2003

Role of Diabatic Potential Vorticity during Hurricane Genesis

Leela Ramaswamy
THE FLORIDA STATE UNIVERSITY
COLLEGE OF ARTS & SCIENCES

ROLE OF DIABATIC POTENTIAL VORTICITY DURING
HURRICANE GENESIS

By
LEELA RAMASWAMY

A Thesis submitted to the
Department of Meteorology
in partial fulfillment of the
requirements for the degree of
Master of Science

Degree Awarded:
Fall Semester, 2003
The members of the Committee approve the thesis of Leela Ramaswamy defended on September 25, 2003.

T.N. Krishnamurti  
Professor Directing Thesis

Albert I. Barcilon  
Committee Member

Paul H. Ruscher  
Committee Member

The Office of Graduate Studies has verified and approved the above named committee members.
ACKNOWLEDGEMENTS

I would like to thank the people who have supported me during my studies including my family, friends, and colleagues. I wish to especially thank Dr. T. N. Krishnamurti for all his support and vast amount of help throughout my research. I would also like to thank my committee members, Dr. Albert Barcilon and Dr. Paul Ruscher, for their comments and suggestions. Finally, I would like to acknowledge those people, come and gone, from the Krishnamurti lab who have assisted me during my time here, especially Dr. Vijay Kumar, Mr. J. Sanjay, and Dr. Arun Chakraborty.
# TABLE OF CONTENTS

LIST OF TABLES ....................................................................................................................... vi

LIST OF FIGURES ..................................................................................................................... vii

ABSTRACT.................................................................................................................................. xi

1. INTRODUCTION ................................................................................................................... 1
   1.1 Storm History ............................................................................................................ 1
   1.2 Historical on Hurricane Formation ........................................................................ 2
   1.3 Objectives and Organization of Thesis ..................................................................... 3

2. POTENTIAL VORTICITY AS A FRAMEWORK ................................................................. 5
   2.1 Overview .................................................................................................................... 5
       2.1.1 Distribution of PV ....................................................................................... 5
       2.1.2 Upper-level PV Anomalies ......................................................................... 7
       2.1.3 Surface Potential Temperature Anomalies ................................................. 8
   2.2 Conservation of Potential Vorticity ........................................................................... 8
       2.2.1 Application to Forecasting .......................................................................... 9
   2.3 Diabatic PV ................................................................................................................ 10
   2.4 Proposed diagnosis ..................................................................................................... 11

3. INFLECTION POINT INSTABILITY ................................................................................... 13
   3.1 Overview .................................................................................................................... 13
   3.2 Rayleigh’s Inflection Point Theorem ........................................................................ 13
   3.3 Combined Barotropic/Baroclinic Instability .............................................................. 14

4. BAROTROPIC EXPERIMENT .............................................................................................. 17
   4.1 Overview of Barotropic Dynamics ........................................................................... 17
   4.2 Barotropic Model ....................................................................................................... 17
   4.3 Results ........................................................................................................................ 18

5. FSU GLOBAL SPECTRAL MODEL ...................................................................................... 21
   5.1 Overview of FSUGSM .............................................................................................. 21
   5.2 Computation of Potential Vorticity and Associated Terms ....................................... 23
## 6. RESULTS FOR POTENTIAL VORTICITY BUDGET

6.1 Development of the Storm

6.1.1 Pressure and Wind Fields

6.1.2 Potential Vorticity Field

6.2 Relative Importance of Each Term During the Formation Stage

6.2.1 Evolution of Terms on Constant Sigma Surface

6.2.2 Vertical Cross-Sections

6.3 Relating Formation to the Diabatic PV

6.3.1 Inner Storm Area Histograms

6.3.2 Outer Storm Area Histograms

6.3.3 Maximum Values

## 7. THREE DIMENSIONAL TRAJECTORIES

7.1 Overview

7.2 Calculating the trajectories

7.3 Parcel trajectories

7.3.1 Trajectory 1

7.3.2 Trajectory 2

7.3.3 Trajectory 3

## 8. SUMMARY AND CONCLUSIONS

8.1 Discussion and Summary

8.2 Future Work

REFERENCES

BIOGRAPHICAL SKETCH
3.1 Calculated terms in the horizontal gradient of mean potential vorticity equation (Eq. 3.6). Terms were evaluated at 850mb between 40°-75°W. Units of column 2 are ms$^{-1}$ and columns 3-6 are m$^2$s$^{-1}$.........................................................15

7.1 Values of 3-D PV, horizontal advection, first diabatic term, second diabatic term, and third diabatic term for trajectory 1 during the 72-hour forecast. Unit of PV in kg$^{-1}$m$^2$s K. Units for components of PV tendency in kg$^{-1}$m$^2$s$^{-2}$K............................65

7.2 Values of 3-D PV, horizontal advection, first diabatic term, second diabatic term, and third diabatic term for trajectory 2 during the 72-hour forecast. Unit of PV in kg$^{-1}$m$^2$s K. Units for components of PV tendency in kg$^{-1}$m$^2$s$^{-2}$K............................71

7.3 Values of 3-D PV, horizontal advection, first diabatic term, second diabatic term, and third diabatic term for trajectory 3 during the 72-hour forecast. Unit of PV in kg$^{-1}$m$^2$s K. Units for components of PV tendency in kg$^{-1}$m$^2$s$^{-2}$K............................77
LIST OF FIGURES

2.1 Vertical cross section of potential temperature (solid contours, units: °K) and potential vorticity (dashed contours, units: PVU) averaged over Jan. 1979-1989. Contour interval of potential temperature is 10°K and PV is 0.5PVU. Summer months do not differ significantly and have same general structure. Figure is taken from http://www.boi.noaa.gov/training/ipv/ipv2.html ................................6

2.2 Idealized structure of a) a positive (trough) PV anomaly and b) a negative (ridge) PV anomaly. Figure is taken from http://www.boi.noaa.gov/training/ipv/ipv2.html......................................................................7

3.1 Barotropic, baroclinic, and combined instability terms averaged over each latitude (40°-75°W) at 850mb. Units: m⁻¹s⁻¹ ..............................................................................................16

4.1 72-hour barotropic forecast streamfunctions (contour interval of 2E+06 m²s⁻¹) at 850mb for a) September 6, 2001 12Z, b) September 7, 2001 12Z, c) September 8, 2001 12Z, and d) September 9, 2001 12Z........................................................................20

6.1 Surface pressure for Hurricane Erin on the s=0.85 surface for a) 7 September 00Z (interval of 1mb), b) 7 September 12Z (interval of 2mb), c) 8 September 00Z (interval of 2mb), d) 8 September 12Z (interval of 2mb), e) 9 September 00Z (interval of 4mb), and f) 9 September 12Z (interval of 4mb). Units: mb..........................28

6.2 Isotachs for Hurricane Erin on the s=0.85 surface for a) 7 September 00Z (interval of 1 ms⁻¹), b) 7 September 12Z (interval of 2 ms⁻¹), c) 8 September 00Z (interval of 2 ms⁻¹), d) 8 September 12Z (interval of 3 ms⁻¹), e) 9 September 00Z (interval of 5 ms⁻¹), and f) 9 September 12Z (interval of 5 ms⁻¹). Units: ms⁻¹ ..........................................................29

6.3 Potential vorticity on the s=0.85 surface for a) 7 September 00Z (interval of 0.5 m²s⁻¹Kkg⁻¹), b) 7 September 12Z (interval of 1E-07 m²s⁻¹Kkg⁻¹), c) 8 September 00Z (interval of 3E-07 m²s⁻¹Kkg⁻¹), d) 8 September 12Z (interval of 3E-07 m²s⁻¹Kkg⁻¹), e) 9 September 00Z (interval of 5E-07 m²s⁻¹Kkg⁻¹), and f) 9 September 12Z (interval of 5E-07 m²s⁻¹Kkg⁻¹). Units: m²s⁻¹Kkg⁻¹...........................................31

6.4a Contour plots of horizontal advection (interval of 3E-12 kg⁻¹m²s⁻²K), first diabatic term (interval of 1E-12 kg⁻¹m²s⁻²K), second diabatic term (interval of 2E-12 kg⁻¹m²s⁻²K), third diabatic term (interval of 5E-13 kg⁻¹m²s⁻²K), and
friction (interval of 5E-15 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K) for 7 September 00Z. Units: kg\(^{-1}\)m\(^2\)s\(^{-2}\)K

6.4b Contour plots of horizontal advection (interval of 1E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), first diabatic term (interval of 5E-12 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), second diabatic term (interval of 1E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), third diabatic term (interval of 5E-13 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), and friction (interval of 2E-14 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K) for 7 September 12Z. Units: kg\(^{-1}\)m\(^2\)s\(^{-2}\)K

6.4c Contour plots of horizontal advection (interval of 1E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), first diabatic term (interval of 5E-12 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), second diabatic term (interval of 1E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), third diabatic term (interval of 1E-12 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), and friction (interval of 1E-13 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K) for 8 September 00Z. Units: kg\(^{-1}\)m\(^2\)s\(^{-2}\)K

6.4d Contour plots of horizontal advection (interval of 5E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), first diabatic term (interval of 2E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), second diabatic term (interval of 2E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), third diabatic term (interval of 5E-12 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), and friction (interval of 2E-13 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K) for 8 September 12Z. Units: kg\(^{-1}\)m\(^2\)s\(^{-2}\)K

6.4e Contour plots of horizontal advection (interval of 5E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), first diabatic term (interval of 2E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), second diabatic term (interval of 5E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), third diabatic term (interval of 3E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), and friction (interval of 1E-12 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K) for 9 September 00Z. Units: kg\(^{-1}\)m\(^2\)s\(^{-2}\)K

6.4f Contour plots of horizontal advection (interval of 1E-10 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), first diabatic term (interval of 2E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), second diabatic term (interval of 5E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), third diabatic term (interval of 2E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), and friction (interval of 2E-12 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K) for 9 September 12Z. Units: kg\(^{-1}\)m\(^2\)s\(^{-2}\)K

6.5 Potential vorticity cross sections for a) 7 September 00Z (interval of 1E-07 m\(^2\)s\(^{-1}\)Kkg\(^{-1}\)), b) 7 September 12Z (interval of 2E-07 m\(^2\)s\(^{-1}\)Kkg\(^{-1}\)), c) 8 September 00Z (interval of 3E-07 m\(^2\)s\(^{-1}\)Kkg\(^{-1}\)), d) 8 September 12Z (interval of 5E-07 m\(^2\)s\(^{-1}\)Kkg\(^{-1}\)), e) 9 September 00Z (interval of 5E-07 m\(^2\)s\(^{-1}\)Kkg\(^{-1}\)), and f) 9 September 12Z (interval of 5E-07 m\(^2\)s\(^{-1}\)Kkg\(^{-1}\)). Units: m\(^2\)s\(^{-1}\)Kkg\(^{-1}\)

6.6. Theta cross sections for a) 7 September 00Z (interval of 5K), b) 7 September 12Z (interval of 5K), c) 8 September 00Z (interval of 5K), d) 8 September 12Z (interval of 5K), e) 9 September 00Z (interval of 5K), and f) 9 September 12Z (interval of 5K). Units: K
6.7a Contour plots of horizontal advection (interval of 5E-12 kg m^{-2} s^{-2} K), first diabatic term (interval of 3E-12 kg m^{-2} s^{-2} K), second diabatic term (interval of 5E-12 kg m^{-2} s^{-2} K), third diabatic term (interval of 2E-12 kg m^{-2} s^{-2} K), and friction (interval of 5E-13 kg m^{-2} s^{-2} K) for 7 September 00Z. Units: kg m^{-2} s^{-2} K ..................................................44

6.7b Contour plots of horizontal advection (interval of 1E-11 kg m^{-2} s^{-2} K), first diabatic term (interval of 1E-11 kg m^{-2} s^{-2} K), second diabatic term (interval of 1E-11 kg m^{-2} s^{-2} K), third diabatic term (interval of 2E-12 kg m^{-2} s^{-2} K), and friction (interval of 2E-12 kg m^{-2} s^{-2} K) for 7 September 12Z. Units: kg m^{-2} s^{-2} K ...........................................................................................................................................45

6.7c Contour plots of horizontal advection (interval of 1E-11 kg m^{-2} s^{-2} K), first diabatic term (interval of 3E-11 kg m^{-2} s^{-2} K), second diabatic term (interval of 3E-11 kg m^{-2} s^{-2} K), third diabatic term (interval of 1E-11 kg m^{-2} s^{-2} K), and friction (interval of 5E-12 kg m^{-2} s^{-2} K) for 8 September 00Z. Units: kg m^{-2} s^{-2} K ...........................................................................................................................................46

6.7d Contour plots of horizontal advection (interval of 3E-11 kg m^{-2} s^{-2} K), first diabatic term (interval of 5E-11 kg m^{-2} s^{-2} K), second diabatic term (interval of 5E-11 kg m^{-2} s^{-2} K), third diabatic term (interval of 2E-11 kg m^{-2} s^{-2} K), and friction (interval of 2E-11 kg m^{-2} s^{-2} K) for 8 September 12Z. Units: kg m^{-2} s^{-2} K ...........................................................................................................................................47

6.7e Contour plots of horizontal advection (interval of 5E-11 kg m^{-2} s^{-2} K), first diabatic term (interval of 1E-10 kg m^{-2} s^{-2} K), second diabatic term (interval of 1E-10 kg m^{-2} s^{-2} K), third diabatic term (interval of 2E-11 kg m^{-2} s^{-2} K), and friction (interval of 3E-11 kg m^{-2} s^{-2} K) for 9 September 00Z. Units: kg m^{-2} s^{-2} K ...........................................................................................................................................48

6.7f Contour plots of horizontal advection (interval of 5E-11 kg m^{-2} s^{-2} K), first diabatic term (interval of 1E-10 kg m^{-2} s^{-2} K), second diabatic term (interval of 1E-10 kg m^{-2} s^{-2} K), third diabatic term (interval of 5E-11 kg m^{-2} s^{-2} K), and friction (interval of 5E-11 kg m^{-2} s^{-2} K) for 9 September 12Z. Units: kg m^{-2} s^{-2} K ...........................................................................................................................................49

6.8 Cross sections of diabatic heating for a) 7 September 00Z (interval of 5E-05 K), b) 7 September 12Z (interval of 2E-04 K), c) 8 September 00Z (interval of 2E-04 K), d) 8 September 12Z (interval of 5E-04 K), e) 9 September 00Z (interval of 5E-04 K), and f) 9 September 12Z (interval of 3E-04 K). Units: K .................................................................................50

6.9 Histogram plots of inner storm area variable means of a) horizontal advection of PV, b) first diabatic term, c) second diabatic term, d) third diabatic term, and e) friction over the 72-hour forecast. Units: kg m^{-2} s^{-2} K .................................................................................53
6.10 Histogram plots of inner storm area variable means of a) horizontal advection of PV, b) first diabatic term, c) second diabatic term, d) third diabatic term, and e) friction over the 72-hour forecast. Units: \( \text{kg}^{-1} \text{m}^2 \text{s}^{-2} \text{K} \)

6.11 Maximum values a) potential vorticity, b) vorticity, c) horizontal advection of PV, d) first diabatic term, e) second diabatic term, and f) third diabatic term on the 308K surface over the 72-hour forecast. Units: \( \text{kg}^{-1} \text{m}^2 \text{s}^{-2} \text{K} \)

7.1 Three-dimensional trajectories (trajectory 1: black, trajectory 2: red, trajectory 3: green) with pressure levels given every six hours during the 72-hour forecast. Solid contours are isotachs (contour interval of 5 ms\(^{-1}\)) on 9 September 12Z. Units: mb

7.2 Values of potential vorticity for 72-hour, 2-D trajectories (trajectory 1: red, trajectory 2: green, trajectory 3: blue) on the 308K surface. Units: \( \text{kg}^{-1} \text{m}^2 \text{s}^{-1} \text{K} \)

7.3 Calculated terms for trajectory 1 during the 72-hour forecast. Represented terms are a) pressure (mb), b) 3-D potential vorticity (m\(^2\)s\(^{-1}\)Kkg\(^{-1}\)), c) horizontal advection of PV (kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), and d) first diabatic term (kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), e) second diabatic term (kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), f) third diabatic term (kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), and g) wind velocity (ms\(^{-1}\))

7.4 Calculated terms for trajectory 2 during the 72-hour forecast. Represented terms are a) pressure (mb), b) 3-D potential vorticity (m\(^2\)s\(^{-1}\)Kkg\(^{-1}\)), c) horizontal advection of PV (kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), and d) first diabatic term (kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), e) second diabatic term (kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), f) third diabatic term (kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), and g) wind velocity (ms\(^{-1}\))

7.5 Calculated terms for trajectory 3 during the 72-hour forecast. Represented terms are a) pressure (mb), b) 3-D potential vorticity (m\(^2\)s\(^{-1}\)Kkg\(^{-1}\)), c) horizontal advection of PV (kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), and d) first diabatic term (kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), e) second diabatic term (kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), f) third diabatic term (kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), and g) wind velocity (ms\(^{-1}\))
ABSTRACT

This study explores potential vorticity budgets from the complete Ertel’s potential vorticity equation during hurricane genesis. The data sets for these experiments are derived from a high-resolution hurricane forecast that provided reasonable simulation on hurricane genesis. The budgets sort out the relative contribution from the horizontal advection of PV (i.e., related to the conservation of PV in isentropic coordinates), vertical advection of PV (a diabatic contribution), differential heating along the vertical, differential heating along the horizontal, and the frictional contributions. This is aimed to sort out the role of conservation versus non-conservation of PV during the formative stage of a hurricane. The main findings of this study are that conservation of PV was not adequate to explain the large increase in PV during genesis of the storm. The vertical differential of heating made by far the most significant contributions to the changes of PV during the length of the forecast. This term contributed to a net generation of PV over a region of stable air with cyclonic vorticity and increased heating with height. The term is akin to the vortex stretching term in the vorticity equation and is the diabatic stretching of the vortex tube. The effect that convergence has on vorticity in pressure coordinates is entirely analogous to the effect that the diabatic stretching term has on PV in isentropic coordinates. The vertical advection of PV and the horizontal differential of heating each made smaller contributions to changes in PV, but neither was a significant contributor as compared to the other terms. Our analysis includes parcel trajectories along which the PV components (adiabatic and diabatic) of the complete equation are calculated at intervals of every three hours. These were also cast on isentropic surfaces to assess the role of the nonlinear advection of PV and of the diabatic contributions. In the inner rain area of the hurricane a jump in the value of diabatic PV was noted (related to the vertical differential of heating) that was roughly several times larger than that of the nonlinear advection of PV (the latter relates to the conservation of PV while the former is a measure of the non-conservation).
CHAPTER 1
INTRODUCTION

1.1 Storm History

Information on the synoptic history of Hurricane Erin presented here is summarized from the National Hurricane Center\(^1\). On 30 August 2001 a tropical wave moved off the coast of western Africa showing immediate signs of formation into a tropical cyclone. For the ensuing two days organization of the system or in the deep convection was not evident. By 1 September 18Z, the National Hurricane Center issued its first report of a tropical depression with a minimum central pressure of 1006mb located at 12.5°N and 34.3°W, approximately 1100 km west-southwest of the Cape Verde Islands. At this time a closed surface circulation developed as the system became better organized. Over the next three days, the tropical depression moved west to west-northwest at approximately 7-9 ms\(^{-1}\) due to a mid-tropospheric ridge north of the system.

