A Demonstration of the Three-Level Hierarchical Generalized Linear Model Applied to Educational Research

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A DEMONSTRATION OF THE THREE-LEVEL HIERARCHICAL GENERALIZED LINEAR MODEL APPLIED TO EDUCATIONAL RESEARCH

By
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This dissertation is dedicated
to my grandfather, Pundit Kashinath Subedi,
who is not in this world today
but
inspired me to be learned since my early childhood…
and
to my father who supported the value of my education all the times…
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ABSTRACT

The purpose of this study is to demonstrate a three-level hierarchical generalized linear model (HGLM) applied to educational research. The sequential steps for developing, analyzing, evaluating, and applying the three-level HGLM are demonstrated. Further, the effects of predictors are interpreted using the simple effect and ANOVA-like approaches. Specifically, interpretation of predictors’ effects using the simple effect approach is of the central importance that provides a unique contribution to the field of study.

This study used NAEP 2000 Reading data for fourth graders in order to demonstrate three-level HGLM using a dichotomous outcome. Reading proficiency or non-proficiency was used as an outcome in the model. A sample of 7,175 students, 1,076 teachers, and 295 schools from 46 states were selected in the study. Student, teacher, and school level data were used as level-1, level-2, and level-3 units respectively in the analysis.

The initial step for model demonstration was provided with a fully unconditional model. Further steps illustrated the logic of selecting potential predictors in level-1, level-2, and level-3 conditional models. Race (minority), and student SES were selected as level-1 predictors. Similarly, class type and school mean SES were selected as level-2, and level-3 predictors respectively. An exploratory analysis was performed in order to select such predictors in level-2 and level-3 conditional models. Thus, the final step demonstrated the selection, analysis, evaluation, and application of the final model. The criteria used for evaluating the final model were the following: the selection of potential predictors, retaining significant variance components, adequate proportion of variance explained, and desirable estimates of reliability. Most of the parameter estimates and all of the variance components were found significant. In level-3 model, all equations, except one, were formulated as random effects model.
Two methods used for effect interpretation were the ANOVA-like approach and the simple effect approach. In an ANOVA-like approach, the odds of reading proficiency for each cell mean as well as main and interaction effects were interpreted. The probability of reading proficiency for a specific group of students associated with different levels of other predictors was interpreted using the main effect approach. The two-way (second-order), and three-way (third-order) interaction effects were also interpreted despite their complexity in interpretation. For example, the two-way interaction effect of minority and school mean SES was 6.9; this effect was interpreted as the factor increase of the odds ratio associated with the difference in minority status of students due to two-standard deviation increase in school SES for a reference teacher. Further, the magnitude of the three-way interaction effect of minority, class type, and school mean SES was 0.03; this effect was interpreted as the factor increase in the second-order interaction between minority and class type due to two-standard deviation increase in school mean SES.

The simple effects on odds of reading proficiency due to different predictors were computed and interpreted. Two effects of school SES, three effects of class type, and two effects of minority were significant. For instance, the school mean SES effect for minority students was defined as the odds ratio associated with a change of two standard deviations in school mean SES for minority students in a non-crowded class. This further was interpreted as an increase in odds of reading proficiency for minority students in non-crowded class by a specific factor (say, 86 based on analysis results) when school mean SES is increased by two standard deviations.

Graphical methods were presented to interpret the effects for class type, and minority effects. Three effects of class type and two effects of minority on odds of reading proficiency were significant. The effect of class type associated with low SES for minority was significant below 0.6 on school mean SES in which range the estimated effect decreased from 2342.7 to 4.2 as school mean SES increased from –1.3 to 0.6. Further, the effects of minority for both the crowded, and non-crowded class types were significant. For crowded class type, the effect was significant from -0.69 to 0.41 on school mean SES, over which range the estimated effect decreased from 15.2 to 4.9 as
school mean SES increased from -0.69 to 0.41. Thus, the graphical presentation was provided in order to facilitate the interpretation of three-level HGLM results in a simpler way.

The research practitioners can replicate the procedural steps of demonstrating a three-level HGLM, and presenting the simple effect as well as ANOVA-like interpretation approaches of predictors. Despite the complexity of the process in computing effects in the simple effect approach, researchers can interpret effects with less complication using this approach compared to the traditional HGLM approach.

The major limitation of this study is that sampling weights associated with students and schools are ignored while analyzing the data. Sampling weights are desired for NAEP data analysis, since the data used in this study are based on stratified complex sampling. Further, the sampling weights are sought not only since each selected school and assessed student represents a fraction of the population of interest, but also NAEP oversamples non-public schools. Thus, sampling weights adjust for disproportionate representation due to such oversampling and make valid inferences between the student samples and the respective populations from which they were drawn. This limitation exists due to the restricted feature of HLM 5 in handling the sampling weights for the three-level HGLM. Nonetheless, the analysis of data without incorporating weights would facilitate the purpose of demonstration of the procedure sought by this study.

The major caveats of this study are the high estimated effects of SES, confidence intervals associated with this effect, and consequently, huge odds ratios produced due to the high effects. It is recommended that research practitioners use the real dichotomous outcome, such as dropout, in order to provide more meaningful implications of study results.
CHAPTER 1
INTRODUCTION

This study sequentially demonstrates a modeling technique using student outcome (proficiency) with hierarchical data structure in educational setting. It is commonly experienced that various student-level factors (independent variables) may affect academic proficiency of a student. For example, reading proficiency of a student (proficiency or non-proficiency status in reading, a dichotomous outcome) may depend on student attributes such as whether a student is a White or non-White or whether a student is participating in free and reduced lunch program. This kind of relationship leads to the single-level modeling (at student-level) of the effect of independent variables (predictors) on dichotomous dependent variable (outcome) forming a nonlinear model.

Hierarchical data structures are present in educational settings where students are nested within a teacher and teachers are nested within a school. The nesting form of data structure generates a Hierarchical Linear Model (HLM). In other words, models at different levels can be built based on specific number of (lower level) units nested within upper level, eventually forming a HLM design. In the situations where such nesting occurs, the relationship between outcome and predictors can be extended to more than one level. Thus, academic proficiency of a student can be predicted not only due to student-level variables but also due to teacher-level predictors in the case where a hierarchy structure of data is present. The student reading proficiency, for example, can be predicted due to teacher-level predictor (along with student-level predictor) such as the class size that a teacher is involved in teaching.

The concept of hierarchy can be further extended to school-level where student reading proficiency can also be predicted due to a school-level predictor (e.g., average school socioeconomic status) including teacher and student-level predictors. In such
modeling practice, the statistical model is developed using the predictors at student, teacher and school levels in order to predict the outcome at student-level. One of the purposes of this study is to demonstrate the modeling approach of three-level nonlinear model (namely, hierarchical generalized linear model or HGLM to be discussed later), in step-by-step basis, using a student-level outcome and student-, teacher-, and school-level predictors.

It is commonly experienced that the interpretation of the predictors’ effects on student reading proficiency becomes more complex as the level of analysis is higher than one level. Specifically, the interpretation becomes complicated based on three-level modeling (HGLM) results. Therefore, another purpose of this study is to describe the effect of predictors, on student reading proficiency, in simpler ways using simple effect and ANOVA-like approaches. Graphical and tabular forms of demonstrations are presented for the descriptions of predictors’ effects.

In order to enable us to perform the simultaneous analysis of multilevel data, the HLM technique has become increasingly popular. Many decisions in educational policy and evaluation are based on the application of HLM analysis. For the purpose of analyzing hierarchical data, researchers such as Raudenbush & Bryk (2002) and Goldstein (1995) have advocated the application of HLM in educational and social science research.

Two features of data, often obtained in the educational setting, present complications for the appropriate analysis of the data. First, a multilevel structure of data leads to a complexity in design and statistical analysis. For example, multilevel structure of data that occurs due to the nesting of students within a teacher and nesting of teachers within a school suggests the use of multilevel statistical design such as HLM. Second, modeling binary response data and interpreting the effects of predictors on the binary outcome is another intricacy. For example, if we consider a hierarchical data structure, the interpretation of the effect of level-2 and level-3 predictors on the level-1 outcome is not straightforward. As the level of design is of a higher order for the given data, the effect interpretation becomes more complicated.
The multilevel structure of data has an important application in exploring the
effects of teachers and schools on the student-level outcome. In an HLM design, for
instance, student achievement can be predicted not only by the student level predictors
but also by the teacher and school level predictors. There has been a growing practice of
measuring the teacher and school effectiveness using multilevel data in educational
research. Researchers, such as Heistad (1999), measured teacher effectiveness in reading

A two-level model with an interval outcome and logistic regression are commonly
used in practice, but an extension to a three-level model with dichotomous outcome is
relatively uncommon. The later model is not particularly straightforward either. Besides,
an appropriate approach for demonstrating the sequential steps of model development,
analysis, evaluation, and application is rarely available. Specifically, the interpretation of
effects of predictors on a dichotomous outcome using ANOVA-like and simple effect
approaches is new for multilevel researchers. Since very limited or inadequate exemplars
are available for presenting a sequential illustration of development, analysis, evaluation,
as well as an application of models and using ANOVA-like and simple effect approaches
of effect interpretation, an effort is made here to present the research practitioners with
such a demonstration.

It is remarkable to note that some assumptions of multiple regressions are violated
in HLM analysis. For example, the assumption about the independence of observations in
a multiple regression may no longer be valid in HLM, because the observations within a
group possess similar characteristics, and the observations between the groups possess
dissimilar attributes. For instance, socioeconomic status (SES) of students in a public
school may exhibit more similar characteristics compared to SES of students from a
private school. Thus, the independence assumption may not always be valid when we
observe the student SES data within a specific private school.

Dichotomous outcomes are commonly applied in educational research employing
hierarchical design (see Bryk and Thum, 1989; McCaul et al., 1992; Rumberger, 1995).
For example, reading proficiency (a binary response indicating the proficiency or non-
proficiency status of a student), drop out, or the pass/fail status of students can be
predicted due to the teacher- and school-level predictors at level-2 and level-3 respectively. The natural dichotomous outcomes, such as drop out and pass/fail, can be meaningfully predicted due to teacher- and school-level predictors. Thus, multilevel binary response models are essential for school accountability purposes. Important educational decisions, such as intervention or retention of non-proficient students and initiation of remedial programs for the students who are “at-risk” of dropping out, can be made based on HLM procedure. Although reading proficiency, a derived dichotomous outcome from reading scores, is used for the purpose of a demonstration in this study, such an outcome can be meaningfully used as a continuous outcome.

Several assumptions validated in the multiple regression model will be violated if we attempt to use a dichotomous outcome in the model. For example, the assumptions of normality of residuals and homogeneity of error variance are violated when the model contains a binary outcome. Thus, in modeling situations like logistic regression model (and hierarchical generalized linear model, which is defined later) where dichotomous outcomes are used, the normality and homoscedasticity assumptions are no longer valid.

In recent years, there has been great progress in modeling educational outcomes using multilevel data with a continuous response variable. The development of HLM, with applications to two and three levels, has facilitated studies relating to students’ academic outcomes. For example, Saxe et al. (1999) employed a two-level HLM to predict student achievement while Hargrove and Mao (1997) used a three-level HLM to model academic and contextual variables related to SAT scores.

Logistic regression (LR) and nonlinear mixed models (NMM, although this terminology is commonly used for multilevel modeling) are being used as a single-level equation to model dichotomous outcomes (see Davidian & Gallant, 1993; Susman et al., 1998). In LR and NMM, we can predict a dichotomous outcome, such as the student reading proficiency, due to one or more student-level predictors in the equation. Although the use of logistic regression in educational research is relatively common, such a technique is not sufficient to model multilevel data.

More recently, a nonlinear version of HLM, known as HGLM, has been developed to model dichotomous outcomes. McCullagh and Nelder (1989) described a
generalized linear model (GLM), and Breslow and Clayton (1993) used generalized linear mixed model to study overdispersion, correlated errors, shrinkage estimation, and smoothing of regression relationships. The HGLM is an extension of GLM in multilevel cases. With the HGLM, we can predict the student reading proficiency (a binary outcome) not only due to student level predictors, but also due to teacher- and school-level predictors. The prediction procedure, using an HGLM approach, facilitates determining the extent of the teacher and school effects on student proficiency.

Most applications of the HGLM thus far have been for data from two levels (see Wong & Mason, 1985; Bryk & Thum, 1989; McCaul et al., 1992; Rumberger, 1995). These studies explored the student-level (dichotomous) outcomes using the variables of interest in a higher level. For example, Bryk and Thum (1989) predicted student dropouts employing a two-level HGLM. McCaul et al. (1992) used a two-level HGLM in order to compare dropping-out with personal and social variables.

Only limited applications of the HGLM for data from three levels are currently available. Raudenbush et al. (2001) presented a technical guide for a three-level HGLM analysis using HLM software written by Raudenbush et al. (2000). Tate (2004) described two- and three-level HGLM with one (continuous) predictor in the model. Raudenbush and Bryk (2002) illustrated two- and three-level HGLMs; however, a simpler interpretation of the parameter’s (predictors’) effects on dichotomous outcome is lacking, specifically in three-level HGLM. The limited work using the three-level HGLM demands a further and broader demonstration of the model and interpretation of the predictors’ effects using specific methods, such as simple effect and ANOVA-like approaches.

Since educational research and accountability studies are commonly concerned with dichotomous outcomes in situations with three levels, it is important that applied educational data analysts learn how to use the HGLM for the appropriate analysis of the resulting data. The demonstration of the application of a three-level HGLM would be of great importance for educational research practitioners in order to learn the appropriate procedures and rationale of the steps in the illustrative process.
Purpose of the Study

The purposes of this study are to demonstrate a three-level HGLM and employ the simple effect, as well as ANOVA-like approaches, to interpret the effects of predictors in educational settings. An illustration of sequential steps is presented to develop the model, select the potential predictors to analyze the model, evaluate the model based on estimated parameters, and apply the model in practice. Further, the effects of predictors are interpreted using two techniques: the simple effect procedure, and an ANOVA-like approach. The interpretation of effects using the simple effect procedure is of central importance of this study.

The NAEP Reading Assessment 2000 data set is used for illustration purposes. The Socioeconomic status (SES), race/minority, and sex at the student level, computer use, class type, and teacher expectation at teacher level, and Title 1 funding as well as school socioeconomic status at the school level are the predictors to be considered in the beginning in this study. However, race/minority and SES at the student level, class type at the teacher level, and school mean SES at the school level were the predictors selected in the final model.

This study aims to demonstrate a three-level HGLM in terms of development, analysis, evaluation, and application of the model. More importantly, an interpretation of the predictors’ effects based on HGLM results are described using simple effect and ANOVA-like approaches. Therefore, the audiences of this study are assumed that they have some familiarity with the procedures related to two- and three-level HLM for continuous outcomes and one-level logistic regression. The HGLM technique of modeling demonstrated in this study is similar to the HLM model formulation, except for the setting of the level-1 model as a nonlinear equation. Although the interpretation of the effect of predictors on dichotomous outcome (i.e., reading proficiency) is different from interpreting the effect of predictors on continuous outcome, the knowledge of linear HLM may help the audience understand the illustrations. In order to aid the reader, HLM-related procedures are reviewed in the second Chapter of this study.

In the third Chapter, study goals, analysis procedure as well as research questions, model development and selection/evaluation of the final model including the method of
HGLM modeling and analysis are illustrated. The description of the NAEP-based situation and variables to be used for the illustration is provided. Additionally, the computation and interpretation of the simple effect of specific predictors on reading proficiency (a dichotomous outcome), and determining how this effect depends on the levels of other predictors in the model are also presented. For example, the procedure to find the simple effect of class type on the log-odds of student reading proficiency, based on the levels of other predictors in the model, is illustrated. The computation and interpretation of the effect of a predictor using an ANOVA-like procedure is also presented in the third Chapter.

In the fourth Chapter of this dissertation, I have presented a detailed result demonstration, providing sequential illustrations of a three-level HGLM procedure at each step with the necessary rationale for any decision, the interpretations of results employing the simple effect approach and an ANOVA-like procedure. I have provided an executive summary at the end of the Chapter. Technical details and annotated computer printouts are documented in the appendices.

Finally, in the fifth Chapter, I have summarized the results, discussed the strengths and weaknesses of the study including costs and complications, and provided recommendations for future researches with more appropriate modeling options.

Significance of Study

A common presence of three-level data with dichotomous outcomes is observed in educational research. Hence, it is important to demonstrate the sequential procedure of three-level HGLM analysis using potential predictors, and then evaluate the models based on different estimators. Such a logical demonstration will be valuable to teach educational research practitioners and data analysts how to use the sequential and logical technique of illustration. Consequently, this type of illustration is presented to help educational research practitioners in terms of developing the appropriate model with potential predictors, using proper analysis technique, evaluating the model based on different estimators, and finally, using such a model in practice.
The major significance of a demonstration of the three-level HGLM with a dichotomous outcome presented here is to teach educational research practitioners an illustrative procedure of development, analysis, evaluation, and application of the model. Since the sequential illustration of three-level HGLM is still rare, a vital need of a step-by-step demonstration of development through evaluation and application of models in educational research is felt more than ever. This type of practical demonstration will aid researchers in initiating, developing, analyzing, evaluating, and applying the three-level HGLM in educational research settings.

The interpretation of the effects of predictors using the simple effect approach deserves equally important significance of the study. Employing such approach we opt to interpret the effect of student-, teacher-, and school-level predictors on reading proficiency in a simpler way compared to traditional HLM-based way of effect interpretation. In addition, an ANOVA-like approach of effect interpretation, with a description of the overall relationship as well as description of the effects of individual predictors, such as main and interaction effects, is also illustrated. Using both of the above approaches (i.e., simple effect and ANOVA-like methods) of effect interpretation, the effect of a predictor on reading proficiency is interpreted based on the levels of other predictors.

Limitations

The NAEP reading assessment employs a complex multistage sampling procedure to select the students (first there is a selection of geographic areas, then schools, and finally a random selection of students within the selected schools). Each selected school that participated in the assessment, and each student assessed, represents a fraction of the population of interest. Therefore, sampling weights are desired in order to make valid inferences between the student samples and the respective populations from which they were drawn. Additionally, NAEP over-samples non-public schools, and those schools in which more than fifteen percent of the student population is non-white. Sampling weights adjust for disproportionate representation due to such over-sampling.
The major limitation of this study is that sampling weights associated with students and schools are ignored while analyzing the data. Consequently, the implications based on the findings are limited due to this restriction. In other words, the study results are applied only to illustrate the statistical techniques rather than using the study results in terms of implementation. This limitation exists due to the restricted feature of HLM 5 in handling the sampling weights for the three-level HGLM. Since the data used in this study are based on stratified complex samplings, however, the analysis of data without incorporating weights would only facilitate the purpose of demonstration of the technique.

The illustrations are limited to a relatively standard formulation of the model incorporating only a dichotomous outcome in the level-1 equation, and the more complex modeling options (e.g., counts as outcomes) are not provided. In addition, this study is limited to a three-level model.

In addition, major caveats of the study are due to the resulting huge odds ratios, and huge magnitude of confidence intervals for student SES associated with crowded and non-crowded class types. The results showing huge values of odds ratio and confidence intervals are simply not credible. However, given primary purpose of demonstrating the sequential procedure of modeling, it was decided that the current results, despite not being credible, would still suffice to fulfill the purpose.
CHAPTER 2
LITERATURE REVIEW

Overview

It is common practice to use a dichotomous outcome for modeling purposes in educational research. In situations where a single-level equation, such as logistic regression, is insufficient to model the data with hierarchical structures, a multilevel model is desired. For example, if the data set consists of multi-level predictors and we opt to predict a level-1 binary outcome using such predictors, a hierarchical generalized linear model (HGLM) would be appropriate. Furthermore, three-level HGLM is necessary in those situations where the application of a two-level HGLM does not enable us to model level-3 predictors.

This study uses large-scale NAEP data with student, teacher, and school demographic and background variables. Research questions associated with the prediction of the log-odds of reading proficiency, employing a simple effect approach, would be addressed by this research. The cross-level interaction effects of the predictors, at each of the three levels (that is, student, teacher, and school levels), would be estimated and interpreted using an ANOVA-like approach as well as a simple effect technique for a three-level HGLM.

This Chapter presents the conceptual framework on logistic regression, two- and three-level HLM, as well as providing a basic conception about two- and three-level HGLM. The simple effect approach, interaction effects, and interpretation of such effects are also described. In addition, teacher and school effects on student reading achievement are also presented in this Chapter.
Logistic Regression

The basic conception of modeling begins with bivariate and multiple regression models in which we consider a continuous dependent variable. Although employing a multiple regression is analogous to modeling logistic regression, several problems occur when we use a dichotomous outcome in a multiple regression model. Menard (1995) addresses problems, such as generating illegitimate probabilities of predicted outcome, that will result in either larger than 1 or less than 0 (i.e., negative). He also indicates the violation of assumptions of normality and homogeneity of error variance in such modeling situations. This leads the researchers to perform a transformation of the continuous dependent variable incorporating binary codes 0 and 1.

Many studies illustrate odds and log of odds for modeling the dichotomous outcome (see Williamson & McNamara, 2002; Cizek & Fitzgerald, 1999; Hosmer & Lemeshow; 1989). For example, let reading proficiency be defined as a binary outcome coded 1 for “proficiency,” and 0 for “non-proficiency” in reading. Then the odds of proficiency for a given student is defined as the ratio of the probability of proficiency (\( \phi \)) to the probability of non-proficiency (1-\( \phi \)), and is expressed as

\[
\text{Odds} = \left( \frac{\phi}{1-\phi} \right)
\]  

(2.1)

where

\[
\phi = \frac{\exp(\eta)}{1 + \exp(\eta)}
\]  

(2.2)

where \( \eta \) is the log-odds of proficiency [see (2.3) for the mathematical expression of log-odds]. It can be evidently seen that the odds has a minimum value of zero, but has no fixed maximum value.

Further the transformation of the odds can be made, and that can be expressed as log-odds. Log-odds is the natural logarithm (symbolized by ln) of the odds, and is given by

\[
\eta = \ln \left( \frac{\phi}{1-\phi} \right)
\]  

(2.3)
It is notable that the value of log-odds, as a continuous variable, ranges from negative infinity to positive infinity. Hosmer and Lemeshow (1989) indicate that the link function is used to measure the relationship between dependent and independent variables in logistic regression. The mathematical expression of link function is given by (2.3).

Researchers at times have also used “logit” for log-odds. Having reviewed relevant literature, Peng et al. (2002) illustrates “logit” in logistic regression as a natural logarithm of odds. Jimenez and Salas-Velasco (2000) use the logit model and MLE procedure to investigate the influence of explanatory variables on binary outcome. Despite the transformation of continuous into dichotomous variables, a linear combination of the independent variables predicts the outcome in the logistic model. By means of a mathematical equation, the logit can be expressed as

\[
\eta = \ln \left( \frac{\varphi}{1 - \varphi} \right) = \beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k \quad (2.4)
\]

Equation (2.4) is analogous to the level-1 structural equation in the hierarchical generalized model (HGLM), which will be discussed in Chapter 3. Raudenbush and Bryk (2002) and Goldstein (1995) present the mathematical expression of the logit and logit link function, expressed by (2.3) and (2.4), which are required when modeling dichotomous outcome in multilevel settings. Fisher (2001) mention that logit models use the conditional probability of the event in binary models where the observed response is a (0, 1) dichotomy, and logit models rely on the standard logistic distribution; the mean is zero as for the standard normal curve, but the variance is \( \pi^2/3 \).

As we have only one possible outcome (0 or 1) at each trial, the binary response in logistic regression model follows the Bernoulli distribution. Hannan and Murray (1996) consider a dichotomous outcome, a Bernoulli random variable, in which the probability of success in a single trial is fixed. However, since there are several possible outcomes at a single occurrence in binomial distribution, Rindskopf (2002) suggests that the binomial model can be expressed as a special case of logistic regression with no predictors in the model.
The effect of independent variables on an outcome in logistic regression is represented by an odds ratio, which is analogous to the slope or regression coefficient in multiple regression. Many authors (e.g., Jonson-Reid et al., 2001; Cohen, 2000; Cizek & Fitzgerald, 1999; Sadler & Hammerman, 1999; and Hosmer & Lemeshow, 1989) use the odds ratio to interpret the effect of predictors. Although the effects of interval and dichotomous predictors on the binary outcome in logistic regression are differently computed and interpreted, the odds ratio is the key basis for interpreting such effects. Odds ratios are computed by using log odds for a meaningful prediction. Cohen (2000) provides a computational definition of odds ratio. In a study, Jonson-Reid et al. (2001) report odds ratios in order to examine whether youths with serous emotional disturbance status were more likely to be incarcerated for violent offenses. Sadler and Hammerman (1999) interpret odds ratios to predict the probability of admission to higher education institutes.

**Interpreting Odds Ratio**

For a continuous predictor with a quantitative difference of c units between two values of predictor, the odds ratio is given by

$$\text{Odds Ratio} = \frac{Odds(k + c)}{Odds(k)}$$

(2.5)

Odds in 2.4, is defined earlier (that is, the ratio of the probability of reading proficiency to the probability of non-proficiency). Expressed exponentially, this effect is computed by taking the exponential of c*β, or, $e^{(c*\beta)}$. This odds ratio for a continuous predictor can be interpreted as the ratio of odds of proficiency for the predictor equal to (k+c) to the odds for the predictor equal to the equal to k, controlling other predictors.

However, the odds ratio for the dichotomous predictor (say, race, coded as 1 for those having minority status and 0 for those not having minority status) is interpreted differently. For example, the effect of a dichotomous predictor on reading proficiency is simply computed by $e^{(\beta)}$. The odds ratio for the dichotomous predictor can be interpreted as how many times the odds of reading proficiency for students with minority status is greater than the odds of non-minority status.
For example, the odds ratio associated with student SES (a continuous predictor) for a given increase of c units in SES, is interpreted as “when SES is increased by c units, the odds of reading proficiency will increase by a factor of \( e^{c\beta_2} \), controlling for other predictors,” where e is an exponential function, and \( \beta_2 \) is the estimated effect for SES (\( e^{c\beta_2} \) can be calculated by using a pocket calculator). However, the effect of a dichotomous predictor, say student minority, can be interpreted as “the odds of reading proficiency for minority students are \( e^{\beta_1} \) times greater than the odds for non-minority, controlling for other predictors,” where \( \beta_2 \) is the estimated effect for student minority.

**Assumptions**

The logistic regression model assumes that the model is correctly specified. By correctly specifying the model, we mean that a) the probability of (reading) proficiency is a logistic function of the predictors, b) no important variables are omitted, c) no extraneous variables are included, and d) the predictors are measured without error. Other assumptions are that the cases are independent, and the independent variables are not linear combinations of each other. Binary response models follow the Bernoulli sampling model with a logit link; logit link is defined in (2.3).

**Applications of Logistic Regression for Measuring Student Outcome**

There have been several applications of logistic regression in estimating the likelihood of success. For example, Zeng et al. (2002) identify teacher-level predictors to predict pass or fail status in a teacher certification test known as the Elementary Professional Development Examination for Certification of Educators in Texas (ExCET). Employing logistic regression, Boe et al. (1999) investigate teacher attrition using national trend and predictor data from public schools. The single-level equation has been popular in attempts to predict student performance. Chen et al. (2001) uses logistic regression to construct a prediction model, with high predictive validity, relating to the pass/fail status of candidates who took the United States Medical Licensure Examination (USMLE).
The single-level (logistic regression) model has been also used for measuring the effects of school on reading achievement. In a school level study involving elementary school students, Slack and St. John (1999) investigate improvement in reading/language arts test performance by non-transient learners, for third and fifth graders in three accelerated schools, using logistic regression.

The limitation of the single-level equation in modeling features, especially for those data nested within a group in the form of hierarchy, led educational researchers to explore an alternative modeling technique, known as hierarchical linear and nonlinear modeling (HLM). Such a modeling approach is beneficial for researchers, since they do not have to analyze individual lower (for instance, student) level and upper (for instance, school) level models separately. Although the methodological problem of measuring change and unit of analysis have distinct, long-standing, and non-overlapping literatures, they, in fact, share a common cause – the inadequacy of traditional statistical techniques for the modeling of hierarchy, and with the recent development of statistical theory of the HLM, the basis for a more appropriate approach now exists (Bryk & Raudenbush, 1988).

Two-Level HLM

*Historical Development*

There are many situations in which we generally encounter the existence of a hierarchy of data, suggesting the use of HLM. In order to address and model the hierarchical structure of data, numerous authors have contributed in initiating and advancing the historical development of the HLM. Aitkin et al. (1981) use a hierarchical structure of data, where students are nested within the classroom, for modeling purposes. Mason et al. (1983) provide a comprehensive and coherent technique for multilevel modeling. Comparing random and nonrandom multilevel models, Tate and Wongbundhit (1983) demonstrate that single-level analysis is problematic in the presence of a multilevel structure. Braun et al. (1983) apply the HLM to the setting of school studies, and Raudenbush and Bryk (1986) demonstrate that the HLM can measure school effects.
Goldstein (1987a) illustrates multilevel modeling applied in educational and social science research.

Raudenbush (1988) develops an important class of models, termed as hierarchical linear models, in which observations within each group vary as a function of group-level or “microparameters.” Further, Bryk and Raudenbush (1988) demonstrate the modeling aspects of the HLM in educational contexts, describing within-school and between-school equations. Goldstein (1995) demonstrates multilevel linear and nonlinear modeling approaches in which he also discusses school effectiveness and the cross-classification of data.

Various researchers have employed the HLM by different names. Some of the popular names are covariance component models (Goldstein, 1987b; Longford, 1987), random-effects and mixed-effects models (Laird & Ware, 1982; Singer, 1998), and multilevel regression models (Hox, 2002). Along with this variety in designation, researchers have also employed a variety of multilevel (statistical) programs for data analysis and modeling purpose.

Bryk and Raudenbush (1988), as well as Raudenbush and Bryk (2002) demonstrate the application of HLM in education and the social sciences. Further, Raudenbush et al. (2001) presents step-by-step analysis procedures. However, Raudenbush and Bryk (2002) provide limited material on the demonstration and interpretation of the effect of predictors in three-level HGLM.

Motivation of Development of Two-Level HLM in Past

A two-level multilevel model has been employed by many researchers (e.g., Young et al., 1996; Goldstein, & Rabash, 1996; Goldstein, 1986; Fitzmaurice, 1993; Zhao and Prentice 1990) in order to predict student level outcomes. By modeling a continuous outcome, Young et al. (1996) shows that the individual measures explain most of the variance where previous achievement shows a preponderant influence on subsequent achievement. Thus, the studies in education address the prediction of a student achievement with the application of a hierarchical model. Muthen (1991)
considers within-class and between-class decomposition of achievement variance using a multilevel model. Others measure the classroom effect on student achievement, employing a hierarchical model. For example, Saxe et al. (1999) present the relationship between student achievement and classroom practices and find that the attachment of classroom practices with reform principles is associated with a student’s achievement in problem solving, but not in computation.

In considering a fixed-effects model with no predictors in both the level-1 and level-2 models, the predicted student outcome for a reference individual in jth group can be given as the average predicted student outcome. In a two-level HLM study with some variables at the lower level having fixed and others having random coefficients, Snijders and Bosker (1993) derive an approximation to the covariance matrix of the estimators of the fixed regression coefficients at both the levels assuming a large sample.

Thum (1997) develops two-level models in order to understand behavioral data; however, Raudenbush and Bryk (2002) indicate that complex situations can arise, contrasted with standard Ordinary Least Squares (OLS), when data are modeled using two-level models with the presence of conditional variance. In such a situation, they suggest that the error terms for a higher level model cannot be independent and constant across the group. To exemplify this situation, let us assume a two-level model where the level-1 model is a student level, and the level-2 model is a teacher level. Suppose $\beta_{0j}$ and $\beta_{1j}$ are the level-1 intercept and slope respectively which become outcomes at the teacher-level model. If we consider several students nested within a teacher, where $u_{0j}$ and $u_{1j}$ are the residuals associated respectively with intercept ($\beta_{0j}$) and slope ($\beta_{1j}$) as outcomes in the teacher-level model, then the residual terms within each teacher are dependent since students nested within a specific teacher $j$ share the common error terms $u_{0j}$ and $u_{1j}$. Further, we can substitute the value of intercept ($\beta_{0j}$) and slope ($\beta_{1j}$) in the level-1 model and formulate a single-equation (see Equation (2.8) for an explanation). In this situation, we will get a factor $[u_{0j} + u_{1j}X_{ij}]$ in the resulting single-equation. The factor $[u_{0j} + u_{1j}X_{ij}]$ is the function of $u_{0j}$, $u_{1j}$, and $X_{ij}$, and varies across both teachers and students. Therefore, the assumption of independence and constant variance is difficult to meet in a multilevel design.
There has been a growing number of studies in cross-classified random effects models. For example, Rabash and Goldstein (1994) generalize two-way cross-classification, considered earlier by Raudenbush (1993) in p-way cross-classifications at any number of levels, by providing an efficient computational procedure. Employing the HLM, Goldstein (1995) uses the cross-classification of teachers by students to predict level-1 (student) outcome, whereas Raudenbush and Bryk (2002) use a cross-classified random effects model to measure the impact of neighborhood and school on educational attainment. Educational researchers have used cross-classified random effects models in order to estimate the effects of lower and higher level units on level-1 outcomes. Based on a multilevel study, Antony (2002) analyzes subject teaching-group effectiveness, and suggests cross-classified hierarchical models with weighted random effects for unraveling student, group, and teacher effects.

Pike and Saupe (2002) evaluate the usefulness of three approaches, namely, the traditional regression model, high-school-effects model, and hierarchical linear model. Their research finds that both the high-school-effects model and hierarchical linear model predict freshman Grade Point Averages (GPA) more accurately, compared to the traditional regression model, specifically for lower ability students.

Deng (1992) measures the effect of the classroom climate on student achievement using the two-level hierarchical model and finds that classroom climate factors affect student achievement differently, depending on student characteristics. Deng further infers that higher and more equitable distributions of achievement exist in classes with higher levels of academic emphases and student satisfaction, as well as in classes with low levels of tension.

