of Montague’s (2003) thinking processes. He comprehended linguistic and numerical information in the problem by listing the important information that was needed to solve the problem. He also read the problem several times. He translated and transformed information by writing what the problem was asking for. Hank also formulated a plan to solve the problem by stating that he would have to list all the possible outcomes in order to find the answer. He regulated the solution path by computing his algorithm. Hank detected and corrected errors by checking his work.

Hank found the correct solution and Adam found an incorrect solution. There were 16 ways to form three piles of 25 dimes. Both students formed systematic lists to solve the problem. The common strategies used by Hank and Adam for this problem includes making a systematic list. The common thinking processes include comprehending the linguistic and numerical information, formulating a plan to solve, regulating the solution path, and detecting and correcting errors.
Problem 2:  *Night of the Howling Dogs*

Shawna liked to jog in the late afternoon. One day she noticed an unusual phenomenon. As she jogged, dogs would hear her and bark. After the first dog barked for about 15 seconds, two other dogs would join in and bark. In about another 15 seconds, it seemed that each barking dog would “inspire” two more dogs to start barking. Of course, long after Shawna passed the first dog, it continued to bark, as dogs are inclined to do. After about 3 minutes, how many dogs were barking?

Figure 5.4 – Johnson, Herr, Kysh (2003, p. 120)

Students’ Work:

Adam read the problem aloud. He read the problem again and said that the seconds increasing by 15 was important. He read the problem again and turned 3 minutes into 180 seconds (Figure 5.5). He subtracted 180 and 15 because of the first dog. He then divided 165 by 15 and got 11 intervals. He multiplied 11 by 2 because there were two dogs added each time. He then added the first dog to get an answer of 23 dogs. Adam checked his answer. He read the problem again and decided that his answer was wrong. After reading the problem again, he underlined the phrase “each barking dog” and “two more dogs”.

Figure 5.5 – Adam’s *Night of the Howling Dogs*
Adam realized that the problem was exponential because the number of dogs was increasing at a multiplicative rate. He read the problem again and developed the exponential function $2^n$ where $n = \text{number of dogs}$. He changed the function to $2^{15^x}$ where $x = \text{number of seconds}$. He used a calculator to evaluate the function for $x = 180$. He found the answer to be 4,096 dogs. He graphed his answer on the calculator which allowed him to check his calculations. He submitted his answer.

Hank read the problem aloud. He read the problem again and underlined some words in the paragraph. He said that he was weeding out the unnecessary information such as ‘Shawna liked to jog late in the afternoon’. He said that he needed to develop an equation because of the intervals. He tried to form a ratio, but he said that he could not remember how to form the ratio.

Figure 5.6 – Hank’s Night of the Howling Dogs
Hank began looking for a pattern. He wrote 1 dog = 15 seconds. Next, he wrote 3 dogs = 15 seconds, and 6 dogs = 15 seconds. Hank kept writing the pattern until he had 60 seconds or 1 minute. He read the problem again and continued with his pattern until he reached 3 minutes. Hank realized that for each interval of 15 seconds, he had to multiply the total number of dogs by 2. Hank found an answer of 6, 144 dogs.

Researcher’s Observations:

One observation is that neither student used a diagram to try to work the problem. Both students believed that an equation or function could be used to solve the problem. Adam eventually found an exponential function which he thought would work. Hank tried to form an equation by forming a ratio; he couldn’t remember how to set up the ratio did not work. He resorted to using a pattern to determine the answer. Adam drew a graph demonstrate his exponential function once he determined his answer.

A second observation is that Hank disregarded unnecessary information in the problem. He felt that he needed to display the unnecessary information by underlining it. Disregarding unnecessary data or information is one of the common problem solving strategies that was used among African American eighth grades in Malloy’s (1994) study. It was interesting to see Hank use this strategy.

Another observation is that both students showed that there was an increase in dogs during each 15 second increase. Hank used a systematic list to show the increase. Adam also used a systematic list to show the increase in dogs during the first minute. Adam’s list helped him develop his exponential function. It took Adam approximately 17 minutes to complete the problem. It took Hank approximately 27 minutes to complete the problem.

Adam used five of Montague’s (2003) thinking processes. He comprehended linguistic and numerical information in the problem by reading the problem several times. He also used an equation to find the answer. Adam observed relationships among the elements of the problem drawing a diagram to show that the dogs were increasing exponentially every 15 seconds. He formulated a plan to solve the problem by stating that he would need an exponential equation to show the increase in dogs in order to solve the problem. He regulated the solution path by calculating his algorithm. Adam detected and correct errors during the problem solution. After
he worked the problem, he checked his work by reading the problem again. He discovered that he had misinterpreted the problem and worked the problem a second time.

Hank used three of Montague’s (2003) thinking processes. He translated information by underlining unnecessary information. He also formulated a plan to solve the problem. He stated that he would need to either find an equation or pattern to determine the answer. Hank also regulated the solution path by computing his algorithm.

Both students’ solutions were incorrect. There were 531,441 dogs barking at the end of 3 minutes. Hank formed a systematic list, looked for a pattern, and disregarded unnecessary information to find the answer. Adam used an equation to determine the answer. They did not have a common strategy. The common thinking processes included formulating a plan to solve the problem and regulating the solution.

**Problem 3: Cascades State Park**

| Emi and Margit had stopped at the bottom of one of the highest waterfalls in Cascades State Park. As Emi looked up at the waterfall, she said, “Wow, I think the top of that fall is about 20 feet more than 3 times the height of that young redwood?” Margit, of course, had a different opinion. She said, “No, I think its about 50 feet less than four times the height of the redwood.” If both are approximately right, about how tall is the redwood and how high is the waterfall? |

Figure 5.7 – Johnson, Herr, Kysh (2003, p. 145)

**Students’ Work:**

Adam read the problem aloud. He read the problem again and decided that he needed to use linear equations. He read the problem again. Adam formed two equations with two unknowns and decided that he needed to use a system of equations (Figure 5.8).
He read the problem again and decided to set his equations equal to each other and solve for the unknown. He found $x = 260$ feet which was the height of the tree. He substituted 260 into one of the equations and solved for the height of the waterfall which was 840 feet. He checked his calculations on the calculator and submitted his answer.

Hank read the problem aloud. He drew a picture of the waterfall and the tree (Figure 5.9). He wrote the names of Emi and Margit at the bottom of the waterfall and tree. He read the problem again and wrote an equation. He read the problem again and wrote a second equation. Hank decided that he needed to use a system of equations. He said that he was thinking back to a problem that he had previously worked that was similar to this one.
Hank then set the equations equal to each other and found \( x = 70 \). He substituted 70 into both of the equations, \( 3x + 20 = y \) and \( 4x - 50 = y \), and found \( y = 230 \). Hank found the tree to be 70 feet and the waterfall to be 230 feet. Hank checked his work by reading the problem again and checking his calculations on the calculator. He submitted his answer.

Researcher’s observations

One observation is that only one student drew a diagram to show the tree and the waterfall. Hank expressed that the diagram helped him to visualize the relationship between the tree, waterfall, and Emi and Margit. Adam did not draw a diagram. He immediately believed that he needed to form linear equations to solve the problem.