On 2 September 06Z the storm strengthened due to weak vertical shear and was classified as a tropical storm with maximum winds of 20 ms\(^{-1}\). The maximum winds strengthened to 25 ms\(^{-1}\) by 3 September 06Z, but soon after vertical shear linked to an upper-level low to the northwest of the system increased and, consequently, weakened the system. The storm re-strengthened to some extent on 4\(^{th}\) as the shear diminished, but soon weakened as it had failed to improve its organization. On 5 September 18Z, the storm was downgraded to a low-pressure area.

By the 6 September 12Z, the storm began to re-developed and by 18Z was again classified as a tropical depression. It reached tropical storm status on 7 September 18Z. The storm then moved to the northwest and north-northwest due to a mid-level ridge that re-

\(^{1}\) Synoptic history of Hurricane Erin is available on the National Hurricane Center website at http://www.nhc.noaa.gov/2001erin.html.
established itself after temporarily being weakened. The storm reached hurricane status on 9 September 00Z with winds of 38 ms\(^{-1}\). Maximum winds of 54 ms\(^{-1}\) were reached by 18 Z. Over the next three hours, the eye of the hurricane passed approximately 165 km to the east-northeast of Bermuda.

By 10 September 18Z the hurricane showed signs of weakening while continuing to move to the north-northwest. On 11 September 12Z, Erin turned to the east-northeast due to a weakening in the Atlantic subtropical ridge. By the 12\(^{th}\) the motion was to the east. The storm then accelerated to the northeast due to a mid to upper-level trough over Canada. Hurricane Erin was downgraded to a tropical storm on 15 September 00Z and six hours later became an extratropical storm that passed over southern Greenland on 16 September. On 17 September the storm merged with cyclonic flow over eastern Greenland.

1.2 Historical on Hurricane Formation

Tropical cyclones form from initial convective disturbances generally between 20°N and 20°S (McBride 1995). Factors affecting the formation of a hurricane include different dynamical and thermodynamical parameters. Among these large-scale conditions are warm sea surface temperatures, weak vertical shear of the horizontal winds, large values of low-level absolute vorticity, a mid-tropospheric moist layer, and the influence of closed lows and upper-level troughs on the incipient disturbance, among others (McBride 1995; Gray 1968; Emanuel 2003).

The warmest sea surface temperatures (SSTs) generally occur in late summer. It has been found that tropical cyclones only form when SSTs exceed 26°C and when a deep thermocline is present (McBride 1995). This deep thermocline is necessary to prevent storm induced mixing and upwelling from significantly cooling the surface layer (Emanuel 2003).

It has generally been accepted that strong vertical shear in the presence of the incipient disturbance prevents it from developing into a tropical storm. Strong vertical shear acts to advect the warm core away from the low-level circulation while weak vertical shear allows temperature and moisture anomalies to grow in the mid-troposphere (McBride 1995).

In the vicinity of an initial convective disturbance, low-level relative vorticity is almost two times as large as the values in a non-developing area (McBride and Zehr 1981). Therefore, McBride (1995) concludes that this is a precursor to tropical cyclone formation. It has been suggested that this spin-up of relative vorticity is due to an enhancement of the flow next to the
monsoon trough, energy dispersion of low-latitude Rossby waves, or due to regions of potential vorticity (PV) maxima interacting (McBride 1995).

Proximity of closed lows and upper-level troughs on the incipient disturbance may also influence the formation of a tropical cyclone. Frank (1982) showed that other storms may develop in the wake of a tropical cyclone as the influence of the storm can extend out to 1000-2000km. Upper-level PV anomalies may also enhance the formation of another storm (Emanuel 2003).

1.3 Objectives and Organization of Thesis

This study will examine the dynamical processes that are involved in hurricane genesis. Specifically, the role of non-conservation versus conservation of PV in hurricane formation will be explored. The FSU global spectral model will be employed to provide a reasonable simulation of hurricane genesis from which the various components of the complete PV equation will be derived. The role of these components in hurricane genesis will be analyzed. A major theme of this study is on the interpretation and importance of the diabatic stretching in Ertel’s complete PV equation for hurricane genesis. Most are familiar with the stretching term (also called the divergence term) in the vorticity equation. The effect of convergence and related stretching results in a generation of vorticity. The new theme of the present work is the term analogous to the stretching term of the vorticity equation, i.e. diabatic stretching term of the complete PV equation, which leads to the generation of PV from a similar argument. This appears to be a vital component during hurricane genesis.

The primary goals of this research are to:

• Establish if the necessary condition for barotropic and combined barotropic/baroclinic instability has been met.
• Assess the role of conservation of absolute vorticity in the genesis of Hurricane Erin through the use of a barotropic model.
• Use the data sets derived from the FSU global spectral model to sort out the relative contribution to PV changes from each component of the complete PV equation.
• Examine the PV changes following the 3-D and 2-D motion of a parcel.

Chapter 2 will outline potential vorticity as a framework for hurricane genesis and examine each of the diabatic terms from the complete PV equation to assess their expected
contribution to PV changes. The necessary condition for barotropic instability and combined barotropic/baroclinic instability in hurricane formation is explored in Chapter 3. Next, the ability of barotropic dynamics to form the storm is investigated in Chapter 4. Chapter 5 gives an overview of the FSU global spectral model and the associated computations used in the following chapters. Results from the FSU global spectral model are given in Chapter 6. Each component of the PV tendency equation is examined to assess the influence of each term on PV changes. Chapter 7 focuses on the 3-D and 2-D trajectories of a parcel of air flowing into the vicinity of the hurricane. Again, each component of the complete PV equation is examined along the trajectory to assess the role of conservation versus non-conservation of PV following the motion.
CHAPTER 2
POTENTIAL VORTICITY AS A FRAMEWORK

2.1 Overview

Ertel defines potential vorticity (PV) in adiabatic, frictionless flow as a product of absolute vorticity and static stability on a constant potential temperature surface. It is defined as:

\[ PV = \zeta_a \times \left( -g \frac{\partial \theta}{\partial p} \right) \]

where \( \zeta_a = (\zeta_\theta + f) \) is the absolute vorticity and \( -g \frac{\partial \theta}{\partial p} \) is the static stability. Potential vorticity has units of PVU (potential vorticity unit) and is derived from typical midlatitude, synoptic scale flow:

\[ PV = -gf \frac{\partial \theta}{\partial p} \]

where \( \frac{\partial \theta}{\partial p} = -\frac{10K}{100hPa} \)

\[ PV = -\left(10ms^{-2}\right)\left(10^{-4}s^{-1}\right)\left(-\frac{10K}{10kPa}\right)\left(\frac{1kPa}{10^3kgms^{-2}m^{-2}}\right) \]

\[ = 10^{-6} \text{ m}^2\text{s}^{-3} \text{Kkg}^{-1} = 1 \text{ PVU} \]  (2.1)

Tropospheric air is associated with values of PV up to 1.5 PVU (Bluestein 1993). Once the tropopause is reached the values show a near discontinuous jump due to the jump in static stability across the tropopause (Smith 1993). Large values of PV are found in the stratosphere and generally exceed 4 PVU (Smith 1993). For adiabatic, frictionless flow PV is a parcel invariant. The theme of this thesis is on the conservation versus non-conservation aspects of PV in a hurricane environment.

2.1.1 Distribution of PV

Given the vertical structure of the isentropic surfaces (Fig. 2.1), it is evident that these surfaces slope down toward the ground moving from the poles to the equator. For example, in Fig. 2.1 the 330K isentropic surface is at lower pressure levels in the tropics compared to the
poles. Simply stated, isentropic surfaces vary with latitude (Bluestein 1993). The figure also shows that while moving from the equator to the poles in the upper levels the values of PV rise quickly due to the change in static stability at the tropopause, hence, high PV air is found at lower heights at the poles. In the lower levels the PV does not change significantly from the equator to the poles. The slight increase in the lower levels is due to the increase in the Coriolis force (Billingsley 2002).

Fig. 2.1. Vertical cross section of potential temperature (solid contours, units: °K) and potential vorticity (dashed contours, units: PVU) averaged over Jan. 1979-1989. Contour interval of potential temperature is 10°K and PV is 0.5PVU. Summer months do not differ significantly and have same general structure. Figure is taken from http://www.boi.noaa.gov/training/ipv/ipv2.html.

Maxima in potential vorticity are formed from areas of this high PV, stratospheric air at the poles that extend equatorward as well as from isolated regions of stratospheric air situated equatorward (Bluestein 1993). These are associated with troughs or closed lows in the height field and cyclonic flow. Minima in potential vorticity occur from areas of low PV, tropospheric air that extend poleward as well as from isolated regions of tropospheric air that sit within the stratospheric reservoir (Bluestein 1993). These are associated with ridges in the height field and
anticyclonic flow. If friction and diabatic heating are neglected, then it has been said that PV is conserved following the motion and these isolated regions and tongues of PV evolve due to the wind field (Bluestein 1993). Therefore, areas of high potential vorticity can be considered as positive PV anomalies and areas of low potential vorticity as negative PV anomalies (Bluestein 1993).

2.1.2 Upper-level PV Anomalies

The idealized structure of positive and negative upper level PV anomalies are given below (Fig. 2.2):

![Fig. 2.2. Idealized structure of a) a positive (trough) PV anomaly and b) a negative (ridge) PV anomaly. Figure is taken from http://www.boi.noaa.gov/training/ipv/ipv2.html.](image)

The figures show statically stable stratospheric air overlaying the relatively less stable tropospheric air. Figure 2.2a shows a positive PV anomaly, which brings statically stable air down to the troposphere. This positive PV anomaly induces a cyclonic vortex and has high static stability (relative to the surrounding tropospheric air) (Bluestein 1993). Figure 2.2b shows a negative PV anomaly in which the less stable tropospheric air pushes up into the stable stratosphere and creates an anti-cyclonic vortex (Bluestein 1993). The static stability above and below the positive PV anomaly is comparatively low and the static stability above and below the negative PV anomaly is comparatively high (Bluestein 1993). This vertical structure is transported along with the PV anomaly as it moves implying that there is vertical motion (Smith 1993). For an advancing positive anomaly, any air parcel ahead of the structure will rise along the isentropic surface, while any air parcel behind it will descend along the isentropic surface.
Smith 1993). The opposite is true for a negative anomaly. It is also known that large-scale, upper-level PV anomalies induce strong wind fields that can be felt at the ground, while small-scale PV anomalies induce weak wind fields felt only in a shallow layer (Bluestein 1993).

### 2.1.3 Surface Potential Temperature Anomalies

Surface potential temperature anomalies have the same structure as those of the upper-level PV anomalies. The dipping down of potential temperature lines at the surface indicates a warm core and corresponds to high static stability and large PV values at the surface. This is equivalent to a cyclonic (or positive) PV anomaly (Hoskins et al. 1985). Likewise, potential temperature lines that bulge up at the surface indicate a cold core, which is equivalent to an anticyclonic (negative) PV anomaly. Generally, horizontal potential temperature gradients at the surface are equivalent to horizontal PV gradients at the surface (Smith 1993). The synoptic-scale, low-level PV anomalies can induce wind fields that can be felt at the troposphere, just as the upper-level PV anomalies can induce wind fields that can be felt at the ground (Bluestein 1993).

### 2.2 Conservation of Potential Vorticity

The quasi-static version of the Ertel potential vorticity equation is expressed by (Bluestein 1993):

\[
\frac{\partial}{\partial t} \left( \frac{-\zeta}{4} \frac{\partial \theta}{\partial p} \right) = -\nabla_s \cdot \nabla \left( \frac{-\zeta}{4} \frac{\partial \theta}{\partial p} \right) - \frac{\partial \theta}{\partial t} \frac{\partial}{\partial p} \left( \frac{-\zeta}{4} \frac{\partial \theta}{\partial p} \right) + \left( \frac{-\zeta}{4} \frac{\partial \theta}{\partial p} \right) \frac{\partial}{\partial t} \theta \frac{\partial \theta}{\partial p} + \left( \frac{\nabla \cdot \left( \mathbf{F} \times \hat{k} \right) \theta}{4} \right) \frac{\partial \theta}{\partial p}
\]

According to conservation of PV, all diabatic and friction terms are zero and the equation then becomes:

\[
\frac{\partial}{\partial t} PV = -\nabla_s \cdot \nabla \theta PV,
\]
i.e., the local change of potential vorticity is due simply to the horizontal advection of PV.

2.2.1 Application to Forecasting

As Bluestein (1993) notes, in order to use potential vorticity as a forecasting tool, basic ideas must be utilized. These include the idea that the atmosphere is made up of a basic state current, upper-level PV anomalies, and lower-level potential temperature anomalies, the atmosphere is in gradient wind balance and is statically stable, conservation of PV holds, and induced wind fields change the distribution of PV, which results in a new distribution of PV inducing new wind fields, and so on.

The invertibility principle for potential vorticity is what allows forecasters to use PV maps to determine the state of the atmosphere from the distribution of PV. That is, wind, pressure, and temperature fields can be deduced given specific boundary conditions, a balance condition, and by solving the problem globally (Hoskins et al. 1985). Assuming the atmosphere is in gradient wind and hydrostatic balance and the wind and temperature fields are circularly symmetric, the thermal wind relation for the gradient wind on an isentropic surface in polar coordinates can be found (Bluestein 1993). Using this thermal wind equation, the potential vorticity equation, and the relative vorticity equation, by rearrangement and substitution of terms the following equation is obtained:

\[
\frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial (rV)}{\partial r} \right) + \zeta_{\theta} \frac{1}{\sigma^* g} \frac{\partial}{\partial \theta} \left( f_{loc} \frac{\partial V}{\partial \theta} \right) + \frac{\partial f}{\partial r} = \sigma^* \frac{\partial P}{\partial r},
\]

where \( r \) is the radius, \( V \) is the wind speed, \( \zeta_{\theta} \) is the absolute vorticity, \( \sigma^* \) is a measure of static stability, \( g \) is gravity, \( \theta \) is the potential temperature, \( f_{loc} = f + \frac{2V}{r} \) is the absolute rotation rate about the local vertical, \( K(p) = \frac{\partial}{\partial p} \left[ C_p \left( \frac{p}{p_0} \right)^{\kappa_{vp}} \right] \), \( f \) is the Coriolis force, and \( P \) is PV (Bluestein 1993). This second-order, partial differential equation can be solved for \( V(r,\theta) \) given the proper boundary conditions (Bluestein 1993). Therefore, according to the principle of conservation of PV, if the PV distribution is known, the wind field can also be deduced (Bluestein 1993).

Using conservation of PV, positive (cyclonic) PV anomalies near the tropopause can be examined. If there is a low-level baroclinic region that an upper-level PV anomaly moves over, a cyclonic circulation is induced near the ground, the upper level circulation being stronger than
the low-level circulation (Smith 1993). A warm temperature anomaly then occurs ahead of the upper-level PV anomaly due to the advection by this low-level circulation (Smith 1993). The warm anomaly in turn induces its own cyclonic circulation to the east of the original circulation, which is added to the circulation induced from the upper levels (Hoskins et al. 1985). This creates a strong low-level cyclone slightly ahead of the upper-level PV anomaly (Hoskins et al. 1985). A positive feedback situation is created in which the there is mutual intensification of each anomaly (Hoskins et al. 1985). Consequently, high PV air is advected equatorward upstream of the system and low PV air is advected poleward downstream of the system, which, in turn, slows down its eastward propagation (Hoskins et al. 1985).

In his paper, Smith (1993) writes that the degree of cyclogenesis as described above depends on certain factors. These include the strength of the upper and lower-level anomalies, the Rossby depth scale as compared to the height of the anomaly, and the effect of strengthening that each anomaly has on the other.

### 2.3 Diabatic PV

As stated, PV is conserved following the 2-D trajectory of a parcel in the absence of diabatic heating and friction. Potential vorticity anomalies in the lower troposphere are generally associated with non-conservative effects such as diabatic heating due to contact with the ground or precipitation and fluxes of sensible heat (Morgan and Nielsen-Gammon 1997). In a hurricane, release of latent heat due to energetic deep convection occurs (Smith 1993). The heating produces a low-level cyclonic PV anomaly throughout the depth of the troposphere with a small outflow layer consisting of an anticyclonic anomaly existing just below the tropopause (Smith 1993). This diabatic heating is an indication that PV is not conserved. In this case, the quasi-static Ertel potential vorticity equation is used in its full form (Eq. 2.2). The diabatic terms include the vertical advection of PV, the vertical differential of heating, and the horizontal differential of heating.

Vertical advection of PV is expressed as $-\frac{d\theta}{dt} \frac{\partial}{\partial \theta} PV$ in the full PV equation. The term $\frac{d\theta}{dt}$ is the diabatic heating which contains convective heating, non-convective heating (such as anvil rain), air-sea interaction, and radiative processes. It is generally positive below
approximately 400mb. As evidenced by Fig. 2.1, PV is generally positive definite in the
Northern Hemisphere, however, $\frac{\partial}{\partial \theta} PV$ will vary depending on the location of PV maximums
and minimums. Consequently, it is expected that if $\frac{\partial}{\partial \theta} PV$ is positive then $-\frac{d\theta}{dt} \frac{\partial}{\partial \theta} PV$ will be
negative below 400mb and positive above and vice versa.