Researchers in education have discussed the drawbacks of the traditional regression model. Suppose that we have student data in level-1, and school data in level-2 for fitting two separate regression models. Researchers argue that in such a situation, the regression model ignores students’ grouping within schools, and the aggregated residuals do not represent the effect of schools as a whole or the effect of school practices and policies (see Aitkin & Longford, 1986; Raudenbush & Willms, 1995; Pituch, 1997).
A general use of the HLM is the testing of hypotheses in cross-sectional and longitudinal growth modeling studies. In a cross-sectional study, Subedi (2003) tests the effect of resources and class size on student achievement, and also examines teacher-to-teacher variation in student achievement. Several researchers use linear and nonlinear growth analyses using HLM (see Seltzer et al., 2002; Shay, 2002; Stage, 2001; Hoekema & Knol, 2001; Brody et al., 2001; Kevan, 1997). Kevan (1997) demonstrates two-level HLM growth modeling in which he finds that initial scores are related to the sex of the respondent, mother’s education, and the number of books in the household. Hoekema and Knol (2001) test a predictive hypothesis in order to demonstrate a multivariate longitudinal growth modeling by linking developmental data to a criterion. In the social sciences, researchers have also tested the effect of different social factors on behavior during childhood. Testing several hypotheses with a two-level model, Brody et al. (2001) measure the effect of social factors, such as neighborhood disadvantage, collective socialization, and parenting on children’s affiliation with deviant peers.

Linear and nonlinear growth modeling are popular for measuring the growth of student achievement based on several time points. Stage (2001) evaluates the growth of ethnically diverse second-grade students in oral reading fluency using curriculum-based measurement and finds that first-grade reading performance significantly predicts initial second-grade reading performance. In settings where we obtain time-series data for each person, this entails examining interactions between treatment and initial status on a rate of change and, by using a hierarchical model, this approach provides the ability to handle the number and spacing of time-series data (Seltzer et al., 2002).

**Model formulation**

The level-1 equation for predicting \( Y_{ij} \), reading achievement for student \( i \) associated with teacher \( j \), can be formulated as follows.

\[
Y_{ij} = \beta_{0j} + \beta_{1j} X_{1ij} + \beta_{2j} X_{2ij} + r_{ij}
\]  

(2.6)

Let the first (dichotomous) predictor in the model, \( X_{1ij} \) (student minority/race), be un-centered, and the second (continuous) predictor in the model, \( X_{2ij} \) (student
socioeconomic status (SES)), be grand mean centered. In (2.5), $\beta_{0j}$ is an intercept, $\beta_{1j}$ is the slope for student minority/race ($X_{1ij}$), and $\beta_{2j}$ is the slope associated with student SES ($X_{2ij}$). The term $r_{ij}$ is the random effect for student $i$ nested in teacher $j$.

The interpretation of the parameters $\beta_{0j}$, $\beta_{1j}$, and $\beta_{2j}$ is important in order to understand the remainder of this study. In the above model, the intercept $\beta_{0j}$ is the expected reading achievement of a student whose values for $X_{1ij}$ (minority), say $X_1$ for brevity, and $X_{2ij}$ (SES), say $X_2$ for brevity, are equal to zero (this is also known as the “reference student,” where $X_1 = 0, X_2 = 0$). The slope $\beta_{1j}$ is interpreted as the predicted reading achievement for minorities ($X_1$) relative to non-minorities (or difference in reading achievement between minority and non-minority), controlling for SES ($X_2$), and the slope $\beta_{2j}$ is interpreted as the expected change in reading achievement associated with a unit change in SES ($X_2$), controlling for minority ($X_1$).

The level-2 model is formulated by using the level-1 intercept ($\beta_{0j}$) and slopes ($\beta_{1j}$ and $\beta_{2j}$) as outcomes in the model. Let $W_{ij}$ be the first teacher-level predictor (computer use by teacher for reading instruction, henceforth known as teacher computer use), which is continuous, and $W_{2j}$ be the second teacher-level predictor (classroom type, i.e., crowded or non-crowded classroom), which is dichotomous.

Consider $W_{ij}$ and $W_{2j}$ as grand-mean centered and uncentered respectively. Then, the level-2 model can be formulated as follows.

$\beta_{0j} = \gamma_{00} + \gamma_{01} W_{ij} + \gamma_{02} W_{2j} + u_{0j}$
$\beta_{1j} = \gamma_{10} + \gamma_{11} W_{ij} + \gamma_{12} W_{2j} + u_{1j}$
$\beta_{2j} = \gamma_{20} + \gamma_{21} W_{ij} + \gamma_{22} W_{2j} + u_{2j}.$

(2.7)

where $u_{0j}$, $u_{1j}$, $u_{2j}$ are the teacher-level random effects associated with teacher $j$.

The single-equation formulation can be expressed, as follows, by substituting (2.7) in (2.6).

$Y_{ij} = \gamma_{00} + \gamma_{01} W_{ij} + \gamma_{02} W_{2j} + \gamma_{10} X_{1ij} + \gamma_{11} W_{ij} X_{1ij} + \gamma_{12} W_{2j} X_{1ij} + \gamma_{20} X_{2ij}$
$+ \gamma_{21} W_{ij} X_{2ij} + \gamma_{22} W_{2j} X_{2ij} + [u_{0j} + u_{1j} X_{1ij} + u_{2j} X_{2ij} + r_{ij}]$  (2.8)

Before we interpret the coefficients in (2.7), it is worth mentioning that the cross-level interaction terms, such as $W_{ij} X_{1ij}$, $W_{2j} X_{2ij}$ in the model, represent the interaction effects of student and teacher level predictors on reading achievement.
In (2.8), $\gamma_{00}$ is the predicted reading achievement for a reference student and a reference teacher (i.e., with the values of all student-level and teacher-level predictors equal to zero). The terms $\gamma_{01}$, $\gamma_{02}$, $\gamma_{10}$, and $\gamma_{20}$ are the simple effects on reading achievement due to teacher computer use, classroom type, student SES, and student race, respectively. The parameters $\gamma_{11}$, $\gamma_{12}$, $\gamma_{21}$, and $\gamma_{22}$ are a two-way interaction effects of student-level and teacher-level predictors on reading achievement.

Due to the complexity of interpreting the parameters present in the single-equation given by (2.8), we prefer to consider the simple effect approach in describing the individual effect of student-level and teacher-level predictors. Such effects are discussed in the following section, representing the effect of an individual predictor on student achievement by using a mathematical differentiation technique.

**Assumptions**

The assumptions for the two-level HLM are given below.

a) The error terms of each level-1 unit should have a mean of zero, and the error terms should be normally distributed. If, for example, we consider level-1 and level-2 units as students and teachers, respectively, then the mean of the error within each teacher should be zero, and the error terms should be normally distributed.

b) It is assumed that the relationship between predictors and outcome variables is linear.

c) Another assumption is the homogeneity of error variance at both levels.

d) Level-1 predictors are independent of the level-1 error term. In other words, the covariance between the level-1 predictors and the error term should equal zero.

e) Level-2 error terms have a mean of zero and follow a multivariate normal distribution.

f) Level-2 predictors are independent of all level-2 error terms. That is, all variables in the level-2 models are not related to any of the error terms on the corresponding level of the model, including the error term for the level-1 intercept, and the error term for any of the slopes of level-1 variables.
g) The level-1 error terms are independent of level-2 error terms. That is, there is no relationship between the error term at level-1 and the error term in the level-2 equation for the level-1 intercept, or the error term in any of the equations used to estimate the slopes of level-1 variables.

Simple Effect Description

The effect of each predictor in the level-1 and level-2 models on reading achievement (Y) formulated in 2.8 can be described by partially differentiating Y with respect to W₁, W₂, X₁, and X₂ at a time. This process, known as the simple effect method, reduces the effects of the student-level predictor X or teacher-level predictor W (given in (2.8)) to four effects depending upon the predictors at other level. Using a continuous outcome, Tate (2004) employs the differentiation approach to interpret the simple effect of a predictor on outcome. The simple effect approach, given below, is the key approach for interpreting the effects of predictors at both levels.

\[
\exp \left( c_{w_1} \frac{\partial Y}{\partial W_1} \right) = \exp \left[ c_{w_1} \left( \gamma_{01} + \gamma_{11} X_{1ij} + \gamma_{21} X_{2ij} \right) \right] = E_{w_1}
\]

\[
\exp \left( c_{w_2} \frac{\partial Y}{\partial W_2} \right) = \exp \left[ c_{w_2} \left( \gamma_{02} + \gamma_{12} X_{1ij} + \gamma_{22} X_{2ij} \right) \right] = E_{w_2}
\]

\[
\exp \left( c_{x_1} \frac{\partial Y}{\partial X_1} \right) = \exp \left[ c_{x_1} \left( \gamma_{10} + \gamma_{11} W_{1ij} + \gamma_{12} W_{2ij} \right) \right] = E_{x_1}
\]

\[
\exp \left( c_{x_2} \frac{\partial Y}{\partial X_2} \right) = \exp \left[ c_{x_2} \left( \gamma_{20} + \gamma_{21} W_{1ij} + \gamma_{22} W_{2ij} \right) \right] = E_{x_2}
\]

In (2.9), the terms \( \frac{\partial Y}{\partial W_1} \) and \( \frac{\partial Y}{\partial W_2} \) represent the simple effects of W₁ (teacher computer use) and W₂ (classroom type), respectively, on student reading achievement.

Similarly, the terms \( \frac{\partial Y}{\partial X_1} \) and \( \frac{\partial Y}{\partial X_2} \) represent the simple effects of X₁ (student SES) and X₂ (student race), respectively, on student reading achievement.
We need to compute the confidence intervals of simple effects in order to reflect precision. Confidence intervals are estimated by using an appropriate critical value, computed point estimates, and standard errors. Standard errors can be computed from the variance-covariance matrix. Tate (2004) provides the computational technique of standard error using four $\gamma$s (parameters) with matrix formulation.

The linear combination of the model parameters, used to compute the standard error of any estimated effect, is given by,  
$$ m = a'\gamma $$

Further, the standard error of estimated linear combination is given by  
$$ S_m = \sqrt{a' V a} $$

where  
$$ a' = (a_{00} a_{01} a_{02} a_{10} a_{11} a_{12} a_{20} a_{21} a_{22}) $$

and $\gamma'$ is defined as  
$$ \gamma' = (\gamma_{00} \gamma_{01} \gamma_{02} \gamma_{10} \gamma_{11} \gamma_{12} \gamma_{20} \gamma_{21} \gamma_{22}) $$

The matrix $V$ is defined below.

$$ V = \begin{pmatrix}
S_{00}^2 & S_{00,01} & S_{00,02} & S_{00,10} & S_{00,11} & S_{00,12} & S_{00,20} & S_{00,21} & S_{00,22} \\
S_{01,00} & S_{01,01}^2 & S_{01,02} & S_{01,10} & S_{01,11} & S_{01,12} & S_{01,20} & S_{01,21} & S_{01,22} \\
S_{02,00} & S_{02,01} & S_{02,02}^2 & S_{02,10} & S_{02,11} & S_{02,12} & S_{02,20} & S_{02,21} & S_{02,22} \\
S_{10,00} & S_{10,01} & S_{10,02} & S_{10,11}^2 & S_{10,12} & S_{10,20} & S_{10,21} & S_{10,22} & \\
S_{11,00} & S_{11,01} & S_{11,02} & S_{11,11} & S_{11,12}^2 & S_{11,20} & S_{11,21} & S_{11,22} & \\
S_{12,00} & S_{12,01} & S_{12,02} & S_{12,11} & S_{12,12} & S_{12,21}^2 & S_{12,22} & \\
S_{20,00} & S_{20,01} & S_{20,02} & S_{20,10} & S_{20,11} & S_{20,12} & S_{20,21}^2 & S_{20,22} & \\
S_{21,00} & S_{21,01} & S_{21,02} & S_{21,11} & S_{21,11}^2 & S_{21,20} & S_{21,21} & S_{21,22} & \\
S_{22,00} & S_{22,01} & S_{22,02} & S_{22,10} & S_{22,11} & S_{22,12} & S_{22,20} & S_{22,21} & S_{22,22}^2
\end{pmatrix} $$

The diagonal elements in the above matrix are variances, and the off-diagonal elements are covariance components of parameter estimates.

The partial derivatives of $Y$ associated with the simple effect of $W_1$, in a two-level HLM, can be given as follows, denoted by $a'$. 

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The partial derivatives of $Y$ associated with the simple effect of $W_2$ are:

$$a' = (0 \ 0 \ 0 \ X_1 \ 0 \ 0 \ X_2 \ 0). \quad (2.15a)$$

Similarly, partial derivatives of $Y$ with respect to $X_1$ are:

$$a' = (0 \ 0 \ 0 \ 1 \ W_1 \ W_2 \ 0 \ 0 \ 0). \quad (2.15b)$$

And, partial derivatives of $Y$ with respect to $X_2$ can be given as:

$$a' = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ W_1 \ W_2) \quad (2.15d)$$

Using the variance and covariance terms from matrix (2.14) and $a'$ from (2.15a), the standard error for the simple effect of $W_1$ can be given as:

$$S_{m, EW_1} = \sqrt{S^2_{01} + 2S_{01,11} + 2S_{01,21} + S^2_{11,1} X_1^2 + S^2_{21,2} X_2^2} \quad (2.16)$$

Similarly, using the variance and covariance terms from matrix (2.14), and $a'$ from (2.15b), the standard error for the simple effect of $W_2$ can be given as follows.

$$S_{m, EW_2} = \sqrt{S^2_{02} + 2S_{02,12} + 2S_{02,22} + S^2_{12,1} X_1^2 + S^2_{22,2} X_2^2} \quad (2.17)$$

We can compute the standard errors for $X_1$ and $X_2$ in a similar fashion.

The confidence interval (in the case of a continuous outcome) for $W_1$, with critical value $C_v$, a simple effect $E_{W1}$, and standard error $S_{m, EW_1}$, is given by

$$CI (E) = E_{W1} \pm C_v S_{m, EW_1} \quad (2.18)$$

Similarly, the confidence interval for $W_2$, with critical value $C_v$, simple effect $E_{W2}$, and standard error $S_{m, EW_2}$, is given by

$$CI (E) = E_{W2} \pm C_v S_{m, EW_2} \quad (2.19)$$

Thus, confidence intervals for $X_1$, and $X_2$ can be given in a similar fashion.

Although it is more complex to form the variance-covariance matrix and to compute standard errors in a three-level HLM/HGLM, a similar matrix formulation and the use of such formulae facilitate the computation process. More discussion about this is provided in Chapter 3.
Centering has been a matter of interest in HLM analysis. Such an interest has been initiated in order to purge the clustering effect, and to obtain more precise estimators. For example, to describe grand-mean centering for the level-1 predictors, the grand-mean is subtracted from the original predictor for the pooled population of students. Similarly, the level-2 predictor can also be grand-mean centered by subtracting the grand-mean of level-2 predictors from the original predictor for the entire population of groups. Presenting a combined equation for a two-level model, Burton (1993) states that centering has the effect of changing the coefficients that are being estimated and cannot be regarded as merely a technical device when, in fact, it changes the research questions that are actually being asked.

In a group-mean centering, we subtract the group-mean from the original predictor. Kreft (1995) clarifies that the practice of centering on the group mean, while not adding the mean back into the model, deletes information from the data and may lead to overestimation of the macro level variables in the model. Raudenbush and Bryk (2002) advocate the usefulness of a grand-mean centering in which the intercept, $\beta_{0j}$ is the expected outcome for a subject whose value on $X_{ij}$ is equal to the grand mean, $\bar{X}_{..}$, a standard choice of location for $X_{ij}$ in the classical ANCOVA model.

By means of HLM analyses using three methods of centering, namely, grand-mean centering, group-mean, and uncentering, to explore the effect of students’ lunch statuses on reading achievement, Schumacker and Bembry (1995) find that the group-mean centering method provides a more reliable estimate compared to the other two techniques. Further Kreft et al. (1995) mention that centering around the group mean amounts to fitting a different model than centering around the grand mean or using raw scores. However, the grand-mean centering method explains most between-school (level-2) variance among the three methods of centering. It seems that research is needed to perform more experimental (HLM) analyses in centering to draw concrete conclusions.
Unconditional and Conditional Models

The unconditional model is expressed with no predictors at either level in the model; however, the conditional model is formulated with relevant predictors at one level in the model. Level-1 and level-2 unconditional and conditional models are given below.

Level –1 Unconditional model: \[ Y_{ij} = \beta_{0j} + r_{ij} \] (2.20)

Level-2 Unconditional model: \[ \beta_{0j} = \gamma_{00} + u_{0jk} \] (2.21)

Level –1 conditional model: \[ Y_{ij} = \beta_{0j} + \beta_{1j} X_{i1j} + \beta_{2j} X_{i2j} + r_{ij} \] (2.22)

Level-2 conditional model:
\[ \beta_{0j} = \gamma_{00} + \gamma_{01} W_{1j} + \gamma_{02} W_{2j} + u_{0j} \]
\[ \beta_{1j} = \gamma_{10} + \gamma_{11} W_{1j} + \gamma_{12} W_{2j} + u_{1j} \] (2.23)
\[ \beta_{2j} = \gamma_{20} + \gamma_{21} W_{1j} + \gamma_{22} W_{2j} + u_{2j} \]

All parameters in the models above are interpreted as in (2.6), and (2.7).

Developing equations, from unconditional to conditional models, is an important aspect of HLM analysis. For illustration, Raudenbush and Bryk (2002) discuss the procedures employed to build conditional from unconditional models. These authors suggest developing fully unconditional models first, which allows the test and estimation of between-group variability; then, conditional models are formulated if any evidence of significant between-group variability is observed.

A residual term associated with level-1, \( r_{ij} \) in (2.10) and (2.12) is the unique effect on reading achievement due to ith student and jth teacher. A residual term associated with level-2, \( u_{0j} \) in (2.10) and (2.12) is the unique effect of teacher j on the reading achievement, holding \( W_1 \) and \( W_2 \) constant. The term \( u_{1j} X_{1ij} \) is the interaction effect of student SES (\( X_1 \)) and the error associated with its slope for student i and teacher j, holding \( W_1 \) and \( W_2 \) constant. Similarly, \( u_{2j} X_{2ij} \) is the interaction effect of student race (minority/non-minority status or \( X_2 \)) and the error associated with its slope, holding \( W_1 \) and \( W_2 \) constant.

It is important to note that the precision of estimated slopes \( \beta \)'s may vary across teachers, due to the variation in the amount of data available for each individual teacher j. According to Bryk and Raudenbush (1988), in estimating the \( \gamma \) coefficients, HLM
methods weigh the contribution of the estimated $\beta_j$s from each level-2 unit, proportional to their precision, where this optimal weighting procedure minimizes the effects of unreliability in the estimated $\beta_j$s on inferences about model parameters.

**Model Estimation Features and Reliability**

Let us consider the following equation with slope as the outcome referred from the set of equations in (2.23).

$$\beta_{ij} = \gamma_{10} + \gamma_{11} W_{1j} + \gamma_{12} W_{2j} + u_{ij}$$  \hspace{1cm} (2.24)

The above model can be characterized by fixed effects (all the terms except $u_{ij}$ on the right hand side of (2.24)), and the random effect term ($u_{ij}$). That is, the slope $\beta_{ij}$ is a function of teacher level covariates (e.g., teacher computer use, class type) and sampling error given by $u_{ij}$ (also known as the random effect). Thus, teacher-level predictors produce an effect on distribution of student reading achievement within teachers, whereas $u_{ij}$ yields a unique effect associated with teacher $j$. It is notable that the estimated slopes ($\beta_{1j}$, and $\beta_{2j}$) are measured with errors, which contain an additional level-1 error term ($e_{ij}$) in (2.24) when we estimate $\beta_{1j}$. The precision of the estimated slopes, $\beta_j$s, depends on the covariance structure (the degree that these slope parameters covary) among them.

Since estimated slope includes slope ($\beta_j$) with the sampling error ($e_{ij}$) in the estimation process, we can differentiate between true teacher effects variation (variance of $\beta_j$), and sampling variation or random effect variation (variance of $e_{ij}$). Thus, the total observed variance can be decomposed into parameter variance and sampling variance. Bryk and Raudenbush (1988) discuss the decomposition of such variances.


Goldstein (1995) indicates that reliability may be lower within population subgroups, defined by social status, than in the population as a whole. Thus, reliability
could be higher for a student population than for a school population, given that our
level-1 units of analysis are students. Further, reliability, as defined by Raudenbush and
Bryk (2002), is the ratio of parameter variance (or true score, given by “$\tau_{00}$”) to the sum
of parameter variance and error variance (i.e., total variance or observed variance of
sample mean, given by “$\tau_{00} + V_j$”). This can be expressed as
\[
\lambda_j = \frac{\text{Var}(\beta_{0j})}{\text{Var}(Y_{j})} = \frac{\tau_{00}}{\tau_{00} + V_j}
\]

\hspace{1cm} (2.25)

**Teacher Effects**

Although teacher effect can be decomposed into fixed and random effects (e.g.,
given by (2.24)), here we are interested only in random effects. Therefore, a random
effect (level-2) model is formulated in order to measure teacher effects. The random
effect pertains to the teacher-to-teacher variation in our sample. Since the factors
associated with the teaching process may vary from teacher-to-teacher, it is important to
explore the extent of this variation. Employing a two-level HLM, Subedi (2003)
measures the unique teacher effects on student achievement, and finds significant
teacher-to-teacher variation.

Another application of HLM is the grading of teachers based on their
performance. Du and Heistad (1999) emphasize the use of hierarchical models to rank
teachers on their effectiveness at improving student reading achievement, and also
evaluate the factors that contribute to teacher effectiveness. Research suggests that the
estimation of teacher effect and interpreting such effects for meaningful purposes are
very important for the teaching/learning process.

**School Effects**

The ranking of schools is essential for a school accountability system. Based
on past literatures, state agencies associated with the K-12 system are increasingly
applying HLM, and considering the random effects model, as a school grading
system. Tate (2000), as well as Du and Heistad (1999), employ HLM to estimate
school effects when studying the school accountability system, and use value-added
models to assess the school effectiveness. As a caveat of school ranking, Weerasinghe & Orsak (1998) state that the school ranking cannot be used all of the time, but when the number of students per school and the distinguishing effect sizes between two schools meet specific criteria, HLM can be effectively used to rank schools. Goldstein and Spiegelhalter (1996) mention that such rankings are of limited use in practice since the fine comparison among schools may be difficult due to large confidence intervals. In addition, the researchers should evaluate the validity of method used in the school ranking process where one should be aware of the possibility of abuse of these methods.

Further researches address the contextual setting of groups in an educational system, where HLM designs would provide appropriate and powerful analyses. Since public schools do not randomly assign students and teachers across schools, multilevel evaluation methods that account for student and school contextual and practice variables in their natural settings provide the most rigorous means for showing empirically what is actually happening in classrooms (Phillips and Adcock, 1997). For example, the effect on student achievement due to contextual variables, such as student’s ethnic group and socioeconomic status (SES), school SES, and classroom type (sparse versus crowded), would be valuable in the assessment of school effectiveness.

Compared to additive models, interactive models in HLM can provide a more meaningful interpretation for school effectiveness using cross-level interaction terms. Aitkin and Zuzovsky (1994) emphasize the need for interactive models for meaningful prediction. Despite the complex nature of modeling and effect interpretation, research has used interactive models with cross-level predictors. Using school-level residuals in a two-level HLM analysis, Pituch (1997) presents techniques that can be used to detect interactions between school practices and student attributes.

Researches such as what have been presented by Draper (1995) and Ferguson (1999) discuss the application of hierarchical models in school effectiveness studies. Ferguson (1999) used a multilevel model to measure the year-round school effect on student achievement, and found that the students’ scores were more consistent, indicating
that these students were less fatigued and frustrated, and had more staying power.
Student-level and school-level models are commonly analyzed in educational research in order to assess the quality of school systems. Schaffer et al. (2000) employs a two-level model at student- and school-level in order to explore school effects and evaluate schools within a particular state, and infers that the HLM approach has higher stability across the test forms.

Three-Level HLM

The two-level model may not be sufficient for meaningful analysis when we encounter hierarchically nested data structures, such as students nested in teachers, and teachers nested in schools. Apparently, a three-level design can be sought in this case, in which level-1, level-2, and level-3 models can be expressed as student-, teacher-, and school-level equations, respectively. In three-level HLM, level-1 coefficients will be the outcomes in the level-2 model, and the level-2 parameters will be modeled as outcomes in the level-3 model. Consequently, student outcomes are predicted by not only level-1 and level-2 predictors, but by level-3 predictors as well.

A three-level HLM is important not only in estimating and interpreting the effects of higher level predictors on level-1 outcome, but also in computing cross-level interaction effects of predictors in a model, regardless of the complexity of interpreting such interaction effects. Several studies in three-level modeling address such issues. Goldstein (1986) discusses the interactions of explanatory variables between levels introducing additional random terms into the model. Goldstein also discusses the partition of variance components, and illustrates random coefficients using a three-level model.

General applications of three-level HLM, as in the case of the two-level model, can be found in contextual analysis and growth modeling studies. Employing three-level contextual analysis, Bryk and Raudenbush (1988) discuss the methodological perspective of analysis in terms of partitioning variance-covariance into within- and between-school components. They also suggest that the sampling variation that arises as a result of the total observed variance comprises parameter variance with sampling error. Kevan (1997)
demonstrates the HLM growth modeling technique comparing reading scores of the American population.

Several situations may arise in which we want to measure the change in reading achievement, and estimate the effect of important student-level characteristics in a multiple nesting design, for instance, time periods nested within students within school. An adequate approach to questions of this sort requires a three-level model that allows us to combine the features of modeling to both individual growth over time and the organizational structure of students nested within different contexts, such as classrooms or schools (Bryk and Raudenbush, 1988).

Relevant literatures demonstrate that HLM analysis employing a three-level model can be more appropriate for NAEP data. The use of HLM on NAEP data facilitates to resolve the problem of sampling error resulting from the multi-stage sampling in NAEP (Arnold, 1993). Further, the multilevel structure of educational systems, the evidence of classroom effects in the NAEP data sets, and prior research suggest that a three-level analysis is appropriate for NAEP since there is evidence that a three-level analysis produces the most efficient estimates (Cheong et al., 2001). Since a single teacher was involved in teaching within a specific classroom, in this study, the classroom effect can be analogous to the teacher effect in NAEP data set.

**Model Formulation**

The level-1 model for three-level HLM, for student i, teacher j, and school k, can be given as follows.

\[ Y_{ijk} = \pi_{0jk} + \pi_{1jk}a_{1ijk} + \pi_{2ijk}a_{2ijk} + e_{ijk} \]  \hspace{1cm} (2.26)

In (2.21), \( Y_{ijk} \) is represented as the reading achievement of student i associated with teacher j, and school k. Predictors \( a_{1ijk} \) (say, \( a_1 \)) and \( a_{2ijk} \) (say, \( a_2 \)) are student minority (race) and student SES respectively. The coefficient \( \pi_{0jk} \) is the intercept, \( \pi_{1jk} \) is the slope for SES, and \( \pi_{2jk} \) is the slope associated with student race. Further, \( a_{1ijk} \) is a continuous predictor, and is grand-mean centered. However, \( a_{2ijk} \) is a dichotomous predictor and is uncentered. Grand-mean centering was defined previously, under two-level HLM. The term \( e_{ijk} \) represents the
random effect for student i, teacher j, and school k, which is normally distributed with mean zero and variance $\sigma^2$.

The level-2 model is formulated by using level-1 intercept ($\pi_{0jk}$) and slopes ($\pi_{1jk}$ and $\pi_{2jk}$) as outcomes. The level-2 equation, where $X_{1jk}$, is a continuous predictor and grand-mean centered, and $X_{2jk}$ is a dichotomous predictor and uncentered can be given as follows.

$$\begin{align*}
\pi_{0jk} &= \beta_{00k} + \beta_{01k} X_{1jk} + \beta_{02k} X_{2jk} + r_{0jk} \\
\pi_{1jk} &= \beta_{10k} + \beta_{11k} X_{1jk} + \beta_{12k} X_{2jk} + r_{1jk} \\
\pi_{2jk} &= \beta_{20k} + \beta_{21k} X_{1jk} + \beta_{22k} X_{2jk} + r_{2jk}
\end{align*}$$

(2.27)

In 2.27, the parameters $\beta_{00k}$, $\beta_{10k}$, and $\beta_{20k}$ are level-2 intercepts. Further, the coefficients $\beta_{01k}$, $\beta_{02k}$, $\beta_{11k}$, $\beta_{12k}$, $\beta_{21k}$, and $\beta_{22k}$ are level-2 slopes. The terms $r_{0jk}$, $r_{1jk}$, and $r_{2jk}$ are random effects for teacher j, and school k.

For school k, the level-3 model can be formulated as follows. In the level-three model, the level-2 intercepts and slopes are used as outcomes.

$$\begin{align*}
\beta_{00k} &= \gamma_{000} + \gamma_{001} W_{1k} + \gamma_{002} W_{2k} + u_{00k} \\
\beta_{01k} &= \gamma_{010} + \gamma_{011} W_{1k} + \gamma_{012} W_{2k} + u_{01k} \\
\beta_{02k} &= \gamma_{020} + \gamma_{021} W_{1k} + \gamma_{022} W_{2k} + u_{02k} \\
\beta_{10k} &= \gamma_{100} + \gamma_{101} W_{1k} + \gamma_{102} W_{2k} + u_{10k} \\
\beta_{11k} &= \gamma_{110} + \gamma_{111} W_{1k} + \gamma_{112} W_{2k} + u_{11k} \\
\beta_{12k} &= \gamma_{120} + \gamma_{121} W_{1k} + \gamma_{122} W_{2k} + u_{12k} \\
\beta_{20k} &= \gamma_{200} + \gamma_{201} W_{1k} + \gamma_{202} W_{2k} + u_{20k} \\
\beta_{21k} &= \gamma_{210} + \gamma_{211} W_{1k} + \gamma_{212} W_{2k} + u_{21k} \\
\beta_{22k} &= \gamma_{220} + \gamma_{221} W_{1k} + \gamma_{222} W_{2k} + u_{22k}
\end{align*}$$

(2.28)

The terms $u_{00k}$, $u_{02k}$, $u_{10k}$ etc. in 2.28 are random effects associated with school k.

The single-equation can be formulated as follows, by substituting (2.28) in (2.27), and then substituting the newly produced (2.27) in (2.26).
In (2.29), the parameter $\gamma_{000}$ is interpreted as the predicted reading achievement for a reference student associated with a reference teacher in a reference school (in this case, we assume all Xs, Ws, and as equal to zero). The terms $\gamma_{001}$, $\gamma_{010}$, $\gamma_{020}$, $\gamma_{100}$, and $\gamma_{200}$ are the simple effects of individual student, teacher, and school level predictors. The parameters $\gamma_{011}$, $\gamma_{012}$, $\gamma_{021}$, $\gamma_{102}$, $\gamma_{110}$, $\gamma_{120}$, $\gamma_{121}$, $\gamma_{201}$, $\gamma_{202}$, and $\gamma_{220}$ represent simple two-way interaction effects of any of the two student, teacher, and school level predictors. The terms $\gamma_{111}$, $\gamma_{112}$, $\gamma_{122}$, $\gamma_{211}$, $\gamma_{212}$, $\gamma_{221}$, and $\gamma_{222}$ represent the three-way interaction effect on reading achievement due to student, teacher, and school level predictors.

Equation (2.29) also consists of residual terms associated with all three levels. The level-1 residual, $e_{ijk}$, is the unique effect of student $i$ on reading achievement, associated with teacher $j$, and school $k$. Similarly, $r_{0jk}$ is the unique effect of teacher $j$ from school $k$, and $u_{00k}$ is the unique school effect for $k$th school for a reference teacher and reference student. The terms $r$’s and $u$’s are level-2 and level-3 residual terms respectively associated with slopes. The interaction terms of random effect and individual predictor or cross-level predictors are also present in the above model. For example, the term $\{r_{2jk}a_{2ijk}\}$ is the interaction of the unique effect associated with student race-slope and student SES, and $u_{12k}$ $\{u_{22k}X_{2jk}a_{2ijk}\}$ is the three-way interaction between classroom type, student race, and the residual term associated with the slope of classroom type.
**Assumptions**

The following assumptions can be made for three-level HLM.

a) The error terms of each level-1 unit should have a mean of zero, and the error terms should be multivariate normally distributed. If, for example, we consider level-1 and level-2 units as students and teachers, respectively, then the mean of the error within each teacher should be zero, and these error terms should be multivariate normally distributed.

b) It is assumed that the relationship between predictors and outcome variables, at all three levels, is linear.

c) Another assumption is the homogeneity of variance. That is, all teachers should have equal variances in the sample.

d) Level-1 predictors are independent of the level-1 error term. In other words, the covariance between the level-1 predictors and the error term should equal zero.

e) Level-2 and level-3 error terms have a mean of zero and follow a multivariate normal distribution.

f) Level-2 predictors are independent of all level-2 error terms and level-3 predictors are independent of all level-3 error terms.

g) The level-1 error terms are independent of (uncorrelated to) level-2, and level-3 error terms in the model. That is, the correlation is zero between the level-1 error term and the level-2 error term in the model for the level-1 intercept, or the error term in any of the equations used to estimate the slopes of level-1 variables.

**Unconditional and Conditional Models**

For the three-level HLM, the unconditional model is formulated by using no predictors in the model, and the conditional model is expressed with appropriate predictors in the equation. The resulting level-1 unconditional and conditional models are the same as those provided for two-level HLM, in Equations (2.20) and (2.22), respectively. The level-2 unconditional and conditional models are expressed as Equations (2.21) and (2.23), respectively. Only a change in notation will take place in three-level HLM (for example, we use π’s, β’s, and γ’s as level-1, level-2, and level-3
coefficients/parameters in three-level HLM). The level-3 conditional model can be expressed by (2.28); however, level-3 unconditional models can be given by deleting W’s in (2.28) and using only intercepts and random effect terms in the model.

Teacher and School Effects

The estimation of unique teacher effects on student achievement can be informative for improving teaching effectiveness. Rowan et al. (2002) estimate teacher effects on student achievement by employing three-level hierarchical modeling. They suggest that a great many studies have decomposed the variance in student achievement into components lying among schools, among classrooms within schools, and among students within classrooms. Partitioning error variance into within- and between-group components allows us effectively, and simultaneously, to measure teacher effects on reading achievement in this study.