A second observation is that both students used the equal sign in their equations correctly. Many students misuse the equal sign, but Adam and Hank used it correctly.

Another observation is the strategies that the students used. Both students created equations as part of their solution. Hank and Adam both formed systems of equations and solved the systems by using the substitution method. Adam used different equations than Hank. This could be due to the understanding of the problem. Adam understood the problem as taking 3 and multiplying it by the height and 20. Hank understood the problem as taking 3 and multiplying it by the height and then adding 20. This misunderstanding caused the students to come up with two different answers. Even though Adam checked his answer, he did not check the reasonableness of a tree being 230 feet.

Neither students used the guess and check strategy, even though this problem was specifically designed for the guess and check method. The guess and check method leads to
using algebraic equations. Hank and Adam decided not the waste time guessing and checking, but to use algebraic equations instead. It took Hank approximately 14 minutes to complete the problem, whereas it took Adam approximately 5 minutes to complete the problem.

Adam used four of Montague’s (2003) thinking processes. He comprehended linguistic and numerical information by reading the problem several times. He formulated a plan to solve the problem. Adam stated that he would use a system of linear equations to solve for the unknown measurements. He also regulated the solution path by computing the equations. Adam detected and corrected errors by checking his calculations.

Hank used four of Montague’s (2003) thinking processes. He observed relationships among the elements of the problem by drawing a diagram. He also formulated a plan to solve the problem by stating that he would use a system of equations to solve for the heights. Hank regulated the solution path by computing his equations. He detected and corrected errors by checking his calculations.

Adam’s solution was incorrect, and Hank’s solution was correct. The height of the redwood tree was 70 feet and the height of the waterfall was 230 feet. The student who got the answer correct drew the diagram. The common strategies used by Adam and Hank for this problem included using algebraic equations. The common thinking processes included formulating a plan, regulating the solution path, and detecting and correcting errors.

Problem 4: Divisors and Reciprocals

The divisors of 360 add up to 1170. What is the sum of the reciprocals of the divisors of 360?

Figure 5.10 (Johnson, Herr, & Kysh, 2003 p. 229)

Students’ Work:

Adam read the problem aloud. He read the problem again. He immediately began looking for an easier way to solve the problem because fractions with different denominators could take a long time to add. Adam decided to list the divisors of 360 (Figure 5.11). He made sure
that they all added up to 1170 by adding them on his calculator. He wrote the summation of $\frac{1}{n} = 1170$ because he knew that he had to add the divisors.

Adam decided to use his calculator to arrive at the answer. He used the list method and made a list of all of the divisors of 360 and stored the list in his calculator. He turned each number stored in the list into a fraction and summed the list of fractions. He reached an answer of $3.25$ which he changed to a fraction of $\frac{13}{4}$.

Hank read the problem aloud. He read the problem again. Hank made a list of the divisors of 360 (Figure 5.12). He used his calculator to determine the divisors. He read the problem again. He wrote $\frac{1}{1170}$. He said that the answer was very simple, but
he wanted to check. He said that math questions tended to ‘play on words’, so he needed to check the wording very carefully. Hank stated that some problems were very tricky because of the manner in which some problems were stated. He submitted his answer.

Researcher’s Observations:

The first observation is that both students listed all of the divisors of 360. Adam checked his list by making sure that the numbers added up to 1170. Hank did not check his list. He was more confident in his list of divisors.

The second observation is that both students immediately began listing the divisors without asking about the definition of the word divisor. Neither student asked about the definition of the word reciprocal. Both Adam and Hank appeared to have knowledge of how to solve the problem without any hesitation. However, Adam took more time looking for a shorter method to solve the problem than he did actually arriving at an answer. Adam spent approximately 17 minutes working on the problem. Hank only spent approximately 6 minutes working on the problem.

Adam used two of Montague’s (2003) thinking processes. He formulated a plan to solve the problem by stating that he would list all divisors and find reciprocals of each and add the reciprocals. He regulated the solution by computing his algorithm.
Hank used three of Montague’s (2003) thinking processes. He formulated a plan to solve the problem. Hank stated that he would need to list all the divisors of 360 and find the reciprocals of each. He regulated the solution by following through with his plan to list the divisors. Hank did not find the reciprocals as he had previously stated as part of his plan. He detected and corrected errors by checking his answer.

Adam submitted a correct solution and Hank submitted an incorrect solution. The correct solution was \( \frac{1170}{360} = \frac{13}{4} \). The common strategy used for this problem solving task was making a list. The students could have found the answer by working an easier problem such as finding the divisors of 10 and finding the sum of the reciprocal. They could have also used the number 16. The students would have realized that the method to solve the problem was the sum of the divisors divided by the number. Adam came up with correct solution with the use of his calculator. Without his calculator, he would have been forced to use another method, perhaps the suggested method for this problem solving task. The common thinking processes included formulating a plan and regulating the solution path.

Problem 5: Dad’s Wallet

Dad went to the ATM on Wednesday of spring break and withdrew some money. On Thursday morning my brother borrowed half of Dad’s money to open a checking account because he was always short of money. On Friday I needed some money for a date, so I borrowed half of what remained. My sister came along next and borrowed half of the remaining money. Dad then went to gas up the car and used half of the rest of his money, and he wondered why he had only $15 left. How much money did he start with in his wallet?

Students’ Work:

Adam read the problem aloud. He read the problem again. He let the variable \( T \) equal the total amount of money Dad started with in his wallet. He divided \( T \) by 2 to represent the first amount of money Dad gave away (Figure 5.14). He divided \( T \) by 4 to represent the next amount of money Dad gave away. He continued to follow the increase by a factor of 2 until he reached \( T \) divided by 16. Adam realized that the amount of money Dad gave away was decreasing exponentially.
Adam wrote \( \frac{T}{16} = \$15 \). He solved the equation for \( T \). He found that \( T = 240 \). He checked his answer by taking $240, reading the problem, and decreasing the total according to the directions in the problem. Adam submitted his answer.

Hank read the problem aloud. He read the problem again. He made a written note that the days were not valuable information (Figure 5.15). Hank listed Wednesday and assigned \( \frac{x}{2} \) as its value. He listed Thursday and assigned \( \frac{x}{4} \) as its value. He listed Friday and assigned \( \frac{x}{6} \) as its value. He listed ‘lil sis’ and assigned \( \frac{x}{8} \) as its value. He finally listed gas and assigned \( \frac{x}{8} \) as its value. He created the equation \( \frac{x}{8} = 15 \). He solve the equation and got \( x = 120 \).
Hank checked his answer by reading the problem. He started with $120 and decreased it according to the directions in the problem. He realized that he did not have the correct answer. Hank read the problem again and he doubled $120 to get $240. He checked $240 by reading the problem and decreasing the total according to the directions in the problem. He submitted his answer of $240.

Researcher’s Observations:

The first observation is that both students developed equations to solve the problem. Both students first defined their variable. Adam defined his verbally and Hank defined his in writing. They also decreased their variable by dividing it according to the directions in the problem. They created their equations and solved the equations for the variable.