The vertical differential of heating is expressed by $PV \frac{\partial}{\partial \theta} \frac{d\theta}{dt}$ in the full PV equation.
Again, the term $\frac{d\theta}{dt}$ is positive below 400mb roughly and $\frac{\partial}{\partial \theta} \frac{d\theta}{dt}$ will vary based on the vertical
distribution of heating. Therefore, if $\frac{\partial}{\partial \theta} \frac{d\theta}{dt}$ is positive then $PV \frac{\partial}{\partial \theta} \frac{d\theta}{dt}$ is positive below
400mb as well and vice versa.

The horizontal differential of heating is expressed as $\left\{ \nabla \frac{d\theta}{dt} \cdot \frac{\partial (V \times \hat{k})}{\partial \theta} \right\} \frac{\partial \theta}{\partial p}$ in the full
PV equation. This can be written in scalar form as $\left( \frac{\partial v}{\partial \theta} \frac{\partial}{\partial x} \frac{d\theta}{dt} - \frac{\partial u}{\partial \theta} \frac{\partial}{\partial y} \frac{d\theta}{dt} \right) \frac{\partial \theta}{\partial p}$. This term is
mathematically complex and its sign varies as a function of horizontal and vertical positions.
For that reason, the influence on the change of PV will not be speculated upon. Rather, its
mathematical result will be analyzed in a later chapter. This term is analogous to the twisting
term of the x-y-p frame of reference.

Based on this reasoning, it is expected that the vertical advection of PV and the vertical
differential of PV will contribute to an increase of PV in the lower troposphere. This in turn will
generate rotation, which will then help to form a storm.

2.4 Proposed Diagnosis

Since approximately 2% of every 100km square in the tropics is occupied by deep
convective elements normally whereas in hurricanes approximately 30% is occupied, it is
suggested that non-conservation of PV was the underlying mechanism that formed Hurricane
Erin. This study will explore PV budgets from the complete Ertel’s PV equation during genesis
of Hurricane Erin. The data sets for these experiments will be derived from the FSU global
spectral model, a high-resolution hurricane forecast that provides reasonable simulation on hurricane genesis. The budgets will sort out the relative contribution to PV changes from the horizontal advection of PV, vertical advection of PV, differential heating along the vertical, differential heating along the horizontal, and the frictional contributions. This is aimed to sort out the role of conservation versus non-conservation of PV during the formative stage of a hurricane. For ease of reading, the vertical advection of PV will be referred to as the first diabatic term, the vertical differential of heating will be referred to as the second diabatic term, and the horizontal differential of heating will be referred to as the third diabatic term.
CHAPTER 3
INFLECTION POINT INSTABILITY

3.1 Overview

Barotropic and combined barotropic/baroclinic instability is examined in order to give a complete picture of the dynamics behind the formation of Hurricane Erin, i.e. if either one of these processes could have been responsible for the formation of the storm. As in the case of Hurricane Erin, African waves develop most frequently in September and October, usually due to barotropic or combined barotropic/baroclinic instability (Rennick 1976; Burpee 1972), when these instability processes are most intense (http://www.nssl.noaa.gov/users/pena/public_html/thesis/Chapter2.html). Barotropic instability occurs in areas of large horizontal curvature in the mean flow, while baroclinic instability occurs in areas of large vertical shear or curvature (Limpasuvan et al. 2000). In this chapter we ask whether Erin’s genesis into a tropical cyclone required that the necessary condition for combined barotropic/baroclinic instability to exist be explored.

3.2 Rayleigh’s Inflection Point Theorem

The existence of combined barotropic/baroclinic instability can be determined by using Rayleigh’s inflection point theorem. This theorem states that in the absence of an inflection point, a parcel remains stable when displaced in the meridional direction. However, if an inflection point is present, a parcel displaced becomes unstable (Ruby Krishnamurti, personal communication, May, 2003). The proof of this theorem begins with the potential vorticity equation:

\[
\left( \frac{\partial}{\partial t} + V_H \cdot \nabla \right) P = 0 ,
\]

(3.1)

where \( V_H \) is the horizontal wind and \( P \) is the potential vorticity defined by:
\[ P = \nabla^2 \psi + f + \frac{\partial}{\partial p} \left( \frac{f^2}{\sigma} \frac{\partial \psi}{\partial p} \right). \]  (3.2)

In this equation, \( \psi \) is the streamfunction and \( \sigma \) is the dry static stability defined by:

\[ \sigma = \frac{\partial \phi}{\partial p} \frac{\partial}{\partial p} \ln \theta, \]  (3.3)

with \( \phi \) as the geopotential and \( \theta \) as the potential temperature (Krishnamurti 1979). After a series of manipulations, we obtain the equation:

\[ c_i \int_{-d}^{d} \int_{p_0}^{p} \left( \psi \frac{\partial}{\partial y} \frac{\partial P}{\partial y} (u - c)^2 \right) dy dp = 0, \]  (3.4)

with the assumption of rigid boundaries at \( y = \pm d \) (Krishnamurti 1979). For unstable conditions \( c_i \neq 0 \), i.e.:

\[ \iint \psi \frac{\partial P}{\partial y} (u - c)^2 dy dp = 0, \]  (3.5)

(Krishnamurti 1979). This leads us to the necessary condition for combined instability:

\[ \frac{\partial P}{\partial y} = \left( \beta - \frac{\partial^2}{\partial y^2} \frac{u}{u} - \frac{\partial}{\partial p} \frac{f^2}{\sigma} \frac{\partial u}{\partial p} \right) = 0, \]  (3.6)

which is that \( P \) changes sign somewhere in the domain, where \( P \) represents the basic state potential vorticity (Krishnamurti 1979). This suggests that the meridional gradient of potential vorticity must vanish somewhere in the region of interest.

### 3.3 Combined Barotropic/Baroclinic Instability

In order to establish that the African wave that formed Hurricane Erin satisfies the necessary condition for combined instability, Eq. (3.6) must be proven true somewhere in the domain of interest. The domain of interest in this case is that of the African wave, approximately 40°-75°W and 15°-28°N. Table 3.1 shows the computed terms from Eq. (3.6). The first column simply represents the latitude at which the terms are computed. The second column indicates the average zonal wind along the latitude. The third and forth columns are the horizontal shear and the barotropic term, respectively, averaged over each latitude. The barotropic term consists of the beta term and the horizontal shear. The fifth column shows the effects of the vertical wind shear averaged over each latitude and is the baroclinic term. The last column is the sum of
columns four and five and is the meridional gradient of potential vorticity averaged over each latitude. The beta term is averaged over the whole domain and is equal to 2.08E-11 s\(^{-1}\). Each term is valid at 850mb.

Table 3.1. Calculated terms in the horizontal gradient of mean potential vorticity equation (Eq. 3.6). Terms were evaluated at 850mb between 40°W-75°W. Units of column 2 are ms\(^{-1}\) and columns 3-6 are m\(^1\)s\(^{-1}\).

<table>
<thead>
<tr>
<th>Latitude (\phi)</th>
<th>(\vec{u})</th>
<th>(-\frac{\partial^2 \vec{u}}{\partial y^2})</th>
<th>(\beta = \frac{\partial^2 \vec{u}}{\partial y^2})</th>
<th>(-\frac{\partial}{\partial p} \frac{f_0}{\sigma} \frac{\partial \vec{u}}{\partial p})</th>
<th>(\frac{\partial P}{\partial y})</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.09</td>
<td>-8.06258</td>
<td>-4.00E-11</td>
<td>-1.92E-11</td>
<td>-1.28E-12</td>
<td>-2.05E-11</td>
</tr>
<tr>
<td>15.79</td>
<td>-7.36953</td>
<td>1.65E-11</td>
<td>3.73E-11</td>
<td>-5.69E-12</td>
<td>3.16E-11</td>
</tr>
<tr>
<td>16.49</td>
<td>-6.77609</td>
<td>5.45E-11</td>
<td>7.53E-11</td>
<td>-4.99E-12</td>
<td>7.03E-11</td>
</tr>
<tr>
<td>17.89</td>
<td>-6.61954</td>
<td>6.03E-11</td>
<td>8.11E-11</td>
<td>-1.47E-11</td>
<td>6.64E-11</td>
</tr>
<tr>
<td>18.6</td>
<td>-7.10194</td>
<td>3.53E-11</td>
<td>5.61E-11</td>
<td>-2.85E-11</td>
<td>2.76E-11</td>
</tr>
<tr>
<td>19.3</td>
<td>-7.79802</td>
<td>-1.92E-11</td>
<td>1.63E-12</td>
<td>-3.76E-11</td>
<td>-3.60E-11</td>
</tr>
<tr>
<td>20</td>
<td>-8.37821</td>
<td>-6.23E-11</td>
<td>-4.15E-11</td>
<td>-3.72E-11</td>
<td>-7.86E-11</td>
</tr>
<tr>
<td>21.4</td>
<td>-8.28776</td>
<td>-7.84E-11</td>
<td>-5.76E-11</td>
<td>-3.26E-11</td>
<td>-9.02E-11</td>
</tr>
<tr>
<td>22.11</td>
<td>-7.50581</td>
<td>-5.73E-11</td>
<td>-3.65E-11</td>
<td>-2.90E-11</td>
<td>-6.55E-11</td>
</tr>
<tr>
<td>22.81</td>
<td>-6.37714</td>
<td>-2.86E-11</td>
<td>-7.84E-12</td>
<td>-2.21E-11</td>
<td>-2.99E-11</td>
</tr>
<tr>
<td>23.51</td>
<td>-5.07532</td>
<td>1.55E-12</td>
<td>2.23E-11</td>
<td>-1.43E-11</td>
<td>8.00E-12</td>
</tr>
<tr>
<td>24.91</td>
<td>-2.69516</td>
<td>4.22E-11</td>
<td>6.30E-11</td>
<td>-8.33E-12</td>
<td>5.47E-11</td>
</tr>
<tr>
<td>25.61</td>
<td>-1.86275</td>
<td>1.78E-11</td>
<td>3.86E-11</td>
<td>-7.18E-12</td>
<td>3.14E-11</td>
</tr>
<tr>
<td>26.32</td>
<td>-1.14109</td>
<td>-1.04E-11</td>
<td>1.04E-11</td>
<td>-3.79E-12</td>
<td>6.59E-12</td>
</tr>
<tr>
<td>27.02</td>
<td>-0.35642</td>
<td>-2.67E-11</td>
<td>-5.95E-12</td>
<td>-3.48E-12</td>
<td>-9.43E-12</td>
</tr>
<tr>
<td>27.72</td>
<td>0.58998</td>
<td>-2.74E-11</td>
<td>-6.57E-12</td>
<td>-9.22E-12</td>
<td>-1.58E-11</td>
</tr>
</tbody>
</table>
A change in sign of the barotropic term and in the sum of the terms as a function of latitude indicates that the necessary condition for barotropic instability and combined instability have been satisfied, respectively, near 23°N where the wave had its largest amplitude. Examining Table 3.1 and Fig. 3.1, it is apparent that the African wave does satisfy both the barotropic and combined instability conditions. However, it is also evident that the baroclinic term by itself does not change sign in the domain of interest. Fig. 3.1 indicates that the baroclinic term stays negative along each latitude. Therefore, it seems that barotropic instability and the combined instability account for the growth of this wave.

![Combined Barotropic/Baroclinic Instability](image)

**Fig. 3.1.** Barotropic, baroclinic, and combined instability terms averaged over each latitude (40°-75°W) at 850mb. Units: m s⁻¹.

In conclusion, it is clear that the criteria for the existence of both barotropic and combined instability has been met. However, the inflection point theorem simply states that the necessary condition has been met; it does not state that this condition is sufficient for the growth of the African wave. In order to completely examine the influence of each instability, further diagnostic studies must be performed.
4.1 Overview of Barotropic Dynamics

Since the necessary condition for barotropic instability to exist is satisfied it is useful to examine if barotropic dynamics alone are able to form the storm. Barotropic dynamics implies conservation of absolute vorticity. Conservation of absolute vorticity is defined by the following relation:

\[ \frac{d\zeta_a}{dt} = 0, \quad (4.1) \]
\[ \zeta_a = S + C + f, \quad (4.2) \]

where \( S \) is the shear vorticity, \( C \) is the curvature vorticity, and \( f \) is the Coriolis parameter.

The sum \( S + C + f \), following a parcel, is an invariant for barotropic dynamics. If \( S + C + f = \) constant, curvature vorticity can increase at the expense of shear vorticity and the Coriolis parameter. This is a three component nonlinear system. During the three day forecast, the African wave that formed Hurricane Erin was moving roughly due west, thus, the Coriolis parameter was not changing significantly. Consequently, the only possibility for the generation and maintenance of a storm would be a conversion from shear vorticity to curvature vorticity within barotropic dynamics. In order to answer the question of whether shear vorticity was increasing curvature vorticity, it was necessary to run a barotropic forecast.

4.2 Barotropic Model

A non-divergent barotropic model, World Meteorological Organization (1996) audited by T.N. Krishnamurti, was used to make a forecast. It has been shown that barotropic models are
useful for tropical wind prediction where the divergence is small (Krishnamurti 1996). The
barotropic model is based on one equation and one unknown, the streamfunction $\psi$:

$$\frac{\partial}{\partial t} \nabla^2 \psi = -J(\psi, \nabla^2 \psi) - \beta \frac{\partial \psi}{\partial x},$$

where $J$ is the Jacobian operator and $\beta = \frac{\partial f}{\partial y}$ is the beta parameter.

In a closed domain, the total kinetic energy, $K$, and the second power of the absolute
vorticity, $\zeta^2$, are the invariants of the barotropic model. In order to conserve these properties,
the finite difference analogs of the advective terms were computed with the 9-point Arakawa
Jacobian (Krishnamurti 1979). Time integration was performed using the Matsuno time
differencing scheme, which consists of a predictor and a corrector step. The boundary conditions
in this model are cyclic in the zonal direction throughout the integration while the southern and
northern boundary values are formed by linear extrapolation outwards (Krishnamurti 1996).

The initial values of $\psi$ were computed by solving the Poisson equation, $\nabla^2 \psi = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$,
in which Gaussian grid values of $u$ and $v$ at 850mb were converted to the spectral coefficients of
vorticity. The spectral equation relating $\psi$ to $\zeta$ was used to obtain the initial $\psi$ field, which
was then interpolated onto a latitude-longitude grid.

Before performing the integration, the model calls a subroutine to define various control
parameters. These are the base constants of the problem. Included are a total forecast length of
72 hours starting at 6 September 12Z, the time step of integration set to 3600 seconds, and a
resolution equivalent to an approximate grid size of 0.70° latitude and longitude.

### 4.3 Results

Figure 4.1 displays the streamfunction, $\psi$, at 850mb in 24-hour intervals during the three-
day forecast. It indicates that the African wave was initially located on the southern side of
closed $\psi$ isolines centered at approximately 58°W on 6 September 12Z (a). By 7 September
12Z, the wave had slightly amplified and moved roughly 6° west, as indicated by Fig. 4.1b. By 8
September 12Z the wave had moved another 3° west (c) and by 9 September 12Z it was centered
at 70°W.
The results of the model run indicate that the movement of the easterly wave from the barotropic model was reasonable as it approximately paralleled the observed movement of the tropical cyclone. However, no curvature vorticity developed as evidenced by a lack of a closed low in the vicinity of the storm as the forecast progressed. Hence, barotropic evolution was small and formation of Erin cannot be attributed to barotropic dynamics alone. The failure to form appears to be due to the lack of convection and associated divergent motions in the model (Krishnamurti 1986). We also note that, although the necessary condition for the existence of barotropic instability was satisfied for the prevailing data sets, the sufficient condition based on this forecast failed to generate a closed low system.
Fig. 4.1. 72-hour barotropic forecast streamfunctions (contour interval of $2E+06 \text{ m}^2\text{s}^{-1}$) at 850mb for a) September 6, 2001 12Z, b) September 7, 2001 12Z, c) September 8, 2001 12Z, and d) September 9, 2001 12Z.
CHAPTER 5
FSU GLOBAL SPECTRAL MODEL

5.1 Overview of FSUGSM

Since barotropic dynamics alone are unable to form the storm it is necessary to run a model that includes full dynamics and physics. The Florida State University Global Spectral Model (FSUGSM) was able to provide a reasonable simulation of hurricane genesis. The FSUGSM is intended for use in tropical numerical weather prediction and climate studies (Kumar 2000). The resolution of the model can be modified to suit individual research needs. It has been found that triangular truncation of spherical harmonics is more capable for tropical prediction in a global model as opposed to rhomboidal truncation (Krishnamurti et al. 1989). In addition, it has approximately the same resolution in the zonal and meridional directions around the globe (Peterson 2000). Hence, in this particular study triangular truncation at wave number 170, i.e. T170, is used. The T170 corresponds to a grid size of approximately 0.70 degrees of latitude near the equator. The model runs on an x, y, σ coordinate system and includes 11 irregularly spaced vertical levels for the moisture variables and 14 irregularly spaced vertical levels for the dependent variables, each vertical level existing between 10mb and 1000mb. In this coordinate system, \( \sigma = \frac{p}{p_s} \) where \( p \) is the pertinent pressure level and \( p_s \) is the surface pressure. From this equation, it is evident that \( \sigma = 0 \) at the top of the atmosphere and \( \sigma = 1 \) at the earth’s surface. The boundary conditions define the vertical velocity equal to zero at the top and bottom of the atmosphere, i.e. \( \partial \sigma = \frac{d\sigma}{dt} \), where \( \partial \) is the vertical velocity (Kumar 2000). The zonal and meridional components of the wind and the geopotential height are defined on the odd \( \sigma \) levels. Temperature and dewpoint depression are defined on the even \( \sigma \) levels. On the horizontal grid, the model consists of latitude and longitude and prognostic variables including
surface pressure, dewpoint depression, relative humidity, and a variable that combines
geopotential height and the log of surface pressure (Kumar 2000).

The FSUGSM utilizes a closed system of equations that governs the atmospheric motion.
This closed system for a global model with s as the vertical coordinate includes the vorticity
equation, divergence equation, mass continuity equation, the first law of thermodynamics,
hydrostatic approximation, and moisture conservation as defined in Kumar (2000).