In a multilevel study for a Local Education Authority, Aitkin and Longford (1986) assess school effectiveness by employing a variety of models and advocate the use of variance component or ‘random parameter’ models for analysis in such studies involving clustered observations. For clarification, the variance component model and hierarchical model are analogous where school, classroom, or teacher effects on student outcome can be estimated. Webster et al. (1998) use HLM analysis to estimate school effects on the student learning process.

Some argue that HLM allows schools to differ in their effects by including student level predictor/s in the model. Rather than simply assuming that a school has a constant effect on all of its students, as in conventional school effects analyses, this model allows us to represent different effects for different students through the inclusion of student characteristics in the equation, and formally, we say that the $\beta_{jk}$ coefficients represent the distributive effects of school j on student achievement (Bryk & Raudenbush, 1988).

Two-Level and Three-Level HGLM

There are certain situations in which the predicted value of the outcome in a linear model cannot be used for meaningful reasons. The predicted value of the outcome in
linear HLM can produce a real value; however, as mentioned at the beginning of this Chapter, one of the major restrictions is that the predicted value for the dichotomous outcome must lie between 0 and 1. Raudenbush and Bryk (2002) indicate that this constraint gives meaning to the effect sizes defined by the model, and a nonlinear transformation of the predicted value, such as a logit or probit transformation, will satisfy this constraint.

Rachman-Moore and Wolfe (1984) propose a nonlinear statistical model in order to determine an educational outcome as a nonlinear function of explanatory variables defined at different levels of a survey data hierarchy, such as students and classes. Thus, a data hierarchy can be modeled with multilevel design when the level-1 model becomes a nonlinear model. Level-2 model outcomes (formed by level-1 intercept and slope/s), however, can be viewed as linear functions of level-2 predictors.

Compared to the HLM, the HGLM does not require several assumptions, such as normality, independence of residual terms, and homogeneity of error variance or homoscedasticity. There may appear several situations in which the outcome is a binary variable, demanding the use of nonlinear modeling. Examples of binary response data include the proficiency or non-proficiency status of students, dropout or non-dropout, repeated or not repeated, and so on. This section aims to discuss previous work in modeling such dichotomous outcomes.

Some studies have considered dropout as a binary outcome to be predicted by using variables of interest (see Bryk & Thum, 1989; McCaul et al., 1992; Rumberger, 1995; Patrick, 2000; Ma, 1999). In order to model binary response data associated with dropouts, Bryk and Thum (1989) uses a two-level hierarchical generalized model and investigates the effects of structural and normative features of schools on dropping out and absenteeism. They find that high levels of internal differentiation within high schools and weak normative environments contribute to the problems of absenteeism and dropping out.

In a study considering the dichotomous outcome in the multilevel model, McCaul et al. (1992) compare dropping out to personal, social, and labor market experiences. They find that dropouts differ from graduates in personal and social adjustments. In a
dropout study for modeling the dichotomous outcome, Rumberger (1995) uses a two-level statistical model with students in level-1 and schools in level-2. The research was conducted using a logistic regression model as the level-1 model, and the coefficients of logistic regression as outcomes in the level-2 model. However, Ma (1999) uses student socioeconomic status and prior achievement in mathematics to predict dropout, employing a two-level HGLM.

Albert (1988) models the binary response variable in order to test the goodness of fit of the model, and uses Bayesian two-stage prior distribution for estimating the mean. Raudenbush and Bryk (2002) present a two-level HGLM example using ‘repetition’ as a binary outcome and different socio-demographic variables as predictors. The authors mention that the intraclass correlation index is less informative in nonlinear model, compared to the linear model, since level-1 variance in nonlinear modeling is heteroscedastic.

Wong and Mason (1985) have considered the hierarchical logistic regression model in relation to model a binary response, using the logit function as an outcome in the level-1 model. These researchers illustrate micro- and macro-level models in which micro observations are embedded within macro observations. However, the interpretive procedures they used in these analyses were analogous to those commonly used in the two-level HGLM, with level-1 (micro) embedded within the level-2 (macro) equations.

Research clarifies the advantages of the HGLM over the single-level logistic model. For example, Guo and Zhao (2000) maintain that multilevel modeling not only enables one to decompose the total variance associated with the outcome variable into the parts of each level, but also facilitates reducing cluster bias, produces correct standard errors and correct confidence intervals. However, the complexity in analysis and interpretation with a large number of predictors in the models, and estimating their simple and interaction effects cannot be ignored.

From methodological standpoint, Raudenbush et al. (2001) indicate that the HGLM produces estimates for both the unit-specific and population-average models, where the population-average results are based on generalized least squares given the variance-covariance estimates from the unit-specific model. Additionally, Zeger et al.
Goldstein (1986) employs a multilevel mixed-effects model and also describes a model for interactions of explanatory variables between levels generating more random terms in the model. Additionally, Goldstein (1995) discusses the random interaction effect in the model, and states that the adequacy of such a model can be tested against an additive model using a likelihood ratio test criterion.

There have been limited studies to date, using binary outcomes, in the three-level HGLM. Although Raudenbush and Bryk (2002) indicate that three-level nonlinear HLM will be the same as three-level linear HLM, the authors do not make sufficient efforts to illustrate and interpret the parameters in three-level HGLM with the binary outcome. Tate (2004) discusses the interpretation of slopes as outcomes in three-level HGLM, with the binary outcome and the single continuous predictor at each level, modeling cross-level interaction effects in the single-equation. However, a more comprehensive step-by-step demonstration procedure of three-level HGLM, with binary outcome in order to model continuous and dichotomous predictors, is felt necessary. More importantly, describing predictors’ effects on the binary outcome in the model by means of graphical presentation is crucial to simplify the effect interpretations in multilevel researches employing ANOVA-like and simple effect techniques.

Simple Effect and ANOVA-Like Interpretations

The simple effect method simplifies the interpretation of specific predictor’s effect on reading proficiency based on the levels of other predictors. Tate (2004) provides the interpretive process using the simple effect approach. Situations may arise when the interpretation of effects in HGLM would be more complex. Particularly when cross-level interaction, based on multiple levels of predictors, is to be interpreted, it is not generally straightforward. However, the simple effect approach facilitates the interpretation of predictor’s effect (point estimate) as well as the ranges of the effects (interval estimate).
Using this approach, we can assert not only the significance of effect, but also the magnitude and direction of the effect of one predictor based on the multiple levels of other predictor/s.

One of the approaches of presenting individual effects and cross-level interaction effects, based on HLM results, is an ANOVA-like approach. It is generally complicated to understand the interpretation of cross-level interaction (using traditional method of interpretation) in a three-level HGLM. However, a simpler technique of interpretation can be followed using the ANOVA-like approach. Through this approach, the description of the overall relationship can be provided by interpreting the effect of a specific predictor based on different levels of other predictors. Further, the interpretations of the main effect, and two-way as well as three-way interaction effects, can be provided using the ANOVA-like approach. Kirk (1995) describes procedural steps in computing main as well as interaction effects in factorial ANOVA, and Tate (2004) briefly presents the interpretations of the main and interaction effects using the ANOVA-like approach in HGLM.

Research on NAEP Data using HLM

Many studies have used NAEP data sets for HLM analysis. Arnold (1995) uses a two-level HLM analysis with 1990 NAEP data in order to identify school and other correlations of student achievement. Arnold found that HLM methods worked well to explain variations in achievement. In order to examine the effects of both student and school characteristics on student achievement, Arnold and Kaufman (1992) employed a two-level HLM with NAEP data. McLaughlin and Drori (2000) showed the potential value of a linkage between the Schools and Staffing Survey (SASS) and student achievement, which included the reading and mathematics achievement.

Raudenbush et al. (1999) employs NAEP data from the Trial State Assessment using a two-level HLM and found state-to-state heterogeneity in student mathematics proficiency. Further, Cheong et al. (2001) uses a three-level HLM and found that the three-level analyses produced sound standard errors when samples were fairly large. In the latter case, the researcher preferred three-level analysis over two-level.
Summary

This Chapter presents a review of literature on the conceptual bases of the logistic regression model, two- and three-level HLM, and HGLM. The modeling features were presented with relevant examples, using appropriate predictors in the models. The limitation of a single-level model and the advantages of the two- and three-level models were discussed. A three-level model is sometime preferred; such a model is necessary when the two-level model is insufficient to measure the effects of predictors in situations where a three-level data hierarchy is present. We also indicated the necessity of the two- and three-level HGLM when we propose to measure the effects on dichotomous outcome due to multilevel predictors in a hierarchical design.

I also discussed centering issues, and applied grand-mean centering for the continuous predictor in all level-1, level-2, and level-3 models. The assumptions required for a nonlinear logistic regression model and a hierarchical linear model were also discussed.

A common approach of interpreting parameters in HLM is by formulating a single-equation with the presence of the individual effect and cross-level interaction effects. The use of a simple effect approach, applying the differentiation technique, is the most straightforward way to interpret predictors’ effects on level-1 outcome.
CHAPTER 3

METHODOLOGY

Demonstration Data

This Chapter consists of a brief introduction to the National Assessment of Educational Progress (NAEP) data, description of the variables used, and the primary research questions addressed in this study. Methodological illustrations of statistical models related to sequential development, analysis, evaluation, and application of such models are presented for demonstration purpose. Finally, the approaches of effect interpretation using ANOVA-like and simple effect techniques are also described.

Study Goals

The goals of this study are to present a sequential demonstration of three-level HGLM to aid individuals in applying such procedures to their researches, and describe the simple effect and the ANOVA-like approaches of interpreting effects. Specifically, the following goals are achieved by means of this study.

a. The illustration of the sequential steps to be followed to develop model, formulate unconditional and conditional models by including potential predictors in the conditional models.

b. The evaluation and application of the models which are based on estimated parameters.

c. The interpretation of the effects of predictors based on simple effect and ANOVA-like approaches.

All the above goals are accomplished, with illustrations, in the fourth Chapter. In this Chapter, the model formulation for two-level and three-level HGLM is presented.
with the appropriate definitions of the symbols or notations used in the models. Procedures of developing, analyzing, evaluating, and applying the formulated models are also presented.

The major significance of this study is to demonstrate the three-level HGLM with the dichotomous outcome in order to teach educational research practitioners an illustrative procedure of development, analysis, evaluation, and application of the model. Another important significance of this study is to describe the simple effect approach in order to interpret the effect of a predictor, based on the levels of other predictors, on reading proficiency. In addition, an ANOVA-like method of effect interpretation, with a description of overall relationship as well as a description of the effects of individual predictors (i.e., main and interaction effects), is illustrated to interpret the effect on reading proficiency due to a specific predictor based on the levels of other predictors.

Information on NAEP

The NAEP 2000 Reading Assessment is a large-scale assessment, and was developed to focus on the nature of reading comprehension. This assessment included three components of reading comprehension, namely, reading for literary experience, reading to gain information, and reading to perform tasks. The assessment contained general background questions, reading comprehension items, reading-specific background items, and items related to student motivation and familiarity with the assessment. The assessment permitted maximum coverage of reading ability at grade four, and minimized the time burden on the examinee. In addition to student test booklets, a teacher questionnaire, a school questionnaire, and a Student with Disabilities/Limited English Proficiency (SD/LEP) questionnaire were administered.

The NAEP 2000 reading assessment of fourth graders is based on a national sample, in which a complex multistage sampling procedure was employed. Data for the 2000 reading assessment were collected from January through March, 2000, with makeup sessions in early April.
Data

This study used a sample of 7,175 fourth graders, from 46 states, provided by the NAEP 2000 Reading Main Assessment data. The data comprises of 1,076 teachers and 295 schools after eliminating the cases with insufficient information. Such cases include those with missing identification numbers either for teachers or for schools, and data with two or fewer students nested within a teacher and two or fewer teachers nested within a school. The rationale for deleting level-1 and level-2 units with two or fewer cases is to avoid the perfect model fit or prevent overestimation of the explained variance ($R^2$) in the model, from multiple regression modeling perspective. It is notable that the level-1, level-2, and level-3 units of analysis are student, teacher, and school level data, respectively.

The data provided 3 and 27 as the minimum and maximum numbers, respectively, of students nested within a teacher. The mean and median numbers of students nested within a teacher were 7 and 6, respectively. Similarly, the minimum and maximum numbers of teachers nested within a school were 3 and 10, respectively. The mean and median numbers of teachers nested within a school were 4 and 3, respectively.

Sample

The data are based on a national representative sample of fourth-grade students. The sample was selected using a three-stage (also called complex multistage) sampling procedure that involved sampling students from selected schools within selected geographic areas across the country. In the first stage, the geographic areas, such as a county, group of counties, or metropolitan area, were chosen. Second, the schools (public and non-public) within the selected geographical area were sampled. In the third stage, students were sampled from the selected schools.

Since the NAEP sampling was based on a complex sampling procedure (unequal number of students drawn from each school and unequal number of schools drawn from each geographical stratum), sampling weights were allocated in the original data set. Due to the technical limitation of HLM software in terms of handling weights, the sampling weights were ignored while analyzing data. Regardless of dealing with sampling weights associated with any levels, this study aims to demonstrate the three-
level HGLM, and interpret the simple effects of predictors based on estimated parameters.

**Variables**

Although the variables to be used for each of the three levels in the model are based on NAEP 2000 Reading Main Assessment data, some variables are recoded into new variables. The construction of recoded new variables are based on the substantial effect of these predictors on reading proficiency. Justification about the considerable effect of selected predictors is discussed above in the review of literature. The variables, which were reverse-coded in the original data set, were recoded in the positive scale.

**Outcome variable**

The reading proficiency status is used as a dichotomous outcome variable. This variable is computed by averaging five reported reading composite scores (ARRPCM1-ARRPCM5). The composite reading scores¹ used were scores for two basic purposes: reading for literary experience and reading to gain information.²

The NAEP composite scores are based on plausible values. The plausible values are some kind of student ability estimates based on the random draws from the distribution of student ability estimates. However, I have used the average of composite scores only for illustration purpose. Note that the resulting estimate of an individual score is not intended by NAEP, but it will fulfill the demonstration purpose of this study.

The mean reading composite score distribution is dichotomized in order to obtain the binary outcome. The median of the average composite scores is used to determine the cut-off score for a student’s reading proficiency status. Buckendahl and Ferdous (2002)

¹ The five reported reading composite scores were: Reporting reading value 1 (ARRPCM1), reporting reading value 2 (ARRPCM2), reporting reading value 3 (ARRPCM3), reporting reading value 4 (ARRPCM4), and reporting reading value 5 (ARRPCM5).

² According to the 2000 Reading Assessment Data Companion, the relative contribution of each reading purpose at grade 4, as specified in the reading framework, is 55 percent for Reading for Literary Experience and 45 percent for Reading to Gain Information.
use a median as the cut-off score for standard setting, since the median can compensate for the extreme or inconsistent values in a distribution. Similarly, Azibo (1983) dichotomizes the black personality scores into two groups using a median value, and Onis (2000) also uses the median to serve as the cut-off point in a set of scores. A brief description of all variables used in this study is presented in the following Table.

**Predictors**

Table 3.1  
Description of Variables Used for Demonstration Purposes

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Student-level:</strong></td>
<td></td>
</tr>
<tr>
<td>Socioeconomic status (SES)</td>
<td>This is a continuous variable derived from three Student-level variables, namely, student lunch eligibility (SLUNCH), Title 1 funding status, and parents’ education (PARED). The minimum and maximum values for SES were 3 and 9, respectively.</td>
</tr>
<tr>
<td>Sex (Male)</td>
<td>This is a dichotomous predictor coded from student’s ‘Sex,’ an original variable, and is recoded 1 for male, and 0 for female.</td>
</tr>
<tr>
<td>Race (Minority)</td>
<td>The ‘minority’ is a dichotomous variable derived from student race, 1 for minority (non-White), and 0 for non-minority (White).</td>
</tr>
<tr>
<td><strong>Teacher-level:</strong></td>
<td></td>
</tr>
<tr>
<td>Teacher expectation</td>
<td>This is a continuous variable showing the range of teacher’s (who taught the course) expectation for student achievement, 1(somewhat negative) to 3 (very positive).</td>
</tr>
</tbody>
</table>
Table 3.1  (Continued)

Use of computer  This continuous variable describes how often the teacher uses the computer for instruction in reading. The ranges of measurement were from never or hardly ever (1) to every day (4).

Class type  This predictor is based on the median number of students (assigned to a specific teacher) in grade 4 reading. Class type is derived as a dichotomous predictor with non-crowded (equal to or less than the median number of students, and is coded as 0) and crowded (larger than the median number, and is coded as 1) for each teacher.

School-level:

School mean SES  This is a continuous variable derived by average SES aggregates from student SES for each school.

School type  This is a school-type dichotomous variable, coded 1 for public and 0 for nonpublic.

Funding status  This is a dichotomous variable providing the information whether the school is included in Title 1 or Chapter 1 federal funding, with 1 (yes) and 0 (no) codes.

Analysis Procedure

*Three-Level HGLM*

**Research Questions for Demonstration**

This study addresses the following major research questions.

1. What is the effect of the individual predictor at each level on the odds of reading proficiency?
2. How does this effect depend on the levels of other predictors?
3. What is the overall relationship of the effect on reading proficiency due to one predictor based on the levels of other predictors?

The simple effect is estimated for predictors. For example, we have estimated the simple effect of school socioeconomic status at level-3 on the odds of student reading proficiency, and we have also determined whether the levels of student- and teacher-level predictors affect such effects.

*Level-1 Link Function*

For illustration purposes in this study, let \( \phi \) be the probability of reading proficiency, and \((1-\phi)\) the probability of non-proficiency. The odds of proficiency is then given by \([\phi/(1-\phi)]\), a ratio of probability of proficiency to the ratio of probability of non-proficiency, and the log of odds is given by

\[
\eta_{ijk} = \ln \left( \frac{\phi}{1-\phi} \right) \quad (3.1)
\]

where \( \eta_{ijk} \) can take any real value and \( \phi \) lies between 0 and 1, and \( \ln \) is the natural logarithm.

In order to adopting consistency for modeling purpose, let us consider student-, teacher-, and school-level models at level-1, level-2, and level-3, respectively. Assume that a dichotomous outcome, \( Y_{ijk} \), measures student’s proficiency in reading. Then, \( \phi_{ijk} \) is the probability of proficiency and \((1-\phi_{ijk})\) is the probability of non-proficiency for student i, teacher j, and school k. We denote the probability of the proficiency equal to 1, and \( \phi_{ijk} \) can be modeled using the logit link function \( (\eta_{ijk}) \). Analogous to (2.2), this can be given as the following.

\[
\phi_{ijk} = \frac{\exp(\eta_{ijk})}{1 + \exp(\eta_{ijk})} \quad (3.2)
\]

Since each level-1 observation related to a student occurs with a single binary outcome per student, the subsequent model will be consistent with Bernoulli. The link function, \( \eta_{ijks} \), is analogous to the log-odds of proficiency, and is expressed in (3.1).
Model Formulation

**Level-1 Structural Model.** The level-1 structural model is formulated by using the log-odds as an outcome, and includes a desired number of predictors in the model. The level-1 model can be given, as below, with \( i=1, 2, \ldots, n_{ijk} \) students nested within each of \( j=1, 2, \ldots, J_k \) teachers within each of \( k = 1, 2, \ldots K \) schools.

\[
\eta_{ijk} = \pi_{0jk} + \pi_{1jk} a_{1ijk} + \pi_{2jk} a_{2ijk}
\]  

(3.3)

where \( a_{1ijk} \) (say, \( a_1 \)) and \( a_{2ijk} \) (say, \( a_2 \)) in the above equation are two student-level predictors, namely, student race and student SES, respectively. Student SES is a continuous predictor, and is grand-mean centered, whereas student race (minority or non-minority status) is a dichotomous predictor that is uncentered. In (3.3), \( \eta_{ijk} \) is the log-odds of reading proficiency for student \( i \), associated with teacher \( j \) and school \( k \). The parameter \( \pi_{0jk} \) is the intercept for log-odds of reading proficiency, associated with the reference student. Further, \( \pi_{1jk} \) and \( \pi_{2jk} \) are the slopes for student SES and student race, associated with teacher \( j \) in school \( k \).

**Level-2 Structural Model.** As in the case of linear HLM, the level-1 intercept \( (\pi_{0jk}) \), and slopes \( (\pi_{1jk} \text{ and } \pi_{2jk}) \) are considered as outcomes in the level-2 model. Let us suppose that the continuous predictor \( X_{1jk} \) (say, \( X_1 \)) is the teacher’s use of computer for reading instruction, and is grand-mean centered, and the dichotomous predictor \( X_{2jk} \) (say, \( X_2 \)) is the classroom type (crowded or non-crowded classroom). To demonstrate, the level-2 model can be formulated as follows.

\[
\begin{align*}
\pi_{0jk} &= \beta_{00k} + \beta_{01k} X_{1jk} + \beta_{02k} X_{2jk} + r_{0jk} \\
\pi_{1jk} &= \beta_{10k} + \beta_{11k} X_{1jk} + \beta_{12k} X_{2jk} + r_{1jk} \\
\pi_{2jk} &= \beta_{20k} + \beta_{21k} X_{1jk} + \beta_{22k} X_{2jk} + r_{2jk}
\end{align*}
\]  

(3.4)

where \( \beta_{00k}, \beta_{10k}, \text{ and } \beta_{20k} \) are level-2 intercepts, and \( \beta_{01k}, \beta_{02k}, \beta_{11k}, \beta_{12k}, \beta_{21k}, \text{ and } \beta_{22k} \) are level-2 slopes. Further, \( r_{0jk}, r_{1jk}, \text{ and } r_{2jk} \) are the random effects associated with teacher \( j \) and school \( k \).
Level-3 Structural Model. Let us consider $W_1$ as the school mean socioeconomic status, and $W_2$ as the school’s Title 1 funding status (getting funded or not funded). In this case, the level-3 model for school $k$, can be formulated by the following set of Equations, given by (3.5). In the level-3 model, the level-2 intercepts and slopes become outcomes.

\[
\begin{align*}
\beta_{00k} &= \gamma_{000} + \gamma_{001} W_{1k} + \gamma_{002} W_{2k} + u_{00k} \\
\beta_{01k} &= \gamma_{010} + \gamma_{011} W_{1k} + \gamma_{012} W_{2k} + u_{01k} \\
\beta_{02k} &= \gamma_{020} + \gamma_{021} W_{1k} + \gamma_{022} W_{2k} + u_{02k} \\
\beta_{10k} &= \gamma_{100} + \gamma_{101} W_{1k} + \gamma_{102} W_{2k} + u_{10k} \\
\beta_{11k} &= \gamma_{110} + \gamma_{111} W_{1k} + \gamma_{112} W_{2k} + u_{11k} \\
\beta_{12k} &= \gamma_{120} + \gamma_{121} W_{1k} + \gamma_{122} W_{2k} + u_{12k} \\
\beta_{20k} &= \gamma_{200} + \gamma_{201} W_{1k} + \gamma_{202} W_{2k} + u_{20k} \\
\beta_{21k} &= \gamma_{210} + \gamma_{211} W_{1k} + \gamma_{212} W_{2k} + u_{21k} \\
\beta_{22k} &= \gamma_{220} + \gamma_{221} W_{1k} + \gamma_{222} W_{2k} + u_{22k} \\
\end{align*}
\]

(3.5)

In (3.5), $u$’s are the random effect terms associated with schools. Substituting (3.5) in (3.4), and then (3.4) in (3.3), the resultant single-equation can be expressed as follows.

\[
\eta_{ijk} = \gamma_{000} + \gamma_{001} W_{1k} + \gamma_{002} W_{2k} + \gamma_{010} X_{1jk} + \gamma_{011} W_{1k} X_{1jk} + \gamma_{012} W_{2k} X_{1jk} + \gamma_{020} X_{2jk} + \gamma_{021} W_{1k} X_{2jk} + \gamma_{022} W_{2k} X_{2jk} + \gamma_{100} A_{1ijk} + \gamma_{101} W_{1k} A_{1ijk} + \gamma_{102} W_{2k} A_{1ijk} + \gamma_{110} X_{1jk} A_{1ijk} + \gamma_{111} W_{1k} X_{1jk} A_{1ijk} + \gamma_{112} W_{2k} X_{1jk} A_{1ijk} + \gamma_{120} X_{2jk} A_{1ijk} + \gamma_{121} W_{1k} X_{2jk} A_{1ijk} + \gamma_{122} W_{2k} X_{2jk} A_{1ijk} + \gamma_{200} A_{2ijk} + \gamma_{201} W_{1k} A_{2ijk} + \gamma_{202} W_{2k} A_{2ijk} + \gamma_{210} X_{1jk} A_{2ijk} + \gamma_{211} W_{1k} X_{1jk} A_{2ijk} + \gamma_{212} W_{2k} X_{1jk} A_{2ijk} + \gamma_{220} X_{2jk} A_{2ijk} + \gamma_{221} W_{1k} X_{2jk} A_{2ijk} + \gamma_{222} W_{2k} X_{2jk} A_{2ijk} + [u_{000} + u_{001} X_{1jk} + u_{002} X_{2jk} + u_{10k} A_{1ijk} + u_{11k} X_{1jk} A_{1ijk} + u_{12k} X_{2jk} A_{1ijk} + u_{20k} A_{2ijk} + u_{21k} X_{1jk} A_{2ijk} + u_{22k} X_{2jk} A_{2ijk} + r_{0jk} + r_{1jk} A_{1ijk} + r_{2jk} A_{2ijk}]
\]

(3.6)

In (3.6), $\gamma_{000}$ is the average log-odds of reading proficiency for a reference student associated with a reference teacher and a reference school. The parameters $\gamma_{001}$, $\gamma_{010}$, $\gamma_{020}$, $\gamma_{100}$, and $\gamma_{200}$ are the reference simple effects of individual student-, teacher-, and
school-level predictors on log-odds of reading achievement. The terms $\gamma_{011}, \gamma_{012}, \gamma_{021}, \gamma_{120}, \gamma_{201}, \gamma_{022}, \gamma_{101}, \gamma_{102}, \gamma_{110}, \gamma_{210}, \gamma_{202}, \gamma_{220}$ symbolize the reference, simple two-way interaction effects of any of the two predictors at the student, teacher, or school level. And, the parameters $\gamma_{111}, \gamma_{112}, \gamma_{121}, \gamma_{122}, \gamma_{211}, \gamma_{212}, \gamma_{221}, \gamma_{222}$ represent the three-way interaction effect on log-odds of reading proficiency.

**Assumptions**

a. The relationship between outcome and predictors at level-1 is nonlinear. However, the relationship between outcome and predictors at level-2 and level-3 is linear.

b. The model is correctly specified, i.e., a) the true conditional probabilities for the level-1 model are a logistic function of the predictors, b) no important variables are omitted, c) no extraneous variables are included, and d) the observed variables are measured without error.

c. The residuals at both level-2 and level-3 are assumed to be independently and multivariate normally distributed with constant variance-covariance matrix.

d. The predictor variables are not linear combinations of each other. Perfect multicolinearity makes estimation impossible, while strong multicolinearity makes estimates imprecise.

**Simple Effect Description**

This study provides the procedure for simple effect interpretation along with an ANOVA-like interpretation in the three-level HGLM. In the simple effect approach, the effect of each individual predictor (for each of the three levels) can be expressed by taking partial derivative of $\eta$ with respect to $W_1, W_2, X_1, X_2, a_1,$ and $a_2$, at a time, and exponentiating the results of the first derivative by multiplying it with a constant associated with that specific predictor. Tate (2004) discusses simple effects interpretation in HGLM, and Long (1997) shows that we can express the discrete change in an outcome in a model that consists of continuous and dichotomous predictors. Ignoring the letter $(i, j, k)$
subscripts, the resulting equations for simple effects can be expressed as follows after partial differentiation of \( \eta \) (log-odds of reading proficiency) in (3.6), with respect to \( W_1 \), \( W_2 \), \( X_1 \), \( X_2 \), \( a_1 \), and \( a_2 \), one at a time, and exponentiating it.

\[
\exp(c_{w1} \frac{\partial \eta}{\partial W_1}) = \exp\left[c_{w1}(\gamma_{001} \cdot X_1 + \gamma_{021} \cdot X_2 + \gamma_{101} \cdot a_1 + \gamma_{111} \cdot a_1 \cdot X_1 + \gamma_{121} \cdot a_1 \cdot X_2 + \gamma_{201} \cdot a_2 + \gamma_{211} \cdot a_2 \cdot X_1 + \gamma_{221} \cdot a_2 \cdot X_2)\right] \quad (3.7)
\]

\[
\exp(c_{w2} \frac{\partial \eta}{\partial W_2}) = \exp\left[c_{w2}(\gamma_{002} \cdot X_1 + \gamma_{022} \cdot X_2 + \gamma_{102} \cdot a_1 + \gamma_{112} \cdot a_1 \cdot X_1 + \gamma_{122} \cdot a_1 \cdot X_2 + \gamma_{202} \cdot a_2 + \gamma_{212} \cdot a_2 \cdot X_1 + \gamma_{222} \cdot a_2 \cdot X_2)\right] \quad (3.8)
\]

\[
\exp(c_{x1} \frac{\partial \eta}{\partial X_1}) = \exp\left[\gamma_{010} + \gamma_{011} \cdot W_1 + \gamma_{012} \cdot W_2 + \gamma_{110} \cdot a_1 + \gamma_{111} \cdot a_1 \cdot W_1 + \gamma_{112} \cdot a_1 \cdot W_2\right] \quad (3.9)
\]

\[
\exp(c_{x2} \frac{\partial \eta}{\partial X_2}) = \exp\left[\gamma_{020} + \gamma_{021} \cdot W_1 + \gamma_{022} \cdot W_2 + \gamma_{120} \cdot a_1 + \gamma_{121} \cdot a_1 \cdot W_1 + \gamma_{122} \cdot a_1 \cdot W_2\right] \quad (3.10)
\]

\[
\exp(c_{a1} \frac{\partial \eta}{\partial a_1}) = \exp\left[\gamma_{100} + \gamma_{101} \cdot W_1 + \gamma_{102} \cdot W_2 + \gamma_{110} \cdot X_1 + \gamma_{111} \cdot X_1 \cdot W_1 + \gamma_{112} \cdot X_1 \cdot W_2 + \gamma_{120} \cdot X_2 + \gamma_{121} \cdot X_2 \cdot W_1 + \gamma_{122} \cdot X_2 \cdot W_2\right] \quad (3.11)
\]

\[
\exp(c_{a2} \frac{\partial \eta}{\partial a_2}) = \exp\left[\gamma_{200} + \gamma_{201} \cdot W_1 + \gamma_{202} \cdot W_2 + \gamma_{210} \cdot X_1 + \gamma_{211} \cdot X_1 \cdot W_1 + \gamma_{212} \cdot X_1 \cdot W_2 + \gamma_{220} \cdot X_2 + \gamma_{221} \cdot X_2 \cdot W_1 + \gamma_{222} \cdot X_2 \cdot W_2\right] \quad (3.12)
\]

In the above equations, the constants (e.g., \( c_{w1} \), \( c_{w2} \), \( c_{x1} \), and so on) are computed using the standard-deviation in the case of continuous predictors; however, the constants for all dichotomous predictors are used as 1 (see Chapter 4 for detail). When the fixed terms in (3.6) are linear (i.e., not with powers) and composed of the products of predictors associated with any of the three levels, we can collect the terms containing predictor of interest (say, \( W \)), factor out this predictor, and use the remaining expression in order to find the simple effect. It is notable that in the case of the dichotomous outcome, the simple effect of \( W_1 \), for instance, associated with a \( c_{w1} \) change in \( W_1 \) is computed by multiplying the right hand side of (3.7) with \( c_{w1} \) and taking an exponential of the result in right side. We can apply the similar procedure in order to calculate the simple effects of any of the predictors in Equations (3.8) through (3.12).
For the determination of confidence intervals for simple effects, computation of a standard error is necessary. We can apply a procedure similar to those given in (2.16) and (2.17) to compute the standard errors, which is applicable in the case of the three-level HLM/HGLM. We can also use (2.18) and (2.19) to estimate the confidence intervals of effects. The variance-covariance matrix $V$ with four $\gamma$s (parameters) and a $9 \times 9$ matrix in (2.14) can be extended to a $27 \times 27$ matrix since there exist altogether 27 $\gamma$s in the three-level HGLM that determine predictors’ effects (assuming that individual equations in level-2 and level-3 models have two predictors along with two predictors in level-1 model). In other words, the matrix $V$ of a $27 \times 27$ order is necessary while modeling two predictors, one continuous and one dichotomous, at each of the levels in three-level HGLM.

The linear combination of the model parameters in (2.10) is given by $m = \mathbf{a}'\gamma$.

Forming the matrix $\mathbf{a}$ of the order of $27 \times 1$, with appropriate partial derivatives for measuring the effect of a specific predictor at a time, we can compute the standard error by applying formula, i.e.,

$$S_m = \sqrt{\mathbf{a}'V\mathbf{a}}.$$  \hspace{1cm} (3.13a)

According to Tate (2004), for $h$ number of parameters in the family of interest, and $\alpha_{fw}$ amount of family-wise error rate, the critical value can be computed by the following formula.

$$C_v = \sqrt{h F(\alpha_{fw}; h, dfe)}$$ \hspace{1cm} (3.13b)

where $h$ is the number of predictors $df_e$ is the error degree of freedom equivalent to $n-k-1$, where $n$ is the lowest sample size among all levels in the study, and $k$ is the number of parameters (including interaction effect but excluding intercept term) in single-equation. The d.f. associated with $h$ would be the number of hypothesis variables to be tested in the study.