The second observation is that both students checked their answers. Hank’s checking method revealed that he had the incorrect answer. He was able to make a correction in order to obtain the correct answer. Both Hank and Adam used working backward to check their answer. Adam spent approximately 5 minutes solving the problem whereas Hank only spent approximately 16 minutes.

Adam used three of Montague’s (2003) thinking processes. He formulated a plan to solve the problem by developing an algorithm to find the answer. Adam also regulated the solution path by computing his algorithm. He detected and corrected errors by checking his calculations.

Hank used five of Montague’s (2003) thinking processes. He translated information by listing unnecessary information. He wrote that the days was invaluable information. Hank observed the relationships among the elements of the problem by making a schedule to show the days and the amount of money deducted during each day. He also formulated a plan to solve the problem by stating that he would form an equation, let the equation equal 15, and solve for the unknown variable. Hank regulated the solution path by computing his algorithm. He detected and correct errors by checking his calculations.

Adam and Hank both submitted a correct answer. Dad started with $240 in his wallet. Both students used the algebraic equations strategy. The students used the working backwards strategy to check their answers. The common thinking processes used were formulating a plan, regulating the solution path, and detecting and correcting errors.
Problem 6: The Pool Deck

Curly used a shovel to dig his own swimming pool. He figured he needed a pool because digging it was hard work and he could use it to cool off after working on it all day. He also planned to build a rectangular concrete deck around the pool that would be 6 feet wide at all points. The pool is rectangular and measures 14 ft by 40 ft. What is the area of the deck?

Students’ Work:

Adam read the problem aloud. He read the problem again. He drew a picture of a smaller quadrilateral inside of a larger quadrilateral (Figure 5.17). He labeled the inside quadrilateral with its measurements. He also labeled the distance between the outside shape and the inside shape as a distance of 6 feet. He wrote the formula for area as

\[ A = lw \]

and he determined that the total area of the pool and deck as \((14 + 6)(40 + 6)\). His calculation revealed the total area of the pool and deck was 920. Adam subtracted 920 and \((40)(14)\) and obtained 360 as the area of the deck. He used his calculator to check his multiplication.

Figure 5.17 – Adam’s The Pool Deck
Hank read the problem aloud. He read the problem again. He drew a picture of a smaller quadrilateral inside of a larger quadrilateral (Figure 5.18). He labeled the inside quadrilateral with its measurements as given in the problem. He also labeled the outside quadrilateral with its measurements of 52 and 26 which he determined by adding 6 feet to each side of the pool. He wrote the formula for area as $A = lw$. Hank also wrote the formula for the area of the deck as the area of whole minus the area of the pool. He substituted the measurements into the formula and obtained $1352 - 560$. Hank found the area of the deck to be 792.

Figure 5.18 – Hank’s *The Pool Deck*

He checked his answer by reading the problem and checking his calculations on the calculator. Hank submitted his answer.

**Researcher’s Observations:**

The first observation is that both students drew a diagram. This problem was designed to be solved by drawing a diagram and both students used that strategy to solve the problem. This was the first diagram drawn by Adam, but the second diagram drawn by Hank.
The second observation is that both students drew very similar diagrams. However, the measurements they determined were different. Adam added 6 to the length and 6 to the width in order to determine the total area of the pool and deck. Hank added 12 to length and 12 to the width in order to determine the total area of the pool and deck.

The third observation is that both students realized the steps involved in finding the area of the deck. Adam and Hank realized that in order to find the area of the deck, the total area of the deck and pool had to be subtracted from the area of the pool. Both students demonstrated an understanding of the problem and neither student had difficulties recalling the formula for finding area. Adam spent approximately 6 minutes completing the problem, and Hank spent approximately 9 minutes completing the problem.

Adam used four of Montague’s (2003) thinking processes. He observed relationships among the elements of the problem by drawing a diagram showing where the pool was located in relation to the deck. Adam formulated a plan to solve the problem by stating that he would take the total area and subtract the area of the pool to find the answer. He also regulated the solution as it was executed by computing his algorithm. Adam detected and corrected his errors by checking his calculations.

Hank used four of Montague’s (2003) thinking processes. He observed relationships among the elements of the problem by drawing a diagram of the pool and the deck. Hank formulated a plan to solve the problem by stating that he would subtract the area of the pool from the total area of both structures. He also regulated the solution path by computing the algorithm. Hank detected and corrected errors by checking his calculations.

Adam submitted an incorrect answer, and Hank submitted a correct answer. The area of the deck was 792 square feet. Both students subtracted the total area of both the pool and the deck from the area of the pool. However, Adam did not realize that the deck was 6 feet wide at all points. He only added six to one of the lengths and one of widths; whereas, Hank added the 6 feet to all four sides which led to the correct answer. The strategies used in this problem solving task included drawing a picture. The common thinking process included observing relationships among elements, formulating a plan to solve, regulating the solution path, and detecting and correction errors.
Strategies and Thinking Processes Common to Both Adam and Hank:

Table 5.1 summarizes the common strategies and thinking processes that both low mathematics anxiety students used to solve the six problems presented to them.

Table 5.1

*Strategies and Thinking Processes Common to Both Adam and Hank*

<table>
<thead>
<tr>
<th>Problem</th>
<th>Strategy</th>
<th>Thinking Processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penny’s Dimes</td>
<td>Making a list</td>
<td>Comprehending linguistic &amp; numerical info</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Formulating plan to solve</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Regulating the solution path</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Detecting &amp; correcting errors</td>
</tr>
<tr>
<td>Night of the Howling Dogs</td>
<td>Looking for patterns Making a list Algebraic equation</td>
<td>Formulating plan to solve</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Regulating the solution path</td>
</tr>
<tr>
<td>Cascades State Park</td>
<td>Algebraic Equations</td>
<td>Formulating plan to solve</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Regulating the solution path</td>
</tr>
<tr>
<td>Divisors and Reciprocals</td>
<td>Making a list</td>
<td>Comprehending linguistic &amp; numerical info</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Formulating plan to solve</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Regulating the solution path</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Detecting &amp; correcting errors</td>
</tr>
<tr>
<td>Dad’s Wallet</td>
<td>Working Backward</td>
<td>Formulating plan to solve</td>
</tr>
<tr>
<td></td>
<td>Algebraic Equations</td>
<td>Regulating the solution path</td>
</tr>
<tr>
<td>The Pool Deck</td>
<td>Drawing a picture</td>
<td>Observing relationships</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Formulating plan to solve</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Regulating the solution path</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Detecting &amp; correcting errors</td>
</tr>
</tbody>
</table>

Mathematics Anxiety Survey

All of the students completed a math anxiety survey (Appendix E) after completing the problem solving tasks. The survey asked students to respond according to their feelings.
Adam experienced an inability to recall material studied for questions 1 and 4. Adam incorrectly answered both questions. Even though Adam answered other questions incorrectly, he did not respond that he noticed any mathematical anxiety symptoms. I noticed that he would tap his pencil on the table when solving the problems. I also noticed that Adam would shift in his chair when he appeared to be trying to decide what step to perform next during the problem solving tasks. During the follow up interview, I asked Adam about the tapping of the pencil and the shifting in the chair and he stated that he was nervous.