The vorticity equation in sigma coordinates is defined as:

$$\frac{\partial \zeta}{\partial t} = \nabla \cdot \left( \zeta + f \right) \mathbf{V} - \hat{k} \cdot \nabla \times \left( RT \nabla q \right) + \alpha \left( \frac{\partial V}{\partial \sigma} - F \right),$$  \hspace{1cm} (6.1)

where the vertical component of relative vorticity is \( \zeta = \hat{k} \cdot \nabla \times \mathbf{V} \), \( f \) is the Coriolis force, \( \nabla \) is the two-dimensional Laplace operator, \( R \) is the gas constant of dry air, \( T \) is the temperature, \( q \) is the log of the surface pressure, \( \mathbf{V} \) is the horizontal wind vector, \( F \) is the frictional force, and \( \sigma \) is as defined above.

The divergence equation is defined as:

$$\frac{\partial \mathbf{D}}{\partial t} = \hat{k} \cdot \nabla \times \left( \zeta + f \right) \mathbf{V} - \nabla \cdot \left( RT \nabla q + \alpha \frac{\partial V}{\partial \sigma} - F \right) - \nabla^2 \left( \phi + \frac{\mathbf{V} \cdot \mathbf{V}}{2} \right),$$  \hspace{1cm} (6.2)

where \( \mathbf{D} = \nabla \cdot \mathbf{V} \) is horizontal divergence and \( \phi \) is the geopotential.

The mass continuity equation is defined as:

$$\frac{\partial q}{\partial t} = -\nabla \cdot \mathbf{V} q - \mathbf{D} - \frac{\partial \alpha \kappa}{\partial \sigma}.$$

The first law of thermodynamics is defined as:

$$\frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{V} T + T \mathbf{D} + \alpha \kappa - \frac{RT}{c_p} \left( \mathbf{D} + \frac{\partial \alpha \kappa}{\partial \sigma} \right) + H_t,$$  \hspace{1cm} (6.4)

where the static stability parameter is given by \( \Gamma = \frac{RT}{c_p \sigma} - \frac{\partial T}{\partial \sigma} \), \( H_t \) is the diabatic heating, and \( c_p \) is the specific heat of air.

The hydrostatic equation is defined as:

$$\sigma \frac{\partial \phi}{\partial \sigma} = -RT,$$  \hspace{1cm} (6.5)

where \( R \) is the universal gas constant.
The moisture conservation equation is defined as:

\[
\frac{\partial S}{\partial t} = -\nabla \cdot (VS) - SD - \frac{\partial \varepsilon}{\partial \sigma} \frac{RT}{c_p} \left[ \frac{RT}{\sigma} - \frac{RT_d^2}{L} \right] \left[ \frac{1}{D} + \frac{\partial \sigma}{\partial \sigma} - \frac{\partial \sigma}{\sigma} \right] + H_t - H_M, \quad (6.6)
\]

where \( S = T - T_d \) is the dewpoint depression, \( \varepsilon \) is the weight of water vapor to dry air, \( L \) is the latent heat of condensation, and all moisture sources and sinks are represented by \( H_M \).

To proceed with the forecast these model equations need to be transformed into spherical coordinates. The Robert wind functions are utilized to remove any discontinuities at the north and south boundaries. The variables are then transformed from real to spectral space using Fourier-Legendre expansion of the variables and the equations are rewritten in their spectral form. Once this is completed, the model can be integrated forward in time using a semi-implicit time differencing scheme where the linear terms are integrated implicitly and the nonlinear terms are integrated explicitly (Krishnamurti et al. 1998). This time differencing scheme uses an Asselin time filter, which significantly reduces any computational errors (Kumar 2000). The horizontal differential terms are solved spectrally and the vertical differential terms are solved using finite-difference analogs (Kumar 2000). Each term can then be projected back onto the space domain to obtain forecast fields.

The physical processes included in the FSUGSM are large scale condensation (Kanamitsu 1975), dry convective adjustment (Kanamitsu 1975), shallow convection (Tiedke 1984), deep moist convection based on modified Kuo scheme (Krishnamurti et al. 1983), surface fluxes based on the similarity theory, Richardson number dependent vertical distribution of fluxes, and physical initialization (Krishnamurti et al. 1991), among others. A more detailed outline of the above processes is given in Krishnamurti et al. (1989).

### 5.2 Computation of Potential Vorticity and Associated Terms

A three-day forecast for Hurricane Erin was run starting on 6 September 12Z and ending on 9 September 12Z. The system initially was a low-pressure area that reached hurricane status in the last 12 hours of the forecast. Model output, including (instantaneous) diagnostic variables, was extracted at three hourly intervals. The full Ertel potential vorticity equation, as given in Chapter 2, was broken down into its component parts and computed using model output. Each
term was computed at each of the 14 odd vertical levels as well as on the 308K isentropic surface. The following equations were utilized in the computations of each term.

- **Coriolis force:**
  \[ f = 2\Omega \sin \phi, \]
  where \( f \) is the Coriolis force, \( \Omega \) is the Earth’s rate of rotation, and \( \phi \) is the latitude.

- **Absolute vorticity:**
  \[ \zeta_a = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + \frac{u \tan \phi}{a} + f, \]
  where \( \zeta_a \) is the absolute vorticity, \( u \) is the zonal wind, \( v \) is the meridional wind, and \( a \) is the radius of the Earth.

- **Sigma coordinate conversion:**
  \[ \sigma = \frac{p}{p_s}, \]
  where \( \sigma \) and \( p \) are the relevant sigma and pressure levels, respectively, and \( p_s \) is the surface pressure.

- **Poisson’s relation:**
  \[ \theta = T \left( \frac{p_s}{p} \right) \frac{g}{R}, \]
  where \( \theta \) is the potential temperature, \( T \) is the temperature, \( R \) is the specific gas constant of dry air, and \( c_p \) is the specific heat capacity of dry air.

- **Potential vorticity:**
  \[ PV = -g \zeta_a \frac{\partial \theta}{\partial p}, \]
  where \( g \) is the gravity and \( -\frac{\partial \theta}{\partial p} \) is the stability.

All terms were computed on odd sigma levels. Variables that were given on even levels were converted to odd levels by taking the average of the variable over adjacent vertical levels at each horizontal grid point. The horizontal boundaries were addressed using global cyclic continuity. All terms on odd levels were found using centered finite differencing methods.
Backward finite differencing methods were employed at the vertical boundaries and for the even levels.

To compute the horizontal advection of PV term, \(- \mathbf{V} \cdot \nabla PV\), it first was written in scalar form, i.e. 
\[ -u \left( \frac{\partial}{\partial x} PV \right) - v \left( \frac{\partial}{\partial y} PV \right). \]
The zonal and meridional wind components were available at each horizontal grid point and at each of the 14 vertical levels. Potential vorticity was computed at each horizontal and vertical grid point. The full term was then computed at each grid point.

The term \(\frac{d\theta}{dt}\), used in both the first diabatic term \((-\frac{d\theta}{dt} \frac{\partial}{\partial \theta} PV\)) and the second diabatic term \((PV \frac{\partial}{\partial \theta} \frac{d\theta}{dt})\), is the diabatic heating which contains convective heating, non-convective heating (such as anvil rain), air-sea interaction, and radiative processes. It was computed from the diagnostic variables from model output. Each full term was then computed by multiplying the appropriate variables at each grid point. To compute the third diabatic term,
\[
\left\{ \nabla \left( \frac{d\theta}{dt} \left( \frac{\partial}{\partial \theta} (\mathbf{V} \times \hat{k})_b \right) \right) \right\} \cdot \nabla \theta \quad \text{in scalar form, i.e.}
\left( \frac{\partial v}{\partial \theta} \frac{\partial}{\partial x} \frac{d\theta}{dt} - \frac{\partial u}{\partial \theta} \frac{\partial}{\partial y} \frac{d\theta}{dt} \right) \frac{\partial \theta}{\partial p}. \]
The components of the horizontal gradient of diabatic heating and the terms \(\frac{\partial v}{\partial \theta}\) and \(-\frac{\partial u}{\partial \theta}\) were found at each grid point. The appropriate variables were then multiplied and subtracted to obtain the full term.

The friction term, \(\left( \nabla \cdot (\mathbf{F} \times \hat{k}) \right) \left( \frac{\partial}{\partial \theta} \right) \frac{\partial \theta}{\partial p}\), was also be written in scalar form, i.e.
\[
- \left( \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \frac{\partial \theta}{\partial p}. \]
The horizontal components of friction, \(F_x\) and \(F_y\), were contained in model output. The full term was then computed by multiplying the components at each grid point.

Each term of the PV tendency equation was also computed on the 308K isentropic surface. This was accomplished by interpolating the data between \(s=0.85\) and \(s=0.99\). Using Poisson’s relation, \(\theta = T \left( \frac{p_s}{p} \right)^{\gamma/\kappa}\), the potential temperature was found at each horizontal grid
point at each of the four sigma levels. Based on these values, a constant potential temperature surface, in this case 308K, was found that existed between the four levels. Linear interpolation was then used to find the relevant term, $Q$, at the 308K surface using the equation

$$ Q = Q_L + \frac{\partial Q}{\partial \theta} \Delta \theta. $$
6.1 Development of the Storm

Results from the potential vorticity budget are analyzed from the three-day forecasts to examine which components of the complete PV equation make the biggest impact during the hurricane genesis. Plots of pressure, wind, and potential vorticity fields were first prepared to illustrate the development of the system. Plots of the components of the PV tendency equation were next analyzed for terms of the complete PV equation and its evolution over time. Each illustration was plotted following the motion of the storm in the horizontal and at the $s=0.85$ vertical level. Cross sections of the components were also produced to examine the vertical structure of the storm and to determine where each component made the greatest impact. Cross sections of PV, $\frac{d\theta}{dt}$, and theta were also created to help identify certain important features in the component cross sections.

6.1.1 Pressure and Wind Fields

On 6 September 12Z, a low-pressure area was centered at approximately 59.5°W and 21.5°N. As the forecast proceeds a deeper closed low forms, as evidenced by Fig. 6.1. The typical horizontal wind pattern associated with a hurricane also develops around the pressure minimum of 984mb as indicated by Fig. 6.2. By 8 September 00Z the storm has moved west-northwest with a pressure minimum of approximately 1002mb and strengthening winds, as indicated by Fig. 6.1c and Fig. 6.2c, respectively. By this time the system is classified as a tropical storm. By 9 September 00Z the storm has continued on its northwest track with its central pressure having reached approximately 988mb (Fig. 6.1e) and maximum winds having reached 40ms$^{-1}$ (Fig. 6.2e). At this time the system is classified as a hurricane. By the last forecast hour (Fig. 6.1f and 6.2f), Erin is just a few hours short of reaching its greatest strength as
Fig. 6.1. Surface pressure for Hurricane Erin on the s=0.85 surface for a) 7 September 00Z (interval of 1mb), b) 7 September 12Z (interval of 2mb), c) 8 September 00Z (interval of 2mb), d) 8 September 12Z (interval of 2mb), e) 9 September 00Z (interval of 4mb), and f) 9 September 12Z (interval of 4mb). Units: mb.
Fig. 6.2. Isotachs for Hurricane Erin on the $\sigma=0.85$ surface for a) 7 September 00Z (interval of 1 ms$^{-1}$), b) 7 September 12Z (interval of 2 ms$^{-1}$), c) 8 September 00Z (interval of 2 ms$^{-1}$), d) 8 September 12Z (interval of 3 ms$^{-1}$), e) 9 September 00Z (interval of 5 ms$^{-1}$), and f) 9 September 12Z (interval of 5 ms$^{-1}$). Units: ms$^{-1}$. 
a Category 3 hurricane with a central pressure of 984mb and maximum winds at approximately 50ms$^{-1}$.

**6.1.2 Potential Vorticity Field**

Potential vorticity following the motion of the storm is plot in Fig. 6.3. A positive PV anomaly is present on 6 September 12Z as the storm develops which is initially located at approximately 59.5°W and 21.5°N, as with the pressure minimum. The PV maximum moves with the pressure minimum on a west-northwest and then northwest track. By 8 September 00Z there is a large gradient of PV with a maximum value of 2.4e-06 m$^2$s$^{-1}$Kkg$^{-1}$, as indicated by Fig. 6.3c. The PV gradient continues to increase during the next 36 hours and by 9 September 12Z (Fig. 6.3f) the maximum value reaches 5.0e-06 m$^2$s$^{-1}$Kkg$^{-1}$ and is located at approximately 66.5°W and 25.5°N. The increase of PV and the PV dynamics (and thermodynamics) during the formation of Hurricane Erin is the central topic of this thesis.

**6.2 Relative Importance of Each Term During the Formation Stage**

In order to understand the processes that lead to the formation of Hurricane Erin, each component of the PV tendency equation is analyzed over the three-day forecast. Contour plots were created to illustrate these components. As with the pressure, wind, and PV diagrams, the same storm centered domain is used. Vertical cross sections were also plotted in order to examine the vertical structure of each component. The cross sections were chosen based on regions exhibiting minimum and maximum heating of the hurricane.

**6.2.1 Evolution of Terms on Constant Sigma Surface**

It is possible to write down the complete diabatic PV equation in almost any frame of reference. Although the best suited coordinate system is x, y, ?, t, we can easily retain all of that information by projecting the information on the x, y, s, t frame of reference. That is what is displayed in this section. Figure 6.4a-f illustrates the five components of the PV tendency equation on the s=0.85 surface over the 72-hour forecast. These are the horizontal advection of PV, the first diabatic term, the second diabatic term, the third diabatic term, and friction. On 7 September 00Z, Fig. 6.4a, the horizontal advection of PV term consisted of a dipole centered around the pressure minimum. The counterclockwise circulation is helping to bring high PV air down to the northwestern
Fig. 6.3. Potential vorticity on the s=0.85 surface for a) 7 September 00Z (interval of 0.5 m$^2$s$^{-1}$Kkg$^{-1}$), b) 7 September 12Z (interval of 1E-07 m$^2$s$^{-1}$Kkg$^{-1}$), c) 8 September 00Z (interval of 3E-07 m$^2$s$^{-1}$Kkg$^{-1}$), d) 8 September 12Z (interval of 3E-07 m$^2$s$^{-1}$Kkg$^{-1}$), e) 9 September 00Z (interval of 5E-07 m$^2$s$^{-1}$Kkg$^{-1}$), and f) 9 September 12Z (interval of 5E-07 m$^2$s$^{-1}$Kkg$^{-1}$). Units: m$^2$s$^{-1}$Kkg$^{-1}$. 

Fig. 6.4a. Contour plots of horizontal advection (interval of 3E-12 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), first diabatic term (interval of 1E-12 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), second diabatic term (interval of 2E-12 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), third diabatic term (interval of 5E-13 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), and friction (interval of 5E-15 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K) for 7 September 00Z. Units: kg\(^{-1}\) m\(^2\) s\(^{-2}\) K.
Fig. 6.4b. Contour plots of horizontal advection (interval of 1E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), first diabatic term (interval of 5E-12 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), second diabatic term (interval of 1E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), third diabatic term (interval of 5E-13 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K), and friction (interval of 2E-14 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K) for 7 September 12Z. Units: kg\(^{-1}\)m\(^2\)s\(^{-2}\)K.
September 8, 2001 00Z
sigma=0.85

Fig. 6.4c. Contour plots of horizontal advection (interval of 1E-11 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), first diabatic term (interval of 5E-12 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), second diabatic term (interval of 1E-11 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), third diabatic term (interval of 1E-12 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), and friction (interval of 1E-13 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K) for 8 September 00Z. Units: kg\(^{-1}\) m\(^2\) s\(^{-2}\) K.
Fig. 6.4d. Contour plots of horizontal advection (interval of \(5\times 10^{-11} \text{ kg}^{-1} \text{m}^2 \text{s}^{-2} \text{K}\)), first diabatic term (interval of \(2\times 10^{-11} \text{ kg}^{-1} \text{m}^2 \text{s}^{-2} \text{K}\)), second diabatic term (interval of \(2\times 10^{-11} \text{ kg}^{-1} \text{m}^2 \text{s}^{-2} \text{K}\)), third diabatic term (interval of \(5\times 10^{-12} \text{ kg}^{-1} \text{m}^2 \text{s}^{-2} \text{K}\)), and friction (interval of \(2\times 10^{-13} \text{ kg}^{-1} \text{m}^2 \text{s}^{-2} \text{K}\)) for 8 September 12Z. Units: \(\text{kg}^{-1} \text{m}^2 \text{s}^{-2} \text{K}\).
Fig. 6.4e. Contour plots of horizontal advection (interval of 5E-11 kg$^{-1}$ m$^2$ s$^{-2}$ K), first diabatic term (interval of 2E-11 kg$^{-1}$ m$^2$ s$^{-2}$ K), second diabatic term (interval of 5E-11 kg$^{-1}$ m$^2$ s$^{-2}$ K), third diabatic term (interval of 3E-11 kg$^{-1}$ m$^2$ s$^{-2}$ K), and friction (interval of 1E-12 kg$^{-1}$ m$^2$ s$^{-2}$ K) for 9 September 00Z. Units: kg$^{-1}$ m$^2$ s$^{-2}$ K.
Fig. 6.4f. Contour plots of horizontal advection (interval of $1 \times 10^{-10}$ kg m$^{-2}$ s$^{-2}$ K), first diabatic term (interval of $2 \times 10^{-11}$ kg m$^{-2}$ s$^{-2}$ K), second diabatic term (interval of $5 \times 10^{-11}$ kg m$^{-2}$ s$^{-2}$ K), third diabatic term (interval of $2 \times 10^{-11}$ kg m$^{-2}$ s$^{-2}$ K), and friction (interval of $2 \times 10^{-12}$ kg m$^{-2}$ s$^{-2}$ K) for 9 September 12Z. Units: kg m$^{-2}$ s$^{-2}$ K.
side of the storm and low PV air to the southeastern side. The first diabatic term is negative to the northeast of the storm, i.e. $\frac{d\theta}{dt} > 0$, and $\frac{\partial PV}{\partial \theta} > 0$, hence, $-\frac{d\theta}{dt} \frac{\partial PV}{\partial \theta} < 0$, as is expected. This term is quite analogous to the vertical advection term in the conventional vorticity equation in the x, y, p frame of reference. The vertical advection term of the isentropic coordinate is a diabatic contribution whose interpolation is identical to that of the conventional vertical advection. The second diabatic term is positive to the northeast of the storm, i.e. $\frac{\partial \frac{d\theta}{dt}}{\partial \theta} > 0$, and $PV \frac{\partial \theta}{dt} > 0$, also as expected. The third diabatic term, like horizontal advection of PV, has a dipole structure that is present to the northeast of the storm. As stated in Chapter 2, this term varies as a function of horizontal and vertical position. Given the scalar form of the third diabatic equation, $\frac{\partial v}{\partial \theta} \frac{\partial d\theta}{dt} - \frac{\partial u}{\partial \theta} \frac{\partial d\theta}{dt} + g \frac{\partial \theta}{\partial p}$, areas of positive values must satisfy $\frac{\partial v}{\partial \theta} \frac{\partial d\theta}{dt} - \frac{\partial u}{\partial \theta} \frac{\partial d\theta}{dt} < 0$ and vice versa since $g \frac{\partial \theta}{\partial p}$ is negative.