Applying the computational formula of standard error from (2.18) and (2.19), the confidence interval for the simple effect of school mean SES or $W_1$ (a continuous predictor), with a change of $c_{w1}$ units, can be computed as follows.
where \( c_{w1} \) is a constant defined as two-standard deviation of school mean SES. Similarly, the confidence interval for the dichotomous predictor \( W_2 \) (Title 1 school funding status), with \( c_{w2} = 1 \), can be computed by (3.15).

\[
\text{C.I.}[E_{W2}] = \exp \left[ c_{w2} \frac{\partial \eta}{\partial W_2} \pm C_v \, c_{w2} \, S_{m,Ew2} \right]
\]  

(3.15)

The confidence interval for an estimated simple effect of teacher computer use, \( X_1 \), with a change of \( c_{x1} \) units (two-standard deviation) in the teacher computer use, can be computed by (3.16).

\[
\text{C.I.}[E_{X1}] = \exp \left[ c_{x1} \frac{\partial \eta}{\partial X_1} \pm C_v \, c_{x1} \, S_{m,Ex1} \right]
\]  

(3.16)

The confidence interval for the dichotomous predictor \( X_2 \) (class type), with \( c_{x2} = 1 \), can be computed by the following formula.

\[
\text{C.I.}[E_{X2}] = \exp \left[ c_{x2} \frac{\partial \eta}{\partial X_2} \pm C_v \, c_{x2} \, S_{m,Ex2} \right]
\]  

(3.17)

The confidence interval for the dichotomous predictor \( a_1 \) (student race, i.e., minority or non-minority student status), with \( c_{a1} = 1 \), can be computed by the following formula.

\[
\text{C.I.}[E_{a1}] = \exp \left[ c_{a1} \frac{\partial \eta}{\partial a_1} \pm C_v \, c_{a1} \, S_{m,EA1} \right]
\]  

(3.18)

Finally, the confidence interval for the estimated simple effect with a change of \( c_{a2} \) units in the student SES, can be computed by the following formula.

\[
\text{C.I.}[E_{a2}] = \exp \left[ c_{a2} \frac{\partial \eta}{\partial a_2} \pm C_v \, c_{a2} \, S_{m,EA2} \right]
\]  

(3.19)

where \( c_{a2} \) is a constant defined as two-standard deviation of student SES. The test of simple effect is conducted by comparing the resulting interval with the null value of one. 

\textit{Presentation Formats}

A Table is presented for the outcomes that include only dichotomous predictor/s in the model. Such Tables contain the columns with the levels of predictors, effect, lower limit, and upper limit demonstrating the results that show the change of each simple
effect across the range of the other predictors. For example, a Table is presented for the simple effect of school mean SES based on different levels of minority and class type. The magnitude of effect on reading proficiency is computed for possible values of 0 and 1 of both the predictors, namely, minority and class type. A similar Table is presented for describing the simple effect of student SES, a function of class type. However, a graphical display is presented for measuring effect if at least one of the predictors is of interval scale (continuous) in the equation. Based on several specific values of the other predictors, the point and interval estimates of the simple effects of specific predictor/s are presented graphically. Thus, the graphs showing the simple effect on odds of reading proficiency due to a specific predictor based on various levels of other predictors are presented.

**Preliminary Analysis**

*Missing Data.* Missing data are reasonably managed by using appropriate statistical measures. Raudenbush and Bryk (2002) and Goldstein (1995) discuss the effect of missing data in the analysis results, and caution that missing data should be eliminated before analysis. The listwise deletion procedure is performed for missing data. Sufficient students are included in the analysis, ensuring proper representation of the population.

*Case Analysis.* Case analysis is performed at each level to examine whether outlier observations exist in the data. The residual analysis is performed in order to determine whether excessively large residuals are present in our data. Standardized residuals with a value equal to or larger than 2.5 are flagged as possible outliers. In level-1, residual statistics are computed using the appropriate approach (e.g., using nonlinear modeling option) and casewise diagnostics of residuals are performed. A graphical method are used for identifying outlier observations. For level-2 and level-3 residual analyses, appropriate multilevel technique is followed.

*Basic Statistics.* Basic statistics, such as mean, standard deviation, minimum, and maximum of the variables are reported. In addition, histograms of within-group standard
deviation are reported (even though a test of homogeneity of level-1 variance is not required).

Examining Validity of Assumptions. Level-2 and level-3 residuals are assumed to be multivariate normal, and homogeneous in variance-covariance. Similarly, a linear form of relationship between outcome and predictors is assumed. Relevant diagrams are presented to verify such assumptions.

Steps for Demonstration of Three-Level HGLM

a. Testing for true variance. A fully unconditional model is formulated using reading proficiency as an outcome and with no predictors in the model. The log-odds of reading proficiency is predicted via a three-level model. The equation contains neither predictor nor an error terms in the level-1 (as the events in level-1 model follow Binomial response model). However, error terms exist in level-2 and level-3 models. The allocation of variability in the log-odds of reading proficiency can be estimated across teachers as well as schools. For example, three-level HGLM can be expressed mathematically as the following equations.

Level- 1: \( \eta_{ijk} = \pi_{0jk} \)  
Level- 2: \( \pi_{0jk} = \beta_{00k} + r_{0jk} \)  
Level- 3: \( \beta_{00k} = \gamma_{000} + u_{00k} \)

where above terms are defined in (3.3), (3.4), and (3.5). Further, the level-1 predictors are included in the model based on the bivariate correlation between outcome and the the predictor.

b. Specifying student-level variables. Conditional level-1, and unconditional level-2 and level-3 models are formulated at this step. Two predictors, one continuous and one dichotomous, are selected based on the correlation between predictors and outcome at level-1. Thus, the correlation index, although the qualitative importance of the relationship should not be solely based on this type correlation, is one of the bases of specifying predictors at level-1 model. However, only the potential predictors are
included in the model. Such a selection is based on several factors: the degree of the parameter’s effect, the significance of the variance components associated each predictor, and the degree of reliability for predicting reading proficiency due to a specific predictor.

The conditional level-1 model and unconditional level-2 and level-3 models can be formulated as follows.

Level-1 conditional model: \[ \eta_{ijk} = \pi_{0jk} + \pi_{1jk}a_{1ijk} + \pi_{2jk}a_{2ijk} \] (3.23)

Level-2 unconditional model:
\[ \begin{align*}
\pi_{0jk} &= \beta_{00k} + r_{0jk} \\
\pi_{1jk} &= \beta_{10k} + r_{1jk} \\
\pi_{2jk} &= \beta_{20k} + r_{2jk}
\end{align*} \] (3.24)

Level-3 unconditional model:
\[ \begin{align*}
\beta_{00k} &= \gamma_{000} + u_{00k} \\
\beta_{10k} &= \gamma_{010} + u_{01k} \\
\beta_{20k} &= \gamma_{020} + u_{02k}
\end{align*} \] (3.25)

The terms in the above models are defined in (3.3), (3.4), and (3.5). It is notable that we can develop a conditional level-2 model if a significant parameter variance is observed in teacher-level (level-2) equations, given by (3.24). An exploratory analysis is carried out in order to select the potential predictor/s at (conditional) level-2 and level-3 model.

c. Specifying teacher-level variables. In this step, conditional level-1 and level-2 equations are formulated. The most potential predictor is included in the model at teacher-level model. The conditional level-1 and level-2 models can be given by (3.3) and (3.4) respectively. The unconditional level-3 model can be given by (3.25), as a randomly varying model. As earlier step, proportion of variance explained and reliability of level-2 and level-3 models are evaluated.

The variance components are tested in the level-3 unconditional model, and if a significant school-to-school variation is found, predictors are included in the model.
d. Final model: Specifying school-level variables. The final model is formulated by using conditional models with potential predictors at each of the levels. Thus, a theoretical example of the conditional level-1, level-2, and level-3 models that produces a final model can be expressed by the set of equations in (3.3), (3.4), and (3.5). The potential predictors in level-3 conditional model are selected based on exploratory analysis.

The model fit for the final model is assessed in which the predictors only with significant effects are retained, and the model is used as a final model. If the effects are not significant, then such predictors are deleted from the model.

The final model is developed, analyzed, evaluated, and applied based on the parsimony (i.e., not having a relatively large number of predictors in the model), and adequacy of fit. The proportion of variance explained in conditional level-3 (final) model relative to unconditional model is evaluated, and the reliability estimates at level-2 and level-3 models are also assessed.

Description of Overall Relationship

The overall relationship is described using a factorial ANOVA-like approach of interpretation. The odds of reading proficiency is predicted, and predicted odds are presented for different levels of predictors, such as student SES and school mean SES, associated with different levels of class type and minority. The log-odds of reading proficiency ($\eta_{ijk}$) can be predicted by using (3.6) without using letter subscripts and error terms in the equation, which can be given as follows.

\[
\eta_{ijk} = \gamma_{000} + \gamma_{001} W_1 + \gamma_{002} W_2 + \gamma_{010} X_1 + \gamma_{011} X_1 W_1 + \gamma_{012} X_1 W_2 + \gamma_{020} X_2 \\
+ \gamma_{021} X_2 W_1 + \gamma_{022} X_2 W_2 + \gamma_{100} a_1 + \gamma_{101} a_1 W_1 + \gamma_{102} a_1 W_2 + \gamma_{110} a_1 X_1 \\
+ \gamma_{111} a_1 X_1 W_1 + \gamma_{112} a_1 X_1 W_2 + \gamma_{120} a_1 X_2 + \gamma_{121} a_1 X_2 W_1 + \\
\gamma_{122} a_1 X_2 W_2 + \gamma_{200} a_2 + \gamma_{201} a_2 W_1 + \gamma_{202} a_2 W_2 + \gamma_{210} a_2 X_1 + \\
\gamma_{211} a_2 X_1 W_1 + \gamma_{212} a_2 X_1 W_2 + \gamma_{220} a_2 X_2 + \gamma_{221} a_2 X_2 W_1 + \\
\gamma_{222} a_2 X_2 W_2
\] (3.26)
The odds of reading proficiency is computed after taking exponential of (3.26). This is shown in Chapter 4 based on HGLM results. Further, the interpretations are provided for different levels of SES \(a_2\), and school mean SES \(W\) associated with the levels of minority \(a_1\) status, and class type \(X\).

**Description of the Effects of Individual Predictors**

**Main and Interaction Effects**

Using ANOVA-like procedure, computation and interpretation of main and interaction effects of predictors, associated with different predictors, are presented in Chapter 4. For this purpose, the main effect (M. E.) of school mean SES \(W\) in (3.26) are calculated by taking the exponential of the coefficient \(\gamma_{001}\) after multiplying it with a constant, \(c_w\), where \(c_w\) is the constant based on the two-standard deviation of school mean SES. This can be expressed as follows.

\[
M. E. = \exp(c_w \times \gamma_{001}) \tag{3.27}
\]

The magnitude of the main effect given in (3.27) are interpreted as the odds ratio for reading proficiency associated with a change of two standard deviations in school mean SES for reference student, and reference teacher. Using a similar procedure, the main effects of other predictors are computed and interpreted. However, the constant “c” is selected as 1 for dichotomous predictors, such as minority, and class type.

Two-way interaction effects are computed using the exponential expression of the terms \(\gamma_{011}\), \(\gamma_{101}\), \(\gamma_{110}\), and \(\gamma_{210}\), and three-way interaction is computed by exponentiating \(\gamma_{111}\) from (3.26). Further, the interaction effects are interpreted as the variation of effect on reading proficiency due to one predictor based on the levels of other predictors. For example, the two-way interaction effect (I.E.) for \(\gamma_{011}\) (i.e., I.E. of class type and school mean SES) can be computed as

\[
\exp(c_x \times c_w \times \gamma_{011}) \tag{3.28}
\]

where \(c_x\) is used as 1 and \(c_w\) is used as two-standard deviation associated with school mean SES.
The three-way interaction, $\gamma_{111}$, is computed as
\[
\exp(c_{a} \times c_{x} \times c_{w} \times \gamma_{111})
\]  
(3.29)

The interaction effect given in (3.29) is interpreted as the factor increase in second order interaction between minority and class type due to two-standard deviation increase in school mean SES.

Interpretation of Final Model

The primary approach of interpreting the estimates of the HGLM-based final model is using simple effect procedure. Theoretically, the simple effects are expressed by Equations (3.7) through (3.12). However, the number of predictors in these Equations will be based on the potentiality of the predictors. The effect of a particular predictor on the odds of reading proficiency is described by the simple effect interpretation approach (using differential calculus and matrix algebra). For example, in order to compute and interpret the simple effect of minority on the odds of reading proficiency, Equation (3.11) is used. The research questions, such as, what is the effect of an individual predictor at each level on the log-odds of reading proficiency, and how does this effect depend on the levels of the other predictors? are addressed by using a simple effect approach.

An ANOVA-like approach is another approach for interpreting predictors’ effects. The overall relationship is described interpreting the effect of predictors on the odds of reading proficiency based on different levels of other predictors. Using an ANOVA-like approach, the main effects as well as two-way and three-way interaction effects are interpreted.
CHAPTER 4
DEMONSTRATION RESULTS

This Chapter provides a sequential demonstration of development, analysis, evaluation, and selection of the final model. For illustration purpose, NAEP Reading Assessment 2000 data are used. An exploratory analysis approach is used to select appropriate predictors in conditional models. Several steps are illustrated to develop the conditional models from unconditional models. The significance of variance components, predictors’ effects, reliabilities etcetera are considered while evaluating the final models.

Since the natural dichotomous outcomes (e.g., pass/fail, drop out status of students) were not available in NAEP Reading Assessment 2000 data, a derived dichotomous outcome--reading proficiency--was used for demonstration purposes. The rationale for changing the scale of “race” from categorical to dichotomous or “minority” is also to demonstrate the effect of a dichotomous predictor on level-1 outcome, i.e., reading proficiency. Several studies in the past have used “minority” as a dichotomous predictor (see Saint-Blancat, 1995; Burton, 1993; Arbuthnot & Wayner, 1982). The modeling choice with a dichotomous outcome incorporating two predictors in the student level, and one predictor in the teacher as well as school levels are merely for demonstration purposes.

Two approaches, a factorial ANOVA approach and a simple effect approach, are used to interpret the effects of predictors. The simple effect approach, which is broadly presented, incorporates confidence bands associated with the effects of predictors. In addition, preliminary analyses are performed, and tabular presentations of data are provided.
Descriptive Statistics

The final usable data set was based on NAEP 2000 fourth grade reading assessment data that contained 7,175 students nested within 1,076 teachers in 295 schools. The summary statistics for student level variables are presented in Table 4.1. The range of student SES, which is derived from three student level predictors, namely, student’s eligibility for free and reduced lunch, Title 1 funding status, and parent’s education, is a continuous predictor and ranged from 4 to 10. After grand mean centering, the mean of student SES was zero. Detailed descriptions about student level statistics are presented in Table 4.1a.

Table 4.1a
Mean and Standard Deviation of Student Level Variables

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student SES</td>
<td>7.18</td>
<td>1.16</td>
<td>4.00</td>
<td>10.00</td>
</tr>
<tr>
<td></td>
<td>(0.00)(^a)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Race (Minority)(^b)</td>
<td>0.49</td>
<td>0.50</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Sex (Male)</td>
<td>0.49</td>
<td>0.50</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Reading Proficiency</td>
<td>0.51</td>
<td>0.50</td>
<td>0.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

\(^a\)Mean of student SES after grand mean centered ,
\(^b\)Minority is coded as 0= Whites, 1= Minority/Non-Whites.

The predictor “minority” is derived from race of students where minority includes all non-White students. All the dichotomous variables, namely race/minority, sex/male, and reading proficiency, have a standard deviation (s.d.) of 0.5, minimum of 0, and maximum of 1. Race/minority and sex/male has a mean of 0.49, whereas reading proficiency has a mean of 0.51. The mean values for dichotomous predictors imply the proportion of the category represented by “1” coding to the total population. For example, a mean value of 0.49 for male signifies that the proportion of males to total population is 0.49.
The summary statistics for teacher level predictors are presented in Table 4.1b.

Table 4.1b
Mean and Standard Deviation of Teacher Level Predictors

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher Expectation</td>
<td>2.56</td>
<td>0.43</td>
<td>2.0</td>
<td>3.33</td>
</tr>
<tr>
<td>Class Type&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.50</td>
<td>0.50</td>
<td>0</td>
<td>1.00</td>
</tr>
<tr>
<td>Computer Use</td>
<td>3.28</td>
<td>0.06</td>
<td>3.10</td>
<td>3.50</td>
</tr>
</tbody>
</table>

<sup>c</sup> Class type is coded as 0 = Non-crowded classroom, and 1 = Crowded classroom

Class type is a dichotomous predictor derived from grade 4 student numbers assigned to a specific teacher. The median score for class size, which was used to determine the crowded and non-crowded class types, was 19. As it can be seen from Table 4.1b, both the mean and s. d. of class type are 0.5, whereas minimum and maximum are 0 and 1 respectively. The teacher expectation, which is a continuous predictor, has mean, s. d., minimum, and maximum of 2.56, 0.43, 2.0, and 3.33 respectively. Similarly, teacher computer use, another continuous predictor, has mean, s. d., minimum, and maximum of 3.28, 0.06, 3.1, and 3.5 respectively.

Table 4.1c
Mean and Standard Deviation of School Level Predictors

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public/Private&lt;sup&gt;d&lt;/sup&gt;</td>
<td>0.60</td>
<td>0.49</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>School mean SES</td>
<td>7.37</td>
<td>(0.00)&lt;sup&gt;f&lt;/sup&gt;</td>
<td>0.64</td>
<td>9.30</td>
</tr>
<tr>
<td>Title 1 Funding&lt;sup&gt;e&lt;/sup&gt;</td>
<td>0.58</td>
<td>0.49</td>
<td>0.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<sup>d</sup>Public/private school status is coded as 0=Nonpublic/private schools, 1= Public schools;

<sup>e</sup>Title 1 funding is coded as 0=Not receiving Title 1 funding, 1=Receiving Title 1 funding;

<sup>f</sup>Mean of school mean SES after grand centered.
The summary statistics for school level predictors are displayed in Table 4.1c. School mean SES, a continuous predictor, has mean, s. d., minimum, and maximum of 7.37, 0.64, 5.30, and 9.30 respectively. The dichotomous predictors, public/private school status, and Title 1 funding, have s. d. of 0.49, minimum of 0 and maximum of 1. Public/private school status has a mean of 0.6, and Title 1 funding has a mean of 0.58.

The bivariate correlations between student level variables are presented in Table 4.2a. Table 4.2a shows that reading proficiency has a positive correlation with SES (0.504), and negative correlations with race (-0.479) and sex (-0.257). Similarly, small negative correlations between SES and race or minority (-0.013) as well as SES and sex (-0.071) are found. A very low positive correlation between race and sex (0.021) is obtained.

Table 4.2a
Bivariate Correlations between Student Level Variables

<table>
<thead>
<tr>
<th>Variables</th>
<th>Teacher Expectation</th>
<th>Class Type</th>
<th>Computer Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher Expectation</td>
<td>1</td>
<td>-0.014</td>
<td>0.021</td>
</tr>
<tr>
<td>Class Type</td>
<td>-0.014</td>
<td>1</td>
<td>-0.012</td>
</tr>
<tr>
<td>Computer Use</td>
<td>0.021</td>
<td>-0.012</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.2b
Bivariate Correlations between Teacher Level Predictors

<table>
<thead>
<tr>
<th>Variables</th>
<th>Teacher Expectation</th>
<th>Class Type</th>
<th>Computer Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher Expectation</td>
<td>1</td>
<td>-0.014</td>
<td>0.021</td>
</tr>
<tr>
<td>Class Type</td>
<td>-0.014</td>
<td>1</td>
<td>-0.012</td>
</tr>
<tr>
<td>Computer Use</td>
<td>0.021</td>
<td>-0.012</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.2b provides bivariate correlations between teacher level predictors. Very weak negative correlations between class type and teacher expectation (-0.014) as well as class type and computer use (-0.012) are obtained. A very low positive correlation between teacher expectation and computer use (0.021) is found.

Table 4.2c represents the bivariate correlations between school level predictors. Negative correlations between school mean SES and public/private (-0.23), as well as
school mean SES and Title 1 funding (-0.26) are found. A positive correlation (0.24) between Title 1 funding and public/private status of school is obtained.

Table 4.2c
Bivariate Correlations between School Level Predictors

<table>
<thead>
<tr>
<th>Variables</th>
<th>Public/Private</th>
<th>School SES</th>
<th>Title 1 Funding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public/Private</td>
<td>1</td>
<td>-0.23</td>
<td>0.24</td>
</tr>
<tr>
<td>School SES</td>
<td>-0.23</td>
<td>1</td>
<td>-0.26</td>
</tr>
<tr>
<td>Title 1 Funding</td>
<td>0.24</td>
<td>-0.26</td>
<td>1</td>
</tr>
</tbody>
</table>

Model Development

During the onset of the study, three predictors are selected for the student, teacher, and school level models. For example, student socioeconomic status (SES), sex (male), and race (minority) are used in the student level model; teacher expectation, computer use, and class type are used in the teacher level; and school mean SES, Title 1 funding status, and public/private status of school are used in school level model. Two of the most potentially useful predictors are used in the level-1 conditional model. However, level-2 and level-3 conditional models included only one predictor in the model. Exploratory analysis was used in order to select the appropriate predictor in the subsequent model, and the t-statistic was considered as a benchmark to retain the predictor in conditional model. Further, insignificant variance components, and predictors, based on p-values, and t-statistics, are dropped from the model.

The following four steps are followed for model development purposes. We follow the steps given below in logical order to build up the proposed models. For the purpose of illustration, let us consider three predictors in the student (level-1), teacher (level-2), and school (level-3) level models. Assume that the log-odds of reading proficiency ($\eta_{ijk}$) can be predicted by socioeconomic status (SES), sex, and race of the student. Thus, the possible predictors to be selected in level-1 conditional model are SES, male (sex), and minority (race). Further, the intercept and slopes in the within-teacher
model can be predicted by the teacher level predictors. The possible predictors in teacher level model are teacher expectation, computer use, and class type. In turn, the school level predictors, such as school mean SES, Title 1 funding, and public/private status of the school, can predict the teacher level coefficients in the within-school model. Starting with the above potential predictors at each level, an exploratory approach is employed in order to select the best two predictors from level-1, the best single predictor from level-2, and the best single predictor from level-3.

**Step One: Testing for True Variance**

**Formulating the Model**

A fully unconditional model is formulated in this step. The general purpose of this stage is to estimate the intercept in a level-1 unconditional model. Thus, the resulting estimate would be the average log-odds of reading proficiency. In addition, this step would be helpful to find the level-2, and level-3 estimates from an intercept-as-outcome model.

In the beginning, the unconditional equations are formulated at all levels, i.e., at level-1, level-2, and level-3 models. A nonlinear model was set up using optional specifications as Bernoulli (0, 1) and nonlinear analysis (with default number of iterations and stopping criteria in HLM).

A model with only intercept as a predictor in the equation was formulated at level-1, the student level, or within-teacher model, where several students are nested within a teacher, with the student’s reading proficiency as an outcome. Such a model can be given by (4.1a) where \( \eta_{ijk} \) is an outcome variable defined as the log-odds of reading proficiency for \( i^{th} \) student nested in \( j^{th} \) teacher in \( k^{th} \) school.

\[
\eta_{ijk} = \pi_{0jk}
\]  

(4.1a)

In (4.1a), \( \pi_{0jk} \) is a level-1 intercept and is defined as the average log-odds of reading proficiency for any student nested in \( j^{th} \) teacher in \( k^{th} \) school.

In level-2, an unconditional model was formulated as given in (4.1b).

\[
\pi_{0jk} = \beta_{00k} + r_{0jk}
\]  

(4.1b)
In (4.1b), $\beta_{00k}$ is the level-2 intercept. The random effect, a residual term, was included in the above (level-2) model, where $r_{0jk} \sim N(0, \tau_\pi)$. The inclusion of the random effect term allows us to test the level-2 variance component. Given the significance of specific variance components, we include corresponding variance terms in a resulting conditional model.

Similarly, the level-3 model is also formulated as an unconditional random effect model as given by (4.1c).

$$\beta_{00k} = \gamma_{000} + u_{00k} \quad (4.1c)$$

In (4.1c), $\gamma_{000}$ is the intercept, and $u_{00k}$ is the variance component at level-3, where $u_{00k} \sim N(0, \tau_\beta)$.

More specifically, $\beta_{00k}$ and $\gamma_{000}$ are the average log-odds of reading proficiency across the fourth grade teachers within schools and across elementary schools respectively in the U. S. Further, $r_{0jk}$ is a random teacher effect (i.e., the deviation of teacher jk’s mean from the school mean), and these effects are assumed normally distributed with mean 0 and variance $\tau_\pi$. The dispersion among teachers is assumed to be constant within each of the k schools. Similarly, $u_{00k}$ is a random school effect (i.e., the deviation of school k’s mean from the grand mean), and these effects are assumed normally distributed with a mean of 0 and variance $\tau_\beta$.

The variance components in the level-2 and level-3 models will be included only after we obtain significant variances. In other words, $r_{00k}$ and $u_{00k}$ are deleted in forthcoming model if the variance components are not significant, i.e., if $p>.05$. Further, an exploratory analysis is conducted in HLM in order to detect the possible potential predictors in the level-2, and level-3 models.

Necessary case analysis and assessment of the assumptions required for level-2 and level-3 models, such as normality and homogeneity of variance, were performed. The case analysis revealed that there were no outliers that exerted excessive influence on the results of the study. It also revealed that there were no violations of statistical assumptions. The details associated with the assessment of assumptions are presented in step four while illustrating the final model.
Results

The nonlinear analysis at level-1 produced the following results of the parameter estimates. Considering reading proficiency (READPROF) as the outcome variable, the estimated values, based on HLM output, are presented in Table 4.3a.

Table 4.3a
Final estimation of fixed effects

<table>
<thead>
<tr>
<th>Fixed Effect</th>
<th>Coefficient</th>
<th>se</th>
<th>t Ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher model for student level intercept, $\pi_{0jk}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>School model for teacher level intercept, $\beta_{00k}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, $\gamma_{000}$</td>
<td>0.417</td>
<td>0.024</td>
<td>17.375</td>
<td>&lt;.001</td>
</tr>
</tbody>
</table>

The intercept (0.417) was found to be significant ($p<.001$) at 5% and 1% level of significance.

The estimation of level-2 and level-3 variance components are given in Table 4.3b and Table 4.3c respectively.

Table 4.3b
Final Estimation of Level-2 Variance Components

<table>
<thead>
<tr>
<th>Random Effect</th>
<th>Variance Component</th>
<th>df</th>
<th>Chi-square</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teachers (level-2), $r_{0jk}$</td>
<td>0.325</td>
<td>781</td>
<td>1247.44</td>
<td>&lt;.001</td>
</tr>
</tbody>
</table>

Table 4.3c
Final Estimation of Level-3 Variance Components

<table>
<thead>
<tr>
<th>Random Effect</th>
<th>Variance Component</th>
<th>df</th>
<th>Chi-square</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schools (level-3), $u_{00k}$</td>
<td>0.878</td>
<td>294</td>
<td>1038.44</td>
<td>&lt;.001</td>
</tr>
</tbody>
</table>
Both the level-2 and level-3 variance components are found to be significant (p<.001). The reliability estimates for level-2 (predicting level-2 outcome), and level-3 (predicting level-3 outcome) models are found to be 0.501 and 0.695 respectively, which are within the range of satisfactory magnitude.

*Evaluating the Model*

The predicted grand mean of the log-odds of reading proficiency was significant (p<.001), and level-2 as well as level-3 variance components were also found significant. Thus, the significance of level-2 and level-3 variances suggested the inclusion of these terms in a subsequent model. In addition, the reliabilities at level-2 are acceptable, and reliabilities at level-3 are found to be reasonably large.

*Step Two: Specifying Student Level Variables*

*Formulating the Model*

In this step, the level-1 conditional, and level-2 and level-3 unconditional models are formulated. We believe that the log-odds of reading proficiency can be predicted by important student level predictors. Based on a meaningful correlation between reading proficiency and three student level predictors, namely student SES, race (minority), and sex, these three predictors are included in student level conditional model. For example, the correlations of reading proficiency with SES, race and sex are found to be 0.504, -0.479, and –0.257 respectively. However, sex was deleted from the model after we did not observe the evidence of a significant slope (effect) associated with sex in our analysis. In other words, only student SES and race produced significant effects.

The level-2 and level-3 variance components are included or excluded in the subsequent model based on the p-values, i.e., based on the significance of the variance components. Such residual terms are included in level-2 and level-3 models only if we observed p-values less than 0.05.

Further, an exploratory analysis was conducted to determine the potential predictors in order to include in level-2 a conditional model. The magnitude of t-statistic,
the predictor with the highest value of t-statistic that are larger than 2.0, produced from exploratory analysis is considered as a basis for the inclusion of the appropriate predictor in the model.

The conditional level-1; unconditional level-2, and level-3 models are given below.

Level-1: Students within the teacher model
\[ \eta_{ijk} = \pi_{0jk} + \pi_{1jk} \text{MINORITY} + \pi_{2jk} \text{SES} \]  
(4.2a)

Level-2: Teachers within the school model
\[ \pi_{0jk} = \beta_{00k} + r_{0jk} \]
\[ \pi_{1jk} = \beta_{10k} + r_{1jk} \]  
(4.2b)
\[ \pi_{2jk} = \beta_{20k} + r_{2jk} \]

Level-3: The between-school model
\[ \beta_{00k} = \gamma_{000} + u_{00k} \]
\[ \beta_{10k} = \gamma_{100} + u_{10k} \]  
(4.2c)
\[ \beta_{20k} = \gamma_{200} + u_{20k} \]

where \( \pi_{0jk} \) is the predicted log-odds reading proficiency for ‘reference student’. In other words, this is the predicted log-odds of reading proficiency for a non-minority American fourth grader of average SES. The parameters \( \pi_{1jk} \) and \( \pi_{2jk} \) are the effects of minority and SES respectively on log-odds of reading proficiency. Minority, a dichotomous predictor, is uncentered, and student SES, a continuous predictor, is grand mean centered.

The terms \( r_{0jk}, r_{1jk}, r_{2jk} \), are random teacher effects, and these effects are assumed to be multivariate normally distributed with a mean vector 0 and variance-covariance matrix \( \Sigma \). Note that the variation and co-variation among teachers is assumed to be the constant within each of the k schools. Similarly, \( u_{00k}, u_{10k}, u_{20k} \) are random school effects, which are assumed to be multivariate normally distributed with mean vector 0 and variance-covariance matrix \( \Sigma \).

Case analysis and assessment of the assumptions required for level-2 and level-3 models are performed using appropriate statistical procedures. The statistical analysis did not reveal the violation of statistical assumptions, and the case analysis did not show any
outliers that exert an excessive influence on the study results. A detailed discussion on the case analysis are presented in the final model.

Results

The estimators of slopes and intercept are presented in Table 4.4a. The final estimations of level-2 and level-3 variance components are presented in Table 4.4b and 4.4c respectively. The estimations of reliabilities for predicting level-2 and level-3 model outcomes are presented in Table 4.4d and 4.4e respectively.

Table 4.4a
Estimation of Intercepts, and Slopes

<table>
<thead>
<tr>
<th>Fixed Effect</th>
<th>Coefficient</th>
<th>se</th>
<th>t Ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher model for student level intercept, $\pi_{0jk}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>School model for teacher level intercept, $\beta_{00k}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, $\gamma_{000}$</td>
<td>1.316</td>
<td>0.047</td>
<td>28.000</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Teacher model for student level effect of minority, $\pi_{1jk}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>School model for teacher level intercept, $\beta_{10k}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, $\gamma_{100}$</td>
<td>-2.545</td>
<td>0.070</td>
<td>-36.357</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Teacher model for student level effect of SES, $\pi_{2jk}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>School model for teacher level intercept, $\beta_{20k}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, $\gamma_{200}$</td>
<td>8.269</td>
<td>0.270</td>
<td>30.626</td>
<td>&lt;.001</td>
</tr>
</tbody>
</table>

The estimation of level-2 variance components is presented in Table 4.4b. Similarly, the estimation of level-3 variance components is provided in Table 4.4c. Estimation of Level-3 Variance Components

All level-2 (provided in Table 4.4b) and level-3 (provided in Table 4.4c) variance components are found to be significant at 0.01 level of significance.
Table 4.4b
Final Estimation of Level-2 Variance Components

<table>
<thead>
<tr>
<th>Random Effect</th>
<th>Variance Component</th>
<th>df</th>
<th>Chi-square</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Odds of reading prof. for ref. students, r_{0jk}</td>
<td>0.08306</td>
<td>732</td>
<td>875.74</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Effect of minority, r_{1jk}</td>
<td>0.09651</td>
<td>732</td>
<td>929.55</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Effect of SES, r_{2jk}</td>
<td>6.00709</td>
<td>732</td>
<td>989.70</td>
<td>&lt;.001</td>
</tr>
</tbody>
</table>

The level-2 (within-teacher) and level-3 (within-school) reliabilities are provided in Table 4.4d and 4.4e respectively.

Table 4.4c
Final Estimation of Level-3 Variance Components

<table>
<thead>
<tr>
<th>Random Effect</th>
<th>Variance Component</th>
<th>df</th>
<th>Chi-square</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average odds of reading prof., u_{00k}</td>
<td>0.23810</td>
<td>294</td>
<td>251.14</td>
<td>0.004</td>
</tr>
<tr>
<td>Average minority effect, u_{10k}</td>
<td>0.18921</td>
<td>294</td>
<td>305.69</td>
<td>0.002</td>
</tr>
<tr>
<td>Average SES effect, u_{20k}</td>
<td>2.63219</td>
<td>294</td>
<td>292.85</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Table 4.4d
Final Estimation of Level-2 Reliabilities

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Reliability estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average odds of within-teacher reading prof.</td>
<td>0.538</td>
</tr>
<tr>
<td>Average minority effect</td>
<td>0.537</td>
</tr>
<tr>
<td>Average SES effect</td>
<td>0.525</td>
</tr>
</tbody>
</table>

The reliabilities of the coefficients for predicting level-2 outcomes are found reasonably large (i.e., > 0.525). The reliabilities of the coefficients for predicting level-3 outcomes are found to be high (i.e., > 0.735).
Evaluating the Model

All level-3 intercepts are found to be significant (p<.001). In other words, the intercepts in school level model for predicting $\beta_{00k}$, $\beta_{01k}$, $\beta_{10k}$, $\beta_{11k}$, $\beta_{20k}$, and $\beta_{21k}$ are found to be significant. Similarly, the significance of all level-2 and level-3 variance components (p<.01) showed significant variations across teachers and schools.