Hank responded that he experienced a queasy stomach on each of the problems. However, he did answer some of his problems correctly. Hank also experienced worry and apprehension on questions 1 and 3, but he did answer those problems correctly. He experienced mental disorganization on question 2 which answered incorrectly. Hank experienced an inability to recall material studied on questions 2 and 3, but he answered question 3 correctly. Hank would tap his pencil on the table and shift consistently in his chair. Hank would also play with his hair while he was solving the problems. At one point, Hank had a rubber band on his wrist which he played with throughout the task-based interview for that session. During the follow up interview, I asked Hank about the behavior and he stated that he was nervous about finding the correct solution. Even though Hank experienced some mathematical anxiety, he was able to answer five of the six questions correctly.

Emerging Themes

Looking at the students’ problem solving tasks and their surveys, I determined several themes that emerged through analyses. The themes were:

1. Exhibiting math anxiety symptoms does not hinder student performance. Hank admitted to being nervous and having a queasy stomach during all the problem solving tasks. However, he successfully completed 4 out of 6 problems.

2. Drawing pictures and diagrams is not important for low mathematics anxiety students, only when the problem requires it. Adam only drew a diagram on the problem that required a diagram. Hank drew a diagram on only two problems. He was successful each time he drew a diagram in his solution.

3. Inability to recall material studied may hinder students’ ability to be successful during problem tasks. Adam admitted to being unable to recall material on two problems. He answered those two problems incorrectly.
4. Mathematical anxiety symptoms can be visibly present even if a student does not notice the symptoms himself. Hank and Adam both tested as students with low mathematics anxiety. However, both students experienced mathematics anxiety. Hank admitted to feeling some symptoms of anxiety. Adam did not, but he did nervously tap his pencil on the table when he was trying to decide how to solve a few of the problems.

5. Failure to use all of thinking processes described by Montague (2003) may not hinder a student’s ability to successfully solve mathematical problems. Hank did not make use of all of the thinking processes, but he was successful on 4 out of 6 problems. Adam also did not make use of all of the thinking processes, but he was successful of 2 out of 6 problems.

I analyzed the low mathematics anxiety students’ problem solving tasks and surveys in order to determine what emerged from their work. The next chapter will analyze the similarities and differences between the strategies and thinking processes of the high and low mathematics anxiety students.
CHAPTER 6

CONCLUSIONS

In this chapter I discuss the similarities and differences in the strategies and thinking processes of the students with high mathematics anxiety and low mathematics anxiety. This chapter also discusses the results and the implications the completed research can have for mathematics educators, curriculum writers, textbook authors, and others with an interest in mathematics education.

The purpose of this research study was to investigate the strategies and thinking processes of African American community college students with high and low mathematics anxiety while performing mathematical problem solving tasks. I wanted to answer the following research questions:

1. What strategies and thinking processes do students with low mathematics anxiety use when they engage in problem solving tasks?
2. What strategies and thinking processes do students with high mathematics anxiety use when they engage in problem solving tasks?
3. What differences and similarities are there, if any, in the strategies and thinking processes of students with high mathematics anxiety and low mathematics anxiety when they engage in problem solving tasks?

Two cases were examined: (1) the case of the high mathematics anxiety students, Kate and Sharon; (2) the case of the low mathematics anxiety students, Adam and Hank. The students completed six mathematical problem solving tasks and a mathematics anxiety symptom survey after each problem solving task. I then analyzed the students’ results using the frameworks of Malloy (1994) and Montague (2003).

The Students’ Strategies

The students completed six problem solving tasks during the course of this research study. After the students completed the problem solving tasks, I identified the strategies and thinking processes the students utilized to complete the problem solving activities and whether they employed the strategies established by Malloy (1994) and the thinking processes by Montague (2003). In this section I provide each student’s strategies and thinking processes for
the six problems. This section addresses the research question: What differences and similarities are there, if any, in the strategies and thinking processes of students with high mathematics anxiety and low mathematics anxiety when they engage in problem solving tasks.

A table summarizing and comparing the strategies and thinking processes of the high mathematics anxiety and low mathematics anxiety students used to solve the six problems presented to them is presented.

**Problem 1: *Penny’s Dimes***

Nick’s daughter penny has 25 dimes. She likes to arrange them into three piles, putting an odd number of dimes into each pile. In how many ways could she do this?

Figure 6.1 – Johnson, Herr, Kysh (2003, p. 30)

Table 6.1

*Penny’s Dimes* Strategies and Thinking Process Common to Each Case

<table>
<thead>
<tr>
<th></th>
<th>High Mathematics Anxiety</th>
<th>Low Mathematics Anxiety</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strategy</strong></td>
<td>Draw a picture</td>
<td>Making a list</td>
</tr>
<tr>
<td></td>
<td>Make a list</td>
<td></td>
</tr>
<tr>
<td><strong>Thinking Process</strong></td>
<td>Observing relationships</td>
<td>Comprehending linguistic &amp; numerical info</td>
</tr>
<tr>
<td></td>
<td>Formulating plan to solve</td>
<td>Formulating a plan to solve</td>
</tr>
<tr>
<td></td>
<td>Regulating the solution path</td>
<td>Regulating the solution path</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Detecting &amp; correcting errors</td>
</tr>
</tbody>
</table>

All of the students used a list to solve the ‘Penny’s Dimes’ problem. The students with high mathematics anxiety drew a picture in addition to using a list. One of the students with low mathematics anxiety made a systematic list. The other students made a list without any structure or organization. Also, all of the students formulated a plan to solve the problem and regulated the solution path as their thinking processes.
Hank, a low mathematics anxiety student, was the only student to get the Problem 1 correct. The low mathematics anxiety students checked their work whereas the high mathematics anxiety students did not.

**Problem 2: Night of the Howling Dogs**

<table>
<thead>
<tr>
<th></th>
<th>High Mathematics Anxiety</th>
<th>Low Mathematics Anxiety</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strategy</strong></td>
<td>Draw a picture</td>
<td>Looking for patterns</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Making a list</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Algebraic Equation</td>
</tr>
<tr>
<td><strong>Thinking Process</strong></td>
<td>Comprehending linguistic info</td>
<td>Formulating a plan to solve</td>
</tr>
<tr>
<td></td>
<td>Observing relationships</td>
<td>Regulating the solution path</td>
</tr>
<tr>
<td></td>
<td>Formulating plan to solve</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Regulating the solution path</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Detecting &amp; correction errors</td>
<td></td>
</tr>
</tbody>
</table>

Shawna liked to jog in the late afternoon. One day she noticed an unusual phenomenon. As she jogged, dogs would hear her and bark. After the first dog barked for about 15 seconds, two other dogs would join in and bark. In about another 15 seconds, it seemed that each barking dog would “inspire” two more dogs to start barking. Of course, long after Shawna passed the first dog, it continued to bark, as dogs are inclined to do. After about 3 minutes, how many dogs were barking?