Friction is positive to the northeast of the storm also as expected. Since the plot is valid at the $s=0.85$ surface, we can expect smaller values of friction as opposed to more significant values near the surface. The heating terms and friction values occur to the northeast of the system, as convection was most likely centered there. With the progression of the storm, convection and, hence, the heating terms will become centered around the pressure minimum. With the exception of friction, the orders of magnitude of the terms are similar. However, the second diabatic term is making the most positive contribution to the PV tendency equation with a value of 1.6e-11 kg$^{-1}$m$^{2}$s$^{-2}$K.

By 7 September 12Z, the horizontal advection of PV, first diabatic term, and second diabatic term have strengthened, as indicated by Fig. 6.4b. The first diabatic term and second diabatic term centers of minimum/maximum are now centered around the pressure minimum. Any positive values in the previous plot of the third diabatic term are replaced by negative values. Two centers of minimum are now present. Friction develops a dipole structure with stronger positive values to the northeast of the pressure minimum and negative values to the southwest. The scalar form of the friction term is given by $-\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right) g \frac{\partial \theta}{\partial p}$ where the
stability term $-\varrho \frac{\partial \theta}{\partial p}$ is nearly always positive. The horizontal components of friction can be approximated using the similarity theory as $F_x \approx c_D \rho u |\mathbf{V}|$ and $F_y \approx c_D \rho v |\mathbf{V}|$, where $c_D$ is the time varying drag coefficient, $\rho$ is the density of air, $u$ and $v$ are the horizontal components of the wind, and $|\mathbf{V}|$ is the total wind speed. This leads to the approximation

$$\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} = c_D \rho \mathbf{V} \zeta_r,$$

where $\zeta_r$ is the relative vorticity and $\mathbf{V}$ is the 2-dimensional wind velocity. In a hurricane, relative vorticity is positive which leads to deciding the sign of this friction term in the complete PV equation. Outside of this cyclonic circulation anticyclonic shear is present leading to negative relative vorticity and, hence, negative values of friction. This is a curl of the wind stress. In this way a frictional dipole can form. At this time in the storm development, the orders of magnitude of the horizontal advection of PV, first diabatic term, and second diabatic term are some of the contributors with the largest positive values again being contributed to the PV tendency equation by the second diabatic term.

In the next 12 hours, each component’s values strengthen once again, as indicated by Fig. 6.4c. The dipole structure of the horizontal advection of PV term, the negative values of the first diabatic term, and the strong positive values of the second diabatic term are still present. The third diabatic term’s structure now has three small centers of minima in values. The friction increases in intensity by an order of magnitude and retains its dipole structure, but with the positive and negative values slightly rotated as the hurricane rotated. At this time, the second diabatic term’s magnitude is almost an order of magnitude larger than the other terms with the exception of friction which is much smaller.

Figure 6.4d indicates that structurally the horizontal advection of PV and the second diabatic term retain the same characteristics, but are stronger. The first diabatic term’s negative values strengthen and it develops positive values to the east of the pressure minimum. This is most likely due to PV values below the $s=0.85$ surface that have increased. The third diabatic term’s strong values remain negative, but weak positive maxima to the northeast and southwest of the minimum are present. The friction term’s dipole structure has weakened and stronger positive values remain with the exception of a small center of negative values located at 65.5ºW.
and 23.5°N. Again, the second diabatic term values remain larger and contributing most to \( \frac{\partial}{\partial t} PV > 0 \) than the other terms.

By 9 September 00Z (Fig. 6.4e), the horizontal advection of PV term and second diabatic term structures once again retain similar magnitudes, but the latter is stronger. The first diabatic term loses its positive values and has stronger negative values which occur to the north and east of the pressure minimum. The third diabatic term assumes a dipole structure with stronger negative values to the north of the pressure minimum and positive values to the south. The friction term strengthens and has negative values flanked by positive values to the southeast and northwest. At this stage in the development of the system, the positive values of the horizontal advection of PV term and second diabatic term are similar and both are contributing to \( \frac{\partial}{\partial t} PV > 0 \). However, the area where the horizontal advection of PV destroys \( \frac{\partial}{\partial t} PV \) is where the second diabatic term acts to increase it.

By 9 September 12Z (Fig. 6.4f), the horizontal advection of PV term retains its dipole structure with slightly stronger positive and negative values. The first diabatic term assumes a dipole structure around the pressure minimum with a slight weakening of its negative values. The second diabatic term begins to show signs of developing a dipole structure around the pressure minimum, but retains its strong positive values. Again, the horizontal advection of PV term and the second diabatic term have similar positive values. The third diabatic term retains its structure with similar positive values, but a strengthening in negative values. Friction preserves the same structure with a slight strengthening of negative values.

Based on these contour plots it is evident that the second diabatic term has the biggest impact on \( \frac{\partial}{\partial t} PV > 0 \) throughout the formation of the storm. Once the storm hits hurricane status, the horizontal advection of PV term helps to contribute to \( \frac{\partial}{\partial t} PV > 0 \) in some areas as well, but also works to destroy it in other areas.
Fig. 6.5. Potential vorticity cross sections for a) 7 September 00Z (interval of $1E-07 \text{ m}^2\text{s}^{-1}\text{Kg}^{-1}$), b) 7 September 12Z (interval of $2E-07 \text{ m}^2\text{s}^{-1}\text{Kg}^{-1}$), c) 8 September 00Z (interval of $3E-07 \text{ m}^2\text{s}^{-1}\text{Kg}^{-1}$), d) 8 September 12Z (interval of $5E-07 \text{ m}^2\text{s}^{-1}\text{Kg}^{-1}$), e) 9 September 00Z (interval of $5E-07 \text{ m}^2\text{s}^{-1}\text{Kg}^{-1}$), and f) 9 September 12Z (interval of $5E-07 \text{ m}^2\text{s}^{-1}\text{Kg}^{-1}$). Units: $\text{m}^2\text{s}^{-1}\text{Kg}^{-1}$.
Fig. 6.6. Theta cross sections for a) 7 September 00Z (interval of 5K), b) 7 September 12Z (interval of 5K), c) 8 September 00Z (interval of 5K), d) 8 September 12Z (interval of 5K), e) 9 September 00Z (interval of 5K), and f) 9 September 12Z (interval of 5K). Units: K.
6.2.2 Vertical Cross-Sections

Vertical cross sections of PV and theta are plotted in Fig. 6.5 and Fig. 6.6, respectively. Figure 6.5a indicates a PV maximum beginning to form in the mid-troposphere. A clear indication of the beginning of a warm core is present in Fig. 6.6a. Over the ensuing 24 hours the PV maximum strengthens to reach a value of 2.7e-06 m²s⁻¹Kkg⁻¹ in the mid-troposphere. Figure 6.6c indicates a warm core forming throughout the depth of the atmosphere at approximately 62°W. The PV maximum continues to strengthen reaching a final maximum value of 5.0e-06 m²s⁻¹Kkg⁻¹ by 9 September 12Z, as indicated by Fig. 6.5f. The large warm anomaly also deepens significantly, as indicated by Fig. 6.6f. The magnitude of this temperature anomaly converted to °C is roughly 5° to 7°C compared to the mean tropical atmosphere.

Vertical cross sections of the components of the PV tendency equation are plotted in Fig. 6.7a-f. It is important to note that some inconsistencies in the precise casting of budgets can arise if 12-hourly displays are presented. The precise budgets can only be accomplished if the entire model data and code for each time step are followed very carefully.

On 7 September 00Z (Fig. 6.7a), the horizontal advection of PV in the cross section indicates cyclonic flow advecting high PV air to the south, indicated by positive values west of 59.5°W, and low PV air to the north, indicated by negative values east of 59.5°W. In the upper atmosphere, an anticyclonic circulation is inducing the opposite effect. The first diabatic term’s vertical structure of maximum and minimum is explained by examination of the vertical cross sections of PV and heating, \( \frac{d\theta}{dt} \) (Fig. 6.8a). The heating term is positive throughout the core of the hurricane. This is largely contributed by the convective heating. The term \( \frac{\partial}{\partial \theta} PV \) is generally positive in the lower atmosphere and negative in the upper atmosphere indicating that the first diabatic term, \(-\frac{d\theta}{dt} \frac{\partial}{\partial \theta} PV\), should be negative in the lower atmosphere and positive in the upper atmosphere. The vertical gradient of heating, \( \frac{\partial}{\partial \theta} \frac{d\theta}{dt} \), is positive until approximately the s=0.4 surface beyond which it is negative. Since PV is positive in the vicinity of the hurricane, we can expect the second diabatic term, \( PV \frac{\partial}{\partial \theta} \frac{d\theta}{dt} \), to be positive up to the s=0.4
Fig. 6.7a. Contour plots of horizontal advection (interval of 5E-12 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), first diabatic term (interval of 3E-12 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), second diabatic term (interval of 5E-12 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), third diabatic term (interval of 2E-12 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), and friction (interval of 5E-13 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K) for 7 September 00Z. Units: kg\(^{-1}\) m\(^2\) s\(^{-2}\) K.
Fig. 6.7b. Contour plots of horizontal advection (interval of 1E-11 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), first diabatic term (interval of 1E-11 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), second diabatic term (interval of 1E-11 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), third diabatic term (interval of 5E-12 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), and friction (interval of 2E-12 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K) for 7 September 12Z. Units: kg\(^{-1}\) m\(^2\) s\(^{-2}\) K.
September 8, 2001 00Z

Fig. 6.7c. Contour plots of horizontal advection (interval of 1E-11 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), first diabatic term (interval of 3E-11 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), second diabatic term (interval of 3E-11 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), third diabatic term (interval of 1E-11 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), and friction (interval of 5E-12 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K) for 8 September 00Z. Units: kg\(^{-1}\) m\(^2\) s\(^{-2}\) K.
Fig. 6.7d. Contour plots of horizontal advection (interval of 3E-11 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), first diabatic term (interval of 5E-11 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), second diabatic term (interval of 5E-11 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), third diabatic term (interval of 2E-11 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K), and friction (interval of 1E-11 kg\(^{-1}\) m\(^2\) s\(^{-2}\) K) for 8 September 12Z. Units: kg\(^{-1}\) m\(^2\) s\(^{-2}\) K.
Fig. 6.7e. Contour plots of horizontal advection (interval of $5\times 10^{-11}\,\text{kg}\,\text{m}^{-2}\text{s}^{-2}\text{K}$), first diabatic term (interval of $1\times 10^{-10}\,\text{kg}\,\text{m}^{-2}\text{s}^{-2}\text{K}$), second diabatic term (interval of $1\times 10^{-10}\,\text{kg}\,\text{m}^{-2}\text{s}^{-2}\text{K}$), third diabatic term (interval of $2\times 10^{-11}\,\text{kg}\,\text{m}^{-2}\text{s}^{-2}\text{K}$), and friction (interval of $3\times 10^{-11}\,\text{kg}\,\text{m}^{-2}\text{s}^{-2}\text{K}$) for 9 September 00Z. Units: $\text{kg}\,\text{m}^{2}\text{s}^{-2}\text{K}$. 
Fig. 6.7f. Contour plots of horizontal advection (interval of 5E-11 kg$^{-1}$ m$^2$ s$^{-2}$ K), first diabatic term (interval of 1E-10 kg$^{-1}$ m$^2$ s$^{-2}$ K), second diabatic term (interval of 1E-10 kg$^{-1}$ m$^2$ s$^{-2}$ K), third diabatic term (interval of 5E-11 kg$^{-1}$ m$^2$ s$^{-2}$ K), and friction (interval of 5E-11 kg$^{-1}$ m$^2$ s$^{-2}$ K) for 9 September 12Z. Units: kg$^{-1}$ m$^2$ s$^{-2}$ K.
Fig. 6.8. Cross sections of diabatic heating for a) 7 September 00Z (interval of 5E-05 K), b) 7 September 12Z (interval of 2E-04 K), c) 8 September 00Z (interval of 2E-04 K), d) 8 September 12Z (interval of 5E-04 K), e) 9 September 00Z (interval of 5E-04 K), and f) 9 September 12Z (interval of 3E-04 K). Units: Ks⁻¹.
surface and negative above. The presence of negative values of the second diabatic term below the \( s=0.9 \) surface is due to the large surface layer heating. Since the ocean surface is very warm and latent and sensible heat fluxes are provided to the atmosphere from there, the heating is decreasing with height in the boundary layer, i.e. \( \frac{\partial}{\partial t} \frac{d\theta}{dt} < 0 \). Since \( PV>0 \), this contributes to \( \frac{\partial}{\partial t} PV < 0 \) in the surface layer. An effect of this heating is to lower the static stability of the surface layer. As the stability decreases somewhat, the absolute vorticity increases in the surface layer as the hurricane forms. The third diabatic term exhibits a dipole structure in the mid to upper-troposphere. Since its sign changes as a function of vertical and horizontal position it is difficult to explain the vertical structure of this term. It should be noted that this term has the smallest contribution among the three heating terms throughout hurricane genesis. The friction term is equal to 0.0 throughout the upper atmosphere, as is expected, and is negative near the surface. At this stage of the forecast the magnitudes of the horizontal advection of PV and the three diabatic terms were of the same order, i.e. small.

By 7 September 12Z (Fig. 6.7b), the horizontal advection of PV term retains its structure with a slight westward shift of the upper anticyclone and an increase in values. The first diabatic and second diabatic terms’ structure also remains the same with a strengthening of values. The third diabatic term no longer has positive values as may be seen from Fig. 6.4b and its negative values have slightly strengthened. The friction term’s negative values strengthen somewhat and positive values are now present west of 61.5°W. The first diabatic term and second diabatic term provide the biggest contributions to \( \frac{\partial}{\partial t} PV > 0 \) at this time.

Over the next twelve hours (Fig. 6.7c), the vertical structure of each component of the PV tendency equation remains the same while continually strengthening. By 8 September 12Z (Fig. 6.7d), the anticyclonic circulation aloft is replaced by cyclonic circulation throughout the depth of the troposphere. Therefore, the lower-tropospheric dipole structure of the horizontal advection of PV now extends through the depth of the troposphere. The first diabatic term and the friction term’s structure remain small, as they were previously. The second diabatic term’s structure also remains the same except for the disappearance of negative values in the boundary layer. Examination of Fig. 6.8d reveals that the decrease in the vertical gradient of heating is extremely small, if present at all. The third diabatic term now exhibits a dipole structure, as
indicated in Fig. 6.4d. The magnitudes of the horizontal advection of PV, first diabatic term, and second diabatic term are the same, however, the horizontal advection of PV and the first diabatic term are acting to increase $\frac{\partial}{\partial t} PV$ in some areas as well as decrease it in others. Only the second diabatic term acts to increase $\frac{\partial}{\partial t} PV$ throughout the troposphere. Over the next 24 hours, the vertical structure of all the terms remains the same.

During the three-day forecast, each component increases in magnitude. Every component of the PV tendency equation acts to increase $\frac{\partial}{\partial t} PV$ in some areas. However, the second diabatic term’s values were generally stronger than any of the other terms and it was clearly the biggest contributor to $\frac{\partial}{\partial t} PV > 0$.

6.3 Relating Formation to the Diabatic PV

Inner storm area and outer storm area histograms were plot to illustrate the tendencies of each component at a specific time step and to determine which term had the greatest impact on the PV tendency. The inner storm area was chosen by looking at each term’s area of influence and choosing a 3 degree latitude by 3 degree longitude box. The outer storm area utilized the full computational domain as those used for the contour plots at each time step minus the values of the inner storm area. Vertically, both the inner and outer storm areas were summed over the $s=0.4$ to $s=0.7$ surfaces, i.e. the area of maximum heating. Maximum values of each component at each time step were also plot to examine the relative magnitudes of each term.

6.3.1 Inner Storm Area Histograms

Evaluation of Fig. 6.9 reveals that over the forecast period the horizontal advection of PV term remains at fairly small, negative values in the inner storm area until the last forecast hour. The first diabatic term remains at very small, positive values throughout the length of the forecast, except on 9 September 00Z when it is slightly negative. The second diabatic term remains at strong positive values, the third diabatic term at strong negative values, and friction at extremely small values throughout the length of the forecast. A comparison of components reveals that the second diabatic term makes the greatest contribution to the PV tendency.
Fig. 6.9. Histogram plots of inner storm area variable means of a) horizontal advection of PV, b) first diabatic term, c) second diabatic term, d) third diabatic term, and e) friction over the 72-hour forecast. Units: kg m\(^{-2}\) s\(^{-2}\) K.
equation during the genesis of the hurricane. Twelve hours after classification of a hurricane, the horizontal advection of PV term took over as the greatest contributor to PV changes. This large contribution from the advection of PV should not be thought of as an importance of conservation of PV since these values of PV advection are instantaneous and they, too, in a full model continually adjust to the heating effects. At no other time did the horizontal advection term contribute to positive PV values. The first diabatic term made small contributions to PV values while the third diabatic term made the least contribution to changes in PV, with the exception of friction.

6.3.2 Outer Storm Area Histograms

Evaluation of Fig. 6.10 reveals that the horizontal advection of PV term is positive on 7 September 00Z and then becomes negative over the next 24 hours. During the final 36 hours it remains positive. The first diabatic term is positive on 7 September 00Z and then becomes negative over the next 36 hours. A slightly positive value occurred on 9 September 00Z, but the term then becomes negative again during the last 12 hours. The second diabatic term remains positive throughout the length of the forecast, whereas the third diabatic term remains at strong negative values. Friction again remains at extremely small values. A comparison of components reveals that the horizontal advection of PV and the first diabatic term in the outer storm area oscillate from positive to negative values over the 72 hours. The second diabatic term’s values oscillate in the positive range as well. The oscillating in values of the components is due to the unstable convection in the outer storm area.