This suggested the development of level-2 conditional model and inclusion of important predictor/s in the model. The level-2 reliabilities are found to be larger than 0.53, and level-3 reliabilities are found to be larger than 0.73 for predicting intercept and slopes at respective levels.

Three teacher level predictors, namely class type, teacher expectation, and computer use, were used in exploratory analysis. However, only the class type is found to be significant (t-statistics larger than 2.0) associated with intercept and slopes (of SES and race/minority) as outcomes. The basis for selecting class type in the level-2 conditional model was the t-statistic larger than 2.0 for this predictor.

In the following step, we have formulated and run the conditional models at level-1 and level-2 as suggested by the analysis.

Step Three: Specifying Teacher-Level Variables

Formulating the Model

In this step, level-1 and level-2 conditional models and level-3 unconditional models were formulated. Level-1 conditional model with two predictors, as in step two,
level-2 conditional model with one predictor (i.e., class type) and residual terms in the model, and level-3 unconditional model only with intercepts and residual terms are run. In the beginning, three teacher level predictors, namely, teacher expectation, teacher computer use, and class type, are considered while performing exploratory analysis. Since the t-statistic of class type, through exploratory analysis, was found to be the largest one among all predictors having significant t-statistics, this predictor (class type) was included in level-2 conditional model. Further, all the level-2 variance components are included in the model based on the evidence that they are found to be significant in the preceding (step two) analysis.

An exploratory analysis is performed within the level-3 model with three predictors, namely, school mean SES, Title 1 funding, and public/private school. The inclusion or exclusion of a specific predictor in level-3 conditional model (final model) is based on the magnitude of t-statistic of corresponding predictor. To demonstrate, we have selected the most potential predictor with highest value of t-statistic (with t>2.0). Like in step three, the p-values of variance components are examined, and only significant variances are included in the subsequent model.

The level-1 and level-2 conditional models, and level-3 unconditional models are formulated as follows, where level-2 and level-3 models are run with variance components in each equation.

Level-1: Students within teacher model
\[ \eta_{ijk} = \pi_{0jk} + \pi_{1jk} \text{(MINORITY)} + \pi_{2jk} \text{(SES)} \]  
(4.3a)

Level-2: Teachers within school model
\[ \pi_{0jk} = \beta_{00k} + \beta_{01k} \text{(CLASTYPE)} + r_{0jk} \]
\[ \pi_{1jk} = \beta_{10k} + \beta_{11k} \text{(CLASTYPE)} + r_{1jk} \]  
(4.3b)
\[ \pi_{2jk} = \beta_{20k} + \beta_{21k} \text{(CLASTYPE)} + r_{2jk} \]
Level-3: Between-school model

\[
\begin{align*}
\beta_{00k} &= \gamma_{000} + u_{00k} \\
\beta_{01k} &= \gamma_{010} + u_{01k} \\
\beta_{10k} &= \gamma_{100} + u_{10k} \\
\beta_{11k} &= \gamma_{110} + u_{11k} \\
\beta_{20k} &= \gamma_{200} + u_{20k} \\
\beta_{21k} &= \gamma_{210} + u_{21k}
\end{align*}
\]  
(4.3c)

where \(\pi_{0jk}\), as in an earlier step, is the predicted log-odds reading proficiency for a non-minority American fourth grader of average SES for teacher j and school k. The parameters \(\pi_{1jk}\) and \(\pi_{2jk}\) are as defined in step two. The term \(\beta_{00k}\) is the between school log-odds of reading proficiency for reference teacher, and reference student. The parameters \(\pi_{ijk}\) (i = 0, 1, 2) and \(\beta_{jk}\) are the slopes in student level and teacher level models respectively. Further, \(\gamma_{s}\) are school level intercepts. For example, \(\gamma_{00}\) is the average log-odds of reading proficiency in fourth grade across U. S. schools. All parameters are defined in step four or the final model. The dichotomous predictors, minority (at level-1), and class type (at level-2) are uncentered, and the continuous predictor, student SES, is grand mean centered.

The terms \(r_{0jk}, r_{1jk}, r_{2jk}\), are residuals or random teacher effects, which are assumed multivariate normally distributed with mean vector 0 and variance-covariance matrix \(T_{\pi}\). Further, \(u_{00k}, u_{01k}, u_{10k}, u_{11k}, u_{20k}, u_{21k}\) are random school effects. These effects are assumed normally distributed with mean vector 0 and variance-covariance matrix \(T_{\beta}\).

As in previous steps, case analysis and assessment of the assumptions required for level-2 and level-3 models were performed. The statistical analysis revealed no violation of statistical assumptions, such as multivariate normality and homoscedasticity. Further, case analysis did not indicate any outliers that exert the excessive influence on the study results.

Results

Table 4.5a provides the estimates of intercepts, and slopes. All the estimates of level-3 parameters are found to be significant at the 0.01 level. All level-2 variance
components are also found to be significant at the 0.01 level, and all level-3 variance components, except \( u_{20k} \), are found to be significant at the 0.01 level of significance.

Table 4.5a
Estimation of Intercepts, and Slopes

<table>
<thead>
<tr>
<th>Fixed Effect</th>
<th>Coefficient</th>
<th>se</th>
<th>t Ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher model for student level intercept, ( \pi_{0jk} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>School model for teacher level intercept, ( \beta_{00k} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, ( \gamma_{000} )</td>
<td>1.363</td>
<td>0.068</td>
<td>20.155</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>School model for teacher level effect of the class type, ( \beta_{01k} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, ( \gamma_{010} )</td>
<td>-0.829</td>
<td>0.368</td>
<td>-2.253</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Teacher model for student level effect of a minority, ( \pi_{1jk} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>School model for teacher level intercept, ( \beta_{10k} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, ( \gamma_{100} )</td>
<td>-2.701</td>
<td>0.103</td>
<td>-26.292</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>School model for teacher level effect of the class type, ( \beta_{11k} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, ( \gamma_{110} )</td>
<td>0.278</td>
<td>0.115</td>
<td>2.417</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Teacher model for student level effect of SES, ( \pi_{2jk} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>School model for teacher level intercept, ( \beta_{20k} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, ( \gamma_{200} )</td>
<td>9.138</td>
<td>0.417</td>
<td>21.903</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>School model for teacher level effect of the class type, ( \beta_{21k} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, ( \gamma_{210} )</td>
<td>-0.607</td>
<td>0.209</td>
<td>-2.904</td>
<td>&lt;.001</td>
</tr>
</tbody>
</table>
Table 4.5b
Estimation of Level-2 Variance Components

<table>
<thead>
<tr>
<th>Random Effect</th>
<th>Variance Component</th>
<th>df</th>
<th>Chi-square</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Odds of reading prof. for ref. students, r_{0jk}</td>
<td>0.0607</td>
<td>437</td>
<td>862.14</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Effect of minority, r_{1jk}</td>
<td>0.0951</td>
<td>437</td>
<td>907.48</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Effect of SES, r_{2jk}</td>
<td>4.6427</td>
<td>437</td>
<td>950.49</td>
<td>&lt;.001</td>
</tr>
</tbody>
</table>

Table 4.5c
Estimation of Level-3 Variance Components

<table>
<thead>
<tr>
<th>Random Effect</th>
<th>Variance Component</th>
<th>df</th>
<th>Chi-square</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average odds of reading prof., u_{00k}</td>
<td>0.16301</td>
<td>266</td>
<td>271.92</td>
<td>0.001</td>
</tr>
<tr>
<td>Class type effect on av. reading prof., u_{01k}</td>
<td>0.16194</td>
<td>266</td>
<td>260.74</td>
<td>0.002</td>
</tr>
<tr>
<td>Average minority effect, u_{10k}</td>
<td>0.13863</td>
<td>266</td>
<td>268.27</td>
<td>0.003</td>
</tr>
<tr>
<td>Effect of class type on minority effect, u_{11k}</td>
<td>0.3933</td>
<td>266</td>
<td>258.74</td>
<td>0.004</td>
</tr>
<tr>
<td>Average SES effect, u_{20k}</td>
<td>2.52757</td>
<td>266</td>
<td>168.88</td>
<td>&gt;.500</td>
</tr>
<tr>
<td>Effect of class type on SES effect, u_{21k}</td>
<td>8.25046</td>
<td>266</td>
<td>249.74</td>
<td>0.004</td>
</tr>
</tbody>
</table>

The estimations of reliabilities for predicting the level-2 and level-3 model outcomes are presented in Table 4.5d and Table 4.5e respectively. All level-2 and level-3 reliabilities are found to be reasonably large.

Table 4.5d
Estimation of Level-2 Reliabilities

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Reliability estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average odds of within-teacher reading prof.</td>
<td>0.552</td>
</tr>
<tr>
<td>Minority effect</td>
<td>0.555</td>
</tr>
<tr>
<td>SES effect</td>
<td>0.559</td>
</tr>
</tbody>
</table>
Comparing level-2’s unconditional model (in step two) with level-2’s conditional model (in step three), the following proportions of variance explained can be computed using level-2 variance components. The proportion of explained variation, and consequently the resulting percent of variance explained, for predicting level-1 intercept ($\pi_0$) can be given as follows.

$$\text{Proportion of variance explained} = \frac{\tau_{\pi_{00}}(unconditional) - \tau_{\pi_{00}}(conditional)}{\tau_{\pi_{00}}(unconditional)}$$

$$= \frac{0.083 - 0.061}{0.083} = .265 = 26.5\%$$

In similar fashion, the proportion of variance explained for predicting the effect of minority and the effect of SES can be computed. The percentages of variance explained are presented in Table 4.6f.

### Table 4.5f
Proportion of Variance Explained for Level-2 Conditional Model Compared to Level-2 Unconditional Model

<table>
<thead>
<tr>
<th>Level-2 outcomes</th>
<th>Percent of variance explained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept, $\pi_0$</td>
<td>26.5</td>
</tr>
<tr>
<td>Effect of minority, $\pi_1$</td>
<td>43.3</td>
</tr>
<tr>
<td>Effect of SES, $\pi_2$</td>
<td>29.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Reliability estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average within-school reading proficiency</td>
<td>0.764</td>
</tr>
<tr>
<td>Class type effect on av. reading prof.</td>
<td>0.761</td>
</tr>
<tr>
<td>Average minority effect</td>
<td>0.773</td>
</tr>
<tr>
<td>Effect of class type on minority effect</td>
<td>0.768</td>
</tr>
<tr>
<td>Average SES effect</td>
<td>0.775</td>
</tr>
<tr>
<td>Effect of class type on SES effect</td>
<td>0.765</td>
</tr>
</tbody>
</table>
Evaluating the Model

All intercept terms in level-3 model are found to be significant either at 5% or 1% level of significance. Three of the intercepts in between-school model are found negative, and the remaining intercepts are found positive. Most of the variance components are found significant ($p<.005$). The reliabilities for level-2 coefficients are found larger than 0.54, and reliabilities for level-3 coefficients are found larger than 0.76. A maximum of 43% proportion of variance is explained, which is for predicting the minority slope ($\pi_1$) in level-1 model.

Although three school level predictors are used in exploratory analysis, two of these (school mean SES and Title 1 funding status) are found to have t-statistics larger than 2.0 associated with intercept and slopes of class type. Public/private school status did not appear to be a potential level-3 predictor. As a note of interest, we opted to include only school mean SES in level-3 conditional model. The rationale for using only one level-3 predictor is due to the ease of demonstration. The basis for selecting school mean SES is the larger magnitudes of t-statistics compared to Title 1 funding while predicting level-2 intercepts and slopes.

Since an improvement in the model with better estimates of intercepts, slopes, and reliabilities is found, the rationale for developing conditional equations in within-school model is justified.

In step four, fully conditional models with relevant predictors in within-teacher, within-school, and between school models are developed as a final model.

Step Four: Specifying School-Level Variables

Formulating the Model

In this step, the final model is formulated with conditional equations at all levels. Apparently, we have already demonstrated level-1 and level-2 conditional models in previous steps. The evidence of significant school-to-school variation, i.e., all significant level-3 variance components except $u_{20k}$, suggested the formulation of the level-3
conditional model with variance terms in the equations. To recall, the inclusion of predictors in level-3 model is based on the exploratory analysis performed in step three.

The result of the exploratory analysis suggested the inclusion of school mean SES in the level-3 model. Since the variance component for the model outcome $\beta_{20k}$ is not found significant, this model is formulated as a fixed effect model. Further, none of the predictors were included in the model with $\beta_{20k}$ and $\beta_{21k}$ as outcomes. This is because the t-statistics based on exploratory analysis are found less than 2.0 (or not significant) for these outcomes. A detailed description of conditional models and interpretations are presented in the subsequent section.

Let $\eta_{ijk}$ be the log-odds of reading proficiency for $i^{th}$ student associated with $j^{th}$ teacher in $k^{th}$ school, where $\phi$ is the probability of proficiency, the following conditional equations can be formulated as the final model.

**Level-1: Students within teacher model**

$$\eta_{ijk} = \log(\phi/1-\phi) = \pi_{0jk} + \pi_{1jk} \text{(MINORITY)} + \pi_{2jk} \text{(SES)} \quad (4.4a)$$

**Level-2: Teachers within school model**

$$\pi_{0jk} = \beta_{00k} + \beta_{01k} \text{(CLASTYPE)} + r_{0jk}$$

$$\pi_{1jk} = \beta_{10k} + \beta_{11k} \text{(CLASTYPE)} + r_{1jk} \quad (4.4b)$$

$$\pi_{2jk} = \beta_{20k} + \beta_{21k} \text{(CLASTYPE)} + r_{2jk}$$

**Level-3: Between-school model**

$$\beta_{00k} = \gamma_{000} + \gamma_{001} \text{(SCHLSES)} + u_{00k}$$

$$\beta_{01k} = \gamma_{010} + \gamma_{011} \text{(SCHLSES)} + u_{01k}$$

$$\beta_{10k} = \gamma_{100} + \gamma_{101} \text{(SCHLSES)} + u_{10k}$$

$$\beta_{11k} = \gamma_{110} + \gamma_{111} \text{(SCHLSES)} + u_{11k}$$

$$\beta_{20k} = \gamma_{200}$$

$$\beta_{21k} = \gamma_{210} + u_{21k} \quad (4.4c)$$
In 4.4a, $\pi_{0jk}$ is the predicted log-odds reading proficiency for a non-minority American fourth grader of average SES associated with $j^{th}$ teacher in $k^{th}$ school. In 4.4c, $\beta_{20k}$ was formulated as a fixed effect since the variance component was not found to be significant in unconditional level-3 model. The school mean SES (a continuous predictor) was grand mean centered.

Case Analysis and Assumptions

Case analyses were performed for level-1, level-2, and level-3 residuals. Since HLM analysis does not provide the data for level-1 residuals, a binary logistic regression approach was adopted for level-1 case/residual analysis by considering reading proficiency as a (dichotomous) dependent variable and minority as well as SES as covariates. In order to detect outliers, a case-wise listing of residuals, outside the 2.5 standard deviation, was analyzed. No outliers were found based on such an analysis. In addition, the values of maximum delta betas (the change in the coefficient for a predictor due to deleting the observation for an individual) for minority and SES were found, and the proportions of maximum delta betas to the corresponding point estimates were computed. Both of the proportions were found almost zero and did not suggest any influence of outliers in study results.

For level-3 residual analyses, the residual plots shown in Figure A4.1h through Figure A4.1k are used for the identification of outliers. Similarly, Figure A4.2e and Figure A4.2f are used for outlier identification of level-2 residuals. No extreme outliers are found in both of the cases. For assessment of level-2, and level-3 linearity and constant variance assumptions, the same Figures noted above are used. No significant violations were observed.

For level-2 and level-3 models, different assumptions applied for linear equations, were assessed after the residual analysis of level-2 and level-3 residual data. The model assumptions, and the rationale for validating such assumptions, are described below. In order to check the necessary assumptions, graphs are presented in Appendix A.

Figures A4.1a through A4.1k assess the assumptions of normality, homogeneity of variance, and linearity for the residuals in level-3 equations. Specifically, Figures
A4.1a through A4.1d represent the histograms for Empirical Bayes residuals for the intercepts and slopes. These histograms with normal curves assess the normality assumption. Moderate violation of normality assumption associated with residuals in level-2 and level-3 models is observed, with some skewed graphs, such as Figure A4.1d. However, the analysis results are found robust to such violations. According to Raudenbush and Bryk (2002), checking for normality at level-2 (and also at level-3 in this study) is complicated by the fact that the level-2 (and at level-3) outcomes are not directly observed, and when the sample size per group is small, the variance estimates will be quite uncertain. Figure A4.1e through Figure A4.1g represents the normal P-P curves of empirical Bayes residual for the intercept and slope of minority, and slope of SES respectively. These scatter plots provide clear indication of homoscedasticity of residuals in level-3 model.

For level-2 residual analyses, Figure A4.2a through Figure A4.2d assess the normality assumption. Although Figure A4.2a and Figure A4.2b appear slightly skewed and a bit non-normal with moderate violation of normality assumption, study results were robust to such violations. Estimation of the fixed effects will not be biased by failure of the normality assumption at level-2 (Raudenbush & Bryk, 2002).

Figures A4.2e and A4.2f provide the graphs homogeneity of variance, and the linearity assumptions respectively. These Figures support the homoscedasticity, and linearity assumptions for residuals in the level-2 model.

**Results**

The estimation of level-3 parameters from final model is given in Table 4.6a. The estimation of level-2 variance components is given in Table 4.6b, and the estimation of level-3 variance components is given in Table 4.6c. All level-2 and level-3 variance components are found to be significant at 0.001 and 0.005 levels respectively.
Table 4.6a
Estimation of Intercepts, and Slopes from the Final Model

<table>
<thead>
<tr>
<th>Fixed Effect</th>
<th>Coefficient</th>
<th>se</th>
<th>t Ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher model for student level intercept, $\pi_{0jk}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>School model for teacher level intercept, $\beta_{00k}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, $\gamma_{000}$</td>
<td>1.373</td>
<td>0.067</td>
<td>20.528</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>School SES, $\gamma_{001}$</td>
<td>1.995</td>
<td>0.824</td>
<td>2.423</td>
<td>0.006</td>
</tr>
<tr>
<td>School model for teacher level effect of class type, $\beta_{01k}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, $\gamma_{010}$</td>
<td>-0.271</td>
<td>0.127</td>
<td>-2.135</td>
<td>0.007</td>
</tr>
<tr>
<td>School mean SES, $\gamma_{011}$</td>
<td>-0.693</td>
<td>0.367</td>
<td>-1.887</td>
<td>0.070</td>
</tr>
<tr>
<td>Teacher model for student level effect of minority, $\pi_{1jk}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>School model for teacher level intercept, $\beta_{10k}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, $\gamma_{100}$</td>
<td>1.697</td>
<td>0.103</td>
<td>16.476</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>School mean SES, $\gamma_{101}$</td>
<td>1.486</td>
<td>0.616</td>
<td>2.411</td>
<td>0.006</td>
</tr>
<tr>
<td>School model for teacher level effect of class type, $\beta_{11k}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, $\gamma_{110}$</td>
<td>0.270</td>
<td>0.143</td>
<td>1.890</td>
<td>0.070</td>
</tr>
<tr>
<td>School mean SES, $\gamma_{111}$</td>
<td>-2.738</td>
<td>1.281</td>
<td>-2.138</td>
<td>0.007</td>
</tr>
<tr>
<td>Teacher model for student level effect of SES, $\pi_{2jk}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>School model for teacher level intercept, $\beta_{20k}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, $\gamma_{200}$</td>
<td>8.975</td>
<td>0.393</td>
<td>22.824</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>School model for teacher level effect of class type, $\beta_{21k}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept, $\gamma_{210}$</td>
<td>-0.424</td>
<td>0.169</td>
<td>-2.509</td>
<td>0.005</td>
</tr>
</tbody>
</table>
Table 4.6b  
Estimation of Level-2 Variance Components

<table>
<thead>
<tr>
<th>Random Effect</th>
<th>Variance Component</th>
<th>df</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Odds of reading prof. for ref. students, r_{0jk}</td>
<td>0.04563</td>
<td>437</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Effect of minority, r_{1jk}</td>
<td>0.08437</td>
<td>437</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Effect of SES, r_{2jk}</td>
<td>4.14029</td>
<td>731</td>
<td>&lt;.001</td>
</tr>
</tbody>
</table>

Table 4.6c  
Estimation of Level-3 Variance Components

<table>
<thead>
<tr>
<th>Random Effect</th>
<th>Variance Component</th>
<th>df</th>
<th>Chi-square</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average odds of reading prof., u_{00k}</td>
<td>0.05422</td>
<td>266</td>
<td>263.43</td>
<td>0.003</td>
</tr>
<tr>
<td>Class type effect on av. reading prof., u_{01k}</td>
<td>0.11381</td>
<td>266</td>
<td>251.52</td>
<td>0.002</td>
</tr>
<tr>
<td>Average minority effect, u_{10k}</td>
<td>0.10127</td>
<td>266</td>
<td>279.07</td>
<td>0.001</td>
</tr>
<tr>
<td>Effect of class type on minority effect, u_{11k}</td>
<td>0.31292</td>
<td>266</td>
<td>261.18</td>
<td>0.003</td>
</tr>
<tr>
<td>Effect of class type on SES effect, u_{21k}</td>
<td>1.81555</td>
<td>266</td>
<td>241.50</td>
<td>0.004</td>
</tr>
</tbody>
</table>

The estimation of level-2 coefficient reliabilities and level-3 coefficient reliabilities are presented in Table 4.6d and 4.6e respectively. These reliability estimates are found to be fairly large.
Table 4.6d
Estimation of Level-2 Reliabilities

<table>
<thead>
<tr>
<th>Level-2 outcomes</th>
<th>Reliability estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average odds of reading prof.</td>
<td>0.570</td>
</tr>
<tr>
<td>Minority effect</td>
<td>0.587</td>
</tr>
<tr>
<td>SES effect</td>
<td>0.577</td>
</tr>
</tbody>
</table>

Table 4.6e
Estimation of Level-3 Reliabilities

<table>
<thead>
<tr>
<th>Level-3 outcomes</th>
<th>Reliability estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average odds of reading prof.</td>
<td>0.771</td>
</tr>
<tr>
<td>Class type effect on av. reading prof.</td>
<td>0.785</td>
</tr>
<tr>
<td>Average minority effect</td>
<td>0.783</td>
</tr>
<tr>
<td>Effect of class type on minority effect</td>
<td>0.779</td>
</tr>
<tr>
<td>Average SES effect on reading prof.</td>
<td>0.775</td>
</tr>
<tr>
<td>Effect of class type on SES effect</td>
<td>0.781</td>
</tr>
</tbody>
</table>

Compared to level-3 unconditional model in step three, the following proportions of variance explained can be given for the level-3 conditional model using level-2, and level-3 variance components.

Using level-3 variance component, the proportion of explained variance for predicting level-2 intercept (β₀₀), i.e., average odds of reading proficiency, can be given as follows.

Proportion of variance explained = \( \frac{\tau_{\beta_{00}}(unconditional) - \tau_{\beta_{00}}(conditional)}{\tau_{\beta_{00}}(unconditional)} \)

= \( \frac{0.163 - 0.054}{0.163} \) = 0.669 = 66.9%

Similarly, the proportion of an explained variation in order to predict other level-3 outcomes can be computed and expressed in a percentage. These proportions of explained variance for the conditional level-3 model compared to the unconditional level-3 model are presented below in Table 4.6f.
Table 4.6f
Proportion of Variance Explained for Level-3 Conditional Model
Compared to Level-3 Unconditional Model

<table>
<thead>
<tr>
<th>Level-3 outcomes</th>
<th>Percent of variance explained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average odds of reading prof., $\beta_{00}$</td>
<td>66.9</td>
</tr>
<tr>
<td>Class type effect on av. reading prof., $\beta_{01}$</td>
<td>29.6</td>
</tr>
<tr>
<td>Average minority effect, $\beta_{10}$</td>
<td>27.3</td>
</tr>
<tr>
<td>Effect of class type on minority effect, $\beta_{11}$</td>
<td>50.1</td>
</tr>
<tr>
<td>Effect of class type on SES effect, $\beta_{21}$</td>
<td>77.9</td>
</tr>
</tbody>
</table>

Evaluating the Model

All of the level-3 coefficients except for $\gamma_{011}, \gamma_{110}$ are found significant either with $p<.001$ or with $p<.01$. All five level-3 variance components are also significant ($p<.005$). The reliabilities of level-2 for coefficients are found to be larger than 0.57, and reliabilities of level-3 coefficients are found larger than 0.77. For predicting the level-3 outcomes, the minimum and maximum proportions of variance explained are found 27% and 78% respectively.

Based on the results of exploratory analysis and for a purpose of a demonstration, we have used school mean SES in level-3 conditional model. Since most of the estimated coefficients and level-2 as well as level-3 variance components are found to be significant, we have decided to use the above model (Equations 4.4a through 4.4c) as the final model. In the following section, the single-equation formulation, and a demonstration of the results based on single-equation formulation are presented.

Interpretations of Final Model

Single-Equation Formulation

The single-equation can be formulated as follows by substituting 4.4c in 4.4b, and then 4.4b in 4.4a.
In order to express the equation in a simpler form, let us substitute MINORITY = a₁, SES = a₂, CLASTYPE = X, and SCHLSES = W. Now, collecting the coefficients together for a specific predictor and dropping the subscripts for predictors, the resulting single-equation can be formulated as follows.

\[ \eta_{ijk} = (\gamma_{000} + r_{0jk} + u_{00k}) + (\gamma_{010} + r_{1jk} + u_{010}) a_1 + (\gamma_{100} + r_{2jk}) a_2 + (\gamma_{011} + u_{01k}) X + \gamma_{101} a_1 W + \gamma_{110} a_1 X \]

\[ + \gamma_{101} a_1 W + \gamma_{110} a_1 X + u_{11k} a_1 X + u_{21k} a_2 X \]  

(4.5b)

In (4.5a) and (4.5b), \( \eta_{ijk} \) is the log-odds of reading proficiency that can be expressed as \( \log(\phi/1-\phi) \). The parameters in (4.5b) and odds ratios associated with these parameters are interpreted in Table 4.7 using ANOVA-like terms.

In (4.5b), the terms \( r_{0jk}, r_{1jk}, \) and \( r_{2jk} \) are the residual terms or variance components in the level-2 model, and the terms \( u_{00k}, u_{10k}, u_{11k}, u_{21k} \) are residual terms in the level-3 model. However, we equate all of the level-2 residual terms to zero (i.e., \( r_{0jk} = r_{1jk} = r_{2jk} = 0 \)) for a “typical teacher”, and all of the level-3 residual terms to zero (i.e., \( u_{00k} = u_{10k} = u_{11k} = u_{21k} = 0 \)) for a “typical school”. Then, we substitute the estimated coefficients in order to compute the simple effects of the predictors in the model. This is illustrated in the following sections.

**Description of Overall Relationship**

A factorial ANOVA-like approach of interpretation can be used to describe the overall relationship by predicting the odds of reading proficiency for different levels of student SES, and school mean SES associated with two levels of class type and minority.
The log-odds of reading proficiency ($\eta_{ijk}$) can be predicted by substituting the estimated coefficients from Table 4.6a in (4.5b), and can be given as follows.

$$
\eta_{ijk} = 1.373 + 1.697a_1 + 8.975a_2 - 0.271X + 1.995W -0.693XW \\
+ 1.486a_1W + 0.27a_1X - 0.424a_2X + 2.738a_1XW
$$

(4.5c)

The predicted level-1 outcome, i.e., log-odds of reading proficiency, can be computed by using single-equation given by (4.5c). However, the odds of reading proficiency can be computed after taking exponential of (4.5c), and presented in Table 4.7. This Table gives the odds of reading proficiency for different levels of SES and school mean SES associated with two levels of minority status and class type.

Table 4.7
Odds of Reading Proficiency for Different Levels of SES, School Mean SES, Minority Status, and Class Type

<table>
<thead>
<tr>
<th>SES</th>
<th>School mean</th>
<th>Class type for non-minority</th>
<th>Class type for minority</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SES</td>
<td>Non-crowded</td>
<td>Crowded</td>
</tr>
<tr>
<td>Low</td>
<td>Low</td>
<td>2.67×10^{-10}</td>
<td>2.11×10^{-10}</td>
</tr>
<tr>
<td>Low</td>
<td>Medium</td>
<td>3.57×10^{-9}</td>
<td>1.95×10^{-8}</td>
</tr>
<tr>
<td>Low</td>
<td>High</td>
<td>4.78×10^{-8}</td>
<td>1.80×10^{-6}</td>
</tr>
<tr>
<td>Medium</td>
<td>Low</td>
<td>0.30</td>
<td>0.23</td>
</tr>
<tr>
<td>Medium</td>
<td>Medium</td>
<td>3.95</td>
<td>21.54</td>
</tr>
<tr>
<td>Medium</td>
<td>High</td>
<td>52.80</td>
<td>1988.83</td>
</tr>
<tr>
<td>High</td>
<td>Low</td>
<td>32.57×10^{7}</td>
<td>25.75×10^{7}</td>
</tr>
<tr>
<td>High</td>
<td>Medium</td>
<td>43.57×10^{8}</td>
<td>23.78×10^{9}</td>
</tr>
<tr>
<td>High</td>
<td>High</td>
<td>58.28×10^{9}</td>
<td>21.95×10^{11}</td>
</tr>
</tbody>
</table>

Note.  a) For SES: Low = -2.32, Medium = 0, High = 2.32.

b) For school mean SES: Low = -1.3, Medium = 0, High = 1.3.
The odds of reading proficiency for a low SES and a high SES, associated with different levels of school mean SES, class type, and minority, are either incredibly small (almost zero) or very high (in the millions). The possible reasons for producing extremely small and extremely high values of odds are discussed in Chapter 5. To illustrate and interpret these results, only the odds for medium SES associated with different levels of school SES, class type, and minority is considered.

Using Table 4.7, we want to predict the odds in terms of happening of proficiency as opposed to non-proficiency. For example, the odds for both medium SES and school mean SES associated with non-crowded class type for minority is $3.01 \approx 3$. This can be interpreted as the probability of reading proficiency for a minority student, in a non-crowded classroom, with medium SES and school mean SES is thrice the size of the probability of non-proficiency in reading. In contrast, the odds of 0.3 for medium SES and low school mean SES associated with the non-crowded class type for minority can be interpreted as the probability of reading proficiency is less than one third that of the probability of non-proficiency, since the odds is obtained from probability of proficiency $(0.23)/1$-probability of non-proficiency $(0.77) = 0.3$. As a rule of thumb, the odds of 1.0 is a neutral value for odds in which both the outcomes, reading proficiency and reading non-proficiency, are equally likely.

The lowest odds ($2.11\times10^{-10}$) was obtained for non-minority students in a crowded classroom with low SES, and the low school mean SES. However, the highest odds ($21.95\times10^{11}$) was obtained for non-minority students in a crowded class type with high SES, and high school mean SES.

**Descriptions of the Effects of Individual Predictors**

**Descriptions Based on Individual Parameters**

*Main effects.* In Equation (4.5b), let us consider the simple effect of a particular (individual) predictor on its own which is analogous to the main effect in ANOVA. For example, the main effect (M. E.) of school mean SES can be calculated by taking the exponential of the coefficient $\gamma_{001}$ (from Table 4.6a) after multiplying it with a constant,
\( c_w \), of 1.3, i.e., twice the standard deviation (s.d.) of school mean SES. This can be computed as

\[
M.E. = \exp(c_w \times \gamma_{001}) = \exp(1.3 \times 1.995) = \exp(2.59) = 13.33.
\]

The main effects (M. E.) for individual predictors are interpreted in Table 4.8.

Table 4.8
Interpretation of Main Effects (M. E.) Based on Single-Equation

<table>
<thead>
<tr>
<th>Main effect</th>
<th>Interpretation of M. E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_{001} = 1.995 )</td>
<td>The odds ratio for reading proficiency associated with a change of two standard deviations in school mean SES for a reference student (non-minority with average SES), and reference teacher (teacher teaching a non-crowded classroom) is estimated as 13.33.</td>
</tr>
<tr>
<td>M.E. = \exp(1.3 \times 1.995) = \exp(2.59) = 13.33**</td>
<td></td>
</tr>
<tr>
<td>( \gamma_{010} = -0.271 )</td>
<td>The predicted odds of reading proficiency for crowded class type associated with non-minority student having average SES (reference student) with average school SES (reference school) is 0.76 units higher than that for non-crowded class type.</td>
</tr>
<tr>
<td>M.E. = \exp(1 \times -0.271) = 0.76**</td>
<td></td>
</tr>
<tr>
<td>( \gamma_{100} = 1.697 )</td>
<td>The predicted odds of reading proficiency for minority associated with non-crowded class type (reference teacher), and average school SES (reference school) is 5.46 units higher than that for non-minority students.</td>
</tr>
<tr>
<td>M.E. = \exp(1 \times 1.697) = \exp(2.59) = 5.46**</td>
<td></td>
</tr>
<tr>
<td>( \gamma_{200} = 8.975 )</td>
<td>The odds ratio for reading proficiency associated with a change of two standard deviations in SES for a reference teacher (i.e., teacher teaching a non-crowded class) and reference school (i.e., school with average SES) is estimated to be 11.04 \times 10^8.</td>
</tr>
<tr>
<td>M.E. = \exp(2.32 \times 8.975) = 11.04 \times 10^8**</td>
<td></td>
</tr>
</tbody>
</table>

\*\* significance at 0.01 level; \*\*\* significance at 0.001 level
The main effect (of school mean SES) can be interpreted as the odds ratio for reading proficiency associated with a change of two standard deviations in school mean SES for a non-minority student with an average SES, i.e., reference student, in a non-crowded classroom taught by a teacher (reference teacher) is estimated to be 13.33.

Using a similar procedure, the main effects of other predictors can be computed by multiplying the estimated coefficient with a constant (associated with specific predictor) equal to two s.d’s for continuous predictors and a constant of 1 for dichotomous predictors and then exponentiating the product. Thus, the main effects for minority, SES, class type, and school mean SES are computed by using the constants of $c_{a_1} = 1$, $c_{a_2} = 2.32$ (twice the s.d. of SES), $c_x = 1$, and $c_w = 1.3$ (twice the s.d. of school mean SES) respectively, and exponentiating the product of the constant and individual coefficient. The effects of school mean SES, minority, and student SES are found to be positive. However, the effect of class type is found negative.