Figure 6.2 – Johnson, Herr, Kysh (2003, p. 120)

<table>
<thead>
<tr>
<th></th>
<th>High Mathematics Anxiety</th>
<th>Low Mathematics Anxiety</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strategy</strong></td>
<td>Draw a picture</td>
<td>Looking for patterns</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Making a list</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Algebraic Equation</td>
</tr>
<tr>
<td><strong>Thinking Process</strong></td>
<td>Comprehending linguistic info</td>
<td>Formulating a plan to solve</td>
</tr>
<tr>
<td></td>
<td>Observing relationships</td>
<td>Regulating the solution path</td>
</tr>
<tr>
<td></td>
<td>Formulating plan to solve</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Regulating the solution path</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Detecting &amp; correction errors</td>
<td></td>
</tr>
</tbody>
</table>

The high mathematics anxiety students drew pictures. The low mathematics anxiety students made lists and used algebraic equations. All of the students arrived at an incorrect solution. For the thinking processes, all of the students formulated a plan to solve the problem and carried out the plan. The high mathematics anxiety students comprehended linguistic information in the problem by reading the problem several times whereas the low mathematics anxiety did not spend a great amount of time rereading the problem.
Problem 3: Cascades State Park

Emi and Margit had stopped at the bottom of one of the highest waterfalls in Cascades State Park. As Emi looked up at the waterfall, she said, “Wow, I think the top of that fall is about 20 feet more than 3 times the height of that young redwood?” Margit, of course, had a different opinion. She said, “No, I think its about 50 feet less than four times the height of the redwood.” If both are approximately right, about how tall is the redwood and how high is the waterfall?

Table 6.3
Cascades State Park Strategies & Thinking Processes Common to Each Case

<table>
<thead>
<tr>
<th>Strategy</th>
<th>High Mathematics Anxiety</th>
<th>Low Mathematics Anxiety</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Draw a picture</td>
<td>Algebraic Equations</td>
</tr>
<tr>
<td></td>
<td>Algebraic Equations</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Thinking Process</th>
<th>High Mathematics Anxiety</th>
<th>Low Mathematics Anxiety</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observing relationships</td>
<td>Formulating a plan to solve</td>
</tr>
<tr>
<td></td>
<td>Formulating plan to solve</td>
<td>Regulating the solution path</td>
</tr>
<tr>
<td></td>
<td>Regulating the solution path</td>
<td>Detecting &amp; correcting errors</td>
</tr>
</tbody>
</table>

Both high mathematics anxiety students drew pictures. The low mathematics students used algebraic equations. The students who arrived at the correct solution drew a picture and used algebraic equations which were Hank (low mathematics anxiety) and Sharon (high mathematics anxiety). For Problem 3, Hank drew a picture whereas Adam did not, thus drawing a picture was not common for the low mathematics anxiety students. All of the students used algebraic equations. Also, all of the students formulated a plan and regulated the solution path as it was executed. Even though Kate (low mathematics anxiety) drew a picture and used algebraic equations, she demonstrated a lack of understanding based upon her low reading skills.

Problem 4: Divisors and Reciprocals

The divisors of 360 add up to 1170. What is the sum of the reciprocals of the divisors of 360?

Figure 6.4 – Johnson, Herr, Kysh (2003, p. 229)
Table 6.4

*Divisors & Reciprocals Strategies & Thinking Processes Common to Each Case*

<table>
<thead>
<tr>
<th></th>
<th>High Mathematics Anxiety</th>
<th>Low Mathematics Anxiety</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strategy</strong></td>
<td>None</td>
<td>Making a list</td>
</tr>
<tr>
<td><strong>Thinking Process</strong></td>
<td>Comprehending linguistic info</td>
<td>Comprehending linguistic info</td>
</tr>
</tbody>
</table>

The high mathematics anxiety students did not use a strategy to solve the problem. The high mathematics anxiety students became frustrated and stopped working on the problem. This was a problem where drawing a picture could not help the high mathematics anxiety students. They all comprehended the linguistic information in the problem by reading the problem several times. The low mathematics anxiety students made a list of the divisors as a strategy, but only Adam found the correct answer. Hank became frustrated that he could not find a shorter method to solve the problem and did not attempt to work the problem using the long method. Adam, who arrived at the correct solution, used his calculator to arrive at the answer. Without the calculator, he would have tried to solve the problem using the long method. All of the students had a calculator, yet only Adam decided to use it to arrive at the correct solution.

**Problem 5: Dad’s Wallet**

Dad went to the ATM on Wednesday of spring break and withdrew some money. On Thursday morning my brother borrowed half of Dad’s money to open a checking account because he was always short of money. On Friday I needed some money for a date, so I borrowed half of what remained. My sister came along next and borrowed half of the remaining money. Dad then went to gas up the car and used half of the rest of his money, and he wondered why he had only $15 left. How much money did he start with in his wallet?

Figure 6.5 – Johnson, Herr, Kysh (2003, p. 283)
Table 6.5

*Dad’s Wallet* Strategies & Thinking Processes Common to Each Case

<table>
<thead>
<tr>
<th></th>
<th>High Mathematics Anxiety</th>
<th>Low Mathematics Anxiety</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strategy</strong></td>
<td>Working Backward</td>
<td>Working Backward</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Algebraic Equation</td>
</tr>
<tr>
<td><strong>Thinking Process</strong></td>
<td>Formulating plan to solve</td>
<td>Formulating a plan to solve</td>
</tr>
<tr>
<td></td>
<td>Regulating the solution path</td>
<td>Regulating the solution path</td>
</tr>
<tr>
<td></td>
<td>Detecting &amp; correction errors</td>
<td></td>
</tr>
</tbody>
</table>

When solving Problem 6, three of the four students used working backwards as a solution strategy. The low mathematics anxiety students also used algebraic equations to solve the problem. All of the students formulated plans and regulated the solution path as it was executed. Kate was the only student with the incorrect solution. This can be attributed to her lack of understanding of the problem. She misread the problem each time which resulted in an error. Only the high mathematics anxiety students checked their work. It appears that the low mathematics anxiety students may have been confident so they did not need to check their answers.

**Problem 6: The Pool Deck**

Curly used a shovel to dig his own swimming pool. He figured he needed a pool because digging it was hard work and he could use it to cool off after working on it all day. He also planned to build a rectangular concrete deck around the pool that would be 6 feet wide at all points. The pool is rectangular and measures 14 ft by 40 ft. What is the area of the deck?

Figure 6.6 – Johnson, Herr, Kysh (2003, p. 15)
Table 6.6

*The Pool Deck* Strategies & Thinking Processes Common to Each Case

<table>
<thead>
<tr>
<th></th>
<th><strong>High Mathematics Anxiety</strong></th>
<th><strong>Low Mathematics Anxiety</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strategy</strong></td>
<td>Draw a picture</td>
<td>Draw a picture</td>
</tr>
<tr>
<td><strong>Thinking Process</strong></td>
<td>Comprehending linguistic info</td>
<td>Observing relationships</td>
</tr>
<tr>
<td></td>
<td>Observing relationships</td>
<td>Formulating a plan to solve</td>
</tr>
<tr>
<td></td>
<td>Formulating plan to solve</td>
<td>Regulating the solution path</td>
</tr>
<tr>
<td></td>
<td>Regulating the solution path</td>
<td>Detecting &amp; correcting errors</td>
</tr>
</tbody>
</table>

All of the students drew a picture as a strategy to solve the problems. All of the students observed relationships among the elements of the problem, formulated a plan, and regulated the solution path as it was executed. All of the students used the same strategies and the same thinking processes. Hank was the only student to arrive at the correct solution. The other students did not have a clear understanding of the problem which led to errors.