6.3.3 Maximum Values

Maximum values of PV, vorticity, horizontal advection of PV, and the three diabatic terms in the storm centered coordinates were plot on the 308K surface in Fig. 6.11. Increase of maximum values is generally a clear indicator of non-conservation. The tendency of the PV and vorticity to increase during the length of the forecast is apparent in Fig 6.11a and Fig. 6.11b, respectively. The horizontal advection of PV (Fig. 6.11c) gradually increases over the first 63 hours of the forecast, with a sharp spike in values occurring at hour 66 to reach 2.8e-10 kg⁻¹ m² s⁻² K, and then a slight decline to approximately 1.25e-10 kg⁻¹ m² s⁻² K. The first diabatic term (Fig. 6.11d) exhibits oscillations in the first 60 hours of the forecast, but then sharply increases to reach 5.9e-10 kg⁻¹ m² s⁻² K at hour 66. It then declines to 1.5e-10 kg⁻¹ m² s⁻² K at the last forecast hour. The second diabatic term steadily increases throughout the 72 hours while oscillating.
Fig. 6.10. Histogram plots of inner storm area variable means of a) horizontal advection of PV, b) first diabatic term, c) second diabatic term, d) third diabatic term, and e) friction over the 72-hour forecast. Units: kg m$^{-1}$ m$^2$ s$^{-2}$ K.
Its maximum value of $3.2\text{e-10 } \text{kg}^{-1}\text{m}^2\text{s}^{-2}\text{K}$ is reached at hour 63. The third diabatic term stays nearly constant until hour 45 after which it oscillates and gradually increases to reach a final value of $5.3\text{e-11 } \text{kg}^{-1}\text{m}^2\text{s}^{-2}\text{K}$. Of the component terms, the first diabatic term reached the highest value at $5.9\text{e-10 } \text{kg}^{-1}\text{m}^2\text{s}^{-2}\text{K}$. However, any significant contribution by this term to PV is sporadic at best. The biggest contributor to the PV tendency is the second diabatic term. The horizontal advection of PV term also contributes to the PV changes, however, its contribution is slightly less than the second diabatic term’s contribution. The third diabatic term made the least significant contribution to the PV tendency.
Maximum Values
308 K

Fig. 6.11. Maximum values a) potential vorticity, b) vorticity, c) horizontal advection of PV, d) first diabatic term, e) second diabatic term, and f) third diabatic term on the 308K surface over the 72-hour forecast. Units: kg\(^1\)m\(^2\)s\(^{-2}\)K.
CHAPTER 7
THREE DIMENSIONAL TRAJECTORIES

7.1 Overview

Trajectory calculations are useful for the understanding of the path of a parcel carrying a certain property through time. The principle behind a trajectory program is that the displacement of a parcel is based on and can be constructed from its u, v, and w wind components (Krishnamurti 1996). Generally, the 3-D wind components are available at specific grid points and time intervals, therefore, interpolation of the data is carried out to obtain the wind components at any point in space and time along the parcel’s path (Krishnamurti 1996). Displacements of the parcel along the x, y, and p axes are obtained using Newton’s law and are given by:

\[
\begin{align*}
Dx &= 0.5(u + u' )Dt \\
Dy &= 0.5(v + v' )Dt \\
Dp &= 0.5(\omega + \omega' )Dt
\end{align*}
\]

where the prime (’) notation denotes the velocity at the end point of the trajectory segment (Krishnamurti 1996). From the parcel’s initial point, the following point is found by using the displacements, \(Dx\), \(Dy\), and \(Dp\), and moving the parcel said distance along the x, y, and p axes, and so on (Krishnamurti 1996). The values of \(u’\), \(v’\), and \(\omega’\) are arrived at by a successive correction technique that utilizes a local three-dimensional objective volume.

7.2 Calculating the Trajectories

Three-day, backward trajectories for Hurricane Erin were computed starting at the 72\textsuperscript{nd} forecast hour grid point positions with the initial placement of the parcel close to or in the area of maximum wind velocity. Available wind data were the u, v wind components on the x, y, and s coordinate system. The data was regularly spaced at 0.703125\(^\circ\) in the x-direction, approximately
0.7º in the y-direction, and across 14 irregularly spaced sigma levels. Consequently, the wind components were vertically interpolated from the 14 sigma levels to 19 regularly spaced pressure levels, i.e. every 50mb from 1000mb to 100mb, to create a regularly spaced 3-D grid. The backward trajectory program read the horizontal wind data starting at the 72nd forecast hour. The vertical velocity was evaluated by employing the kinematic method from 1000mb to 100mb using input consisting of u, v, dx (east-west grid distance), dy (north-south grid distance), and p. Data were interpolated to a temporal resolution of 10 minutes, which is the required time step for the computation (Krishnamurti 1996). The program then converted the starting and ending points of each trajectory to the closest i, j, and k indices (east-west, north-south, and vertical indices, respectively), computed the distance from the trajectory point to this data point, used a trilinear interpolation scheme to interpolate the 3-D wind at the starting point of the trajectories, and then computed the displacement of the parcel to determine the next point (Krishnamurti 1996a). The domain for the parcel trajectories was approximately 50W-80W, 12N-36N, and 100-10000mb. Since the 3-D trajectory included vertical motion it could not be used to imply conservation of PV, since vertical motions invoke the heating \( \frac{d\theta}{dt} \) in the isentropic frame of reference. Therefore, two-dimensional trajectories were also composed to assess if PV was conserved following the motion. In order to carry out this exercise, the program was run with no vertical motion, giving a purely horizontal trajectory on the 308K isentropic surface.

Values of potential vorticity (from the 2-D and 3-D trajectories), horizontal advection of PV (from the 2-D trajectories), the first diabatic term, the second diabatic term, and the third diabatic term were evaluated over 3-hour intervals along each trajectory. Using the output from the trajectory program the latitude and longitude of the parcel was found at every third hour, which determined the closest grid points to the parcel. From these grid points, values of the above variables were found by horizontally and vertically interpolating the data along the trajectory.

### 7.3 Parcel Trajectories

Numerous trajectories were constructed, each of which were centered in or near the center of maximum wind velocity of the storm at the 72nd forecast hour. Three trajectories were chosen to illustrate the properties of a small volume of air traveling into a hurricane.
Specifically, how the properties changed over the course of trajectory was examined as the parcel moved near, around, and in the vicinity of Hurricane Erin. We also examined how the horizontal advection of PV (conservation) and the diabatic terms (non-conservation) influenced the values of PV, as well as asking which of the terms made the biggest contribution to PV changes.

3-D Trajectories

Fig. 7.1. Three-dimensional trajectories (trajectory 1: black, trajectory 2: red, trajectory 3: green) with pressure levels given every six hours during the 72-hour forecast. Solid contours are isotachs (contour interval of 5ms\(^{-1}\)) on 9 September 12Z. Units: mb.
2-D Potential Vorticity

Fig. 7.2. Values of potential vorticity for 72-hour, 2-D trajectories (trajectory 1: red, trajectory 2: green, trajectory 3: blue) on the 308K surface. Units: kg\(^{-1}\)m\(^2\)s\(^{-1}\)K.
7.3.1 Trajectory 1

On 6 September 12Z the parcel is located at approximately 17.8ºN and 54.3ºW as indicated by Fig. 7.1. Although these are backward trajectories we shall here follow the trajectory forward in time. The parcel moves west-northwest toward the storm. Once it moves into the vicinity of the storm, it follows the surrounding winds on a counterclockwise trajectory and then moves into the center of the storm. The parcel’s trajectory ends on 9 September 12Z at 26ºN and 66ºW, on the western edge of the center of maximum winds.

Initially, the pressure along the trajectory is 838mb as indicated by Fig. 7.3a. Over the first 57 forecast hours, overall there is tendency for the pressure to rise. During that time there are slight fluctuations, but the parcel pressure remains within 25mb of its initial value. After hour 57, within six hours the parcel pressure rises by 100mb to hit 958mb by passing through an area of descending motion. Within another six hours, the parcel pressure drastically drops by approximately 100mb to reach 850mb at the last forecast hour as it passes through areas of strong ascending motion moving toward the center of the storm.

Figure 7.3b reveals that for the first 51 hours along the trajectory, the values of the 3-D PV remain at a fairly constant value, approximately 1.0E-07 m²s⁻¹Kkg⁻¹. Between hours 51 and 63, there are slight oscillations with the value remaining positive. Hours 63 to 69 see a decrease to nearly 0.0 and then a sharp increase of 3.2E-06 m²s⁻¹Kkg⁻¹ within three hours as the parcel moves into the center of the storm. The values of PV from the 2-D trajectories were calculated and are illustrated in Fig. 7.2. Trajectory 1’s 2-D PV is initially 3.1E-07 m²s⁻¹Kkg⁻¹. It gradually decreases until hour 33 after which it shows larger oscillations, finally ending at a value of 4.2E-07 m²s⁻¹Kkg⁻¹ at the last forecast hour. The oscillations and the overall trend of the PV to increase establish that potential vorticity is not conserved following the 2-D motion of trajectory 1. It should be noted that we are using the output of a fully diabatic model here, thus this non-conservation is something that can be expected.

Values of horizontal advection of PV (from the 2-D trajectory) remain constant at 0.0 through hour 33, as indicated by Fig. 7.3c. Between hours 36 and 39, a dip and then a rise back to the initial value occurs, while the PV shows the opposite pattern during this time. These opposite patterns are consistent with the definition of advection, \(- \mathbf{V} \cdot \nabla (PV)\). Another decrease in values occurs at hour 63 to \(-8.0E-11 \text{ kg}^{-1}\text{m}^2\text{s}^{-2}\text{K}\), followed by a sharp increase to 8.0E-11 kg⁻¹m²s²K and then a drop back to \(-3.0E-11 \text{ kg}^{-1}\text{m}^2\text{s}^{-2}\text{K}\) by the final forecast hour. Again, the
values of PV exhibit the opposite pattern. The oscillations in the horizontal advection term arise due to the flow field encountering these PV maxima and minima. As the parcel moves into a PV maximum it horizontally advects low-PV air into the area, hence, the dip in values of the horizontal advection of PV and the rise in values of PV and vice versa. As can be seen, the horizontal advection of PV tries to counteract the increasing or decreasing values of PV. However, as was already mentioned, PV is not conserved following the 2-D trajectory and, therefore, the horizontal advection of PV cannot alone explain the changes in potential vorticity.

Like the values of 3-D PV, the first diabatic term stays constant near 0.0 for the first 48 hours of the trajectory, as indicated by Fig. 7.3d. From hours 48 to 66, there are slight oscillations in values. The increase and decrease from hours 51 to 57 appear to have contributed to the slight increase and decrease in values of PV, however at all other times the first diabatic term decreases whereas the PV increases. This may be related to other features of the model. A moderate rise from negative to positive values of the first diabatic term occurs between hours 63 and 69. On the other hand, PV decreases during hours 63 to 69. The first diabatic term exhibits the same tendency as the horizontal advection of PV during the last three forecast hours. It drops rapidly by 8.3E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K while the values of PV rise rapidly. The same reasoning used to explain the dips and rises in values of horizontal advection can be used here as well. As the parcel moves into a PV maximum \(\frac{\partial}{\partial \theta} PV\) is positive (meaning PV increases upwards), hence,

\[-\frac{d\theta}{dt} \frac{\partial}{\partial \theta} PV\]

is negative. Consequently, it can be concluded that the first diabatic term makes some small contributions to values of PV, but does not contribute to the major PV change in the last three forecast hours when the hurricane formed.

Figure 7.3e illustrates that the second diabatic term stays constant near 0.0 until hour 60, after which it dips into negative values. During the last three hours there is a drastic increase from 0.0 to 6.0E-10 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K. Comparing Figs. 7.3b and 7.3e, the drop in values of the second diabatic term during hours 60 to 63 does not correspond to the PV, which increases. Hours 63 to 69 again show opposing tendencies, but although the second diabatic term is increasing the values are still negative and contributing to the drop of PV. During the last three hours, \(\frac{\partial}{\partial \theta} \frac{d\theta}{dt}\) is positive leading to a positive second diabatic term. This is shown in Fig. 7.3e, as the term dramatically increases during that time. The values of PV exhibit a similar drastic increase.
Consequently, it can be concluded that the second diabatic term makes contributions to the PV changes between hours 60 and 69 and makes a fairly significant contribution to the changes in PV in the last three forecast hours. Also, it can be concluded that the second diabatic term makes the most significant contribution to the PV compared to the other terms, as it is an order of magnitude larger than the others.

The initial value of the third diabatic term is $2.5 \times 10^{-12} \text{ kg}^{-1} \text{m}^2 \text{s}^{-2} \text{K}$, but then drops to 0.0 where it remains constant for the first 60 hours, as indicated by Fig. 7.3f. There is a small oscillation between 0.0 and $1.0 \times 10^{-11} \text{ kg}^{-1} \text{m}^2 \text{s}^{-2} \text{K}$ between hours 60 to 69. During the last three hours, a sharp decline to $-1.05 \times 10^{-10} \text{ kg}^{-1} \text{m}^2 \text{s}^{-2} \text{K}$ occurs. This can be explained by using the scalar form of the third diabatic term, 

$$
\left( \frac{\partial v}{\partial \theta} \frac{\partial d \theta}{\partial x} \frac{dt}{dt} - \frac{\partial u}{\partial \theta} \frac{\partial d \theta}{\partial y} \frac{dt}{dt} \right) g \frac{\partial \theta}{\partial p}.
$$

Based on Fig. 7.1, it can be assumed that the parcel is moving north. The zonal wind component is very small, i.e. $u=0$. Examination of vertical cross-sections of the meridional wind reveals that $\frac{\partial v}{\partial \theta}$ is positive. The term $\frac{\partial}{\partial x} \frac{d \theta}{dt}$ is positive as well, while $g \frac{\partial \theta}{\partial p}$ is known to be negative. Therefore, the third diabatic term in scalar form becomes 

$$
\left( \frac{\partial v}{\partial \theta} \frac{\partial d \theta}{\partial x} \frac{dt}{dt} \right) g \frac{\partial \theta}{\partial p}
$$

and is negative. This term does not contribute to the values of PV at any time other than between hours 66 and 69. Therefore, it can be safely concluded that the third diabatic term makes no significant contribution to values of PV during the three day forecast.

Figure 7.3g displays the wind velocity along the parcel trajectory. As the parcel initially moves toward the storm the velocity stays somewhat constant. After hour 27 the velocity increases as it moves in toward the central area of the storm, reaching just shy of 30 ms$^{-1}$ at hour 60. The velocity then decreases to 22 ms$^{-1}$ by hour 66, most likely due to the wide trajectory it takes around the storm. As the parcel reaches the center of the storm the wind velocity increases drastically to 58 ms$^{-1}$. This in, out, and in configuration of the trajectory can also explain the behavior of the first diabatic term that exhibits an increase, decrease, and an increase.

Overall, each term makes some contributions to the change in values of PV, as indicated by Table 7.1. The first and third diabatic terms make the least significant contribution to PV. The horizontal advection term tries to conserve the values of PV and makes no significant
contribution to the changes in PV. During the last three hours of the forecast, when PV makes a significant jump in value, the only term to influence it is the second diabatic term. Moreover, this term’s contributions are an order of magnitude larger than the contributions from any other term. Hence, it can be concluded that the second diabatic term has the most influence on the PV values.

Table 7.1. Values of 3-D PV, horizontal advection, first diabatic term, second diabatic term, and third diabatic term for trajectory 1 during the 72-hour forecast. Unit of PV in kg\(^{-1}\)m\(^2\)s\(^{-1}\)K. Units for components of PV tendency in kg\(^{-1}\)m\(^2\)s\(^{-2}\)K.