Interaction effects. Two-way and three-way interaction effects, which imply the variations of the effect on reading proficiency due to one predictor based on the levels of other predictors, are complex to interpret. Tate (2004) mentioned the ANOVA-like interpretations of interaction and main effects. For example, the two-way interaction effect (I. E.) for $\gamma_{110} = 0.270$ can be computed by multiplying this term with the constants $c_{a_1} = 1$, and $c_x = 1$ associated with minority and class type respectively, and taking the exponential of the product. This can be expressed as the following.

Two-way I. E. of minority and class type $= \exp(c_{a_1} \times c_x \times \gamma_{110})$

$$= \exp(1 \times 1 \times 0.27) = 1.31 \quad (4.5d)$$

The interpretation of the two-way and three-way interactions is presented in Table 4.9. The second order (two-way) interaction given by (4.5d) is defined as ratio of odds ratios, which is interpreted as the factor increase of the odds ratio associated with the interaction of the difference in the effect between minority and non-minority, and the difference in effect between crowded and non-crowded class types for a reference school is estimated to be 1.31. Table 4.9 provides the computations and interpretations of two- and three-way interaction effects.
### Table 4.9
Computation and Interpretation of Two-Way and Three-Way Interaction Effects (I.E.)

<table>
<thead>
<tr>
<th>Interaction Effect</th>
<th>Interpretation of interaction effect</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Two-way</strong></td>
<td></td>
</tr>
<tr>
<td>$\gamma_{011} = -0.693$</td>
<td>For reference student, there is 0.89 factor increase in the odds ratio reflecting the effect of class type that is associated with two-standard deviation increase in school mean SES, or, equivalently, there is a 0.89 factor increase in the odds ratio due to two-standard deviation increase in school mean SES that is associated with class type contrast.</td>
</tr>
<tr>
<td>$\gamma_{101} = 1.486$</td>
<td>For reference teacher, there is 6.89 factor increase in the odds ratio reflecting the effect of minority that is due to two-standard deviation increase in school mean SES, or, equivalently, there is a 6.89 factor increase in the odds ratio due to two-standard deviation increase in school mean SES that is associated with minority contrast.</td>
</tr>
<tr>
<td>$\gamma_{110} = 0.27$</td>
<td>For a reference school, there is a 1.31 factor increase in the odds ratio reflecting the effect of minority that is associated with the class type contrast, or, equivalently, there is a 1.31 factor increase in the odds ratio reflecting the effect of class type that is associated with the minority contrast.</td>
</tr>
<tr>
<td>$\gamma_{210} = -0.424$</td>
<td>For a reference school, there is a 0.98 factor increase in the odds ratio due to two-standard deviation of SES that is associated with the class type contrast, or, equivalently, there is a 0.98 factor increase in the odds ratio reflecting the effect of minority that is associated with the minority contrast.</td>
</tr>
</tbody>
</table>
Table 4.9 (Continued)

Three-way

\( \gamma_{111} = -2.738 \)  

There is a 0.03 decrease in the two-way interaction between any two explanatory variables that is associated with a change in the third variable.

* significance at 0.05 level;  ** significance at 0.01 level  
*** significance at 0.001 level

The alternative approach to this complex procedure for interpreting the effects of predictors is described employing the simple effect approach in the following section.

**Simple Effect Description: Computational Procedure**

The simple effects of school mean SES (W), class type (X), minority \((a_1)\), and student SES \((a_2)\) on odds of reading proficiency can be described by (4.6a), (4.6b), (4.6c), and (4.6d) respectively after partially differentiating (4.5b). Exponentiating the derivatives after multiplying it by a constant associated with a specific predictor, following equations, including random effect terms in the model, are produced.

\[
E_w (X, a_1) = \exp \left[ c_w \frac{\partial \eta}{\partial W} \right] = \exp \left[ c_w \left( \gamma_{001} + \gamma_{101} a_1 + \gamma_{011} X + \gamma_{111} a_1 X \right) \right] \quad (4.6a)
\]

\[
E_x (W, a_1, a_2) = \exp \left[ c_x \frac{\partial \eta}{\partial X} \right] = \exp \left[ c_x \left( \gamma_{010} + \gamma_{011} W + \gamma_{110} a_1 + \gamma_{111} a_1 W + \gamma_{210} a_2 + u_{01k} + u_{21k} a_2 + u_{11k} a_1 \right) \right] \quad (4.6b)
\]

\[
E_{a_1} (W, X) = \exp \left[ c_{a_1} \frac{\partial \eta}{\partial a_1} \right] = \exp \left[ c_{a_1} \left( \gamma_{100} + \gamma_{101} W + \gamma_{110} X + \gamma_{111} XW + r_{1jk} + u_{10k} + u_{11k} X \right) \right] \quad (4.6c)
\]

\[
E_{a_2} (X) = \exp \left[ c_{a_2} \frac{\partial \eta}{\partial a_2} \right] = \exp \left[ c_{a_2} \left( \gamma_{210} X + r_{2jk} + u_{21k} X \right) \right] \quad (4.6d)
\]
It is noted that the simple effect of $W$ (based on the above partial derivation procedure) is fixed. However, the simple effect of $X$ (class type) depends on school level random terms, and the simple effects of $a_1$ (minority) and $a_2$ (SES) depend on both the school level and teacher level random terms ($r$’s and $u$’s).

The simple effect for any of the predictors in (4.6a) through (4.6d) can be computed by multiplying the right hand side terms in the equation with a constant associated with a particular predictor, and exponentiating the result. For demonstration purposes, let us estimate the change in odds of reading proficiency with an increase of ±2SD units in student SES and ±2SD units in school mean SES. However, since the other predictors, namely minority (at student level) and class type (at teacher level) are dichotomous, we use a constant of 1 for these predictors. The resulting equations for the simple effects of predictors by using standard deviation (SD) of 1.16 for student SES and SD of 0.64 for school mean SES, and using 2SD as constant (c) for these continuous predictors, and substituting the estimated parameters from Table 4.6a for a “typical teacher” (i.e., all $r_{jk} = 0$), and a “typical school” (i.e., all $u_k = 0$) are presented below in exponential forms.

\[
E_w(X, a_1) = \exp[1.28 (1.995 + 1.486 a_1 - 0.693X - 2.738a_1X)] \quad (4.7a)
\]
\[
E_x(W, a_1, a_2) = \exp[-0.271 - 0.693W + 0.270 a_1 - 2.738a_1W - 0.424a_2] \quad (4.7b)
\]
\[
E_{a_1}(W, X) = \exp[1.697 + 1.486 W + 0.270 X - 2.738WX] \quad (4.7c)
\]
\[
E_{a_2}(X) = \exp[2.32 (8.975 - 0.424X)] \quad (4.7d)
\]

Thus, the simple effects are computed by substituting the values of appropriate student, teacher, or school level parameters in (4.7a) through (4.7d). The resulting value on the right side of above equations are exponentiated in order to obtain the simple effects of individual predictors.

The matrix of fixed parameters ($\gamma$s) with a 10x1 order as well as its variance-covariance matrix with a 10x10 order is presented below. The order of elements in the matrix $V$ is corresponding to the order of $\gamma$s given below in (4.8a). The right hand side matrix in (4.8a) is based on the values of estimates of final model, which is provided earlier in Table 4.6a.
The following matrix gives the variance-covariance structure of Gamma matrix.

\[
\begin{pmatrix}
\gamma_{000} & = & 1.373 \\
\gamma_{001} & & 1.995 \\
\gamma_{010} & & -0.271 \\
\gamma_{011} & & -0.693 \\
\gamma_{100} & & 1.697 \\
\gamma_{101} & & 1.486 \\
\gamma_{110} & & 0.270 \\
\gamma_{111} & & -2.738 \\
\gamma_{200} & & 8.975 \\
\gamma_{210} & & -0.424 \\
\end{pmatrix}
\]

(4.8a)

\[
V = \begin{pmatrix}
0.449 \\
0.801 & 67.898 \\
-0.443 & -0.804 & 1.613 \\
-0.812 & -15.734 & 1.741 & 13.469 \\
-0.504 & -0.211 & 0.501 & 0.211 & 1.061 \\
-1.211 & -11.203 & 1.201 & 114.31 & -1.311 & 37.946 \\
0.503 & 0.211 & -1.023 & -0.803 & -1.091 & 1.321 & 2.045 \\
0.999 & -8.941 & -1.013 & 8.942 & -1.811 & 0.592 & 1.812 & -0.589 & 15.445 \\
-0.992 & 8.931 & 1.785 & -1.066 & 1.799 & -0.595 & -3.403 & 1.535 & -1.543 & 2.856 \\
\end{pmatrix} \times 10^{-2}
\]

(4.8b)
Using the above matrix $V$, the standard error can be computed by the following formula.

$$S_m = \sqrt{a' V a} \quad (4.9)$$

Where $a'$ is a 10x1 matrix obtained from 4.6 after a partial derivation. For example, for $W$ the matrix $a'$ can be formulated after partial derivation of $\eta$ with respect to $W$ equating all constants (i.e., without any $W$ term in the equation) to zero, and equating all $\gamma$s multiple of $W$ to 1. In other words, $\gamma_{000}$ will be zero since this is a constant while taking partial derivative of $W$ with respect to $\eta$, and $\gamma_{001} W$, which yields $\gamma_{001}$ after partial differentiation, can be equated to 1. Using this rule of thumb, we can generate the matrix $a'$ for the simple effect of $W$ as follows.

$$a' = (0 \ 1 \ 0 \ X \ 0 \ a_1 \ 0 \ a_1X \ 0 \ 0) \quad (4.10a)$$

The $\gamma$ matrix is formulated from (4.4c), following the sequential order given by Table 4.6a or in (4.8a), where we consider $\gamma$s in a logical order from the first equation to the last equation. Using (4.9), the standard error for $W$ (school mean SES) can be given by the following equation.

$$S_{m,E_w} = \left[0.67898 - 0.22406 \ a_1 + 0.31468X + 0.13469X^2 + 0.37946 \ a_1^2 + 4.52136 \ a_1X \\
-1.94186 \ a_1X^2 - 0.38184 \ a_1^2X + 1.64096 \ a_1^2X^2 \right]^{1/2} \quad (4.10b)$$

The $a'$ matrix for the effect of class type can be given as follows.

$$a' = (0 \ 0 \ 1 \ W \ 0 \ 0 \ a_1 \ W \ 0 \ a_2) \quad (4.11a)$$

Then using the formula given by (4.9), the standard error for the effect of class type can be computed by the following equation.

$$S_{m,E_x} = \left[0.01613 - 0.02046a_1 + 0.0348W - 0.06872a_1W - 0.06806a_1a_2 + 0.0307a_1a_2W \\
+ 0.0357a_2 - 0.0214a_2W + 0.02045a_1^2 - 0.00198a_1^2W - 1.94186 \ a_1W^2 \\
+ 0.02856a_2^2 + 0.13469W^2 + 1.64096a_1^2W^2 \right]^{1/2} \quad (4.11b)$$

The standard errors for the effect of minority and student SES can be computed in a similar fashion. For this purpose, we can use $a'$ matrix for minority as follows.

$$a' = (0 \ 0 \ 0 \ 1 \ W \ X \ XW \ 0 \ 0) \quad (4.12a)$$

Using the formula given by (4.9), the standard error for the effect of minority can be computed by the following equation.
Further, the matrix \( a' \) for SES can be formulated as below.

\[
a' = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad X)
\]

(4.13a)

Applying the formula given by (4.9), the standard errors for SES effects can be given as follows.

\[
S_m, E_{a_1} = [0.01061 - 0.02182X - 0.02620W + 0.05282XW + 0.02041X^2 - 0.00198X^2W + 0.37946W^2 - 0.38184XW + 1.64096X^2W^2]^{1/2}
\]

(4.12b)

The critical value for the simple effects with school mean SES or student SES in the equation (i.e., for \( E_x \) and \( E_{a_1} \)) needs an infinite family of tests, which is given by Scheffe’s test of multiple comparisons. The rationale for the necessity of infinite family of tests is based on the test associated with the results from continuous ranges of predictors (e.g., student SES and school mean SES) in Equations (4.6b) and (4.6c). It is assumed that the analyst wishes to provide simultaneous inference for the infinite family of simple effects for one independent variable as a continuous function of the other variable (Tate, 2004). The d.f. associated with \( h \) would be the number of hypothesis variables being tested in this study, which is 4 since the total number of predictors used in the models is four. Thus, using critical F Table for \( \alpha = .05 \) with hypothesis d.f. of 4 and error d.f. of infinity (\( \infty \)), i.e.,

\[
F(\alpha_{fw}; h, df_e) = F(0.05; 4, \infty) = 2.37
\]

The critical value, \( c_v \), is given by

\[
c_v = \sqrt{hF(\alpha_{fw}; h, df_e)} = \sqrt{4 \times 2.37} = 3.08
\]

(4.14a)

However, critical values associated with the simple effects that do not contain continuous predictors, such as school mean SES and student SES in the equation (i.e., for \( E_w \) and \( E_{a_2} \)), are given by t Table. This is because the error is related to family-wise error rate (of all pairwise comparisons), and we wish to compute the simple effect for one predictor as a discrete function of the other predictor. Thus, using critical t Table for \( \alpha = .05 \) and infinity error d.f., the critical value can be given by (4.14b).

\[
t(.05, \infty) = 1.96
\]

(4.14b)
The confidence interval (CI) can be computed by using the effect and standard error associated with specific predictors of interest, and critical value (Cv) computed above. For example, the confidence interval for the school mean SES (W) is given by

\[
\text{CI (E}_W\text{)} = \exp[(c_w \frac{\partial \eta}{\partial W}) \pm C_v c_w S_{m,Ew}]
\]  

(4.15a)

In (4.15a), \(S_{m,Ew}\) is the standard error for school mean SES (W) provided in (4.10b), and \(c_w\) is a constant as defined earlier. The critical value for school mean SES and student SES is 1.96, given by (4.14b), and the critical value for minority and class type is 3.08, given by (4.14a).

Similarly, the confidence interval for the effect of class type (E_x) associated with a constant \(c_x\), can be given by (4.15c) by using the effect (E_x) from (4.7b) after taking the exponential of the computed effect terms, standard error of E_x from (4.11b) and a critical value of 3.08 as of above.

\[
\text{CI (E}_X\text{)} = \exp[(c_x \frac{\partial \eta}{\partial X}) \pm C_v c_x S_{m,Ex}]
\]  

(4.15b)

Confidence interval for minority can be computed as follows.

\[
\text{CI (E}_{a1}\text{)} = \exp[(c_{a1} \frac{\partial \eta}{\partial a_1}) \pm C_v c_{a1} S_{m,Ea1}]
\]  

(4.15c)

Confidence interval for student SES can be computed as follows.

\[
\text{CI (E}_{a2}\text{)} = \exp[(c_{a2} \frac{\partial \eta}{\partial a_2}) \pm C_v c_{a2} S_{m,Ea2}]
\]  

(4.15d)

Simple Effect Description: Results and Interpretations

Effect of school mean SES. The simple effect of school mean SES on odds of reading proficiency can be computed by (4.7a). The simple effect can be described as the effect of one independent variable (predictor) for different levels of the other independent variables (predictors). In (4.7a), for example, the effect of school mean SES depends on the level of minority (i.e., \(a_1 = 0\) and \(a_1 = 1\)), and the level of class type (\(X = 0\), and \(X = 1\)).
Table 4.10 represents the simple effects and confidence intervals for school mean SES associated with different levels of minority ($a_1$) and class type ($X$). Only two effects, those for minority with non-crowded class type (86.12) and non-minority with non-crowded class type (12.85), were significant since the confidence intervals in both of these cases do not capture 1. The effects of school mean SES for minority and non-minority students, associated with crowded and non-crowded class types, are found positive (greater than one). The simple effect of 86.12 is defined as the odds ratio associated with a change of two standard deviations in school mean SES for ‘minority students in a non-crowded class’. For example, this effect can be interpreted as an increase in odds of reading proficiency for this group, i.e., for minority students in non-crowded class, by a factor of 86.12 when the school mean SES increases by two standard deviations.

<table>
<thead>
<tr>
<th>$a_1$</th>
<th>$X$</th>
<th>Effect</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>5.294</td>
<td>0.368</td>
<td>76.059</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>12.85*</td>
<td>1.626</td>
<td>101.581</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>86.115*</td>
<td>8.706</td>
<td>851.780</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1.066</td>
<td>0.001</td>
<td>768.212</td>
</tr>
</tbody>
</table>

* Significant at 0.05 level.

The constant ($c_{a1}$) associated with minority ($a_1$) was 1.

The constant ($c_X$) associated with class type ($X$) was 1.

A positive effect of 12.85 is found for the school mean SES associated with ‘non-minority in non-crowded class.’ This effect is interpreted as an increase in the odds of reading proficiency for this group, i.e., for non-minority in non-crowded class, by a factor of 12.85 when the school mean SES increases by two standard deviations.
Effect of student SES. The simple effect of student SES on odds of reading proficiency can be computed by (4.7d) based on two levels of class type (i.e., X = 0, 1). The effect of SES for a non-crowded classroom (X = 0) is found $11.03 \times 10^8$ whereas the effect for crowded classroom (X = 1) is obtained $4.13 \times 10^8$. Apparently, the effects of student SES for crowded and non-crowded class types are found positive.

The resulting lower and upper limits are found to be high for the effect of student SES. The lower and upper limits for SES with a crowded class type are found to be $1.3 \times 10^{17}$ and $7.3 \times 10^{22}$ respectively. Similarly, the lower and upper limits for SES associated with non-crowded class are found $1.6 \times 10^{20}$ and $5.7 \times 10^{21}$ respectively. Both of the above confidence intervals are found to be significant.

The above results for the effects and associated confidence intervals of student SES, being huge, are not credible. This is discussed in detail in Chapter 5.

Effect of class type. The graphical presentation of the simple effect of class type as well as its confidence intervals associated with the effect is provided below. It is notable that this effect depends on one or more continuous predictors. The total range of school mean SES (displayed in horizontal axis) is considered as four times its standard deviation (i.e., $4 \times 0.64 = 2.56 \approx 2.6$). Thus referring 4.7b, the simple effect of class type on odds of reading proficiency is a function of three predictors: minority, student SES and school mean SES.

The point estimates of class type effect for minority and non-minority with low SES are given in Figure 4.1a, and confidence intervals, associated with this effect, are given in Figures 4.1b and 4.1c. The class type effect for non-minority is not significant throughout the range of school mean SES (Figure 4.1b). For non-minority, the effect is found positive from -1.3 to 1.0, and negative from 1.1 to 1.3 on school mean SES. The effect for minority is significant below 0.6 on school mean SES (Figure 4.1c). Over this range, the estimated effect decreases from 2342.7 to 4.2 as school mean SES increases from -1.3 to 0.6. For minority, the effect is found positive from -1.3 to 0.9, and negative from 1.0 to 1.3 on school mean SES.
Figure 4.1a. Effect of class type for minority with low SES \((a_1 = 1, a_2 = -2.3)\) and non-minority with low SES \((a_1 = 0, a_2 = -2.3)\) associated with different levels of school mean SES \((W)\).

Figure 4.1b. Confidence bands for the effect of class type for non-minority \((a_1 = 0)\) and low SES \((a_2 = -2.32)\) associated with different levels of school mean SES \((W)\). No region is found significant since the values of CI capture 1.
Figure 4.1c. Confidence bands for the effect of class type for minority ($a_1 = 1$) and low SES ($a_2 = -2.32$) associated with different levels of school mean SES ($W$). The region of significance lies between –1.3 and 0.59.

The point estimates of class type effect for minority and non-minority with average SES are given in Figure 4.2a, and confidence intervals, associated with this effect, are given in Figures 4.9b and 4.9c. The class type effect for non-minority is not significant throughout the range of school mean SES (Figure 4.2b). For non-minority, the effect is found positive from –1.3 to -0.4, and negative from –0.39 to 1.3 on school mean SES. The effect for minority is significant from -1.3 to 0.15, and 0.1 to 1.3 on school mean SES (Figure 4.2c). Over this range, the estimated effect decreases from 86.4 to 2.0 as the school mean SES increases from –1.3 to 0.15, and the estimated negative effect increases in the strength from 0.71 to 0.01 as school mean SES increases from 0.1 to 1.3. For minority, the effect is found positive from –1.3 to –0.1, and negative from 0 to 1.3 on school mean SES.
The effect of minority is out of range of the scale.

Figure 4.2a. Effect of class type for minority with average SES ($a_1=1$, $a_2=0$), and non-minority with medium SES ($a_1=0$, $a_2=0$) associated with different levels of school mean SES ($W$).

The upper limit is out of range of the scale.

Figure 4.2b. Confidence bands for the effect of class type for non-minority ($a_1 = 0$) and medium SES ($a_2 = 0$) associated with different levels of school mean SES ($W$). No region was significant since all CI values capture 1.
Figure 4.2c. Confidence bands for the effect of class type for minority ($a_1 = 1$) and average SES ($a_2 = 0$) associated with different levels of school mean SES ($W$). The regions of significance lie: a) between $-1.3$ and $0.15$, b) between $0.1$ and $1.3$.

Figure 4.3a. Effect of class type for minority with high SES ($a_1=1$, $a_2 = 2.32$) and non-minority with high SES ($a_1=0$, $a_2 = 2.32$) associated with different levels of school mean SES ($W$).
The point estimates of class type effect for minority and non-minority with high SES are given in Figure 4.3a, and confidence intervals, associated with this effect, are given in Figures 4.10b and 4.10c. The class type effect for non-minority is not significant for any part of the range of school mean SES (Figure 4.3b). For non-minority, the effect is found negative throughout the range of school mean SES. The class type effect for minority is significant from -1.3 to -0.65, and 0.0 to 1.3 on school mean SES (Figure 4.3c). Over this range, the estimated effect decreases from 32.3 to 3.8 as the school mean SES increases from –1.3 to -0.65, and the estimated negative effect increases in strength from 0.37 to 0.01 as school mean SES increases from 0.0 to 1.3. For minority, the class type effect is found positive from –1.3 to –0.3, and negative from 0.2 to 1.3 on school mean SES.

Figure 4.3b. Confidence bands for the effect of class type for non-minority ($a_1 = 0$) and high SES ($a_2 = 2.32$) associated with different levels of school mean SES (W). No region was significant since all the values in lower band were less than 1.
Figure 4.3c. Confidence bands for the effect of class type for minority ($a_1 = 1$) and high SES ($a_2 = 2.32$) associated with different levels of school mean SES (W). The regions of significance lie between: a) –1.3 and -0.65, and b) 0.0 to 1.3.

In summary, the class type effects for minority students were found to be significant for low, average (medium), and high SES. The effects of non-minority were not found to be significant for any levels of SES. For all levels of SES, the range of school mean SES included a major portion of significant effect of class type for minority.

Effect of minority. Point estimates of the minority effect are given in Figure 4.4a, and confidence intervals, associated this effect, are given in Figures 4.11b and 4.11c. The minority effect for crowded class type is found significant from -0.69 to 0.41 on school mean SES (Figure 4.4b). In this range, the estimated effect decreases from 15.2 to 4.9 as school mean SES increases from -0.69 to 0.41. For the crowded class type, the effect is found positive throughout the range of school mean SES.

The minority effect for non-crowded class type is significant above -0.46 on school mean SES (Figure 4.4b). In this range, the estimated effect increases from 2.6 to 37.7 as school mean SES increases from -0.46 to 1.3. The effect of minority for non-crowded class type is found negative from –1.3 to –1.2, and positive from –1.1 to 1.3 on school mean SES.
Figure 4.4a. Effect of minority for crowded (X=1) and non-crowded (X=0) class types associated with different levels of school mean SES (W).

Figure 4.4b. Confidence bands for the effect of minority for crowded class type (X =1) associated with different levels of school mean SES (W). The region of significance lies between −0.69 and 0.41.
In summary, the effects of minority for both the crowded and non-crowded class types were found significant. This is displayed in Figure 4.4c. In general, the effect of minority for crowded class type was found to be decreasing as school mean SES increases, whereas the reverse was the case for non-crowded class type.

Example Executive Summary

In this Chapter, we provided a sequential demonstration of development, analysis, evaluation, and application of the model in educational research using three-level HGLM. The illustration was based on NAEP reading data with student, teacher, and school as level-1, level-2, and level-3 units respectively. Preliminary analyses supported the use of the data for the model demonstration purpose. For effect interpretation purposes, two approaches, namely, the ANOVA-like method and the simple effect approach were illustrated.
Several steps associated with model development, analysis, and evaluation were illustrated. The initial step with a fully unconditional model and the last step for the final model with fully conditional model were demonstrated. The final model was selected as the best model based upon the use of potential predictors, desirable estimated values of coefficients, and adequate proportion of variance explained. The level-2 and level-3 estimates of reliabilities were found to be fairly high for the final model. An exploratory analysis was used in order to select the potential predictors in conditional models during the model building process. Thus, an illustration for development, analysis, and evaluation of the three-level (nonlinear) model to be applied in educational research was presented.

An ANOVA-like approach of interpreting the effect of predictors was presented as a description of an overall relationship between the odds of reading proficiency and all of the explanatory variables. The lowest odds ($2.11 \times 10^{-10}$) was obtained for non-minority students in a crowded classroom with low SES and low school mean SES. However, the highest odds ($21.95 \times 10^{11}$) was obtained for non-minority students in a crowded class type with high SES, and high school mean SES.

In the simple effect description approach, the effect on odds of reading proficiency due to individual predictor, based on various levels of other predictor(s), was computed and interpreted. Different mathematical procedures, such as partial differentiation and matrix manipulation, were used while computing the simple effects.

Two simple effects for school mean SES, combined with different levels of other predictors, were significant. These effects were for minority students located in non-crowded class (86.1), and non-minority students located in non-crowded class (12.9). For instance, the school mean SES effect for minority students was defined as the odds ratio associated with a change of two standard deviations in school mean SES for minority students in non-crowded class. This was interpreted as an increase in odds of reading proficiency for minority students in a non-crowded class by a factor of 86.1 when school mean SES is increased by two standard deviations. Similarly, the school mean SES effect of 12.9 for non-minority student was interpreted as an increase in the odds of reading proficiency for non-minority in a non-crowded class by a factor of 12.9 when school
mean SES was increased by two standard deviations. It is remarkable that the simple effect of student SES was extremely high.

The effects of student SES for crowded and non-crowded class types are found positive. Similarly, the effects of school mean SES for minority and non-minority students, associated with crowded and non-crowded class types, are also found positive. For a low SES, a) the class type effect is found positive from -1.3 to 1.0, and negative from 1.1 to 1.3 on school mean SES for non-minority students, and b) the class type effect is found positive from -1.3 to 0.9, and negative from 1.0 to 1.3 on school mean SES for minority students. For an average SES, a) the class type effect is found positive from -1.3 to -0.4, and negative from -0.39 to 1.3 on school mean SES for non-minority students, and b) the class type effect is found positive from -1.3 to -0.1, and negative from 0 to 1.3 on school mean SES for minority students. For a high SES, a) the class type effect is found negative throughout the range of school mean SES for non-minority students, and b) the class type effect is found positive from -1.3 to -0.3, and negative from 0.2 to 1.3 on school mean SES for minority students. The effect of minority for crowded class type is found positive throughout the range of school mean SES. However, the effect of minority for non-crowded class type is found negative from -1.3 to -1.2, and positive from -1.1 to 1.3 on school mean SES.

In order to facilitate the interpretations, the effects and confidence intervals for class type and minority were computed and presented graphically. Three effects of class type, on odds of reading proficiency, were significant. First, the effect of class type associated with low SES for minority was significant below 0.6 on school mean SES in which range the estimated effect decreased from 2342.7 to 4.2 as school mean SES increased from -1.3 to 0.6. Second, the effect associated with medium SES for minority was significant in the region from -1.3 to 0.15, and 0.1 to 1.3 on school mean SES. Within this region, the estimated effect decreased from 86.4 to 2.0 as the school mean SES increased from -1.3 to 0.15, and the estimated effect decreased from 0.71 to 0.01 as the school mean SES increased from 0.1 to 1.3. Third, the effect of class type associated with a high SES for minority was significant in the region from -1.3 to -0.65, and 0.0 to 1.3 on school mean SES. Within this region, the estimated effect decreased from 32.3 to
3.8 as the school mean SES increased from −1.3 to -0.65, and the estimated effect decreased from 0.37 to 0.01 as school mean SES increased from 0.0 to 1.3.

The effects of minority for both the crowded and non-crowded class types were significant. For the crowded class type, the effect was significant from -0.69 to 0.41 on school mean SES. Over this range of significance, the estimated effect decreased from 15.2 to 4.9 as the school mean SES increased from -0.69 to 0.41. For a non-crowded class type, the minority effect was significant above -0.46 on the school mean SES. In this region of significance, the estimated effect increased from 2.6 to 37.7 as the school mean SES increased from -0.46 to 1.3.

A note of caveat worth to mention here is that some of the indicators used in this study may not perfectly reflect the explanatory variables. For example, student SES, which was derived from student lunch eligibility, Title 1 funding status, and parents’ education, may not perfectly measure student SES. In other words, parents’ education level may not be a ideal measure of student SES given the reality that many students who participate in free and reduced lunch program (low economic status) may have the parent/s with higher level academic qualification (or many students with just high school graduate parents may not participate in a free and reduced lunch program). Thus, low economic condition of a student may not necessarily reflect low social status or vice-versa.
CHAPTER 5
SUMMARY, DISCUSSION AND CONCLUSIONS

This study demonstrated a three-level hierarchical generalized linear model (HGLM) using NAEP 2000 Reading Assessment data. A sequential procedure was illustrated for development, analysis, application, and evaluation of the final model with two predictors at level-1, and one predictor at level-2 and level-3. The models were evaluated on the basis of model development until the selection of final model.

Two ways of describing the effects of the individual predictors were discussed in this study. First, an ANOVA-like procedure based on an interpretation of individual model parameters, with emphasis on effects analogous to “main effect” and “interaction effect” in factorial ANOVA, was illustrated. Second, a description of simple effects, showing how the effect of each predictor varies over levels of the other predictors, was presented.

Summary

The logical steps demonstrated in this study are important for the model building process. In order to build the models sequentially, we started the process with fully unconditional equations, where three components, namely, estimated parameters, variance components, and reliabilities, were evaluated based on the analysis. For example, the predicted mean of the log-odds of reading proficiency ($\gamma_{000}$) as well as level-2, and level-3 variance components were significant ($p<.001$) in fully unconditional model in step one.

In the sequential progression of model development, race (minority), SES, and sex/male were included as predictors in the level-1 conditional model. This decision was based on the medium high correlation between reading proficiency and these predictors.
However, sex was deleted afterwards from the model since the effect of this predictor was not significant.

The major criterion for developing a conditional model from an unconditional model was the evidence of significant variance components in the pertaining model. For example, the significant variance components in level-2 model suggested constructing the level-2 conditional model. However, the identification and selection of specific predictors to be included in the conditional model was determined by an exploratory analysis.

Throughout the model building process, the exploratory analysis not only provided the basis for selecting potential predictors in the model but it also indicated the significance of the effects of selected predictors. For example, step two in Chapter 4 involved the selection of potential predictors in the level-2 conditional model through an exploratory analysis in which the value of t-statistic was larger than 2.0 (i.e., significant) only for class type. Therefore, only class type was included in the level-2 conditional model, and consequently, the p-value was found significant associated with the slope of this predictor.

The evaluation of models enabled to attain the best models based on analysis results and provided a judgment on the selection of the successive model. In the level-2 conditional model, for instance, all estimated parameters as well as level-2 and level-3 variances components, except $u_{20k}$, were significant. This suggested the formulation of a level-3 conditional model associated with significant variance components. This kind of sequential model development and evaluation facilitated the researcher to apply appropriate models in the study.

The logical procedure of model development facilitated not only attained the final model but also aided in a comparison of the estimators, such as the variance terms. Such a comparison was useful in providing the information about the percent of variance explained compared to the previous model. Thus, the intermediate steps from step-one to step-four were important in analyzing the improvement in different estimated parameters during the model building process.
One of the options of interpreting the parameters’ effects we could have followed is the traditional approach of interpretation. Such an approach is based on interpreting the effect of level-3 coefficient on the effect (slope) of level-2 on the effect (slope) of level-1 coefficient. However, this approach appears to be more complex compared to the interpretations based on ANOVA-like and simple effect description procedures. The later technique of interpretations adopted in this study was analogous to the interpretation of coefficients in logistic regression and facilitated the interpretations in a more straightforward way.

By fitting the final model not only aided incorporating the right predictor in level-3 model in step-four but also obtained better estimates of effects, variance components, and reliability coefficients. Thus, most of the effect estimates and variance components in the final model were significant. Level-two and level-3 reliability coefficients were larger than 0.57 and 0.77 respectively. The minimum and maximum proportions of variance explained for the level-3 conditional model compared to the level-3 unconditional model were found to be 27% and 78% respectively.

To summarize the process, this study demonstrated a sequential procedure of model development and illustrated a three-level HGLM approach involving four steps. The complete steps included building fully unconditional models through developing fully conditional models and incorporating appropriate predictors in the models. During the analysis process, the exploratory method was adopted as a tool for identifying the proper variables in the models. The succeeding models were also compared in terms of improvement in estimated coefficients, proportion of variance explained, and reliabilities. The development, analysis, evaluation and application of the final model facilitated the research practitioners in terms of replicating similar models in future.

Comparing the levels of predictors in HGLM with the levels of factors in the factorial ANOVA, this study used an ANOVA-like interpretation approach to interpret the odds of reading proficiency due to the effect of individual predictors. The odds of reading proficiency for each cell was predicted for different levels of SES and school mean SES associated with two levels of minority status and class type. In order to facilitate the interpretations, the continuous predictors such as SES and school mean SES
were classified in three levels or categories, namely low, medium, and high. Citing the results, the odds of reading proficiency for medium SES and school mean SES associated with non-crowded class type for minority was 3.0. This was interpreted as the probability of reading proficiency for this group (i.e., for minority students in a non-crowded classroom with medium SES and school mean SES) is thrice the size of the probability of non-proficiency in reading.