**Synopsis of Themes**

As I analyzed the students’ work, themes emerged. The themes which emerged through triangulation of two or more problems for the high and low mathematics anxiety students were:

1. Exhibiting mathematics anxiety symptoms may not completely hinder student performance. (High and Low Mathematics Anxiety Students)
2. Drawing pictures or diagrams maybe important for high mathematics anxiety students when solving problems, but not necessarily meaningful. (High and Low Mathematics Anxiety Students)
3. Mathematics anxiety may lead to an inability to recall previously studied material which hinders student performance. (High and Low Mathematics Anxiety Students)
4. Failure to use all of the thinking processes described by Montague (2003) does not hinder the student’s ability to successfully solve mathematical problems. (High and Low Mathematics Anxiety Students)
5. Difficulty with reading skills may hinder the problem solving process. (High Mathematics Anxiety Students)
6. Students must be able to check their calculations and check for the use of correct procedures. (High Mathematics Anxiety Students)

7. Mathematical anxiety symptoms can be visibly present even if a student does not notice the symptoms himself. (Low Mathematics Anxiety Students)

Again, I concluded that these categories were significant because they appeared in at least two problem solutions or they were apparent in the survey the students completed.

**Implications**

Students’ problem solving strategies and thinking processes in this study offer suggestions for the improvement of mathematics instruction for all students but specifically African American community college students. The results of this study has implications to address changes in curriculum development, teacher preparation and teacher evaluation.

**Exhibiting Mathematics Anxiety Symptoms:**

Throughout the study, I observed all of the students exhibiting some type of mathematics anxiety. Even though the students did exhibit mathematics anxiety behavior, it did not hinder their performance on all of the problems. Sharon exhibited mathematics anxiety in several problems, but she successful answered two problems. Hank also exhibited mathematics anxiety behavior, but he also successful answered several problems. Norwood (1994) found that mathematics anxiety affected community college students’ ability to successfully solve mathematical problems. In this study, mathematics anxiety did not always affect the students’ ability to successfully solve mathematical problems. Some of the students were able to overcome their mathematics anxiety and successfully solve the mathematical problems. This contradiction deserves further investigation.

An implication for exhibiting mathematics anxiety symptoms is to teach students how to overcome mathematics anxiety. Students need to have a positive attitude. This positive attitude comes with quality teaching for understanding and confident teachers. Students should also be encouraged to ask questions until the students have determined that they understand the mathematics. It is also important to encourage students to practice mathematics regularly and to be persistent. Teach students to consider multiple alternative solution methods when frustration arises. Based on the analysis of the results, the African American community college students were willing to consider multiple alternative solution methods. Some of the students tried to understand and work the problem using a variety or approaches. For example, the Cascades
The State Park problem could have been solved using the guess and check method. The students chose to solve the problem using drawing diagrams and using algebraic equations. None of the students used the guess and check method.

**Drawing Pictures or Diagrams:**

Hembree (1992) found that the use of diagrams contributed to success in problem solving. The high mathematics anxiety students relied upon their drawings to clarify the problems, even though they were not very successful in their problem solving. One of the low mathematics anxiety students relied upon his drawings in only one problem which was the only problem that had drawing a diagram as a dominant strategy. He was not successful with that the problem. The other low mathematics anxiety student relied upon his drawings on two problems and solved both of those problems successfully. Some of the students stated that they were taught to always draw a diagram or picture in order to visualize the problem. The high mathematics anxiety students appeared confused and frustrated on the problems where they could not draw a diagram.

An implication for drawing pictures or diagrams is to teach students the value of using diagrams as a problem solving strategy. It is also important to teach students that a mathematical problem can be solved using different methods. There are several different approaches to mathematical problems, and students have to be encouraged to evaluate the quality of the approaches. Students with high mathematics anxiety should practice solving the same problem in many different ways. The high mathematics anxiety students admitted in the follow up interview that their teachers had taught them to always draw a picture or diagram. Teachers should also show all students multiple approaches to solving a problem instead of just showing them one approach.

**Inability to Recall Previously Studied Material:**

One factor that was seen in both the high and low mathematics anxiety students was the inability to recall material studied. The students admitted that they could not recall material studied in prior mathematics courses. The inability to recall material studied is a symptom of mathematics anxiety. Ashcraft and Kirk (2001) suggested that increased mathematics anxiety is associated with decreased working memory. In a study conducted by Ashcraft and Kirk, they found that students had less working memory space to effectively deal with math problems because their math anxiety was using working memory space that could be used to solve math
problems. In my study of the African American community college students, several of the concepts that the students struggled with were concepts that were covered in their mathematical courses during their matriculation at the community college. During follow up interviews, the students admitted that they could not remember certain material, but they knew that they had previously studied the material. There were also occasions where students admitted that they could not recall previously studied material, but they were successful with their problem solution. This fact shows that there maybe a contradiction to mathematics anxiety contributing to the inability to recall information hindering student performance in mathematics problem solving that may lead to a future study.

An implication for mathematics anxiety contributing to students’ inability to recall previously studied material is to teach techniques for coping with mathematic anxiety. Teachers should provide possible relaxation techniques for all students regardless of whether they test with high mathematics anxiety or admit to having mathematics anxiety. The strategies could be listed in the syllabus that the teacher provides to the students. The teacher can also refer to that list of strategies when presenting new material. Teachers can also incorporate previously studied concepts with new concepts to make sure that students actually learned the material and not just memorized the information for the test. Students should be encouraged to continue to review old concepts and practice using those concepts.

Failure to Use All of Montague’s Thinking Processes:

Cognitive processes are integral to successful problem solving. They help to increase a student’s opportunity for success in problem solving, but it does not guarantee that a student will always be successful with problem solving. The cognitive process is unfamiliar to students, but they do implement some of process during problem solving. I observed several occasions where the students in this study did not use all seven of Montague’s thinking processes, and their successfulness was not hindered. I also observed that the high mathematics anxiety students used a combination of three of Montague’s seven thinking processes in most of their problem solution. The high mathematics anxiety students used observing relationships, formulating a plan, and regulating the solution path in all of their problem solutions except for Problem 4. The low mathematics anxiety students used formulating a plan and regulating the solution path in all of their problem solutions except for Problem 4. All of the students used formulating a plan and regulating the solution path.
An implication for the failure to use all of Montague’s thinking processes is to recognize that students process information in several different ways. All of the students formulated a plan to solve and all of the students regulated the solution path regardless of their level of mathematics anxiety. It is important that mathematics educators strive to understand their students’ thinking processes in order to provide an avenue for students to become better problem solvers. This can be accomplished by administering a cognitive process diagnostic exam in order to determine how students process information. It is also important to teach students how to “think” and how to “process information”. This can be accomplished by modeling the thinking process in class so that students can become accustomed to the process.