<table>
<thead>
<tr>
<th>Forecast hour</th>
<th>PV</th>
<th>Horizontal advection</th>
<th>First diabatic term</th>
<th>Second diabatic term</th>
<th>Third diabatic term</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00E-07</td>
<td>1.57E-12</td>
<td>8.13E-13</td>
<td>-1.82E-14</td>
<td>2.47E-12</td>
</tr>
<tr>
<td>3</td>
<td>1.03E-07</td>
<td>-3.62E-13</td>
<td>1.52E-12</td>
<td>-2.53E-13</td>
<td>-9.42E-14</td>
</tr>
<tr>
<td>6</td>
<td>1.12E-07</td>
<td>-3.21E-13</td>
<td>4.47E-13</td>
<td>6.87E-13</td>
<td>2.85E-15</td>
</tr>
<tr>
<td>9</td>
<td>9.66E-08</td>
<td>1.05E-12</td>
<td>3.34E-13</td>
<td>6.05E-13</td>
<td>-4.04E-15</td>
</tr>
<tr>
<td>12</td>
<td>1.03E-07</td>
<td>2.37E-12</td>
<td>5.45E-13</td>
<td>5.01E-13</td>
<td>3.43E-14</td>
</tr>
<tr>
<td>15</td>
<td>1.06E-07</td>
<td>1.70E-12</td>
<td>7.31E-13</td>
<td>5.56E-13</td>
<td>1.75E-14</td>
</tr>
<tr>
<td>18</td>
<td>9.94E-08</td>
<td>4.87E-13</td>
<td>7.81E-13</td>
<td>5.61E-13</td>
<td>1.53E-14</td>
</tr>
<tr>
<td>21</td>
<td>1.07E-07</td>
<td>-3.34E-14</td>
<td>7.87E-13</td>
<td>1.41E-13</td>
<td>1.12E-14</td>
</tr>
<tr>
<td>24</td>
<td>9.75E-08</td>
<td>-4.02E-13</td>
<td>4.61E-13</td>
<td>1.79E-13</td>
<td>-1.13E-14</td>
</tr>
<tr>
<td>27</td>
<td>7.90E-08</td>
<td>-7.07E-13</td>
<td>5.06E-13</td>
<td>4.71E-13</td>
<td>1.09E-14</td>
</tr>
<tr>
<td>30</td>
<td>8.02E-08</td>
<td>-3.26E-12</td>
<td>5.27E-13</td>
<td>3.92E-13</td>
<td>-6.09E-14</td>
</tr>
<tr>
<td>33</td>
<td>7.65E-08</td>
<td>-7.45E-12</td>
<td>3.76E-13</td>
<td>5.26E-13</td>
<td>-2.44E-14</td>
</tr>
<tr>
<td>36</td>
<td>7.87E-08</td>
<td>-5.81E-11</td>
<td>9.56E-13</td>
<td>3.23E-13</td>
<td>1.35E-14</td>
</tr>
<tr>
<td>39</td>
<td>7.90E-08</td>
<td>-2.65E-12</td>
<td>1.03E-12</td>
<td>3.65E-13</td>
<td>-7.64E-15</td>
</tr>
<tr>
<td>42</td>
<td>1.12E-07</td>
<td>2.61E-12</td>
<td>1.06E-12</td>
<td>7.19E-13</td>
<td>-2.09E-13</td>
</tr>
<tr>
<td>45</td>
<td>9.52E-08</td>
<td>1.82E-12</td>
<td>9.05E-13</td>
<td>5.51E-13</td>
<td>-8.52E-14</td>
</tr>
<tr>
<td>48</td>
<td>6.93E-08</td>
<td>2.51E-12</td>
<td>2.96E-13</td>
<td>9.66E-14</td>
<td>-2.31E-13</td>
</tr>
<tr>
<td>51</td>
<td>8.72E-08</td>
<td>4.30E-13</td>
<td>-3.07E-12</td>
<td>2.47E-12</td>
<td>-1.45E-13</td>
</tr>
<tr>
<td>54</td>
<td>1.23E-07</td>
<td>9.83E-13</td>
<td>9.91E-13</td>
<td>1.49E-13</td>
<td>2.03E-13</td>
</tr>
<tr>
<td>57</td>
<td>6.24E-08</td>
<td>5.90E-12</td>
<td>-4.76E-13</td>
<td>-2.97E-14</td>
<td>2.35E-15</td>
</tr>
<tr>
<td>60</td>
<td>1.01E-07</td>
<td>-6.67E-12</td>
<td>-3.71E-12</td>
<td>-5.05E-12</td>
<td>1.03E-12</td>
</tr>
<tr>
<td>63</td>
<td>1.50E-07</td>
<td>-8.24E-11</td>
<td>-4.31E-12</td>
<td>-1.83E-11</td>
<td>1.02E-11</td>
</tr>
<tr>
<td>66</td>
<td>8.17E-08</td>
<td>7.70E-11</td>
<td>1.32E-13</td>
<td>-1.65E-11</td>
<td>-7.61E-13</td>
</tr>
<tr>
<td>69</td>
<td>2.20E-08</td>
<td>6.54E-11</td>
<td>6.07E-12</td>
<td>1.63E-12</td>
<td>1.64E-12</td>
</tr>
<tr>
<td>72</td>
<td>3.16E-06</td>
<td>-2.67E-11</td>
<td>-7.71E-11</td>
<td>6.05E-10</td>
<td>-1.04E-10</td>
</tr>
</tbody>
</table>
Fig. 7.3. Calculated terms for trajectory 1 during the 72-hour forecast. Represented terms are a) pressure (mb), b) 3-D potential vorticity (m$^2$s$^{-1}$Kkg$^{-1}$), c) horizontal advection of PV (kg$^{-1}$m$^2$s$^{-2}$K), and d) first diabatic term (kg$^{-1}$m$^2$s$^{-2}$K).
Fig. 7.3, cont. Calculated terms for trajectory 1 during the 72-hour forecast. Represented terms are e) second diabatic term (kg m$^{-1}$ m$^2$ s$^{-2}$ K), f) third diabatic term (kg m$^{-1}$ m$^2$ s$^{-2}$ K), and g) wind velocity (ms$^{-1}$).
7.3.2 Trajectory 2

On 6 September 12Z a parcel is located at approximately 24ºN and 60ºW as indicated by Fig. 7.1. Following the trajectory forward in time, the parcel moves southwest, loops around, and heads northwest toward the storm. It then proceeds to move around the periphery of the storm following the surrounding winds on a counterclockwise trajectory. The parcel’s trajectory ends on 9 September 12Z at 26.5ºN and 66ºW, on the northern edge of the center of maximum winds.

Initially, the pressure along the trajectory is 848mb as indicated by Fig. 7.4a. There is a slight drop to 844mb in the subsequent 6 hours. For the next 60 hours, a gradual increase in the parcel’s pressure occurs as the parcel moves through an area of descending motion. Only one change in pressure occurs, a drop of 6mb at forecast hour 42. By hour 66 the pressure of the parcel is 946mb, a drop of approximately 100mb in 60 hours. As it moves through an area of ascending motion and into the storm center over the next 6 hours, the pressure decreases by approximately 100mb reaching 850mb.

The parcel’s initial 3-D PV value is $2.5 \times 10^{-7} \text{ m}^2 \text{s}^{-1} \text{Kkg}^{-1}$. During the first 12 hours of the trajectory, Fig. 7.4b reveals that the values of PV drop slightly and rise back to the initial value. Potential vorticity values then steadily decline to 0.0, but stay positive, by forecast hour 48. From hour 48 on, the values begin to increase, at first gradually and then a sharp increase. After hour 63, there is an increase until hour 66, after which PV values drop and then increase dramatically by $1.0 \times 10^{-6} \text{ m}^2 \text{s}^{-1} \text{Kkg}^{-1}$ within the last three forecast hours. Examining the 2-D trajectory’s PV in Fig. 7.2, it reveals that PV is nearly constant until hour 42, after which it increases, decreases, sharply increases, sharply decreases, and then stays constant in the last three hours to end at a value of $3.0 \times 10^{-7} \text{ m}^2 \text{s}^{-1} \text{Kkg}^{-1}$. Once again, it is obvious that PV is not conserved following the 2-D motion for the trajectory 2.

Values of horizontal advection of PV (from the 2-D trajectory) remain fairly constant near 0.0 through hour 39, as indicated by Fig. 7.4c. Thereafter a slight decrease in values between hours 39 to 45 followed by oscillations is noted. By hour 66, the value has reached nearly $5.3 \times 10^{-11} \text{ kg}^{-1} \text{m}^2 \text{s}^{-2} \text{K}$ followed by a sharp decrease to $-8.0 \times 10^{-11} \text{ kg}^{-1} \text{m}^2 \text{s}^{-2} \text{K}$ by the last forecast hour. Comparing the horizontal advection of PV to the values of 2-D PV, a dip in values of horizontal advection occurs between hours 39 and 45 while a small increase occurs in PV values. From hours 45 to 51, horizontal advection increases and decreases as PV continually...
decreases. Then, PV shows a steep rise in values after hour 54 while the horizontal advection term simply oscillates. From hours 63 to 66, the horizontal advection term decreases while the PV increases. During hours 66 to 69, both the PV and the horizontal advection term decrease, but during the last three forecast hours the PV values stay constant while the horizontal advection term continues to decrease. It can be concluded that horizontal advection of PV does have some impact on values of PV during the length of the trajectory. However, during the last three hours once the parcel reached the inner core of the storm the horizontal advection term does not contribute to the changes in PV noted from the model output.

The first diabatic term stays nearly constant at small, slightly positive values for the first 39 hours of the trajectory, as indicated by Fig. 7.4d. After hour 39, values of the first diabatic term begin to oscillate around 1.0E-11 kg⁻¹ m² s⁻² K of its constant value. The tendency during the last 33 hours, besides oscillating, is a slight drop in values to reach a final value of −7.0E-12 kg⁻¹ m² s⁻² K at hour 72. This term contributes to PV between hours 39 to 54 and 67 to 63. During the rest of the forecast hours, the first diabatic term does not contribute to the changes of PV.

Figure 7.4e indicates that the second diabatic term stays nearly constant near 0.0 through hour 54. A slight dip to negative values occurs from hours 54 to 66 with a rise back to 0.0 by hour 69. During the last three hours, there is a drastic increase of 4.9E-10 kg⁻¹ m² s⁻² K due to a positive \( \frac{\partial}{\partial \theta} \frac{d\theta}{dt} \) term leading to a positive second diabatic term. This can be seen Fig. 7.4e, as the term dramatically increases during that time. The term’s negative values between hours 54 and 66 do not contribute to changes in PV as the values of PV are increasing during that time period. However, both terms significantly increase during the last three forecast hours and it can be concluded that the second diabatic term makes the most significant contribution to the changes in PV during that time.

Figure 7.4f shows that the third diabatic term stays constant at 0.0 until hour 39. For the following nine hours the values show small oscillations around 0.0, but then return to 0.0 at hour 48 until hour 57. An extremely small rise and drop of values back to 0.0 occurs over the next nine hours that leaves the values in the positive range during that time. From hours 66 to 69, the third diabatic term rises to 1.0E-11 kg⁻¹ m² s⁻² K and within the last three hours drops by 8.5E-11 kg⁻¹ m² s⁻² K. The last three hours may appear to be a small period, however, a parcel flow covers a distance of nearly 100km in that time. The same reasoning used in trajectory 1 to explain the
negative values during the last three forecast hours can be used here. Based on Fig. 7.1, it can be assumed that the parcel is moving north, therefore, the zonal wind component is small, i.e. $u=0$.

Examination of vertical cross-sections of the meridional wind reveals that $\frac{\partial v}{\partial \theta}$ is positive. The term $\frac{\partial \theta}{\partial x} \frac{d\theta}{dt}$ is positive as well, while $g \frac{\partial \theta}{\partial p}$ is known to be negative. Therefore, the third diabatic term in scalar form becomes $\left( \frac{\partial v}{\partial \theta} \frac{\partial \theta}{\partial x} \frac{d\theta}{dt} \right) g \frac{\partial \theta}{\partial p}$ and is negative. The third diabatic term appears to have made a very small contribution to the changes in PV between hours 57 to 66, but makes no other contribution at any other time. Therefore, it can be safely concluded that the third diabatic term makes no significant contribution to changes in PV during the three day forecast.

Figure 7.4g displays the wind velocity along the parcel trajectory. The parcel begins its movement in the area of strengthening winds associated with the storm, as can be seen in the first 12 hours of Fig. 3g. For the next 16 hours the parcel moves just outside of the area of strong winds and its velocity decreases to 2 ms$^{-1}$. After hour 30, it once again moves toward the center of the storm and its velocity increases to 35 ms$^{-1}$. The parcel is then pulled outside of the central storm area and has a decrease in wind velocity. By hour 63, it once again begins to move into the central storm area and by the last forecast hour it is located in the area of maximum winds with a velocity of 55 ms$^{-1}$.

As far as the influence of the terms on the values of PV, trajectory 2 is similar to trajectory 1, as indicated by Table 7.2. However, the horizontal advection term has a larger influence on the values of PV as compared to trajectory 1, but it is still the second diabatic term that has the greatest impact on changing the values of PV. This is especially true during the last three forecast hours of the trajectory (when the parcel covers a distance of nearly 100km, i.e. the inner rain area of the storm) as it is an order of magnitude larger than the rest of the terms.
Table 7.2. Values of 3-D PV, horizontal advection, first diabatic term, second diabatic term, and third diabatic term for trajectory 2 during the 72-hour forecast. Unit of PV in kg m\(^{-1}\) m\(^2\) s\(^{-1}\) K. Units for components of PV tendency in kg m\(^{-1}\) m\(^2\) s\(^{-2}\) K.

<table>
<thead>
<tr>
<th>Forecast hour</th>
<th>PV</th>
<th>Horizontal advection</th>
<th>First diabatic term</th>
<th>Second diabatic term</th>
<th>Third diabatic term</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.53E-07</td>
<td>-9.77E-13</td>
<td>7.57E-13</td>
<td>1.98E-14</td>
<td>-8.79E-14</td>
</tr>
<tr>
<td>3</td>
<td>2.65E-07</td>
<td>-7.40E-13</td>
<td>6.57E-13</td>
<td>-1.15E-14</td>
<td>-1.77E-13</td>
</tr>
<tr>
<td>6</td>
<td>2.17E-07</td>
<td>-5.94E-13</td>
<td>6.77E-13</td>
<td>-1.72E-13</td>
<td>-2.08E-14</td>
</tr>
<tr>
<td>9</td>
<td>2.39E-07</td>
<td>-3.95E-13</td>
<td>1.88E-12</td>
<td>1.19E-12</td>
<td>2.83E-14</td>
</tr>
<tr>
<td>12</td>
<td>2.76E-07</td>
<td>-2.55E-13</td>
<td>2.85E-12</td>
<td>1.18E-12</td>
<td>4.30E-14</td>
</tr>
<tr>
<td>15</td>
<td>2.49E-07</td>
<td>-9.86E-14</td>
<td>2.71E-12</td>
<td>1.09E-12</td>
<td>5.24E-14</td>
</tr>
<tr>
<td>18</td>
<td>2.55E-07</td>
<td>-3.22E-13</td>
<td>2.45E-12</td>
<td>2.44E-13</td>
<td>1.18E-14</td>
</tr>
<tr>
<td>21</td>
<td>2.08E-07</td>
<td>-4.83E-13</td>
<td>1.56E-12</td>
<td>1.34E-13</td>
<td>-6.88E-14</td>
</tr>
<tr>
<td>24</td>
<td>2.22E-07</td>
<td>-6.12E-13</td>
<td>1.52E-12</td>
<td>3.24E-13</td>
<td>-6.44E-15</td>
</tr>
<tr>
<td>27</td>
<td>2.00E-07</td>
<td>-1.16E-12</td>
<td>1.26E-12</td>
<td>4.01E-13</td>
<td>3.31E-14</td>
</tr>
<tr>
<td>30</td>
<td>1.71E-07</td>
<td>-1.63E-12</td>
<td>1.30E-12</td>
<td>3.43E-13</td>
<td>-2.53E-14</td>
</tr>
<tr>
<td>33</td>
<td>1.60E-07</td>
<td>-1.78E-12</td>
<td>1.77E-12</td>
<td>5.83E-13</td>
<td>2.01E-14</td>
</tr>
<tr>
<td>36</td>
<td>9.19E-08</td>
<td>-8.77E-13</td>
<td>8.22E-13</td>
<td>5.14E-13</td>
<td>2.47E-14</td>
</tr>
<tr>
<td>39</td>
<td>6.17E-08</td>
<td>1.24E-13</td>
<td>7.21E-14</td>
<td>5.62E-13</td>
<td>1.20E-13</td>
</tr>
<tr>
<td>42</td>
<td>1.35E-07</td>
<td>-3.80E-12</td>
<td>8.91E-12</td>
<td>1.41E-12</td>
<td>-1.28E-12</td>
</tr>
<tr>
<td>45</td>
<td>1.36E-08</td>
<td>-5.68E-12</td>
<td>2.41E-12</td>
<td>-1.14E-12</td>
<td>2.11E-12</td>
</tr>
<tr>
<td>48</td>
<td>5.96E-09</td>
<td>6.91E-12</td>
<td>-4.68E-12</td>
<td>-2.86E-12</td>
<td>-2.98E-13</td>
</tr>
<tr>
<td>51</td>
<td>2.94E-08</td>
<td>-1.92E-11</td>
<td>-4.21E-12</td>
<td>-3.19E-12</td>
<td>1.55E-14</td>
</tr>
<tr>
<td>54</td>
<td>7.46E-08</td>
<td>1.09E-11</td>
<td>-2.83E-12</td>
<td>-5.30E-12</td>
<td>1.87E-13</td>
</tr>
<tr>
<td>57</td>
<td>1.64E-07</td>
<td>1.05E-12</td>
<td>-6.38E-12</td>
<td>-1.18E-11</td>
<td>3.47E-13</td>
</tr>
<tr>
<td>60</td>
<td>2.09E-07</td>
<td>1.33E-11</td>
<td>-4.96E-12</td>
<td>-1.67E-11</td>
<td>1.68E-12</td>
</tr>
<tr>
<td>63</td>
<td>2.24E-07</td>
<td>1.84E-11</td>
<td>-3.72E-12</td>
<td>-2.08E-11</td>
<td>1.07E-12</td>
</tr>
<tr>
<td>66</td>
<td>4.55E-07</td>
<td>5.32E-11</td>
<td>-6.01E-12</td>
<td>-3.82E-11</td>
<td>3.20E-13</td>
</tr>
<tr>
<td>69</td>
<td>2.93E-07</td>
<td>-2.96E-12</td>
<td>-1.08E-12</td>
<td>3.33E-12</td>
<td>1.03E-11</td>
</tr>
<tr>
<td>72</td>
<td>1.38E-06</td>
<td>-8.05E-11</td>
<td>-7.14E-12</td>
<td>4.90E-10</td>
<td>-7.51E-11</td>
</tr>
</tbody>
</table>
Fig. 7.4. Calculated terms for trajectory 2 during the 72-hour forecast. Represented terms are a) pressure (mb), b) 3-D potential vorticity (m$^2$s$^{-1}$Kkg$^{-1}$), c) horizontal advection of PV (kg$^{-1}$m$^2$s$^{-2}$K), and d) first diabatic term (kg$^{-1}$m$^2$s$^{-2}$K).
Fig. 7.4, cont. Calculated terms for trajectory 2 during the 72-hour forecast. Represented terms are e) second diabatic term (kg$^{-1}$m$^2$s$^{-2}$K), f) third diabatic term (kg$^{-1}$m$^2$s$^{-2}$K), and g) wind velocity (ms$^{-1}$).
7.3.3 Trajectory 3

On 6 September 12Z a parcel is located at approximately 16.8°N and 54.8°W as indicated by Fig. 1. Following the trajectory forward in time, the parcel moves west and then northwest towards the storm. It moves counterclockwise around the perimeter of the storm, moving into the center during the last 12 hours. The parcel’s trajectory ends on 9 September 12Z at 25.5°N and 65.5°W, on the southern edge of the center of maximum winds.

Initially, the parcel pressure along the trajectory is 866mb as indicated by Fig. 7.5a. During the first 42 hours the pressure increases by 140mb reaching 909mb. From this point, the parcel travels along a path through ascending air increasing the pressure by approximately 90mb over twelve hours. The pressure then rises to 946mb by hour 63. Over the last nine hours, the parcel moves west into ascending air and its pressure increases by approximately 100mb to finally reach 850mb.

Figure 7.5b reveals that for the first 60 hours along the trajectory, the values of 3-D PV remain constant with a slightly positive value. Over the next three hours, there is a slight rise to 2.3E-07 m²s⁻¹Kkg⁻¹ and then a drop over the next six hours to approximately 0.0. Within the last three hours, the PV rises dramatically to values around 2.1E-06 m²s⁻¹Kkg⁻¹. Examining the 2-D trajectory’s PV in Fig. 7.2, it reveals that values oscillate along the whole trajectory. As the forecast progresses, the oscillations grow in amplitude and during the last three forecast hours the value of PV increases finally reaching 5.2E-07 m²s⁻¹Kkg⁻¹. As a result, it is concluded that the PV is not conserved following the 2-D motion of trajectory 3 along an isentropic surface.