The main and interaction effects were computed and interpreted using an ANOVA-like approach. The main effect of 13.33 for the school mean SES was interpreted as the odds ratio for reading proficiency associated with a change of two standard deviations in school mean SES for a reference student (non-minority student with average SES), and a reference teacher (teacher teaching in a non-crowded classroom) was estimated to be 13.33. Further, the second order interaction of predictors was interpreted as the change in the effect of the first predictor on odds of reading proficiency associated with a change in the ratio of odds ratios. The third order interaction was interpreted as the change in any of the second order interactions associated with a change in the ratio of ratios of odds ratios.

The illustration of an ANOVA-like approach presented in this study was advantageous because of its familiarity. It is worth mentioning here that the extreme values of odds of low and high SES are, in fact, associated with the probability of reading proficiency and non-proficiency (since odds is defined as a ratio of the probability of reading proficiency to non-proficiency). In case of extremely low value of odds, a ratio of very small probability of proficiency to very large probability of non-proficiency is expected. Unlikely, a reverse situation arises when resulting values of odds are very high. Although many of the odds values of reading proficiency presented in Table 4.7 are not credible, the values in this Table suffice our demonstration purpose.

An important aspect of this study is using the simple effect approach to interpret results. A partial derivative technique was employed in order to measure the simple effect of a particular predictor on odds of reading proficiency. Point estimates, as well as interval estimates of the effects associated with each predictor, were computed.
The presence of interaction effects signifies that effect of each predictor varies with levels of other predictors. The simple effect approach describes this variation.

If the variables consisted of only dichotomous predictor(s) in the equation, result Tables were presented to describe the simple effects and confidence intervals of such predictors. For example, the simple effect of the school mean SES for minority associated with non-crowded class type was significant. Similarly, the effect of the school mean SES for non-minority students associated with non-crowded class type was also significant.

Graphical presentations of simple effects and confidence intervals associated with these effects were provided only for those variables which included either only continuous or a combination of continuous and dichotomous predictors in the equation. Such presentations provided not only discernible descriptions of the magnitude, direction, and comparison of the effects but also facilitated to conclude the region of significance for the effects. For example, the estimated effect of minority drops when the value of school mean SES increases, and the region of significance lies within –0.69 and 0.41 of the school mean SES for this effect.

Despite the complexity of the simple effect approach in terms of mathematical derivation, using matrix algebra and differential calculus, this approach provided the simpler interpretations of effects compared to the traditional approach. The effects of the levels of each predictor on odds of reading proficiency can be interpreted without any complexity. This approach is efficient in terms of providing the simple interpretation of the effects of several levels even though multiple predictors are included in the model. More importantly, the effects and confidence intervals of the effects can be represented by line graphs. These graphs are easily comprehensible despite the complexity of multiple predictors with several levels incorporated in the equations. The magnitude, direction, and the significance region associated with the effect of a particular predictor can be determined simply by inspecting the graphs.
Discussion

Cost and Complications

Concepts and skills of multiple regression, logistic regression, and ANOVA are essential for the audiences of this study. More skills are desirable for research practitioners in order to work with modeling aspects, including model development, analysis, evaluation, and model selection, and interpreting the effects using an ANOVA-like and simple effect description approaches. The outcome variable used in this study is dichotomous. Since the outcome is dichotomous, familiarity with the changes in dichotomous outcome associated with a unit change in a given predictor is fundamental. Because different statistical terms, such as odds, log-odds, and odds ratios, are commonly used in this study, concepts and familiarity with such terminologies are desired in order to interpret the results. Likewise, familiarity with a change in outcome associated with multiple levels of predictors is also necessary. For example, in the case of describing the simple effect approach to interpret predictors’ effects, one should be able to handle multiple levels for several predictors. Skills associated with matrix manipulation, finding derivatives, plotting, and presenting complex graphs are also required.

Complications of the study may arise due to different factors. Since the simple effect approach used here is a new and complex technique with complicated mathematical procedures incorporating several predictors with multiple levels in an equation, the computation of effect may be complex for research practitioners. In addition, the sample used in this study is based on stratified complex sampling at each level. So, the complexity may arise because appropriate weighting for complex sample is unclear.

As a note of caveat, some of the indicators used in this study may not perfectly reflect the predictors. As mentioned earlier, student SES, which was derived from student lunch eligibility, Title 1 funding status, and parents’ education, may not perfectly measure student SES. For example, parents’ education level may not be an ideal measure of student SES given the reality that many students who do not participate in free and reduced lunch program (high economic status) may have their parent/s with low
level of academic qualification. Thus, high economic condition of a student may not necessarily reflect high social status or vice-versa.

Limitations of the Study

The major limitation of this study (as discussed earlier in Chapter 1) is the use of data for analysis without assigning weights for the variables used in the study. Note that the appropriate approach to using sampling weights for HLM is not yet available. Since the data used in this study is based on stratified complex sampling, the analysis of data without incorporating weights would only facilitate the purpose of a demonstration of the procedure.

Major caveats of the study are the results that produced huge amounts of odds ratios (4.13x10^8 and 11.03x10^8), and huge magnitude of confidence intervals for student SES associated with crowded (1.3x10^{17} and 7.3x10^{22}) and non-crowded (1.6x10^{20} and 5.7x10^{21}) class types. In addition, a huge main effect (11.04x10^8) is found for student SES. An estimated coefficient of 8.975 for SES produced such an enormous odds ratio, confidence intervals, and main effects. The revised analysis did not suggest any specific reasons for high estimates of SES effects. Appendix F provides the details of revised analysis. The results showing huge values of odds ratio and confidence intervals are simply not credible. In hindsight, it is likely that mistakes were made in the preparation of the data set for input into HGLM. The review did not suggest what these mistakes might be. Given the primary purpose of demonstration of the sequential procedure of modeling and then interpreting the predictors’ effects employing simpler techniques, it was decided that the current results, despite not being credible, would still suffice to fulfill the purpose.

Recommendations for Future Research

Although this study outlined the sequential demonstrations of three-level HGLM using national data set, research practitioners are recommended to continue the following works.
1. Using the count data as an outcome in the model, the researchers can demonstrate the three-level HGLM and present the sequential illustrations of developing, analyzing, evaluating and applying the models.

2. Research practitioners are suggested to use the real dichotomous outcome, such as dropout, in order to provide more meaningful implications of the study results.

3. Research practitioners are recommended to work further in extending simple effect and ANOVA-like approaches using more predictors in the model and using a large-scaled data.

Conclusions

This study presented a sequential procedure of model development, analysis, evaluation, and application. The research practitioners can replicate the procedural steps applying the similar techniques demonstrated in this study. The logical procedure of developing models, analyzing the models by including appropriate predictors and random effect terms, evaluating the estimators, and applying the model with best estimators will help research practitioners to replicate the similar modeling process. Specifically, researchers can replicate this study for final model selection purpose, and for predicting level-1 dichotomous outcome based on level-1, level-2, and level-3 predictors.

An ANOVA-like approach of interpretation addressed the issues of determining and interpreting the odds of reading proficiency due to the effect of individual predictors depending on various levels of other predictors in the model. In addition, main and interaction effects were computed and interpreted based on HGLM results. The application of HGLM results in terms of ANOVA-like interpretation would facilitate research practitioners to replicate similar modeling procedures.

Although the process of computation involves a series of complex mathematical techniques in simple effect approach, researchers can interpret effects with fewer complications using this approach compared to traditional method. Using the procedure illustrated in this study, the effects of a certain predictor, with a combination of different levels of other predictors, can be determined in the graphical form. In addition, the confidence intervals associated with the effects of predictors are represented graphically.
APPENDIX A

PRELIMINARY ANALYSES

Level-3 Residual Analysis

Figure A4.1a. Histogram with normal curve for Empirical Bayes residual analysis for predicting school model for teacher level intercept, $\beta_{00k}$ (EB00).

Figure A4.1b. Histogram with normal curve for Empirical Bayes residual analysis for predicting school model for teacher level effect of class type, $\beta_{01k}$ (EB01).
Figure A4.1c. Histogram with normal curve for Empirical Bayes residual analysis for predicting school level model for average effect of minority, $\beta_{10k}$ (EB10).

Figure A4.1d. Histogram with normal curve for Empirical Bayes residual analysis for predicting school model for teacher level effect of class type, $\beta_{11k}$ (EB11).
Figure A4.1e. Normal P-P curve of Empirical Bayes residual analysis for effect of class type associated with level-1 intercept, $\beta_{01k}$ (EB01).

Figure A4.1f. Normal P-P curve of Empirical Bayes residual analysis for average minority effect (intercept) in teacher level model, $\beta_{10k}$ (EB10).
Figure A4.1g. Normal P-P curve of Empirical Bayes residual analysis for the effect of class type associated with minority slope, $\beta_{11k}$ (EB11).

Figure A4.1h: Scatter diagram for residual versus fitted values showing homogeneity of variance for EB00.
Figure A4.1i: Scatter diagram for residual versus fitted values showing homogeneity of variance for EB01.

Figure A4.1j: Scatter diagram for residual versus fitted values showing homogeneity of variance for EB10.
Figure A4.1k: Scatter diagram for residual versus fitted values showing homogeneity of variance for EB11.

Figure A4.1l: Residual plots for average minority slope (intercept) versus school mean SES for linearity test.
Figure A4.1m: Residual plots for average minority slope versus school mean SES for linearity test.

Level-2 Residual Analysis

Figure A4.2a: Histogram with normal curve of residual for intercept associated with minority slope
Figure A4.2b: Histogram with normal curve for the Empirical Bayes residual for the slope of minority.

Figure A4.2c. Normal P-P curve of empirical Bayes residual for the slope of minority.
Figure A4.2d. Normal P-P curve of empirical Bayes residual for the slope of SES.

Figure A4.2e. Plots for residuals for intercept associated with minority effect versus fitted value for homogeneity of variance test.
Figure A4.2f. Plots for residuals for school level intercept versus school mean SES for linearity test.
APPENDIX B

HLM OUTPUT

Step One: Analysis for Testing for True Variance

Program: HLM Hierarchical Linear and Nonlinear Modeling
Authors: Stephen Raudenbush, Tony Bryk, & Richard Congdon
Publisher: Scientific Software International, Inc. (c) 2000
techsupport@ssicentral.com
www.ssicentral.com

Module: HLM3.EXE (5.04.21205.1)
Date: 1 November 2003, Saturday
Time: 12:16:19

SPECIFICATIONS FOR THIS NONLINEAR HLM3 RUN
Sat Nov 01 12:16:19 2003

Problem Title: NO TITLE

The data source for this run = C:\Windows\Profiles\BIDYA\MY DOCUMENTS\DISSERTATION\DATA\READNEW4.SSM
The command file for this run = whelmtemp.hl m
Output file name = C:\Windows\Profiles\BIDYA\MY DOCUMENTS\DISSERTATION\DATA\HLM3.OUT
The maximum number of level-2 units = 1076
The maximum number of level-3 units = 295
The maximum number of microiterations = 100
Method of estimation: full PQL
Maximum number of macroiterations = 100

Distribution at Level-1: Bernoulli

The outcome variable is READPROF

The model specified for the fixed effects was:

Level-1 Coefficients Level-2 Predictors Level-3 Predictors
INTERCEPT1, P0 INTERCEPT2, B00 INTERCEPT3, G000

Summary of the model specified (in equation format)

Level-1 Model
Prob(Y=1|B) = P
log[P/(1-P)] = P0

Level-2 Model
P0 = B00 + R0

Level-3 Model
B00 = G000 + U00

129
Random level-1 coefficient  Reliability estimate

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Random level-2 coefficient  Reliability estimate

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</tbody>
</table>

Final estimation of fixed effects: (Unit-specific model)

<table>
<thead>
<tr>
<th>Fixed Effect</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-ratio</th>
<th>d.f.</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>For INTROPT1, P0</td>
<td>0.4167</td>
<td>0.023747</td>
<td>1.176</td>
<td>294</td>
<td>0.000</td>
</tr>
<tr>
<td>For INTROPT2, B00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>INTROPT3, G000</td>
<td>0.4167</td>
<td>0.023747</td>
<td>1.176</td>
<td>294</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Final estimation of level-1 and level-2 variance components:

<table>
<thead>
<tr>
<th>Random Effect</th>
<th>Standard Deviation</th>
<th>Variance Component</th>
<th>df</th>
<th>Chi-square</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTROPT1, R0</td>
<td>0.57028</td>
<td>0.32522</td>
<td>781</td>
<td>1247.44057</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Final estimation of level-3 variance components:

<table>
<thead>
<tr>
<th>Random Effect</th>
<th>Standard Deviation</th>
<th>Variance Component</th>
<th>df</th>
<th>Chi-square</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTROPT1/INTROPT2, U00</td>
<td>0.93710</td>
<td>0.87816</td>
<td>294</td>
<td>1038.25704</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Step Two: Analysis for Specifying Student-Level Variables

Program: HLM 5 Hierarchical Linear and Nonlinear Modeling
Authors: Stephen Raudenbush, Tony Bryk, & Richard Congdon
Publisher: Scientific Software International, Inc. (c) 2000
techsupport@ssicentral.com
www.ssicentral.com

Module: HLM. EXE (5.04.21205.1)
Date: 1 November 2003, Saturday
Time: 12:25:37

SPECIFICATIONS FOR THIS NONLINEAR HLM RUN Sat Nov 01 12:25:37 2003

Problem Title: NO TITLE

The data source for this run = C: \WINDOWS\PROFILES\BIDYA\MY DOCUMENTS\DISSERTATION\DATA\READNEW4.SSM
The command file for this run = whl t emp.hlm
Output file name = C: \WINDOWS\PROFILES\BIDYA\MY DOCUMENTS\DISSERTATION\DATA\HLM3.OUT
The maximum number of level-2 units = 1076
The maximum number of level-3 units = 295
The maximum number of micro iterations = 100
Method of estimation: full PQL
Maximum number of macro iterations = 100

Distribution at Level-1: Bernoulli
The outcome variable is READPROF

The model specified for the fixed effects was:

Level-1 Model
Prob( Y=1|B) = P
log[P/(1-P)] = P0 + P1*(M.NORIT) + P2*(SES)

Level-2 Model
P0 = B00 + R0
P1 = B10 + R1
P2 = B20 + R2

'%' - This variable has been centered around its grand mean

Summary of the model specified (in equation format)
Level -3 Model

\[ B_{00} = G_{000} + U_{00} \]
\[ B_{10} = G_{100} + U_{10} \]
\[ B_{20} = G_{200} + U_{20} \]

Random level -1 coefficient Reliability estimate

<table>
<thead>
<tr>
<th>Intercept, P0</th>
<th>0.525</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minority, P1</td>
<td>0.537</td>
</tr>
<tr>
<td>SES, P2</td>
<td>0.519</td>
</tr>
</tbody>
</table>

Random level -2 coefficient Reliability estimate

<table>
<thead>
<tr>
<th>Intercept1/Intercept2, B00</th>
<th>0.735</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minority/Intercept2, B10</td>
<td>0.759</td>
</tr>
<tr>
<td>SES/Intercept2, B20</td>
<td>0.758</td>
</tr>
</tbody>
</table>

Final estimation of fixed effects: (Unit-specific model)

<table>
<thead>
<tr>
<th>Fixed Effect</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-ratio</th>
<th>d.f.</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>For Intercept1, P0</td>
<td>1.316407</td>
<td>0.046960</td>
<td>28.032</td>
<td>294</td>
<td>0.000</td>
</tr>
<tr>
<td>For Intercept2, B00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>For Intercept3, G000</td>
<td>-2.544617</td>
<td>0.070056</td>
<td>36.323</td>
<td>294</td>
<td>0.000</td>
</tr>
<tr>
<td>For Minority slope, P1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>For Intercept2, B10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>For Intercept3, G100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>For SES slope, P2</td>
<td>8.269674</td>
<td>0.270489</td>
<td>30.573</td>
<td>294</td>
<td>0.000</td>
</tr>
<tr>
<td>For Intercept2, B20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>For Intercept3, G200</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Final estimation of level -1 and level -2 variance components:

<table>
<thead>
<tr>
<th>Random Effect</th>
<th>Standard Deviation</th>
<th>Variance Component</th>
<th>df</th>
<th>Chi-square</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept1, P0</td>
<td>0.28820</td>
<td>0.08306</td>
<td>732</td>
<td>875.73645</td>
<td>0.000</td>
</tr>
<tr>
<td>Minority slope, P1</td>
<td>0.31067</td>
<td>0.09651</td>
<td>732</td>
<td>929.55052</td>
<td>0.000</td>
</tr>
<tr>
<td>SES slope, P2</td>
<td>2.45094</td>
<td>6.00709</td>
<td>732</td>
<td>989.69767</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note: The chi-square statistics reported above are based on only 1076 units that had sufficient data for computation. Fixed effects and variance components are based on all the data.
**Final estimation of level-3 variance components:**

<table>
<thead>
<tr>
<th>Random Effect</th>
<th>Standard Deviation</th>
<th>Variance Component</th>
<th>df</th>
<th>Chi-square</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTROPT1 / INTROPT2, U00</td>
<td>0.48795</td>
<td>0.23810</td>
<td>294</td>
<td>251.14038</td>
<td>0.004</td>
</tr>
<tr>
<td>MINORITY / INTROPT2, U10</td>
<td>0.43498</td>
<td>0.18921</td>
<td>294</td>
<td>305.68570</td>
<td>0.002</td>
</tr>
<tr>
<td>SES / INTROPT2, U20</td>
<td>1.15420</td>
<td>2.63219</td>
<td>294</td>
<td>292.84557</td>
<td>0.003</td>
</tr>
</tbody>
</table>

**Exploratory Analysis: estimated level-2 coefficients and their standard errors obtained by regressing EB residuals on level-2 predictors selected for possible inclusion in subsequent runs**

<table>
<thead>
<tr>
<th>Level-1 Coefficient</th>
<th>Potential Level-2 Predictors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TCHEXPCT</td>
</tr>
<tr>
<td></td>
<td>COMPUSE</td>
</tr>
<tr>
<td></td>
<td>CLASTYPE</td>
</tr>
<tr>
<td>INTROPT1, P0</td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>0.012</td>
</tr>
<tr>
<td>Standard Error</td>
<td>-0.049</td>
</tr>
<tr>
<td>t value</td>
<td>-2.653</td>
</tr>
<tr>
<td></td>
<td>TCHEXPCT</td>
</tr>
<tr>
<td></td>
<td>COMPUSE</td>
</tr>
<tr>
<td></td>
<td>CLASTYPE</td>
</tr>
<tr>
<td>MINORITY, P1</td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>-0.013</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.014</td>
</tr>
<tr>
<td>t value</td>
<td>2.763</td>
</tr>
<tr>
<td></td>
<td>TCHEXPCT</td>
</tr>
<tr>
<td></td>
<td>COMPUSE</td>
</tr>
<tr>
<td></td>
<td>CLASTYPE</td>
</tr>
<tr>
<td>SES, P2</td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>0.101</td>
</tr>
<tr>
<td>Standard Error</td>
<td>-0.412</td>
</tr>
<tr>
<td>t value</td>
<td>-2.780</td>
</tr>
</tbody>
</table>
Step Three: Analysis for Specifying Teacher-Level Variables

Program: HLM 5 Hierarchical Linear and Nonlinear Modeling
Authors: Stephen Raudenbush, Tony Bryk, & Richard Congdon
Publisher: Scientific Software International, Inc. (c) 2000
techsupport@ssicentral.com
www.ssicentral.com

Module: HLM3.EXE (5.04.21205.1)
Date: 1 November 2003, Saturday
Time: 12:39:05

SPECIFICATIONS FOR THIS NONLINEAR HLM3 RUN Sat Nov 01 12:39:05 2003

Problem Title: NO TITLE

The data source for this run = C:\WINDOWS\PROFILES\BIDYA\MY DOCUMENTS\DISSERTATION_DATA\READNEW4.SSM
The command file for this run = whl mtemp.hl m
Output file name = C:\WINDOWS\PROFILES\BIDYA\MY DOCUMENTS\DISSERTATION_DATA\HLM3.OUT
The maximum number of level-2 units = 1076
The maximum number of level-3 units = 295
The maximum number of micro iterations = 100
Method of estimation: full PQL
Maximum number of macro iterations = 100

Distribution at Level-1: Bernoulli

The outcome variable is READPROF

The model specified for the fixed effects was:

<table>
<thead>
<tr>
<th>Level-1 Coefficients</th>
<th>Level-2 Predictors</th>
<th>Level-3 Predictors</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRCPT1, P0</td>
<td>INTRCPT2, B00</td>
<td>INTRCPT3, G000</td>
</tr>
<tr>
<td></td>
<td>CLASTYPE, B01</td>
<td>INTRCPT3, G100</td>
</tr>
<tr>
<td>MNCRTY slope, P1</td>
<td>INTRCPT2, B10</td>
<td>INTRCPT3, G100</td>
</tr>
<tr>
<td>%</td>
<td>CLASTYPE, B11</td>
<td>INTRCPT3, G110</td>
</tr>
<tr>
<td>SES slope, P2</td>
<td>INTRCPT2, B20</td>
<td>INTRCPT3, G200</td>
</tr>
<tr>
<td></td>
<td>CLASTYPE, B21</td>
<td>INTRCPT3, G210</td>
</tr>
</tbody>
</table>

'%' - This variable has been centered around its grand mean

Summary of the model specified (in equation format)

Level-1 Model

\[ \Pr ob(Y=1|B) = P \]

\[ \log[P/(1-P)] = P0 + P1*(MNCRTY) + P2*(SES) \]
Level - 2 Model

\[ P_0 = B_{00} + B_{01} \cdot (CLASTYPE) + R_0 \]
\[ P_1 = B_{10} + B_{11} \cdot (CLASTYPE) + R_1 \]
\[ P_2 = B_{20} + B_{21} \cdot (CLASTYPE) + R_2 \]

Level - 3 Model

\[ B_{00} = G_{000} + U_{00} \]
\[ B_{01} = G_{010} + U_{01} \]
\[ B_{10} = G_{100} + U_{10} \]
\[ B_{11} = G_{110} + U_{11} \]
\[ B_{20} = G_{200} + U_{20} \]
\[ B_{21} = G_{210} + U_{21} \]

Random level - 1 coefficient

\begin{tabular}{lcl}
INTROPT1, P0 & 0.552 \\
MINORITY, P1 & 0.555 \\
SES, P2 & 0.559 \\
\end{tabular}

Random level - 2 coefficient

\begin{tabular}{lcl}
INTROPT1/INTROPT2, B00 & 0.764 \\
INTROPT1/CLASTYPE, B01 & 0.761 \\
MINORITY/INTROPT2, B10 & 0.773 \\
MINORITY/CLASTYPE, B11 & 0.768 \\
SES/INTROPT2, B20 & 0.775 \\
SES/CLASTYPE, B21 & 0.765 \\
\end{tabular}

Final estimation of fixed effects: (Unit-specific model)

\begin{tabular}{lclllll}
Fixed Effect & Coefficient & Standard Error & T-ratio & d.f. & P-value \\
For INTROPT1, P0 & & & & & \\
For INTROPT2, B00 & & & & & \\
INTROPT3, G00 & 1.363158 & 0.067634 & 20.155 & 294 & 0.000 \\
For CLASTYPE, B01 & & & & & \\
INTROPT3, G010 & -0.829174 & 0.368101 & -2.253 & 294 & 0.004 \\
For MINORITY slope, P1 & & & & & \\
For INTROPT2, B10 & & & & & \\
INTROPT3, G100 & -2.701166 & 0.102737 & -26.292 & 294 & 0.000 \\
For CLASTYPE, B11 & & & & & \\
INTROPT3, G110 & 0.277872 & 0.115359 & 2.414 & 294 & 0.003 \\
For SES slope, P2 & & & & & \\
For INTROPT2, B20 & & & & & \\
INTROPT3, G200 & 9.137612 & 0.417194 & 21.903 & 294 & 0.000 \\
For CLASTYPE, B21 & & & & & \\
INTROPT3, G210 & -0.607419 & 0.209012 & -2.904 & 294 & 0.002 \\
\end{tabular}

Final estimation of level - 1 and level - 2 variance components:

\begin{tabular}{lcccc}
Random Effect & Standard Deviation & Variance Component & df & Chi-square & P-value \\
INTROPT1, R0 & 0.24832 & 0.06067 & 437 & 862.14445 & 0.000 \\
MINORITY slope, R1 & 0.30838 & 0.09510 & 437 & 907.48303 & 0.000 \\
SES slope, R2 & 2.15470 & 4.64271 & 437 & 950.49337 & 0.000 \\
\end{tabular}
Final estimation of level-3 variance components:

<table>
<thead>
<tr>
<th>Random Effect</th>
<th>Standard Deviation</th>
<th>Variance Component</th>
<th>df</th>
<th>Chi-square</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRCPT1/INTRCPT2, U00</td>
<td>0.40374</td>
<td>0.16301</td>
<td>266</td>
<td>271.92075</td>
<td>0.001</td>
</tr>
<tr>
<td>INTRCPT1/CLASSTYPE, U01</td>
<td>0.40242</td>
<td>0.16194</td>
<td>266</td>
<td>260.73905</td>
<td>0.002</td>
</tr>
<tr>
<td>MINORITY/INTRCPT2, U10</td>
<td>0.37233</td>
<td>0.13863</td>
<td>266</td>
<td>268.26628</td>
<td>0.003</td>
</tr>
<tr>
<td>MINORITY/CLASSTYPE, U11</td>
<td>0.62714</td>
<td>0.39330</td>
<td>266</td>
<td>258.74382</td>
<td>0.004</td>
</tr>
<tr>
<td>SES/INTRCPT2, U20</td>
<td>2.87236</td>
<td>8.25046</td>
<td>266</td>
<td>249.73704</td>
<td>&gt;.500</td>
</tr>
</tbody>
</table>

Exploratory Analysis: estimated level-3 coefficients and their standard errors obtained by regressing EB residuals on level-3 predictors selected for possible inclusion in subsequent runs

<table>
<thead>
<tr>
<th>Level-2 Predictor</th>
<th>Potential Level-3 Predictors</th>
</tr>
</thead>
<tbody>
<tr>
<td>PUBPRIV SCHLSES TITLE1</td>
<td>INTRCPT2, B00</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.040</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.012</td>
</tr>
<tr>
<td>t value</td>
<td>-3.324</td>
</tr>
<tr>
<td>PUBPRIV SCHLSES TITLE1</td>
<td>INTRCPT1/CLASSTYPE, B01</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.049</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.017</td>
</tr>
<tr>
<td>t value</td>
<td>2.890</td>
</tr>
<tr>
<td>PUBPRIV SCHLSES TITLE1</td>
<td>MINORITY/INTRCPT2, B10</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.095</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.011</td>
</tr>
<tr>
<td>t value</td>
<td>2.625</td>
</tr>
<tr>
<td>PUBPRIV SCHLSES TITLE1</td>
<td>MINORITY/CLASSTYPE, B11</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.107</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.032</td>
</tr>
<tr>
<td>t value</td>
<td>3.312</td>
</tr>
<tr>
<td>PUBPRIV SCHLSES TITLE1</td>
<td>SES/INTRCPT2, B20</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.446</td>
</tr>
<tr>
<td>Standard Error</td>
<td>1.368</td>
</tr>
<tr>
<td>t value</td>
<td>0.326</td>
</tr>
<tr>
<td>PUBPRIV SCHLSES TITLE1</td>
<td>SES/CLASSTYPE, B21</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.334</td>
</tr>
<tr>
<td>Standard Error</td>
<td>1.877</td>
</tr>
<tr>
<td>t value</td>
<td>0.178</td>
</tr>
</tbody>
</table>
Step Four: Analysis for Specifying Student-Level Variables

Program: HLM 5 Hierarchical Linear and Nonlinear Modeling
Authors: Stephen Raudenbush, Tony Bryk, & Richard Congdon
Publisher: Scientific Software International, Inc. (c) 2000

techsupport@ssicentral.com
www.ssicentral.com

Module: HLM3.EXE (5.04.21205.1)
Date: 1 November 2003, Saturday
Time: 12:57:55

Specifications for This Nonlinear HLM3 Run Sat Nov 01 12:57:55 2003

Problem Title: NO TITLE

The data source for this run = C:\WINDES\PROFILES\BIDYA\MY DOCUMENTS\DISSERTATION_DATA\READNEW4.SSM
The command file for this run = whlmt emperor.hlml
Output file name = C:\WINDES\PROFILES\BIDYA\MY DOCUMENTS\DISSERTATION_DATA\HLM3.OUT
The maximum number of level-2 units = 1076
The maximum number of level-3 units = 295
The maximum number of microiterations = 100
Method of estimation: full PQL
Maximum number of microiterations = 100

Distribution at Level-1: Bernoulli

The outcome variable is READPROF

The model specified for the fixed effects was:

<table>
<thead>
<tr>
<th>Level -1 Coefficients</th>
<th>Level -2 Predictors</th>
<th>Level -3 Predictors</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT1, P0</td>
<td>INTERCEPT2, B00</td>
<td>INTERCEPT3, G000</td>
</tr>
<tr>
<td></td>
<td>QLASTYPE, B01</td>
<td>INTERCEPT3, G010</td>
</tr>
<tr>
<td></td>
<td></td>
<td>% SCHLSES, G011</td>
</tr>
<tr>
<td>MINORITY slope, P1</td>
<td>INTERCEPT2, B10</td>
<td>INTERCEPT3, G100</td>
</tr>
<tr>
<td></td>
<td>QLASTYPE, B11</td>
<td>% SCHLSES, G110</td>
</tr>
<tr>
<td></td>
<td></td>
<td>% SCHLSES, G111</td>
</tr>
<tr>
<td>% SES slope, P2</td>
<td># INTERCEPT2, B20</td>
<td>INTERCEPT3, G200</td>
</tr>
<tr>
<td></td>
<td>QLASTYPE, B21</td>
<td>INTERCEPT3, G210</td>
</tr>
</tbody>
</table>

' #' - The residual parameter variance for the parameter has been set to zero
' % ' - This variable has been centered around its grand mean
Summary of the model specified (in equation format)

Level - 1 Model
\[
\Pr(Y=1|B) = P
\]
\[
\log\left[\frac{P}{1-P}\right] = P_0 + P_1 \times (\text{MINORITY}) + P_2 \times (\text{SES})
\]

Level - 2 Model
\[
P_0 = B_{00} + B_{01} \times (\text{CLASTYPE}) + R_0
\]
\[
P_1 = B_{10} + B_{11} \times (\text{CLASTYPE}) + R_1
\]
\[
P_2 = B_{20} + B_{21} \times (\text{CLASTYPE}) + R_2
\]

Level - 3 Model
\[
B_{00} = G_{000} + G_{001} \times (\text{SCHLSES}) + U_{00}
\]
\[
B_{01} = G_{010} + G_{011} \times (\text{SCHLSES}) + U_{01}
\]
\[
B_{10} = G_{100} + G_{101} \times (\text{SCHLSES}) + U_{10}
\]
\[
B_{11} = G_{110} + G_{111} \times (\text{SCHLSES}) + U_{11}
\]
\[
B_{20} = G_{200}
\]
\[
B_{21} = G_{210} + U_{21}
\]

Level - 1 variance = \(1/[P(1-P)]\)

<table>
<thead>
<tr>
<th>Random level - 1 coefficient</th>
<th>Reliability estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTROPT1, P0</td>
<td>0.570</td>
</tr>
<tr>
<td>MINORITY, P1</td>
<td>0.587</td>
</tr>
<tr>
<td>SES, P2</td>
<td>0.577</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Random level - 2 coefficient</th>
<th>Reliability estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTROPT1/INTROPT2, B00</td>
<td>0.771</td>
</tr>
<tr>
<td>INTROPT1/CLASTYPE, B01</td>
<td>0.785</td>
</tr>
<tr>
<td>MINORITY/INTROPT2, B10</td>
<td>0.783</td>
</tr>
<tr>
<td>MINORITY/CLASTYPE, B11</td>
<td>0.779</td>
</tr>
<tr>
<td>SES/CLASTYPE, B21</td>
<td>0.781</td>
</tr>
</tbody>
</table>

The outcome variable is READPROF
Final estimation of fixed effects: (Unit-specific model)

<table>
<thead>
<tr>
<th>Fixed Effect</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-ratio</th>
<th>Approx. d.f.</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>For INTRCPT1, P0</td>
<td>1.372730</td>
<td>0.066872</td>
<td>20.528</td>
<td>293</td>
<td>0.000</td>
</tr>
<tr>
<td>SCHLSES, Q001</td>
<td>1.995368</td>
<td>0.823530</td>
<td>2.423</td>
<td>293</td>
<td>0.006</td>
</tr>
<tr>
<td>For CLASTYPE, B01</td>
<td>-0.270891</td>
<td>0.126881</td>
<td>-2.135</td>
<td>293</td>
<td>0.007</td>
</tr>
<tr>
<td>SCHLSES, G011</td>
<td>-0.693132</td>
<td>0.367319</td>
<td>-1.887</td>
<td>293</td>
<td>0.070</td>
</tr>
<tr>
<td>For MINORITY slope, P1</td>
<td>1.995368</td>
<td>0.823530</td>
<td>2.423</td>
<td>293</td>
<td>0.006</td>
</tr>
<tr>
<td>SCHLSES, G001</td>
<td>1.485707</td>
<td>0.616152</td>
<td>2.411</td>
<td>293</td>
<td>0.006</td>
</tr>
<tr>
<td>For CLASTYPE, B01</td>
<td>0.270332</td>
<td>0.143025</td>
<td>1.890</td>
<td>293</td>
<td>0.070</td>
</tr>
<tr>
<td>SCHLSES, G111</td>
<td>-2.738311</td>
<td>1.280552</td>
<td>-2.138</td>
<td>293</td>
<td>0.007</td>
</tr>
<tr>
<td>For SES slope, P2</td>
<td>8.975498</td>
<td>0.393251</td>
<td>22.824</td>
<td>294</td>
<td>0.000</td>
</tr>
<tr>
<td>SCHLSES, G200</td>
<td>2.03477</td>
<td>4.14029</td>
<td>731</td>
<td>978.49535</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Final estimation of level -1 and level -2 variance components:

<table>
<thead>
<tr>
<th>Random Effect</th>
<th>Standard Deviation</th>
<th>Variance Component</th>
<th>df</th>
<th>Chi-square</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRCPT1, R0</td>
<td>0.21361</td>
<td>0.04563</td>
<td>437</td>
<td>860.71893</td>
<td>0.000</td>
</tr>
<tr>
<td>MINORITY slope, R1</td>
<td>0.29047</td>
<td>0.08437</td>
<td>437</td>
<td>901.26241</td>
<td>0.000</td>
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<tr>
<td>SES slope, R2</td>
<td>2.03477</td>
<td>4.14029</td>
<td>731</td>
<td>978.46898</td>
<td>0.000</td>
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</tbody>
</table>

Final estimation of level -3 variance components:

<table>
<thead>
<tr>
<th>Random Effect</th>
<th>Standard Deviation</th>
<th>Variance Component</th>
<th>df</th>
<th>Chi-square</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRCPT1/INTRCPT2, U00</td>
<td>0.23286</td>
<td>0.05422</td>
<td>266</td>
<td>263.42869</td>
<td>0.003</td>
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<tr>
<td>INTRCPT1/CLASTYPE, U01</td>
<td>0.33735</td>
<td>0.11381</td>
<td>266</td>
<td>251.51829</td>
<td>0.002</td>
</tr>
<tr>
<td>MINORITY/INTRCPT2, U10</td>
<td>0.35195</td>
<td>0.10127</td>
<td>266</td>
<td>279.06874</td>
<td>0.001</td>
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<tr>
<td>MINORITY/CLASTYPE, U11</td>
<td>0.55939</td>
<td>0.31292</td>
<td>266</td>
<td>261.17748</td>
<td>0.003</td>
</tr>
<tr>
<td>SES/CLASTYPE, U21</td>
<td>1.34742</td>
<td>1.81555</td>
<td>266</td>
<td>241.49535</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Exploratory Analysis: estimated level-3 coefficients and their standard errors obtained by regressing EB residuals on level-3 predictors selected for possible inclusion in subsequent runs

<table>
<thead>
<tr>
<th>Level-2 Predictor</th>
<th>Potential Level-3 Predictors</th>
</tr>
</thead>
<tbody>
<tr>
<td>PUBPRI V</td>
<td>TI TLE1</td>
</tr>
<tr>
<td>INTRCPT1/INTRCPT2, B00</td>
<td>Coefficient</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.006</td>
</tr>
<tr>
<td>t value</td>
<td>-0.171</td>
</tr>
<tr>
<td>PUBPRI V</td>
<td>TI TLE1</td>
</tr>
<tr>
<td>INTRCPT1/CLASTYPE, B01</td>
<td>Coefficient</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.009</td>
</tr>
<tr>
<td>t value</td>
<td>0.112</td>
</tr>
</tbody>
</table>
APPENDIX C

HLM COMMAND FILES

Step One: Analysis for Testing for True Variance

#WHLM CMD FILE FOR C:\WINDOWS\Profiles\Bidya\My Documents\Dissertation_DATA\READNEW4.SSM
nonlin:binomial
microit:100
macroit:100
stopmicro:0.00000000000
stopmacro:0.00000000000
level1:READPRF=INTRCPT1+RANDOM
level2:INTRCPT1=INTRCPT2+random/
level3:INTRCPT2=INTRCPT3+random/
fixsigma2:1.000000
fixtau2:3
fixtau3:3
accel:5
resfil2:n
resfil3:n
hypo:n
CONSTRAIN:N
LAPLACE:N,50
LVR-BETA:N
title:NO TITLE
output:C:\WINDOWS\Profiles\Bidya\My Documents\Dissertation_DATA\hlm3_1.out
fishertype:2
Step Two: Analysis for Specifying Student-Level Variables

#WHLM CMD FILE FOR C:\WINDOWS\Profiles\Bidya\My Documents\Dissertation_DATA\READNEW4.SSM
nonlin:binomial
microit:100
macroit:100
stopmicro:0.0000010000
stopmacro:0.0001000000
level1:READPROF=INTRCPT1+MINORITY+SES,2+RANDOM
level2:INTRCPT1=INTRCPT2+random/TCHEXPCT,COMPUSE,CLASTYPE
level3:INTRCPT2=INTRCPT3+random/
level2:MINORITY=INTRCPT2+random/TCHEXPCT,COMPUSE,CLASTYPE
level3:INTRCPT2=INTRCPT3+random/
level2:SES=INTRCPT2+random/TCHEXPCT,COMPUSE,CLASTYPE
level3:INTRCPT2=INTRCPT3+random/
fixsigma2:1.000000
fixtau2:3
fixtau3:3
accel:5
resfil2:n
resfil3:n
hypo:n
CONSTRAIN:N
LAPLACE:N,50
LVR-BETA:N
title:no title
output:C:\WINDOWS\Profiles\Bidya\My Documents\Dissertation_DATA\hlm3.out
fishertype:2
Step Three: Analysis for Specifying Teacher-Level Variables

#WHLM CMD FILE FOR C:\WINDOWS\Profiles\Bidya\My Documents\Dissertation_DATA\READNEW4.SSM
nonlin:binomial
microit:100
macroit:100
stopmicro:0.0000010000
stopmacro:0.0001000000
level1:READPROF=INTRCPT1+MINORITY+SES,2+RANDOM
level2:INTRCPT1=INTRCPT2+CLASTYPE+random/TCHEXPCT,COMPUSE
level3:INTRCPT2=INTRCPT3+random/SCHLSES,PUBPRIV,TITLE1
level3:CLASTYPE=INTRCPT3+random/SCHLSES,PUBPRIV,TITLE1
level2:MINORITY=INTRCPT2+CLASTYPE+random/TCHEXPCT,COMPUSE
level3:INTRCPT2=INTRCPT3+random/SCHLSES,PUBPRIV,TITLE1
level3:CLASTYPE=INTRCPT3+random/SCHLSES,PUBPRIV,TITLE1
level2:SES=INTRCPT2+CLASTYPE+random/TCHEXPCT,COMPUSE
level3:INTRCPT2=INTRCPT3+random/SCHLSES,PUBPRIV,TITLE1
level3:CLASTYPE=INTRCPT3+random/SCHLSES,PUBPRIV,TITLE1
fixsigma2:1.000000
fixtau2:3
fixtau3:3
accel:5
resfil2:n
resfil3:n
hypoth:n
CONSTRAIN:N
LAPLACE:N,50
LVR-BETA:N
title:NO TITLE
output:C:\WINDOWS\Profiles\Bidya\My Documents\Dissertation_DATA\hlm3_3.out
fishertype:2
Step Four: Analysis for Specifying Student-Level Variables

#WHLM CMD FILE FOR C:\WINDOWS\Profiles\Bidya\My Documents\Dissertation_DATA\READNEW4.SSM
nonlin:binomial
microit:100
macroit:100
stopmicro:0.0000010000
stopmacro:0.0001000000
level1:READPROF=INTRCPT1+MINORITY+SES,2+RANDOM
level2:INTRCPT1=INTRCPT2+CLASTYPE+random/TCHEXPCT,COMPUSE
level3:INTRCPT2=INTRCPT3+SCHLSES,2+random/PUBPRIV,TITLE1
level3:CLASTYPE=INTRCPT3+SCHLSES,2+random/PUBPRIV,TITLE1
level2:MINORITY=INTRCPT2+CLASTYPE+random/TCHEXPCT,COMPUSE
level3:INTRCPT2=INTRCPT3+SCHLSES,2+random/
level3:CLASTYPE=INTRCPT3+SCHLSES,2+random/
level2:SES=INTRCPT2+CLASTYPE+random/TCHEXPCT,COMPUSE
level3:INTRCPT2=INTRCPT3+SCHLSES,PUBPRIV,TITLE1
level3:CLASTYPE=INTRCPT3+random/
fixsigma2:1.000000
fixtau2:3
fixtau3:3
accel:5
resfil2:n
resfil3:n
hypoth:n
CONSTRAIN:N
LAPLACE:N,50
LVR-BETA:N
title:NO TITLE
output:C:\DOCUMENTS AND SETTINGS\BIDYA SUBEDI\MY DOCUMENTS\DISSERTATION - NEW\DISSERTATION_DATA\HLM3.OUT
fishertype:2
APPENDIX D

Variance-Covariance Matrix

<table>
<thead>
<tr>
<th></th>
<th>1.37273</th>
<th>1.99543</th>
<th>-0.27142</th>
<th>-0.69313</th>
<th>1.48571</th>
<th>0.27033</th>
<th>-2.73831</th>
<th>8.97543</th>
<th>-0.42375</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00449</td>
<td>0.00801</td>
<td>-0.00443</td>
<td>-0.00812</td>
<td>-0.00504</td>
<td>-0.01211</td>
<td>0.00503</td>
<td>0.01212</td>
<td>0.00999</td>
<td>-0.00992</td>
</tr>
<tr>
<td>0.00801</td>
<td>0.67898</td>
<td>-0.00804</td>
<td>-0.15734</td>
<td>-0.00211</td>
<td>-0.11203</td>
<td>0.00211</td>
<td>1.11758</td>
<td>-0.08941</td>
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<tr>
<td>-0.00443</td>
<td>-0.00804</td>
<td>0.01613</td>
<td>0.01741</td>
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<td>0.01201</td>
<td>-0.01023</td>
<td>-0.02633</td>
<td>-0.01013</td>
<td>0.01785</td>
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<td>-0.15734</td>
<td>0.01741</td>
<td>0.13469</td>
<td>0.00211</td>
<td>1.1431</td>
<td>-0.00803</td>
<td>-0.09259</td>
<td>0.08942</td>
<td>-0.01066</td>
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<tr>
<td>-0.00504</td>
<td>-0.00211</td>
<td>0.00501</td>
<td>0.00211</td>
<td>0.01061</td>
<td>-0.01311</td>
<td>-0.01091</td>
<td>0.01321</td>
<td>-0.01811</td>
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<td>-0.01211</td>
<td>-0.11203</td>
<td>0.01201</td>
<td>1.1431</td>
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<td>0.37946</td>
<td>0.01321</td>
<td>-0.19092</td>
<td>0.00592</td>
<td>-0.00595</td>
</tr>
<tr>
<td>0.00503</td>
<td>0.00211</td>
<td>-0.01023</td>
<td>-0.00803</td>
<td>-0.01091</td>
<td>0.01321</td>
<td>0.02045</td>
<td>-0.00099</td>
<td>0.01812</td>
<td>-0.03403</td>
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<td>-0.02633</td>
<td>-0.09259</td>
<td>0.01321</td>
<td>-0.19092</td>
<td>-0.00099</td>
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<td>-0.01013</td>
<td>0.08942</td>
<td>-0.01811</td>
<td>0.00592</td>
<td>0.01812</td>
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<td>0.15445</td>
<td>-0.01543</td>
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<td>0.01799</td>
<td>-0.00595</td>
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<td>0.01535</td>
<td>-0.01543</td>
<td>0.02856</td>
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</tbody>
</table>

Note: The first row pertains to the estimated coefficients.
### APPENDIX E

Level-1, Level-2, and Level-3 Descriptive Statistics

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>N</th>
<th>MEAN</th>
<th>SD</th>
<th>MINIMUM</th>
<th>MAXIMUM</th>
</tr>
</thead>
<tbody>
<tr>
<td>MALE</td>
<td>7175</td>
<td>0.49</td>
<td>0.5</td>
<td>0</td>
<td>1</td>
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<tr>
<td>READPROF</td>
<td>7175</td>
<td>0.51</td>
<td>0.5</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>MINORITY</td>
<td>7175</td>
<td>0.49</td>
<td>0.5</td>
<td>0</td>
<td>1</td>
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<tr>
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<td>1.16</td>
<td>4</td>
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</table>

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>N</th>
<th>MEAN</th>
<th>SD</th>
<th>MINIMUM</th>
<th>MAXIMUM</th>
</tr>
</thead>
<tbody>
<tr>
<td>TCHEXPCT</td>
<td>1076</td>
<td>2.56</td>
<td>0.43</td>
<td>2</td>
<td>3.33</td>
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<td>COMPUSE</td>
<td>1076</td>
<td>3.28</td>
<td>0.06</td>
<td>3.1</td>
<td>3.5</td>
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<td>0.5</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>N</th>
<th>MEAN</th>
<th>SD</th>
<th>MINIMUM</th>
<th>MAXIMUM</th>
</tr>
</thead>
<tbody>
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<td>0.6</td>
<td>0.49</td>
<td>0</td>
<td>1</td>
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<tr>
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<td>7.37</td>
<td>0.64</td>
<td>5.3</td>
<td>9.3</td>
</tr>
<tr>
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<td>0.58</td>
<td>0.49</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
APPENDIX F

Outputs for Crosstabs, Logistic Regression, and HLM based on Revised Analysis

i. Crosstabs of reading proficiency versus SES, and minority versus SES

### READPROF * SES Crosstabulation

<table>
<thead>
<tr>
<th>SES</th>
<th>READPROF .00 Count</th>
<th>% within READPROF.00</th>
<th>READPROF 1.00 Count</th>
<th>% within READPROF 1.00</th>
<th>Total Count</th>
<th>% within READPROF Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.00</td>
<td>215</td>
<td>6.1%</td>
<td>68</td>
<td>1.9%</td>
<td>283</td>
<td>3.9%</td>
</tr>
<tr>
<td>5.00</td>
<td>473</td>
<td>13.5%</td>
<td>75</td>
<td>2.0%</td>
<td>548</td>
<td>7.6%</td>
</tr>
<tr>
<td>6.00</td>
<td>1084</td>
<td>30.9%</td>
<td>313</td>
<td>8.5%</td>
<td>1397</td>
<td>19.5%</td>
</tr>
<tr>
<td>7.00</td>
<td>1101</td>
<td>31.4%</td>
<td>861</td>
<td>23.5%</td>
<td>1962</td>
<td>27.3%</td>
</tr>
<tr>
<td>8.00</td>
<td>532</td>
<td>15.2%</td>
<td>1134</td>
<td>30.9%</td>
<td>1666</td>
<td>23.2%</td>
</tr>
<tr>
<td>9.00</td>
<td>89</td>
<td>2.5%</td>
<td>878</td>
<td>23.9%</td>
<td>967</td>
<td>13.5%</td>
</tr>
<tr>
<td>10.00</td>
<td>10</td>
<td>.3%</td>
<td>342</td>
<td>9.3%</td>
<td>352</td>
<td>4.9%</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### MINORITY * SES Crosstabulation

<table>
<thead>
<tr>
<th>SES</th>
<th>MINORITY .00 Count</th>
<th>% within MINORITY .00</th>
<th>MINORITY 1.00 Count</th>
<th>% within MINORITY 1.00</th>
<th>Total Count</th>
<th>% within MINORITY Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.00</td>
<td>130</td>
<td>3.6%</td>
<td>153</td>
<td>4.3%</td>
<td>283</td>
<td>3.9%</td>
</tr>
<tr>
<td>5.00</td>
<td>233</td>
<td>6.4%</td>
<td>315</td>
<td>8.9%</td>
<td>548</td>
<td>7.6%</td>
</tr>
<tr>
<td>6.00</td>
<td>650</td>
<td>17.8%</td>
<td>747</td>
<td>21.1%</td>
<td>1397</td>
<td>19.5%</td>
</tr>
<tr>
<td>7.00</td>
<td>957</td>
<td>26.3%</td>
<td>1005</td>
<td>28.4%</td>
<td>1962</td>
<td>27.3%</td>
</tr>
<tr>
<td>8.00</td>
<td>866</td>
<td>23.8%</td>
<td>800</td>
<td>22.6%</td>
<td>1666</td>
<td>23.2%</td>
</tr>
<tr>
<td>9.00</td>
<td>571</td>
<td>15.7%</td>
<td>396</td>
<td>11.2%</td>
<td>967</td>
<td>13.5%</td>
</tr>
<tr>
<td>10.00</td>
<td>235</td>
<td>6.5%</td>
<td>117</td>
<td>3.3%</td>
<td>352</td>
<td>4.9%</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ii. Logistic regression based outputs

### Variables in the Equation

<table>
<thead>
<tr>
<th>Step</th>
<th>B</th>
<th>S.E.</th>
<th>Wald</th>
<th>df</th>
<th>Sig.</th>
<th>Exp(B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SES</td>
<td>1.055</td>
<td>.028</td>
<td>1437.142</td>
<td>1</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Constant</td>
<td>-7.669</td>
<td>.204</td>
<td>1412.707</td>
<td>1</td>
<td>.000</td>
</tr>
</tbody>
</table>

a. Variable(s) entered on step 1: SES.

### Variables in the Equation

<table>
<thead>
<tr>
<th>Step</th>
<th>B</th>
<th>S.E.</th>
<th>Wald</th>
<th>df</th>
<th>Sig.</th>
<th>Exp(B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SES</td>
<td>1.118</td>
<td>.030</td>
<td>1377.834</td>
<td>1</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>MINORITY</td>
<td>-1.516</td>
<td>.060</td>
<td>637.475</td>
<td>1</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Constant</td>
<td>-7.371</td>
<td>.215</td>
<td>1171.706</td>
<td>1</td>
<td>.000</td>
</tr>
</tbody>
</table>

a. Variable(s) entered on step 1: SES, MINORITY.
iii. In the revised analysis, the sample size for HLM-based analysis was 7175 students, 1076 teachers, and 295 schools. The sample size for SPSS-based analysis was also 7175 students, 1076 teachers, and 295 schools. There was no difference in sample sizes in HLM-based and SPSS-based analyses.

iv. Level-1, Level-2, and Level-3 descriptive statistics based on revised HLM analysis are as follows.

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>N</th>
<th>MEAN</th>
<th>SD</th>
<th>MINIMUM</th>
<th>MAXIMUM</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEVEL-1 DESCRIPTIVE STATISTICS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MALE</td>
<td>7175</td>
<td>0.49</td>
<td>0.5</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>READPROF</td>
<td>7175</td>
<td>0.51</td>
<td>0.5</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>MINORITY</td>
<td>7175</td>
<td>0.49</td>
<td>0.5</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>SES</td>
<td>7175</td>
<td>7.18</td>
<td>1.16</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>LEVEL-2 DESCRIPTIVE STATISTICS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TCHEXPCT</td>
<td>1076</td>
<td>2.56</td>
<td>0.43</td>
<td>2</td>
<td>3.33</td>
</tr>
<tr>
<td>COMPUSE</td>
<td>1076</td>
<td>3.28</td>
<td>0.06</td>
<td>3.1</td>
<td>3.5</td>
</tr>
<tr>
<td>CLASTYPE</td>
<td>1076</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>LEVEL-3 DESCRIPTIVE STATISTICS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PUBPRIV</td>
<td>295</td>
<td>0.6</td>
<td>0.49</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>SCHLSES</td>
<td>295</td>
<td>7.37</td>
<td>0.64</td>
<td>5.3</td>
<td>9.3</td>
</tr>
<tr>
<td>TITLE1</td>
<td>295</td>
<td>0.58</td>
<td>0.49</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
v. SPSS-based Level-1, Level-2, and Level-3 descriptive statistics are as follows.

Student Level (Level-1) Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>MALE</td>
<td>7175</td>
<td>0</td>
<td>1</td>
<td>.49</td>
<td>.50</td>
</tr>
<tr>
<td>READPROF</td>
<td>7175</td>
<td>0</td>
<td>1</td>
<td>.51</td>
<td>.50</td>
</tr>
<tr>
<td>MINORITY</td>
<td>7175</td>
<td>0</td>
<td>1</td>
<td>.49</td>
<td>.50</td>
</tr>
<tr>
<td>SES</td>
<td>7175</td>
<td>4</td>
<td>10</td>
<td>7.18</td>
<td>1.16</td>
</tr>
<tr>
<td>Valid N (listwise)</td>
<td>7175</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Teacher Level (Level-2) Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>TCHEXPCT</td>
<td>1076</td>
<td>2.00</td>
<td>3.33</td>
<td>2.5601</td>
<td>.4301</td>
</tr>
<tr>
<td>COMPUSE</td>
<td>1076</td>
<td>3.10</td>
<td>3.50</td>
<td>3.2802</td>
<td>6.011E-02</td>
</tr>
<tr>
<td>CLASTYPE</td>
<td>1076</td>
<td>0</td>
<td>1</td>
<td>.50</td>
<td>.50</td>
</tr>
<tr>
<td>Valid N (listwise)</td>
<td>1076</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

School Level (Level-3) Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>PUBPRIV</td>
<td>295</td>
<td>0</td>
<td>1</td>
<td>.60</td>
<td>.49</td>
</tr>
<tr>
<td>SCHLSES</td>
<td>295</td>
<td>5.30</td>
<td>9.30</td>
<td>7.37</td>
<td>.64</td>
</tr>
<tr>
<td>TITLE1</td>
<td>295</td>
<td>0</td>
<td>1</td>
<td>.58</td>
<td>.49</td>
</tr>
<tr>
<td>Valid N (listwise)</td>
<td>295</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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New HLM Output for final model based on revised analysis
=============================================================

Program: HLM 5 Hierarchical Linear and Nonlinear Modeling
Authors: Stephen Raudenbush, Tony Bryk, & Richard Congdon
Publisher: Scientific Software International, Inc. (c) 2000
techsupport@ssicentral.com
www.ssicentral.com

---

Module: HLM.EXE (5.05.2330.2)
Date: 27 February 2005, Sunday
Time: 12:47:19
---

SPECIFICATIONS FOR THIS NONLINEAR HLM3 RUN Sun Feb 27 12:47:19 2005
---

Problem Title: NO TITLE

The data source for this run = C:\WINDOWS\PROFILES\BI DYA\MY DOCUMENTS\DISSERTATION_DATA\READG 4.SSM
The command file for this run = whlm temp.hl m
Output file name = C:\WINDOWS\PROFILES\BI DYA\MY DOCUMENTS\DISSERTATION_DATA\HLM3.OUT
The maximum number of level-2 units = 1076
The maximum number of level-3 units = 295
The maximum number of microiterations = 100
Maximum number of macroiterations = 100

Distribution at Level-1: Bernoulli

The outcome variable is READPROF

The model specified for the fixed effects was:

<table>
<thead>
<tr>
<th>Level-1 Coefficients</th>
<th>Level-2 Predictors</th>
<th>Level-3 Predictors</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRCEPT1, P0</td>
<td>INTRCEPT2, B00</td>
<td>INTRCEPT3, G000</td>
</tr>
<tr>
<td></td>
<td>CLASTYPE, B01</td>
<td>INTRCEPT3, G010</td>
</tr>
<tr>
<td></td>
<td>M NORI TY slope, P1</td>
<td>INTRCEPT2, B10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CLASTYPE, B11</td>
</tr>
<tr>
<td>% SES slope, P2</td>
<td># INTRCEPT2, B20</td>
<td>INTRCEPT3, G200</td>
</tr>
<tr>
<td></td>
<td>CLASTYPE, B21</td>
<td>INTRCEPT3, G210</td>
</tr>
</tbody>
</table>

' #' - The residual parameter variance for the parameter has been set to zero
' %' - This variable has been centered around its grand mean
Summary of the model specified (in equation format)

Level - 1 Model

\[ \text{Pr}(Y=1|B) = P \]

\[ \log \left[ \frac{P}{(1-P)} \right] = P_0 + P_1 \times (\text{MINORITY}) + P_2 \times (\text{SES}) \]

Level - 2 Model

\[ P_0 = B_{00} + B_{01} \times (\text{CLASTYPE}) + R_0 \]
\[ P_1 = B_{10} + B_{11} \times (\text{CLASTYPE}) + R_1 \]
\[ P_2 = B_{20} + B_{21} \times (\text{CLASTYPE}) + R_2 \]

Level - 3 Model

\[ B_{00} = G_{000} + G_{001} \times (\text{SCHLSES}) + U_{00} \]
\[ B_{01} = G_{010} + G_{011} \times (\text{SCHLSES}) + U_{01} \]
\[ B_{10} = G_{100} + G_{101} \times (\text{SCHLSES}) + U_{10} \]
\[ B_{11} = G_{110} + G_{111} \times (\text{SCHLSES}) + U_{11} \]
\[ B_{20} = G_{200} \]
\[ B_{21} = G_{210} + U_{21} \]

Level - 1 variance = \( \frac{1}{P(1-P)} \)

Random level - 1 coefficient

<table>
<thead>
<tr>
<th>Intercept, P0</th>
<th>Reliability estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.570</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MINORITY, P1</th>
<th>Reliability estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.587</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SES, P2</th>
<th>Reliability estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.577</td>
<td></td>
</tr>
</tbody>
</table>

Random level - 2 coefficient

<table>
<thead>
<tr>
<th>Intercept1/Intercept2, B00</th>
<th>Reliability estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.771</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Intercept1/CLASTYPE, B01</th>
<th>Reliability estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.785</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MINORITY/Intercept2, B10</th>
<th>Reliability estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.783</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MINORITY/CLASTYPE, B11</th>
<th>Reliability estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.779</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SES/CLASTYPE, B21</th>
<th>Reliability estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.781</td>
<td></td>
</tr>
</tbody>
</table>

The outcome variable is READPROF

Final estimation of fixed effects: (Unit-specific model)

<table>
<thead>
<tr>
<th>Fixed Effect</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-ratio</th>
<th>Approx. d.f.</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept1, P0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>For Intercept2, B00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept3, G000</td>
<td>1.372731</td>
<td>0.066872</td>
<td>20.528</td>
<td>293</td>
<td>0.000</td>
</tr>
<tr>
<td>SCHLSES, G001</td>
<td>1.995357</td>
<td>0.823530</td>
<td>2.423</td>
<td>293</td>
<td>0.006</td>
</tr>
<tr>
<td>For CLASTYPE, B01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept3, G010</td>
<td>-0.270892</td>
<td>0.126881</td>
<td>-2.135</td>
<td>293</td>
<td>0.007</td>
</tr>
<tr>
<td>SCHLSES, G011</td>
<td>-0.693129</td>
<td>0.367319</td>
<td>-1.887</td>
<td>293</td>
<td>0.070</td>
</tr>
<tr>
<td>For MINORITY slope, P1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>For Intercept2, B10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept3, G100</td>
<td>1.697172</td>
<td>0.103104</td>
<td>16.461</td>
<td>293</td>
<td>0.000</td>
</tr>
<tr>
<td>SCHLSES, G101</td>
<td>1.485801</td>
<td>0.616049</td>
<td>2.412</td>
<td>293</td>
<td>0.006</td>
</tr>
<tr>
<td>For CLASTYPE, B11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept3, G110</td>
<td>0.270299</td>
<td>0.143039</td>
<td>1.890</td>
<td>293</td>
<td>0.070</td>
</tr>
<tr>
<td>SCHLSES, G111</td>
<td>-2.737791</td>
<td>1.280549</td>
<td>-2.138</td>
<td>293</td>
<td>0.007</td>
</tr>
<tr>
<td>For SES slope, P2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>For Intercept2, B20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept3, G200</td>
<td>8.975137</td>
<td>0.393299</td>
<td>22.820</td>
<td>294</td>
<td>0.000</td>
</tr>
<tr>
<td>For CLASTYPE, B21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept3, G210</td>
<td>-0.423707</td>
<td>0.169211</td>
<td>-2.504</td>
<td>294</td>
<td>0.005</td>
</tr>
</tbody>
</table>
Final estimation of level-1 and level-2 variance components:

<table>
<thead>
<tr>
<th>Random Effect</th>
<th>Standard Deviation</th>
<th>Variance Component</th>
<th>df</th>
<th>Chi-square</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRCPT1, R0</td>
<td>0.21361</td>
<td>0.04563</td>
<td>437</td>
<td>860.71893</td>
<td>0.000</td>
</tr>
<tr>
<td>MINORITY slope, R1</td>
<td>0.29047</td>
<td>0.08437</td>
<td>437</td>
<td>901.26241</td>
<td>0.000</td>
</tr>
<tr>
<td>SES slope, R2</td>
<td>2.03477</td>
<td>4.14029</td>
<td>731</td>
<td>978.46898</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Final estimation of level-3 variance components:

<table>
<thead>
<tr>
<th>Random Effect</th>
<th>Standard Deviation</th>
<th>Variance Component</th>
<th>df</th>
<th>Chi-square</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRCPT1/INTRCPT2, U00</td>
<td>0.23286</td>
<td>0.05422</td>
<td>266</td>
<td>263.42869</td>
<td>0.003</td>
</tr>
<tr>
<td>INTRCPT1/CLASTYPE, U10</td>
<td>0.33735</td>
<td>0.11381</td>
<td>266</td>
<td>251.51829</td>
<td>0.002</td>
</tr>
<tr>
<td>MINORITY/INTRCPT2, U10</td>
<td>0.35195</td>
<td>0.10127</td>
<td>266</td>
<td>279.06674</td>
<td>0.001</td>
</tr>
<tr>
<td>MINORITY/CLASTYPE, U11</td>
<td>0.55939</td>
<td>0.31292</td>
<td>266</td>
<td>261.17748</td>
<td>0.003</td>
</tr>
<tr>
<td>SES/CLASTYPE, U21</td>
<td>1.34742</td>
<td>1.81555</td>
<td>266</td>
<td>241.49535</td>
<td>0.004</td>
</tr>
</tbody>
</table>
APPENDIX G
Approved Forms

Office of the Vice President For Research
Human Subjects Committee
Tallahassee, Florida 32306-2763
(850) 644-8673  FAX (850) 644-4392

APPROVAL MEMORANDUM

Date: 3/10/2005

To: Bidya Raj Subedi
2301 S. Congress Ave., Apt. 1711
Boynton Beach, FL 33426

Dept.: EDUCATIONAL PSYCHOLOGY AND LEARNING SYSTEMS

From: Thomas L. Jacobson, Chair

Re: Use of Human Subjects in Research
A Demonstration of the Three-Level Hierarchical Generalized Linear Model Applied to Educational Research

The forms that you submitted to this office in regard to the use of human subjects in the proposal referenced above have been reviewed by the Secretary, the Chair, and two members of the Human Subjects Committee. Your project is determined to be Exempt per 45 CFR § 46.101(b) 4 and has been approved by an accelerated review process.

The Human Subjects Committee has not evaluated your proposal for scientific merit, except to weigh the risk to the human participants and the aspects of the proposal related to potential risk and benefit. This approval does not replace any departmental or other approvals, which may be required.

If the project has not been completed by 3/8/2006 you must request renewed approval for continuation of the project.

You are advised that any change in protocol in this project must be approved by resubmission of the project to the Committee for approval. Also, the principal investigator must promptly report, in writing, any unexpected problems causing risks to research subjects or others.

By copy of this memorandum, the chairman of your department and/or your major professor is reminded that he/she is responsible for being informed concerning research projects involving human subjects in the department, and should review protocols of such investigations as often as needed to insure that the project is being conducted in compliance with our institution and with DHHS regulations.

This institution has an Assurance on file with the Office for Protection from Research Risks. The Assurance Number is IRB00000446.

Cc: Richard Tate
HSC No. 2005.088
AFFIDAVIT OF NONDISCLOSURE

Graduate Student  
Technical Staff for NEAP Project  
(Job Title)

Florida State University  
(Organization, State or local agency or institutionality)

307 Stone Building, Tallahassee,  
FL 32306-4453  
(Organization or agency Address)

NEAP Reading, Mathematics and  
Science Data Base  
(NCES Data Base or File Containing Individually Identifiable Information*)

August 26, 2002  
(Date of Assignment to NCES Project)

I, Bidya Raj Subedi,  
do solemnly swear (or affirm) that when given access  
to the subject NCES data base or file, I will not -

(i) use or reveal any individually identifiable information [including “schools” in the National Assessment of Educational Progress (NAEP)] furnished, acquired, retrieved or assembled by me or others, under the provisions of Sections 408 and 411 of the National Education Statistics Act of 1994 (20 U.S.C. 9001 et seq.) for any purpose other than statistical purposes specified in the NCES survey, project or contract;

(ii) make any disclosure or publication whereby a sample unit or survey respondent (including “schools” in NAEP) could be identified or the data furnished by or related to any particular person or NAEP school under these sections could be identified; or

(iii) permit anyone other than the individuals authorized by the Commissioner of the National Center for Education Statistics to examine the individual reports.

Bidya Raj Subedi  
(Signature)

[The penalty for unlawful disclosure is a fine of not more than $250,000 (under 18 U.S.C. 3571) or imprisonment for not more than five years (under 18 U.S.C. 3559), or both. The word “swear” should be stricken out when a person elects to affirm the affidavit rather than to swear to it.]

* Request all subsequent followups that may be needed. This form cannot be amended by NCES, so access to databases not listed will require submitting additional notarized Affidavits.

City/County of Leon  
Commonwealth/State of Florida  
Sworn to and subscribed before me this 4th day of September, 2002.

James L. Kaye  
(Notary Public/Seal)

My commission expires  
11/14/95

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REFERENCES


Weerasinghe, D., & Orsak, T. (1998). Can hierarchical linear modeling be used to rank schools: A simulation study with conditions under which hierarchical linear modeling is applicable. Dallas Independent School District, Dallas, TX.


BIOGRAPHICAL SKETCH

Bidya Raj Subedi, born in Nepal, completed his master degree in Statistics from Tribhuvan University, Nepal in 1991, and began his teaching career in the same year as a lecturer in Statistics at Tribhuvan University, Nepal. During 1996, he completed his master degree in Educational Psychology from Michigan State University, Michigan. Then he resumed his job as a lecturer back home from 1996 to 2000. In fall 2000, he received an opportunity to pursue his doctoral degree in Educational Measurement and Statistics at the Florida State University, Florida.

His work experience includes teaching, research, and evaluation. He was involved in working as a teaching assistant as well as research assistant in Florida State University, and as a research assistant in Michigan State University while working as a graduate student. In past, he has published and presented several papers. He also worked as a research assistant in Florida Department of Education where he was involved in test score analysis, and performing researches using Hierarchical Linear Modeling (HLM) technique. He also worked as a school evaluator and researcher in Nepal. Since July 2000, he has been working as a Specialist in Evaluation and Test Development in the School District of Palm Beach County (SDPBC), West Palm Beach, Florida, where he is involved in HLM related research and evaluation works.

His research interests focus on the application of linear and nonlinear HLM in educational evaluation and research. His current research in the SDPBC focuses on the application of HLM technique to measure the school and teacher effectiveness.