Difficulty with Reading Skills:

Difficulty with reading skills was a factor that inhibited the high mathematics anxiety students from being successful with the problem solving task. Even though the students reread the problems, some of them were unable to understand the problem. Students should be taught reading comprehension. Kate, a high mathematics anxiety student, completed a remedial reading course upon entry to the community college due to her reading comprehension skills displayed on the college placement test (CPT). She passed the remedial reading course, but it was very apparent during her task based interviews that she still had problems with reading and understanding. Kate did not answer any of the problems successfully. Hank, a low mathematics anxiety student, also completed a remedial reading course upon entry to the community college due to his reading comprehension skills displayed on the CPT. He did not openly display reading problems. He answered 4 of the 6 problems successfully. It appears that even though Kate passed the remedial reading course, she continues to have difficulty the reading skills. This raises the question of the how the student passed the course and still had difficulty with reading skills. Was the student given a grade instead of earning it? This requires further investigation of the remedial reading course.

An implication for students’ inability to comprehend the problem solving task includes incorporating reading comprehension strategies into mathematics courses. Teachers should spend a few minutes when introducing a new concept to review important reading comprehension skills. I think that reading comprehension would have made a difference in the way the students performed. Teachers should also require their students to read aloud in their mathematics course in order to detect any reading difficulties. Students should be encouraged to
use their experiences for understanding problems. It has been suggested by Willis (1992) that African-American students’ learning includes a harmonious relationship between learning and living, where knowledge is sought for practical, utilitarian, and relevant purposes. Some of the students attempted to use contextual relationships from their experiences to help understand problems. For example, Kate used her experience with opening a savings account to understand the Dad’s Wallet problem. She made several statements about how much money she needed to open her savings account in an attempt to answer the problem successfully. It may be important that teachers stress to African Americans the importance of using their experiences to understand mathematical problems. Teachers can accomplish this by using more real world examples in class. Use examples that incorporate students in the course which may help students to make the connection to their experiences.

Checking Calculations and Solution Process:

Checking the problem entails verifying both the process as well as the product. I observed that detecting and correcting errors was not common with the high mathematics anxiety students when compared with the low mathematics anxiety students. There were also instances where the low mathematics anxiety students did not check their answers. I also observed that even though the students checked their calculations, they did not check their problem solving process.

An implication is that teachers should ask students more questions about the reason their answer is correct. Teachers could ask students to explain why their result answers the problem they have solved. This would force the student to reflect on what they have done. Teachers should also model the checking the process. If students see the teacher checking their work when completing examples in class, then the students will become accustomed to checking their work.

Visible Mathematics Anxiety Symptoms:

Mathematics anxiety symptoms can be visibly present even if a student does not notice the symptoms himself. There were instances where the low mathematics anxiety students stated on their survey that they had no signs of mathematics anxiety symptoms, but the students did exhibit some symptoms. The symptoms were not fatal enough to cause a disruption in the students’ success. However, the symptoms were very distracting while observing the student.
An implication for visible or even not so visible mathematics anxiety symptoms is to help all students overcome mathematics anxiety. It is important to encourage students to look upon mathematics in a positive light. Humor may be a way to establish positivism in mathematics. Cartoons may be used to introduce new concepts for class discussion. Manipulatives may also be used to teach a concept instead of using lecture and textbooks. Teachers should engage students in exploring and thinking rather than rote learning of rules and procedures.

**Limitations**

My comments come from the study that was completed and my experience as an educator. All students are different and have various learning styles and the comments expressed in this study should be adapted to fit the students of the person interpreting it. There are limiting factors which should be mentioned before concluding. This study has limited generalizability to the population of African American students. The population of interest for this study was students who have an interest in pursuing a degree in Science, Technology, Engineering, or Mathematics. The students who participated in this study were from a select group of students. Another limiting factor was the researcher bias. In an attempt to limit my biases, I had colleagues read the analyses to ensure fidelity.

**Considerations for Future Research**

From this research new ideas for future studies have emerged:

1. The study presented here only used two students representing low and high mathematics anxiety; a future study would use more students. The students used would focus on various types of students in the community college instead of the African Americans with interests in Science, Technology, Engineering, and Mathematics.

2. The study presented here focused on the students’ problem solving strategies and thinking processes; a future study could focus on the factors that encourage African American community college students to demonstrate the use of various representations in solving mathematical problems. What representations are used in solving mathematical problems and what influences their decision when choosing various representations?
3. Does mathematics anxiety contribute to the inability to recall information and does it hinder student performance in mathematics problem solving? What types of students are affected by this inability to recall information influenced by mathematics anxiety?

4. An investigation of incorporating reading comprehension skills into the mathematics curriculum. What type of reading skills should be taught in the mathematics curriculum? Will this incorporation increase student success in mathematical problem solving?

5. Does exhibiting some type of mathematics anxiety hinder all students’ performances on mathematical problem solving?

There are many studies that have emerged in addition to the ones listed above. This study did not identify if the key variable was mathematics anxiety or African Americans and their lack of strategies. A future study could investigate African Americans without testing their level of mathematics anxiety in order to focus on African Americans and their strategies. A future study could also become more generalizable by investigating more ethnic groups. For example, a future study could investigate Hispanics, Whites, and African Americans and compare their strategies and thinking processes.

Another future study could incorporate more information about the K-12 schools which were attended by the participants. This study did not focus on the grade of the schools in which the students attended. A future could investigate the schools in order to determine if the students’ lack of strategies evolved while in the K-12 program. A future study could also focus on the reading problems in connection to the K-12 programs. Does the reading problem begin in the K-12 program? Could the reading problem be corrected throughout the K-12 program instead of dealing with the problem in college?

This study did not address the issues involved in gender and mathematics. A future study could address the gender issues in mathematical problem solving. The results of this study revealed that the females had high mathematics anxiety and the males had low mathematics anxiety. Is mathematics anxiety related to gender?

Many researchers have described problem solving strategies and thinking processes as mentioned in the literature. This study intended to provide insight to the strategies and thinking processes used by African American community college students with varying levels of
mathematics anxiety. The strategies and thinking processes have been outlined above. Because not much research has examined the problem solving strategies and thinking processes of African American community college students, this research serves as evidence to motivate educators in creating enrichment programs or learning materials to enhance the mathematical skills of African American community college students.
APPENDIX A

MATHMATICS ANXIETY RATING SCALE: SHORT VERSION

The items in the questionnaire refer to things that may cause fear or apprehension. For each item, place a check in the box under the column that describes how much you are frightened by it nowadays. Work quickly but be sure to consider each item individually.