Values of horizontal advection of PV remain constant at 0.0 through hour 33, as indicated by Fig. 7.5c. At hour 39, there is a decrease of 7.7E-11 kg⁻¹m²s⁻²K that corresponds to a decrease in values of 2-D PV. After hour 39, values of horizontal advection return to 0.0 until hour 60. Once again there is a sharp decrease to -2.7E-10 corresponding to a rise in PV to 7.2E-07 kg⁻¹m²s⁻²K at hour 63. During the last three forecast hours the horizontal advection term drops to -7.8E-11 kg⁻¹m²s⁻²K, while the PV increases. As with trajectory 1, the oscillations in the horizontal advection term arise due to the flow field of the PV maximums and minimums. As the parcel moves into a PV maximum it horizontally advects low-PV air into the area, hence, the dip in values of the horizontal advection of PV and the rise in values of PV and vice versa. The horizontal advection of PV tries to conserve PV, however, as was already mentioned, PV is not
conserved following the 2-D trajectory. Therefore, the horizontal advection of PV cannot alone explain the changes in potential vorticity noted here.

The first diabatic term stays constant at 0.0 for the first 60 hours of the trajectory, as indicated by Fig. 7.5d. For the following nine hours, the values oscillate between 3.0E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K and –3.5E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K. During the last three hours as the parcel moves into a PV maximum \(\frac{\partial}{\partial \theta} PV\) is positive, hence, \(- \frac{d\theta}{dt} \frac{\partial}{\partial \theta} PV\) is negative and the term’s value drops by 1.8E-10 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K. This term contributes to values of PV around hour 60 to 66, but makes no other contribution at any other time and does not contribute to the major PV changes of the last three forecast hours.

Figure 7.5e illustrates that the second diabatic term stays constant at 0.0 until hour 60, after which it decreases to –7.6E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K. From hours 63 and on, the second diabatic term increases by 2.4E-10 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K to positive values. During the last three hours, \(\frac{\partial}{\partial \theta} \frac{d\theta}{dt}\) is positive leading to a positive second diabatic term, ending at a value of 1.7E-10 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K. This term’s change in values exhibits the same pattern as that of PV during the last three forecast hours and contributes to the decrease in PV between hours 63 to 69 as it is negative during that time. Hence, it can be concluded that the second diabatic term significantly contributes to the change in PV during the last nine forecast hours. In 12 hours, at 5ms\(^{-1}\), the parcel easily traverses a distance of 160km, thus this diabatic term covers a large area.

The third diabatic term, as with the other terms, stays constant near 0.0 for the first 57 hours and then increases slightly during the subsequent three hours, as indicated by Fig. 7.5f. Between hours 60 and 63, a sharp decrease to –1.12E-10 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K occurs, while values of PV increase slightly. During the next 3 hours, a sharp increase to 1.0E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K occurs, while PV values decrease. The last 6 hours brings the third diabatic term’s value down by 5.5E-11 kg\(^{-1}\)m\(^2\)s\(^{-2}\)K, while PV slightly decreases and then increases in value during the same time frame. Again, the reasoning used to explain negative values of the third diabatic term in trajectory 1 and 2 during the last three forecast hours can be applied here. Hence, it can safely be concluded that the third diabatic term makes no significant contribution to values of PV during the three day forecast.
Figure 4g displays the wind velocity along the parcel trajectory. As the parcel moves toward the storm and around the perimeter of it, the velocity gradually increases. By hour 60, the parcel begins to move into the central area of the storm, as indicated by the increasing wind velocity after hour 60. There is a dip in wind velocity at hour 69 most likely caused by the movement of the parcel outside the central storm area. As the parcel reaches the center of maximum winds its velocity increases to 52 ms\(^{-1}\).

The difference between this trajectory and the other trajectories is that the orders of magnitude of the horizontal advection, first diabatic, and second diabatic terms are all similar, as indicated by Table 7.3. The horizontal advection term tries to conserve the values of PV and makes no significant contribution to the changes in PV. Besides the last three forecast hours, the three diabatic terms make contributions to the values of PV. Nevertheless, it can still be seen that the second diabatic term makes the most significant contribution to PV in the last three hours of the trajectory when the parcel is inside the center of the storm.

In conclusion, the horizontal advection term tried to conserve the values of PV in trajectory 1 and 3 and it made no significant contribution to the changes in PV. In trajectory 2, the horizontal advection term did have an impact on values of PV during the length of the trajectory. However, once the parcel reached the inner core of the storm the term did not contribute to the changes. The first diabatic term made some contributions to PV, but it was the second diabatic term that had the greatest impact on PV values. It was the only term to contribute to the PV during the last three forecast hours. During that time, it continually increased while the other terms showed a decrease in values. In three hours, a parcel traveling into the inner core of a storm can easily cover a large distance. In all three trajectories, the parcel traveled nearly 400km in the last three hours (trajectory 3 was slightly less than this). This indicates that the second diabatic term is the term that is most significantly influencing the potential vorticity at the center of the storm. The third diabatic term seemed to make the least contribution to PV.
Table 7.3. Values of 3-D PV, horizontal advection, first diabatic term, second diabatic term, and third diabatic term for trajectory 3 during the 72-hour forecast. Unit of PV in kg\(^{-1}\) m\(^2\) s\(^{-1}\) K. Units for components of PV tendency in kg\(^{-1}\) m\(^2\) s\(^{-2}\) K.

<table>
<thead>
<tr>
<th>Forecast hour</th>
<th>PV</th>
<th>Horizontal advection</th>
<th>First diabatic term</th>
<th>Second diabatic term</th>
<th>Third diabatic term</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8.05E-08</td>
<td>-1.31E-13</td>
<td>-1.11E-13</td>
<td>3.50E-13</td>
<td>-3.67E-13</td>
</tr>
<tr>
<td>3</td>
<td>7.87E-08</td>
<td>-1.76E-13</td>
<td>3.82E-13</td>
<td>3.73E-13</td>
<td>-3.52E-14</td>
</tr>
<tr>
<td>6</td>
<td>6.22E-08</td>
<td>-2.46E-13</td>
<td>3.02E-13</td>
<td>3.19E-13</td>
<td>1.60E-15</td>
</tr>
<tr>
<td>9</td>
<td>6.09E-08</td>
<td>-4.98E-13</td>
<td>4.51E-13</td>
<td>3.42E-13</td>
<td>8.25E-15</td>
</tr>
<tr>
<td>12</td>
<td>5.53E-08</td>
<td>9.97E-14</td>
<td>5.31E-13</td>
<td>3.28E-13</td>
<td>1.43E-14</td>
</tr>
<tr>
<td>15</td>
<td>6.23E-08</td>
<td>9.92E-13</td>
<td>5.90E-13</td>
<td>3.35E-13</td>
<td>1.31E-14</td>
</tr>
<tr>
<td>18</td>
<td>4.63E-08</td>
<td>2.09E-13</td>
<td>5.29E-13</td>
<td>2.05E-13</td>
<td>6.00E-15</td>
</tr>
<tr>
<td>21</td>
<td>4.79E-08</td>
<td>-3.35E-12</td>
<td>4.68E-13</td>
<td>1.30E-13</td>
<td>-3.51E-15</td>
</tr>
<tr>
<td>24</td>
<td>3.71E-08</td>
<td>-3.06E-12</td>
<td>2.64E-13</td>
<td>1.23E-13</td>
<td>-1.41E-13</td>
</tr>
<tr>
<td>27</td>
<td>4.54E-08</td>
<td>-2.40E-12</td>
<td>2.62E-13</td>
<td>-2.18E-14</td>
<td>-5.49E-14</td>
</tr>
<tr>
<td>30</td>
<td>3.84E-08</td>
<td>-1.19E-13</td>
<td>2.24E-13</td>
<td>3.08E-14</td>
<td>-3.82E-14</td>
</tr>
<tr>
<td>33</td>
<td>3.58E-08</td>
<td>-4.60E-12</td>
<td>3.66E-13</td>
<td>1.96E-14</td>
<td>2.66E-14</td>
</tr>
<tr>
<td>36</td>
<td>3.92E-08</td>
<td>-1.86E-11</td>
<td>3.98E-13</td>
<td>7.80E-14</td>
<td>2.33E-14</td>
</tr>
<tr>
<td>42</td>
<td>5.85E-08</td>
<td>-9.32E-12</td>
<td>4.26E-13</td>
<td>1.14E-13</td>
<td>3.10E-13</td>
</tr>
<tr>
<td>45</td>
<td>9.83E-08</td>
<td>1.87E-12</td>
<td>2.59E-13</td>
<td>-7.49E-13</td>
<td>1.23E-14</td>
</tr>
<tr>
<td>48</td>
<td>1.12E-07</td>
<td>1.76E-12</td>
<td>-7.15E-14</td>
<td>1.09E-12</td>
<td>-1.81E-13</td>
</tr>
<tr>
<td>51</td>
<td>7.86E-08</td>
<td>-7.27E-13</td>
<td>-1.19E-13</td>
<td>6.84E-13</td>
<td>8.50E-14</td>
</tr>
<tr>
<td>54</td>
<td>8.35E-08</td>
<td>-3.46E-12</td>
<td>8.87E-13</td>
<td>3.14E-13</td>
<td>-7.88E-14</td>
</tr>
<tr>
<td>57</td>
<td>1.07E-07</td>
<td>4.43E-12</td>
<td>5.18E-14</td>
<td>8.40E-13</td>
<td>2.16E-13</td>
</tr>
<tr>
<td>60</td>
<td>1.03E-07</td>
<td>1.41E-12</td>
<td>-1.74E-12</td>
<td>-1.33E-12</td>
<td>1.80E-12</td>
</tr>
<tr>
<td>63</td>
<td>2.30E-07</td>
<td>-2.68E-10</td>
<td>3.02E-11</td>
<td>-7.65E-11</td>
<td>-1.13E-10</td>
</tr>
<tr>
<td>69</td>
<td>-1.60E-08</td>
<td>6.21E-11</td>
<td>1.29E-11</td>
<td>7.30E-12</td>
<td>4.06E-12</td>
</tr>
<tr>
<td>72</td>
<td>2.11E-06</td>
<td>-7.81E-11</td>
<td>-1.72E-10</td>
<td>1.68E-10</td>
<td>-4.48E-11</td>
</tr>
</tbody>
</table>
Fig. 7.5. Calculated terms for trajectory 3 during the 72-hour forecast. Represented terms are a) pressure (mb), b) 3-D potential vorticity (m$^2$s$^{-1}$Kkg$^{-1}$), c) horizontal advection of PV (kg$^{-1}$m$^2$s$^{-2}$K), and d) first diabatic term (kg$^{-1}$m$^2$s$^{-2}$K).
Fig. 7.5, cont. Calculated terms for trajectory 3 during the 72-hour forecast. Represented terms are e) second diabatic term (kg⁻¹m²s⁻２K), f) third diabatic term (kg⁻¹m²s⁻２K), and g) wind velocity (ms⁻¹).
CHAPTER 8
SUMMARY AND CONCLUSIONS

8.1 Discussion and Summary

Through the diagnosis of Ertel’s full PV equation, specifically the contribution from each component to hurricane formation, this study aims to assess the role of non-conservation versus conservation of PV in hurricane genesis. In Chapter 1, historical hurricane formation mechanisms are explored. Included among these mechanisms are the influence of warm sea surface temperatures, the absence of extreme vertical shear of the horizontal wind, the influence of closed lows and upper-level troughs on the incipient disturbance, and internal dynamical processes.

Potential vorticity as a framework for hurricane genesis is examined in Chapter 2. Since approximately 30% of every 100km square within a hurricane is occupied by deep convective elements it is proposed that non-conservation of PV, specifically the second diabatic term which is a diabatic stretching term for the generation of PV, is the underlying mechanism that forms Hurricane Erin.

The role of barotropic instability and combined barotropic/baroclinic instability in hurricane formation is explained in Chapter 3. The vanishing of the meridional gradient of PV somewhere in the domain of interest infers that the necessary condition for combined instability has been met. It is shown that the incipient African wave does satisfy the conditions for both barotropic and combined instability.

Chapter 4 details a barotropic experiment that examines the role of conservation of absolute vorticity in the genesis of Hurricane Erin. The primary mechanism for formation under barotropic dynamics, in this case, is a conversion from shear vorticity to curvature vorticity. Results from the experiment indicate that the movement of the easterly wave approximately parallels the observed movement of the hurricane and is, therefore, a reasonable approximation
of the motion. However, the wave fails to develop due to the failure of curvature vorticity to develop and it is concluded that barotropic dynamics alone does not succeed in forming the storm.

An overview of the FSU global spectral model and the computation methods of PV and associated terms is given in Chapter 6. The FSUGSM is a model that contains full dynamics and physics including the effects from diabatic heating and provides reasonable simulation on hurricane genesis. From the model output the horizontal advection of PV, vertical advection of PV, differential heating along the vertical, differential heating along the horizontal, and the frictional components are derived.

Chapters 7 and 8 focus on the data sets derived from the FSU global spectral model. In particular, the budgets sort out the relative contribution to PV changes from each component of the complete PV equation. Trajectories were plot in Chapter 8 to examine the PV changes following the 3-D and 2-D motion of a parcel.

The results demonstrate that in the inner rain area of the storm (r<100km) a gradual increase of PV from 6.5E-07 m^2 s^-1 Kg^-1 to 5.5E-06 m^2 s^-1 Kg^-1 occurs on the s=0.85 surface during the 72-hour forecast. Examining the dry static stability during the forecast, we observe that this term only increases from 2.6E-03 m^2 Kg^-1 to 6.5E-03 m^2 Kg^-1 on the s=0.85 surface leading us to conclude that the increase in PV is due to the marked increase in absolute vorticity. Given that the Coriolis force does not significantly change during the 72-hour forecast we can also conclude that this increase in PV is due mainly to the relative vorticity, which increases from 9.0E-05s^-1 to 7.0E-04s^-1. The corresponding increase in the wind speed is from 10ms^-1 to 50 ms^-1.

The analysis clearly shows that during hurricane genesis the substantial increase in PV can be largely attributed to the second diabatic term, which is a product of PV and the vertical derivative of the net heating. This term contributes to a net generation of PV over a region of stable air with cyclonic vorticity and increased heating with height. The second diabatic term,

\[ PV \frac{\partial}{\partial \theta} \frac{d \theta}{dt} \]

is akin to the vortex stretching term, \( \zeta_a \frac{\partial}{\partial p} \omega \), in the vorticity equation and is the diabatic stretching of the vortex tube. Additionally, the effect that convergence has on vorticity in pressure coordinates is entirely analogous to the effect that the diabatic stretching term has on PV in isentropic coordinates. That is, convergence produces vorticity that in turns forms a storm

81
while diabatic stretching produces PV, which translates to vorticity, that again translates to formation of a storm.

Trajectory calculations show that the values of PV and each component of the PV tendency equation increase or decrease drastically once reaching the inner core of the storm, i.e. \( r < 300 \text{ km} \). In each trajectory calculation it is found that diabatic stretching is the only term to contribute systematically to the PV in the last three forecast hours as it continually increases while other terms decrease. In three hours, a parcel traveling into the inner core of a storm can easily cover a large distance indicating that diabatic stretching is most significantly influencing the potential vorticity at the center of the storm.

### 8.2 Future Work

The TRMM satellite provides proxies for the vertical distribution of heating from the microwave radiometer as well as the TRMM radar data sets. It may be useful to conduct an experiment using multiple aircraft to get a detailed picture of the circulation and the satellite based heating all derived from observations alone since the use of models incorporates model biases into the results. Such a possibility exists during CAMEX 5 and TEXMEX II.

The Convection and Moisture Experiment (CAMEX) is a sequence of field research investigations that focus on the study tropical cyclone genesis, tracking, landfall impacts, and intensification. Planning for CAMEX 5 is now underway for 2005 and it will examine formation and intensification issues. The CAMEX mission employs aircraft that flies over, through, and around tropical cyclones and storms before landfall occurs.

TEXMEX II, the Texas Mexico Experiment, (Emanuel 2003) is a field experiment proposed for the year 2005 by Dr. Kerry Emanuel of MIT. This experiment is being proposed to go on during the same time as those of the CAMEX 5 operation. These are eastern Pacific hurricanes of the northern summer months. The central scientific issue is genesis of hurricanes. Observational resources for this study include Doppler radar, dynamic measurements, thermodynamic measurements, and cloud physical measurements. It will work to resolve in time the evolution of a hurricane by employing aircraft deployed every 12 hours. The environment in which the storms are occurring will also be examined also with the use of dropsondes deployed by high-altitude aircraft in 24-hour intervals. This is another possible opportunity for a fully observational look at the diabatic PV issues that are studied here with model data sets.
If several storms are monitored during the formative stage then we can reassess the role of diabatic PV by carrying out detailed data assimilation and diagnosis without referencing to a forecast where model biases can abound.

In a forthcoming paper by Krishnamurti et al. (2003), a discussion of scale interactions among deep convective cloud scales and the hurricane scale are addressed. That study was motivated from the findings reported here.

Some important interpretation of PV, the invertibility principle, and the use of moist static stability within the definition of PV have been used by Schubert et al. (2001), Molinari et al. (1998), and Davis and Bosart (2002). These studies open up future avenues for the extension of our work presented here.
REFERENCES

<http://www.boi.noaa.gov/training/ipv/ipv2.html>.


Kanamitsu, M., 1975.  On numerical prediction over a global tropical belt.  Report No. 75-1, Department of Meteorology, Florida State University, Tallahassee, FL 32306, 142 pp.


Krishnamurti, T.N., A. Kumar, and X. Li, 1986: Results of extensive integrations with simple NWP models over the entire tropics during FGGE. *Tellus*, 39A, 152-160.


<http://kingfish.coastal.edu/physics/var/paper/Limp_Et_Al_I.pdf>.


BIOGRAPHICAL SKETCH

Leela Ramaswamy was born on June 9th, 1976 in Longmeadow, Massachusetts. In 1994, she graduated from Longmeadow High School and later that year enrolled at the University of Miami from which she graduated in 1998. Leela obtained her Bachelor’s degree in Environmental Science and later worked as an Environmental Scientist at EnviroCare, Inc. in Ft. Lauderdale, FL. After graduation, she also took time off to travel throughout Europe and Costa Rica. Leela began her graduate education in meteorology at Florida State University in the fall of 2001. In 2003, she was accepted into Chi Epsilon Pi, the meteorology honor society.