<table>
<thead>
<tr>
<th></th>
<th>Not at all</th>
<th>A little</th>
<th>A fair amount</th>
<th>Much</th>
<th>Very much</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Taking an examination (final) in a math course.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>2. Thinking about an upcoming math test one week before.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>3. Thinking about an upcoming math test one day before.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>4. Thinking about an upcoming math test one hour before.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>5. Thinking about an upcoming math test five minutes before.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>6. Waiting to get a math test returned in which you expected to do well.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>7. Receiving your final math grade in the mail.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>8. Realizing that you have to take a certain number of math classes to fulfill the requirements in your major.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>9. Being given a “pop” quiz in a math class.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>10. Studying for a math test.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>11. Taking the math section of a college entrance exam.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>12. Taking an examination (quiz) in a math course.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>13. Picking up the math text book to begin working on a homework assignment.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>14. Being given a homework assignment of many difficult problems which is due the next class meeting.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>15. Getting ready to study for a math test.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Not at all</td>
<td>A little</td>
<td>A fair amount</td>
<td>Much</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>------------</td>
<td>---------</td>
<td>---------------</td>
<td>------</td>
</tr>
<tr>
<td>16.</td>
<td>Dividing a five digit number by a two digit number in private with pencil and paper.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.</td>
<td>Adding up 976 + 777 on paper.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18.</td>
<td>Reading a cash register receipt after your purchase.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19.</td>
<td>Figuring the sales tax on a purchase that costs more than $1.00.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20.</td>
<td>Figuring out your monthly budget.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21.</td>
<td>Being given a set of numerical problems involving addition to solve on paper.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22.</td>
<td>Having someone watch you as you total up a column of figures.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23.</td>
<td>Totaling up a dinner bill that you think overcharged you.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24.</td>
<td>Being responsible for collecting dues for an organization and keeping track of the amount.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25.</td>
<td>Studying for a driver’s license test and memorizing the figures involved, such as the distances it takes to stop a car going at different speeds.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26.</td>
<td>Totaling up the dues received and the expenses of a club you belong to.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27.</td>
<td>Watching someone work with a calculator.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>28.</td>
<td>Being given a set of division problems to solve.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>29.</td>
<td>Being given a set of subtraction problems to solve.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30.</td>
<td>Being given a set of multiplication problems to solve.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX B

TASK-BASED INTERVIEW QUESTIONS

1. The purpose of this section is to see how you solve some problems. Please work as hard as you can to find the answer to the question, but I am more interested in how you solve the problem than if you can find the correct answer. I want you to tell me what you are thinking as you solve the problems. I will not help you with the problems. After you complete the problem, I will ask you some questions about you did. Do you have any questions?

2. Here is a pencil and calculator and you can use if you want. Write anything on the paper to help you solve the problem. Do not erase anything you write down, just draw a line through anything you feel is not correct. Read the problem aloud…Tell me what you are thinking as you solve it.

3. If the student becomes silent while working the problem, say: Tell me what you are thinking.

Penny’s Dimes
Nick’s daughter Penny has 25 dimes. She likes to arrange them into three piles, putting an odd number of dimes into each pile. In how many ways could she do this?

Night of the Howling Dogs
Shawna liked to jog in the late afternoon. One day she noticed an unusual phenomenon. As she jogged, dogs would hear her and bark. After the first dog barked for about 15 seconds, two other dogs would join in and bark. In about another 15 seconds, it seemed that each barking dog would “inspire” two more dogs to start barking. Of course, long after Shawna passed the first dog, it continued to bark, as dogs are inclined to do. After about 3 minutes, how many dogs were barking?

Cascades State Park
Emi and Margit had stopped at the bottom of one of the highest waterfalls in Cascades State Park. As Emi looked up at the waterfall, she said, “Wow, I think the top of that fall is about 20 feet more than 3 times the height of that young redwood?” Margit, of course, had a different opinion. She said, “No, I think its about 50 feet less than four times the height of the redwood.” If both are approximately right, about how tall is the redwood and how high is the waterfall.
**Divisors and Reciprocals**
The divisors of 360 add up to 1170. What is the sum of the reciprocals of the divisors of 360?

**Dad’s Wallet**
Dad went to the ATM on Wednesday of spring break and withdrew some money. On Thursday morning my brother borrowed half of Dad’s money to open a checking account, because he was always short of money. On Friday I needed some money for a date, so I borrowed half of what remained. My sister came along next and borrowed half of the remaining money. Dad then went to gas up the car and used half of the rest of his money, and he wondered why he had only $15 left. How much money did he start with in his wallet?

**The Pool Deck**
Curly used a shovel to dig his own swimming pool. He figured he needed a pool because digging it was hard work and he could use it to cool off after working on it all day. He also planned to build a rectangular concrete deck around the pool that would be 6 feet wide at all points. The pool is rectangular and measures 14 feet by 40 feet. What is the area of the deck?

Adapted from Johnson, Kerr, Kysh (2003)
## APPENDIX C

### CODING SHEET FOR STRATEGIES

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Draw a picture</td>
<td></td>
</tr>
<tr>
<td>Looking for patterns</td>
<td></td>
</tr>
<tr>
<td>Make a list, table or chart</td>
<td></td>
</tr>
<tr>
<td>Guess and check</td>
<td></td>
</tr>
<tr>
<td>Work backwards</td>
<td></td>
</tr>
<tr>
<td>Logical deduction</td>
<td></td>
</tr>
<tr>
<td>Disregard unnecessary information</td>
<td></td>
</tr>
<tr>
<td>Solve a simpler problem</td>
<td></td>
</tr>
<tr>
<td>Read the problem carefully</td>
<td></td>
</tr>
<tr>
<td>No strategy</td>
<td></td>
</tr>
<tr>
<td>Understanding</td>
<td></td>
</tr>
<tr>
<td>Looking Back (Reflecting)</td>
<td></td>
</tr>
<tr>
<td>Implementing</td>
<td></td>
</tr>
<tr>
<td>Planning</td>
<td></td>
</tr>
</tbody>
</table>

Adapted from Watson (1980) and FCAT
## APPENDIX D

### MATH ANXIETY SURVEY

Circle the appropriate response to demonstrate your feelings.

<table>
<thead>
<tr>
<th>Symptom</th>
<th>YES</th>
<th>NO</th>
<th>SOMEWHAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Queasy stomach (butterflies)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clammy hands and feet</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Increased or irregular heartbeat</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Muscle tension, clenched fists</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tight shoulders</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Feeling faint, shortness of breath</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Headache</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shakiness</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dry mouth</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cold sweat, excessive perspiration</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Negative self-talk</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panic or fear</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worry and apprehension</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Desire to flee the situation or avoid it altogether</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A feeling of helplessness or inability to cope</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mental disorganization, incoherent thinking</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Feelings of failure or worthlessness</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Extreme tension and nervousness</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inability to recall material studied</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Adapted from Haralson (2003)
APPENDIX E

INTERVIEW QUESTIONS

1. What were you thinking when you first read the problem?

2. Was there anything that you did not understand about the problem? And/or Did you understand the problem right away?

3. Have you ever solved other problems like this one?

4. Could you have found the answer to the problem another way?

5. How did you decide to solve the problem the way that you did?

6. If the student draws a picture or diagram, ask, can you show it to me and tell me about it?

7. Did you check your answer? If yes, How did you do that? If no, Why not?

8. How do you know that your answer is correct?

9. What more would you like to know about this problem?

10. Is there anything else you would like to add?
REFERENCES


Flavell, J.H. (1979). Metacongition and cognitive monitoring: A new area of cognitive-


BIOGRAPHICAL SKETCH

Calandra Moorman Walker graduated from Paine College in Augusta, GA with a BS in Applied Mathematics. She graduated from the University of Central Florida with a MS in Applied Mathematics. She completed her PhD. in Mathematics Education in Fall 2007. Her research interest include mathematics problem solving with African American community college students.